Should Bank Reserves Earn Interest?

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> The case for payments of interest on reserves applies not only to the 100% reserve system, but equally to our present fractional reserve system. Accordingly, even if reserves are not raised to 100%, Reserve Banks should be required to pay interest on their deposit liabilities.
>
> —Milton Friedman
> *A Program for Monetary Stability*

As the introductory quote indicates, Milton Friedman (1959), among others, has advocated paying interest on reserves.[1] In the United States and many other countries, banks and other financial intermediaries are required to hold a fraction of their assets as fiat money—unbacked, interest-free bills of the central bank. In the absence of interest on these reserves, the average return to assets held by banks must lie below the market rate of return. This implies that banks must pay their depositors a return below the market rate of interest, unnecessarily discouraging the holding of bank deposits. Because such intervention into the business of banking is so common, basic questions about the desirability of such requirements may easily be overlooked. For instance, by forcing banks to hold unbacked assets paying no interest, might the central bank be discouraging banking and the accumulation of capital?

But where would the interest come from? As with any government expenditure, interest paid on reserves must (at least eventually) come from taxes, raising two questions: Wouldn't wealth be reduced by the rise in taxes? Wouldn't taxation introduce its own economic distortions, possibly worse than those that result from the absence of interest?

Paying interest on reserves would increase the demand for deposits and thus for reserves. This, in turn, would raise the value of existing reserves, increasing the wealth of those who own bank deposits at the time that interest payments are initiated. Bruce Smith (1991) shows that this windfall gain to those holding deposits at the time the policy is enacted comes at the expense of future generations; that is, future generations must pay the taxes to finance the interest payments but do not receive all of the resulting benefits. Thus, Smith shows that the transfer of wealth created by the payment of interest makes future generations worse off.

In this article, we propose a means of eliminating this transfer. We begin by discussing the role reserve requirements play in a simple economy. People finance the next period's consumption by holding deposits. The key feature

of the model is that reserve requirements force banks to hold fiat money as fractional backing for deposits. The merits of paying interest on reserves will be clear if the government offsets the wealth transfer identified by Smith.

Our idea for an offsetting transfer is adapted from a policy proposed by Leonardo Auernheimer (1974).[2] When interest on reserves is initiated, the central bank should expand the stock of nominal reserves to keep the price level from decreasing. If the central bank uses this increase in the money stock to purchase interest-bearing assets (an open market operation), the interest generated by these assets can help pay for the interest paid on reserves, lowering the tax burden on future generations. We argue that paying interest on reserves, when accompanied by the appropriate open market operation, can make every future generation better off without hurting initial deposit holders.

We also take up the second of our nettlesome questions: Would taxation introduce its own economic distortions? The taxes available to government in the real world are generally ad valorem taxes; the amount of tax collected is set at some fraction of an economic variable, such as income or sales. Ad valorem taxes artificially discourage the taxed activity, just as the absence of interest on reserves discourages deposits at banks. We show that despite this tax-induced distortion, we can make people unambiguously better off by paying interest on reserves. This improvement occurs even if the interest must be funded by a tax used in the real world—a distorting ad valorem tax on capital—if this tax is accompanied by a price-stabilizing open market purchase.

In sum, our questions about the costs of paying interest on reserves are fairly straightforward to resolve. Both capital taxation and open market operations are widely used real-world policy options. Therefore, there exists a way to finance the payment of interest on reserves that will make the public unambiguously better off.

A model of reserve requirement banking

To address these questions, let us examine a simple model adapted from the framework shared by David Romer (1985), Thomas Sargent and Neil Wallace (1985), Scott Freeman (1987), and Smith (1991) in which financial intermediaries that mobilize capital are subject to a reserve requirement.

In each period, starting from some initial period 1, N people who live two periods are born. Each produces y goods when young and nothing when old, but wishes to consume in both periods of life. The problem facing these people is the means of financing consumption in the second period of life. There is also a generation that lives and consumes only in the initial period, hereafter referred to as the "initial old."

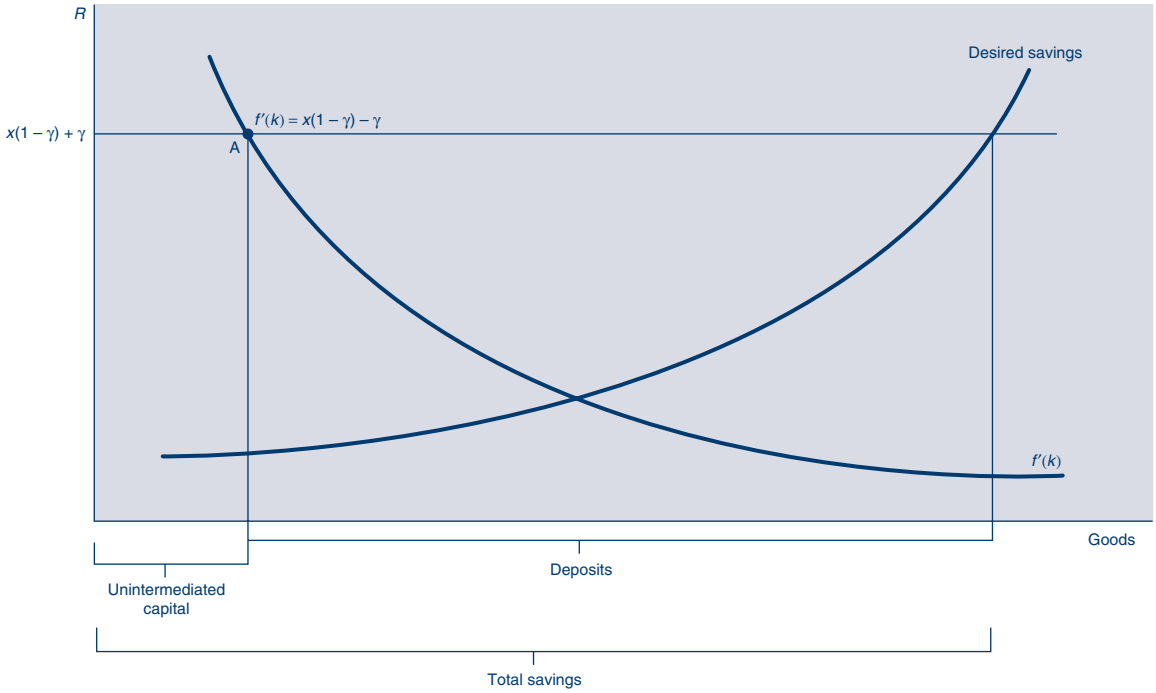
In the first period of this model economy, there is a fixed stock of M (divisible) pieces of paper called *fiat money*. In addition to money, there are also two forms of capital. The first form is available to any individual in isolation. An investment of k_t goods in period t will produce $f(k_t)$ consumption goods in period $t + 1$. The marginal product of capital, which we express as $f'(k)$, is positive but decreasing. The second form of capital produces a constant x consumption goods ($x > 1$) in period $t + 1$ for each good invested at t. This latter form of capital can be made only in amounts greater than y so that no individual alone has the resources to finance capital. Both forms of capital produce consumption goods only once.

Note that the second form of capital is illiquid in this economy because it cannot be divided into small units. It is easy to see how an intermediary can overcome this illiquidity by simply pooling the deposits of many individuals to an amount greater than y. We assume for simplicity that the intermediation services are costlessly and competitively provided by entities referred to as "banks."[3]

In this economy, we assume that a reserve requirement is imposed: for each good deposited, a bank must hold fiat money worth γ goods but is free to invest the remaining $1 - \gamma$ goods in the illiquid, or intermediated, capital good.[4] (We assume throughout this analysis that the initial old hold positive quantities of both unintermediated capital and deposits.) If fiat money's rate of return is less than that of capital, banks will hold no more than the required balances of fiat money. Suppose, for now, that banks do not hold any excess reserves. (We will verify shortly that this is a wise decision.) If s_t denotes deposits per young person, then banks will hold fiat money balances worth $\gamma N s_t$ goods. Those required reserves represent the total *demand* for fiat money measured in goods. The *supply* of fiat money is M dollars or, when measured in goods, $v_t M$, where v_t represents the goods that can be purchased by a single dollar. The goods value of a dollar is simply the inverse of the dollar price (p_t) of one good, or $v_t = 1/p_t$. Furthermore, the gross real rate of return from holding fiat money is the ratio of goods purchased by a single dollar

Figure 1

Determining Savings, Deposits, and Unintermediated Capital for a Given Reserve Requirement



in period $t + 1$ to the goods purchased by a single dollar in the current period, or v_{t+1}/v_t.

For the demand for fiat money to equal its supply,

$$\gamma N s_t = v_t M. \tag{1}$$

Notice that when deposits, s_t, are constant over time, the demand for fiat money is constant over time. Therefore, when the stock of fiat money is also constant over time, the value of a dollar and the price level will both be constant over time. It follows that the gross real rate of return of a dollar, v_{t+1}/v_t, equals 1.[5]

What, then, will be the rate of return offered by competitive banks? Assuming for simplicity that intermediation services are costlessly provided by banks in a competitive market, then banks will offer depositors the rate of return that the banks can earn on the assets they hold. This (gross, real) rate of return (call it R) is

$$R = (1 - \gamma)x + \gamma \tag{2}$$

because for each good deposited, the bank can invest $(1 - \gamma)$ in capital paying the rate of return x and purchase γ in fiat money paying the rate of return 1. Notice in equation 2 that increasing the reserve requirement lowers the rate of return on deposits by forcing banks to hold more low-return fiat money per deposit. Clearly, every subsequent generation suffers from this lower rate of return on their deposits. Moreover, with $x > 1$, equation 2 indicates that the bank best serves its depositors by not holding reserves in excess of those required.

People will invest in the asset paying the better rate of return. This implies that the people who hold both deposits and unintermediated capital will invest in unintermediated capital up to the point that its marginal rate of return just equals the rate of return offered by intermediaries:

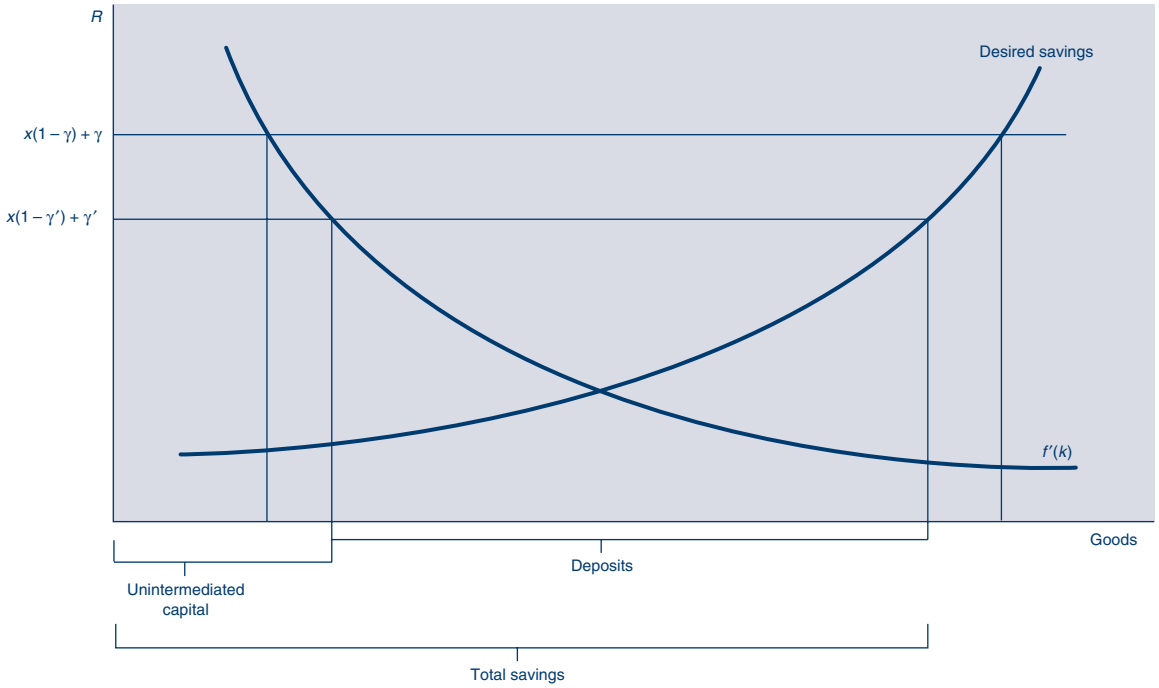
$$f'(k) = (1 - \gamma)x + \gamma. \tag{3}$$

Because an increase in the reserve requirement lowers the return on intermediated capital, people switch from deposits to unintermediated capital. This switching occurs until the rate of return on unintermediated capital falls to equal the new lower rate of return on deposits.

Figure 1 illustrates the basic point made in equation 3. The desired savings curve plots the quantity of savings for next-period consumption at different rates of return. The rate of return on the vertical axis is equal to the rate offered by competitive banks and is determined by the returns on intermediated capital and reserves. The horizontal line emanating from the value $x(1 - \gamma) + \gamma$ on the vertical axis in Figure 1 is the return on deposits. For a given

Figure 2

The Effect of an Increase in the Reserve Requirement from γ to γ'



level of savings, the distribution between intermediated and unintermediated capital depends on the assumption that the return on unintermediated capital falls with each additional unit of this form of capital. Figure 1 captures this feature by representing the $f'(k)$ curve as a downward sloping line. From equation 3, people add units of unintermediated capital up to the point at which the return offered by banks equals the return on unintermediated capital. This occurs at point A in Figure 1. The horizontal distance between the vertical axis and point A measures how much unintermediated capital people will choose. The difference between desired savings and unintermediated capital—the horizontal distance between total goods saved and point A—measures the quantity of deposits.

An additional implication of equation 3 seen in Figure 1 is that with reserve requirements, the output produced by one more unit of unintermediated capital, $f'(k)$, is less than the output from a unit of intermediated capital, x.[6] Therefore, by encouraging people to switch from intermediated capital to unintermediated capital, a reserve requirement reduces output for each good switched. More generally, higher reserve requirements discourage total savings because of the lower rates of return offered on both unintermediated capital and deposits. Figure 2 illustrates the effects an increase in reserve requirements has on savings and each form of capital.

There is, we should note, a group that benefits from the imposition of a reserve requirement: the initial old. By assumption, this group starts with a portfolio of assets that include fiat money. If reserve requirements were removed, this fiat money would have no value. Consequently, the value of the initial old's portfolio would fall. Alternatively, increasing the reserve requirement increases the demand for fiat money, making each dollar more valuable and raising the welfare of the initial old by raising the value of fiat money (see equation 1). In short, the reserve requirement transfers wealth from all future generations to the initial old.

The central bank can increase the rate of return on deposits if it increases the rate of return on fiat money by, for example, paying interest on required reserves. This will increase the rate of return paid to depositors, but will it make them better off? As we demonstrate in the next section, the answer depends on how this higher rate of return is financed.

A case with interest payments on reserves

In this section, we consider how different financing schemes affect the desirability of paying interest on reserves. Paying interest on reserves will be deemed desirable if at least one group is made better off while no other group is

harmed. In the model outlined above, the groups can be identified using the date at which the policy is implemented as the reference point; thus, the two groups that come to mind are those already holding money when the policy is implemented (the initial old) and the future generations.

The central bank as an intermediary. Consider first a policy that would have the government pay the interest from interest-bearing assets of the central bank. Suppose that instead of leaving the initial stock of central bank money in the hands of the initial old, the central bank takes it and uses it to purchase (intermediated) capital. This gives the central bank ownership of a stock of capital. (We focus our attention here on an equilibrium in which the stocks of reserves, central bank capital, and the value of money are constant over time.) Formally, the central bank's balance sheet constraint is

$$K^g = vM, \tag{4}$$

where K^g is an interest-bearing asset that represents the value of capital held by the central bank.[7] The central bank will pay interest on its liability, reserves (central bank money), using the return on this capital net of its replacement cost, $xK^g - K^g = (x - 1)K^g$. If ρ denotes the nominal net interest paid on a dollar of reserves, then the interest paid on reserves equals ρvM, implying that each period the central bank's budget requires

$$(x - 1)K^g = \rho vM. \tag{5}$$

Because the central bank owns capital exactly equal to the value of reserves ($K^g = vM$), the central bank can offer interest on reserves equal to the net return of capital:

$$\rho = (x - 1), \tag{6}$$

which implies that the gross rate of return on reserves, $1 + \rho$, is x. Because the central bank backs its money with capital, reserves pay the same rate of return as other interest-bearing assets owned by private banks. Under this plan, the central bank has become an intermediary paying market interest rates to its depositors (private banks). Therefore, depositors at private banks will no longer care what fraction of their deposits is required to go into reserves. The gross rate of return on deposits is now

$$\begin{aligned} R &= (1 - \gamma)x + \gamma(1 + \rho) \\ &= (1 - \gamma)x + \gamma x = x, \end{aligned} \tag{7}$$

which equals the gross rate of return on capital regardless of the size of the reserve requirement. For any positive reserve requirement, all future generations are made better off by this plan to pay interest on reserves because they are offered a higher rate of return on their deposits.

Would anyone oppose such a plan to pay interest on reserves from central bank capital? Yes, the initial old would. Notice that this financing scheme begins with the central bank confiscating the initial old's money balances without any compensation. Such a tax collection scheme reduces the wealth holdings of the initial old, reducing their consumption.

The payment of interest on reserves from central bank capital has the same welfare effects as abandoning reserve requirements. In both cases, future generations receive a better rate of return (x) on their deposits, but the initial old lose the value of their initial balances of fiat money.

There are three differences between abandoning reserve requirements and confiscating the initial old's money balances. First, when reserve requirements are simply abandoned, all fiat money becomes worthless.[8] Under central bank intermediation, however, there is still a demand for reserves, and the value of a dollar is again determined by the equality of supply and demand for reserves set forth in equation 1. As we have shown, banks will hold zero excess reserves, so that

$$\gamma Ns = vM. \tag{8}$$

Second, the model economy described above specifies that intermediated capital always returns x units of the consumption good for every one unit invested. It is not difficult to imagine a situation in which returns are related to the quality of the investment decisions made. Under central bank intermediation, we entrust a governmental body, the central bank, with investment decisions. The central bank may not be motivated by maximization of profits. Consequently, if the central bank does not choose as wisely as private banks, the return offered on reserves may be below the market rate of return. Of course, one way to remove the investment decision from the purview of the central bank is to open the discount window. Banks could borrow funds at the market rate of return and make the investment decisions. Then the central bank's only responsibility would be to restrict its lending to sound banks.

Third, we have thus far assumed that intermediation services are costlessly provided. A

more realistic assumption recognizes that costs, such as those of record-keeping, are associated with creating private intermediary services. With central bank intermediation, there is a second level of record-keeping; people make deposits at banks and then banks make deposits (hold reserves) at the central bank. If it is costly to keep records and otherwise manage deposits, the total of these costs will be higher under this two-level system of intermediation than under the one-level setup.

Tax-financed interest on reserves. Suppose that the central bank wants to finance interest on reserves without hurting the initial old. It can do so if each future generation is taxed to pay the interest.[9] Would the benefits of the increased rate of return exceed the cost of the taxation? Increasing the rate of return on deposits would increase deposits and thus capital, as desired. The increased deposits, however, also increase the demand for reserves. Greater demand for fiat money increases the value of the initial reserves owned by the initial old. (Note from equation 1, the equality of supply and demand for reserves implies that $v = \gamma Ns/M$. Clearly, an increase in s will increase v.) In effect, the taxes paid by future generations go to pay interest on reserves and to increase the wealth of the initial old. Smith (1991) demonstrates that this transfer of wealth from the future generations to the initial generation lowers the welfare of the future generations despite the greater rate of return on deposits. To understand this result, note that the taxes paid by people in the future generations are exactly equal to the value of the interest payments received on reserves. These two changes to lifetime wealth, therefore, exactly cancel each other out. However, the policy has a side effect: the reserves that the initial generation owns and that subsequent generations need have been made more expensive. This transfers wealth from subsequent generations to the initial old. Therefore, the central bank cannot increase the welfare of the future generations simply by financing interest on reserves through pay-as-you-go taxation.

Auernheimer (1974) suggests a way to finance the payment of interest on reserves without hurting or helping the initial old.[10] The initial old gain under the tax plan just described because of an increase in the value of their initial money balances. The value of money can be brought back to its initial level if the central bank prints more money, such that the increased demand for money is exactly matched by an increased supply of money. How can a plan featuring taxes and accommodating money supply increases help the future generations? Let the central bank use the increase in the stock of its money to buy capital. The central bank's exchange of (intermediated) capital for fiat money is an open market purchase. The additional capital can then be used to help finance the payment of interest on reserves, lessening the tax burden of future generations. Freeman and Haslag (forthcoming) show that this tax-financed interest on reserves makes the future generations better off. (A formal proof is also presented in the appendix.) The higher rate of return on deposits encourages savings through banks at its optimal level, without a transfer of wealth from the future generations to the initial owners of money. In short, the future generations pay enough taxes to finance the interest payments on reserves but do not pay for a transfer to the initial old.

A case with distortionary taxes

The financing scheme outlined above is based on a lump-sum tax. The desirability of taxing to pay interest on reserves may no longer hold if the tax, like many real-world taxes, itself distorts individual incentives. An income tax, for example, may well reduce incentives to work and invest, therefore causing more economic distortion than the absence of interest on reserves. To address this concern, we now examine the payment of interest on reserves financed by a tax commonly used in the real world, a tax on capital. We show that people are better off with interest paid on reserves, even if it must be financed by a tax that discourages the holding of capital.

Consider, in particular, a tax applied against the return from both types of capital; that is, the payment of interest on reserves is to be financed by a tax of α times the return to both intermediated and unintermediated capital. As with the lump-sum case described above, we assume that the government conducts an open market purchase that keeps the price level constant. Thus, the net interest on the government's capital goods plus revenue from the capital tax is equal to the government's net interest on reserves.

The question is whether interest-bearing required reserves are welfare-improving when financed with a distortionary capital tax. Freeman and Haslag (forthcoming) and the appendix to this article show that the total net return to the future generations is increased when the government pays interest on required reserves, even if the interest is financed by a tax on capital.

The intuition behind this result is fairly straightforward. If reserves pay no interest, a reserve requirement directly distorts the return to intermediated capital. In this way, the reserve requirement is like a tax on the return to intermediated capital, while unintermediated capital is not directly taxed. Paying the market rate of interest on reserves means that deposits earn the same return as unintermediated capital, ending the discouragement of deposits resulting from the lower return from required reserves. Taxing both intermediated and unintermediated capital at the same rate spreads the distortion equally, and thus efficiently, across the two types of capital. In short, people do not make investment choices between the two forms of capital based on after-tax returns. When taxes are applied equally, both the pre- and after-tax returns are equalized. The gain from the increased return on deposits more than offsets the lower after-tax return on unintermediated capital. Consequently, future generations have a higher total return than when the return of only one type of capital is distorted.[11]

The payment of interest on reserves encourages people to marginally substitute intermediated capital for unintermediated capital. For each extra unit of intermediated capital, x goods are produced, while an extra unit of unintermediated capital produces $f'(k)$ goods. We have seen that when intermediated capital is subject to reserve requirements without interest, $f'(k) = (1 - \gamma)x + \gamma = x - (x - 1)\gamma$, which is less than x. Therefore, when people switch one unit of savings from unintermediated capital to intermediated capital, more output is gained (x) from the increase in intermediated capital than is lost [$f'(k)$] from the drop in unintermediated capital. Therefore, there is more overall output and greater welfare when interest is paid on reserves. Output and welfare would be even greater if the interest could be funded by lump-sum taxes, but stuck as we are with distorting taxes, the payment of interest on reserves is still an improvement.

Conclusions

In this article, we demonstrate how alternative schemes to finance interest payments on required reserves will affect people. We consider four different schemes: directly taxing initial required reserves, a lump-sum tax on future generations, and two financing schemes that are accompanied by open market purchases—lump-sum taxes and capital taxes.

We show that using lump-sum taxes accommodated by an open market purchase to finance interest-bearing reserves will avoid some of the pitfalls associated with either directly taxing initial required reserves or the lump-sum tax alone. When open market purchases accompany the payment of interest on reserves, members of the future generations are better off while the initial old are unaffected. Clearly, this makes society better off. We further show that paying interest on reserves is strictly better than not paying interest, even if the taxes are distortionary. This last result underscores the distortionary effect associated with reserve requirements. Spreading the distortion across both types of capital—in the spirit of the Ramsey rule of efficient taxation—raises welfare.

A key feature of the welfare improvement is the accommodating open market purchase suggested by Auernheimer. The payment of interest on reserves effects a transfer from future generations to the initial old. This transfer can be exactly offset by an open market purchase. The assets thus purchased can then be used to help finance the payment of interest. Such an accommodation is not beyond the central bank's normal operations. Indeed, Haslag and Hein (1995, 1989) provide evidence that the Federal Reserve systematically accommodates changes in reserve requirements with open market operations.

Overall, the main purpose of this article is to demonstrate that paying interest on reserves improves welfare in a broader class of model economies than previously believed. We extend the class of economies along two distinct lines. For some time, people have recognized the improvement that is possible in Friedman's setting with infinitely lived people and lump-sum taxes. Smith raises questions about the desirability of paying interest on reserves when the initial (finite-lived) money holders benefit but are not taxed. Our first extension shows that welfare improvement is still possible in this economy if a simple coordinated financing scheme is adopted. The second extension shows that paying interest on reserves can improve people's welfare, even if the interest is funded through distortionary taxes.

Notes

[1] George Tolley (1957) also argues that the central bank should pay interest on reserves. Joshua Feinman (1993) traces the historical evolution of reserve requirements in the U.S. banking system. Feinman also notes that the Federal Reserve has explicitly supported legislation authorizing the payment of interest on reserves since the 1970s.

[2] Auernheimer's proposal is designed to offset changes

in the demand for money induced by inflation rate changes. Also see Phillippe Bacchetta and Ramon Caminal (forthcoming), who apply the idea to reserve requirement changes.

[3] Certainly there are many other services provided by banks, but this one is simple to model and adequate to illustrate the points of this article. Other services of banks are implicitly included in x.

[4] In the United States, the requirement for checkable deposits at large banks is currently 10 percent, or $\gamma = 0.10$.

[5] More generally, if the economy is growing at the gross rate n (that is, $N_t = nN_{t-1}$) and the fiat money stock is growing at the gross rate z (that is, $M_t = zM_{t-1}$), the gross rate of return on a dollar will be n/z.

[6] From equation 3, $f'(k) = x - (x-1)\gamma < x$.

[7] The central bank could also buy bonds from private banks, which would then use these funds to invest in intermediated capital. This scheme is closer to actual open market purchases but is equivalent in its effects to the direct purchases of capital by the central bank.

[8] This would not be true if there were an additional demand for fiat money as currency (negotiable notes passed from hand to hand). In most modern economies, the government retains a monopoly on the issuance of currency by outlawing its issuance by private banks backed by bank holdings of capital. This is exactly equivalent to a reserve requirement of 100 percent on currency.

[9] This is the financing scheme associated with Friedman's (1959) proposal and investigated by Smith (1991).

[10] Auernheimer (1974) describes just such a monetary policy accommodation scheme in describing the revenue-maximizing rate of inflation.

[11] The idea that taxing all goods improves welfare is discussed in Frank Ramsey's (1927) rule for efficient taxation. According to Ramsey, the government can raise welfare by setting distortionary taxes such that the percentage reduction in the quantity demanded of each commodity is the same. In our setting, Ramsey's rule is implemented by taxing both types of capital as opposed to taxing only one type. This result is demonstrated by Peter Diamond and James Mirrlees (1971) in a general setting. Diamond and Mirrlees demonstrate that taxing an intermediate input is not part of an optimal policy plan. In a monetary economy, Kent Kimbrough (1989) shows that the Ramsey tax rule applied to final goods improves welfare relative to a case in which intermediate goods were taxed.

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Appendix

In this appendix, we show more formally that paying interest on reserves will make people better off, even if the interest is financed with a distortionary tax on capital. To do so, we must first calculate the tax rate that would be needed to pay the market rate of interest on reserves. We let S represent total savings—deposits plus unintermediated capital—per young person and use asterisks to indicate values of variables in the absence of interest on reserves. The government must finance net interest on reserves of $(x - 1)\gamma(S - k)$ from taxes on the return from savings, $\tau x(S - k) + \tau f(k)$, and from the interest on the capital it acquires from the open market purchase in the initial period, $(x - 1)\gamma[(S - k) - (S^* - k^*)]$.

Altogether, this implies the government budget constraint is

$$(A.1) \qquad (x - 1)\gamma(S - k) = \tau x(S - k) + \tau f(k) + (x - 1)\gamma[(S - k) - (S^* - k^*)],$$

or

$$(A.2) \qquad (x - 1)\gamma(S^* - k^*) = \tau x(S - k) + \tau f(k).$$

Paying interest on reserves makes future generations better off if for any given level of savings, $S = S^*$, the total return net of taxes is greater when interest is paid on reserves:

$$(A.3) \qquad (1 - \tau)x(S - k) + (1 - \tau)f(k) > [x(1 - \gamma) + \gamma](S - k^*) + f(k^*).$$

We can now use the government budget constraint (equation A.2) to cancel several of the tax terms with terms on the right-hand side of equation A.3, leaving us with

$$(A.4) \qquad -x k + f(k) > -x k^* + f(k^*),$$

or

$$(A.5) \qquad x(k^* - k) > f(k^*) - f(k).$$

We know that $k^* > k$ because unintermediated capital is taxed when interest is paid on reserves. Because $f(.)$ is a concave function (capital has a diminishing marginal product),

$$(A.6) \qquad f'(k)(k^* - k) > f(k^*) - f(k).$$

When interest is paid on reserves, we know that the two forms of capital must offer the same marginal rate of return; that is, $f'(k) = x$. It follows that the inequality (equation A.5) is satisfied, proving that future generations are better off with interest paid on reserves, even if it must be financed through a distorting capital tax.