# Taxation, Growth And Welfare: A Framework For Analysis And Some Preliminary Results 

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A<br>shift away from the taxation<br>of capital toward taxation of consumption is potentially welfare<br>enhancing... .[E]ven larger welfare<br>gains might be obtained by substituting<br>consumption taxes for taxes<br>on labor income.

The belief that the U.S. tax code ought to treat capital more favorably than it currently does has become a perennial of political discourse in recent years. Proposed changes in the tax code have ranged from the relatively modest, such as reducing the tax rate on capital gains, to the more radical, such as a switch to a flat tax (which would eliminate double taxation of capital income) and "ending the IRS as we know it." Reform proposals are usually accompanied by claims that the reforms, if implemented, would yield a myriad of benefits to taxpayers generally, foremost among these benefits being an increase in the economy's long-run growth rate. Yet it is only in the past decade or so that economists have developed the tools that allow us to assess the validity of these claims. The objectives of this article are, first, to present some comparative statistics on how heavily capital is taxed in the United States relative to the other major industrialized countries; second, to outline a simple model that allows us to address some of the claims about how changes in the capital tax rate would affect the economy's long-run growth rate and the well-being of the average household; and third, to perform some simple "reform experiments" illustrating the potential magnitude of the gains that could accompany tax reforms.

The basic principles of optimal taxation were first worked out by Ramsey (1927). One of Ramsey's contributions was to demonstrate formally that under certain circumstances the optimal tax rates on different commodities would be inversely proportional to their elasticities of supply and demand. That is, if the supply or demand for some commodity is absolutely inelastic, then all tax revenue should be raised by taxing that commodity because doing so will entail no loss of welfare. In the short run, the quantity of capital supplied to productive activities is in fixed supply, and arguably, therefore, one should raise as much revenue as possible from taxes on capital. ${ }^{1}$ However it was not until the 1970s that economists developed a deeper understanding of the welfare costs of financing govemment expenditures with taxes on capital.

Feldstein (1978, 1974a, and 1974b) presented the pioneering analysis that challenged the notion that we could safely abstract from the decision to accumulate capital when evaluating the welfare costs of capital taxation, albeit in environments that restricted the response of either households or markets to policy changes. The first general equilibrium analysis of the effects of capital taxation was presented by Chamley (1981), who studied the welfare con-
sequences of eliminating a tax on capital. A more detailed analysis was presented by Judd (1987), who compared the welfare cost associated with the taxation of capital and labor income.

All these studies (and a substantial number of others) found that there would be significant welfare gains associated with the elimination of capital income taxation or, more realistically, the replacement of such taxation with higher taxes on consumption and labor. However, none of these studies allowed for any feedback to the long-run growth rate of the economy. All the analyses were conducted in the context of models in which the long-run growth rate was determined exogenously, or by factors outside the model. Insofar as the elimination or reduction of tax rates on capital income had any growth effects, they were transitory as the economy adjusted to a new long-run growth path. It was not until the development of models of endogenous growth in the 1980s that economists could begin to ask whether tax policy had any effect on the economy's long-run growth rate. These models, which were pioneered by Romer (1986), Lucas (1988), and Rebelo (1991), were in part motivated by the desire to construct a framework within which we could begin to meaningfully address the very long-run consequences of certain policies (this was especially true in the case of Rebelo 1991).

The seminal study of tax policy in the context of an endogenous growth model is by Lucas (1990, 293 and 314), who writes:

When I left graduate school, in 1963, I believed that the single most desirable change in the U.S. tax structure would be the taxation of capital gains as ordinary income. I now believe that neither capital gains nor any of the income from capital should be taxed at all....The supply-side economists... have delivered the largest genuinely free lunch I have seen in 25 years in this business, and I believe we would have a better society if we followed their advice.

Lucas $(1990,314)$ estimates the gain in welfare from the elimination of all capital taxation in the United States to be around 1 percent of annual consumption and notes for comparison that "it is about twice the welfare gain that I have elsewhere estimated would result from eliminating a 10 percent inflation, and something like 20 times the gain from eliminating post-war-
sized business fluctuations. It is about 10 times the gain Arnold Harberger... once estimated from eliminating all product-market monopolies in the U.S." Subsequent studies by, among others, King and Rebelo (1990) and Jones, Manuelli, and Rossi (1993) have tended to reinforce Lucas' findings about the benefits of eliminating capital income taxation.

In what follows, I examine a prototypical multiple-sector model of endogenous growth and use it to explore the welfare consequences of some fairly simple tax reforms. One key difference from existing analyses is that I use recently constructed estimates of average marginal tax rates on capital and labor income and consumption to calibrate the model. The "reforms" I consider consist of a halving of the tax rates on capital income, labor income, and consumption from their average levels over the past thirty years. I find that there would be a significant welfare gain associated with a reduction in the tax rate on capital income and its replacement with a consumption tax. However, I also show that there could be an even larger gain associated with a reduction in the tax rate on labor income. This possibility arises because of the importance of human capital accumulation for the growth process.

The findings in this article reinforce the general principle of efficient taxation that factors that are supplied inelastically should be taxed relatively more than factors that are supplied elastically. In the analysis below, the only factor that is supplied inelastically is raw, unimproved labor. The return to raw labor is inextricably tied to the return on human capital, which is supplied elastically in the long run, and so efficiency dictates that the burden of taxation be shifted toward consumption purchases as a proxy for taxing raw labor. In the model economy studied below, the first-best tax scheme would be to raise all revenue by taxing consumption and to exempt both labor and capital income from taxation.

## Factor income taxation in the United States

Research on the aggregate implications of changes in tax policy has long been hindered by the lack of data on measures of the tax rates on labor and capital income that correspond to the relevant concepts suggested by economic theory. For the United States, a number of authors have attempted to construct measures of average marginal tax rates on total income using data on individual tax returns (for example, Barro and Sahasakul 1983). However, these estimates do not distinguish between income derived from
labor and income derived from physical capital. As for international comparisons, the problem is compounded by differences in tax laws across countries.

Recently, however, Mendoza, Razin, and Tesar (1994) have tried to remedy this problem by constructing estimates of tax rates on capital and labor income, as well as on consumption expenditure, for the G-7 industrial countries for the period 1965-89. Their estimates aggregate all the various deductions, allowances, and so forth in a single measure and are the relevant empirical counterparts to the tax variables considered in dynamic economic models of the sort I examine below. Their measures of tax rates are constructed as follows.

The average effective tax rate on sales of consumption goods, $\tau^{\mathrm{C}}$, is defined as the ratio of the sum of tax revenues from general taxes on goods and services plus revenues from excise taxes to the consumption tax base. The consumption tax base is measured as the sum of private final consumption expenditures and government final consumption expenditures, less compensation of employees paid by producers of government services and tax revenues from general taxes on goods and services and excise taxes.

To construct a measure of the effective tax rate on labor income, Mendoza, Razin, and Tesar start by constructing a measure of the average tax rate on total income that households receive. This tax rate, $\tau^{\mathrm{H}}$, is defined as the ratio of the total revenue from taxation of the income, profits, and capital gains of individuals to the sum of the operating surplus ${ }^{2}$ of private unincorporated enterprises, households' property, entrepreneurial income, and wage and salary payments received by the household sector.

The estimate of the tax rate on labor income, $\tau^{\mathrm{W}}$, is then constructed as the product of the tax rate on total household income and wage and salary earnings, plus total social security contributions and taxes on payroll and workforce (which do not exist in the United States), expressed as a fraction of the tax base for labor income taxes. The tax base for labor income taxes is measured as the sum of wage and salary payments and employers' contributions to social security.

The numerator of tax rate on capital income, $\tau^{\mathrm{R}}$, is constructed as the product of the average tax rate on total income and the operating surplus of private unincorporated enterprises plus households' property and entrepreneurial income, to which is added the taxes on the income, profits, and capital gains of corporations;
recurrent taxes on immovable property; and taxes on financial and capital transactions. The denominator is the base for capital taxation, which is simply the operating surplus of the economy.

Figures 1 through 3 present Mendoza, Razin, and Tesar's tax rate estimates, which I have extended through 1994 using data from recent issues of the Organization for Economic Cooperation and Development's National Accounts and Revenue Statistics publications. The figures reveal a number of interesting differences in tax policy across the major industrialized countries. First, note that the tax rate on capital income in the United States is relatively high in comparison with those of other G-7 countries. In many years, the tax rate on capital income in the United States is exceeded only by the ludicrous levels of capital income taxation in the U.K., although note that toward the end of the sample, capital tax rates in Canada and Japan overtake those of the United States. There is no noticeable trend in the tax rate on capital income in the United States, and the only country for which such a trend (toward higher taxation of capital income) is apparent is Japan, the country that posted the most impressive growth performance over this period. However, even in Japan this trend seems to have reversed itself in the late 1980s.

Second, note that for all the countries there is a noticeable trend toward heavier taxation of labor income over most of this period, with the overall level of taxation on labor income in the United States being around the middle of the pack. Note that this trend seems to reverse itself in the early 1980s in the U.K., in the late 1980s in the United States, and in the early 1990s in Japan.

Figure 1
Capital Tax Rates in G-7 Countries


Figure 2


Finally, note that there is no discernible trend in the rate of taxation of consumption expenditures. The lowest tax rates on consumption spending are in Japan and the United States (both in the 4.5 percent to 6.5 percent range), and the highest rates of consumption taxation are in France. Comparing the levels of the three types of taxes, it is striking how much more heavily all the countries tax capital, as opposed to either consumption or labor.

A model with endogenous growth
To get a handle on some of the issues raised in the introduction, it is necessary to lay out a model that allows tax policy to affect the economy's long-run growth rate. In this section, I develop such a model. I consider an economy in which households divide their time among three different production activities: producing goods that are consumed, producing capital goods for use in production activities, and producing human capital that augments the productivity of raw effort. I assume that factors (labor and physical capital) supplied to each of these activities are subject to taxation and that the tax rate is the same regardless of the sector to which factors are supplied. I also assume that the household sector owns all physical capital and that human capital is embodied in individuals and cannot be supplied independently of effort. Government activity will be restricted to levying distortionary taxes on labor and capital income and on consumption purchases, with the proceeds from these taxes distributed to the household sector in a lump-sum manner.

The representative household is assumed to have preferences over consumption of final goods and leisure, as summarized by the following functional:

$$
\begin{equation*}
\sum_{t=0}^{\infty} \beta^{t} U\left(C_{t}, L_{t}\right), \tag{1}
\end{equation*}
$$

where $C_{t}$ denotes consumption at date $t, L_{t}$ denotes leisure or time devoted to nonmarket activities at date t , and I assume that the discount factor satisfies $1>\beta>0$. Note I am abstracting from consumer durables here: consumption services that yield utility are identical to purchases of consumer goods. ${ }^{3}$

I assume that the point-in-time utility function takes the following specific functional form:

$$
\begin{equation*}
\mathrm{U}\left(\mathrm{C}_{\mathrm{t}}, \mathrm{~L}_{\mathrm{t}}\right)=\log \left(\mathrm{C}_{\mathrm{t}}\right)+\theta \log \left(\mathrm{L}_{\mathrm{t}}\right) \tag{2}
\end{equation*}
$$

with $\theta>0$. The representative household is assumed to choose a lifetime plan for consumption and leisure at each date that maximizes utility subject to the following budget constraint:

$$
\begin{align*}
& \left(1+\tau_{t}^{\mathrm{C}}\right) \mathrm{C}_{\mathrm{t}}+\mathrm{P}_{\mathrm{t}}^{\mathrm{K}} \mathrm{I}_{\mathrm{t}}^{\mathrm{K}}+\mathrm{P}_{\mathrm{t}}^{\mathrm{H}} \mathrm{I}_{\mathrm{t}}^{\mathrm{H}}  \tag{3}\\
\leq & \mathrm{W}_{\mathrm{t}}\left(1-\tau_{\mathrm{t}}^{\mathrm{W}}\right) \mathrm{H}_{\mathrm{t}} \mathrm{~N}_{\mathrm{t}}+\mathrm{R}_{\mathrm{t}}\left(1-\tau_{\mathrm{t}}^{\mathrm{R}}\right) \mathrm{K}_{\mathrm{t}}+\mathrm{T}_{\mathrm{t}},
\end{align*}
$$

where $\tau_{t}^{c}$ denotes the tax on consumption purchases at date $\mathrm{t}, \mathrm{P}_{\mathrm{t}}^{\mathrm{K}}$ denotes the relative price of physical capital in terms of consumption goods at date $t, I_{t}^{K}$ denotes purchases of new physical capital goods at date $\mathrm{t}, \mathrm{P}_{\mathrm{t}}^{\mathrm{H}}$ denotes the relative price of human capital in terms of consumption goods at date $\mathrm{t}, \mathrm{I}_{\mathrm{t}}^{\mathrm{H}}$ denotes purchases of new human capital at date $t, W_{t}$ denotes the wage rate in terms of consumption goods at date $t, \tau_{t}^{W}$ denotes the tax rate on labor income at date $\mathrm{t}, \mathrm{H}_{\mathrm{t}}$ denotes the total stock of human capital available for use in production at date $t, N_{t}$ denotes the total number of hours devoted to market production at date $t$, $R_{t}$ denotes the rental rate on physical capital at

Figure 3
Consumption Tax Rates in G-7 Countries

date $\mathrm{t}, \tau_{\mathrm{t}}^{\mathrm{K}}$ denotes the tax rate on income from physical capital at date t , $\mathrm{K}_{\mathrm{t}}$ denotes the total stock of physical capital available for use in production activities at date $t$, and $T_{t}$ denotes transfer payments from the government received at date t .

I also assume that the amount of time available for market and nonmarket (or leisure) activities is nomalized to 1 , yielding the following constraint on the allocation of time across activities:

$$
\begin{equation*}
\mathrm{L}_{\mathrm{t}}+\mathrm{N}_{\mathrm{t}}^{\mathrm{C}}+\mathrm{N}_{\mathrm{t}}^{\mathrm{K}}+\mathrm{N}_{\mathrm{t}}^{\mathrm{H}} \leq 1, \tag{4}
\end{equation*}
$$

where $N_{t}^{c}$ denotes hours devoted to production of consumption goods at date $\mathrm{t}, \mathrm{N}_{\mathrm{t}}^{\mathrm{K}}$ denotes hours devoted to the production of physical capital at date t , and $\mathrm{N}_{\mathrm{t}}^{\mathrm{H}}$ denotes hours devoted to the production of human capital at date $t$. Obviously, $\mathrm{N}_{\mathrm{t}}=\mathrm{N}_{\mathrm{t}}^{\mathrm{C}}+\mathrm{N}_{\mathrm{t}}^{\mathrm{K}}+\mathrm{N}_{\mathrm{t}}^{\mathrm{H}}$. Finally, I assume the following constraints on the accumulation of physical and human capital:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{t}+1} \leq\left(1-\delta^{\mathrm{K}}\right) \mathrm{K}_{\mathrm{t}}+\mathrm{I}_{\mathrm{t}}^{\mathrm{K}} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{H}_{\mathrm{t}+1} \leq\left(1-\delta^{\mathrm{H}}\right) \mathrm{H}_{\mathrm{t}}+\mathrm{I}_{\mathrm{t}}^{\mathrm{H}}, \tag{6}
\end{equation*}
$$

where $1 \geq \delta^{\mathrm{K}} \geq 0$ denotes the rate of depreciation of physical capital and $1 \geq \delta^{\mathrm{H}} \geq 0$ denotes the rate of depreciation of human capital.

A few comments are in order. I am assuming that the representative household is infinitely lived, although this assumption is not really crucial for what follows. An alternative, the dynastic interpretation, has those making decisions today taking into account the welfare of future generations. ${ }^{4}$ I assume that the household must divide its time between leisure (or nonmarket) activities and three different market activities - namely, the production of consumption goods, the production of physical capital, and the production of human capital. I assume that time spent in each of these market activities is equally distasteful from the perspective of the representative household and, furthermore, that labor income generated in each of these activities is taxed at the same rate, $\tau^{\mathrm{w}}$. Total (pretax) labor income equals WHN, where $\mathrm{H}=\mathrm{H}^{\mathrm{C}}+\mathrm{H}^{\mathrm{K}}+\mathrm{H}^{\mathrm{H}}$ and $\mathrm{N}=\mathrm{N}^{\mathrm{C}}+\mathrm{N}^{\mathrm{K}}+\mathrm{N}^{\mathrm{H}}$. Because effort can be costlessly reallocated among the three market activities, the real wage will be the same in all three. Note that W is the wage per efficiency hour of effort in each sector. I assume that physical capital is accumulated by households and leased to firms at the prevailing rental rate, R.

Again, because physical capital can be costlessly reallocated among the three market activities, the rental rate will be the same in all three sectors. Finally, note that I assume that the household receives a lump-sum transfer payment from the government equal to T .

The technologies for producing the three types of goods are as follows:

$$
\begin{align*}
& C_{t} \leq A^{c}\left(K_{t}^{c}\right)^{\alpha^{c}}\left(H_{t}^{c} N_{t}^{c}\right)^{1-\alpha^{c}},  \tag{7}\\
& I_{t}^{K} \leq A^{K}\left(K_{t}^{K}\right)^{\alpha^{K}}\left(H_{t}^{K} N_{t}^{K}\right)^{1-\alpha^{K}} \text { and }  \tag{8}\\
& I_{t}^{H} \leq A^{H}\left(K_{t}^{H}\right)^{\alpha^{H}}\left(H_{t}^{H} N_{t}^{H}\right)^{1-\alpha^{H}}, \tag{9}
\end{align*}
$$

where $\mathrm{A}^{\mathrm{C}}, \mathrm{A}^{\mathrm{K}}, \mathrm{A}^{\mathrm{H}}>0$ denotes the level of total factor productivity in each of the sectors, $1>\alpha^{i}$ $>0$ for $\mathrm{i}=\mathrm{C}, \mathrm{K}, \mathrm{H}, \mathrm{K}_{\mathrm{t}}^{\mathrm{C}}$ denotes physical capital devoted to the production of consumption goods at date $\mathrm{t}, \mathrm{H}_{\mathrm{t}}^{\mathrm{c}}$ denotes human capital devoted to the production of consumption goods at date $t$, $\mathrm{K}_{\mathrm{t}}^{\mathrm{K}}$ denotes physical capital devoted to the production of capital goods at date $\mathrm{t}, \mathrm{H}_{\mathrm{t}}^{\mathrm{K}}$ denotes human capital devoted to the production of physical capital at date $\mathrm{t}, \mathrm{K}_{\mathrm{t}}^{\mathrm{H}}$ denotes physical capital devoted to the production of human capital at date t , and $\mathrm{H}_{\mathrm{t}}^{\mathrm{H}}$ denotes human capital devoted to the production of human capital at date t . Obviously, $\mathrm{K}_{\mathrm{t}}=\mathrm{K}_{\mathrm{t}}^{\mathrm{C}}+\mathrm{K}_{\mathrm{t}}^{\mathrm{K}}+\mathrm{K}_{\mathrm{t}}^{\mathrm{H}}$ and $\mathrm{H}_{\mathrm{t}}=\mathrm{H}_{\mathrm{t}}^{\mathrm{C}}$ $+\mathrm{H}_{\mathrm{t}}^{\mathrm{K}}+\mathrm{H}_{\mathrm{t}}^{\mathrm{H}}$. Note that with the various technologies specified as above, the quantity of hours N and the quality of hours H are assumed to be perfect substitutes in production, in that only the combination NH matters in determining output. The key feature of these technologies that allows this model to generate endogenous steadystate growth is the existence of constant retums to scale in the factors that can be accumulated, K and H. Note, however, that this condition is sufficient, not necessary (see Mulligan and Sala-i-Martin 1993).

I assume a particularly simple government sector. Specifically, I assume that the government balances its budget each period and uses its tax proceeds to make lump-sum transfer payments to the household sector:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{t}} \leq \tau_{\mathrm{t}}^{\mathrm{c}} \mathrm{C}_{\mathrm{t}}+\tau_{\mathrm{t}}^{\mathrm{w}} \mathrm{~W}_{\mathrm{t}} \mathrm{H}_{\mathrm{t}} \mathrm{~N}_{\mathrm{t}}+\tau_{\mathrm{t}}^{\mathrm{R}} \mathrm{R}_{\mathrm{t}} \mathrm{~K}_{\mathrm{t}} \tag{10}
\end{equation*}
$$

Note that if I assume that both capital and labor income are taxed at the same rate $\tau_{t}^{\gamma}$ (i.e., there is just a generic "income" tax), this expression collapses to

$$
\begin{equation*}
\mathrm{T}_{\mathrm{t}} \leq \tau_{\mathrm{t}}^{\mathrm{c}} \mathrm{C}_{\mathrm{t}}+\tau_{\mathrm{t}}^{\gamma} \mathrm{Y}_{\mathrm{t}} \tag{11}
\end{equation*}
$$

where $Y_{t}=W_{t} H_{t} N_{t}+R_{t} K_{t}$.

The representative household takes the paths of factor prices $\left.\left\{W_{t}\right\}_{t=0}, \mathbb{R}_{t}\right\}_{t=0}$, relative prices
 and transfers $\mathbb{T}_{t} b_{t=0}$ as given in forming an optimal plan for consumption, work effort, and physical and capital accumulation over its expected (infinite) lifetime. The optimality conditions for the household include the following:

$$
\begin{gather*}
\frac{1}{\mathrm{C}_{\mathrm{t}}}=\lambda_{\mathrm{t}}\left(1+\tau_{t}^{\mathrm{C}}\right),  \tag{12}\\
\theta \frac{\mathrm{N}_{t}}{1-\mathrm{N}_{\mathrm{t}}}=\lambda_{\mathrm{t}}\left(1-\tau_{\mathrm{t}}^{\mathrm{W}}\right) \mathrm{W}_{\mathrm{t}} \mathrm{H}_{\mathrm{t}} \mathrm{~N}_{\mathrm{t}},  \tag{13}\\
\lambda_{\mathrm{t}} \mathrm{P}_{\mathrm{t}}^{\mathrm{K}}=\beta \lambda_{\mathrm{t}+1}\left[\left(1-\tau_{\mathrm{t}+1}^{\mathrm{R}}\right) \mathrm{R}_{\mathrm{t}+1}+\mathrm{P}_{\mathrm{t}+1}^{\mathrm{K}}\left(1-\delta^{\mathrm{K}}\right)\right], \tag{14}
\end{gather*}
$$

(15) $\lambda_{t} P_{t}^{H}=\beta \lambda_{t+1}\left[1-\tau_{t+1}^{W}\right) W_{t+1} N_{t+1}+P_{t+1}^{H}\left(1-\delta^{H}\right]$,
where $\lambda_{t}$ denotes the marginal utility of income at date t . The additional conditions are initial conditions ( $\mathrm{K}_{0}, \mathrm{H}_{0}$ given), transversality conditions for the two types of capital and the relevant budget and time constraints.

I combine the first two of these conditions as

$$
\begin{equation*}
\theta \frac{\mathrm{N}_{\mathrm{t}}}{1-\mathrm{N}_{\mathrm{t}}}=\frac{\left(1-\tau_{\mathrm{t}}^{W}\right)}{\left(1+\tau_{\mathrm{t}}^{\mathrm{C}}\right)} \frac{\mathrm{W}_{\mathrm{t}} \mathrm{H}_{\mathrm{t}} \mathrm{~N}_{\mathrm{t}}}{\mathrm{C}_{\mathrm{t}}}=\frac{\left(1-\tau_{\mathrm{t}}^{W}\right)}{\left(1+\tau_{\mathrm{t}}^{\mathrm{c}}\right)} \mathrm{s}_{\mathrm{L}}, \tag{16}
\end{equation*}
$$

where $\mathrm{s}_{1}$ denotes the share of labor or wage income in aggregate output and $\mathrm{s}_{\mathrm{C}}$ denotes the share of private consumption in aggregate output. This condition determines the con-sumption-leisure trade-off within each time period, with the terms of the trade-off dictated by the preference parameter $\theta$; the prevailing real wage rate, $\mathrm{W}_{\mathrm{t}}$; the available stock of human capital, $\mathrm{H}_{\mathrm{t}}$; the tax rates on consumption purchases, $\tau_{\mathrm{t}}^{c}$; and wage income, $\tau_{\mathrm{t}}^{\mathrm{W}}$. Inspection of this condition suggests that there might be an equivalence between taxes on labor income and taxes on consumption spending when tax rates are constant: specifically, a wage income tax rate of $\tau_{t}^{W}$ is equivalent to a tax on consumption purchases of $\tau_{t}^{c}=\tau_{t}^{W} /\left(1+\tau_{t}^{W}\right)$, and a tax on consumption purchases equal to $\tau_{t}^{c}$ is equivalent to taxing labor income at a $\tau_{t}^{\mathbb{W}}=\tau_{t}^{C} /\left(1+\tau_{t}^{C}\right)$ rate. ${ }^{5}$ However, this equivalence does not hold in this model because the return to human capital accumulation is realized through labor income.

The second pair of equations above governs the optimal accumulation of physical and human capital. The first equation states that along an optimal path, the utility cost of forgoing a unit of consumption to purchase $P_{t}^{K}$ units of physical capital must just equal the gain in utility from doing so. An additional unit of capi-
tal will generate $\left(1-\tau_{t+1}^{\mathrm{R}}\right) \mathrm{R}_{\mathrm{t}+1}$ additional units of after-tax capital income next period and will have a market value of $P_{t+1}^{K}\left(1-\delta^{K}\right)$. The return in utility terms is then obtained by multiplying by the marginal utility of consumption next period, $\lambda_{t+1}$. To express the return in terms of period t utility, simply multiply by the discount factor, $\beta$. The second equation can be interpreted analogously.

Price-taking behavior on the part of firms, along with profit maximization and our assumption that factors can move freely between different productive activities, implies that the real wage will be equated to the marginal product of labor in each sector and the rental rate on physical capital will be equated to the marginal physical product of capital. Thus, we have

$$
\begin{align*}
\mathrm{W}_{\mathrm{t}} & =\left(1-\alpha^{\mathrm{C}}\right) \mathrm{A}^{\mathrm{C}}\left(\mathrm{Z}_{\mathrm{t}}^{\mathrm{C}}\right)^{\alpha^{\mathrm{C}}}  \tag{17}\\
& =\left(1-\alpha^{\mathrm{K}}\right) \mathrm{P}_{\mathrm{t}}^{\mathrm{K}} \mathrm{~A}^{\mathrm{K}}\left(\mathrm{Z}_{\mathrm{t}}^{\mathrm{K}} \alpha^{\alpha^{\mathrm{K}}}\right. \\
& =\left(1-\alpha^{\mathrm{H}}\right) \mathrm{P}_{\mathrm{t}}^{\mathrm{H}} \mathrm{~A}^{\mathrm{H}}\left(\mathrm{Z}_{\mathrm{t}}^{\mathrm{H}}\right)^{\alpha^{\mathrm{H}}}
\end{align*}
$$

and

$$
\begin{align*}
R_{t} & =\alpha^{C} A^{C}\left(Z_{t}^{C}\right)^{\alpha^{C}-1}  \tag{18}\\
& =\alpha^{K} P_{t}^{K} A^{K}\left(Z_{t}^{\mathrm{K}}\right)^{\alpha^{K}-1} \\
& =\alpha^{H} P_{t}^{H} A^{H}\left(Z_{t}^{H}\right)^{\alpha^{H}-1},
\end{align*}
$$

where $Z_{t}^{i} \equiv K_{t}^{i} / H_{t}^{i} N_{t}^{i}$ for $i=C, K, H$.
Balanced growth paths. To keep things tractable, I focus on the behavior of this economy along a balanced growth path. Along such a path, all the aggregate variables (with the exception of hours of work) grow at the same rate, which I denote by $\gamma$. Additionally, all tax rates are constant. The balanced growth path is characterized by the following equations: ${ }^{6}$

$$
\begin{align*}
\gamma= & \beta\left(\left(1-\tau^{\mathrm{W}}\right)\left(1-\alpha^{\mathrm{H}}\right) \mathrm{A}^{\mathrm{H}}\left(\mathrm{Z}^{\mathrm{H}}\right)^{\alpha^{\mathrm{H}}} \mathrm{~N}\right.  \tag{20}\\
& \left.+\left(1-\delta^{\mathrm{H}}\right)\right) .
\end{align*}
$$

Along the balanced growth path, the aggregate capital- "labor" ratio, Z , is given by
(21) $\quad Z=\frac{\left(1-\tau^{\mathrm{R}}\right)}{\left(1-\tau^{\mathrm{W}}\right)} \frac{\gamma-\beta\left(1-\delta^{\mathrm{H}}\right)}{\gamma-\beta\left(1-\delta^{\mathrm{K}}\right)} \frac{1-\mathrm{s}_{\mathrm{L}}}{\mathrm{S}_{\mathrm{L}} \mathrm{N}}$,
which, of course, collapses to the familiar $\mathrm{Z}=\alpha /(1-\alpha)$ in a one-sector setting with no taxation and inelastic labor supply (see, for example, Barro and Sala-i-Martin 1995).

Welfare. The gain or loss of welfare associated with a particular tax policy change can be calculated as the number $\omega$ that satisfies the following equation: ${ }^{7}$

$$
\begin{align*}
\sum_{\mathrm{t}=0}^{\infty} \beta^{\mathrm{t}} & \left\{\log \left(\mathrm{C}_{\mathrm{t}}\left(\tau^{0} ; .\right)(1-\omega)\right)\right.  \tag{22}\\
& \left.+\theta \log \left(1-\mathrm{N}_{\mathrm{t}}\left(\tau^{0} ; .\right)\right)\right\} \\
= & \sum_{\mathrm{t}=0}^{\infty} \beta^{\mathrm{t}}\left\{\operatorname { l o g } \left(\mathrm{C}_{\mathrm{t}}\left(\tau^{1} ; .\right)\right.\right. \\
& \left.+\theta \log \left(1-\mathrm{N}_{\mathrm{t}}\left(\tau^{1} ; .\right)\right)\right\}
\end{align*}
$$

where $C_{t}\left(\tau^{0}\right.$..) denotes the level of consumption at date $t$ under the old tax policy and $C_{t}\left(\tau^{1},.\right)$ denotes the consumption level under the new tax policy. We can interpret the number $\omega$ as the welfare cost of a tax reform. If $\omega>0$, the representative household is better off under the old tax regime than under the new tax regime and would be willing to pay a fraction of its annual consumption up to $\omega$ to remain under the old regime. If $\omega<0$, the representative household is better off under the new tax regime, and the tax reform has a value equal to the fraction $\omega$ of initial consumption. Note that I impose a balanced budget condition on all the tax reform experiments to be conducted below: I assume that any tax cut must be matched by tax increases sufficient to leave the size of the government relative to economic activity unchanged.

If we are willing to focus on comparisons of steady-state balanced growth paths and ignore transitional effects, it is straightforward to show that, with the particular specification of preferences employed above, the number $\omega$ is given by the following:

$$
\begin{equation*}
\omega=1-\left(\frac{\mathrm{s}_{\mathrm{C}}^{\prime}}{\mathrm{s}_{\mathrm{C}}}\right)\left(\frac{\gamma^{\prime}}{\gamma}\right)^{\beta /(1-\beta)}\left(\frac{1-\mathrm{N}^{\prime}}{1-\mathrm{N}}\right)^{\theta}\left(\frac{\mathrm{Y}^{\prime}}{\mathrm{Y}}\right) \tag{23}
\end{equation*}
$$

where we use primes " " " to denote the values of different variables after the policy change. Inspection of this expression reveals that any policy change that increases the share of consumption in final output, increases the growth rate, frees time for nonmarket or leisure activities, or increases the scale of activity will be welfare-improving. Note also that this expression suggests that policy changes that have only modest effects on the growth rate can potentially have very large effects on welfare, depending on the value of the discount factor, $\beta$. In what follows, we will see that tax reforms that lower the tax rate on capital will typically cause consumption's share of output to fall, but this decline will generally be offset by an increase in the growth rate.

Before proceeding, I need to emphasize that by ignoring transitional effects I obtain estimates of welfare gains or losses associated with tax reforms that are best interpreted as upper
bounds on what would occur in reality. Thus, a tax reform that lowers tax rates on, say, physical capital will typically lead to greater investment during the transition to the new balanced growth equilibrium. During the transitional period, consumption will generally be lower and work effort higher, acting to reduce the total welfare improvement from the reform. ${ }^{8}$ The numbers reported below are thus best interpreted as showing how much better or worse off the average household would be living in economies characterized by different tax policies.

Calibration. To analyze the quantitative implications of tax reforms in this model, I need to assign values to the various parameters that characterize tastes and technology. The values for the key parameters were chosen to be consistent with some key features of the U.S. economy. The discount factor was set equal to 0.98, which implies a pure rate of time preference of just over 2 percent per annum. The parameter $\theta$ was chosen so as to generate a fraction of the time endowment devoted to market activities equal to one-third. The parameters $\mathrm{A}^{\mathrm{C}}, \mathrm{A}^{\mathrm{K}}, \mathrm{A}^{\mathrm{H}}$ were all set equal to 0.34 : this generates a steady-state growth rate equal to 1.7 percent per annum in the baseline economy, which is approximately the long-run growth rate of per capita GDP in the United States over the past fifty years. The sectoral classifications used in the model do not map easily into those used in the National Income and Product Accounts, making it difficult to obtain estimates of the elasticities $\alpha^{\mathrm{C}}, \alpha^{\mathrm{K}}, \alpha^{\mathrm{H}}$ using the standard factor share approach. An alternative is to simply assume that the elasticities are about the same in each sector and use the observation that labor typically accounts for about two-thirds of aggregate output. Thus, the parameters $\alpha^{\mathrm{C}}, \alpha^{\mathrm{K}}, \alpha^{\mathrm{H}}$ were set equal to $0.36,0.35$, and 0.37 , respectively. The depreciation rate for capital, $\delta^{\mathrm{K}}$, was set equal to 5 percent somewhat arbitrarily. This is lower than the 10 percent rate of depreciation for both physical and human capital assumed by King and Rebelo and employed in much of the real business cycle literature but generates a more realistic steady-state output share of investment in physical capital. Absent any detailed information on the depreciation rate for human capital, I set $\delta^{\mathrm{H}}$ equal to 1 percent. ${ }^{9}$ This value is a lot lower than the 10 percent value used by King and Rebelo (1990) and Jones, Manuelli, and Rossi (1993) but close to the zero value used by Lucas (1990).

Finally, the steady-state tax rates were set using the estimates reported in Mendoza, Razin, and Tesar (1994). I simply set $\tau^{R}, \tau^{W}$, and $\tau^{C}$

Table 1
Some Simple Tax Reforms

|  | Tax rate on capital | Tax rate on labor | Tax rate on consumption | Share of consumption | Growth rate | Fraction of time worked | Welfare cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\tau^{R}$ | $\tau^{w}$ | $\tau^{C}$ | $S_{C}$ | $1-\gamma$ | $N$ | $\omega$ |
| Baseline | . 429 | . 247 | . 057 | . 572 | 1.7 | . 330 | - |
| Cut capital income taxes, finance with higher consumption taxes | . 215 | . 247 | . 201 | . 477 | 2.5 | . 341 | -. 307 |
| Cut capital income taxes, finance with higher labor income taxes | . 215 | . 366 | . 057 | . 575 | 1.3 | . 291 | . 120 |
| Cut capital income taxes, finance with higher consumption and labor income taxes | . 215 | . 312 | . 122 | . 530 | 1.8 | . 313 | -. 051 |
| Cut labor income taxes, finance with higher consumption taxes | . 429 | . 124 | . 205 | . 477 | 2.8 | . 376 | -. 385 |
| Cut labor income taxes, finance with higher capital income taxes | . 646 | . 124 | . 057 | . 596 | 1.5 | . 355 | . 165 |
| Cut labor income taxes, finance with higher consumption and capital income taxes | . 517 | . 124 | . 145 | . 522 | 2.3 | . 367 | -. 154 |
| Cut consumption taxes, finance with higher capital income taxes | . 471 | . 247 | . 029 | . 592 | 1.5 | . 328 | . 062 |
| Cut consumption taxes, finance with higher labor income taxes | . 429 | . 271 | . 029 | . 590 | 1.5 | . 322 | . 059 |
| Cut consumption taxes, finance with higher capital and labor income taxes | . 444 | . 262 | . 029 | . 590 | 1.5 | . 324 | . 059 |

equal to the means of their estimated tax rates on capital, labor, and consumption, which yield values for these parameters equal to $0.429,0.247$, and 0.057 , respectively. Note that the estimate for the tax rate on labor is probably a bit on the low side, as it does not take into account the trend toward higher taxation of labor income over the past thirty years.

## Tax reforms

Table 1 illustrates the growth and welfare effects of a series of simple (but dramatic) tax reforms in the context of this model. The first row of the table reports the levels of tax rates in the baseline economy, along with the share of consumption, the growth rate, and the fraction of time devoted to market activities. Note that the share of consumption seems rather small, but this is because I am using a measure of output more comprehensive than GDP. Private consumption expenditures account for about two-thirds of U.S. GDP, while in the baseline economy consumer spending accounts for less than 60 percent of aggregate output. The aggregate output concept employed here is broader than GDP in that it includes the output of the human-capital-producing sector. Thus, Barro and Sala-i-Martin (1995) note that GDP fails to include the value of time forgone by students and at least some of the time expended in on-the-job training: they quote estimates that up to half of the value of investment in human capital is excluded from GDP. The remaining nine rows in the table report the consequences of various
tax reforms, where the reform in each case consists of a halving of the relevant tax rate.

Starting with a reduction in the capital tax rate that is financed by higher consumption taxes, we see that the value of such a reform to the representative household is equal to just under 31 percent of initial consumption. Furthermore, such a reform adds almost a full percentage point to the economy's growth rate (boosting it to 2.5 percent per annum) and is accompanied by an increase in employment. However, the reduction in the tax rate on capital income requires an almost fourfold increase in the tax rate on consumption to maintain budget balance. By contrast, the same reduction in the tax rate on capital income when financed by higher labor income taxes is welfare-reducing: the economy's growth rate slows to 1.3 percent per annum, and the representative household would be willing to pay up to 12 percent of its annual consumption to avoid such a tax reform. If instead the reduction in the capital tax rate is financed by equal increases in consumption and labor income tax rates, the growth rate of the economy rises by a trivial 0.1 of a percent, but the representative household is nevertheless better off, to the tune of about 5.1 percent of initial annual consumption.

The second set of experiments considers the implications of reductions in the tax rate on labor income. A halving of the tax rate on labor income financed by increased taxes on consumption boosts the economy's growth rate to 2.8 percent per annum and has a value to the
representative household equal to just under 40 percent of initial annual consumption! Furthermore, this tax reform requires an increase in the tax rate on consumption expenditures only marginally higher than that required to finance a halving of the tax rate on capital (to 20.5 percent as opposed to 20.1 percent). The same reduction in the tax rate on labor income when financed by higher capital tax rates is welfarereducing and slows the economy's growth rate from 1.7 percent per annum to 1.5 percent. If the labor income tax cut is financed by equally sized increases in the tax rates on capital income and consumption, I again get higher growth, and the value of the reform to the representative household is equal to 15.4 percent of its initial annual level of consumption.

Finally, the third set of experiments considers the implications of reductions in the tax rate on consumption expenditures financed by higher taxes on either capital or labor income or both. In all cases, the result is to slow the growth rate to 1.5 percent per annum, with a welfare cost associated with the reform equal to about 6 percent of initial consumption.

The results in Table 1 are in some cases quite dramatic and lend support to the idea that a shift in the burden of taxation toward heavier taxation of consumption spending and away from taxation of capital and labor could have beneficial effects on the economy's long-run rate of growth. More generally, the results illustrate the principle that factors that are supplied elastically should receive more favorable tax treatment (from an efficiency perspective) than factors that are supplied inelastically. In the model studied here, the endowment of raw time that each household has is the factor that is

Figure 4
Cut Capital Income Tax Rates
Welfare cost
(Percent of steady-state consumption)


Figure 5
Cut Labor Income Tax Rates

supplied inelastically in the long run, whereas the factors that can be accumulated (physical and human capital) are supplied elastically. Taxing the flow of consumption services relatively heavily is in some sense equivalent to taxing the endowment of unimproved time.

Figures 4 through 7 illustrate the welfare and growth effects of reductions in capital and labor taxes for reductions ranging from zero to complete elimination of the tax. Starting with Figure 4, we see that marginal reductions in the tax rate on capital income that are financed by higher labor income taxes will have a very small effect on welfare, but larger reductions (greater than 10 percent or so) will cause welfare to decline. Complete elimination of the tax on capital income, if financed by higher labor income taxes, would have a welfare cost equal to just under 40 percent of steady-state consumption. By contrast, even relatively modest reductions in capital taxes financed by higher consumption taxes produce welfare gains immediately. Figure 5 shows what happens when we replace labor income taxes with either consumption or capital income taxes. Mirroring what we see in Figure 4, marginal reductions in labor income taxes that are financed by higher capital taxes lead to negligible welfare changes. A reform that completely eliminates the labor income tax and replaces it with higher taxes on capital income has a welfare cost of about 60 percent of steady-state consumption. Again, replacing the labor income tax with a consumption tax produces welfare gains for even relatively small changes.

Figures 6 and 7 show us what happens to the economy's growth rate when we cut capital and labor taxes by progressively larger amounts.

In both cases, the biggest increase in the growth rate is achieved when we replace capital or labor income taxes with consumption taxes. Replacing capital income taxes with higher labor income taxes causes an immediate decline in the growth rate, but marginal changes in labor income taxes that are financed by higher capital income taxes leave the growth rate unchanged.

## Conclusions and caveats

This article has presented a preliminary analysis of the welfare and growth effects of some simple tax reforms using a relatively standard three-sector endogenous growth model. I have shown that a shift away from the taxation of capital toward taxation of consumption is potentially welfare-enhancing, and that even larger welfare gains might be obtained by substituting consumption taxes for taxes on labor income.

However, a number of caveats surround these findings. It would be incorrect to interpret my results as indicative of the likely welfare consequences of a real-world tax reform, as I have abstracted from transitional dynamics. Elimination or reduction of taxes on capital income would be followed by a period of higher investment and increased work effort that would tend to reduce (but not offset) the gains from the tax cut. Also, I have focused on a very stylized economy. It is by now well known that the welfare and growth rate effects of reducing or eliminating taxation of capital are very sensitive to some model parameters (particularly the elasticity of substitution between capital and labor in production), and it would be important to carry out a sensitivity analysis of the model before deriving specific policy recommendations.

Finally, I have abstracted from the question of the credibility of the various hypothetical

Figure 6
Cut Capital Income Tax Rates


Figure 7
Cut Labor Income Tax Rates

tax reforms. More favorable tax treatment of capital will generally encourage the accumulation of capital, but only if investors believe that the lower tax rates will remain in place. However, once the private sector has built up capital stock in response to lower tax rates, the government faces an incentive to raise the tax rates on capital to confiscatory levels. Investors will realize that the government is likely to face this incentive and so will be wary of betting too much on the persistence of lower capital tax rates into the future.

My analysis also has some implications for the Hall-Rabushka flat tax proposal and the national sales tax proposal. Recognizing that human capital is a factor of production that can be accumulated just like physical capital means that, from a tax perspective, human capital ought to receive similar treatment. Thus, under the Hall-Rabushka flat tax proposal, firms would get to expense purchases of capital equipment. My analysis suggests that efficiency would dictate that households should be able to expense investments in human capital. Proposals to replace income taxes with sales taxes typically would exempt business purchases of capital. Again, my analysis suggests that we would also want to exempt from taxation household expenditures on education that augment human capital.

## Notes

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${ }^{1}$ Ramsey, however, does not make this argument.
${ }^{2}$ Operating surplus measures the income earned by tan-
gible and intangible entrepreneurships and other factors of production from their participation in production.
${ }^{3}$ It is also worth noting at this point that this measure of consumption differs in important regards from the figure for private consumption expenditures that is reported in the national income and product accounts (NIPA). The NIPA measure of consumption expenditures includes purchases of durable goods, from which I am abstracting in this analysis. But more importantly, the NIPA measure includes as part of consumption spending on education. Insofar as such expenditures augment the stock of human capital, they ought to be treated as investment expenditures.
${ }^{4}$ Use of the infinitely lived representative household construct precludes consideration of issues of intraand intergenerational equity.
${ }^{5}$ See, for example, Becsi (1993) for a discussion of such equivalences. It is perhaps worth noting that such equivalences are implicit in various tax reform proposals.
${ }^{6}$ The complete set of equations characterizing the balanced growth path is presented in the appendix.
${ }^{7}$ See, for example, King and Rebelo (1990).
${ }^{8}$ Thus, for example, Lucas (1990) estimates that the welfare gains associated with the elimination of all capital taxes in the United States would equal about 6 percent of annual consumption when transition effects are ignored but less than 1 percent when transitional costs are taken into account.
${ }^{9}$ Jorgenson and Fraumeni (1989) report estimates of investment in human capital in the United States that are at least four times the magnitude of investment in nonhuman or physical capital and estimate that the value of the stock of human capital is over eleven times the value of the stock of physical capital. In the baseline economy studied here, the stock of human capital is about five times the stock of physical capital.

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## Appendix <br> Complete Set of Equations Characterizing the Balanced Growth Path

The full set of equations characterizing the balanced growth path is as follows. From the intertemporal efficiency condition for physical capital accumulation, we have
(A.1) $\quad \gamma=\beta\left(\left(1-\tau^{R}\right) \alpha^{K} A^{K}\left(Z^{K}\right)^{\alpha^{K}-1}+\left(1-\delta^{K}\right)\right)$.

The intertemporal efficiency condition for human capital accumulation is
(A.2) $\quad \gamma=\beta\left(\left(1-\tau^{W}\right)\left(1-\alpha^{H}\right) A^{H}\left(Z^{H}\right)^{\alpha^{H}} N+\left(1-\delta^{H}\right)\right)$.

Equating rental rates on capital across sectors gives us the conditions

$$
\begin{equation*}
Z^{K}=\frac{1-\alpha^{C}}{1-\alpha^{K}} \frac{\alpha^{K}}{\alpha^{C}} z^{C} \text { and } \tag{A.3}
\end{equation*}
$$

(A.4)

$$
Z^{H}=\frac{1-\alpha^{C}}{1-\alpha^{H}} \frac{\alpha^{H}}{\alpha^{C}} Z^{C} .
$$

From the labor-leisure trade-off we obtain

$$
\begin{equation*}
\theta \frac{N}{(1-N)}=\frac{\left(1-\tau^{W}\right)}{\left(1+\tau^{C}\right)} \frac{s_{L}}{s_{C}}, \tag{A.5}
\end{equation*}
$$

while the resource constraint for the consumption goods sector can be written as

$$
\begin{equation*}
s_{C}=\frac{\phi}{\left(1-\alpha^{C}\right)} s_{L}, \tag{A.6}
\end{equation*}
$$

From the capital goods sector resource constraint we have

$$
\begin{equation*}
P^{K}\left(\gamma+\delta^{K}-1\right)\left(\frac{K}{Y}\right)=\frac{\phi^{K} S_{L}}{1-\alpha^{K}}, \tag{A.7}
\end{equation*}
$$

and from the human-capital-producing sector we obtain
(A.8)

$$
P^{H}\left(\gamma+\delta^{H}-1\right)\left(\frac{H}{Y}\right)=\frac{\phi^{H} s_{L}}{1-\alpha^{H}} .
$$

The economy-wide aggregate resource constraint can be written as
(A.9) $1=s_{C}+P^{K}\left(\gamma+\delta^{K}-1\right)\left(\frac{K}{Y}\right)+P^{H}\left(\gamma+\delta^{H}-1\right)\left(\frac{H}{Y}\right)$. By definition,

$$
\begin{equation*}
1=\phi^{C}+\phi^{K}+\phi^{H} . \tag{A.10}
\end{equation*}
$$

The human capital to output ratio is given by

$$
\begin{equation*}
\left(\frac{H}{Y}\right)=\frac{\beta\left(1-\alpha^{C}\right)\left(1-\tau^{W}\right)}{\gamma-\beta\left(1-\delta^{H}\right)} \frac{s_{C}}{\phi^{C}} . \tag{A.11}
\end{equation*}
$$

The physical capital to output ratio is given by

$$
\begin{align*}
\left(\frac{K}{Y}\right)= & \frac{\beta\left(1-\alpha^{C}\right)\left(1-\tau^{R}\right)}{\gamma-\beta\left(1-\delta^{K}\right)} \frac{s_{C}}{\phi^{C}}\left(\frac{\alpha^{C}}{1-\alpha^{C}} \phi^{C}\right.  \tag{A.12}\\
& \left.+\frac{\alpha^{K}}{1-\alpha^{K}} \phi^{K}+\frac{\alpha^{H}}{1-\alpha^{H}} \phi^{H}\right) .
\end{align*}
$$

Finally, we have the definitions of relative prices:

$$
\begin{gather*}
P^{K}=\frac{\left(1-\alpha^{C}\right)}{\left(1-\alpha^{\kappa}\right)} \frac{A^{C}}{A^{K}} \frac{\left(Z^{C}\right)^{\alpha}}{\left(Z^{K}\right)^{\alpha^{K}}} \text { and }  \tag{A.13}\\
P^{H}=\frac{\left(1-\alpha^{C}\right)}{\left(1-\alpha^{H}\right)} \frac{A^{C}}{A^{H}} \frac{\left(Z^{C}\right)^{\alpha^{C}}}{\left(Z^{H}\right)^{\alpha^{H}}} . \tag{A.14}
\end{gather*}
$$

I define $s_{C}$, the share of consumption in total output, and $s_{L}=$ WHN/Y, the share of labor compensation in total output. Note that the measure of total output used here is more comprehensive than the usual GDP-type measure: GDP is generally thought to undermeasure investment in human capital. The above is a system of fourteen equations in fourteen unknowns: $\gamma, Z^{C}, Z^{K}, Z^{H}, N$, $\phi^{C}, \phi^{K}, \phi^{H}, s_{C}, s_{L},(H / Y),(K / Y), P^{K}, P^{H}$.

