Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy

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We present a model embodying moderate amounts of nominal rigidities that accounts for the observed inertia in inflation and persistence in output. The key features of our model are those that prevent a sharp rise in marginal costs after an expansionary shock to monetary policy. Of these features, the most important are staggered wage contracts that have an average duration of three quarters and variable capital utilization.

I. Introduction

This paper seeks to understand the observed inertial behavior of inflation and persistence in aggregate quantities. To this end, we formulate and estimate a dynamic, general equilibrium model that incorporates staggered wage and price contracts. We use our model to investigate the mix of frictions that can account for the evidence of inertia and persistence. For this exercise to be well defined, we must characterize

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Household

- $S$: Shopping, that is, searching for an appropriate goods supplier
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**Household**

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- **R**: Buying under an established relationship with a supplier
- **U**: Seeking work
Household

- \( S \): Shopping, that is, searching for an appropriate goods supplier
- \( R \): Buying under an established relationship with a supplier
- \( U \): Seeking work
- \( J \): Working at a job
Success rate for shopper: \( \frac{\theta_R}{1 + \theta_R} \)
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Success rate for job-seeker: \( \frac{\theta_J}{1 + \theta_J} \)
Success rate for shopper: \( \frac{\theta_R}{1+\theta_R} \)

Success rate for job-seeker: \( \frac{\theta_J}{1+\theta_J} \)

Success rate for retailer attracting customer: \( \frac{1}{1+\theta_R} \)
Success rate for shopper: \( \frac{\theta_R}{1+\theta_R} \)

Success rate for job-seeker: \( \frac{\theta_J}{1+\theta_J} \)

Success rate for retailer attracting customer: \( \frac{1}{1+\theta_R} \)

Success rate for producer recruiting worker: \( \frac{1}{1+\theta_J} \)
Household Bellman system

\[ H_S = \beta \left( \frac{\theta_R}{1 + \theta_R} H_R + \frac{1}{1 + \theta_R} H_S \right) \]
**Household Bellman System**

\[
H_S = \beta \left( \frac{\theta_R}{1 + \theta_R} H_R + \frac{1}{1 + \theta_R} H_S \right)
\]

\[
H_R = \alpha (\tilde{p} - p) + \beta [s_R H_S + (1 - s_R) H_R]
\]
**Household Bellman System**

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H_R = \alpha (\tilde{p} - p) + \beta \left[ s_R H_S + (1 - s_R) H_R \right]
\]

\[
H_U = \beta \left( \frac{\theta_J}{1 + \theta_J} H_J + \frac{1}{1 + \theta_J} H_U \right)
\]
Household Bellman system

\[ H_S = \beta \left( \frac{\theta_R}{1 + \theta_R} H_R + \frac{1}{1 + \theta_R} H_S \right) \]

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\[ H_U = \beta \left( \frac{\theta_J}{1 + \theta_J} H_J + \frac{1}{1 + \theta_J} H_U \right) \]

\[ H_J = w - z\tilde{p} + \beta [s_J H_U + (1 - s_J) H_J] \]
Household optimization

\[ H_S = H_U. \]
Retailer’s Bellman system

\[ F_O = -k_R y + \beta \left( \frac{1}{1 + \theta_R} F_R + \frac{\theta_R}{1 + \theta_R} F_O \right) \]
Retailer’s Bellman system

\[ F_O = -k_R y + \beta \left( \frac{1}{1 + \theta R} F_R + \frac{\theta_R}{1 + \theta R} F_O \right) \]

\[ F_R = \alpha (p - y) + \beta (1 - s_R) F_R \]
**Producer’s Bellman System**

\[ F_V = -k_J y + \beta \left( \frac{1}{1 + \theta_J} F_J + \frac{\theta_J}{1 + \theta_J} F_V \right) \]
Producer’s Bellman system

\[ F_V = -k_J y + \beta \left( \frac{1}{1 + \theta_J} F_J + \frac{\theta_J}{1 + \theta_J} F_V \right) \]

\[ F_J = y x - w + \beta \left( 1 - s_J \right) F_J \]
Free entry to retailing and production

\[ F_O = 0 \]

and

\[ F_V = 0. \]
Calibration

$s_R$ and $s_J =$ known value for the labor market of 3 percent per month
\textbf{Calibration}

\[ s_R \text{ and } s_J = \text{known value for the labor market of 3 percent per month} \]

\[ \theta_R = \theta_J = 1, \text{ implies supplier-finding and job-finding rates of 0.5, and an unemployment rate of 5.7 percent} \]
Calibration

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$\theta_R = \theta_J = 1$, implies supplier-finding and job-finding rates of 0.5, and an unemployment rate of 5.7 percent

$\alpha$, the number of units of output purchased each period by a buyer, =4.6, from American Time Use Survey
Disamenity of work $z = 0.6$
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$\beta = 0.95^{1/12}$
Disamenity of work \( z = 0.6 \)

\[ \beta = 0.95^{1/12} \]

Normalize productivity at \( x = 1 \) and the intermediate product price at \( y = 1 \).
Calibration, continued

Disamenity of work $z = 0.6$

$\beta = 0.95^{1/12}$

Normalize productivity at $x = 1$ and the intermediate product price at $y = 1$.

Households and sellers have equal shares of the transaction surplus and where workers and producers have equal shares of the employment surplus.
Implications

Value of a shopper or a job-seeker is $H_S = H_U = 73.0$
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Value of a buyer or a worker is $H_R = H_J = 73.6$
IMPLICATIONS

Value of a shopper or a job-seeker is $H_S = H_U = 73.0$

Value of a buyer or a worker is $H_R = H_J = 73.6$

Value of a customer relationship or employment relationship $F_R = 0.625$. 
Value of a shopper or a job-seeker is $H_S = H_U = 73.0$

Value of a buyer or a worker is $H_R = H_J = 73.6$

Value of a customer relationship or employment relationship $F_R = 0.625$.

Wage $w = 0.979$
Implications

Value of a shopper or a job-seeker is $H_S = H_U = 73.0$

Value of a buyer or a worker is $H_R = H_J = 73.6$

Value of a customer relationship or employment relationship $F_R = 0.625$.

Wage $w = 0.979$

Product price $p = 1.005$
Implications

Value of a shopper or a job-seeker is $H_S = H_U = 73.0$

Value of a buyer or a worker is $H_R = H_J = 73.6$

Value of a customer relationship or employment relationship $F_R = 0.625$.

Wage $w = 0.979$

Product price $p = 1.005$

Shadow price of the product delivered to the home is $\tilde{p} = 1.077$. 
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Cost of maintaining a consumer opening $k_R = 0.063$
**Implications**

Value of a shopper or a job-seeker is $H_S = H_U = 73.0$

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Wage $w = 0.979$

Product price $p = 1.005$

Shadow price of the product delivered to the home is $\tilde{p} = 1.077$.

Cost of maintaining a consumer opening $k_R = 0.063$

Cost of maintaining a vacancy $k_J = 0.313$. 
Homogeneity

If \((p, y, w)\) is an equilibrium, so is \((\lambda p, \lambda y, \lambda w)\) for any positive \(\lambda\)

Normalize as \((p/y, w/y)\)
Equilibrium wage and price

Equilibria given $p$ and $w$

Slope = $z$

Highest viable wage

Lowest viable price

Price

Wage
**Equilibrium wage and price, magnified**
Static equilibrium set

\[ z \leq \frac{w}{y} \leq x - \frac{1 - \beta(1 - s_J)}{\beta} k_J \]
Static equilibrium set

\[ z \leq \frac{w}{y} \leq x - \frac{1 - \beta(1 - s_J)}{\beta} k_J \]

\[ \frac{1 - \beta(1 - s_R)}{\alpha \beta} k_R \leq \frac{p}{y} \leq \frac{w}{yz}. \]
Real wage

\[ z \leq \frac{w}{p} \leq \frac{x - \frac{1 - \beta (1 - s_J)}{\beta} k_J}{\frac{1 - \beta (1 - s_R)}{\alpha \beta} k_R - 1} \]
\[
\left(1 + \frac{s_R(1 + \theta_R)}{\theta_R}\right) \frac{q}{\alpha} + \left(1 + \frac{s_J(1 + \theta_J)}{\theta_J}\right) \frac{q}{x} = 1
\]
Output and Price

- Intermediate product price, \( y \)
- Output, \( q \)
- Lowest viable, \( p/y \)
- Highest viable \( w/y \)

![Graph showing the relationship between output and price.](#)
 Allocation of time

Share of household time

Working

Job-seeking

Buying

Shopping

Intermediate product price, $y$
**Dynamic model**

\[
H_{S,i} = H_{U,i}. \\
H_{S,i} = \beta E \left( \frac{\theta_{R,i}}{1 + \theta_{R,i}} H_{R,i'} + \frac{1}{1 + \theta_{R,i}} H_{S,i'} \right) \\
H_{R,i} = \alpha (\tilde{p}_i - p_{L(i)}) + \beta E \left[ s_R H_{S,i'} + (1 - s_R) H_{R,i'} \right] \\
H_{U,i} = \beta E \left( \frac{\theta_{J,i}}{1 + \theta_{J,i}} H_{J,i'} + \frac{1}{1 + \theta_{J,i}} H_{U,i'} \right) \\
H_{J,i} = w_{L(i)} - z\tilde{p} + \beta E \left[ s_J H_{U,i'} + (1 - s_J) H_{J,i'} \right] \\
0 = -k_R y_i + \beta E \left( \frac{1}{1 + \theta_{R,i}} F_{R,i'} \right) \\
F_{R,i} = \alpha (p_{L(i)} - y_i) + \beta E \left[ (1 - s_R) F_{R,i'} \right] \\
0 = -k_J y_i + \beta E \left( \frac{1}{1 + \theta_{J,i}} F_{J,i'} \right) \\
F_{J,i} = y_i x_{C(i)} - w_{L(i)} + \beta E \left[ (1 - s_J) F_{J,i'} \right]
\]
Response to productivity
## Alternative Outcomes

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</tbody>
</table>
Central bank policy

\[ \bar{y}_i = E(y_i | C(i) = \iota) \]
HYPOTHETICAL ECONOMY

\[ H_{R,\iota} - H_{S,\iota} = F_{R,\iota}, \]
\[ H_{J,\iota} - H_{U,\iota} = F_{J,\iota}, \]

Let the price and wage in this hypothetical economy be \( \hat{p}_{\iota} \) and \( \hat{w}_{\iota} \)
Hawk and dove policies

Hawk: Solve the hypothetical model for the policy $y_{H,i}$ that keeps the price level $p_i$ constant across the fundamental states.
Hawk and Dove Policies

Hawk: Solve the hypothetical model for the policy $y_{H,i}$ that keeps the price level $p_i$ constant across the fundamental states.

Dove: solve for the policy $y_{D,i}$ that keeps $\theta_J$ constant.
Policy frontier

\[ y_i = h y_{H,i} + (1 - h) y_{D,i}. \]
Volatility of tightness

Hawkishness, \( h \)

Coefficient of variation

Labor-market tightness

Product-market tightness
VOLATILITIES OF PRICE AND WAGE

![Graph showing the relationship between Hawkishness, h, and the Coefficient of variation for Price level and Wage level. The graph illustrates how the price and wage levels vary with changes in Hawkishness.]