Monetary Policy Arithmetic: Some Recent Contributions

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This article explores some of the recent contributions to the literature on deficit financing and the unpleasant monetarist arithmetic.

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Standard undergraduate textbooks often cast monetary policy and fiscal policy as separable undertakings. Such a split does seems natural; after all, the players involved are different entities. In the United States, for instance, monetary policy decisions are made by the Federal Reserve, while fiscal policies come under the purview of the federal government. A direct consequence of this "split personality" view of policy action is that it gives monetary policy sole authority over short-term nominal interest rates and/or money growth rates, while fiscal policy gets the final say on tax rates and transfer payment schedules. Indeed, in the monetarist–Keynesian debate, this separatist tradition asks which class of policies is more effective at managing economic activity.[1]

Almost two decades ago, Sargent and Wallace (SW) burst the bubble on this dichotomy in a pathbreaking 1981 article, arguing that neither policy is conducted in a vacuum.[2] Although it may be appropriate to think of monetary and fiscal policy actions as separate ventures, it is important to understand that the two interact. According to SW, monetary–fiscal policy distinctions are at best arbitrary; monetary policy actions have repercussions for fiscal policy settings and vice versa. Because governments, like private citizens, face budget constraints, SW show that both monetary and fiscal actions interact in a single, unified government budget constraint. Actions taken by the government while wearing the fiscal policy hat, for instance, eventually affect the actions it takes while sporting the monetary policy cap.

A convenient way to understand these monetary–fiscal policy interactions is to think of the central bank and the treasury as engaged in a game of chicken, from which, at most, one winner can emerge.[3] SW consider a setting in which the fiscal wing of the government dominates[4] and focus on a policy in which the treasury finances an increase in government spending by selling interest-bearing debt to the public.[5] Suppose that current money growth, at least initially, is unaffected by this fiscal policy action. In time, the real interest obligations of the treasury would rise. It is even possible that the revenue from new bond sales would be insufficient to pay the outstanding interest on past bond sales. When this rollover option fails, the government can potentially avoid bankruptcy by printing money to pay off the deficit.[6] SW show that this increased interest expense forces the government to print money at a faster rate than would have been necessary had it chosen at the outset to finance the deficit by printing money.

As a consequence, financing the deficit with bonds could ultimately be more inflationary than financing it by printing money. Throughout this article, we refer to unpleasant monetarist arithmetic (UMA)—a term SW coined—as an outcome in which money growth must rise to finance a permanent increase in government debt.[7]

Three conditions are needed to obtain this spectacular result in the SW setup: (1) the central bank is subservient to the fiscal authority, (2) the real interest rate on government debt is higher than the economy's real rate of growth, and (3) the central bank is in a position to raise revenue by printing money. For the SW result (which is contrary to conventional wisdom) to have empirical bite, it is important that some real-world economies share the three features of the SW model economy. If the original Sargent–Wallace UMA result is to serve as a cautionary note for policymakers, all three conditions must hold in the real world.

This article explores some of the recent contributions to the literature on deficit financing and the unpleasant monetarist arithmetic. Although government surpluses—not deficits—are currently making headlines, we believe it is premature to pronounce deficit financing dead, just as it is premature to declare the business cycle dead. After all, government surpluses are neither permanent nor universal. Where this discussion of the UMA may be most illuminating is in expanding our understanding of the deficit financing issues some developing countries face. Government bond sales (to finance deficits) in Russia, and more recently in Brazil, have coincided with a faster rise in inflation than money growth could explain. For countries with surpluses, the UMA's predictions may say something about disinflation.

This article begins with a brief statement and derivation of the UMA result, then reviews the evidence used to refute the UMA predictions. In particular, we present evidence on the real interest rate for the United States and Canada to check whether condition 2 is satisfied. We examine whether, in fact, condition 2 is necessary for the UMA result by considering a case in which we increase the number of assets people can use to transfer income across time. Could it be that with an asset structure less restricted than SW's, condition 2 is no longer necessary? Finally, we extend the SW analysis to consider the deficit-financing consequences of other monetary policy tools. Since central banks have more than one way to raise revenue, does it matter for the UMA which way is chosen?

THE ECONOMY

The starting point for our analysis is a stripped-down description of the economy in which a government is operating. Time is broken into discrete periods and indexed by $t = 1,2,\ldots$. At each date $t = 1$, N_t young people are born. Population grows according to the rule $N_{t+1} = nN_t$, where $n > 1$ is the gross rate of population growth. Each person lives two periods and lifetimes of agents overlap, so that a young person lives at the same time as an old person. The latter dies at the end of the period; the former moves into old age, and a new generation of young people is born. One group (the "initial old") enters date $t = 1$ with only one period of life left.

This economy has a single perishable commodity. Each person receives an endowment of y units of this consumption good when young (the period in which they are born) and nothing when old (the second, and last, period of their life). In this setup, population and aggregate income grow at the same rate.[8]

People in this economy wish to consume something when they are young and something when they are old. Because young people receive some of the consumption good only when young, each forges a plan that will maximize well-being from consumption over the course of a lifetime. These plans will require that each person consume a part of the endowment in the first period of life. What happens to the remainder of the endowment? Since the good itself is nonstorable, each person needs to purchase stores of value, which can be used to finance old-age consumption.[9] Let c_1 (c_2) denote the quantity of goods consumed when the person is young (old). Note that the young pay the government a lump-sum tax of τ goods. Therefore, the division of the endowment by the young person can be represented as

$$y - \tau = c_1 + s,$$

where s is the remainder that is used to buy stores of value. Let r denote the gross real return on the stores of value, so that the product, rs, represents the total goods a young person can consume when old. This means

$$rs = c_2.$$

A typical young person has access to two stores of value, money and bonds. The real purchasing power of the money held, vm, will, of course, change as monetary policy settings change. In the product, vm, m represents the pieces of paper money each young person holds and v is the quantity of the endowment that can be acquired with one unit of paper money.[10] Hence, vm is the value of paper money (per young person) measured in terms of the consumption good. The young person may also buy treasury bonds, giving the treasury b endowment units in return for R/n units of the consumption good when the bonds mature (when the person is old). For simplicity, we assume the person consumes a fixed fraction of y when young and chooses a portfolio of both money and treasury bonds.[11] Put differently, the combined savings, $s = b + vm$, is independent of R/n.

Assume that the government taxes people only when they are young, collecting τ units of the consumption good. The government costlessly transforms the tax collected into units of a government good, denoted by g_t, that are useless to people. If the government wishes to acquire more units of the consumption good, it can borrow, issuing riskless interest-bearing bonds, B, that are repaid one period later. Alternatively, the government could print fiat money, M.[12] For each good borrowed from young people at date $t-1$, the government pays R_t goods at date t. Thus, at any date t the government's budget constraint is

$$(1) \qquad N_t g_t + R_t B_{t-1} = N_t \tau_t + B_t + v_t (M_t - M_{t-1}).$$

Equation 1 captures the required balance between what the government spends and what it collects. The total number of goods the government purchases is $N_t g_t$. If the government sells B_{t-1} bonds at $t-1$, at t it has to pay bondholders $R_t B_{t-1}$ in the form of principal and interest payments. The right-hand side of Equation 1 lists the various sources of revenue. The government collects taxes worth $N_t \tau_t$ and borrows B goods. The last term on the right-hand side of Equation 1 is seigniorage. M_t represents the total stock of fiat money in the economy at date t. Seigniorage, therefore, is the quantity of goods the government purchases by printing money—that is, $M_t - M_{t-1}$.

The monetary policy in this model economy is identifiable. The central bank controls the money growth rate, at least ostensibly. The nominal money stock evolves over time according to the rule $M_t = \lambda_t M_{t-1}$, where λ is the gross rate of money growth. Using this, seigniorage can be rewritten as

$$v_t M_t \left(1 - \frac{1}{\lambda_t}\right).$$

Thus, unless noted otherwise, we define monetary policy as actions taken by the central bank to change the rate of money growth.

It is helpful to simplify the government's budget constraint by rewriting it in terms of quantities per young person. This involves dividing the expression in Equation 1 by the number of young people each period. Dividing by N_t and using the population growth rule yields

$$(2) \qquad g_t + \frac{R_t}{n_t} b_{t-1} = \tau_t + b_t + v_t m_t \left(1 - \frac{1}{\lambda_t}\right),$$

where $b_t = B_t / N_t$ and $m_t = M_t / N_t$.

Three conditions need to be satisfied for this economy to be in equilibrium at any point t: (1) people consume and acquire money and bonds so as to maximize their lifetime well-being; (2) they take the prices for the consumption good, government bonds, and the value of money as given; and (3) markets clear, in that the supply and demand for consumption goods are equal (likewise for the money and bonds), and the government budget constraint is satisfied.

THE UNPLEASANT MONETARIST ARITHMETIC RESULT

We now describe the simple arithmetic of government budget deficits in the spirit of SW. For convenience, our discussion focuses on the long run, or on steady states, which are equilibrium situations in which government purchases, taxes, bonds, real money balances, and so on (per young person) are invariant with respect to time.[13] In steady states, violation of the government budget constraint at any arbitrary date t results in the present value of government spending, including interest obligations, differing from the present value of government revenues. In a steady state, $b_t = b_{t-1} = b$, and g, τ, R, vm, n, and λ are likewise time-invariant. Equation 2 may be rewritten as

$$(3) \qquad g = \tau + \left(1 - \frac{R}{n}\right) b + vm \left(1 - \frac{1}{\lambda}\right)$$

to form the basis for the SW result.

This article focuses on the case in which the central bank is subservient to the treasury.[14] The government's budget constraint in this steady state is satisfied—that is, Equation 3 holds. A steady-state representation of the unpleasant monetarist arithmetic is as follows: with taxes unchanged, a permanent increase in outstanding government bonds requires a permanent increase in the inflation rate to ensure the government budget constraint is satisfied.

Consider an increase in government purchases of the consumption good, g (holding taxes, τ, and money growth, λ, constant), funded by an increase in bonds, b, sold to the public. However, the bonds also have interest costs, which in steady state equal $(R/n)b$. The consequences of a bond-financed increase in government spending depend crucially on R/n, the ratio of the real interest rate to the economy's growth rate.[15] The question is whether the costs are less than, equal to, or greater than the revenue the bond sale generates.

First, consider a situation where $R/n > 1$—that is, the real interest rate on government debt exceeds the economy's growth rate. $(R/n)b > b$, or, in plain English, the interest on existing debt exceeds the revenue from the sale of fresh debt, resulting in a revenue shortfall. In equilibrium, the budget balance holds (Equation 3). With τ fixed, the central bank must raise the revenue needed to make up this shortfall. The central bank responds by increasing λ, which raises both seigniorage, $vm\,(1 - 1/\lambda)$, and the inflation rate.

What is the intuition behind this result? With a permanent increase in b, the steady-state comparison indicates that a revenue shortfall will occur. The government must pay for its initial purchases and cover the additional interest expense. Because the economy grows slower than the gross real return on government debt, steady-state interest expenses exceed revenue from the debt issue. Hence, some other revenue source is needed. SW establish that if the necessary condition $R/n > 1$ is satisfied, either higher taxes or more seigniorage is required to cover the increase in government purchases. In the absence of the tax option, unpleasant arithmetic necessarily follows.[16]

To further understand the role played by the $R/n > 1$ stricture, consider the opposite case, in which the government's revenue from the bond sale is large enough to cover the (steady-state) interest expense.[17] Because net interest payments are growing slower than the economy, the government can glean revenue from this bond issue to pay for the government purchases.[18] The excess revenue from the bond sale, defined as $b - (R/n)b$, allows seigniorage requirements and/or taxes to be lowered.

Having established the importance of R/n, we now review the evidence on the relationship between the real interest rate and the growth rate.

HISTORICAL EVIDENCE ON *R/n*

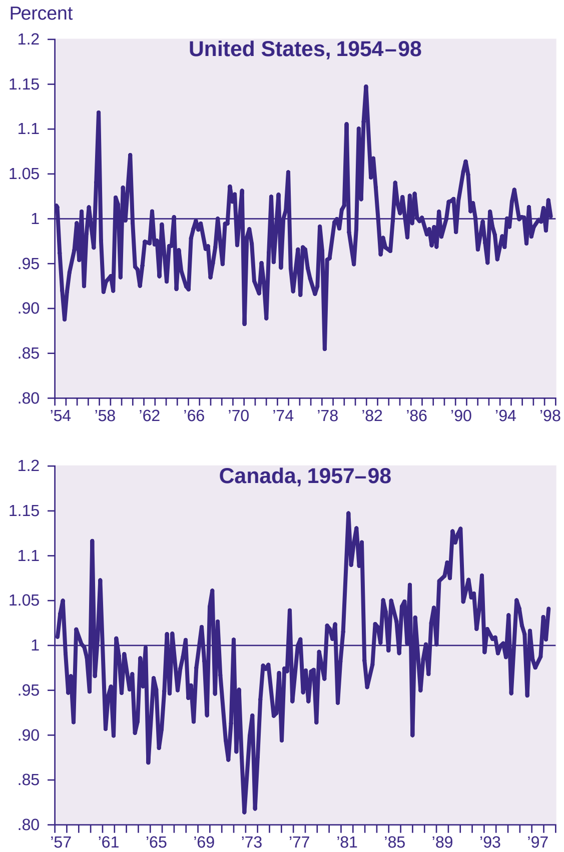
Some argue that the SW result is a theoretical curiosity, that the unpleasant monetarist arithmetic's key prediction is irrelevant because the gross real return on treasury debt is lower than the economy's growth rate. We review the historical evidence on R versus n, since such a comparison is the primary means used to cast doubt on the relevance of the UMA. We then examine the merits of the criticisms.

The researcher confronts a number of thorny issues in trying to measure the real interest rate. The principal difficulty is that the rates are generally unobservable. With more countries issuing indexed government debt, some of these measurement issues are mitigated, but the time series on these securities are generally quite short. Hence, real interest rates have to be computed using an observable measure, such as the nominal interest rate, combined with the inflation rate. The question that then arises is whether the GDP deflator or the Consumer Price Index (CPI) is the more appropriate measure of inflation—a question that cannot be conclusively answered. Despite the nettlesome measurement issues, our approach yields the real interest rate on a treasury security held for a specific length of time. In other words, we opt for a measure of the ex post real return paid on a treasury security.

Along the same lines as Champ and Freeman (1994), we plot the ratio R/n for the United States and Canada (*Figure 1*).[19] The ex post real interest rate is measured using the short-term (three month) nominal interest rate and the GDP deflator. We compute the inflation rate for the period in which the short-term government security is outstanding. Figure 1 shows that the real GDP growth rate is usually greater than the real interest rate (the ratio R/n is less than 1). The most notable exceptions occur in Canada from 1988 through 1992, when the real interest rate exceeds the economy's growth rate. Espinosa-Vega and Russell (1998) argue that the appropriate real return is an after-tax real interest rate. According to them, since World War II the after-tax real rate of return on U.S. government debt has been about –0.4 percent, while the average real growth rate of GDP has been about 3.2 percent.

Based on the average gross real return on government debt and the economy's growth rate, one might conclude the unpleasant monetarist arithmetic would have little predictive bite. It is debatable whether the historical evidence from the United States and Canada bears directly on the question Sargent and Wallace raise, which is whether the government can finance a government purchase with a permanent increase in government debt. The United States has never conducted the SW experiment, so drawing inferences about the UMA from the real interest rate and the economy's growth rate

Figure 1
Real Interest Rate and Real GDP Growth



SOURCE: *International Financial Statistics*, various issues.

could be invalid. Suppose, for example, that a permanent increase in government debt were to result in a higher real (after tax) interest rate. In that case, SW's conditions for infeasibility may become likely.

Thus, the historical record on R and n may not be the most damning evidence against the SW result. By this reckoning, the unpleasant monetarist arithmetic is an intellectual curiosity because the experiment is never part of policy, not because its predictive content is invalid. Insofar as the empirical evidence sheds light on the SW predictions, it does so for a setting in which $R > n$ is a necessary condition for obtaining their result. In the next section, we examine whether $R > n$ is in the set of necessary conditions. While the $R > n$ condition may be necessary in the SW setup to generate the UMA, it may not be necessary under slightly more general model specifications.

ADDING STORES OF VALUE

This section examines an economy in which people can hold stores of value other than government bonds and currency. This extension to the basic model economy permits the assessment of the role a restricted asset structure plays in obtaining SW's results. We establish that $R/n > 1$ is a sufficient but not a necessary condition to obtaining the UMA result; unpleasant monetarist arithmetic may result even if $R/n < 1$ holds. Bhattacharya, Guzman, and Smith (1998) develop this analysis more thoroughly.

Consider an economy with a store of value that yields known, fixed units of the consumption good. For concreteness, we refer to this store of value as an investment project. One unit of the consumption good invested in this project yields a units of the consumption good the following period; thereafter, its scrap value is zero.[20] Further assume that investment projects are large, with a minimum size at which they can operate. This minimum is large enough that no individual can finance an investment project.

It seems natural to assume that in such an environment banks would be created to pool people's savings into amounts large enough to fund the investment projects, thereby giving each person an additional store of value. We assume that a bank collects deposits and transforms the goods into an investment project costlessly and that banks operate in a perfectly competitive environment. The bank promises to pay depositors a return one period after the goods are deposited. This return is the same as could be earned by a person with enough resources to invest directly in the project. Since no single person is rich enough to invest directly, we can assume the bank intermediates all investments.

Because banks in this economy are legally required to hold a fraction $(0 < \theta < 1)$ of all deposits to meet reserve requirements, the bank's portfolio of assets is divided between money and the investment project. We assume the investment project offers a real return that exceeds the real return of money, that is, $a > n/\lambda$. It is clear the bank will hold an exact fraction θ (never more) of its deposit base in the form of money.

Knowing the returns to these assets (and because providing these banking services costs nothing), it is straightforward to calculate the return to deposits by the bank. The gross real return to currency held by the bank is n/λ, which receives a weight θ in the bank's portfolio. The return to the investment project is a, with corresponding weight $(1 - \theta)$. Hence, the return to deposits is a weighted sum of the returns to each of the bank's asset holdings:

$$\theta \frac{n}{\lambda} + (1-\theta)a. \tag{4}$$

We assume that people can continue to purchase riskless government debt directly and that there are no reserve requirements on debt holdings. A person's savings will be divided in two: part will be used to buy government bonds (b), and the remainder will be deposited with the bank (d)—that is, $s = b + d$. For people to hold government debt willingly, its return must be at least as great as the return to deposits. Likewise, people will hold deposits if their return is at least as great as the return to government debt. If both government debt and bank deposits are to be held,

$$R = \theta \frac{n}{\lambda} + (1-\theta)a \tag{5}$$

must result. Equation 5 is what is often called a no-arbitrage condition. More generally, multiple riskless assets will be held only if they bear identical rates of return.[21]

We proceed in two steps. The first step requires that the returns to government bonds, the investment project, and money be ranked. If $a > n > n/\lambda$ holds, $R > n/\lambda$ must also hold.[22] To verify this, suppose $a > n > n/\lambda$, but $R < n/\lambda$. It follows from Equation 5 that $\theta(n/\lambda) + (1 - \theta)a < n/\lambda$ must also hold. This expression reduces to $a < n/\lambda$, which contradicts our original assumption that $a > n/\lambda$. Hence, $a > n/\lambda$ implies $R > n/\lambda$. If the investment project pays a higher real return than money, the real return to government debt must exceed the return to fiat money. Thus, government debt is more expensive (to the government) than money as a means of deficit financing. Treasury debt requires an explicit (nominal) interest payment while money balances do not. (Note that $a > n > n/\lambda$ does not imply $R > n$.)

Our second step examines the case in which the government permanently increases the ratio of government bonds to cash balances. In this instance, the government is selling government debt and buying money—an open market sale. To determine what this means for the government's unified budget constraint, we return to Equation 3, setting $\tau = 0$ to get

$$g = \left(1 - \frac{R}{n}\right)b + vm\left(1 - \frac{1}{\lambda}\right).$$

With $a > n > n/\lambda$, we know $R/n > n/\lambda$ or $(1 - R/n) > (1 - 1/\lambda)$. We have completed our task if we can show that the UMA holds even if $R/n < 1$ holds. Suppose $R/n < 1$, and recall that $s = d + b$. If the government uses an open market sale (denoted Δb) to increase b, and if these bonds are held by the public, d must fall for given s. Since a fraction θ of d constitutes real money demand, we know vm (the seigniorage tax base) must fall by Δvm. So an increase in b raises revenue by the amount $(1 - R/n)\Delta b$ (recall that we are assuming $R/n < 1$). However, for fixed λ, the revenue from money creation, $\Delta vm(1 - 1/\lambda)$, falls. Bhattacharya, Guzman, and Smith (BGS) prove that when $(1 - R/n) < (1 - 1/\lambda)$, the loss in seigniorage exceeds the revenue from bonds.[23] This revenue shortfall has to be made up somehow. One possibility is to raise the money growth, λ, which would, of course, raise the inflation rate. Here is what is happening: the government is raising revenue from the sale of bonds but is losing seigniorage (because the bond sale crowds out money holdings and reduces the inflation tax base). With money growth constant, it is possible the net effect is that the bond sale reduces overall steady-state revenues.

Thus, the two steps establish that a permanent increase in government debt cannot be financed by a permanent increase in government bonds. The first step establishes that the real return on government bonds exceeds the real return on fiat money. With $R > n/\lambda$ and $a > n$, BGS show these two results are sufficient for the UMA. In plain English, the unpleasant monetarist arithmetic could hold even if the real return to government bonds is lower than the economy's growth rate. The key proviso is that there exists a store of value with a real return higher than the economy's growth rate. The open market sale means the government must cover the net interest expenses of the larger stock of debt while reducing the quantity of real money balances. Higher inflation is necessary to pay for these expenses, potentially even when the real return on the debt is lower than the economy's growth rate.

The BGS finding is important because it means the UMA result can be obtained with a set of necessary conditions that does not include $R/n > 1$ (condition 2). Indeed, in the BGS framework, the UMA is a possibility as long as there is an intermediated asset with a real return exceeding the economy's growth rate. Since this last condition does not require that R/n be greater than 1, BGS may have eliminated its need. This undercuts the criticisms leveled by Darby (1984) and others against the "unrealistic" $R/n > 1$ condition.

In the BGS economy, the gross real return to the investment project, a, is constant. Bhattacharya and Kudoh (1998) consider a neoclassical production economy in which the in-

A Seigniorage Laffer Curve

The Laffer curve was originally developed to show that income tax rates can get so high people start to choose nonmarket activities over working. If the rate at which people drop out of work is fast enough relative to the rate at which income taxes are raised, income tax revenue—the product of people's income (the tax base) and the tax rate on it—could decline.

Monetary economists have used similar reasoning to explore the effect increasing the inflation tax has on seigniorage. Recall, the revenue earned from money creation is

$$vm\left(1-\frac{1}{\lambda}\right);$$

vm is the seigniorage tax base, and $1 - 1/\lambda$ is the tax rate.

First, consider the relationship between changes in the money growth rate and the level of real seigniorage. The idea is that faster money growth is associated with higher inflation rates. Holding everything else (especially the seigniorage tax base) constant, higher inflation would produce higher real seigniorage. However, the inflation rate is inversely related to the gross real return to holding money. If people hold less money in response to the higher inflation, it is clear the tax base is declining in the face of a higher tax rate. Inflation rates can get so high that people start to eschew money. In short, vm declines.

The Laffer curve description fits the following scenario. When the inflation tax rate is low, the decline in the seigniorage tax base is small and the product (real seigniorage) rises with increases in the inflation rate. However, it is possible that at high money growth rates (and hence at sufficiently high inflation rates), people will reduce their money holdings so much that real seigniorage may fall with further increases in the inflation rate. When money growth rates and real seigniorage are positively related, they are on the good side of the Laffer curve. Conversely, when a decrease in real seigniorage accompanies an increase in the money growth rate, they are on the bad side of the Laffer curve.

Second, consider the relationship between reserve requirements and real seigniorage. Again, holding everything else constant, an increase in the reserve requirement will raise the seigniorage tax base, resulting in greater seigniorage. However, if the rate of return to other stores of value exceeds that of money, an increase in the reserve requirement drives a greater wedge between the return to money relative to other stores of value. If reserve requirements rise enough, people have an incentive to move their savings from banks to stores of value that do not face reserve requirements. This action could result in a smaller seigniorage tax base. As with the money growth rate, therefore, the relationship between the reserve requirement ratio and real seigniorage may be inverse-U shaped.

vestment project exhibits diminishing marginal returns.[24] As the government increases its reliance on bonds, investment in the project gets crowded out, raising the return on the project and thereby raising the return to bonds, R. Because of the no-arbitrage condition, government bond financing becomes costlier as interest expenses rise with R. Bhattacharya and Kudoh find that even when the real return to capital (analogous to a) is lower than the economy's growth rate, the UMA is still a possibility. As such, they show that the SW/BGS results extend to a more general economic model.

DIFFERENT MONETARY POLICY TOOLS

Sargent and Wallace restrict the central bank to a single policy tool, the money growth rate. However, real-world central banks have other means of raising seigniorage. For example, many directly control the reserve requirement ratio. This gives rise to a broader question: faced with an increase in treasury debt, is it better for the central bank to change the reserve ratio or change the money growth rate? For the purpose of the UMA, does it matter how monetary policy is implemented? Is it possible that the UMA may not result if the central bank changes the reserve ratio (instead of the money growth rate) to raise the required seigniorage?

Freeman (1987) identifies the optimal way for the central bank to raise seigniorage. Abstracting from government debt, he shows that monetary policy could mimic a lump-sum tax. Set the reserve requirement equal to the ratio of government purchases to output. By confiscating this amount of real money balances, the government could fund its purchases. Confiscation would be achieved by making these money balances worthless—that is, by letting money grow at an infinite rate.[25] Thus, Freeman shows that a combination policy using both money growth rates and the reserve requirement ratio would be best.

Bhattacharya and Haslag (1999) study a production economy, similar to the Bhattacharya and Kudoh (1998) economy, in which the central bank controls two monetary policy tools. The central bank applies either the reserve requirement or the money growth rate, holding the other constant, to affect the level of seigniorage. By changing the reserve requirement, the central bank alters the seigniorage tax base, while keeping the seigniorage tax rate constant. Bhattacharya–Haslag quantify the change in the steady-state level of real seigniorage following a change of central bank tool.

People store for future consumption by holding money, government bonds, and investment projects. The central bank could raise the seigniorage needed to cover a permanent increase in government bonds by raising the money growth rate or by lowering the reserve requirement ratio. Because a person's lifetime consumption increases as reserve requirements are lowered, the reserve requirement policy is a pleasant monetarist arithmetic.

In addition to addressing the possibility that monetary policy may be implemented in multiple ways, the Bhattacharya–Haslag exercise underscores the importance of the seigniorage Laffer curve in this discussion. SW's unpleasant monetarist arithmetic occurs because the economy is on the "good"' side of the Laffer curve with respect to the money growth rate: an increase in the money growth rate generates an increase in seigniorage. (The box entitled "A Seigniorage Laffer Curve" discusses the Laffer curve as it applies to monetary policy.) In contrast, the Bhattacharya–Haslag results indicate the economy is on the "bad" side of the Laffer curve with respect to the reserve requirement: a decrease in the reserve requirement raises seigniorage.

For the Bhattacharya–Haslag result, the intuition is straightforward. With higher reserve requirements, fewer of the deposited goods are put into the investment project.[26] As such, investment projects are crowded out of the bank's portfolio. Diminishing marginal returns play a crucial role here. The gross real return on the investment project would rise. If government bonds are held, their return would rise, too. It follows that the government's interest expense would rise: there are more government bonds, and the interest rate on each bond is higher. The increase in the reserve requirements drives up interest expenses, which will require more seigniorage, which will, in turn, require a further increase in the reserve requirement, and so on. Bhattacharya–Haslag find that lowering reserve requirements increases investment in the project enough that the gross real return on government debt declines, permitting the smaller tax base to finance the increase in government expenses. Thus, their findings suggest that how monetary policy is implemented does impact the unpleasant monetarist arithmetic, in the sense that people prefer lower reserve requirements to faster money growth when the central bank must pay for a permanent increase in government bonds. Put differently, a more "pleasant" monetarist arithmetic may be observed if the central bank reduces the reserve ratio to raise the required revenue.

CONCLUDING REMARKS

This article reviews some recent developments in the unpleasant monetarist arithmetic literature, with a focus on the studies that adopt the Sargent–Wallace approach of making the central bank subservient to the treasury. We ignore the literature that explores the game of chicken between these two entities.

This survey highlights two main developments in the literature. The SW result seems to rest squarely on the proviso that the real interest rate on government debt is greater than the economy's growth rate. However, the data appear to relegate this result to that of a theoretical curiosity; for most of the postwar period, the real interest rate has been below the growth rate in both the United States and Canada. One recent development in the literature shows that for the SW result to hold, it is not necessary that the real return on government debt exceed the economy's growth rate. If there is an asset with a real return that exceeds the economy's growth rate, and if government debt offers a positive nominal interest rate, the SW result is possible even if the real return on government debt is less than the economy's growth rate. Both these sufficient conditions seem empirically plausible; for many countries, the average real return on equities is, on average, above the economy's growth rate. Thus, a subservient central bank could still be required to support the treasury's financing needs, even if the real return on government debt is quite low.

Second, we examine a case in which the central bank controls more than one policy tool, to determine whether the unpleasant monetarist arithmetic depends on which tool is used. Numerical analyses indicate that a permanent increase in government debt requires faster money growth, at least for low to medium money growth rates, but lower reserve requirements. The findings are consistent with the notion that reserve requirements are a blunt instrument. Movements in the reserve requirement ratio directly crowd out capital from a bank's portfolio. Movements in the money growth rate do not have such a direct impact on the distribution of the bank's assets. Because the rates of return on these assets are affected, our analysis suggests that people would prefer lower reserve requirements, and thus lower real rates of return, to faster money growth to finance the government's bond issue.

A caveat is necessary. In this article, we provide a status report on the unpleasant monetarist arithmetic, under the explicit assumption the central bank is completely subservient to the treasury. This leaves unanswered the question of to what degree (if any) the central bank should be subservient to the treasury. Answering such a question would require delving into the myriad strategic considerations that determine the degree to which the central bank is independent of the treasury. Doubtless, this is interesting material for future work.[27]

NOTES

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1. Of course, well-defined normative criteria (objective functions) are needed for judging efficacy. The appropriateness of government objective function(s) is outside the scope of this article.
2. The basic ideas had already been presented by Metzler (1951), Patinkin (1965), and especially Christ (1968).

[3] A standard game of chicken involves the following. Two players, each in a car, face each other, separated by a distance of, say, 100 yards. Someone blows a whistle and the cars start rushing toward each other. If neither player moves out of the other's way, both would die. So one player has to give. The one that gives way to the other is the "chicken"; the other player wins. Sargent (1987, 176) attributes the idea of viewing monetary–fiscal policy interactions as a game of chicken to Wallace.

[4] Thinking about these issues as a game of chicken is enlightening in regard to Europe's recent move toward a single currency. The issue of deficit financing arises because the treasuries in the eleven countries would appear to lose the game of chicken against the European central bank.

[5] This policy experiment was popular among researchers embroiled in the monetarists vs. Keynesians debate.

[6] There is no reason, a priori, to believe that an independent central bank would raise seigniorage to meet the treasury's interest obligations. An alternative is for the treasury to raise taxes. A good analogy is the example of the Federal Reserve and the state of Texas. There is no evidence the Fed creates money to meet the state's obligations, although the state is clearly a passive beneficiary of unexpected increases in money growth rates brought about by the Fed.

[7] In this article, we stick to the version of unpleasant monetarist arithmetic Sargent (1987) presents. In particular, we restrict our focus to a comparison of steady states (see the section titled "The Unpleasant Monetarist Arithmetic Result"), which differs slightly from SW (1981). SW compare the inflation rate for two cases: one in which the government finances its deficit with money creation today and one in which the government issues bonds to finance the deficit and is eventually forced to monetize the deficit. The inflation rates differ in the two cases; indeed, SW derive conditions in which the inflation rate is higher in the bond-finance case.

We follow the approach adopted in Sargent (1987). There, the comparisons are conducted on stationary equilibria. Sargent writes: "The higher the stationary value of interest-bearing government debt *b*, the lower the rate of return on currency, that is, the higher the inflation rate. This is the foundation of Sargent and Wallace's result" (147). As such, we derive conditions in which higher inflation is part of the policy package accompanying an increase in government bonds.

[8] To see this, let population growth in this economy be represented by $N_t = nN_{t-1}$, so that the gross population growth rate is n. That is, if n were equal to 1 at date $t - 1$, it would mean the adult population replaced itself one for one with children that period or the population did not grow between dates $t - 1$ and t. Aggregate income (GDP) is the product of the number of young people and their endowments; that is, $N_t y$. Thus, the aggregate income growth rate is given by $N_t y/N_{t-1} y$. With a constant endowment per young person, y, the income growth rate reduces to n, the same as the population growth rate.

[9] We have eliminated the possibility of intergenerational loans. The old would never loan anything to the young because repayment would occur after the old have died. (There are no dynasties that could enforce repayment.) The young would never loan anything to the old because there is no way to enforce contracts with a dead person. See Samuelson (1958) for details.

[10] Put differently, v_t is the inverse of the price level at date t in this economy.

[11] If the choice is between holding interest-bearing bonds or holding non-interest-bearing money, no person will hold money unless its real rate of return is the same as the real return on bonds. In other words, we need to specify the reason people hold money even when better stores of value are available. We return to these issues in the section "Adding Stores of Value."

[12] Fiat money is nothing but intrinsically worthless pieces of paper money that are inconvertible—that is, the government does not promise to convert the money into the consumption good. See Wallace (1980).

[13] Since everything in the economy (except the price level) in a steady state is time-invariant, the subscript t loses any meaning and is therefore dropped.

[14] Central bank independence can be defined in terms of the game of chicken between the treasury and the central bank described above. Suppose the fiscal authority chooses it policies first (τ and b), independent of the central bank. The central bank, having lost the game of chicken, sets λ to ensure that current and future money creation pays for all the treasury's future interest obligations and the government's expenditures. This is our definition of a subservient central bank. Aiyagari and Gertler (1985) label this a non-Ricardian regime. See also Canzoneri, Cumby, and Diba (1998) for alternative classifications in terms of fiscal-dominant and money-dominant regimes. By their definition, the postwar United States has a money-dominant regime. It is important to note that a fiscal-dominant regime in the sense of Canzoneri, Cumby, and Diba (1998) is not the same as a fiscal leadership regime (a term SW use), which in turn differs from a non-Ricardian regime.

[15] Abel (1992) also discusses the consequences of government financing when the real return on government debt exceeds the economy's growth rate. Abel focuses on deficit financing's impact on the capital stock.

[16] Sargent and Wallace go one step further, to show that if people are forward-looking, they will know future money creation is necessary to fund the increase in the government's debt; as a result, current inflation will rise.

[17] Sargent and Wallace (1981), Darby (1984), and Miller and Sargent (1984) also recognize this case and discuss both sides of the debt-financing issue.

[18] Describing the transition from one steady state to the other can be thought of as a case in which the ratio of

public debt to GDP is declining over time. With a declining ratio of debt to aggregate income, principal and interest payments associated with the bond issue become a smaller fraction of the economy. It follows that the growing economy can absorb the debt obligations without having to rely on additional taxes or seigniorage to pay for the initial purchase.

[19] We focus on the United States and Canada because it is the data for these countries that have been used to argue against the SW result. We also have data for Russia and Brazil, although there are only four years of data from Russia and there is a five-year gap in Brazil's R/n ratio. The data are available from the authors upon request.

[20] A real-world analog of this would be a time deposit or certificate of deposit.

[21] It is important to note that the reserve requirement on money holdings is singularly responsible for creating a wedge between the return to government debt and the return to the investment project.

[22] This combines two previously discussed stipulations: investment projects must yield returns that are higher than both the economy's growth rate and the return on money.

[23] The important point here is that this is true even though in this regime $R/n < 1$ (condition 3) is not satisfied. This is also the sense in which printing money is the cheaper option.

[24] Taking this to its natural limit, if the economy could put an infinite quantity of goods into the investment project, the return on the last unit would be zero. Conversely, if the quantity of goods in the investment project is close to its minimum-size requirement, an additional unit of the good put into the project will offer returns that greatly exceed the economy's growth rate.

[25] A technical consideration arises when the money growth rate is set equal to infinity. The Freeman policy prescription works if the money growth rate is some very large, finite number so that the value of money is close to zero.

[26] Recall that people save the same amount regardless of the rate of return. Thus, total saving is taken as given.

[27] In an interesting paper, Carlstrom and Fuerst (forthcoming) examine the rules in the game of chicken in terms of how changes in timing affect the relationship between the central bank and the treasury.

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