Reviewing the Leverage Cycle

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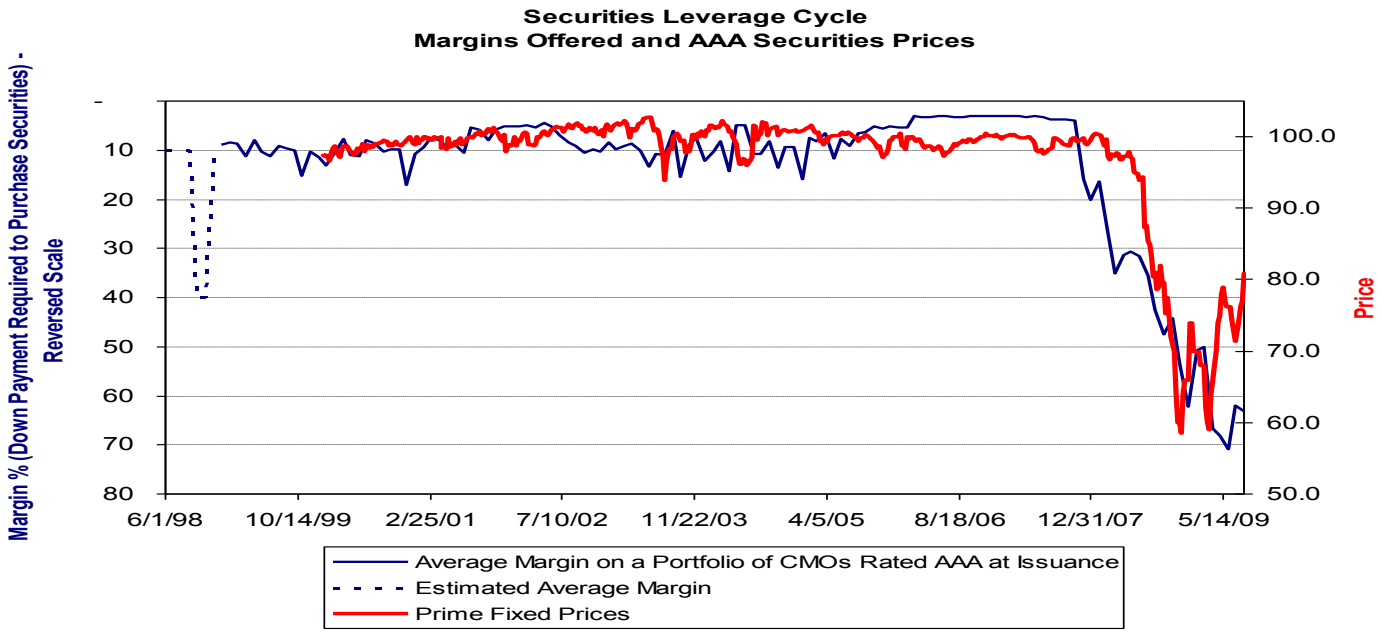
Outline

1. Introduction
2. A Simple Model
3. Simple Example
4. Results
5. Leverage Cycle
6. Multiple Leverage Cycles
7. Volatility

- We review the theory of leverage developed in collateral equilibrium models with incomplete markets.
  - Geanakoplos (1997) Collateral Equilibrium
  - Geanakoplos (2003) Leverage Cycle
  - Fostel and Geanakoplos (2008) Multiple Leverage Cycles
- We explain how leverage tends to boost asset prices, and create bubbles.
- We show how leverage can be endogenously determined in equilibrium, and how it depends on volatility.

- Time Series properties: Leverage Cycle.
  - leverage $\Longleftrightarrow$volatility $\Longleftrightarrow$ asset prices.

Repo Market Leverage



Note: The chart represents the average margin required by dealers on a hypothetical portfolio of bonds subject to certain adjustments noted below. Observe that the Margin % axis has been reversed, since lower margins are correlated with higher prices.

The portfolio evolved over time, and changes in average margin reflect changes in composition as well as changes in margins of particular securities. In the period following Aug. 2008, a substantial part of the increase in margins is due to bonds that could no longer be used as collateral after being downgraded, or for other reasons, and hence count as 100% margin.

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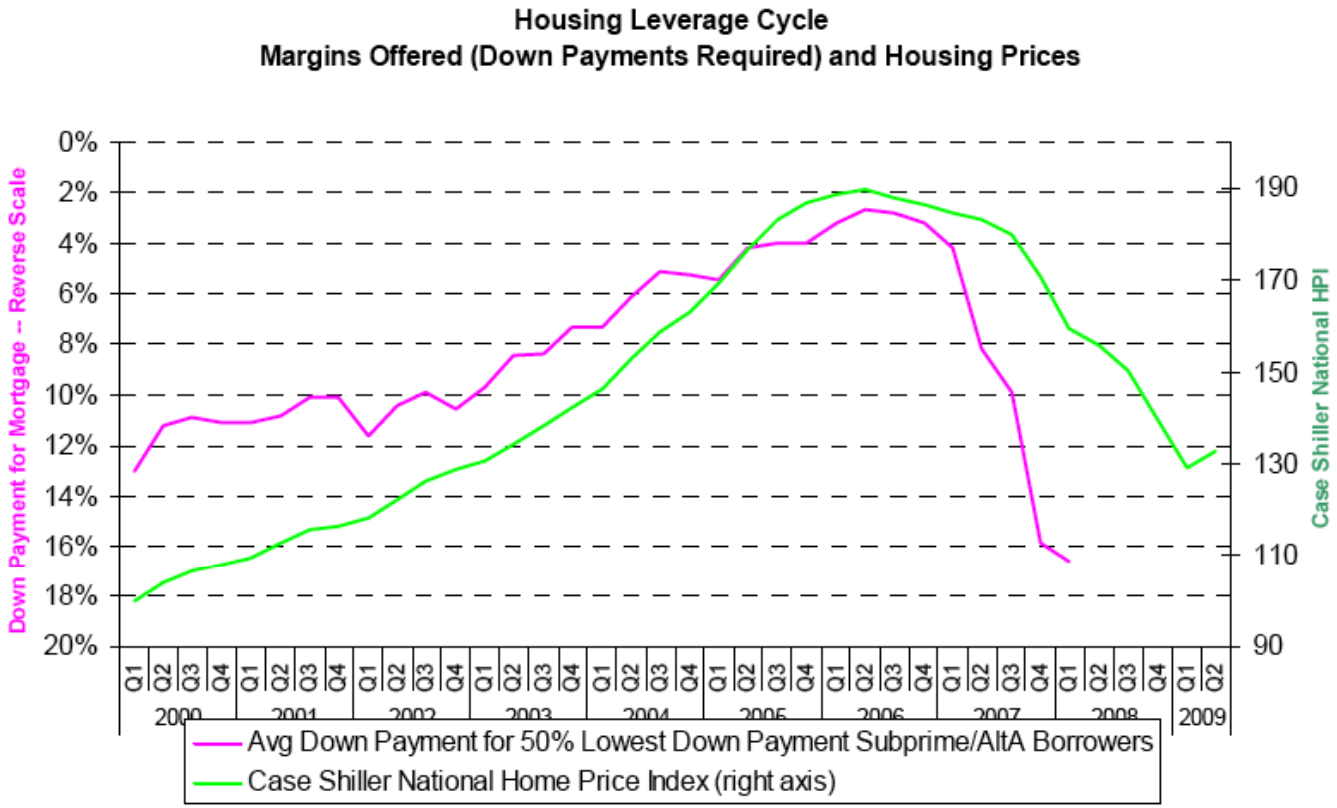
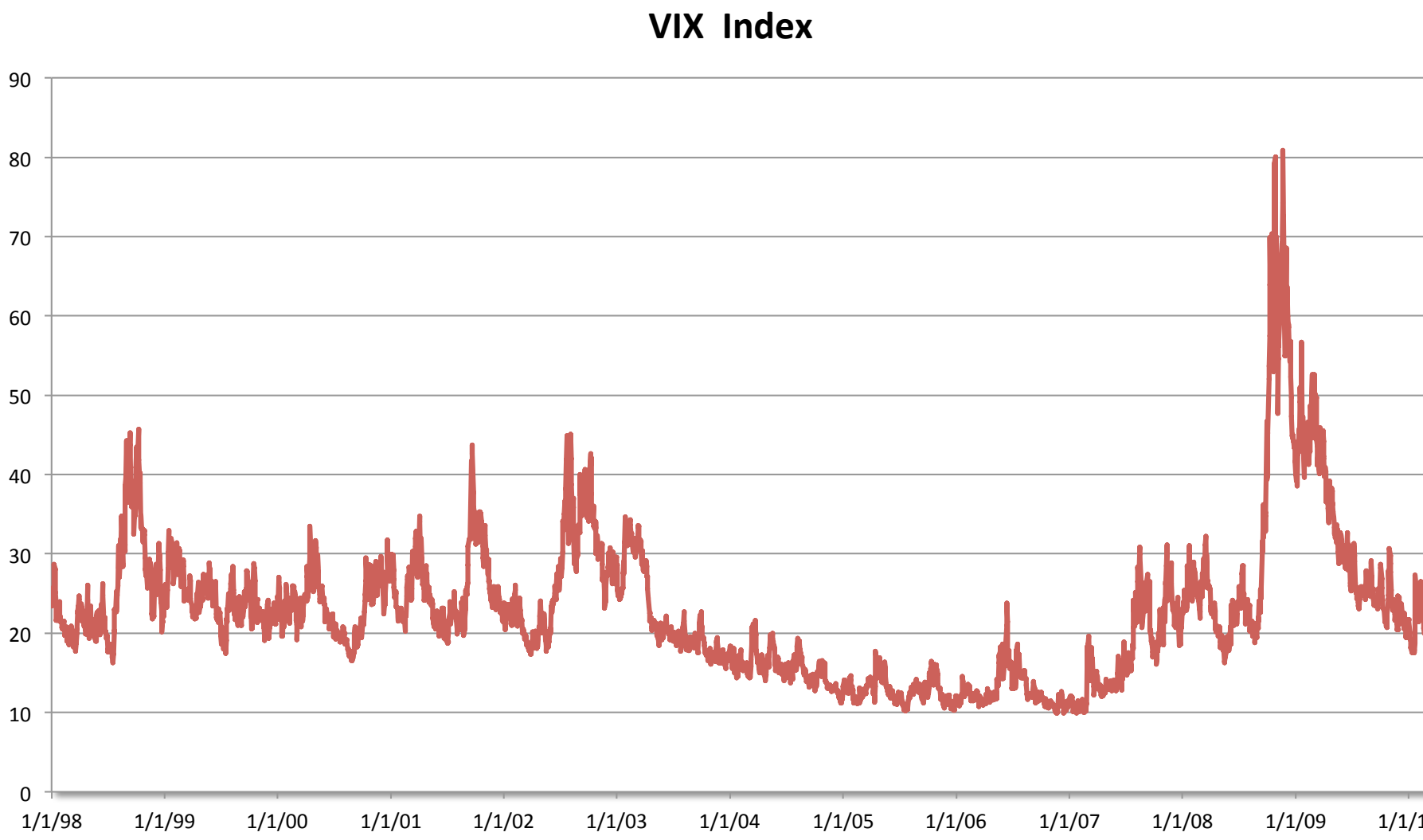


Observe that the Down Payment axis has been reversed, because lower down payment requirements are correlated with higher home prices.

Note: For every AltA or Subprime first loan originated from Q1 2000 to Q1 2008, down payment percentage was calculated as appraised value (or sale price if available) minus total mortgage debt, divided by appraised value. For each quarter, the down payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the diagram. This number is an indicator of down payment required: clearly many homeowners put down more than they had to, and that is why the top half is dropped from the average. A 13% down payment in Q1 2000 corresponds to leverage of about 7.7, and 2.7% down payment in Q2 2006 corresponds to leverage of about 37.

Note Subprime/AltA Issuance Stopped in Q1 2008.


VIX Index
90
80
70
60
50
40
30
20
10
0
1/1/98
1/1/99
1/1/00
1/1/01
1/1/02
1/1/03
1/1/04
1/1/05
1/1/06
1/1/07
1/1/08
1/1/09

- Cross Sectional Properites: Multiple Leverage Cycles:
  - Flight to Collateral
  - Contagion
  - Drastic swings in the volume of trade of high quality assets.

Lessons from the Leverage Cycle Theory

1. Increasing leverage on a broad scale can increase asset prices.
2. Leverage is endogenous and fluctuates with the fear of default.
3. Leverage is therefore related to the degree of uncertainty or volatility of asset markets.
4. The scarcity of collateral creates a collateral value that can lead to bubbles in which some asset prices are far above their efficient levels.
5. Booms and busts of the leverage cycle can be smoothed best not by controlling interest rates, but by regulating leverage.
6. Multiple leverage cycles can explain important phenomena like Flight to Collateral, Contagion and violent swings in volume of trade

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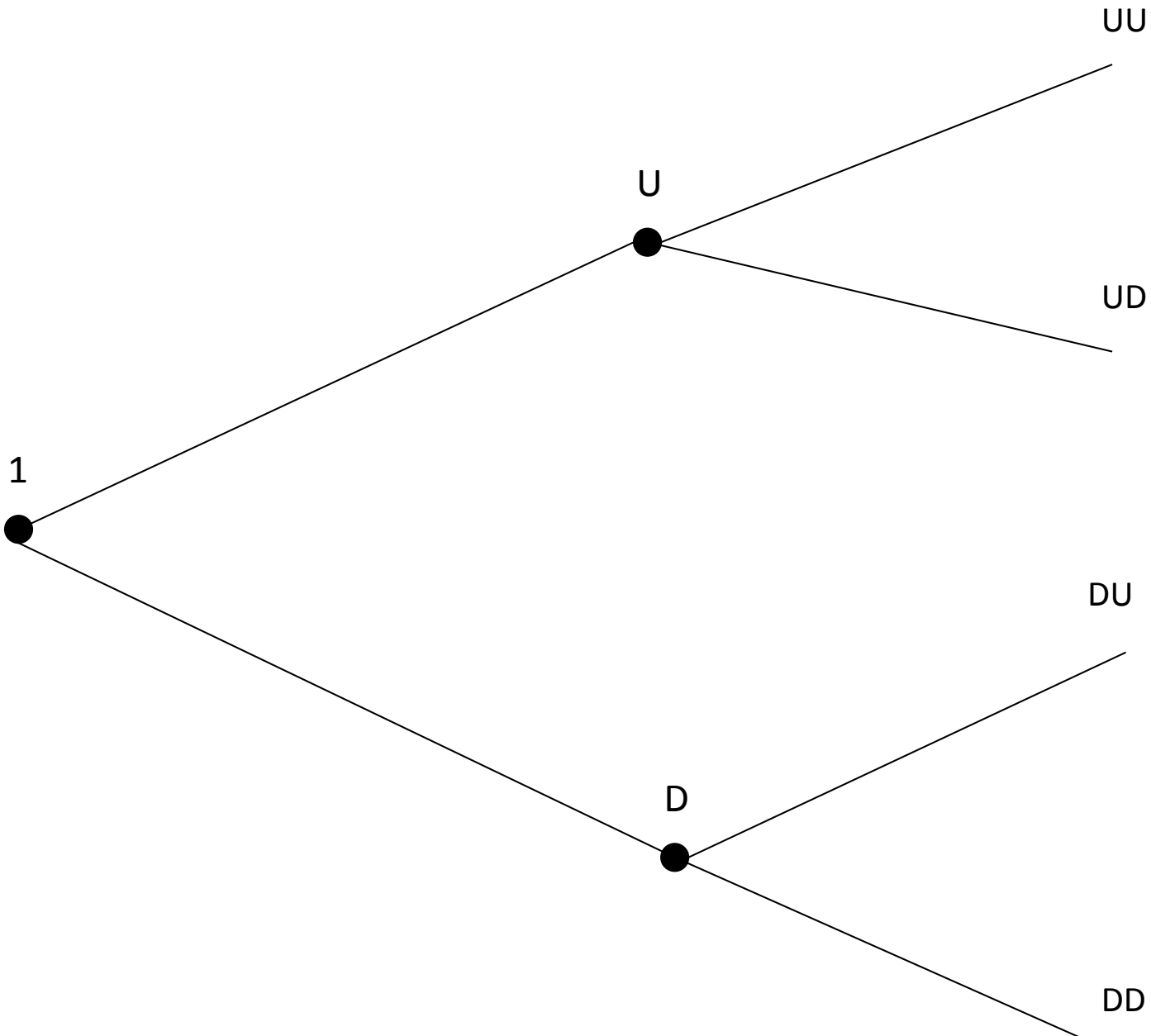
Time and Uncertainty

- Time $t = 0, ..., T$.
- Uncertainty: $s \in S$ including a root $s = 0$. S_T, the set of terminal nodes of S.
- Binomial tree:
  - each state $s \neq 0$ has an immediate predecessor s^*, and each nonterminal node $s \in S \setminus S_T$ has a set $S(s) = \{sU, sD\}$ of immediate successors.
  - binomial tree is simplest model in which uncertainty plays a role in determining leverage; also general theorems can be proved.


UU
U
UD
1
DU
D
DD

Goods and Assets

- There is a single perishable consumption good c. Numeraire.
- $K = \{1, ..., K\}$ assets k which pay dividends d_s^k of the consumption good in each state $s \in S \setminus \{0\}$. Price p_{sk}.
- Financial assets:
  - it gives no direct utility to investors, and
  - it pays the same dividends no matter who owns it.

Financial Contracts: Collateral and Debt

A *debt contract* $j \in J$ is a one-period non contingent bond

- Issued in state $s(j) \in S$
- Promise: $b(j) > 0$ units of the consumption good in each immediate successor state state $s' \in S(s)$
- Collateral: one unit of asset $k(j) \in K$ as collateral
- We denote the set of contracts with issue state s backed by one unit of asset k by $J_s^k \subset J$; we let $J_s = \bigcup_k J_s^k$ and $J = \bigcup_{s \in S \setminus S_T} J_s$.

Financial Contracts: Collateral and Debt

- Price of contract j is π_j.
- An investor can borrow π_j today by selling the debt contract j in exchange for a promise of $b(j)$ in each $s' \in S(s(j))$.
- Actual delivery of debt contract j in each state $s' \in S(s(j))$ is (no-recourse loan)

$$min\{b(j), p_{s'k(j)} + d^k_{s'}\}$$

- The rate of interest promised by contract j in equilibrium is

$$(1 + r_j) = b(j)/\pi_j.$$

  If promise is small enough, the same formula defines a riskless rate of interest.
- Let $\varphi_j > 0 (< 0)$ be the number of contracts j sold (bought).

Leverage

- The *Loan-to-Value* LTV_j associated to contract j in state $s(j)$ is given by

$$LTV_j = \frac{\pi_j}{p_{s(j)k(j)}}.$$

Endogenous Leverage

- We follow Geanakoplos (1997) methodology.
- Agents have access to a menu of contracts J.
- In equilibrium every contract, as well as the asset used as collateral, will have a price. Each contract has a well defined LTV.
- The key is that even if all contracts are priced in equilibrium, because collateral is scarce, only a few will be actively traded. In this sense, leverage becomes endogenous.

Asset and Investor Leverage

- *Leverage for asset k in state s*, LTV_s^k, as the trade-value weighted average of LTV_j across all actively traded debt contracts $j \in J_s^k$ by all the agents $h \in H$

$$LTV_s^k = \frac{\sum_h \sum_{j \in J_s^k} \max(0, \varphi_j^h) \pi_j}{\sum_h \sum_{j \in J_s^k} \max(0, \varphi_j^h) p_{sk}}.$$

- *Leverage for investor h in state s*, LTV_s^h, is defined analogously as

$$LTV_s^h = \frac{\sum_k \sum_{j \in J_s^k} \max(0, \varphi_j^h) \pi_j}{\sum_k \sum_{j \in J_s^k} \max(0, \varphi_j^h) p_{sk}}.$$

Investors

- Each investor $h \in H$ is characterized by a utility function

$$U^h = u^h(c_0) + \sum_{s \in S \setminus 0} \delta_h^{t(s)} \bar{\gamma}_s^h u^h(c_s).$$

- Endowment of the consumption good: $e_s^h \in R_+$, $s \in S$.
- Endowment of assets: $a_s^h \in R_+^K$,$s \in S$.

Budget Set

Given asset prices and contract prices (p, π), each agent $h \in H$ choses consumption, c, asset holdings, y, and contract sales/purchases φ in order to maximize utility (4) subject to the budget set defined by

$$B^h(p,\pi) = \{(c, y, \varphi) \in R^S_+ \times R^{SK}_+ \times (R^{J_s})_{s \in S \setminus S_T} : \forall s$$

$$(c_s - e^h_s) + p_s \cdot (y_s - y_{s^*} - a^h_s) \leq$$
$$\leq \sum_{k \in K} d^k_s y_{s^* k} + \sum_{j \in J_s} \varphi_j \pi_j - \sum_{k \in K} \sum_{j \in J^k_{s^*}} \varphi_j min(b(j), p_{sk} + d^k_s);$$

$$\sum_{j \in J^k_s} max(0, \varphi_j) \leq y^k_s, \forall k\}.$$

Collateral Equilibrium

A *Collateral Equilibrium* is $((p,\pi),(c^h,y^h,\varphi^h)_{h\in H}) \in (R^K_+ \times R^{J_s}_+)_{s\in S\setminus S_T} \times (R^S_+ \times R^{SK}_+ \times (R^{J_s})_{s\in S\setminus S_T})^H$ such that

1. $\sum_{h\in H}(c^h_s - e^h_s) = \sum_{h\in H}\sum_{k\in K} y^h_{s^*k} d^k_s, \forall s.$
2. $\sum_{h\in H}(y^h_s - y^h_{s^*} - a^h_s) = 0, \forall s.$
3. $\sum_{h\in H} \varphi^h_j = 0, \forall j \in J_s, \forall s.$
4. $(c^h,y^h,\varphi^h) \in B^h(p,\pi), \forall h$
   $(c,y,\varphi) \in B^h(p,\pi) \Rightarrow U^h(c) \leq U^h(c^h), \forall h.$

Geanakoplos and Zame (1998) equilibrium exists.

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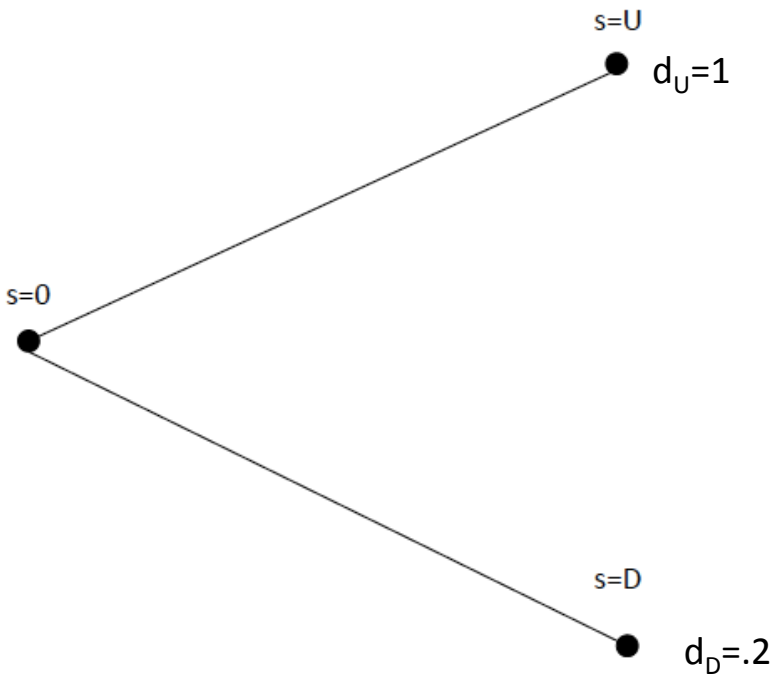
The Economy

- $T = 2$, and $S = \{0, U, D\}$.
- There is one financial asset Y which pays dividends only in the final period: $d_U = 1$ and $d_D = .2$.
- The set of contracts is $J = \{1, ..., 1000\}$, with $b(j) = j/100$.



The Economy

- Two types of agents $H = \{O, P\}$ with logarithmic utilities who do not discount the future.
- Agents differ in their beliefs and wealth:
  - Beliefs: $\gamma_U^O = .9$ and $\gamma_U^P = .4$.
  - Endowments: $a_0^h = 1, h = O, P$, $e_0^O = e_D^O = 8.5, e_U^O = 10$ and $e_s^P = 100, \forall s$.

Equilibrium

Table 1: Equilibrium Static Economy.

States	$s = 0$	$s = U$	$s = D$
Prices and Leverage			
p	0.708		
$b(j^*)$	0.2		
π_{j^*}	0.199		
r_{j^*}	0.1%		
LTV	0.282		
Asset Y			
Optimists	2		
Pessimists	0		
Consumption			
Optimists	8.2	11.6	8.5
Pessimists	100.3	100.4	100.4

Lesson 1: Absence of Default

1. In equilibrium there is not just one interest rate but a menu of interest rates depending on the promise per unit of collateral.
2. The example shows that only one contract is traded, the *max min* contract j^* satisfying $b(j^*) = d_D = .2$.
3. This is the maximum amount optimists can promise while guaranteeing they will not default in the future.

Lesson 2: Constrained Borrowing

There is no default, so the Optimists are paying the riskless interest rate of .1%. To get an extra penny of loan they would be willing to pay a rate of 37%, even if they were obliged, on penalty of death, to pay back the whole loan. But no lender will give them that deal, because there is no penalty.

Lesson 2: Constrained Borrowing

1. One might have thought that optimists would be so eager to borrow money that they would want to promise more than .2 per asset, happily paying a default premium in order to get more money at time 0.

2. According to the equilibrium, this is not the case. The threat of default is so strong, it causes the lenders to constrain the borrowers. More precisely, the offered interest rate rises too fast as a function of $b(j)$ for the borrowers to be willing to take on more debt.

Lesson 3: Endogenous Leverage

1. When only one contract is traded in equilibrium, this uniquely pins down the leverage in the economy.
2. Leverage can then be characterized by $LTV = \frac{\pi_{j^*}}{p} = \frac{d_D/p}{1+r_{j^*}}$.
3. Thus LTV is given by the ratio between the worst case rate of return on the asset and the riskless rate of interest.

Lesson 4: Leverage Raises Asset Prices

1. The collateral equilibrium asset price $p = .708$ is much higher than its price in Arrow-Debreu equilibrium (.539). Thus leverage can dramatically raise asset prices above their efficient levels.

2. What would happen if we dropped leverage, but still prohibited short selling? Get ($p = .609$), much lower than leverage price. As leverage (exogenously) increases from 0 to maxmin level, the asset price rises. One reason is that when less borrowing is allowed, period 0 consumption by the optimists would need to be lower if they continued to buy all the assets. But there is also another reason.

Lesson 4: Leverage Raises Asset Prices

1. The payoff value of the asset for the optimist is $PV^O = \frac{\sum_{s=U,D} \delta^O \gamma_s^O d_s du^O(c_s^O)/dc}{du^O(c_0^O)/dc} = .655.$
2. Yet the price is $p = .708 > PV^O = .655.$
3. The reason the optimists are willing to pay more for the asset than its payoff value to them is that holding more of the asset enables them to borrow more money. This is what Fostel and Geanakoplos (2008) called Collateral Value.
4. This Collateral Value can create bubbles.

Lesson 4: Leverage Raises Asset Prices

1. Collateral Value can create bubbles.
2. Mathematical reason, as explained in Geanakoplos (1997), is that owner does not set asset price equal to marginal utility of its dividends, but rather the downpayment equal to the marginal utility of the dividends net of loan repayments.

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Main Results

1. Collateral equilibrium can determine endogenously which contracts are traded.
2. In Binomial Economies: absence of default. But borrowing limit set by fear of default.
3. LTV given by simple formula: low tail risk
4. LTV moves with volatility.
5. New asset pricing: price=payoff value +collateral value.

Absence of Default

Binomial No-Default Theorem:

Suppose that S is a binomial tree, that is, $S(s) = \{sU, sD\}$, *for each* $s \in S \setminus S_T$. *Suppose that all assets are financial assets and that every contract is a one period debt contract. Let* $((p, \pi), (c^h, y^h, \varphi^h)_{h \in H})$, *be an equilibrium. Suppose that for any state* $s \in S \setminus S_T$ *and any asset* $k \in K$, *the maxmin contract* $j^*(s, k)$ *defined by* $b(j^*(s, k)) = min\{p_{sUk} + d^k_{sU}, p_{sDk} + d^k_{sD}\}$ *is available to be traded, i.e.* $j^*(s, k) \in J_s$.
Then we can construct another equilibrium $((p, \pi), (c^h, \bar{y}^h, \bar{\varphi}^h)_{h \in H})$ *with the same asset and contract prices and the same consumptions, in which only maxmin contracts are traded.*
Proof: See Fostel-Geanakoplos (2013).

Absence of Default

- The theorem provides a hard limit on borrowing.
- It shows that there must be a robust class of economies in which agents would like to borrow more at going riskless interest rates but cannot, even when their future endowments are more than enough to cover their debts.

Absence of Default

- The hard limit on borrowing is caused by the specter of default, despite the absence of default in equilibrium.
- The hard limit is endogenous.
- Binomial economies and their Brownian motion limit are special cases. But they are extensively used in finance. They are the simplest economies in which one can begin to see the effect of uncertainty on credit markets.

Endogenous Leverage

Binomial Leverage Theorem:

Under the assumptions of the Binomial No-Default theorem, equilibrium leverage can always be taken to be

$$LTV_s^k = \frac{d_{sD}^k / p_{sk}}{1 + r_s} = \frac{\textit{worst case rate of return}}{\textit{riskless rate of interest}}.$$

Proof: See Fostel-Geanakoplos (2013).

Endogenous Leverage

- Our second theorem provides a simple formula for leverage.
- Equilibrium LTV is given by

$$LTV = \frac{worst\ case\ rate\ of\ return}{riskless\ rate\ of\ interest}$$

Endogenous Leverage

Though simple and easy to calculate, our formula provides interesting insights:

- it explains which assets are easier to leverage (the ones with low tail risk).
- it explains why changes in the bad tail can have such a big effect on equilibrium even if they hardly change expected payoffs: they change leverage.
- the formula also explains why (even with rational agents who do not blindly chase yield), high leverage historically correlates with low interest rates.

Endogenous Leverage

Collateralized loans always fall into two categories:

- When an agent uses all his assets as collateral.
  - In this case, debt is explained by traditional models of demand for loans without collateral.
- When an agent does not use all his assets as collateral.
  - Debt is determined by the maximum debt capacity of the assets, independent of agent's preferences.
- Theorem says we can always use the same LTV. (If maxmin contract gives 80% LTV and you only want to borrow \$40 on \$100 asset, can instead borrow \$40 using only half the asset as collateral, Still safe loan and with 80% LTV.

Endogenous Leverage

- The distinction between plentiful and scarce capital all supporting loans at the same LTV suggests that it is useful to keep track of a second kind of leverage that we call *diluted* leverage:

$$DLTV_s^k = \frac{\sum_h \sum_{j \in J_s^k} \max(0, \varphi_j^h) \pi_j}{\sum_h y_{sk}^h p_{sk}} \leq LTV_s^k.$$

- Similarly one can define *diluted investor* leverage

$$DLTV_s^h = \frac{\sum_k \sum_{j \in J_s^k} \max(0, \varphi_j^h) \pi_j}{\sum_k y_{sk}^h p_{sk}} \leq LTV_s^h.$$

Leverage and Volatility

Binomial Leverage-Volatility Theorem:

Under the assumptions of the Binomial No-Default theorem, for each state $s \in S \backslash S_T$, and each asset $k \in K$, there are risk neutral pricing probabilities $\alpha = p_U(s,k)$ and $\beta = 1 - \alpha = p_D(s,k)$ such that the equilibrium price p_{sk} and equilibrium margin $m_s^k = 1 - LTV_s^k$ can be taken equal to

$$p_{sk} = \frac{\alpha(p_{sUk} + d_{sU}^k) + \beta((p_{sDk} + d_{sD}^k)}{1 + r_s}$$

$$m_s^k = \sqrt{\frac{\alpha}{\beta}} \frac{Vol_{\alpha,\beta}(k)}{(1 + r_s)p_{sk}}$$

where $Vol_{\alpha,\beta}(k) = \sqrt{\alpha\beta}(p_{sUk} + d_{sU}^k - p_{sDk} - d_{sD}^k)$.
Proof: See Fostel-Geanakoplos (2013). (State prices depend on asset)

Leverage and Asset Pricing I: Liquidity Value and Credit

- The *Payoff Value* of contract j to agent h at state s is

$$PV_{sj}^h = \frac{\sum_{\sigma\in\{U,D\}} \delta_h \gamma_{s\sigma}^h min\{b(j), p_{s\sigma k(j)} + d_{s\sigma}^{k(j)}\} du^h(c_{s\sigma}^h)/dc}{du^h(c_s^h)/dc}$$

- The *Liquidity Value* LV_{sj}^h associated to contract j to agent h at s is

$$LV_{sj}^h = \pi_j - PV_{sj}^h.$$

- The liquidity value represents the surplus a borrower can gain by borrowing money today selling a contract j backed by collateral k.

Leverage and Asset Pricing II: Liquidity Wedge and Discount Factor

- The *Liquidity Wedge* ω_{sj}^h associated to contract j for agent h at state s is
$$1 + \omega_{sj}^h = \frac{\pi_j}{PV_{sj}^h}$$
- In the case that contract j fully delivers, ω_{sj}^h defines the extra interest a potential borrower would be willing to pay above the going riskless interest rate if he could borrow an additional penny and was committed (under penalty of death) to fully deliver.
- This extra interest is called the liquidity wedge; it gives a measure of how tight the contract j credit market is.

Leverage and Asset Pricing II: Liquidity Wedge and Discount Factor

Discount Theorem:

Define the risk adjusted probabilities for agent h in state s by

$$\mu^h_{sU} = \frac{\gamma^h_{sU} du^h(c^h_{sU})/dc}{\gamma^h_{sU} du^h(c^h_{sU})/dc + \gamma^h_{sD} du^h(c^h_{sD})/dc},$$

$$\mu^h_{sD} = \frac{\gamma^h_{sD} du^h(c^h_{sD})/dc}{\gamma^h_{sU} du^h(c^h_{sU})/dc + \gamma^h_{sD} du^h(c^h_{sD})/dc} = 1 - \mu^h_{sU}.$$

If agent h is taking out a riskless loan in state s, then his payoff value in state s for a tiny share of arbitrary cash flows consisting of consumption goods $x = (x_{sU}, x_{sD})$ *is given by*

$$PV^h(x) = \frac{\mu^h_{sU} x_{sU} + \mu^h_{sD} x_{sD}}{(1 + r_s)(1 + \omega^h_s)}.$$

Leverage and Asset Pricing III: Asset Pricing and Collateral Value

- The *Payoff Value* of asset k to agent h at state s is

$$PV_{sk}^h \equiv \frac{\sum_{\sigma \in \{U,D\}} \delta_h \gamma_{s\sigma}^h (p_{s\sigma k} + d_{s\sigma}^k) du^h(c_{s\sigma}^h)/dc}{du^h(c_s^h)/dc}$$

- The *Collateral Value* of asset k in state s to agent i is

$$CV_{sk}^h \equiv p_{sk} - PV_{sk}^h$$

- The collateral value stems from the added benefit of enabling borrowing that some durable assets provide.
- Collateral values distort pricing and typically destroy the efficient markets hypothesis.

Leverage and Asset Pricing III: Asset Pricing and Collateral Value

Collateral Value = Liquidity Value Theorem:

Suppose that $y^h_{sk} > 0$ *and* $\varphi^h_j > 0$ *for some agent* h *and some* $j \in J^k_s$. *Then, in equilibrium the following holds,*

$$LV^h_{sj} = CV^h_{sk}$$

The liquidity value associated to any contract j *that is actually issued using asset* k *as collateral equals the collateral value of the asset.*

Proof: See Fostel-Geanakoplos (2008) and Geanakoplos-Zame (2013).

Liquidity and Endogenous Contracts

- Since one collateral cannot back many competing loans, the borrower will always select the loan that gives the highest liquidity value among all loans with the same collateral.
- The following holds

$$LV_{sj}^{h} = PV_{sj}^{h}\omega_{sj}^{h}$$

- All loans that deliver for sure will have the same liquidity wedge. If this wedge is positive, the borrower will naturally choose the biggest loan, since that has the highest payoff value and therefore the highest liquidity value.
- Formula also shows why, holding liquidity wedge constant, the collateral value and thus the price rises when an asset can be leveraged more.

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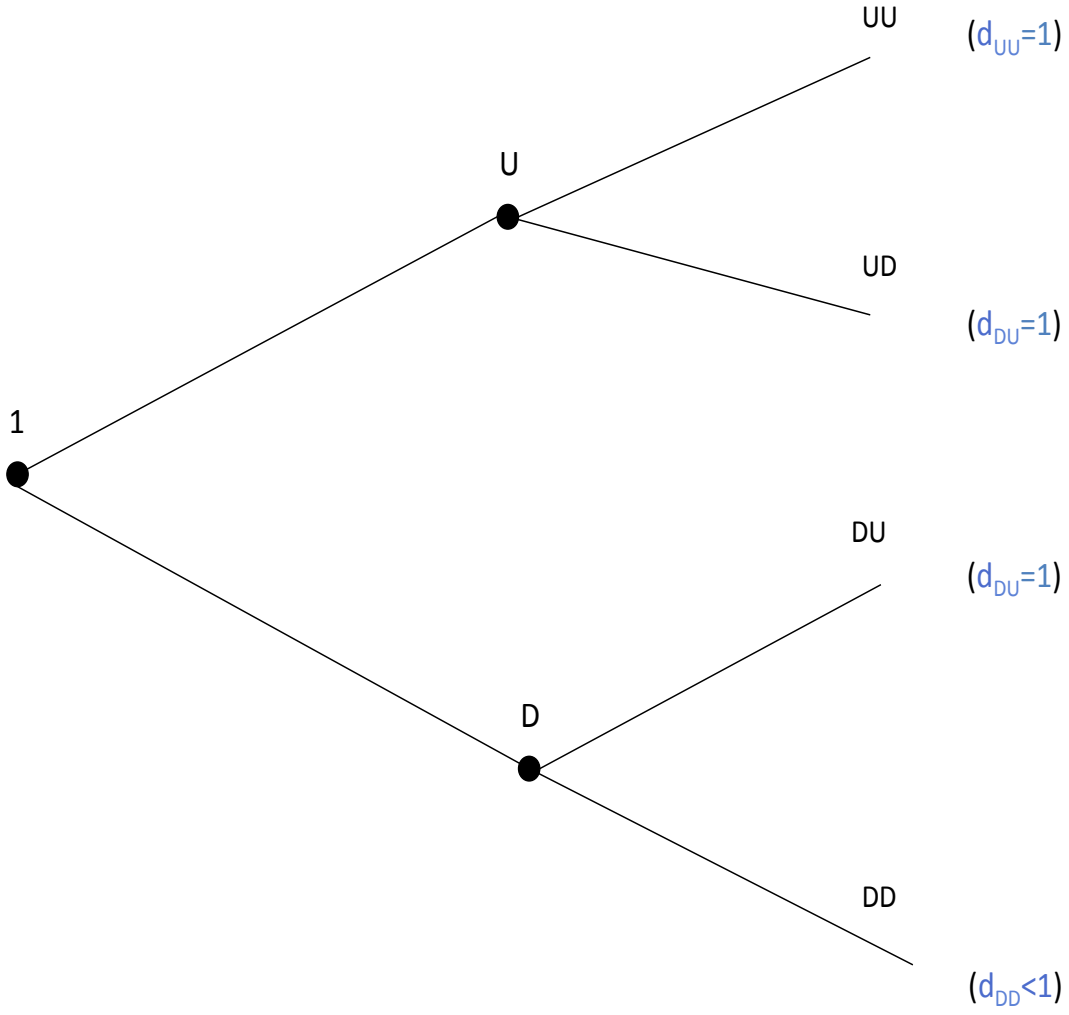
The Economy

- $T = 2$, and $S = \{0, U, D, UU, UD, DU, DD\}$.
- There is one financial asset Y which pays dividends only in the final period: $d_{UU} = d_{UD} = 1$ and $d_{DU} = 1$ and $d_{DD} = .2$.
- Good news reduces uncertainty about the payoff value and bad news increases uncertainty about the payoff value of the asset.



The Economy

- Two types of agents $H = \{O, P\}$ with logarithmic utilities who do not discount the future.
- Agents differ in their beliefs and wealth:
  - Beliefs: $\gamma^O_{sU} = .9$ and $\gamma^P_{sU} = .4$ for all $s \in S \setminus S_T$.
  - Endowments: $a^h_0 = 1, h = O, P$, $e^O_0 = e^O_D = 8.5, e^O_s = 10, s \neq 0, D$ and $e^P_s = 100, \forall s$.

Equilibrium

Table 2: Equilibrium Leverage Cycle.

States	$s = 0$	$s = U$	$s = D$
Prices, Leverage and Liquidity Wedge			
p	0.909	0.982	0.670
j^*	0.670		0.2
π_{j^*}	0.664		0.201
LTV	0.730		.299
PV	0.850	0.98	0.602
CV	0.059	0	0.068
ω	0.097	0	0.518
Asset Holdings			
Optimists	2	.408	2
Pessimists	0	1.591	0
Debt Contract Trades			
Optimists	2		2
Pessimists	-2		-2

Ebullient Times

- When volatility is low, as at $s = 0$, the existing scarce collateral can support large amounts of borrowing to buy assets that are acceptable collateral.
- A bubble can emerge in which the prices of the assets that can be used as collateral rise to levels far above their "Arrow-Debreu" Pareto efficient levels, even though all agents are rational. In this example, leverage at time 0 is almost 4 to 1 ($LTV = .73$), and the asset price at time 0 is .91. In Arrow-Debreu equilibrium, the asset price would only be .71.

Ebullient Times

- On top of all that, the optimists are willing to pay a collateral value of .06 above and beyond the asset payoff value of .85 to them, because holding it enables them to borrow more money.
- The combination of high prices and low volatility creates an illusion of prosperity. But in fact the seeds of collapse are growing as the assets get more and more concentrated in the hands of the most enthusiastic and leveraged buyers.

The Crash

- Leverage cycle crashes always occur because of a coincidence of three factors:
  - The bad news itself lowers the prices.
  - The reduction in wealth of the leveraged buyers.
  - If the bad news also creates more uncertainty, then credit markets tighten and leverage will be reduced.

The Crash:Bad News

- The price of the asset in our example goes down from .91 at 0 to .67 at D after bad news, a drop of 24 points.
- At both 0 and D, the optimists are the only agents holding the asset, and in their view the expected payoff of the asset drops only 7 points, from .99 to .92, after the bad news.
- So there is something much more important than the bad news which explains the drop in asset price. This is the downward path of the leverage cycle.

The Crash:Reduced Wealth and Plummeting LTV

- First notice that the optimists, though still buying all the asset in the economy, lose wealth after bad news. They are forced to consume less.
- The higher volatility at D reduces the amount they can leverage. Leverage plummets from 4 at 0 to 1.4 at D (equivalently, the LTV goes from .73 to .29). Quantitatively the most important factor reducing price.

The Crash:Increased Liquidity Wedge

- As a result their liquidity wedge, which is a measure of how much they are willing to pay above the riskless interest rate, increases dramatically, from 0.1 to 0.52.
- By the Discount Theorem, they then discount all future cash flows at a much higher rate than the riskless rate: the payoff value of the assets sinks all the way to .60.
- Of course there is still a collateral value of .07. But despite the high liquidity wedge, the collateral value of the assets is limited by the small amount of borrowing they support.

Margin Calls

- The most visible sign of the crash is the margin call.
- After the bad news at D starts to reduce asset prices, optimists who want to roll over their loans need to put up more money to maintain the same LTV on their loans.
- They could do that either by selling assets or by reducing their consumption. In our example here, they choose to reduce their consumption. (Geanakoplos 2003, they sell assets).
- They then effectively get a second margin call because the new LTV is much lower than before, forcing them to reduce consumption further.

Volatility

- The signature of the leverage cycle is rising asset prices in tandem with rising leverage, followed by falling asset prices and leverage.
- But the underlying cause of the change in leverage is a change in volatility, or more generally, in some kind of bad tail uncertainty.
- In our example, the volatility of the asset's value is .126 at time 0, when leverage is almost 4, and increases to .394 at D, when leverage plummets to 1.4.

Smoothing the Leverage Cycle

- Asset prices are much too high at 0 (compared to Arrow Debreu first best prices) and then they crash at D, rising and falling in tandem with leverage.
- But interest rates over the cycle in the Leverage Cycle example barely move.
- The leverage cycle suggests that it might be more effective to stabilize leverage than to stabilize interest rates.

Smoothing the Leverage Cycle

- Regulation limiting leverage at time 0 will lower asset price at time 0 and raise it at D, smoothing the cycle. Goes up at D because Optimists have higher marginal propensity to buy the asset, and become richer when owe less debt.
- Will not cause Pareto improvement because no trade in assets at D: Optimists retain them all.
- If modify example by giving Pessimists endowment of assets at U and D, and letting discount of Pessimists be .95, then taxing leverage at time 0 and redistributing revenue to Pessimists will Pareto improve.

Smoothing the Leverage Cycle and Default

- Main reason why limiting leverage can cause Pareto improvement is that it raises asset prices in future, reducing number of defaults.
- If owners who are about to default forego cheap but important repairs to assets, then a deadweight loss results. Limiting leverage then can easily lead to Pareto improvement.

Credit Cycle vs Leverage Cycle

- The Leverage Cycle is not the same as a Credit Cycle.
- A Leverage Cycle is a feedback between asset prices and leverage, whereas a Credit Cycle is a feedback between asset prices and borrowing.
- Of course a leverage cycle always produces a credit cycle. But the opposite is not true.

Credit Cycle vs Leverage Cycle

- Classical macroeconomic models of financial frictions such as Kiyotaki and Moore (1997) produce credit cycles but not leverage cycles. (counter-cyclical leverage despite the fact that borrowing goes down after bad news).
- The reason for the discrepancy is that to generate leverage cycles, uncertainty is needed, and a particular type of uncertainty: one in which bad news is associated with an increase in future volatility.

Credit Cycle vs Leverage Cycle

- Classical macroeconomic models of financial frictions such as Kiyotaki and Moore (1997) ignore huge swings in asset prices that come from procyclical leverage.
- They ignore importance of changes in volatility.
- They take for granted that collateral constraints restrict borrowing and so reduce asset prices and investment. But they miss collateral value. Collateral constraints can actually raise asset prices and investments and cause bubbles and overinvesment.

Agent Heterogeneity

- The leverage cycle relies crucially on agent heterogeneity. In the example, heterogeneity was created by differences in beliefs. But there are many other sources of heterogeneity.
- It is very important to understand that the connection between leverage and asset prices does not rely on differences in beliefs.
- We can change endowments of consumption goods and assume that both agents have identical beliefs and reproduce all the trades and prices in our example.

Lessons from the Leverage Cycle

1. Increasing leverage on a broad scale can increase asset prices.
2. Leverage is endogenous and fluctuates with the fear of default.
3. Leverage is therefore related to the degree of uncertainty or volatility of asset markets.
4. The scarcity of collateral creates a collateral value that can lead to bubbles in which some asset prices are far above their efficient levels.
5. Booms and busts of the leverage cycle can be smoothed best not by controlling interest rates, but by regulating leverage.
6. The amplitude of the cycle depends on the heterogeneity of the valuations of the investors.

Outline

1. Introduction
2. A Simple Model
3. Simple Example
4. Results
5. Leverage Cycle
6. Multiple Leverage Cycles
7. Volatility

Multiple Leverage Cycles

- Many kinds of collateral exist at the same time, hence there can be many simultaneous leverage cycles.
- Collateral equilibrium theory not only explains how one leverage cycle might evolve over time, it also explains some commonly observed cross sectional differences and linkages between cycles in different asset classes.

Multiple Leverage Cycles

- Multiple co-existing leverage cycles can explain:
  - Flight to collateral,
  - Contagion and
  - Drastic swings in the volume of trade of high quality assets.

Multiple Leverage Cycles: Flight to Collateral

- When similar bad news hits two different asset classes, one asset class often preserves its value better than another.
- This empirical observation is traditionally given the name Flight to Quality, because it is understood as a migration toward safer assets that have less volatile payoff values.
- Fostel and Geanakoplos (2008) emphasized a new channel which they called Flight to Collateral: After volatile bad news, collateral values widen more than payoff values, thus giving a different explanation for the diverging prices.

Multiple Leverage Cycles: Flight to Collateral

- Consider the same economy except that now there are two financial assets.
- Asset Y pays $d_s^Y = 1, s = UU, UD, DU$ and $d_s^Y = .2, s = DD$.
- Asset Z is perfectly correlated with asset Y and pays $d_s^Z = 1, s = UU, UD, DU$ and $d_s^Z = .1, s = DD$.
- Agents start with asset endowments of .5 units of each asset, $a_0^h = (.5, .5), h = O, P$ at the beginning.

Multiple Leverage Cycles: Flight to Collateral

Table 4: Equilibrium Flight to Collateral.

	0	$s = U$	$s = D$	$s = 0$	$s = U$	$s = D$
		Asset Y			Asset Z	
p	0.906	0.982	0.664	0.897	0.982	0.621
PV	0.844	0.982	0.593	0.838	0.98	0.586
CV	0.063	0	0.071	0.059	0	0.035
π_{j^*}	.658		0.201	0.615		0.100
LTV	0.726		0.303	0.686		0.162
ω	0.105		0.541			

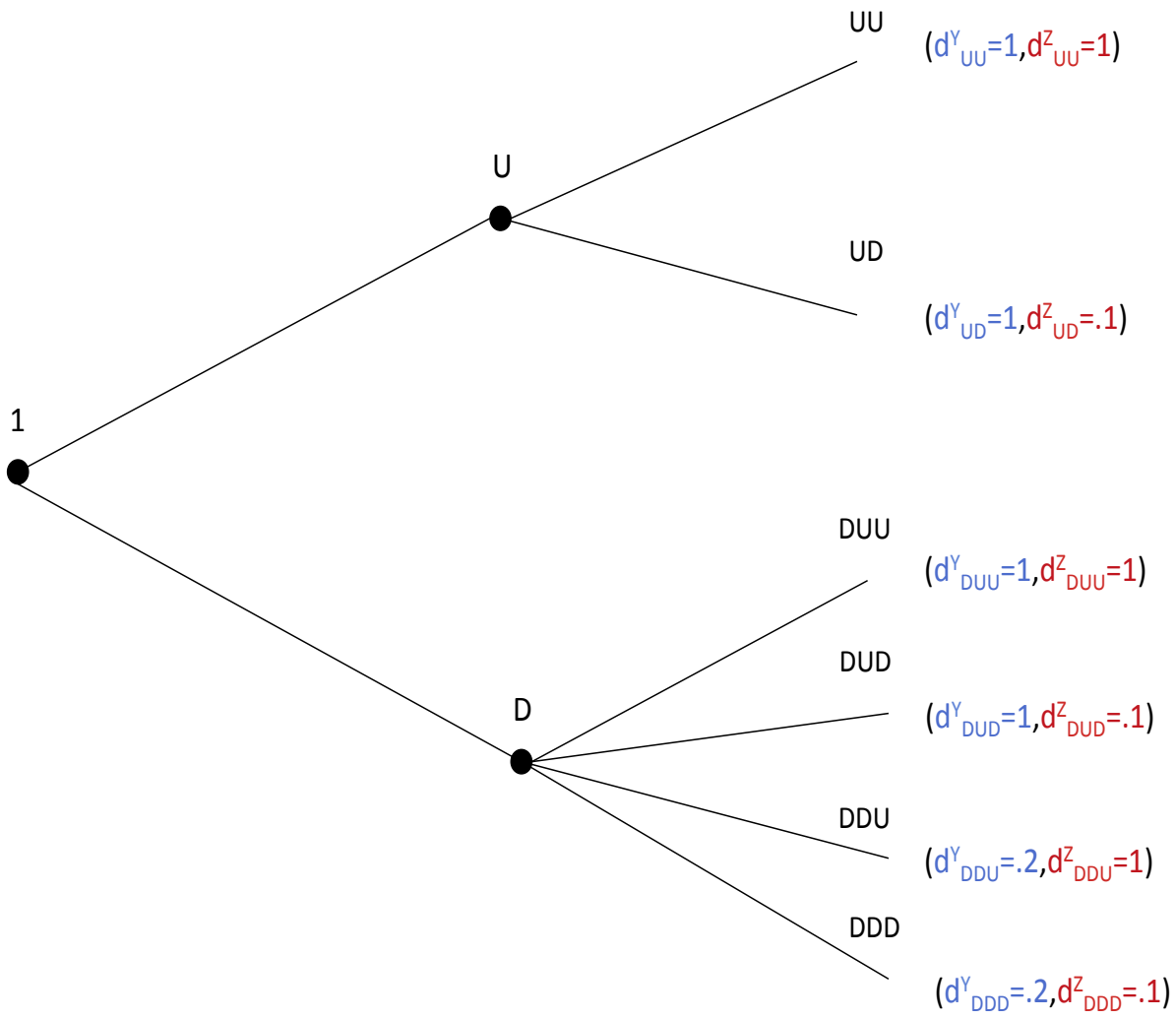
Multiple Leverage Cycles: Flight to Collateral

- Each asset experiences a leverage cycle.
- However, something interesting happens when we look at the cross section variation of all the variables. The gap between asset prices widens after bad news by more than the gap in expected payoffs.
- The price of Y falls from .906 at 0 to .664 at D, while the price of Z falls from .897 to .621. After bad news, both payoff values go down, but gap increases from .009 to .043.
- However, their collateral values move in opposite directions. The widening spread of .034 in prices is almost entirely explained by the widening of collateral values by .034.

Multiple Leverage Cycles: Flight to Collateral

- Flight to collateral occurs when the liquidity wedge is high and the dispersion of LTVs is high.
- During a flight to collateral, investors would rather buy those assets that enable them to borrow money more easily (higher LTVs).
- The other side of the coin is that investors who need to raise cash get more by selling those assets on which they borrowed less money because the sales revenues net of loan repayments are higher.

Multiple Leverage Cycles: Contagion



Multiple Leverage Cycles: Contagion

- Bad news is about Y only. So we should expect the price of Y to go down after bad news due to a deterioration of its expected payoff value.
- But we should not expect the price of asset Z to go down after bad news about Y.

Multiple Leverage Cycles: Contagion

Table 5: Equilibrium Contagion.

States	$s = 0$	$s = U$	$s = D$	$s = 0$	$s = U$	$s = D$
		Asset Y			Asset Z	
p	0.925	0.991	0.667	0.789	0.827	0.624
π_{j^*}	0.660		0.201	0.617	0.099	0.100
LTV	0.721		0.299	0.792	0.119	0.160
ω	0.054		0.544			

Multiple Leverage Cycles: Contagion

- As expected, asset Y experiences a leverage cycle: Its price rises from .925 to .991 after good news and crashes after bad news by more than its payoff value, going down from .925 to .667.
- Surprisingly, the price of Z also goes up from .789 to .827 after good news about Y, and goes down by more than 20% from .789 to .624 after bad news about Y.
- The leverage cycle on Y migrates to asset class Z, producing a leverage cycle on this market as well. In short, we see contagion in equilibrium.

Multiple Leverage Cycles: Contagion

- Fostel-Geanakoplos (2008) showed that contagion is generated by a change in the liquidity wedge.
- The Y leverage cycle lowers the liquidity wedge at U and sharply increases the liquidity wedge at D, as we have seen in our previous examples. A leverage cycle in one asset class alone can move the liquidity wedge. But, the liquidity wedge is a universal factor in valuing all assets.
- Portfolio Effect, amplifying the movements of the liquidity wedge at U and D. At U optimists do not see any advantage in giving up consumption to invest in Y. At D they buy all of Y, reducing their consumption, further increasing the liquidity wedge.

Multiple Leverage Cycles: Swings in High Quality Volume

- Following Dubey-Geanakoplos (2002) and Fostel-Geanakoplos (2008), we now extend the model to encompass asymmetric information: owners of the assets know their quality, but investors do not.
- Multiple leverage cycles can generate violent swings in the volume of trade.

Multiple Leverage Cycles: Swings in High Quality Volume

- We suppose that assets Y and Z are owned by a new class of agents we call issuers.
- Investors cannot distinguish the assets, but their issuers can.
- We can combine perfect competition with quantity signaling by defining equilibrium in terms of a quantity-price schedule. In each state $s \in S$, there are many different markets, each characterized by a quantity limit (which a seller in that market cannot exceed) and its associated market clearing price:

$$\vec{p}_s = \{(x_s, p_s(x_s)); x_s \in (0, 1], p_s \in \Re_+\}. \tag{1}$$

Multiple Leverage Cycles: Swings in High Quality Volume

- Investors who buy assets in market $(x_s, p_s(x_s))$ get a pro rata share of the deliveries of all assets sold in that market.
- With this interpretation there is room for *signaling* as well as *adverse selection* without destroying market anonymity.
- Firms may (falsely) signal more reliable deliveries by publicly committing to (small) quantity markets where the prices are high because the market expects only good types to sell there. The quantity limit characterizing each asset market is exogenous and the associated price is set endogenously as in any traditional competitive model.
- Dubey-Geanakoplos (2002) proved that in models like the one considered in this section, there is a unique equilibrium that is robust to perturbations.

Multiple Leverage Cycles: Swings in High Quality Volume

Table 6: Equilibrium Adverse Selection.

States	$s = 0$	$s = U$	$s = D$	$s = 0$	$s = U$	$s = D$
		Asset Y			Asset Z	
p	0.878	0.981	0.665	0.866	0.981	0.622
π_{j^*}	0.654		0.201	0.612		0.100
LTV	0.745		0.303	0.707		0.162
			Issuance			
	0.923	1	0.300	1	1	0.78
ω	0.208		0.536			

Multiple Leverage Cycles: Swings in High Quality Volume

- There are two coexisting leverage cycles and flight to collateral.
- The new thing in this simulation comes from the supply side.
- In order to signal that their assets are good (so that investors will pay more for them and be able to borrow more using them as collateral), the Y owners always sell less than they would if their types were common knowledge.
- However, after bad news at D, the drop in volume of their sales is huge. The bad Z type issuance goes down 22% from $x_0^Z = 1$ at 0 to $x_D^Z = .78$ at D, whereas the good type Y issuance goes down 67% from $x_0^Y = .92$ all the way to $x_D^Y = .30$.

Multiple Leverage Cycles: Swings in High Quality Volume

- It is not surprising that with the bad news and the corresponding fall in prices, equilibrium issuance falls as well, because issuers are optimists and do not want to sell at such low prices.
- The interesting thing is that flight to collateral combined with informational asymmetries generates such a big drop in good issuance, even though the news is almost equally bad for both assets.
- The explanation is that the bigger price spread between types caused by the flight to collateral requires a smaller good type issuance for a separating equilibrium to exist.

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Volatility

- Thurner and Farmer and Geanakoplos (2013) showed that in an agent based model with nearly Gaussian shocks, prices could display very non Gaussian fat tails and clustered volatility.
- Assume that agents (funds) who can leverage have more stable opinion of value. Other agents (noise traders) think last periods price plus noise is value.
- Big banks and hedge funds use similar models and hire same kinds of people.
- As funds bets win, they get richer and own more assets, stabilizing prices. Lower vol allows them to borrow more, further stabilizing prices. Unlucky shock from noise traders reduces funds wealth and thus raises vol and lowers leverage etc.