Minimally Altruistic Wages and Unemployment in a Matching Model

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Capture institutional reality of unilateral wage posting with employees having a choice whether they stay or go.

To get away from standard market clearing: monopsony.

Allow fairness to play a role in wages.

Hopefully expands institutional routes to nominal wage rigidity.
The facts to fit

The puzzle of real wages: they are somewhat procyclical.

Procyclical suggests “labor demand" shocks important over business cycle.

Low correlation implies that there are also other shocks.

Look for parameters that fit these facts while maintaining perfect correlation between unemployment and vacancies.
Cyclical Log Employment and Real Wage
In US Business Sector

Employment

Real Wage
“Vacancies” and Employment in the United States
Model with posted wages and monopsony.

Add minimum altruism as a fairness constraint.

Variations in MRPL: parameters that make real wages mildly procyclical.

Variations in both minimal altruism and in MRPL: imperfect correlation.

Adding training costs (Silva and Toledo 2006)
Material welfare of worker $j$ employed at $k$

$$E_t \sum_{\tau=0}^{\infty} \rho^\tau w u_{t+\tau}^j,$$

$$u_t^j = \frac{(C_t^j)^{1-\gamma}}{1-\gamma} + x_t^{kj}$$

$x_t^{kj}$ is worker specific (i.i.d.) nonpecuniary utility of this worker on the job.

$$C_t^j = \left[ \sum_i (C_t^i)^{\epsilon_t-1} \epsilon_t^{-1} \right]^{\epsilon_t}$$

Assume workers neither borrow nor lend.

An attractive extension: worker loss at nominal wage cuts.
Material welfare of worker $j$ employed at $k$

$$E_t \sum_{\tau=0}^{\infty} \rho_w^{\tau} u_{t+\tau}^j,$$  
$$u_t^j = \frac{(C_t^j)^{1-\gamma}}{1-\gamma} + x_{t}^{kj}$$

$x_{t}^{kj}$ is worker specific (i.i.d.) nonpecuniary utility of this worker on the job.

$$C_t^i = \left[ \sum_{i} \left( c_t^{ji} \right)^{\epsilon_t \frac{\epsilon_t - 1}{\epsilon_t}} \right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$$

Assume workers neither borrow nor lend.

An attractive extension: worker loss at nominal wage cuts.
Firm’s owners material welfare function

\[ E_t \sum_{\tau=0}^{\infty} \rho^{\tau} C_t^i \]

Leads to maximizing PDV of profits

\[ \Pi_t^i = E_t \sum_{\tau=0}^{\infty} \rho^{\tau} \pi_{t+\tau}^i \]

Real revenue is given by

\[ R_t^i(h_t^i, Y_t, z_t, \epsilon_t) = Y_t^{1/\epsilon_t} [z_t f(h_t^i)]^{1-1/\epsilon_t} \]
Marginal revenue product of labor is

\[ \frac{dR^i_t}{dh^i_t} = Y_t^{1/\epsilon_t} z_t^{1-1/\epsilon_t} \left( 1 - \frac{1}{\epsilon_t} \right) f(h_t^i)^{-1/\epsilon_t} f'(h_t^i) \]

Two key properties

- Rises with both technology and elasticity of demand
- Is made concave both by diminishing returns and by low \( \epsilon \)
Profits and recruitment

Profits:  \[ \pi^i_t = R^i_t - w^i_t h^i_t - \kappa(v^i_t) \]

Vacancies \( v \) help recruit.

Meetings of firm \( i \) with unemployed people

\[ \tilde{m}^{ui}_t = \frac{v^i_t}{v_t} u_{t-1} \left( \frac{v_t}{u_{t-1}} \right)^\eta = v_t^i \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} \]

Meetings of firm \( i \) with employed people

\[ \tilde{m}^{ai}_t \approx \frac{v^i_t}{v_t} \bar{m}_t h_{t-1} \left( \frac{v_t}{h_{t-1}} \right)^\ell = v_t^i \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} \]

Meetings of firm \( i \) employees with other firms

\[ \tilde{m}^{di}_t = \frac{h^i_t}{h_{t-1}} \bar{m}_t h_{t-1} \left( \frac{v_t}{h_{t-1}} \right)^\ell \]
Labor market dynamics

\[ h_t + u_t = 1 \]

\[ h_t = (1 - s)h_{t-1} + u_{t-1} \left( \frac{v_t}{u_{t-1}} \right)^\eta \]

Ensures that Beveridge curve holds.
Job acceptance decision

Nonpecuniary benefits $x$ are i.i.d. across firms and workers.

Unemployed workers always accept their one job offer if minimum $x^e$ satisfies

$$\frac{(w^i_t)^{1-\gamma}}{1-\gamma} + x^e > \frac{(C_u)^{1-\gamma}}{1-\gamma} + x_u$$

Worker $k$ with potential job at $j$ stays at $i$ if

$$\frac{(w^i_t)^{1-\gamma}}{1-\gamma} + x^{ik}_t \geq \frac{(w^j_t)^{1-\gamma}}{1-\gamma} + x^{jk}_t$$

Let $F$ be pdf of $[x^{jk}_t - x^{ik}_t]$. From firm’s point of view, probability a worker with an alternative stays is

$$F\left(\frac{(w^i_t)^{1-\gamma} - (w^j_t)^{1-\gamma}}{1-\gamma}\right)$$
Evolution of firm employment

\[ h_t^i = (1 - s)h_{t-1}^i + \tilde{m}_t^{ui} - \tilde{m}_t^{di} + (\tilde{m}_t^{di} + \tilde{m}_t^{ai})F \left( \frac{(w_t^i)^{1-\gamma} - (\tilde{w}_t)^{1-\gamma}}{1 - \gamma} \right) \]

\[ = h_{t-1}^i (1 - s) - \frac{h_{t-1}^i}{h_{t-1}} (1 - F_t^i) m_t^h + \frac{v_t^i}{v_t} (F_t^i m_t^h + m_t^u). \]

Two methods for increasing \( h_t^i \): raising \( w_t^i \) or \( v_t^i \).

Must be indifferent between them for given \( h_{t+1}^i \).
Perturbation method

Note that

\[ v_{t+1}^i = v_{t+1} \left( \frac{h_{t+1}^i - h_t^i(1-s) - \left( h_t^i/h_t \right) (1 - F_{t+1}^i) m_{t+1}^h}{F_{t+1}^i m_{t+1}^h + m_{t+1}^u} \right) \]

and rewrite PDV of profits as

\[ \Pi_t^i = R_t^i(h_t^i) - w_t^i h_t^i - \kappa(v_t^i) + E_t \rho^2 \Pi_{t+2}^i(h_{t+1}^i) + \]

\[ E_t \rho \left( R_{t+1}^i - w_{t+1}^i h_{t+1}^i - \right. \]

\[ \kappa \left( v_{t+1} \frac{h_{t+1}^i - h_t^i(1-s) - \left( h_t^i/h_t \right) (1 - F_{t+1}^i) m_{t+1}^h}{F_{t+1}^i m_{t+1}^h + m_{t+1}^u} \right) \)
First order conditions

\[
\frac{d \Pi_t^i}{d h_t^i} \frac{d h_t^i}{d v_t^i} - \kappa_t' = 0 \\
\frac{d \Pi_t^i}{d h_t^i} \frac{d h_t^i}{d w_t^i} - h_t^i = 0
\]

\[
\frac{d \Pi_t^i}{d h_t^i} = \frac{d R_t^i}{d h_t^i} - w_t^i + E_t \rho \kappa' \frac{1 - s - \bar{m}(1 - F_{t+1}^i)(v_{t+1}/h_t)^\ell}{F_{t+1} \bar{m}(v_{t+1}/h_t)^{\ell-1} + (v_{t+1}/u_t)^{\eta-1}}
\]

\[
\frac{d h_t^i}{d v_t^i} = F_t^i \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1}
\]

\[
\frac{d h_t^i}{d w_t^i} = (w_t^i)^{-\gamma} F_t^{i'} \bar{m} h_{t-1} \left( \frac{v_t}{h_{t-1}} \right)^\ell \left( \frac{h_{t-1}^i}{h_{t-1}} + \frac{v_t^i}{v_t} \right)
\]
Second order condition with respect to $\nu$

Differentiating \( \frac{d\Pi_i}{dh_t} \frac{dh_t}{dv_t} - \kappa'_t \)

\[
\frac{d^2 \Pi_t}{dv_t^2} = \frac{d^2 \Pi_t}{dh_t^2} \left( \frac{dh_t}{dv_t} \right)^2 + \frac{d\Pi_t}{dh_t} \frac{d^2 h_t}{dv_t^2} - k''_t < 0
\]

At symmetric equilibrium this requires

\[
\frac{d^2 R_t^i}{(dh_t^i)^2} < k'' \left[ 1 + \rho [1 - s - (\bar{m}/2)(v_{t+1}/h_t)^{\ell}]^2 \right]
\]

\[
\left[ ((\bar{m}/2)(v_t/h_{t-1})^{\ell-1} + (u_{t}/h_{t-1})^{\eta-1}]^2 \right]
\]
Symmetric equilibrium - recursive from $h$

Given exogenous evolution of $h$, evolution of $v$ and $u$ follows from MP block.

Wage can then be obtained from indifference between $v$ and $w$ as recruiting tools

$$\frac{\bar{m}}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} = 2\kappa'_t (w_t)^{-\gamma} \bar{F}' \bar{m} \frac{h_{t-1}}{h_t} \left( \frac{v_t}{h_{t-1}} \right)^{\ell}$$

d$R/dh$ that rationalizes this can be obtained from indifference to small changes in $v$

$$\frac{dR_t}{dh_t} - w_t - \frac{\bar{m}}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} + E_t \rho \kappa'_{t+1} \left( 1 - s - \frac{\bar{m}}{2} \left( \frac{v_{t+1}}{h_t} \right)^{\ell} \right) = 0$$
Advantages and disadvantages relative to bargaining approach

Worker’s reservation wages do not play a role in wage setting, so unemployment insurance rate is unimportant.

On the other hand, $\gamma$ is crucial: Increases in $\gamma$ make real wage less procyclical.

If $\gamma$ is zero, wages always equally good at attracting people so increase in tightness leads to unbounded wages increases (so it does not occur).

Note: Declines in marginal utility of income (as consumption rises) discourage wage increases rather than usual opposite.
An altruistic firm

Suppose firm maximizes

\[ \tilde{\Pi}_t^i = E_t \sum_{\tau=0}^{\infty} \rho^\tau \tilde{\pi}_{t+\tau}^i \]

\[ \tilde{\pi}_t^i = \pi_t^i + \lambda^A \chi_t^{A_i} \]

Total workers’ material benefit from being at \( i \)

\[ \chi_t^{A_i} = \left\{ \left[ 1 - s - \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell} \right] h_{t-1}^i + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} v_t^i \right\} \psi_{1i}^t \]

\[ + \left[ \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell} h_{t-1}^i + \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} v_t^i \right] \psi_{2i}^t \]
Expected material benefit for employees with no offers

\[
\psi_{t}^{1i} = \frac{(w_{t}^{i})^{1-\gamma} + Ex - (C_{u})^{1-\gamma} - x_{u}}{1 - \gamma}
\]

Expected material benefit for ones with offers

\[
\psi_{t}^{2i} = \psi_{t}^{1i} / F_{t}^{i}
\]

\[
\psi_{t}^{2i} = \int_{y}^{\infty} g(y^{j}) \int_{y^{j} + \bar{w}_{t}^{1-\gamma} - (w_{t}^{i})^{1-\gamma}}^{\infty} \frac{(w_{t}^{i})^{1-\gamma} + y^{i} - \bar{w}_{t}^{1-\gamma} - y^{j}}{1 - \gamma} g(y^{i}) dy^{i} dy^{j}
\]

Define material gain to employees when \( v_{t}^{i} \) rises

\[
\omega_{t}^{i} = \frac{d\chi_{t}^{Ai}}{dv_{t}^{i}} = \left( \frac{v_{t}}{u_{t-1}} \right)^{\eta-1} \psi_{t}^{1i} + \bar{m} \left( \frac{v_{t}}{h_{t-1}} \right)^{\ell-1} \psi_{t}^{2i}
\]
First order conditions with altruism

\[
\frac{d\tilde{\Pi}_i^t}{dh_t^i} \frac{dh_t^i}{dv_t^i} - \kappa_t' + \lambda^A \omega_t^i = 0
\]

\[
\frac{d\tilde{\Pi}_i^t}{dh_t^i} \frac{dh_t^i}{dw_t^i} - h_t^i(1 - \lambda^A(w_t^i)^{-\gamma}) = 0
\]

which implies

\[
\frac{\tilde{m}}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1}
\]
For $\omega$ small relative to $hw^{-\gamma}$, more altruistic firms pay higher wages - so wage is a signal.

Altruism of worker at $i = \xi(\hat{\lambda}^i, \bar{\lambda}_t)$.

$\hat{\lambda}^i$ is information about firm altruism.

$\bar{\lambda}$ is minimal altruism.

$\xi$ big negative number if hypothesis that firm altruism equals at least $\bar{\lambda}$ can be rejected (in a test of a certain size).

$\xi = 0$ otherwise.
Suppose some firms do have requisite altruism.

Some of these firms are naive (so they ignore repercussions from paying a different wage)

Unique equilibrium with potential anger has all imitating naive altruistic firms.
Symmetric equilibrium with altruism:

\[
\frac{\bar{m}}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} = \frac{2(\kappa'_t - \lambda^A \omega_t) w_t^{-\gamma}}{h_t h_{t-1}} \bar{F}' \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^\ell \left( \frac{h_t}{h_{t-1}} \right) (1 - \lambda^A w_t^{-\gamma})
\]

\[
\frac{dR_t}{dh_t} - w_t + E_t \rho \lambda^A \left( (1 - s) \psi^1_{t+1} + \bar{m} \left( \frac{v_{t+1}}{h_t} \right)^\ell \left( \psi^2 - \psi^1_{t+1} \right) \right) -
\]

\[
\frac{(\kappa'_t - \lambda^A \omega_t)}{\bar{m}} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} + E_t \frac{\rho(\kappa'_{t+1} - \lambda^A \omega_{t+1})}{\bar{m}} \left( 1 - s - \bar{m} \frac{u_{t+1}}{h_t} \right)^\ell = 0
\]
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Alts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ): Discount rate</td>
<td>.996</td>
<td></td>
</tr>
<tr>
<td>( s ): Steady state separation rate into unemployment</td>
<td>.034</td>
<td></td>
</tr>
<tr>
<td>( (v/u)^\eta ): Steady state finding rate for unemployed</td>
<td>.45</td>
<td></td>
</tr>
<tr>
<td>( \eta ): Elasticity of finding rate with respect to ( v/u )</td>
<td>.54</td>
<td></td>
</tr>
<tr>
<td>( \ell ): Elasticity of finding rate with respect to ( v/h )</td>
<td>.54</td>
<td></td>
</tr>
<tr>
<td>( \bar{m}(v/h)^\ell ): Steady state finding rate for employed</td>
<td>.136</td>
<td></td>
</tr>
<tr>
<td>( \lambda^A, \lambda^I ): Steady state altruism</td>
<td>0</td>
<td>.4</td>
</tr>
<tr>
<td>( \psi^1 ): Average welfare gain for unemployed</td>
<td>.1</td>
<td></td>
</tr>
<tr>
<td>( 2\psi^2 ): Welfare gain from new offer</td>
<td>.1</td>
<td></td>
</tr>
<tr>
<td>( \kappa'/w(dh/dv) ): St.st. recruitment cost in wage units</td>
<td>.12</td>
<td></td>
</tr>
<tr>
<td>( \zeta_v ): Elasticity of recruiting costs</td>
<td>1</td>
<td>.66</td>
</tr>
<tr>
<td>( \gamma ): Elasticity of ( du/dC ) with respect to income</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>( \epsilon ): Steady state elasticity of demand</td>
<td>3</td>
<td>10,000</td>
</tr>
<tr>
<td>( \alpha ): Exponent on labor in the production function</td>
<td>.75</td>
<td></td>
</tr>
<tr>
<td>( \Phi/h^\alpha ): Index of returns to scale in production</td>
<td>0</td>
<td>.33</td>
</tr>
</tbody>
</table>
The moments to match

Look at aggregate employment and average hourly earnings of production workers.

Detrend using smooth trend that is "uncorrelated" with cycle

Correlation of two series = .41

S.D($w$) = .017  \hspace{1cm} S.D($h$) = .014

Regression coefficient of $w$ on $h$ is .49

Two approaches: Single shock to $dR/dh$, captured by

$$\tilde{h}_t = .97\tilde{h}_{t-1} + e_t^h$$
## Elasticities with respect to employment

<table>
<thead>
<tr>
<th>Spec.</th>
<th>Parameters</th>
<th>Elasticity of $w$</th>
<th>Elasticity of $dR/dh$</th>
<th>$F'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$\lambda^A = 0, \gamma = 1, \text{Linear } \kappa$</td>
<td>15.38</td>
<td>15.47</td>
<td>28.5</td>
</tr>
<tr>
<td>(2)</td>
<td>$\lambda^A = 0, \gamma = 8, \zeta_v = 1$</td>
<td>1.92</td>
<td>2.18</td>
<td>28.5</td>
</tr>
<tr>
<td>(3)</td>
<td>$\lambda^A = 0, \gamma = 1, \zeta_v = .66$</td>
<td>10.64</td>
<td>10.63</td>
<td>28.5</td>
</tr>
<tr>
<td>(4)</td>
<td>$\lambda^A = 0, \gamma = 8, \zeta_v = .66$</td>
<td>1.33</td>
<td>1.44</td>
<td>28.5</td>
</tr>
<tr>
<td>(5)</td>
<td>$\lambda^A = .4, \gamma = 1, \text{Linear } \kappa$</td>
<td>5.82</td>
<td>4.06</td>
<td>19.7</td>
</tr>
<tr>
<td>(6)</td>
<td>$\lambda^A = .4, \gamma = 8, \zeta_v = 1$</td>
<td>1.15</td>
<td>1.05</td>
<td>19.7</td>
</tr>
<tr>
<td>(7)</td>
<td>$\lambda^A = .4, \gamma = 1, \zeta_v = .66$</td>
<td>3.92</td>
<td>2.67</td>
<td>19.7</td>
</tr>
<tr>
<td>(8)</td>
<td>$\lambda^A = .4, \gamma = 8, \zeta_v = .66$</td>
<td>.78</td>
<td>.64</td>
<td>19.7</td>
</tr>
</tbody>
</table>
Letting \( \bar{\lambda} \) vary

Second approach, suppose that

\[
\bar{\lambda}_t = .96 \bar{\lambda}_{t-1} + e_t^\lambda
\]

\[
\frac{dR_t}{dh_t} \approx \frac{\hat{dR}_t}{dh_t} + \frac{dR}{dh} \frac{\hat{dR}_t}{dh_t}
\]

Actual With Steady Caused
constant state by \( z \) and \( \epsilon \)

\[
\frac{dR_t}{dh_t} = .96 \frac{dR_{t-1}}{dh_{t-1}} + e_t^R
\]

Question: can choices of \( \sigma_\lambda \) and \( \sigma_R \) assure match with second moments of \( w \) and \( h \).
Matching low correlation and standard deviations

\[ \lambda^A = 0.4 \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \sigma_\lambda )</th>
<th>( \sigma_R )</th>
<th>S.D. (h)</th>
<th>S.D. (w)</th>
<th>Corr (h,w)</th>
<th>Frac.(h) due to R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Data</td>
<td>.014</td>
<td>.017</td>
<td>.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma = 1, \zeta_v = 1 )</td>
<td>.090</td>
<td>.025</td>
<td>.007</td>
<td>.034</td>
<td>.41</td>
<td>.58</td>
</tr>
<tr>
<td>( \gamma = 8, \zeta_v = .66 )</td>
<td>.020</td>
<td>.014</td>
<td>.011</td>
<td>.020</td>
<td>.41</td>
<td>.9998</td>
</tr>
<tr>
<td>( \gamma = 6.5, \zeta_v = 1 ) Selfish hiring</td>
<td>.018</td>
<td>.024</td>
<td>.014</td>
<td>.017</td>
<td>.41</td>
<td>.62</td>
</tr>
</tbody>
</table>
Training Costs

Ignoring altruism, profits can be written as

\[ \Pi_t^i = R_t^i(h_t^i) - w_t^i h_t^i - \kappa(v_t^i) + E_t \rho^2 \Pi_{t+2}^i(h_{t+1}^i) \]

\[- \tau \left( h_t^i - h_{t-1}^i \left[ 1 - s - (1 - F_t^i)\bar{m}\left(\frac{v_t}{h_{t-1}}\right)\right] \right) \]

\[- \rho \tau \left( h_{t+1}^i - h_t^i \left[ 1 - s - (1 - F_{t+1}^i)\bar{m}\left(\frac{v_{t+1}}{h_t}\right)\right] \right) + \]

\[ E_t \rho \left( R_{t+1}^i - w_{t+1}^i h_{t+1}^i - \kappa \left( v_{t+1} \frac{h_{t+1}^i - h_t^i(1 - s) - (h_t^i/h_t)(1 - F_{t+1}^i)m_t^{h}}{F_{t+1}^{i}m_t^{h} + m_t^{u}} \right) \right) \]
First order conditions with training costs

\[
\frac{d\Pi^i_t}{dh^i_t} \frac{dh^i_t}{dv^i_t} - \kappa'_t = 0
\]

\[
\frac{d\Pi^i_t}{dh^i_t} \frac{dh^i_t}{dw^i_t} - h^i_t + \tau'_t(w^i_t)^{-\gamma} F^i_t h^i_{t-1} \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^\ell = 0
\]

while

\[
\frac{d\Pi^i_t}{dh^i_t} = \frac{dR^i_t}{dh^i_t} - w^i_t + E_t \rho \kappa'_{t+1} \left( 1 - s - \bar{m}(1 - F^i_{t+1}) \left( \frac{v_{t+1}}{h_t} \right)^\ell \right)
\]

\[
-\tau'_t + E_t \rho \tau'_{t+1} \left( 1 - s - \bar{m}(1 - F^i_{t+1}) \left( \frac{v_{t+1}}{h_t} \right)^\ell \right).
\]
Symmetric equilibrium with training costs

\[
\frac{\bar{m}}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} = \frac{2\kappa'_t(w_t)^{-\gamma} \bar{F}' \bar{m}h_{t-1} \left( \frac{v_t}{h_{t-1}} \right)^{\ell}}{h_t - \tau'(w_t)^{-\gamma} \bar{F}' h_{t-1} \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell}}
\]

\[
\frac{dR_t}{dh_t} - w_t - \frac{\bar{m}}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} + E_t \frac{\rho \kappa'_{t+1} \left( 1 - s - \frac{\bar{m}}{2} \left( \frac{v_{t+1}}{h_t} \right)^{\ell} \right)}{\bar{m}} = 0
\]
Employment elasticities with training costs
\((\tau' = 13\kappa'(dv/dh))\)

<table>
<thead>
<tr>
<th>Spec.</th>
<th>Parameters</th>
<th>Elasticity of (w)</th>
<th>Elasticity of (dR/dh)</th>
<th>(F')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(\lambda^A = 0, \gamma = 1, \zeta_v = 1)</td>
<td>8.29</td>
<td>7.78</td>
<td>3.80</td>
</tr>
<tr>
<td>(2)</td>
<td>(\lambda^A = 0, \gamma = 8, \zeta_v = 1)</td>
<td>1.04</td>
<td>1.67</td>
<td>3.80</td>
</tr>
<tr>
<td>(3)</td>
<td>(\lambda^A = 0, \gamma = 1, \zeta_v = .66)</td>
<td>7.66</td>
<td>7.11</td>
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<tr>
<td>(4)</td>
<td>(\lambda^A = 0, \gamma = 8, \zeta_v = .66)</td>
<td>.96</td>
<td>1.46</td>
<td>3.80</td>
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<tr>
<td>(5)</td>
<td>(\lambda^A = .4, \gamma = 1, \zeta_v = 1)</td>
<td>4.78</td>
<td>3.43</td>
<td>2.30</td>
</tr>
<tr>
<td>(6)</td>
<td>(\lambda^A = .4, \gamma = 8, \zeta_v = 1)</td>
<td>.62</td>
<td>1.15</td>
<td>2.30</td>
</tr>
<tr>
<td>(7)</td>
<td>(\lambda^A = .4, \gamma = 1, \zeta_v = .66)</td>
<td>4.41</td>
<td>3.09</td>
<td>2.30</td>
</tr>
<tr>
<td>(8)</td>
<td>(\lambda^A = .4, \gamma = 8, \zeta_v = .66)</td>
<td>.57</td>
<td>.98</td>
<td>2.30</td>
</tr>
</tbody>
</table>
Matching correlation and standard deviations with training costs \((\lambda^A = .4, \tau' = 13\kappa'(dv/dh))\)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(\sigma_\lambda)</th>
<th>(\sigma_R)</th>
<th>S.D. ((h))</th>
<th>S.D. ((w))</th>
<th>Corr ((h,w))</th>
<th>Frac.((h)) due to (R)</th>
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<tbody>
<tr>
<td>U.S. Data</td>
<td></td>
<td></td>
<td>.014</td>
<td>.017</td>
<td>.41</td>
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<tr>
<td>(\gamma = 1)</td>
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<td>.022</td>
<td>.007</td>
<td>.029</td>
<td>.41</td>
<td>.63</td>
</tr>
<tr>
<td>(\gamma = 6)</td>
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<td>.97</td>
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<tr>
<td>Selfish hiring</td>
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<td>.018</td>
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<td>.77</td>
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<tr>
<td>(\gamma = 4)</td>
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<td>.028</td>
<td>.014</td>
<td>.018</td>
<td>.41</td>
<td>.77</td>
</tr>
</tbody>
</table>
Conclusions

Five effects contribute quantitatively to less procyclical real wages:

- Increasing marginal utility of income as income declines.
- Increasing returns in vacancies.
- Firm altruism that makes wage increases less attractive when their marginal utility to workers is low.
- Training costs.
- Variations in required firm altruism.

Still missing: Reluctance to pain workers with nominal wage cuts.
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