Monetary Policy with Interest on Reserves

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Abstract

I analyze monetary policy in the upcoming regime, with interest on reserves and a large balance sheet. I argue for the desirability of this regime on financial stability grounds. I show that conventional theories do not determine inflation in this regime, so I base the analysis on the fiscal theory of the price level. I find that monetary policy – buying and selling government debt with no effect on surpluses – can peg the nominal rate, and determine expected inflation. With sticky prices, monetary policy can also affect real interest rates and output, though not with the usual signs in this model. I address theoretical controversies, and how the fiscal backing of monetary policy was important for the 1980s disinflation. A concluding section reviews the role of central banks.

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1 Introduction

During the last few years, the Federal Reserve policy has made two changes that will fundamentally affect monetary policy going forward. First, the Fed now pays interest on reserves[1]. Second, the Fed has amassed an enormous balance sheet. Before the crisis, the Fed held about a trillion dollars of U.S. government bonds. Corresponding to these assets, the Fed's liabilities were about $950 billion dollars of currency and $50 billion dollars of bank reserves. Through its various quantitative easing programs, the Federal Reserve has, as of July 2014, accumulated $4.4 trillion of assets, almost all treasuries and mortgage-backed securities, and created about $2.8 trillion of additional reserves in return. Required reserves – the amounts banks must hold at the Fed corresponding to deposits – are about $80 billion, so about $2.7 trillion of those reserves are "excess reserves," held voluntarily by banks.

When interest rates rise, it is pretty clear that the Fed will not return to the previous monetary configuration, a small quantity of non-interest-paying reserves, but will instead maintain a large balance sheet and pay market interest rates on reserves. Indeed, the Fed will attempt to control short term rates primarily by changing the rate it pays on abundant reserves, rather than by controlling the quantity of reserves via open market operations. This plan is articulated in Chairman Bernanke's (2010) testimony and most recently reinforced in the July 2014 FOMC minutes (Federal Reserve (2014)).

My analysis below endorses this choice. A large balance sheet, with reserves that pay market interest, is a desirable configuration of monetary affairs, most of all for its beneficial impact on financial stability.

However, market interest on reserves and a large balance sheet, together with the spread of interest-paying electronic money, deeply challenges standard monetary policy analysis. We will be satiated in liquidity. Reserves and short term treasuries are and will remain essentially perfect substitutes at the margin. In this case, reserve demand and money demand become indeterminate. The money multiplier neither controls nor limits bank lending or deposit creation. Standard answers to fundamental questions, such as how inflation is determined, how the Fed controls real and nominal interest rates, what are the channels by which monetary policy affects the economy and the banking system, all fall apart in this regime.

It is important for theory to catch up to practice, so this new world can come to fruition without learning by mistakes. Old habits die hard, and clear thinking is needed to dispel errors. For example, reserves that pay market interest, in arbitrary quantities, are not inflationary. Period. Up to second-order institutional constraints, the financial system would be perfectly happy to absorb another trillion in reserves and in exchange for a trillion less of short-term treasury debt. When money pays market interest, MV = PY ceases to control PY because velocity V absorbs any change in the supply of M. Yet commentator after commentator in the last five years has noticed

[1] Definitions: Reserves are accounts that banks have at the Federal Reserve. They are used to clear transactions between banks. If Bank A owes bank B $100, for example to settle checks of its customers, and bank B owes bank A $50, bank A will ask the Fed to move $50 of reserves from its account to bank B's account. Reserves are really the numeraire in our economy, as every transaction ultimately involves a promise to deliver reserves. Reserves are also the means of final payment in our economy – though note as in the example that many payments are simply netted with nothing changing hands.

Since reserves are electronic book entries, it is said that the Fed creates reserves out of thin air. This is not true. The Fed creates reserves by buying Treasuries or Mortgage-backed securities. Thus, for every dollar of reserves, which are a liability of the Fed, the Fed has a dollar of assets. This simple fact alone reveals a lot of fallacies and solves a lot of large-balance sheet controversies below. The "balance sheet" is a statement of these assets along with the liabilities – reserves and currency of the Federal Reserve.

the quantity labeled "money" (reserves) shooting up 5,600%, from $50 billion to $2.8 trillion and warned of hyperinflation.

Most deeply, standard theory, which I review below, predicts that inflation is not determined at all in an interest on reserves regime, when money pays market interest rates and monetary frictions disappear. Experience in the U.S. and abroad suggests that inflation is quite stable under an interest on reserves regime, which includes the zero bound. We need different theory, that can operate in a world without monetary frictions, to reassure ourselves that inflation is determined and how it can be controlled.

A main contribution of this essay is to outline such a theory. I revisit classic questions as posed by Friedman (1968). What can monetary policy do, when we are satiated in liquidity? A surprising amount, it turns out. What can't monetary policy do? How should an effective monetary policy work?

Theory is important, as we cannot necessarily rely on experience. One might say that the Fed has been manipulating interest rates for decades, and we have lots of experience and impulse-response functions to tell us what its effects are. But all that experience was accumulated with a small quantity of non-interest-paying reserves, interest rate control through open market operations rationing reserves, and apparently binding reserve requirements. The frictionless mechanisms I outline below and that will certainly operate in the future may have been operating all along, and economists telling a fundamentally false story, but it is not obvious that this is so.

Some monetary frictions will remain. For example, traditionally only banks could hold reserves, so if banks are less than perfectly competitive the interest paid on reserves might not be exactly the same as interest on treasuries and deposits. In fact, we have seen interest on reserves a 10 - 20 bp higher than interest on treasuries – treasuries are *more* "money-like" than reserves, because everyone can hold treasuries. But I think it would be a mistake to base our analysis of big questions of monetary policy – can monetary policy affect GDP and the price level, and if so how – on the remaining small frictions of the interest-on-reserves and large-balance-sheet regime. Instead, we should base our analysis on a theory that is valid in a world with no monetary frictions at all, and then add back the remaining small monetary frictions.

Analyzing a model without monetary frictions, I find that monetary policy – defined as buying and selling government debt with no change in taxes or spending – can set the nominal interest rate and can determine expected inflation. With short-term debt, monetary policy cannot affect unexpected inflation, but with long term debt it has some power to smooth even those shocks. I solve a model with sticky prices, retaining the assumption of no monetary frictions. Here, I find that monetary policy can also affect output and the real rate of interest. However, I find neo-Fisherian signs: an interest rate increase raises real interest rates, and raises consumption, before raising inflation. Comparing to a standard new-Keyenesian model, I find the "monetary policy shock" in the latter changes fiscal policy as well, and the fiscal contraction produces its disinflation.

I find that ties between fiscal and monetary policy are and will remain more important than conventionally acknowledged. For example, the presence of a large stock of outstanding Treasury debt, of relatively short maturity, means that interest rate changes will have large impacts on the Federal budget. The mark-to-market losses on the Fed's portfolio, which monetary analysts have worried about, are tiny in comparison. Fiscal considerations will limit monetary policy in ways that the Federal Reserve is barely thinking about at all. And my analysis points to more explicit and better-considered fiscal and monetary policy coordination.

2 Why market interest on reserves and a big balance sheet are good things

Why does it matter whether the Fed pays interest on reserves with a large balance sheet, or reverts to a traditional small balance sheet with non-interest-paying reserves? The short answers: optimal quantity of money, and financial stability.

2.1 Optimal quantity of money

Milton Friedman (1969) described the "optimal quantity of money:" zero nominal interest rate. In that case, money and bonds pay the same rate of return. People can transact with an asset they hold in abundance for savings purposes. "Shoe leather" costs of cash management vanish. Since there is no social cost to this liquidity, satiation is the optimal quantity of money. Lucas (2000) estimates the welfare benefits of adopting the optimal quantity of money, the area under the money demand curve, at around 1% of GDP, a not inconsiderable sum.

Friedman advocated a slow steady deflation, so that money and bonds could pay a positive real rate of interest. People objected to Friedman's proposal, that prices and wages might be sticky downward, so perpetual deflation might not be a good thing. Moreover, some object that living at the zero bound means the Fed cannot lower interest rates further in response to various shocks.

If money pays a market rate of interest, however, we can have the same liquidity benefits of the optimal quantity of money without deflation. Economists since Friedman (1960) already called for interest-paying reserves, on the grounds that there is no point to inducing banks artificially to economize on reserves.

2.2 Financial stability

In my view, financial stability is an uheralded benefit of market interest on reserves and a large balance sheet, more important than the optimal quantity of money.

The financial crisis was, centrally, a run in the shadow banking system. In the last three decades, the vast majority of the legal economy, and the financial system especially, had developed interest-paying money: extremely liquid fixed-value interest-paying accounts that can be sold in milliseconds to make transactions, such as interest-paying bank accounts, money-market funds, overnight repurchase agreements, short-term commercial paper, and auction-rate securities. Businesses favored the latter shadow-banking arrangements, since deposit insurance does not guarantee large accounts.

Alas, this interest-paying money was largely inside money, circulating private promises to deliver reserves. On the eve of the financial crisis, we were holding something like \$10 trillion of promises to pay the then roughly \$50 billion dollars of reserves, an enormous multiplier. These promises are backed by private assets. If lenders want to be paid, each issuer plans to sell assets such as mortgage-backed securities or commercial paper to raise the needed reserves. But asset values can fall, and issuers can then fail. Worse, when people fear failures, they demand payment at the same time. The issuers cannot collectively fulfill all their promises to deliver reserves, and the financial system fails in a systemic run. Interest-paying inside money, backed by assets such as mortgage-backed securities, is prone to the same systemic run as non-interest-paying banknotes, backed by loans, were in the 19th century.

Now, we have a more elastic money supply than the pre-Fed gold standard era, so the sudden demand for reserves does not run up against a completely fixed supply. The Fed bought outright or lent against many of the dodgy assets that banks and shadow-banks were trying to sell, vastly increasing the supply of reserves. But there are limits to what the Fed can do. In particular, the Fed cannot buy or lend against many assets. So, we still had a classic systemic run; a failure of fractional-reserve shadow-banking.

But suppose that people and businesses wanting interest-paying electronic money invested in accounts fully backed by interest-paying reserves, in interest-paying reserves directly, or in money-market funds invested, as reserves are, in short-term treasuries. We would then have fully outside, run-proof, interest-paying money. As the Federal Government displaced private banknote issue in the 19th and early 20th century, so it can displace private interest-paying money in the 21st, with similar benefits to financial stability. Backing short-term money-like debt by the present value of future government surpluses is arguably better than backing such debt by the present value of loans. It's not perfect, as sovereigns inflate and default as well, but at least for a country like the U.S. it is arguably a lot more stable.

This is modern "narrow banking." Cochrane (2014) explains its possibility in detail, and anticipates many objections. The most obvious: that widespread narrow banking will starve the economy of funds for investment; that all those dollars invested in interest-paying reserves will not be available to build houses. This criticism is already made of the Fed's large balance sheet.

It's wrong. This argument forgets that reserves are Federal reserve liabilities, backed by assets. The Fed buys a dollar of treasury or agency debt for every dollar of reserves it issues. With a large balance sheet, the private sector on net holds no more or less government debt, including reserves, and no less private debt, than it does with a small balance sheet. It holds the same treasuries and mortgage-backed securities, through the Fed, that it held before. All the Fed does, really, is to enhance the liquidity of a *given* stock of Treasury and Agency debt.

Narrow banking is a long way away. But the first step, before we get grandiose and think about *forcing* narrow banking, regulating financial institutions away from the temptation to take some risk of a run in return for higher interest rates, we can at least *allow* narrow banking, and let government-provided interest-bearing money compete and contribute to financial stability.

A Federal Reserve with $2.8 Trillion of interest-paying excess reserves outstanding is a financial system with $2.8 trillion of completely run-proof narrow-banking deposits, a great advance in financial stability!

2.3 Other voices

Kashyap and Stein (2012) advocate something similar: using interest on reserves to raise and lower interest rates in order to "manage the inflation-output tradeoff," while using balance sheet and reserve requirement control to "regulate the externalities created by socially excessive short-term debt issuance on the part of financial intermediaries." Similarly, Keister, Martin, and McAndrews (2008) advocate interest on reserves so the Fed can expand the balance sheet to buy up lots of assets in a crisis.

Rather than actively manipulate the externalities posed by short-term debt or focus on crisis firefighting, I regard paying full interest on reserves as the first step to eliminating those externalities by eliminating any substantial short-term debt issuance. So, we both want to emphasize financial stability, but I think it comes from narrowing the spread between reserves and treasuries with a large balance sheet, and they think it comes from widening that spread, controlling quantities, and a controlled balance sheet.

Goodfriend (2002, 2011) advocates interest on reserves, stating the optimal quantity points eloquently. (2011, p. 4, bottom.) He writes additionally that interest on reserves "frees monetary policy to fund credit policy independently of interest rate policy." In other words, interest on reserves allows the Fed can buy a trillion dollars of mortgage backed securities (credit policy) without driving interest rates to zero (interest rate policy).

In a series of thoughtful speeches, Charles Plosser (2009,2010,2012,2013) argues that tightening should be accompanied by selling off the balance sheet and returning to a pre-2007 configuration with small reserves that do not pay interest. In Plosser (2010), for example, "The Fed will need to shrink the size of its balance sheet toward pre-crisis levels and return its composition to all Treasuries."

His argument is however primarily about the political-economy dangers of a large balance sheet, not economic dangers. For example, Plosser (2010) writes (p.8) "the duration of the portfolio is now exposed to a great deal more interest rate risk." That is true, but the Fed's loss is the Treasury's gain. The consolidated government budget constraint is unaffected.

Plosser is more concerned about credit risk on the Fed's balance sheet. Plosser (2010, p.8) writes: "the composition of the portfolio has changed for the explicit purpose of supporting a particular sector of the economy – housing – which breaks entirely new ground. The public and market participants may believe that the Fed can and will use its purchases to pursue other sorts of credit policies than has been its practice in the past." In Plosser's view, these are properly operations of the Treasury. This bug is exactly Goodfriend's feature – interest on reserves allows the Fed to buy credit risk to fight crises.

However, Plosser also writes (2010 p. 10) that "paying IOR ties together the central bank's balance sheet and the government's budget constraint, since the interest is financed by government revenues," and "If the balance sheet is perceived as a "slack" variable for policymakers, someone will want to put it to use." (2010, p. 11). Plosser basically argues here that a large balance sheet amounts to monetization of government debt.

Here I disagree. Interest on reserves is not financed by government revenues. Interest on $2.6 trillion of reserves is paid by the interest on $2.6 trillion of Treasuries. If the Fed were to sell off the balance sheet, the Treasury would pay the same interest to the new owners of the debt. The only potential loss to the government is the interest that will now be paid on the roughly $80 billion of required reserves, couch change (at 5%, $400 million) in the Federal budget.

The Fed cannot maintain a large balance sheet without paying interest. Thus, the option of maintaining a $2.6 trillion balance sheet, not paying interest on reserves, and rebating the interest on the corresponding Treasury assets, without hyperinflation, simply does not exist. We cannot count that as a "cost."

With interest on reserves, the balance sheet is *economically,* if not politically, meaningless, so it cannot be a temptation or a slack variable. Plosser's view contradicts my assertion that with full interest on reserves, reserves and treasuries are perfect substitutes.

Plosser does not argue that the Fed *cannot* raise interest rates without shrinking the balance sheet, or that the government as a whole will lose control of the price level if it does so, which are the central economic points of my argument.

However, his warning is important. Reserves are *only* non-inflationary and they are *only* immune from temptations, if they pay the same interest as short-term Treasuries. So, I think these concerns in the end emphasize the importance of always paying full market interest on reserves. If the Fed adds discretion to pay less than market interest on reserves, and to use the spread between reserves and treasuries as a second policy variable, as Kashyap and Stein (2012) suggest, all of Plosser's warnings take hold.

There are many, larger, political economy issues involved with a large balance sheet and interest-paying reserves, which I do not mean to diminish, yet will not treat in any great detail. Let's get the economics straight first.

2.4 Fed Motives

Neither optimum quantity of money nor financial stability are, as far as I can tell, even among the reasons for the Fed's decisions, nor does anyone at the Fed share my long-run vision. The Fed wants to raise interest rates without selling off its huge balance sheet, both for fear that those sales would upset bond markets, and because the Fed would then have to recognize mark-to-market losses on its portfolio of long-term securities. Such losses are not irrelevant from an economic or monetary policy point of view. But they might be politically embarrassing.

Actions taken for different reasons can have good consequences anyway. But the good consequences can disappear if they are not eventually appreciated. Most of all, the Fed does not seem committed to maintaining a large balance sheet, and is likely to let its longer term investments mature and the balance sheet slowly melt away. Many at the Fed argue for a smaller balance sheet, though mostly on political rather than monetary economic reasons. The Fed is also tempted to use the interest on reserves as a second policy tool, and try to manipulate the spread between interest on reserves and treasuries, as well as to manipulate the spread between interest offered on reserves to non-banks. These actions would revive shadow-banking and undo all the financial stability benefits. Since setting up shadow-banking structures and modifying cash management habits takes time, a firm commitment to keep interest on reserves equal to Treasury rates forever, and a large balance sheet, are important for the optimal quantity of money and financial stability goals. A Fed that wandered in to these desirable policies in pursuit of other goals will not have that commitment.

3 Inflation and interest rate targets in a frictionless model

I start with the simplest possible model, to answer the most fundamental questions: Can the Fed control nominal interest rates, and how will inflation be determined in the interest on reserves regime?

I start with a completely frictionless model – neither monetary frictions nor pricing frictions. The absence of monetary frictions is crucial to any analysis of this question. The absence of pricing frictions lets us start with a simple benchmark, and gives some sense of standard "long run neutrality" predictions. I add pricing frictions below.

3.1 Valuation formula

I base this analysis on the valuation formula for government debt, which states that the real value of nominal debt equals the present value of the primary surpluses that will pay off that debt,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{1}$$

Here, B_{t-1} is the nominal value of government debt outstanding at the beginning of time t, P_t is the price level, $\beta = 1/(1+r)$ is a constant real interest rate, and s_t are real primary surpluses. The quantity B_{t-1} includes interest-paying reserves, and I ignore cash.

Formula (1) essentially says that money is backed by the fact that the government accepts it for tax payments. There are two theories of the value of money: Money can be inherently worthless but gain value from its use in exchange, or money can have value because it is backed; because it promises conversion into something real. With full interest on reserves, a big balance sheet, and interest-paying private money, money loses any scarcity value in exchange. So we need to analyze the value of money in terms of its backing.

In fact, the fiscal backing expressed by equation (1) really is the *only* available theory for analyzing the interest-on-reserves regime. When monetary frictions disappear and all theories that assume away fiscal backing fail to determine the price level or inflation rate. I review this point extensively below, and I address the most common misunderstandings and objections to the use of equation (1). I also exhibit a complete general-equilibrium model to verify that simply using (1) does not lead one to error. But it's better to get on with a theory that does work first, in its simplest form, and outline why standard theories don't work, and address complaints later. (Stein 2012 also analyzes the interest on reserves regime using the fiscal theory for price determination.)

The fiscal theory of the price level represented by (1) has a long tradition, starting with Adam Smith, who wrote that paper currency might be given value if a Prince accepted that money for taxes. The modern theory has its roots in Sargent and Wallace (1981) and then pure statements and elaboration in Leeper (1991), Woodford 1995, Cochrane (1998, 2001, 2005, 2011b) and Sims (2001, 2005). Sims (2013) is an excellent treatment covering issues similar to those covered here.

3.2 Monetary policy and inflation in the simple model

Examine the expected and unexpected components of (1),

$$\frac{B_{t-1}}{P_{t-1}} (E_t - E_{t-1}) \left(\frac{P_{t-1}}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{2}$$

$$\frac{B_{t-1}}{P_{t-1}} E_{t-1} \left(\frac{P_{t-1}}{P_t} \right) = E_{t-1} \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{3}$$

Equation (2) shows us that *unexpected inflation is determined entirely by fiscal policy*, where by "fiscal policy" I mean changes in current and expected surpluses $\{s_t\}$. The face value of debt B_{t-1} is known ahead of time, so the price level must adjust if there is an unexpected shock to the present value of surpluses.

By contrast, Equation (3) shows that *the government can entirely determine expected inflation by nominal bond sales B_{t-1}, even with no change at all in fiscal policy, $\{E_{t-1}s_{t+j}\}$.* If we think of "monetary policy" as manipulating government debt without any change in taxes or spending – surpluses – then "monetary policy" can control expected inflation in this completely frictionless model.

To clarify the effect, write (3) as

$$\frac{B_{t-1}Q_{t-1}}{P_{t-1}} = E_{t-1}\sum_{j=0}^{\infty}\beta^{j+1}s_t, \tag{4}$$

where Q_{t-1} is the one-period bond price,

$$Q_{t-1} = \beta E_{t-1}\left(\frac{P_{t-1}}{P_t}\right). \tag{5}$$

Now, think about the government's decision to sell additional debt B_{t-1} at time $t-1$. P_{t-1} is already determined by (1) at time $t-1$, independently of B_{t-1}. Therefore the real value that the government raises by debt sales $B_{t-1}Q_{t-1}$ is fixed by the time $t-1$ present value of real surpluses. Thus, if the government sells more B_{t-1}, it faces a unit-elastic demand curve; the nominal bond price Q_{t-1} falls one for one, because inflation $E_{t-1}(P_{t-1}/P_t)$ rises one for one. This is just like a share split or a currency revaluation. Like those examples, it shows that the government has complete power over units in this frictionless model, which means it can control expected inflation without changing anything real.

Furthermore, writing (4) as

$$\frac{B_{t-1}}{P_{t-1}}\frac{1}{1+i_{t-1}} = E_{t-1}\sum_{j=0}^{\infty}\beta^{j+1}s_{t+j} \tag{6}$$

we can interpret a government decision of the quantity of debt to sell B_{t-1} to a much more realistic-sounding interest rate target. To set an interest rate target, the government auctions bonds – it says "the nominal interest rate is 5%, and we are selling nominal bonds at 1/0.95 dollars per face value. We will sell any amount demanded at that price." Equation (6) then is a simple reading of private demand – if the government targets nominal interest rates at a level i, that equation tells us how many bonds B_{t-1} will be demanded.

In sum, then,

- *In this frictionless model, the government can set a nominal interest rate target; it can set that target without any adjustments to fiscal policy $\{E_t s_{t+j}\}$, and by setting nominal interest rates, the government can control expected inflation.*

The combination (3) and (2) address classic issues in monetary economics. The Fisher relationship $i_{t-1} = r + E_{t-1}\pi_t$ already says that by controlling nominal interest rates, the government can control expected inflation. But it is not clear by that simple statement just *how* the government controls nominal interest rates in a frictionless world – rationing non-interest-paying reserves doesn't work. More importtntly, targeting nominal interest rates left open the Sargent-Wallace (1981) indeterminacy result, that unexpected inflation $\pi_t - E_{t-1}\pi_t$ could be anything. Now we see that the government *can* set the nominal interest rate, and the action it must do to achieve that result: auction nominal bonds with fixed surpluses. We also see that

- *Recognizing the fiscal backing of nominal debt in the government valuation equation solves the indeterminacy problem of interest rate targets.*

Ex-post inflation and deflation are pinned down by the valuation of government debt. Even a completely fixed interest rate target need not lead to inflation instability or indeterminacy. A Taylor-type rule reacting to inflation is not needed. (Woodford 200x makes this point in analyzing pre-accord monetary policy.)

3.3 Mapping the simple model to our world

Now, let us step back a bit to apply this model to monetary policy and current institutions a bit better. It's pretty straightforward to think of $\{s_t\}$ choices as "fiscal policy."

In this model, there is no difference at all between interest-paying reserves and Treasury debt held directly by the public. The symbol B_{t-1} refers to the sum of the two quantities. So without further frictions, "monetary policy" of open-market purchases of one-period debt in exchange for interest-paying reserves has no effect at all.

But if we understand "monetary policy" as "changes in the quantity and structure of government debt with no change in taxes or spending," then monetary policy is in fact quite effective here. This "monetary policy" *can* set nominal interest rates, and by doing so *can* control expected inflation. (It's not so obvious. Fama 2013 challenges the first assertion empirically.)

And this is a good definition of monetary policy. The heart of central banking is that central banks are forbidden to undertake direct fiscal actions. For example, despite many commenters' desire for helicopter drops, central banks may not undertake them. Helicopter drops, writing checks to voters, are fiscal policy.

Controlling interest rates.

Can the Fed even control interest rates? If so, how? If you said in the past, in response to such questions, "Sure, the Fed controls interest rates. It rations the supply of money, and then we work down the money demand curve," that reply is *completely* irrelevant now. We remain in the region where money demand is undefined, because we are satiated in liquidity and money (reserves) and bonds are perfect substitutes.

New-Keynesian models such as Woodford (2005) specify interest rate control with no money. But as Woodford makes clear, such models consider a limiting case of the standard mechanism. Woodford (2005) advocates the sensible limit that reserves ($50 billion) are so small as to be effectively zero for seigniorage and budget constraint purposes. But in Woodford's limit the supply of reserves is effectively zero as well. Woodford imagines that interest rate control amounts to rationing the very small remaining supply of reserves.

In this model, though the government as a whole can control the nominal interest rate, the government as a whole cannot separately control the nominal interest rate and the quantity of nominal debt. To set the nominal rate here, the government must sell whatever amount of nominal debt B_{t-1} the market demands at that rate. (Or, it must use (6) to successfully forecast that demand.) It cannot auction a fixed nominal quantity.

However, if this regime is successful, the Treasury is not likely to notice, and no unusual quantities of debt will be sold. If the Fed announces an interest rate, Treasury rates rise by arbitrage, then the Treasury will sell exactly the amount of nominal debt needed to finance the deficit.

The question then is whether the *Fed* can control nominal rates, and can the Fed separately control interest rates and the size of its balance sheet?

The Fed certainly can control the rate it pays on reserves. The question is whether and how those rates will spread to other rates.

Now, if the Fed simply announced a higher interest rate on reserves *and* an unlimited quantity, "we pay 5% on reserves, come and get it. Give us your treasuries, we will create reserves and pay 5% interest," then clearly the interest rate on treasuries must rise to 5% by arbitrage. But then the Fed would potentially loses control of the balance sheet. A "tightening" might well imply a large *increase* in the balance sheet, opposite the usual sign.

But our Federal Reserve plans to pay interest on reserves *and* to control the size of the balance sheet, by determining the quantity of bonds it will buy and sell at market prices. The question is whether, in this arrangement, a higher interest rate on reserves will affect treasury and other rates.

Arbitrage relationships should allow the Fed to control all rates. If the Fed pays 5% on reserves, banks should compete to attract depositors, and end up raising deposit rates to 5% minus costs. Depositors should then try to sell Treasuries to get bank deposits until Treasuries rise to that level. Banks may also try to dump treasuries to hold more reserves. They collectively cannot do this, but their efforts to sell will help to drive down Treasury prices.

But it's not clear how strong these arbitrage relationships are. Just paying your nanny $50 will not, by arbitrage, raise low-skill wages to $50 tomorrow morning. The Fed's introduction of the repo program, essentially allowing non-bank financial companies to transfer bank deposits directly to reserves, is pretty much precisely aimed at the Fed being unsure how strong this competition and arbitrage mechanism is. If banks won't pass through the interest on reserves quickly enough, the Fed will do it directly. (See Singh 2014 for a good overview of financial "plumbing" and limits to arbitrage under IOR.)

In sum, the completely frictionless model says that yes, the government can set the nominal interest rate, and the Fed can be the agency of the government that does it. Furthermore, in the frictionless model, the size of the balance sheet is indeterminate, so the Fed can separately control the balance sheet and the nominal interest rate. However that statement relies on perfect competition and arbitrage relationships in banking. In fact, the Fed may find it has to *expand* the balance sheet when trying to raise rates.

Expectations and the separation between Treasury and Fed.

The expected surplus terms in all these equations suggests a reason for the strong institutional separation between Treasury and Fed. When the Treasury sells more debt, it *wants* to raise more real revenue, and it does not want to cause inflation or cause an adverse move in interest rates. The Treasury wants to communicate a simultaneous rise in promised real surpluses $\{E_{t-1}s_{t+j}\}$ with the new debt sale precisely to avoid these effects.

By contrast, the "Fed" wants to communicate the opposite expectations: To control nominal rates, it wants to communicate that changes in debt have no implications about future surpluses or changes in government revenue.

Isolating the debt sales in two distinct branches of the government, one with no legal authority to do anything fiscal, is a great way to communicate different expectations of future surpluses of otherwise identical debt sales. In the same way, corporations market share splits – fully-diluting increases in shares outstanding with no changes in earnings – and public offerings – increases in shares outstanding that are intended to fully correspond to changes in earnings with no dilutions–

in ways that convey the right expectations. In the first case, the corporation wants to change prices only, and in the second case it wants to raise money with as little price impact as possible. Governments market currency reforms or unions – fully-diluting changes in nominal debt with no change in future real surpluses, designed to affect nominal prices and raise no revenue – very differently from debt sales – changes in nominal debt with one for one changes in promised real surpluses, designed to raise revenue with no change in nominal prices – in ways designed to convey the desired implicit promises about surpluses. The increase in debt B_{t-1} is the same in all cases. As we think about better institutional design for monetary policy – which we should really call coordinated monetary-fiscal policy – communicating the intended promises about future surpluses is a central issue.

3.4 Long-term debt and quantitative easing

The discussion over quantitative easing has focused on various "frictions" or "segmentations," in the face of the Modigliani Miller theorem which says that the term structure of interest rates is invariant to the issued maturity structure of debt. However that theorem refers to real effects. Changes in the maturity structure of government debt relative to changes in the maturity structure of future surpluses, can affect the path of nominal inflation.

So far, I assumed that the government only issues one-period nominal debt. The U.S. maturity structure is in fact pretty short, with most debt rolling over in less than two years. So, we can apply these simple equations as a first approximation if we think of the "period" as at least two years. But thinking about long-term debt quite substantially changes the possibilities posed even by this very simple frictionless model. Cochrane (2001) undertakes a deeper analysis. I present a simple example here.

Consider a three-period model with $r = 0$. At time $t = 0$ the government issues one and two year debt, B_0^1, B_0^2. It will retire this debt with surpluses s_1, s_2. At time 1, the government sells or repurchases some additional $t = 2$ debt without changing current or promised surpluses. This will be our "quantitative easing." Denote the amount of time $t = 2$ debt outstanding at the end of time one, after the purchase and sale, B_1^2, so the purchase or sale is in the quantity $B_1^2 - B_0^2$.

The flow equation money in = money out for time $t = 2$ is simply

$$B_1^2 = P_2 s_2, \tag{7}$$

which tells us what P_2 will be. The nominal bond price Q_1^2 at time 1 for bonds that come due at time 2 is

$$Q_1^2 = P_1 E_1 \left(\frac{1}{P_2} \right) = \frac{P_1}{B_1^2} E_1 \left(s_2 \right).$$

The flow equation at time 1 is

$$B_0^1 = P_1 s_1 + Q_1^2 \left(B_1^2 - B_0^2 \right)$$

and substituting Q_1^2,

$$\frac{B_0^1}{P_1} = s_1 + \frac{B_1^2 - B_0^2}{B_1^2} E_1(s_2). \tag{8}$$

Together, (8) and (7) tell us what the equilibrium price level will be at time 1 and 2, as a function of debt sold and surpluses.

Again, we take expected values,

$$E_0\left(\frac{1}{P_2}\right) = Q_0^2 = E_0\left(\frac{s_2}{B_1^2}\right) = E_0\left[\frac{s_2}{B_0^2 + \left(B_1^2 - B_0^2\right)}\right]$$
$$E_0\left(\frac{1}{P_1}\right) = Q_0^1 = \frac{E_0\left(s_1\right)}{B_0^1} + \frac{1}{B_0^1}E_0\left[\left(\frac{B_1^2 - B_0^2}{B_1^2}\right)s_2\right]$$

Fixing the surpluses s_1 and s_2, the government can achieve whatever values on the left hand side it desires by the choice of debt $\{B\}$.

- *The maturity structure of debt, together with expected future purchases and sales, can control the time path of expected inflation and the nominal term structure of interest rates.*

Expected future sales and purchases are not even needed here. If there are no time-1 sales and $B_1^2 = B_0^2$, then the maturity structure at time 0 simply sets the time path of inflation and the nominal term structure of interest rates.

$$E_0\left(\frac{1}{P_2}\right) = Q_0^2 = \frac{E_0\left(s_2\right)}{B_0^2}$$
$$E_0\left(\frac{1}{P_1}\right) = Q_0^1 = \frac{E_0\left(s_1\right)}{B_0^1}$$

So, if we again somewhat expansively define "monetary policy" to be control of debt with no control of surpluses, then monetary policy can, in fact, fully control the nominal term structure of interest rates.

As before, the Fed could just as easily directly target the long rates, and let the quantities follow. The Fed could target the entire term structure. (Why it does not do so is a bit of a puzzle to me. If the Fed wants the 10 year rate to be 50 bp lower, why does it just not say "we buy and sell Treasuries at a 2.0% ten year yield. Come and get them." If the Fed can control long term rates via QE, why are we fighting so much about its ability to do so, 15 bp announcement effects that vanish when the Fed actually starts buying bonds and so on.)

Taking unexpected values, we obtain again at $t = 2$ that fiscal policy fully determines the unexpected time 2 price level. But at time 1, we find

$$B_0^1\left(E_1 - E_0\right)\left(\frac{1}{P_1}\right) = \left(E_1 - E_0\right)s_1 + \left(E_1 - E_0\right)\left\{\frac{B_1^2 - B_0^2}{B_1^2}s_2\right\}. \tag{9}$$

Now, equation (9) offers an exciting new opportunity: By unexpectedly selling more time-2 debt, the government dilutes existing claims to time-2 surpluses. This action raises revenue and that revenue can be used to increase the payoff to period 1 bondholders, lowering inflation at time 1.

There is a catch however: Selling additional long-term debt raises inflation at time 2,

$$\left(E_1 - E_0\right)\left(\frac{B_1^2}{P_2}\right) = \left(E_1 - E_0\right)E(s_2).$$

In sum,

- *"Monetary policy" – a change in the maturity structure of government debt with no change in fiscal stance – can affect unexpected inflation in the presence of long-term debt. It does so by rearranging the path of inflation, delaying inflation or bringing inflation forward.*

We may read current "quantitative easing" as the opposite policy. By unexpectedly (relative to when the debt was sold) *buying* long-term debt, the Fed is trying to "stimulate," i.e. to *increase* inflation today, in exchange for less inflation later on.

This result also points to a stabilization role for quantitative easing. Extra debt sales $B_1^2 - B_0^2$ in (9) can be used to offset surplus shocks $(E_1 - E_0)\, s_1$ to insulate the price level $(E_1 - E_0)\,(1/P_1) = 0$ from shocks. Active management of the debt – or interest rate or price-level targets that imply fluctuating quantities of government debt of various maturities – emerge as appropriate policies to stabilize the price level in the face of shocks to surpluses, not just to create inflation or disinflation.

The general case of these formulas is quite complex (Cochrane 2001), suggesting a very interesting job of maturity management for governments that want to stabilize inflation and, with pricing frictions, output.

4 Real rates and sticky prices

The simple frictionless models got us quite far in describing the potential and limits for monetary policy to affect inflation, but leave out any effects on output and real interest rates.

Again, it is not at all obvious that monetary policy in an interest on reserves regime will have the traditional effects. The mechanism for interest rate increase, and the mechanism for transmission to the price level are utterly different. So we need to explore potential real rate and output effects.

4.1 A simple sticky-price model

I maintain a model without *monetary* frictions – reserves still are perfect substitutes for overnight Treasury debt, and people hold no non-interest-bearing money overnight. But I add *pricing* frictions. I add pricing frictions in the simplest possible way by forcing producers to state prices ahead of time, and then sell whatever quantity people want at those prices. The point here is not to create an empirically successful model of pricing dynamics, but to explore the basic signs and mechanisms by which monetary policy might work in the absence of monetary frictions.

For monetary policy to affect real output and interest rates at all, we need some friction. While one can have many standard objections to sticky-price models, I think it is better to innovate along as few dimensions as possible. So I embed a simple standard sticky-price mechanism into a cash-free model with fiscal price level determination.

I set the model out in detail in the Appendix. It is a simplification of Galí (1999). Households consume a CES composite good of many varieties. Each household i uses labor n_{it} to produce one variety y_{it} with production function $y_{it} = An_{it}$, and must set its price one period in advance.

The general conclusions of the formal model are as one would guess. The government debt valuation equation remains, unsurprisingly,

$$u'(c_t)\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j u'(c_{t+j}) s_{t+j} \tag{10}$$

where now P_t is determined at time $t-1$. When there is an unexpected surplus shock, P_t cannot now adjust. Therefore, either $u'(c_t)$ or $E_t u'(c_{t+j})$ must adjust, and real interest rates and output will be affected. The point of the model is to determine which of these possibilities holds, and how P_t is determined.

Since prices are only sticky for one period, marginal utility can only be expected to diverge from the frictionless value for one period. With $\bar{c}$ equal to the frictionless level of consumption and output, we have

$$E_{t-1}\left[u'(c_t)\right] = u'(\bar{c}).$$

This also means that the real interest rate

$$1 + r_t = 1/E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\right] = \frac{u'(c_t)}{\beta u'(\bar{c})} \tag{11}$$

can only diverge from its expected value for one period, $1 + E_{t-1}(r_t) = 1 + \bar{r} = 1/\beta$.

These one-period deviations mean that we can write the basic valuation formula as[2]

$$\frac{u'(c_t)}{u'(\bar{c})}\frac{B_{t-1}}{P_t} = \frac{u'(c_t)}{u'(\bar{c})}s_t + E_t\sum_{j=1}^{\infty}\beta^j\left[s_{t+j} + rp\right]. \tag{12}$$

where rp is a risk premium generated by the covariance of surplus shocks with marginal utility,

$$rp = cov_{t+j-1}\left[\frac{u'(c_{t+j})}{u'(\bar{c})}, s_{t+j}\right]$$

I specialize to a constant rp as I will not analyze changes in that risk premium.

4.2 Surplus shocks

Again, we examine expected and unexpected components of (12), as in (2) and (3). The unexpected component reads

$$\frac{B_{t-1}}{P_t}\left(E_t - E_{t-1}\right)\frac{u'(c_t)}{u'(\bar{c})} = \left(E_t - E_{t-1}\right)\left[\frac{u'(c_t)}{u'(\bar{c})}s_t\right] + \left(E_t - E_{t-1}\right)\sum_{j=1}^{\infty}\beta^j s_{t+j}. \tag{13}$$

Think of a shock to future surpluses $\left(E_t - E_{t-1}\right)\sum_{j=1}^{\infty}\beta^j s_{t+j}$ that does not affect s_t. This shock produced a one time jump in the price level in the frictionless model. But now, P_t is no longer inside the $E_t - E_{t-1}$. Prices cannot adjust to the surplus shock as they did in (2). *All the adjustment to the surplus shock now comes by adjustment to $u'(c_t)$ and hence to the real interest rate, since the price level cannot adjust.*

[2] Algebra: For $j \geq 1$

$$\begin{aligned} E_t\left[u'(c_{t+j})s_{t+j}\right] &= E_t\left\{E_{t+j-1}\left[u'(c_{t+j})s_{t+j}\right]\right\} \\ &= E_t\left\{u'(\bar{c})E_{t+j-1}\left[s_{t+j}\right] + cov_{t+j-1}\left[u'(c_{t+j}), s_{t+j}\right]\right\} \\ &= u'(\bar{c})E_t\left\{E_{t+j-1}\left[s_{t+j}\right] + cov_{t+j-1}\left[\frac{u'(c_{t+j})}{u'(\bar{c})}, s_{t+j}\right]\right\} \\ &= u'(\bar{c})\left[E_t\left[s_{t+j}\right] + rp\right]. \end{aligned}$$

A negative (inflationary) shock to expected future surpluses $(E_t - E_{t-1}) \sum_{j=1}^{\infty} \beta^j s_{t+j}$ lowers $u'(c_t)$, i.e. raises c_t, so *an inflationary fiscal shock produces a temporary output expansion. The inflationary shock also lowers the real interest rate,*

$$1 + r_t = \frac{1}{q_t} = \frac{1}{\beta} \frac{u'(c_t)}{u'(\bar{c})},$$

where q_t denotes the real bond price.

These are the "usual" signs – inflation is preceded by higher output and lower real interest rates. In this model, that course of events is out of the Fed's control. But surplus expectations are not directly observable. Observers accustomed to thinking the Fed controls real interest rates might well think that the Fed lowered real rates, induced the output expansion, and later the inflation; they might think that the ex-post observed fall in surpluses $\sum_{j=1}^{\infty} \beta^j s_{t+j}$ represented a "Ricardian" reaction by the Treasury. Little in the data could falsify this impression.

To get some insight into how the real rate change absorbs the surplus shock in place of inflation, write (13) in the form

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{(E_t - E_{t-1})(1 + r_t)} (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+1+j}. \tag{14}$$

(Remember we simplified by assuming no change in s_t.) The right-most term is the present value of surpluses from date $t+1$ on, and is also equal to the real value of debt that the government sells in the afternoon of date t. Now, with a constant real interest rate, the real value of debt sold in the evening declined when expected future surpluses declined. To match that decline, the real value of debt coming due in the morning declined, as P_t on the left hand side of (14) rose. But that mechanism is now absent. The real value of debt coming due in the morning B_{t-1}/P_t cannot decline. How can the real value of debt paid off in the morning stay the same, in the face of a decline in expected surpluses? The answer is that the real interest rate also declines, the bond price rises, so the lower expected surpluses now have the same real value.

The same mechanism in nominal terms. We can imagine the government printing up money B_{t-1} to pay off debt at the beginning of period t, money which must be soaked up by current surpluses $P_t s_t$ or by bond sales by the end of time t. (I say "imagine" because the exchange of nominal debt for money is not necessary; this economy works the same way if transactions and tax payments are mediated directly with maturing nominal debt. But money makes a better story.) With flexible prices and a constant real interest rate, the real value of surpluses $E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$ was fixed, and at the same nominal price level P_t these would no longer soak up all the dollars. So at that price level, people tried to buy more goods with their excess dollars. They pushed up goods prices until the same real surplus s_t and lower real quantity of debt soaked up the excess nominal dollars brought in by B_{t-1}.

But now prices cannot rise. People still have more newly-printed money in their pockets B_{t-1} than will be soaked up by surpluses s_t and debt sales. What happens? First, they try to buy more goods and services as before. With prices fixed one period in advance, this extra "aggregate demand" now leads to greater output, not higher prices. But the greater output does not soak up any money in aggregate. More money spent by the buyer is received by the seller, and at the end of the day the excess money B_{t-1} relative to its sponges is still there. So, if money holders cannot bid up the price of *goods,* they bid up the price of *bonds* instead. "Asset price inflation," takes the place of goods inflation. The real interest rate decline / real bond price rise continues until the excess cash is now all soaked up by bond sales at an unchanged price level.

Given that real interest rate rise, the output increase is determined by the intertemporal first order condition $1/(1+r_t) = E_t\left[\beta u'(c_t)/u'(c_t)\right]$. In words, extra "aggregate demand" or the extra wealth of government bonds drives a demand for lifetime consumption, not just consumption today. So people only go out and try to buy more today until they are back on the right tradeoff of consumption today vs. consumption in the future.

Though now (I hope) obvious in terms of the model, these are unconventional predictions. Without the model, we might have thought that a decline in expected future surpluses, a decline in the government's ability to service its debt, would lead to an *increase* in the interest rate, and a *reduction* in the value of government debt. Instead, in equilibrium, real interest rates rise and there is no change in the real value of government debt. This observation may help to make sense of many paradoxes in the data, in which economists note bad news about future surpluses, but interest rates decline and government bond prices rise anyway.

4.3 Debt sales and interest rate policy with one-period stickiness

Next consider "monetary policy," debt sales with no change in surplus, deriving from analysis of the expected value of E_{t-1} of (12). Remembering $E_{t-1}\left[u'(c_t)\right] = u'(\bar{c})$ and remembering that P_t is known at time $t-1$, we obtain

$$\frac{B_{t-1}}{P_t} = E_{t-1}\sum_{j=0}^{\infty}\beta^j\left[s_{t+j} + rp\right]. \tag{15}$$

This equation functions much as its flexible price counterpart (3). By varying debt B_{t-1}, the government can control the actual price level at time t, P_t with sticky prices, just as it controlled the expected price level at time t, $E_{t-1}(1/P_t)$, with flexible prices. Again, this action is like a share split or currency reform.

However, this monetary policy cannot affect the real interest rate. The real interest rate r_{t-1} does not enter in to (15), and the choice of B_t does not enter in to the determination in (14) of r_t. Though B_{t-1} is in the same information set as c_{t-1}, changes in B_{t-1} have no effect on c_{t-1}.

As before, the government can set the price rather than the quantity, and follow a nominal interest rate target. The nominal rate and real rate are related by

$$(1+r_{t-1}) = (1+i_{t-1})\frac{P_{t-1}}{P_t}, \tag{16}$$

the difference being that P_t is fixed at $t-1$, not expected.

However, with the real rate r_{t-1} determined by fiscal shocks at time $t-1$, (and, in a fuller model, real shocks), a fixed nominal rate target $i_{t-1} = \bar{\imath}$ will result in price level volatility: If the real rate r_{t-1} rises and the Fed holds the nominal rate i_{t-1} constant, the price level P_t must decline in (16). Hence, *If the Fed wants to reduce price volatility, it should move the nominal rate target one-for-one with the real rate.* This advice has much of the flavor of current advice to move the nominal rate target (Taylor rule intercept) to follow rises and falls in the "natural" rate.

4.4 Interest rate policy with real effects

When prices are sticky for more than one period, however, monetary policy can affect real quantities.

Suppose now that prices must be set k periods in advance. As there are no asset market distortions, the government debt valuation equation remains,

$$u'(c_t)\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j u'(c_{t+j}) s_{t+j}. \tag{17}$$

Now, P_t must be determined at time $t-k$. Marginal utility and the real interest rate can then diverge from the frictionless value for k periods. Now, a decision to change B_{t-1} can affect $E_{t-1}u'(c_t)$, since it can no longer affect P_t.

The general algebra for this case does not yield much intuition, so I present a simple example. Start at a steady state $P_t = \bar{P}$ with interest rate target

$$\frac{1}{1+i} = \beta = \frac{1}{1+\delta}.$$

Suppose that at time $t = 1$ the government unexpectedly raises the interest rate target $1+i$ to $(1+i)\,\Delta$ and leaves it there. Prices P_1, $P_2, ...P_k = \bar{P}$ cannot respond. Prices P_{k+1} and beyond can respond. Consumption c_1, $c_2, ..c_k$ can respond, but $c_{k+1} = \bar{c}$ and beyond cannot respond.

Our job is to solve (17) together with

$$\frac{1}{1+i_t} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{P_t}{P_{t+1}} \tag{18}$$

for $u'(c_1), u'(c_2), ..u'(c_k), B_1, B_2, ..., P_{k+1}, P_{k+2}, ...$given this nominal interest rate path. From consumption, we can find the real rate of interest.

To fill in the fiscal end in the simplest way, suppose that surpluses are $s_t = 0,\ t < k$,and will be constant $s_t = \bar{s}$, $t \geq k$. The presence of surpluses s_t during the price-sticky period leads to small variations in the value of these surpluses $u'(c_t)s_t$ at these dates, which cloud the basic story. Letting the debt be paid off by far in the future surpluses simplifies later algebra. It does not matter that surpluses start just when price stickiness ends; any date after k gives the same results. Denote $\bar{S} = \sum_{j=0}^{\infty} \beta^j \bar{s} = \bar{s}/(1-\beta)$. The steady state implies from (17) that nominal debt $B_{t-1} = \beta^{k-t}\bar{P}\bar{S}$ for $t \leq k$ and $B_{t-1} = \bar{P}\bar{S}$ for $t \geq k$. Denote these values as $\bar{B}_t$.

Table 1 gives the evolution of each variable in this scenario. I present the algebra below.

t:	0	1	2	...	k	k+1	...
$1+i_t$:	$1+\delta$	$(1+\delta)\,\Delta$	$(1+\delta)\,\Delta$	...	$(1+\delta)\,\Delta$	$(1+\delta)\,\Delta$	...
$u'(c_t)/u'(\bar{c})$:	1	1	$1/\Delta$	...	$1/\Delta^{k-1}$	1	...
$1+r_t$:	$1+\delta$	$(1+\delta)\,\Delta$	$(1+\delta)\,\Delta$	...	$(1+\delta)\,/\Delta^{k-1}$	$(1+\delta)$	...
P_t :	$\bar{P}$	$\bar{P}$	$\bar{P}$	...	$\bar{P}$	$\Delta^k\bar{P}$	...
B_t :	$\bar{B}_t$	$\Delta\bar{B}_t$	$\Delta^2\bar{B}_t$	...	$\Delta^k\bar{B}_t$	$\Delta^{k+1}\bar{B}_t$	...

Table 1. Evolution of variables when the government raises the interest rate target from $1+i_t$ to $(i+i_t)\,\Delta$ at time $t=1$.

More clearly, Figure 1 shows the effects of a 1 percentage point rise in interest rates at time 1, when prices are sticky for $k = 4$ periods.

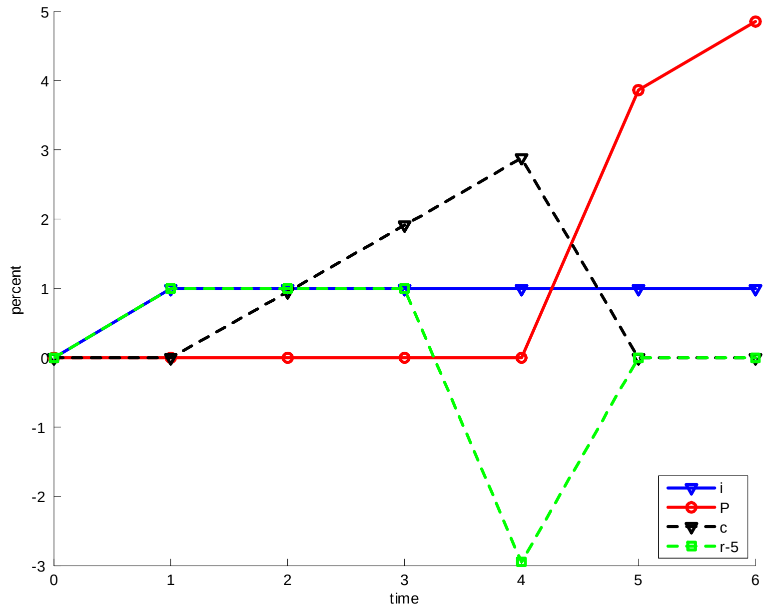


Figure 1: Effects of an interest rate rise in a simple sticky-price model. At time 1, the government unexpectedly raises the nominal insterest rate one percentage point. Prices are fixed four periods in advance. The steady state interest rate is $\delta = 0.05$.

In the model without pricing frictions, this change would just raise inflation to $P_t/\bar{P} = \Delta^{t-1}$ immediately, with no change to consumption or the real interest rate. Now the price level cannot move for four periods. When it is finally free at $t = 5$, it immediately jumps to the frictionless level. All the repressed inflation arrives at once.

During the period of price-stickiness, the real interest rate rises by exactly the rise in the nominal rate. This rise in real rate sets off a boomlet in consumption. Consumption growth rises to match the higher real interest rate. However, at the end of the price stickiness period, consumption will jump back to its frictionless value, prices will jump up to the frictionless value, and there is therefore a period of strong negative real interest rate.

The jumps at the end of the sticky-price period are of course not realistic at all. In a more realistic, Calvo-style model, price stickiness would evaporate gradually. So we should expect a consumption boom with little inflation, then consumption to revert to normal slowly as inflation picks up.

In sum, we see that *in a model with price frictions, but no monetary frictions at all, and relying on fiscal backing to determine the price level, "monetary policy" construed as variation in a nominal interest rate target (or the quantity of debt), with no change in surpluses whatsoever, can induce real interest rate and output dynamics.*

The scenario plotted in Figure 1 does not conform to the usual story told about interest-rate based monetary policy, because there is no period in which the rise in real interest rate lowers the level of current consumption. This monetary policy is expansionary throughout – first consumption rises, then inflation rises. The quantity of debt rises throughout. Consumption and inflation look a lot like you might imagine Friedman (1968) to describe a monetary expansion. Friedman's monetary expansion would have started with a period of lower interest rates. But Friedman's expansion would have produced those rates by working down a money demand curve, and this is a frictionless model where there is no money demand curve.

Thus, though the nature and timing of output and real interest rate effects depend sensitively on the timing of the sticky-price mechanism, and this exercise is simple rather than realistic, this model alerts us to the possibility that perhaps, with interest on reserves and no monetary frictions, the sign will change, and *raising* interest rates is expansionary, not the opposite.

Keep in mind as well that the pure separation between "monetary policy" with no change in surpluses and "fiscal policy" that only changes surpluses is convenient for analysis, but misleading in analyzing actual policy or historical events. Historical events and policy interventions mix monetary and fiscal shocks.

4.4.1 Algebra

To derive the path shown in Table 1 and Figure 1, express equations (17) and (18) at each date, substituting in $\bar{c}, \bar{P}, \bar{B}$, where appropriate. For $t = 1$,

$$u'(c_1)\frac{\beta^{k-1}\bar{P}\bar{S}}{\bar{P}} = \beta^{k-1}u'(\bar{c})\bar{S} \tag{19}$$

$$\beta\frac{1}{\Delta} = \beta\frac{u'(c_2)}{u'(c_1)}\frac{\bar{P}}{\bar{P}}. \tag{20}$$

For $t = 2$,

$$u'(c_2)\frac{B_1}{\bar{P}} = \beta^{k-2}u'(\bar{c})\bar{S} \tag{21}$$

$$\beta\frac{1}{\Delta} = \beta\frac{u'(c_3)}{u'(c_2)}\frac{\bar{P}}{\bar{P}}. \tag{22}$$

For $t = k$,

$$u'(c_k)\frac{B_{k-1}}{\bar{P}} = \beta u'(\bar{c})\bar{S} \tag{23}$$

$$\beta\frac{1}{\Delta} = \beta\frac{u'(\bar{c})}{u'(c_k)}\frac{\bar{P}}{P_{k+1}}. \tag{24}$$

For $t = k+1$,

$$\frac{B_k}{P_{k+1}} = \bar{S} \tag{25}$$

$$\beta\frac{1}{\Delta} = \beta\frac{P_{k+1}}{P_{k+2}}, \tag{26}$$

and similarly for $t = k+2, t = k+3, ...$

Equation (19)tells us right away that consumption at time 1 is not affected.

$$u'(c_1) = u'(\bar{c}).$$

Equation (20) then tells us that time 2 consumption must rise,

$$u'(c_2) = \frac{u'(\bar{c})}{\Delta}.$$

Equation (22) implies

$$u'(c_3) = \frac{1}{\Delta^2} u'(\bar{c})$$

and (21) implies

$$B_1 = \Delta \beta^{k-2} \bar{P} \bar{S} = \Delta \bar{B}_1.$$

Continuing, for $t < k$

$$u'(c_t) = \frac{1}{\Delta^{t-1}} u'(\bar{c})$$

and in particular,

$$u'(c_k) = \frac{1}{\Delta^{k-1}} u'(\bar{c})$$

along with

$$B_{t-1} = \Delta \beta^{k-t} \bar{P} \bar{S} = \Delta \bar{B}_{t-1}.$$

Equations (23)-(24) imply

$$P_{k+1} = \Delta^k \bar{P}$$
$$B_{k-1} = \Delta^{k-1} \beta \bar{P} \bar{S} = \Delta^{k-1} \bar{B}_{k-1}.$$

5 Comparison with a new-Keynesian model, the importance of fiscal anchoring.

A natural reaction at this point is, wait a minute. We have a whole range of models which specify the reaction of the economy to interest-rate policy, with no mention of monetary frictions or fiscal backing: The whole New-Keynesian Taylor-rule DSGE literature, epitomized by Woodford (2005). Why not just reference those and go on to other questions?

In fact, however, this class of models *does* rely heavily on fiscal backing. When you look at them, these models in fact generate inflation predictions by imagining that monetary policy leads to some strong fiscal policy responses. The results of those models depend crucially on the nature of the assumed fiscal response to monetary policy.

This issue is important in practice. Consider again Figure 1. When real interest rates rise, consumption growth must rise. In the conventional new-Keynesian model, however, the level of consumption would be anchored at the right end of the graph, so the rise in consumption *growth* produces a decline in the *level* of consumption at $t = 1$. So the standard new-Keynesian model produces the conventional idea that raising interest rates contracts output, but without money demand. How? The answer turns out to be fiscal policy. The conventional new-Keynesian model pairs the interest rate rise with an assumed shock to the present value of surpluses, which *together* predict a contraction. If I pair the interest rate rise Figure 1 with such a fiscal shock, this model too can produce a contraction. After all, this model *is* pretty much the standard new-Keynesian model, except I use a simpler price-stickiness rule.

The point is *not* that one should evaluate monetary policy holding fiscal policy $\{s_t\}$ constant. That is an artificial, though useful, conceptual exercise. The point is, that one should evaluate monetary and fiscal policy together.

5.1 Fiscal backing in a very simple New-Keynesian model

To make these points, consider the absolutely simplest new-Keynesian model, as presented in Woodford (2005) (and, in detail, in Cochrane 2011): A Fisher equation (first order condition for intertemporal allocation of consumption, in an endowment economy, as above); a Taylor-type rule by which the Fed sets the nominal rate, and a serially correlated monetary policy shock, with which we can evaluate the effects of monetary policy,

$$i_t = r + E_t \pi_{t+1} \tag{27}$$

$$i_t = r + \phi_\pi \pi_t + x_t$$

$$x_t = \rho x_{t-1} + \varepsilon_t. \tag{28}$$

The equilibrium condition for this model is

$$E_t \pi_{t+1} = \phi_\pi \pi_t + x_t.$$

There are multiple equilibria. Any

$$\pi_{t+1} = \phi_\pi \pi_t + x_t + \delta_{t+1};\ E_t(\delta_{t+1}) = 0 \tag{29}$$

is a valid solution.

The New-Keynesian tradition sets $\phi_\pi > 1$. All but one solution now explodes, $\|E_{t+1}\pi_{t+j}\| \to \infty$. Ruling out nominal explosions, one selects the unique locally-bounded solution

$$\pi_t = -\frac{1}{\phi_\pi - \rho} x_t$$

and interest rates thus follow:

$$i_t = -\frac{\rho}{\phi_\pi - \rho} x_t$$

Equivalently, this equilibrium chooses the shock

$$\delta_t = -\frac{\varepsilon_t}{\phi_\pi - \rho}. \tag{30}$$

The new-Keynesian model in this sense affects inflation by inducing the economy to jump to a different equilibrium.

Figure 2 presents the response to a one percentage point monetary tightening, $\varepsilon_1 = 1$, in this simple canonical model[3]. (The plot uses the borderline case $\phi_\pi = 1$ for all solutions. This saves a lot of plots and discussions exploring both the $\phi_\pi > 1$ and $\phi_\pi < 1$ cases. The response functions are a smooth function of ϕ_π, so are visually indistinguishable for ϕ_π slightly above or below one.) The monetary policy shock x_t is positive and slowly declines following the AR(1) pattern. In the lower lines marked "New-Keynesian" plot the response of interest rates and inflation to this shock. Inflation jumps down; the tightening lowers inflation as the standard story says. The actual nominal interest rate also falls, which seems like a strange sort of "tightening." But the actual interest rate falls *less* than ϕ_π times inflation. This represents "tightening" relative to the Taylor

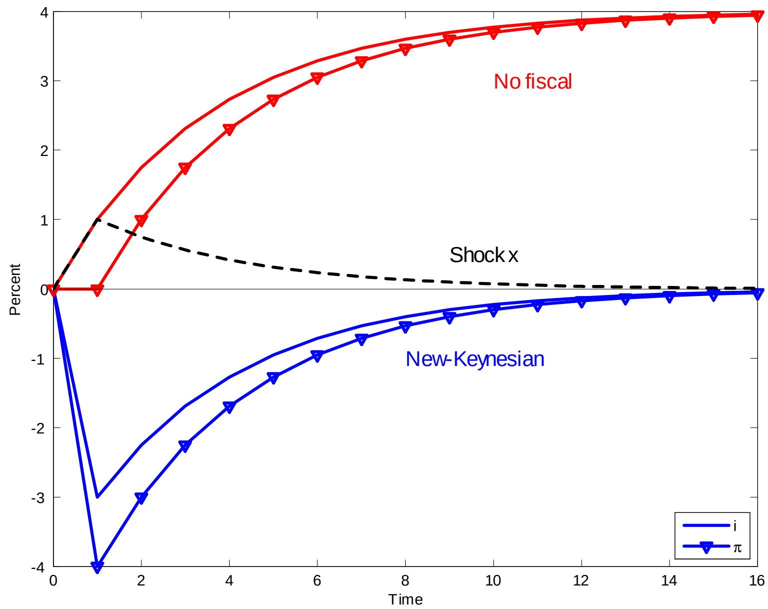


Figure 2: Responses to a monetary tighteing in the standard and fiscally-constrained solutions of a simple new-Keynesian model. $\rho = 0.75, \phi_\pi = 1$.

rule. The dynamics come entirely from the mean-reversion of the shock. A permanent 1% shock leads to an immediate and permanent decline of interest rates and inflation.

The valuation equation for government debt

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{31}$$

is part of this model. It just got brushed in to the footnotes with an assumption that the Treasury will always pass lump sum taxes $\{s_t\}$ to validate whatever solution $\{P_t\}$ emerges. The inflation drop at time $t = 1$ is an *unexpected* drop, as (30) makes clear. As we have seen, with one-period debt the only way to produce an unexpected drop in inflation by (31) is to imagine a change in fiscal policy,

$$\frac{B_{t-1}}{P_{t-1}} (E_t - E_{t-1}) \left(\frac{P_{t-1}}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{32}$$

Thus, to produce the unexpected -4% inflation in response to a monetary policy shock shown in Figure 2, we *must* also think that fiscal policy produces a 4% increase in the net present value of primary surpluses, to validate a 4% increase in the real value of government debt, and that people know this and expect it. The inflation response is thus really a response to two, simultaneous, shocks: an Taylor rule shock and an expected surplus shock. Real interest rates are constant in this model, so the change in present value must come from actual taxing or spending, not changes

[3] The family of response functions are given by

$$\left(\pi_t + \frac{1}{\phi_\pi - \rho} x_t \right) = \phi_\pi^{t-1} \left(\pi_1 + \frac{1}{\phi_\pi - \rho} x_1 \right).$$

in the present value of given surpluses. In the US context, with \$12 billion dollars of outstanding public debt, that means that the Treasury must be expected to come up with about \$500 billion of extra tax increases or spending cuts, in present value terms, to validate the desired inflationary effects of a 1% interest rate rise.

The fiscal coordination is crucial. From the point of view of (31), the *mechanism* by which "monetary policy" produces the downward jump in "aggregate demand" or increased demand for government debt, and thus the *mechanism* by which it produces disinflation, is by inducing this fiscal reaction. Monetary policy is just the carrot in front of the horse, which pulls the cart.

5.2 Same model, no fiscal backing

What if that fiscal backing is not forthcoming? Or, what if people just stop *expecting* it when they see a monetary policy shock?

Equations (32) and (29) allow a nice view of this conundrum: *we can index all the multiple solutions to the new-Keynesian model by their implied fiscal backing.* For example, the case of no fiscal response, $(E_t - E_{t-1})\sum_{j=0}^{\infty}\beta^j s_{t+j} = 0$, that I studied above, is the case $\delta_{t+1} = 0$.

Figure 2 also includes this "fiscal-neutral" solution to the model, plotted in red. This solution is simply computed as $\pi_1 = 0$, $\pi_t = \phi_\pi \pi_{t-1} + x_t$, $x_t = \rho^{t-1}1$. In this solution, inflation does not jump in the period of the shock – that's how we identified the equilibrium choice. This actually sounds pretty reasonable given the data, in which inflation seems pretty sluggish, rather than seeing price level jumps on the same day as FOMC announcements. Then interest rates follow obvious dynamics generated from the shock and inflation.

Now, the fiscal solution gives positive inflation in response to monetary tightening. Isn't this bad –aren't we supposed to see lower inflation in response to monetary tightening, as the new-Keynesian solution showed (granted that measuring "tightening" might be hard given data from that model)? No. This is a purely frictionless model. Real rates are constant, so there is no mechanism for real rates to lower "demand." In a totally frictionless model, all the Fed can do when it raises the nominal rate is to raise expected inflation. So of course raising the nominal rate raises inflation. The mystery here is, how did the new-Keynesian solution produce a downward jump in inflation from a completely frictionless model, with fixed real rate, output, and super-neutrality, yet somehow raising the nominal rate lowers inflation? The answer is now clear: The new-Keynesian solution assumed that the monetary change would *also* be accompanied with an important fiscal tightening, and this fiscal tightening produced the inflation decline.

Monetary and fiscal policy must be coordinated. The new-Keynesian model in fact describes a complex Sargent-Wallace (1981) game of chicken between Treasury and Federal Reserve. With $\phi_\pi > 1$, the Federal Reserve threatens to induce hyperinflationary paths unless the economy jumps to one particular equilibrium, one particular choice of δ_{t+1}. Just why the economy does so has always been a bit of a mystery. Perhaps this threat helps to "coordinate expectations" (Woodford 2005). But once we resurrect equation (32) from the footnotes, a much better answer appears. The game of chicken is not with private-sector expectations. The game of chicken is with the Treasury. Unless the *Treasury* adopts a sufficiently restrictive fiscal policy, which is what actually produces disinflation, the Fed will hyperinflate the economy. There still is the question whether the Fed actually makes a threat to hyperinflate or hyperdeflate to win the game of chicken. But the model really is a game of chicken with the Treasury as much as it is a game of chicken with private sector expectations.

I do not advocate the response function with no fiscal change as the "right" choice. My point is that in our new world without monetary frictions, we ultimately have to anchor the value of money in its fiscal backing. Historical events seem to combine "monetary" – changes in debt – and "fiscal" – changes in surpluses. We only make progress by examining both together.

5.3 The three-equation model

The system (27)-(28) may seem too simple to examine this issue. But the same point holds – all new-Keynesian models imagine that the Fed picks one of multiple solutions by engineering a nominal explosion for all the others; all of those solution choices can also be indexed by the implied fiscal commitment; jumps in inflation imply jumps in fiscal policy; and model predictions are very sensitive to the assumed fiscal backing.

To demonstrate this point, I examine solutions to the standard three-equation new-Keynesian model,

$$y_t = E_t y_{t+1} - \sigma \left(i_t - E_t \pi_{t+1}\right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t$$
$$i_t = \phi_\pi \pi_t + x_{it}$$

$$x_{it} = \rho_i x_{it-1} + \varepsilon_{it}$$

Figure 3 presents the results. (The algebra is in the Appendix.) As one might expect, and similarly to the simple model of Figure 2, the monetary tightening lowers inflation and output. Again, interest rates actually jump down, but less than inflation so this does represent the contemporaneous reaction to tightening. Again, the solution depends on a jump downwards in inflation, which requires a fiscal tightening.

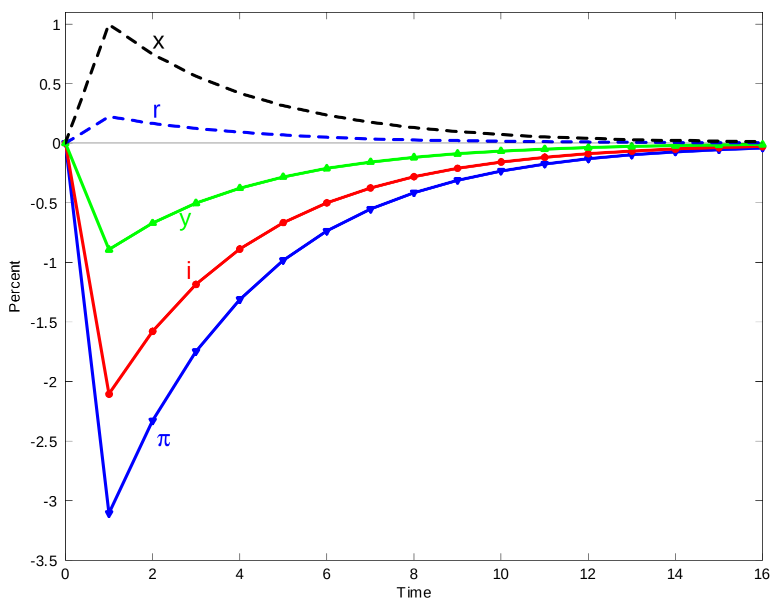


Figure 3: Response of standard new-Keynesian model to a 1% monetary policy shock. $\rho = 0.75$, $\gamma = \sigma = \phi_\pi = 1$.

Figure 4 presents a "fiscal-neutral" solution of the same model. Here again, I just picked the equilibrium in which $(E_t - E_{t-1})\,\pi_t = 0$.

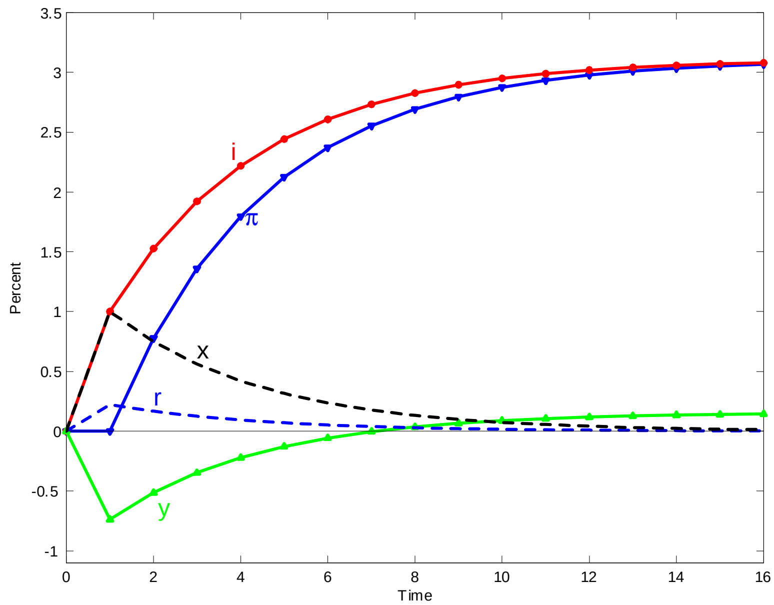


Figure 4: "Fiscal-neutral" response to a 1% monetary policy shock in a new-Keynesian model. I choose the equilibrium with no shock to inflation, $\pi_{t+1} - E_t\pi_{t+1} = 0$.

This change produces radically different inflation and interest-rate responses. Inflation cannot now "jump" down during the period of the shock. The tightening now produces an actual rise in nominal rates. Nominal rates and inflation then chase each other into positive territory, much as they did in Figure 2. Real rates rise, and the real rate and output responses are not that different from the standard new-Keynesian case.

This issue is one that one can settle empirically. In this model, with one-period debt, the contemporaneous response of inflation to any shock measures the fiscal response – the response of $E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$ – to that shock. That measurement is, of course, conditional on the model. In models in which inflation is determined one or more periods in advance, inflation itself cannot move – $(E_t - E_{t-1})\,\pi_t = 0$. But other state variables do jump, and the corresponding jump in fiscal response should be measurable as well.

A disclaimer: properly integrating fiscal backing into models of this sort is more complex than simply adding the frictionless valuation equation, as I have implicitly done here to make a clear illustrative calculation. Two issues are most important. First, interest rate changes (and later, risk premium changes) must be reflected in the value of government debt. The correct measure is $E_t \sum_{j=0}^{\infty} u'(c_{t+j})/u'(c_t)s_{t+j}$, and real interest rates can change the value of government debt without any change (or even with contrary changes) in expected surpluses. In the data, interest rate changes account for large fractions of the change in present value of surpluses. My calculations of responses that produce no inflation shock at all implicitly come with a small rise in actual expected surpluses, to offset the decline in present value occasioned by the small real interest rate rises of those models. As I have emphasized, since historical fiscal and monetary policy typically comes together there is nothing special about no change in surpluses, so this isn't "wrong," and remains a convenient exercise to see how important the fiscal-backing assumption is to standard new-Keynesian model predictions.

Second, the details of asset markets, budget constraints, and the nature of price stickiness, need to be specified explicitly, as I did in the Appendix for the model with prices stuck two periods in advance, along with the maturity structure of government debt and state-contingent changes in that maturity structure. The point here is not to construct a second fully fleshed out model, but to show that the standard new-Keynesian model also stands firmly on fiscal foundations, and that changing those foundations fundamentally changes the model's predictions.

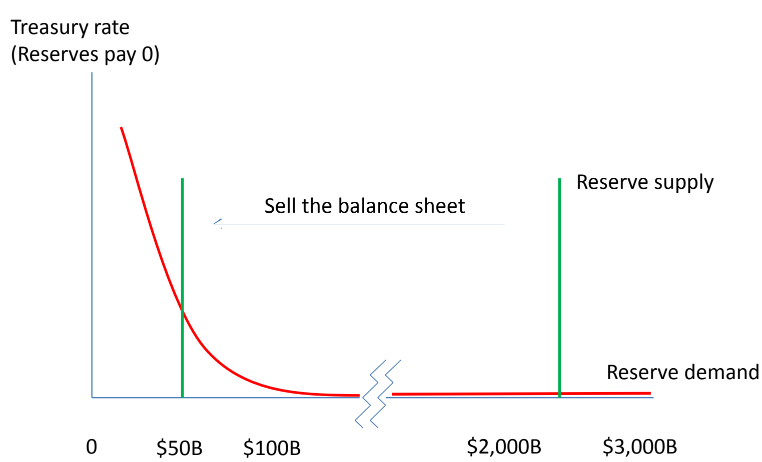
6 Theories

The fiscal backing approach I have taken is controversial. Here, I allay some common theoretical and empirical objections. I also argue that we have no choice. With full interest on reserves and a large balance sheet, without monetary frictions, when we are satiated in liquidity, there simply is no other theory that determines the price level and inflation. It's not this theory or some other – this is the only one we have, and our job has to be to figure out how it works.

6.1 A simple reserve demand analysis

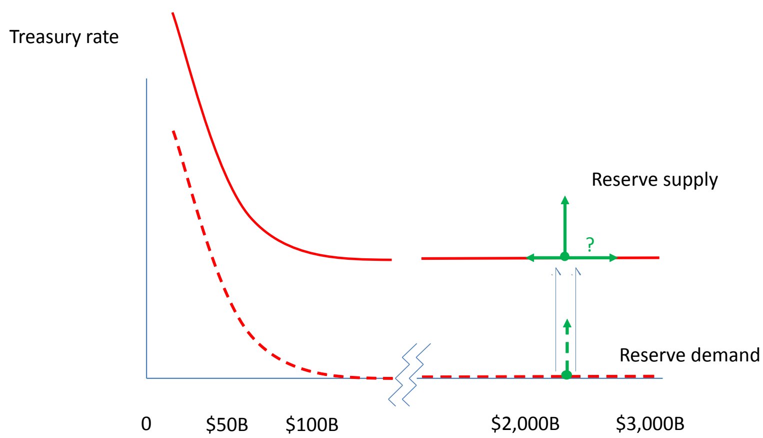
At an informal level, Figure 6.1 illustrates the standard story for monetary policy, and one option for the Fed when it wants to raise rates. In this story, the Fed controls interest rates by rationing the amount of non-interest-paying reserves. Banks must hold reserves in proportion to their deposits. If the Fed sells bonds, taking back reserves, the banks must get along with fewer reserves. They bid up the Federal Funds rate they pay to borrow reserves from each other. Treasury rates and other rates rise by arbitrage with the Federal Funds rate. So all interest rates rise.

Required reserves were about \$50 billion before the crisis and are about \$80 billion now. Banks might be willing to hold a few more billions voluntarily despite a small interest rate spread. But it's unlikely they would hold a trillion of excess reserves while suffering any interest spread relative to Treasuries. So, in order to tighten at all by this mechanism, the Fed would have to sell at least \$2 Trillion of securities before having the slightest impact on short-term rates.



In turn, the standard story goes, banks with fewer reserves must lend less and reduce deposits through the money multiplier. To some, the cut in lending reduces real activity; to others the reduction in the money supply is the crucial channel. Finally, in this story, money including bank deposits eventually determines the price level via MV=PY and some long and variable lags. So, eventually, the price level falls.

Now, consider the interest-on-reserves channel. Figure 6.1 presents the analogous graph. The Fed doesn't buy or sell anything, but simply raises the interest rate it pays on reserves, as shown by the vertical rise in reserve supply. The demand for reserves is a function of the *spread* between Treasuries and reserves. I assume here that Treasuries rise by exactly the same amount as the interest rate on reserves rises, by the usual arbitrage mechanisms. Then, the demand for reserves as a function of the *level* of the Treasury rate rises by the increase in the interest on reserves. Read this demand curve as, "What must the equilibrium rate on Treasuries be so that banks are willing to hold a given quantity and rate of supplied reserves?" In the liquidity satiation range, Treasuries must pay the same amount as reserves.



We conclude, tentatively, that all interest rates rise when the Fed raises interest on reserves. (The strength, speed, and reliability of these arbitrage mechanisms is a bit in doubt, given how uncompetitive our heavily regulated banks have become. This observation motivates the Fed's new program opening up reserves to other financial institutions. But for this level of discussion, we can put off that quibble.)

Already we overturn standard ideas. To raise interest rates, the Fed need do nothing to the balance sheet. In fact, it can raise interest rates while increasing the size of the balance sheet, if it so desires! Making this point, Friedman (2013) writes "Given today's institutions, therefore, the central bank can choose both the quantity of its outstanding liabilities and their market price (that is, the interest rate at which they trade). Monetary policy has not one independent instrument but two." Yes, but the question is, do these two, and especially the second, have any effects on output and inflation, and if so, how?

This is a puzzle, because *the rest of the standard monetary transmission mechanism is completely missing.* Banks still hold massive excess reserves – excess of reserve requirements or internal reserve demands corresponding to lending or deposits. Thus, the interest rate rise has *no* direct impact on bank lending or deposit creation, no effect on the money stock, and thus no effect on inflation by the standard supply channel.

Even if the Fed changed the size of the balance sheet, in this standard analysis that change has no effect at all so long as we stay comfortably in the range of satiated liquidity. At the margin, reserves and Treasuries are perfect substitutes. The money multiplier is zero. Deposits and loans are unhinged from reserves.

The Fed already pretty much recognizes this fact. In describing how quantitative easing – open market operations with zero spread between reserves and treasuries – works, the Fed has emphasized the effects on *bond* supply, not *money* supply, with a theory of segmented markets for bond duration. This is a radical change and a complete reversal. Historically – at least since Friedman (1968) – the effects of monetary policy have been supposed to come from changes in the supply of money which is issued, not changes in the supply of bonds which are bought, which are usually a drop in the bucket of overall bonds. MV = PY states that it does not matter what assets are bought, or if any assets are bought at all – MV = PY states the equivalence of an open market operation to a helicopter drop.

Not only does the Fed's ability to induce changes through the money multiplier and lending fail, but with abundant excess reserves, the Fed loses its ability to *control* deposit and loan changes initiated by banks. If a bank wants to lend an extra dollar, it can simply create the corresponding deposit. A trillion dollars in to excess reserves, that constraint is completely absent. Banks might be limited by capital requirements or regulation – which will be strong temptations for the Fed once it realizes monetary policy has lost control completely – but not by monetary policy.

If the Fed is to affect anything in the interest-on-reserves regime, it must be through prices, not quantities. But the absence of monetary frictions means interest rate spreads should all stay the same, so we need the level of interest rates to matter. Now, assuming expected inflation does not also rise one for one with the interest rate rise, through some sticky price mechanism, the Fed will raise real interest rates by raising interest on reserves. Higher real rates may affect the *demand* for loans by businesses and consumers, if not the supply through the multiplier mechanism. But seeing if that really is the case requires a full model. And that mechanism requires price stickiness. It would seem in this simple analysis that without price frictions as well as monetary frictions, expected inflation will simply move one for one with the interest rate target and the Fed loses all power, absent fiscal backing. The deeper theory discussed below attempts to circumvent this conundrum, but I conclude without success.

6.2 Loss of control with interest on reserves

Next, I survey alternative theories of inflation determination, and show that none of them can apply in the interest on reserves regime. I also survey the literature warning of this fact. My informal discussion summarizes a lengthy academic literature warning already warning that with market interest on reserves, the Fed loses control of inflation. In fact, a number of authors in the monetarist tradition have already advocated limiting financial innovation so that MV=PY can be brought back again and the price level controlled. But that cat is out of the bag

6.2.1 Monetary theory

The basic point is straightforward. If money demand $MV(r - r^{TB}) = PY$, where r is the return on money and r^{TB} is the return on treasury bills, that demand becomes a correspondence where any amount of M above a satiation point will be held as long as $r - r^{TB} = 0$, as I have graphed in Figures 6.1 and 6.1. (In some models of money, we are never completely satiated, but in those models the interest on reserves can never completely approach, let alone exceed as has recently been the case, the interest on treasuries.) In this case, then even if the Fed were to control M, it would have no effect on PY. Models embed that point in general equilibrium to make sure it still holds, and it does.

Sargent and Wallace (1985) is central paper, warning that, "Indeterminacy of equilibrium is a possibility because the proposal eliminates the interest differential between...reserves, and other assets. ... it tends to produce an indeterminate demand for reserves and hence for the monetary base....This source of indeterminacy is widely recognized." In the relevant case (interest paid on reserves comes from earnings on the Fed's portfolio of Treasuries, nominal interest rates are above zero), Sargent and Wallace show that there is no equilibrium. They survey (section 5) alternative models including cash in advance and money in the utility function, and again find that interest on reserves leads to price-level indeterminacy.

Sargent and Wallace, like most well-posed monetary models, does contain a version of the government debt valuation equation (1), and their paper focuses on the financing of interest on reserves as a result. They note that there is a continuum of r (interest rate) and v (tax rate) pairs that generate an equilibrium. Their indeterminacy result (Proposition 1) is that for any r, there exists a v that makes it an equilibrium. But therefore fixing v, there is only one r, and thus one equilibrium. In the notation of this paper, if $B_{t-1}/P_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$, then for any P_t there exists a $\{s_t\}$ such that that P_t is an equilibrium. But fixing $\{s_t\}$, only one P_t is an equilibrium. To get an absence of equilibrium, they assumed the "Ricardian regime" special case that wipes out the government debt valuation equation – appropriately, to their point. Restoring that equation, we restore determinacy. (The rest of their indeterminacy results do not apply to the case we observe, that interest on reserves is financed by interest earnings on the Fed's holdings of Treasuries.)

What about cash?

Cash does not pay interest. Fama (1983) finds price level determinacy with interest on reserves, by anchoring price level determination in a demand for non-interest-bearing currency and control of that currency.

Cash still exists in rather surprising quantity – about a trillion dollars, or more than $3,000 per capita, 77% of it in hundred-dollar bills[4]. But you and I, corporate businesses, and finance don't use cash. The legal, and especially corporate and financial economies, have moved to electronic, interest-bearing money. Almost all of us pay by credit cards or debit cards, linked to accounts that will, when interest rates rise, pay interest, and are mostly settled by bilateral netting between our banks – an essentially electronic accounting system. Cash really is only used in any substantial quantity for illegal transactions, undocumented people, and store of value in foreign mattresses.

For this reason, as a modeling approximation, it seems wiser to think of cash holdings as disconnected from nominal (legal) GDP, than to found control of nominal GDP on control of cash balances not used for most of it. Empirically, cash holdings just trundle along disconnected from the economy and, especially, the financial system. Unredeemed coupons, unused subway cards, sock-drawer change, that stack of receipts you've been putting off submitting for reimbursement, and, more seriously, invoices, and some trade credit are non-interest paying claims. But they're not important for output or price level determination. Sure, the "demand" (quantity held) of unredeemed coupons may track PY well. But that doesn't mean that controlling the inventory of unredeemed coupons would control the price level. In my view, cash has achieved that status in the legal U.S. economic and financial system.

The Fed also does not control the quantity of cash, as Fama prescribes. For MV = PY of anything to control PY, the Fed must control the M (and V must be defined and stable). The Fed allows banks freely to exchange cash for reserves.

[4] http://federalreserve.gov/paymentsystems/coin_data.htm

Cash still has an enormous benefit, anonymity, which is important for all sorts of privacy, and political-freedom reasons. Not all "illegal" transactions are economically or politically injurious. I cheer the development of anonymous electronic currency. But that issue is unrelated to the questions here.

For the rest of this essay, then, I will ignore cash – along with unredeemed coupons and the rest of my humorous list of non-interest-bearing claims – and think of a monetary system based entirely on interest-paying reserves, and consisting entirely of interest-paying electronic money. Reserves, not cash, are really our fundamental numeraire. Private debts and government debts ultimately promise reserves. Anything settled by a bank is such a promise. We certainly don't want to embark on the alternative abstraction – that the functioning of monetary policy and the control of inflation centrally revolves around the demand for cash for illegal purposes.

6.2.2 Interest rate targets

This discussion about money may seem quaint, because our Federal Reserve has not targeted money in generations, except possibly for a short-lived experiment in the early 1980s. It's been following interest rate targets, and obviously will continue to do so. So, the central class of theory needed is a theory that describes how Fed manipulation of interest rate targets controls the price level. I conclude in this survey that we do not have such a theory.

Even without interest on reserves – when the Fed controls interest rates through open market operations as in Figure 6.1 – how pure interest rate targeting controls inflation is controversial. Friedman (1968) warned verbally that an interest rate target would lead to unstable inflation. Sargent and Wallace (1975) show that inflation is indeterminate with an interest rate target. The Fisher relation $i_t = r + E_t\pi_{t+1}$ means that controlling the interest rate can determine expected inflation, but unexpected inflation $\pi_{t+1} - E_t\pi_{t+1}$ can be anything. Sargent and Wallace show that this basic logic survives in a carefully specified general equilibrium model – considering all the equations of the model except a valuation equation (1), $\pi_{t+1} - E_t\pi_{t+1}$ is still not tied down. (I'm being careful with language. Indeterminate is a different object than unstable. $\pi_{t+1} = 1.5\pi_t$ is determinate but unstable. $\pi_{t+1} = \delta_{t+1}$,$E_t\delta_{t+1}$ is stable but indeterminate.)

McCallum (1981) and Hall (1984, 2002) suggest that an interest rate target that varies more than one for one with inflation $i_t = r + \phi\pi_t,\ \phi > 1$, is sufficient to determine inflation, if not quite the price level. The Taylor (1993) rule also specifies $\phi > 1$, so this suggestion is now known as the proposition that the Taylor principle $\phi > 1$ gives determinacy with interest rate targets. This idea is formalized in New-Keynesian models summarized in Woodford (2005) and described above. In this model, the Fed deliberately introduces *in*stability to the economy so that all but one path explode. Choosing the one non-explosive path, we obtain determinacy.

However, Cochrane (2011) argues that this line doesn't work either, and that Sargent-Wallace indeterminacies remain even with $\phi > 1$. The assumption that our Fed deliberately *de*-stabilizes the economy seems strained, and is loudly not how our Fed describes its role. It also rests on a rule that only locally-bounded equilibria are valid, which is not usually part of economics.

Woodford explicitly recognizes that the government valuation equation (1) is part of the model, but he assumes that the Treasury adjusts surpluses $\{s_t\}$ to validate any price level, so that equation has no force in inflation determination. Since the resulting model doesn't determine inflation, my reinterpretation here of Woodford's equations may be attractive. If we regard surpluses in (1) as fixed, so they do determine the price level, but that surpluses respond to the Fed's desired inflation target, then we obtain a theory that does determine the price level. The Fed wins the game of chicken with the Treasury. But that interpretation is solidly fiscal-theoretic, and then describes a fiscal-monetary coordination by which the Treasury allows the Fed to make and communicate its fiscal commitments, and the fiscal commitments ultimately drive the price level. We're looking for a theory in which active movement of interest rate targets alone determine inflation or the price level, without reference to the government valuation equation (1), or in a fully "Ricardian" regime in which surpluses would adjust to validate any price level path, one intended by the Fed or not.

The one model in my (Cochrane 2011) survey which works – in which interest rate targets alone determine inflation – is Taylor's (1999) marriage of an ad-hoc old-Keynesian backward-looking model with an active $\phi > 1$ Taylor rule. This model produces both determinacy and stability. But even Taylor admits that the old-Keynesian equations are hardly an economic "model." Surely we need something more reliable, even if very simplified, that can determine the price level in the interest-on-reserves regime.

6.2.3 Big picture

Fundamentally, there are two possibilities for price-level determination with paper money. Money might be valued because it is uniquely useful in transactions and limited in supply; the quantity theory; MV=PY. But under the interest on reserves regime, the whole point is that money is not scarce. We will be satiated in liquidity and hold far more than needed for transactions. And already, interest rate targets do not limit money supply.

Or money might be valued because it is backed. Under the idealized Gold standard, money is valued by the promise to convert paper notes to gold on demand, whether or not notes have value in exchange. The government debt valuation equation reveals that apparently fiat money is in fact backed by the present value of surpluses which will retire government debt. Since this theory does not require any frictions, it survives intact as monetary frictions disappear.

The new-Keynesian model represents an attempt to construct a third kind of theory, in which money is valued and inflation determined based on active interest-rate setting alone. My survey concludes that this interpretation of the equations is unsuccessful. It too is really a theory of fiscal backing, with a particular monetary-fiscal coordination mechanism by which Fed actions lead the Treasury to adjust surpluses as the Fed wishes.

Ad-hoc backward-looking and mostly static Keynesian ISLM equations do give stable and determinate inflation responses to interest rate targets, while ignoring the government debt valuation equation. This kind of analysis remains popular in policy circles, and underlies most of the verbal explanations the Fed gives of its actions and their effects on the economy. However, it doesn't anymore qualify as an "economic" theory, and for predicting how an economy will work, out of sample, with profoundly new institutions, it is better to start with something a bit more structural.

I conclude that an analysis of inflation based on the government debt valuation equation (1) is the only currently available framework for understanding inflation in the interest-on-reserves regime, i.e. at the limit that monetary frictions vanish.

6.3 Fiscal-theory controversies

Use of the valuation formula (1) to think about inflation is clouded in myriad unnecessary controversies. (See also Cochrane 2005, 2011b on these points.)

It is helpful to derive (1) in a fully-specified model, which I do in the Appendix. The representative consumer maximizes $E\sum_t \beta^t u(c_t)$ and has a constant endowment y. This specification produces a constant real interest rate $1+r=1/\beta$. The government sells one-period nominal debt with face value B_{t-1} at the end of time $t-1$. It redeems debt with money at the beginning of time t, then soaks up that money at the end of time t with lump-sum real surpluses s_t and bond sales with value Q_tB_t, where Q_t is the one-period bond price. Interest is paid overnight, and people do not want to hold money overnight, so money printed in the morning must be soaked up in the afternoon,

$$B_{t-1} = P_t s_t + \beta E_t \left(\frac{P_t}{P_{t+1}}\right) B_t,$$

or, in real terms.

$$\frac{B_{t-1}}{P_t} = s_t + \beta E_t \left(\frac{B_t}{P_{t+1}}\right). \tag{33}$$

Iterating forward and applying the consumer's transversality condition, we obtain the basic equilibrium condition (1)

Equation (1) is not a "budget constraint." It is a valuation equation, an equilibrium condition. It works the same was as the valuation equation by which stock prices adjust the present value of expected dividends. There is no "budget constraint" that *forces* the government to respond to a deflation in P_t by raising surpluses, any more than a stock price "bubble" forces a company to raise earnings to justify the stock price. And just as well, because there is a Laffer curve limiting surpluses, but there is no limit to deflation, so there must be some price at which (1) is violated while budget constraints can never be violated.

Equation (1) has a natural "aggregate demand" interpretation. (Woodford 1995). If the real value of nominal debt is less than the present value of surpluses, then people try to spend their debt and money on goods and services. But collectively, they can't, so this "excess aggregate demand" just pushes up prices until the real value of debt is again equal to the present value of surpluses.

Aggregate demand is nothing more or less than demand for government debt, as by the private-sector budget constraint the only way to spend more on everything else is to spend less on government debt. This equation also expresses a "wealth effect" of government debt.

Though the literature spends a lot of time thinking about "regimes" and testing for them, there is really not much point to that exercise. As a simple example, suppose we modify the model to add a demand

$$M_t V = P_t y \tag{34}$$

for money held overnight that does not pay interest. Equation (1) now includes a seignorage term,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \left[s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right], \tag{35}$$

or equivalently

$$\frac{B_{t-1} + M_{t-1}}{P_t} = \sum E_t \left[m_{t,t+j} \left(s_{t+j} + \frac{i_{t+j}}{1+i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right) \right].$$

Both equations (34) and (35) must hold in equilibrium.

Now, following Leeper (1991) we often talk of a money-dominant "regime" as one in which the Fed sets M_t, P_t follows from (34), and then the Treasury sets $\{s_t\}$ in (35) to validate the Fed-chosen P_t, and a fiscal-dominant "regime" as the opposite case. One might say that in the absence of monetary frictions, the government must "switch" to a "fiscal-dominant regime."

But both equations (34) and (35) hold in both regimes, so there is no testable content to the regime specification from observations of $\{M_t, B_t, P_t, s_t\}$, (Cochrane 2011a). This observation should already alert us to the sterility of the "regime" investigation.

One can read the equations of a "Ricardian" or supposedly fiscal-passive regime as verifying the power and necessity of the fiscal backing. Monetary policy only affects inflation *because, and only because* it induces changed expectations of fiscal surpluses. The change in "aggregate demand" that ultimately affects the price level comes only from the induced change in fiscal surpluses. Is it the foot on the gas pedal, or the engine which ultimately causes the car to go? If a man (Fed) induces a horse (Treasury) to pull a cart by putting a carrot under the horse's nose, does that mean the man pulls the cart?

The same points hold if the Fed follows an interest rate target. Again, a valuation equation like (1) holds, and the Treasury is assumed to adjust $\{s_t\}$ to validate the model's price-level predictions. If the Treasury will not or cannot follow through, the hypothesized price level won't happen. The interest rate only affects the price level because of the induced fiscal response. There is no testable content to whether the Treasury or Fed drives the "regime."

Money and fiscal policy must always be coordinated. Monetary contractions without fiscal support and coordination fail. Fiscal contractions with loose money stop inflations (Sargent and Wallace 1981). If the Fed were to try a 50% deflation now, this would mean doubling the real value of publicly-held debt from \$12 trillion to \$24 trillion, and the value of the government's credit guarantees by additional trillions. A call to the Treasury to double taxes would quickly discover the top of the Laffer curve in the rear-view mirror.

As these examples emphasize, for (1) to hold and play a central role in price determination, one does not have to, and one should not, think of surpluses $\{s_t\}$ as being "exogenous," or set without regard to other variables, including prices. Equation (1) tells us what the equilibrium price level must be, conditioned on the equilibrium $\{B_t\}$ and $\{s_t\}$. That is all.

Some economists regard fiscal price determination as a matter for extremes; currency crashes and hyperinflations maybe, but not normal times. But even in "normal times" monetary and fiscal policy must be coordinated; monetary policy only works if the fiscal backing – the response of surpluses $\{s_t\}$ – is there, even when that response is "small." And cyclical variations in aggregate demand, the right hand side of (1), are not usually thought of as being that "small."

Perhaps the "exogenous" confusion is behind this point. Equation (1) holds even when the surpluses s are within the government's control, not just when the top of the Laffer curve or other disaster means the government loses control of surpluses.

It's tempting and useful for comparative-statics exercises to think about fixing $\{s_t\}$ or $\{B_t\}$ holding the others constant, as I have done above. However, real monetary and fiscal policy is always coordinated, and most events contain large movements in both quantities at the same time. For example, wars and recessions feature big increases in debt B_t with big negative current surpluses s_t. But these events come with big increases in expected future surpluses $E_t s_{t+j}$, because governments want to raise real revenue, not cause inflation. So $\{s_t\}$ follows a response that is negative now, and positive later, to such a shock, nothing like an AR(1), and $\{s_t\}$ and $\{B_t\}$ move together in response to such typical economic shocks.

7 Monetary-fiscal coordination in the great disinflation

The inflation of the 1970s and the disinflation of the 1980s are conventionally analyzed in purely monetary terms, without any attention to fiscal coordination. For example, Sargent (2001) analyzed how the Fed learned about the effects of monetary policy. Clarida, Galí and Gertler (2000) argued that inflation was made determinate by the Fed adopting $\phi > 1$. Taylor (1991, 1999) argued that inflation was made stable by the Fed adopting $\phi > 1$. analysis. These are of course the tip of the iceberg of papers written on monetary policy and the great inflation and disinflation, few papers breathing a word about fiscal coordination.

Now, as I have analyzed it, understanding fiscal-monetary coordination is essential in our nearly frictionless world with interest on reserves, as it is the only surviving model. But fiscal-monetary coordination should always have been an important part of inflation determination. And the 1980s may have already been closer to the frictionless approximation than is usually acknowledged. Even in the 1980s, there were already many liquid interest-bearing assets, including interest-paying bank accounts and money-market funds, as well as short-term commercial paper and other liquid short-term debt. The Fed was already targeting interest rates, which in my analysis vitiates completely monetary control of inflation. The quantity of reserves was already tiny, leading many to wonder if such a small tail could really wag the dog. Arguably, we were already approaching the frictionless limit, just in another way. Interest on reserves just forces us to face that fact.

So, if fiscal-monetary coordination is important now, we should also have a clear understanding of how fiscal-monetary coordination operated previously, and especially in this episode. (Bianchi and Ilut 2014 provide an extended analysis of fiscal-monetary coordination in the postwar period.)

Sargent and Wallace (1981) are the famous exception to the rule, perhaps the most famous analysis of fiscal-monetary coordination. They pointed out that disinflation with high deficits could not work. Using a framework similar to what I have used above, they claimed that the large deficits would inevitability lead to seigniorage and a return to inflation. (Cochrane 2005 compares Sargent and Wallace to pure fiscal theory.)

Their analytical framework is perhaps one of the most famous innovations in monetary economics of the last 40 years, inaugurating the renewed attention to fiscal-monetary coordination. But the disinflation stuck. How did their analysis play out ex-post?

Figures 5, 6 and 7 lay out some important features of the episode. Figure 5 shows interest rates and inflation. Prior to 1980, the 3 month treasury bill rate was almost always below the inflation rate. For a brief period in 1980-1981, the 3 month rate rose above inflation. Inflation promptly fell like a stone in 1982. A period of high ex-post real interest rates followed throughout the 1980s, interrupted only by the quasi-recession of 1987, and the three subsequent actual recessions. Long-term rates indicated an expectation of return to high real rates, or a large risk premium. In any case, long-term rates moved little over the recessions. The conquest of inflation entailed a long period of high real interest rates.

Next, Figure 6 presents some of the fiscal history of the period. I measure deficits as the amount the Federal government actually borrows in credit markets. The picture makes one point very clear: *most of the "Reagan deficits" were due to higher interest costs on the debt rather than unusually large primary deficits.* Figure 7 makes the same point in another way, plotting real primary deficits (net of interest costs) along with detrended GDP. The Reagan era primary deficits were not that large. Moreover, Figure 7 emphasizes that the largest driver of primary deficits is the state of the business cycle. As GDP falls, tax receipts fall and some spending kicks in automatically. The

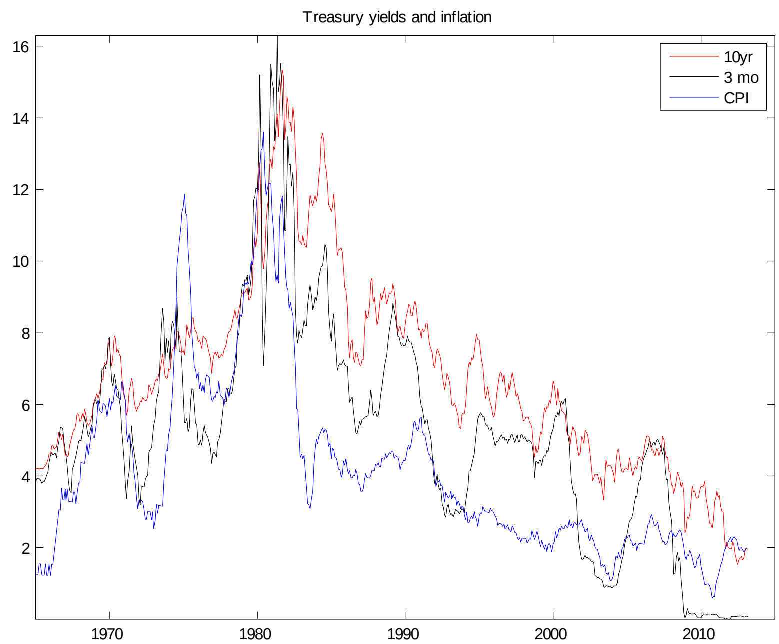


Figure 5: Treasury yields and inflation. Inflation is change from a year ago in log CPI.

Reagan deficits were not that large compared to the size of the recession.

Expectations of long-run surpluses $E_t \sum_{t=0}^{\infty} m_{t,t+j} s_{t+j}$ matter for fiscal backing of monetary policy. So the issue for deficits is not their current size; it is whether current deficits spark a re-evaluation of the long-run sustainability of fiscal policy. Wars are financed with large negative current surpluses, but an understanding that large positive future surpluses will pay off the debt. Consequently, so long as the outcome of the war looks positive, deficits can be huge with little inflation. And conversely. The fact that Reagan-era primary, cyclically-adjusted deficits (at least by this eyeball cyclical adjustment) were not large makes it plausible that they did not spur a change in evaluation of long-run fiscal policy.

Figures 5 and 6 are revealing of the fiscal costs of disinflation. As Figure 5 shows, investors who bought long-term bonds at high interest rates in the late 1970s, expecting large inflation, got a windfall as their bonds were paid off with much lower inflation. The P_t in B_{t-1}/P_t declined unexpectedly. Taxpayers paid for this windfall in higher subsequent surpluses.

Figure 6 reveals a second, larger, fiscal cost of disinflation. In the conventional narrative of a stabilization, the high real interest rates were the crucial monetary policy intervention needed to bring down inflation. (I say "conventional narrative" because my theory section does not include a model with this result. But when the vast majority of analysts think a period of high real rates is key to bringing down inflation, perhaps it is the theory section that needs more development.) But much US debt is relatively short term, so must be rolled over every few years. As the US rolled over a (then) fairly large 20% of GDP stock of debt, the real interest costs of financing the deficit rose. Figure 6 suggests that these interest costs were about 2% of GDP for a decade, so the fiscal costs of disinflation were something like a cumulative 20% of GDP. Those costs came from somewhere as well. Interest costs not financed with current surpluses result in more debt being sold, and that debt too must be paid with higher future surpluses.

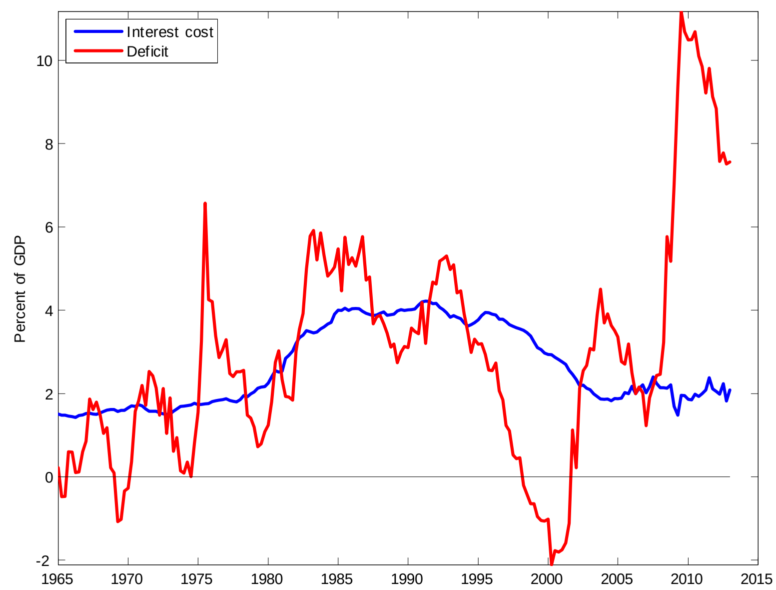


Figure 6: Interest costs and Federal Deficit. BEA table 3.2 federal govenerment current receipts and expenditures. Interest payments and net lending or net borrowing. Both series as a percent of GDP.

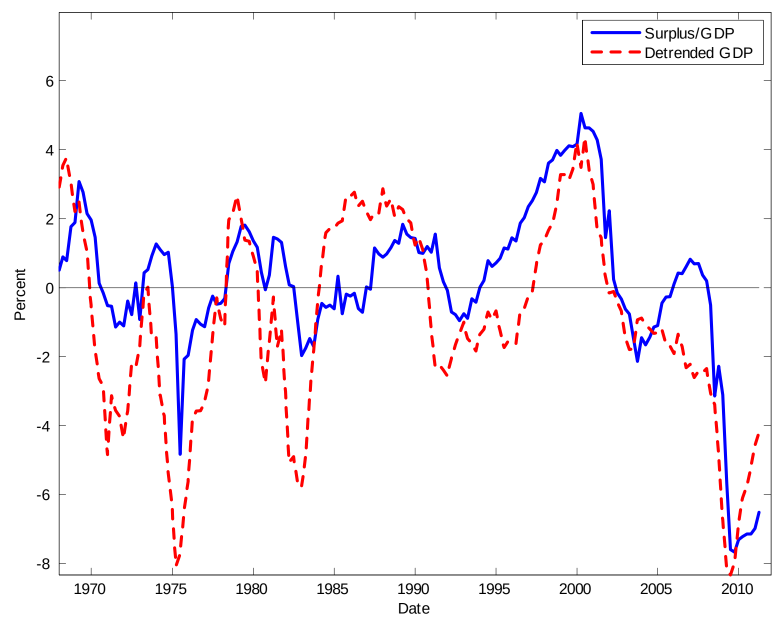


Figure 7: Real primary surplus / real GDP, and detrended real GDP. GDP is detrended with a log-linear trend fit through the previous 15 years.

So, how did the fiscal end of Sargent and Wallace (1981) work out? Figure 7 shows that in fact the US did generate substantial primary surpluses, and these surpluses did pay off the Reagan deficits, both primary and the extra deficits induced by tight monetary policy. That's not too surprising – using ex-post returns to discount, the present value relation is an identity and the ex-post returns were positive. The 1980s also featured tax reforms, a social security reform (important for long-run deficits) and a campaign of deregulation. Starting in the late 1980s, and suggestively coincident with these reforms, the US economy started to grow much faster than before. As Figure 7 emphasizes, surpluses in the U.S. are largely produced by economic growth rather than direct results of higher tax rates or spending policies.

That the debt was paid off does not mean it was costless. That 20% of GDP are real resources, which could have been put to other uses.

The 1980 disinflation was a *joint monetary-fiscal* stabilization. Ending inflation cost something on the order of magnitude of 20% of GDP. The US was able in the short-run to borrow that money, and bondholders somehow had faith they would be paid back and it would not be inflated away. The bond buyers were right. It was paid back, and they enjoyed an unprecedented decade of good returns.

But it did not have to happen that way. Absent the fiscal coordination; absent the large increase in $\sum \beta^j s_{t+j}$ that in fact occurred, the 1980 stabilization could have and would have failed. Just as so many other purely monetary stabilizations without fiscal reforms have failed.

8 Fiscal limits to current monetary policy

This stylized history leads to a central concern for our interest-on-reserves regime. Suppose our Fed decides to tighten, and wishes to raise interest rates to 5%. Suppose further, that as in the 1980s, this is a rise in real interest rates; that inflation does not just jump to 5%, that the contraction is, as intended, effective in limiting or decreasing inflation. In such a scenario, the 2% of GDP interest costs we see in the 1980s are small potatoes. For the debt-to-GDP ratio was only 20% in 1980. Now it is 100%. And that number only counts official Federal debt, not including credit guarantees and many unfunded promises. If the US government refinances $18 trillion of debt at 5% real interest rates, that means $900 billion of deficit annually. Those additional interest costs must either be paid now, with $900 billion per year of current taxes or less spending, or by accumulating debt that much faster, adding $900 per year present value of future higher taxes or less spending, which will somehow be paid it off in present value terms as we did in the 1980s.

Footnotes about treasury being sent a bill for lump sum taxes to validate a money-dominant regime are likely to be put to a severe test with this experiment. I think it more likely that Congress simply refuses, and takes away the Fed's independent authority to set interest rates, and thereby to impose interest costs on the budget.

This outcome happened before, the last time the US had a debt of this magnitude, at the end of World War II. Congress simply told the Fed to target long term rates at 2.5%, in order to lower financing costs. Congress could have the same response to a Fed-imposed $900 bill for interest costs. This fiscal-monetary interaction, not mark-to-market losses on the Fed balance sheet, or reduction Fed transfer payments, strikes me as the most important and generally disregarded fiscal limit to monetary policy at the moment.

Now, this argument is not fully fleshed out. I have alluded to a decade of high real interest rates as a necessary part of disinflation, in deference to the common view of the 1980s. But I have not put forward models that have such dynamics. Perhaps the story that high real rates are important to monetary stabilization are wrong. Perhaps the past really was a $MV(r) = PY$ regime, and high interest rates were necessary to squeeze down M, but that in the new interest-on-reserves regime, where we are never rationed in liquidity and banks never constrained in lending, a high period of real rates will not be necessary.

But historical experience more broadly suggests that all serious monetary stabilizations are joint fiscal and monetary stabilizations. Real fiscal resources are necessary to pay off the windfall to long-term debt holders in any case. And approaching a stabilization with a large stock of nominal debt in hand is more difficult than without.

Another important question is the source of whatever inflation needs controlling in the first place. The western world lives on the edge, with debts of 100% of GDP, emerging slow growth – bad for surpluses – and aging populations who will make claims on unfunded social insurance promises. If the short-term debt "bubble" bursts in a shock to expectations of long-run surpluses, there will be little that monetary policy can do about it.

I conclude that the fiscal *limits* on monetary policy may loom large in the interest-on-reserves era, because it coincides with the era of high sovereign debt.

9 Communication, anchoring, inflation targets and policy rules

The biggest problem we face in thinking about fiscal backing for inflation is the nebulousness of the present value of future surpluses $E_t \Sigma m_{t,t+j} s_{t+j}$. As in financial economics, the difficulty of independently measuring present values leads to interminable arguments whether asset prices vary in a way that accords with the present value formula. But surpluses backing nominal debt are, presumably, chosen, not the maximum the government can extract from the economy as profits are, presumably, maximized. A well-designed fiscal backing regime should give a clearer precommitment and communication of how much the fiscal backing is.

Likewise, if the government *wants* to inflate, how does it communicate a *reduction* in future surpluses? One can read the struggles of Japan in the last decade or two as a measure of just how difficult it is to reduce expectations of future surpluses! People buy Japanese debt expecting it to be paid off in real terms, no matter how much debt Japan racks up. The slow decline in recent U.S. and Euro inflation despite low interest rates and large budget deficits poses a similar conundrum.

Though I have emphasized a very stylized "monetary policy" consisting of debt sales or interest rate targets with no fiscal response, in fact coordinated fiscal responses will make overall policy much more effective. If only the government could announce or communicate a commitment to what $E \sum_{j=1}^{\infty} \beta^j s_{t+j}$ would be, then inflation control would be much easier.

In fact, many monetary institutions can be read precisely as such commitment devices. A review of some past devices, an interpretation of inflation targeting as a current device suggests how we might construct better communication and commitment devices in the future

9.1 Gold standard, exchange rate pegs and currency boards

The gold standard is often thought of as a monetary device, a way to give notes value by backing. Already this story is questionable, because gold itself plausibly derived most of its value by scarcity and use in exchange rather than by its frictionless industrial value. The gold standard is, I think, better seen as a fiscal commitment.

Few governments on a gold standard backed even 100% of their note issue with gold reserves, and no government backed the entire face value of its outstanding debt. There would be no point to debt if the governments had that much gold. Furthermore, a government in fiscal trouble could be sure to grab the gold reserves and inflate newly unbacked notes.

So what happens when a gold-standard government must pay off debt that comes due, in quantity bigger than the gold reserves, or must defend against a run of partially backed note issues? It must either raise current taxes less spending, to obtain gold, or it must credibly commit to raising future taxes less spending so that it can borrow gold.

A gold standard is thus a *fiscal* commitment. It is a way of saying, "we promise to raise surpluses as necessary to pay off our debt at the price level of 1.0 relative to gold, no more and no less."

Like all promises, this one is too easily broken. The gold standard era is as much a history of crashes and runs as it is a history of centuries of happy price stability. But it at least shows the possibility of making a commitment or promise that communicates expectations of the present value of surpluses rather than think of them as a nebulous present value somewhat like the present value of stock earnings.

A currency board or exchange-rate peg operate in the same way. They seem to be purely monetary devices, a way to give money value by its backing rather than scarcity in exchange. Likewise, they leave unanswered where the value of the targeted money comes from. But in fact they are fiscal commitment devices. Exchange rate pegs do not operate with 100% reserves, and even more so do not operate with 100% backing of all government nominal debt. Thus, if there is a test of the peg, or if government debt comes due, the government must raise taxes less spending, in this case raising from its citizens claims abroad that can be used to get foreign currency, now or in present value form. Currency boards operate with 100% *money* reserves, but not 100% *debt* reserves. Thus, again, the government's ability to tax its citizens when debt comes due fundamentally drives the viability of the regime. And of course the government can always grab the board's reserves if surpluses are insufficient, as Argentinians found out to their dismay.

The gold standard, currency boards, and exchange rate pegs are particularly useful when the objective is to stoke rather than to contain inflation. By cutting the gold price or devaluing the currency, the government is able to very clearly communicate that the fiscal underpinnings of monetary policy $E\sum\beta^j s_{t+j}$ are lower, in a way that fiat-currency countries such as Japan, the US, and the EU have not been able to do. It's almost as good as the clearest possible such device, a currency reform.

In sum, the gold standard, currency pegs, and currency boards are at heart fiscal *communication* devices, ways to communicate to money holders just how much surplus, and no more, will be used to redeem nominal debt B_{t-1}. They are somewhat less successful *commitment* devices, as the frequent inabilities to make good on promises in inflationary environments proves. But they are commitment devices nonetheless, because the costs of abandoning the gold standard, exchange rate peg, currency board, or union are much larger than the costs of silently inflating.

9.2 Inflation targeting and anchoring

It is generally thought that inflation expectations are now "anchored," allowing the government latitude for current inflation to go up and down without changes in expected future inflation, or anchoring the expectational component of the Phillips curve which wandered so unfortunately in the 1970s. Countries that have adopted formal inflation targets view that announcement as an important part of this "anchoring"

The question is, what does "anchoring" mean and what institutions have achieved it?

The standard Keynesian and new-Keynesian story is that the target π^* in a Taylor rule $i_t = r_t + \pi^* + \phi_\pi(\pi_t - \pi^*)$, or equivalently the intercept $\bar{\imath}$ in $i_t = r_t + \bar{\imath} + \phi_\pi \pi_t$, where r_t is a time-varying "natural rate," provides the anchoring. People believe that the central bank will react to inflation above its target with higher interest rates and vice versa.

I have argued elsewhere (Cochrane (2011a, 2013)) against this view. It's a bit sensible in an old-Keynesian framework since in that framework higher interest rates lead to lower future inflation, and the coefficient ϕ_π is learnable by experience. Alas, the old-Keynesian framework fails the test of being an economic model, despite a half-century search for micro-foundations. In the new-Keynesian framework, the Taylor rule is a believed commitment by the central bank to blow up the economy, to introduce instability, to raise future inflation in response to high past inflation, as a device to get the economy to jump to one of many equilibria. I have argued this is neither credible nor theoretically successful. Furthermore, there is no experience of $\pi_t - \pi^* \neq 0$ in such a model, so the belief in ϕ_π cannot come from experience.

But none of that matters here. Here it is sufficient to point to the Ricardian footnote. In any well-specified model, we must have fiscal policy coordination. So π^* is in the models a *fiscal commitment* just like the fiscal commitment underlying the gold standard.

This insight suggests a different interpretation of explicit and implicit inflation targeting regimes. When government and central bank agree on an inflation target, this is not just a one-way commitment, constraint, and communication device for the central bank. It is also a commitment, constraint, and communication device for the *fiscal* authorities as well. It says "we will raise taxes and lower spending as required to pay off government debt at the inflation target, no more and no less."

A 2% inflation target says that the treasury will raise surpluses as necessary to pay off debt at 2% inflation, and only 2% inflation. The central bank sets $\{B_t\}$ or a nominal interest rate, which controls expected inflation, and also allows the interest rate to respond to "natural rate shocks." The Treasury commits that if inflation gets out of control it will raise taxes, and vice versa.

Though in principle the "central bank" could control expected inflation entirely with $\{B_{t-1}\}$ variation, for any surplus path, it is surely much easier for the surplus path to be anchored, as much smaller debt variation is required. Conversely, the Treasury's commitment ensures that unexpected inflation is small, and controlling unexpected inflation helps to control expected inflation and to anchor expectations.

Figure 5 then poses a challenge to the standard stylized view of monetary influence, and somewhat supportive of the view expressed in the simple models here that raising nominal rates raises inflation. Beyond the 1980 episode, the more striking feature of Figure 5 is a strong positive correlation between nominal rates and inflation, both at secular and business cycle frequencies. It looks a lot more like the Fisherian view that our frictionless models suggested, guided by a slow and declining inflation target. The government and Fed announce (implicitly) a lower inflation target, the Fed lowers interest rates, and it is understood that the government will raise surpluses as necessary to validate the lower inflation target.

This analysis suggests an endorsement of ideas such as Blanchard, Dell'Ariccia, Mauro (2010) and Woodford (2012) to announce a higher inflation target to that end, but by a quite different mechanism. The usual story is that the higher inflation target, if believed, signals that the Fed will keep interest rates low for longer. In this analysis, the target is a statement that the *Treasury* will work to pay back debt at inflation above the target, but not so for inflation below the target. If believed, inflation would then rise when the target rises, and nominal interest rates would quickly *rise*, not decline.

9.3 The next step

Accept for a moment this view: our institutions have already adapted to the world of interest-paying money, liquidity satiation, nearly frictionless money though sticky prices, and ultimately fiscal backing, with a combination of nominal interest rate targets and implicit or explicit inflation targets providing a communication of underlying fiscal policy. Having understood our institutions this way, how can they be made better?

The fiscal nature of the inflation target, the fiscal communication, and the fiscal commitment, could all be improved. The following are some suggestive ideas.

A fiscal Taylor rule. As part of a budget process, the government could follow a fiscal Taylor rule, promising to raise surpluses when inflation (or, better, the price level) increases, and to decrease it when inflation decreases. Though the idea of any coherent budget policy – or any budgets at all – may seem quaint to American readers, other countries are able to follow more disciplined budget processes. Switzerland, for example, has a debt limit that actually leads to surpluses as it is approached.

A CPI standard. A gold standard is unrealistic for many reasons. Most of all, we want the CPI to be stable, not the price of gold. Now it is infeasible for the Federal Reserve to buy and sell the CPI basket, as it might buy and sell physical gold. But it can buy and sell CPI futures. Alternatively, it can target the spread between nominal and real debt, which accomplishes essentially the same thing in markets that are currently more liquid. By standing ready to buy and sell real and indexed debt at a fixed spread, the Fed or Treasury guarantees that something nominal (nominal debt) can be exchanged for something real (indexed debt). This action has a similar fiscal commitment to the gold standard, as the real debt cannot be inflated away without default.

This is an important change however. By targeting only the *spread* between real and nominal debt, the Fed gets out of the business of managing the level of real and nominal rates. Though such management has a long tradition, in my analysis we still don't have a good theory why interest rate management stabilizes inflation or expected inflation.

Whether the Fed or the Treasury is in charge of such a peg is not that important. The peg has important fiscal consequences, and the Treasury was in charge of the gold standard.

Variable-coupon debt. The Achilles heel of the gold standard and currency pegs – the Achilles heel of all government-provided money, really – is the occasional breakout of inflation when governments run in to fiscal problems and fiscal promises become fiscal defaults.

For this reason, there is a case against making explicit fiscal promises – when they are broken, it looks more like a default rather than a mechanical reduction in the value of equity. On this basis, for example, Sims (2002) argued against dollarizaton for Mexico. Equity is a useful buffer, and nominal debt is equity when it inflates.

On the other hand, why should the government throw noise into every private contract by inflation when it wishes to default on its debt, if explicit default is costly?

Some form of "government equity" could ride to the rescue. Shiller and Kamstra (2010) have advocated "Trills," GDP-linked debt, that would automatically rise and fall as GDP, and hence potential government surpluses, rises and falls. I am not convinced that a tight rule is necessary. If the government issued long-term debt, ideally perpetuities, and had the explicit right to adjust coupons as necessary, the same buffer could be achieved. When the government is in trouble, it can lower or temporarily eliminate coupons, without needing to inflate. When the government is back to fiscal health, say at the end of war, recession, or reform, it will want to raise coupons again, to restore its access to low-cost financing.

This mechanism is really quite similar to the suspension of convertibility that governments, especially the British, followed during wars under the classical gold standard. In a war, the government suspended convertibility to gold. Inflation was moderate. After the war, the government restored convertibility at par, giving relief to those who held nominal debt during the war, profits to those who bought the debt, and faith to those who would hold the debt during the next war. In this reading, Keynes' advice to inflate the WWI debt was contingent on the idea that there would never be another war in which the government would want to borrow and suspend convertibility. Similarly, Velde (2009) tells a "Chronicle of a deflation unforetold" in which France intentionally raised the coupon rate of its bonds, just to enhance its reputation in bond markets.

Promised defaults. To the issue of the moment, if (if!) they want to create some inflation, the US, Japan, and Europe face a conundrum in how to do it at the zero bound. Interest rates can't go below zero, few are Fisherian enough to think that raising nominal rates and leaving them there would result in inflation. There seems to be no way to undo fiscal commitments just a little bit. Government debts are, unlike currency reforms, delicately set up to form expectations that higher future surpluses will pay them off. Setting up exchange rate pegs and devaluing works individually, but leads to collective retaliation. Central banks, unable to speak softly and carry a big stick, are left with speaking loudly because they have no stick, trying various "managing expectations" and "open-mouth" operations. If they are powerless now, but adopt some new target such as nominal GDP, what do they *do* if nominal GDP falls below the target? Adopt a bigger target still?

Absent a move to an explicit fiscally-backed inflation target with something like a fiscal Taylor rule underlying it, governments could, however, make inflationary fiscal promises. For example, they could simply promise to default on debt in the future, proportional to inflation achieved between now and then. If inflation is insufficient, they will default on a proportional amount of debt. For example, a government could say its inflation target is 3%. If inflation comes in below 3%, say 0%, it will default on 3% of its debt, i.e. each bondholder will receive 97 cents on the dollar. Now the value of government debt must reflect 3% expected inflation. People will try to sell government debt, driving up the value of goods and services until the 3% inflation is achieved.

Default is messy, of course, and this is more a conceptual exercise than a realistic proposal. Like a helicopter drop, the trick is to design an intervention that communicates the desired fiscal backing, and then commits the government to following through.

9.4 A new Fed-Treasury Accord

The strong interactions between monetary and fiscal policy, as well as some peculiarities of the interest on reserves regime, suggest a new Fed-Treasury accord is in order, as Plosser (2009) recommends.

The Fed is now buying just about exactly the same amount of long-term debt as the Treasury is issuing. Why, one thinks, can they not agree on this matter? If the Treasury Bureau of Public Debt were simply to issue short-term debt in the first place, then the Fed would not have to buy up long-term debt in exchange, and then the Fed would not have interest rate risk on its balance sheet. Yet each of Fed and Treasury takes the others' decisions as if made on another planet.

The Fed worries about mark-to-market loss on its balance sheet and potential reduction of payments to the Treasury if interest rates rise. But from a consolidated balance sheet point of view – my view as a taxpayer – it makes not the slightest difference which branch of the government holds interest rate risk.

So, clearly, the new Fed-Treasury accord needs to specify who is in charge of the maturity structure of government debt! That maturity structure has important implications for the timing and stabilization of inflation, as I made clear above, which puts it somewhat in the Fed's corner. But the maturity structure also has important implications for the exposure of the US Treasury to interest rate shocks. By keeping a very short maturity structure in public hands, the US is exposed to roll-over risk when interest rates rise, and has missed the opportunity to lock in amazingly cheap funding for the next 30 years, which so many corporations are doing. A longer maturity structure makes inflation less sensitive to surplus shocks (Cochrane (2001)), again arguing that the Fed (or the agency in charge of inflation) should control it. Last on the list, though top on the list of the Bureau of Public debt, to the extent that there are violations of the expectations hypothesis, or liquidity premiums for specific maturities, appropriate choice of the maturity structure of debt can lower overall borrowing costs. Likewise, the Fed believes in its QE operations that it can exploit violations of the Modigliani-Miller theorem as well as of the expectations hypothesis, to affect the yields of specific Treasury maturities, which runs counter to a Treasury philosophy of simply issuing where rates are cheap.

As above, the appearance interest rate risk on the Fed's balance sheet doesn't cause any economic problem. The Fed's loss is the Treasury's gain. A central bank can have a negative mark-to-market value up to the value of currency outstanding. The Treasury can recapitalize the Fed if needed. But such events have great political if little economic importance, so removing interest rate risk from the Fed's balance sheet is important. Once the Fed and Treasury agree who *decides* the maturity structure in private hands, the Fed should not *hold* interest rate risk.

For both Fed and Treasury, the use of interest-rate swaps would allow a lot more flexibility. If the Fed cannot sell securities back to the Treasury, and does not want to recognize the mark-to-market loss that selling securities on the open market, with coordinated Treasury repurchases, would imply, the Fed and Treasury could engage, possibly through a third party in large fixed-for-floating swaps. Similarly, if the Treasury wants to issue in specific "cheap" maturities without thereby taking on interest rates risk, it can cheaply adjust the interest rate risk independently of the maturity structure by engaging in swap transactions.

Plosser (2009) emphasizes credit risk, and advocates a new Fed-Treasury accord that puts all credit risk, credit subsidization, and credit allocation squarely in the Treasury's hands. I might add interest rate risk as well. But such an accord does not argue against an arbitrarily large balance sheet composed of short-term Treasuries.

These are small matters really, compared with the big one. Fiscal and monetary policy are no longer independent. The big "Fed-Treasury coordination" that is needed is the coordination of monetary and fiscal policy. Implementing an inflation target, understood as a fiscal commitment; implementing a fiscal Taylor rule; implementing a CPI standard regime, all require detailed fiscal as well as monetary plans.

10 Concluding comments: the role of central banks

The interest-on-reserves regime with a big balance sheet is an attractive extension of 30 years of financial and monetary innovation. It gives us interest-paying money, the end of monetary frictions, and the foundation of a more stable financial system in which government short-term debt drives out private short term debt, much as government notes drove out banknotes in the 19th century. But this apparently small extension of our institutions challenges the core of traditional monetary theory.

Some of the questions and doctrines I have addressed: There is no need to fear that the government loses control of inflation in this regime. We *can* have price level control with no control of "money," no rationing of liquidity, no limit on central bank balance sheets, no limit of private intermediation, and under interest rate targets, even targets that violate the Taylor principle. We can enjoy full interest on "monetary" assets. We can be satiated in liquidity. The Federal Reserve has the power to target nominal interest rates in this regime, though whether it can simultaneously control the size of its balance sheet is more open to question. Fortunately the size of its balance sheet is also irrelevant. Interest-paying reserves are not inflationary. The money multiplier, the link between open market operations and lending, and velocity will all become meaningless.

I have explored these issues with extremely simple models. Obviously more realistic models with more realistic pricing frictions, some idea of liquidity frictions between various classes of assets, and producing more interesting dynamics, are all in need of exploration. In particular, though I have shown how purely "monetary" policy without fiscal coordination can produce changes in the real interest rate and output, I have not produced a model with the classical sign, that to tighten the Fed first raises nominal rates, real rates rise, then output and inflation fall. Or perhaps these dynamics are not there in the real world. In the real world, monetary and fiscal policy are always coordinated – changes in expectations of $\sum \beta^j s_{t+j}$ accompany all monetary moves, discount rates loom large in the present value of future surpluses, and there is a complex and state-contingent maturity structure. Mapping to data or policy predictions needs to address all these issues.

We started with what seemed like a minor and rather technical issue – in order to raise interest rates, does the Fed need to sell off its balance sheet, or can it just raise interest on reserves and keep the huge balance sheet. We have ended up, really, at once per century redefinition of role and nature of monetary policies, and of the institutions that generate price stability and financial stability, the proper role of a central bank, the question of what monetary policy can do, what it can't do, what it should do, and what it shouldn't do. The looming reevaluation is as large as Friedman (1968), the last time our views of central banks changed so much.

As a simple example, Friedman (1968) wrote that central banks *can* determine the price level, and their primary mandate *should* be to determine the price level. Plosser (2013 p. 6) echoes this view: "..in a regime with fiat currency, only the central bank can ensure price stability. Indeed, it is the one goal that the central bank can achieve over the longer run." My analysis, following in the footsteps of Sargent and Wallace (1981), denies this claim. As Sargent and Wallace wrote, "Friedman's list of the things that monetary policy cannot permanently control may have to be expanded to include inflation" (as well as output and unemployment). Perhaps they should have said "monetary policy *alone* cannot permanently control," for the central point is that once monetary frictions recede – and even before they recede in Sargent and Wallace's analysis – monetary policy cannot control the price level without fiscal backing. And as monetary frictions vanish, like the Cheshire cat, only the fiscal backing remains.

Friedman's view of the power of monetary policy, was novel and revolutionary at the time. In the preceding Keynesian heyday, inflation was held to come from bargaining or wage-price spirals, or other somewhat mysterious forces, but not, centrally, from actions of the central bank. Monetary policy was considered extremely weak, either to do good or thus to do much bad either. Under the previous gold standard, the price level was determined by that standard, operated primarily by the Treasury, and as we have seen really part of fiscal policy. The central bank had some short term role, manipulating interest rates to manage gold flows, but nobody would have thought the central bank necessary or even primarily important for price-level determination. The U.S. didn't even have a central bank or a monetary policy through much of the 19th century, yet we did have a price level!

As a stylized, and thus surely incorrect, history, central banks started with the Bank of England. Its primary role was to intermediate government debt, as well as to organize bondholders so that debt was more likely to be repaid. By buying government debt and issuing bank liabilities, the Bank of England lowered the sovereign's borrowing costs and provided a useful liquid asset. Its first role was thus simply to manage the liquidity and maturity structure of government debt. In the 19th century, the bank of England evolved its role as lender of last resort and liquidity firehose for banking crises. The Federal Reserve was founded to be a lender of last resort and something of a bank regulator as well. Its monetary mandate was to "provide an elastic currency," rather the opposite of controlling M to control PY. The abandonment of the gold standard, and the move to fiat money changed all that, of course. But it was only with Friedman (1968) that our current conception of a powerful central bank was born.

As I have analyzed it, the role of the central bank will revert to something like what it was under the gold standard. Long-run price stability is a function of the structure of government debt, fiscal promises, and fiscal commitments. The central bank has only a short-run smoothing role, as it did under the gold standard. Like it or not, the central bank retains its role as crisis preventer, on a massive scale. I have argued that by keeping a large balance sheet and encouraging 100% backed institutions to drive out run-prone inside money, the Federal Reserve could do a lot more for financial stability than its current massive regulation and crisis lending, bailing out, and asset-price propping up activities. But like it or not, that will be a role for the central bank in the years ahead. More contentious, the central bank can use its regulatory power to micromanage the economy, channel credit, determine which and which kinds of financial businesses survive, and attempt to influence asset prices. Given that conventional monetary policy is weaker and weaker, ones view on the wisdom of this sort of thing depends on one's view whether the economy and financial system need such extensive dirigisme. I think not, but that is also an issue outside the purely monetary-policy scope of this analysis. As Plosser (2013) writes, "Assigning unachievable goals to organizations is a recipe for failure.. I fear that the public has come to expect too much from its central bank and too much from monetary policy, in particular"

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12 Appendix

12.1 Frictionless Model

This section sets out a very simple, but complete, model of frictionless price determination, to verify that analysis using (1) is not incomplete. This is a simplified version of Cochrane (2005).

The representative household maximizes

$$E\sum_{t=0}^{\infty}\beta^t u(c_t).$$

The household receives a constant endowment $y_t = y$.

It is easier to imagine a sequence of events in each period or day, though that sequencing is not important to the model. Each evening, the government sells a face value B_{t-1} of nominal debt due the next period. Each morning t then, the government prints up B_{t-1} new dollars to pay off the outstanding debt. Households receive the dollars, sell their endowments y for dollars and buy goods c_t for dollars. At the end of the day, they must pay lump sum taxes less transfers $P_t s_t$ in dollars.

I fix the real value of net taxation. This is realistic: With standard income taxes, the nominal amount of taxes are a rate times nominal income, $P_t s_t = \tau P_t y_t$, and if the price level doubles so does the nominal amount of taxes. However, I wish to leave tax distortions out of the model.

The government also sells new debt B_t at a nominal bond price Q_t. The government burns all the cash it has received. The government sets $\{B_t, s_t\}$. The household chooses $\{c_t\}$ and along the way demand for bonds and money, and the price level and asset prices clear markets.

The household period budget constraint is

$$B_{t-1} + P_t y = P_t c_t + P_t s_t + Q_t B_t + M_t$$

where M_t is money held overnight. I assume that the nominal interest rate is positive, so the household chooses zero overnight money holdings, $M_t = 0$ and thus

$$B_{t-1} + P_t y = P_t c_t + P_t s_t + Q_t B_t.$$

The household's first order condition with respect to c_t and c_{t+1} yield

$$Q_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\frac{P_t}{P_{t+1}}\right] = \beta E_t\left[\frac{P_t}{P_{t+1}}\right]$$

where in the second equality I have used the equilibrium condition $c_t = y$. Dividing by P_t and substituting, the money in = money out condition reads

$$\frac{B_{t-1}}{P_t} + y = c_t + s_t + \beta E_t\frac{B_t}{P_{t+1}}$$

Iterating forward,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty}\beta^j\left(c_t - y + s_t\right) + \lim_{j\to\infty}\beta^j E_t\frac{B_{t+j}}{P_{t+j+1}}$$

I impose the usual condition that the last term is zero. This transversality condition is a condition for *household* optimality, and a constraint on household borrowing from the government. If it is positive, then the household can increase consumption over income by simply not buying so much government debt. If it were negative, then the household could roll over debt forever. If we just assume that the government borrows but never lends and the price level is positive, that condition is assured.

Then, the equilibrium condition $c_t = y$ at every date implies

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_t.$$

I took some time to derive this equation in order to emphasize that it is *not* an "intertemporal government budget constraint." It combines the household budget constraint, the household's desire not to hold money, the household optimality condition, and equilibrium in the goods market. If the household wished to die holding money, the government could print money and leave money outstanding. If the household wished to hold ever increasing amounts of government debt, the government would never have to pay its debts, and the transversality condition would not hold. If the household were a growing set of overlapping generations, that condition would not, in fact hold.

While I described a "day," and nominal debt exchanged for money and back again, that feature is clearly inessential. People can transact directly with maturing government debt, and pay taxes or buy new debt by delivering maturing government debt B_{t-1}. While this looks like a cash-in-advance economy it is not by one crucial difference: the securities market is always open, and people can hold zero cash overnight.

12.2 Sticky-Price Model

In this section I build an explicit model in which prices are set one period in advance. The sticky-price setup is a simplification of Galí (1999). The contribution is to combine that price-stickiness framework in a model of fiscal price determination.

Each household derives utility from the consumption of a range of goods j. Its objective is

$$\max_{\{c_{jt}, n_t\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - n_t \right]; \; c_t = \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}.$$

The households' period budget constraint is

$$B_{t-1} + \pi_t = \int_{j=0}^{1} p_{jt} c_{jt} dj + S_t + Q_t B_t$$

and a transversality condition I will describe below. The household enters the period with B_{t-1} face value of government debt, receives profits from selling goods,described below, purchases a range of goods from other households, pays nominal taxes less transfers S_t, and buys new bonds B_t at price Q_t.

Demand for varieties.

We can solve the household problem in two steps: First, find the allocation across goods c_{jt} conditional on the overall level of purchases c_t, and then find the optimal allocation across time

c_t and labor supply decision n_t. We can find the first step by the associated cost minimization problem,

$$\min_{\{c_{jt}\}} \int_{j=0}^{1} p_{jt} c_{jt} dj \text{ s.t. } c_t = \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

The first order conditions for buying good j are

$$p_{jt} = \lambda \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{1}{\sigma-1}} c_{jt}^{-\frac{1}{\sigma}}$$

$$p_{jt} = \lambda \left(\frac{c_{jt}}{c_t} \right)^{-\frac{1}{\sigma}} \tag{36}$$

where λ is the Lagrange multiplier. Raising both sides to the $1-\sigma$ power and integrating to evaluate the multiplier, we have

$$\int p_{jt}^{1-\sigma} dj = \lambda^{1-\sigma} \left(\frac{1}{c_t} \right)^{\frac{\sigma-1}{\sigma}} \int c_{jt}^{\frac{\sigma-1}{\sigma}} dj$$

$$\left[\int p_{jt}^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} = \lambda^{-\sigma} \left(\frac{1}{c_t} \right) \left[\int c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

$$\left\{ \left[\int p_{jt}^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \right\}^{-\frac{1}{\sigma}} = \lambda.$$

Defining the price index

$$p_t \equiv \left[\int p_{jt}^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}},$$

and substituting λ in to (36), we obtain the conditional (on c_t) demand curve for each good,

$$\frac{c_{jt}}{c_t} = \left(\frac{p_{jt}}{p} \right)^{-\sigma}. \tag{37}$$

Total expenditure is

$$\int p_{jt} c_{jt} dj = \frac{c_t}{p^{-\sigma}} \int p_{jt}^{1-\sigma} dj = \frac{c_t}{p^{-\sigma}} p_t^{1-\sigma} = p_t c_t.$$

This lovely result allows us to express the consumer's problem in terms of aggregates. Now, the consumer's problem simplifies to

$$\max_{\{c_t, n_t\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - n_t \right]$$

with budget constraint

$$B_{t-1} + \pi_t = p_t c_t + S_t + Q_t B_t$$

Production.

Each household also owns a firm, which produces only one variety of good using the household's labor, with production function

$$y_{it} = An_t$$

and facing the demand curve given by (37) from all the other households. The household earns $\pi_t = p_{it}y_{it}$. The household's problem is then

$$\max_{\{c_t, n_t, p_{it}\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - n_t \right] \text{ s.t.}$$

$$B_{t-1} + p_{it}y_{it} = p_t c_t + S_t + Q_t B_t$$
$$y_{it} = An_t$$
$$\frac{y_{it}}{y_t} = \left(\frac{p_{it}}{p_t} \right)^{-\sigma}$$

I use y_t in the last equation to emphasize that each household takes the aggregate consumption = output and all the other household's pricing decisions as fixed when making its own output and consumption decisions.

Given the constraints, we can let the household choose price, quantity or labor supply. This being a "sticky price" model, I express the decision in terms of price

$$\max_{\{c_t, n_t, p_{it}\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \frac{y_t}{A} \left(\frac{p_{it}}{p_t} \right)^{-\sigma} \right] \text{ s.t.}$$

$$B_{t-1} + p_{it}y_t \left(\frac{p_{it}}{p_t} \right)^{-\sigma} = p_t c_t + S_t + Q_t B_t \tag{38}$$

Flexible prices.

In the flexible price case, the household can set its price at time t. The first order condition for p_{it} is then

$$\frac{y_t}{A} \sigma \left(\frac{p_{it}}{p} \right)^{-\sigma} \frac{1}{p_{it}} = -\lambda_t (1 - \sigma) y_t \left(\frac{p_{it}}{p} \right)^{-\sigma} \tag{39}$$

where λ_t is the Lagrange multiplier on the nominal period t budget constraint (38), the value of a dollar at time t. Simplifying,

$$p_{it} = \frac{1}{A\lambda_t} \frac{\sigma}{\sigma - 1}$$

This optimal price is the same for all households, so all prices are identical, and

$$p_t = \frac{1}{A\lambda_t} \frac{\sigma}{\sigma - 1} \tag{40}$$

With all prices equal, we have $y_{it} = y_t$ and $n_t = y_t/A$.

Substituting this result in the remaining household problem, we obtain

$$\max_{\{c_t, B_t\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \frac{y_t}{A} \right]$$

$$B_{t-1} + p_t y_t = p_t c_t + S_t + Q_t B_t.$$

Now we can solve the household problem and equilibrium condition. The first order condition with respect to c_t gives

$$u'(c_t) = p_t \lambda_t = \frac{1}{A} \frac{\sigma}{\sigma - 1}. \tag{41}$$

The latter equality comes from the pricing decision (40). This is a frictionless economy, so as in our endowment economy consumption is constant with no real shocks, no matter what happens to nominal quantities.

The first order condition with respect to B_t gives

$$Q_t \lambda_t = E_t \lambda_{t+1}$$

so the bond price satisfies

$$\begin{aligned} Q_t &= E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right] \\ &= \beta E_t \left[\frac{p_t}{p_{t+1}} \right] \end{aligned}$$

(p_t is known at time t, but this is prettier.)

Substituting, the flow budget constraint becomes

$$B_{t-1} + p_t y_t = p_t c_t + S_t + \beta E_t \left[\frac{p_t}{p_{t+1}} \right] B_t.$$

Taxes. The government charges net lump-sum *real* taxes in the amount s_t so $S_t = p_t s_t$. This is not an unnatural assumption. For example, if the government charged a rate τ on nominal income $S_t = \tau p_t y_t$, then the real tax revenue would be fixed $s_t = \tau y_t$. I specify lump sum taxes to avoid dealing with distortions.

Present values. Dividing by p_t

$$\frac{B_{t-1}}{p_t} + y_t = c_t + s_t + \beta E_t \left[\frac{B_t}{p_{t+1}} \right]$$

and iterating forward,

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j \left(c_{t+j} - y_{t+j} + s_{t+j} \right) + \beta E_t \left[\frac{B_{t+k}}{p_{t+k+1}} \right]$$

I impose that the limit of the term on the right hand side is zero. In the positive direction, this is a condition for *consumer* optimality. If not, the consumer could increase consumption and hence utility. In the negative direction, this is a standard no-ponzi condition preventing the consumer from borrowing larger and larger amounts. $B > 0$ and $p > 0$ – the government does not lend, and prices must be positive – serve the same purpose.

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j \left(c_{t+j} - y_{t+j} + s_{t+j} \right)$$

Finally, we impose the equilibrium condition,

$$c_t = y_t.$$

This condition determines the overall price level,

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j s_{t+j}$$

just as in the endowment-economy model.

Prices set one period in advance.

To create a sticky price version of this model, I require that each household set its price p_{it} one period in advance. The household is committed to supply whatever demand there is at the posted price. The demand curve faced by each household producer is still

$$\frac{y_{it}}{y_t} = \left(\frac{p_{it}}{p_t}\right)^{-\sigma}.$$

so, with all prices still equal, individual demand will equal aggregate demand. But now aggregate demand y_t can vary over time.

The first order condition of the problem (38), which I repeat here,

$$\max_{\{c_t, n_t, p_{it}\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \frac{y_t}{A} \left(\frac{p_{it}}{p_t}\right)^{-\sigma} \right] \text{ s.t.}$$

$$B_{t-1} + p_{it} y_t \left(\frac{p_{it}}{p_t}\right)^{-\sigma} = p_t c_t + S_t + Q_t B_t$$

with respect to p_{it} in this case becomes, in place of (39),

$$E_{t-1} \frac{y_t}{A} \sigma \left(\frac{p_{it}}{p}\right)^{-\sigma} \frac{1}{p_{it}} = -E_{t-1} \left[\lambda_t (1-\sigma) y_t \left(\frac{p_{it}}{p}\right)^{-\sigma} \right]$$

which we simplify to

$$p_{it} = \frac{1}{A E_{t-1}(\lambda_t)} \frac{\sigma}{\sigma - 1}.$$

again all prices are identical, and

$$p_t = \frac{1}{A E_{t-1}(\lambda_t)} \frac{\sigma}{\sigma - 1} \tag{42}$$

Output now is $y_{it} = y_t$ and thus labor supply $n_t = y_t/A$. The household problem simplifies then to

$$\max_{\{c_t, B_t\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \frac{y_t}{A} \right]$$

$$B_{t-1} + p_t y_t = p_t c_t + S_t + Q_t B_t.$$

The first order condition with respect to c_t still gives

$$u'(c_t) = p_t \lambda_t.$$

However, the new pricing rule (42) now means (41) becomes

$$E_{t-1}\left[u'(c_t)\right] = p_t E_{t-1}(\lambda_t) = \frac{1}{A}\frac{\sigma}{\sigma-1}. \tag{43}$$

This is really the crucial difference. *Expected* marginal utility is constant. But nominal shocks will have real effects. A too low price will induce too much output, and too much consumption.

The first order condition with respect to B_t gives

$$Q_t \lambda_t = E_t \lambda_{t+1}$$

as before, and thus

$$Q_t = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\frac{p_t}{p_{t+1}}\right].$$

However, from (43), we can no longer conclude that c_t and the real interest rate are constant.

The flow budget constraint becomes

$$B_{t-1} + p_t y_t = p_t c_t + p_t s_t + E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\frac{p_t}{p_{t+1}}\right] B_t$$

$$\frac{B_{t-1}}{p_t} + y_t = c_t + s_t + \beta \frac{E_t u'(c_{t+1})}{u'(c_t)}\frac{B_t}{p_{t+1}}$$

where I have used the fact that p_{t+1} is known at time t. Using (43),

$$u'(c_t)\frac{B_{t-1}}{p_t} = u'(c_t)\left[c_t - y_t + s_t\right] + \beta E_t\left[u'(c_{t+1})\right]\frac{B_t}{p_{t+1}}$$

$$u'(c_t)\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} \beta^j u'(c_{t+j})\left[c_{t+j} - y_{t+j} + s_{t+j}\right]$$

equilibrium $c_t = y_t$ requires that p_t obey

$$u'(c_t)\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} \beta^j \left[u'(c_{t+j}) s_{t+j}\right]$$

Now, in this simple model with one-period price stickiness, we have from (43) that $E_t u'(c_{t+j}) = \frac{1}{A}\frac{\sigma}{\sigma-1}$ for $j \geq 1$. If the covariance between marginal utility and surpluses is zero, then

$$u'(c_t)\frac{B_{t-1}}{p_t} = u'(c_t) s_t + \frac{1}{A}\frac{\sigma}{\sigma-1} E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$$

$$\frac{u'(c_t)}{u'(\bar{c})}\frac{B_{t-1}}{p_t} = \frac{u'(c_t)}{u'(\bar{c})} s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$$

in this case, marginal utility $u'(c_t)$ must do all the adjusting when there is a surplus shock, as the price level cannot move.

12.3 Three-equation model

This section sets out the algebra for the three equation model. Cochrane (2011a, online appendix) has a more extensive treatment.

The model is

$$\begin{aligned} y_t &= E_t y_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) + x_{dt} \\ \pi_t &= \beta E_t \pi_{t+1} + \gamma y_t + x_{\pi t} \\ i_t &= \phi_\pi \pi_t + x_{it} \end{aligned}$$

In vector form,

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ x_{dt+1} \\ x_{\pi t+1} \\ x_{it+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta}\left(\beta+\sigma\gamma\right) & -\frac{\sigma}{\beta}\left(1-\beta\phi_\pi\right) & -1 & \frac{\sigma}{\beta} & \sigma \\ -\frac{\gamma}{\beta} & \frac{1}{\beta} & 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & \rho_d & 0 & 0 \\ 0 & 0 & 0 & \rho_\pi & 0 \\ 0 & 0 & 0 & 0 & \rho_i \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ x_{dt} \\ x_{\pi t} \\ x_{it} \end{bmatrix} + \begin{bmatrix} \delta_{yt+1} \\ \delta_{\pi t+1} \\ \varepsilon_{dt+1} \\ \varepsilon_{\pi t+1} \\ \varepsilon_{it+1} \end{bmatrix}$$

$$X_{t+1} = AX_t + \varepsilon_{t+1}$$

The central issue in this class of models is that the model only determines $E_t y_{t+1}$ and $E_t \pi_{t+1}$. Older Keynesian models had lagged values on the right hand side, and thus no indeterminacy issues.

The solution can be found by eigenvalue decomposing the transition matrix,

$$X_{t+1} = Q\Lambda Q^{-1} X_t + \varepsilon_{t+1}$$

$$Q^{-1} X_{t+1} = \Lambda Q^{-1} X_t + Q^{-1}\varepsilon_{t+1}$$

$$\begin{aligned} Z_{t+1} &= \Lambda Z_t + V_{t+1} \\ z_{jt+1} &= \lambda_j z_{jt} + v_{jt+1} \end{aligned}$$

New Keynesian models overcome indeterminacy with the rule that we pick nonexplosive solutions $\lim_{k\to\infty} E_t Z_{t+k} \to 0$. This rule means that for $\lambda_j > 1$, we must have $z_{it} = 0$. Then, the dynamics of the X variables can be written in terms of the first $K < N$ nonzero z as

$$\begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{Kt} \\ \vdots \\ x_{Nt} \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & \cdots & q_K \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \\ \vdots \\ z_{Kt} \end{bmatrix}$$

$$\begin{bmatrix} z_{1t+1} \\ z_{2t+1} \\ \vdots \\ z_{Kt+1} \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_K \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \\ \vdots \\ z_{Kt} \end{bmatrix} + \begin{bmatrix} v_{1t+1} \\ v_{2t+1} \\ \vdots \\ v_{Kt+1} \end{bmatrix}$$

where q_j denote the columns of Q.

Z is a linear combination of X, so $z_{jt} = 0$ is a relationship linking endogenous variables π_t, y_t to shocks x_t. Equivalently, $z_{jt+1} = 0$ means $v_{jt+1} = 0$. v_{jt+1} is a linear combination of δ and ε shocks, so this requirement picks the shocks δ that index alternative equilibria.

The eigenvalues of the transition matrix are

$$\lambda = \lambda_-, \lambda_+, \rho_d, \rho_\pi, \rho_i$$
$$\lambda_\pm = \frac{1}{2\beta}\left(1+\beta+\sigma\gamma \pm \sqrt{(1+\beta+\sigma\gamma)^2 - 4\beta(1+\sigma\gamma\phi_\pi)}\right)$$

The eigenvectors of the first two (model) eigenvalues are

$$\begin{bmatrix} 1-\beta-\sigma\gamma+\sqrt{(1+\beta+\sigma\gamma)^2-4\beta(1+\sigma\gamma\phi_\pi)} \\ 2\gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \lambda_-,$$

$$\begin{bmatrix} 1-\beta-\sigma\gamma-\sqrt{(1+\beta+\sigma\gamma)^2-4\beta(1+\sigma\gamma\phi_\pi)} \\ 2\gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \lambda_+$$

The eigenvectors of the shock eigenvalues are

$$\begin{bmatrix} 1-\rho_d\beta \\ \gamma \\ (1-\rho_d)(1-\beta\rho_d)+\sigma\gamma(\phi_\pi-\rho_d) \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \rho_d$$

$$\begin{bmatrix} \sigma(\rho_\pi-\phi_\pi) \\ 1-\rho_\pi \\ 0 \\ (1-\rho_\pi)(1-\beta\rho_\pi)+\sigma\gamma(\phi_\pi-\rho_\pi) \\ 0 \end{bmatrix} \leftrightarrow \rho_\pi$$

$$\begin{bmatrix} -\sigma(1-\rho_i\beta) \\ -\sigma\gamma \\ 0 \\ 0 \\ (1-\rho_i)(1-\rho_i\beta)+\sigma\gamma(\phi_\pi-\rho_i) \end{bmatrix} \leftrightarrow \rho_i$$

In the standard new-Keynesian equilibrium selection, we assume $\phi > 1$.Then both $\lambda_+ > 1$ and $\lambda_- > 1$, two z are equal to zero so we determine both y and π. The model dynamics can then be written

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1-\rho_d\beta & \sigma(\rho_\pi-\phi_{\pi,0}) & -\sigma(1-\rho_i\beta) \\ \gamma & 1-\rho_\pi & -\sigma\gamma \end{bmatrix} \begin{bmatrix} z_{dt} \\ z_{\pi t} \\ z_{it} \end{bmatrix}$$

$$\begin{bmatrix} z_{dt} \\ z_{\pi t} \\ z_{it} \end{bmatrix} = \begin{bmatrix} \rho_d & 0 & 0 \\ 0 & \rho_\pi & 0 \\ 0 & 0 & \rho_i \end{bmatrix} \begin{bmatrix} z_{dt-1} \\ z_{\pi t-1} \\ z_{it-1} \end{bmatrix} + \begin{bmatrix} v_{dt} \\ v_{\pi t} \\ v_{it} \end{bmatrix}$$

where the z and the x are related by

$$\begin{aligned} x_{dt} &= \left[(1-\rho_d)(1-\rho_d\beta) + \sigma\gamma(\phi_\pi - \rho_d)\right] z_{dt} \\ x_{\pi t} &= \left[(1-\rho_\pi)(1-\rho_\pi\beta) + \sigma\gamma(\phi_\pi - \rho_\pi)\right] z_{\pi t} \\ x_{it} &= \left[(1-\rho_i)(1-\rho_i\beta) + \sigma\gamma(\phi_\pi - \rho_i)\right] z_{it}. \end{aligned}$$

Similarly the shocks v are related to fundamental shocks x by

$$\begin{aligned} \varepsilon_{dt} &= \left[(1-\rho_d)(1-\rho_d\beta) + \sigma\gamma(\phi_\pi - \rho_d)\right] v_{dt} \\ \varepsilon_{\pi t} &= \left[(1-\rho_\pi)(1-\rho_\pi\beta) + \sigma\gamma(\phi_\pi - \rho_\pi)\right] v_{\pi t} \\ \varepsilon_{it} &= \left[(1-\rho_i)(1-\rho_i\beta) + \sigma\gamma(\phi_\pi - \rho_i)\right] v_{it}. \end{aligned}$$

It's interesting to carry along the i response. From

$$i_t = \phi_\pi \pi_t + x_{it},$$

we can simply append the i to the response variables as

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1-\rho_d\beta & \sigma(\rho_\pi - \phi_{\pi,0}) & -\sigma(1-\rho_i\beta) \\ \gamma & 1-\rho_\pi & -\sigma\gamma \\ \gamma\phi_\pi & (1-\rho_\pi)\phi_\pi & -\sigma\gamma\rho_i + (1-\rho_i)(1-\rho_i\beta) \end{bmatrix} \begin{bmatrix} z_{dt} \\ z_{\pi t} \\ z_{it} \end{bmatrix}$$

For the response to a monetary policy shock, we only need the last column.

In the end, then, I plot the response to a monetary policy shock by simulating forward

$$\begin{aligned} \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} &= \begin{bmatrix} -\sigma(1-\rho_i\beta) \\ -\sigma\gamma \\ -\sigma\gamma\rho_i + (1-\rho_i)(1-\rho_i\beta) \end{bmatrix} \begin{bmatrix} z_{it} \end{bmatrix} \\ z_{it} &= \rho_i z_{it-1} + v_{it} \\ v_{it} &= \frac{\varepsilon_{it}}{(1-\rho_i)(1-\rho_i\beta) + \sigma\gamma(\phi_\pi - \rho_i)} \\ z_{it} &= \frac{x_{it}}{(1-\rho_i)(1-\rho_i\beta) + \sigma\gamma(\phi_\pi - \rho_i)} \end{aligned}$$

Fiscal solution.

In the fiscal solution, we pick the inflation shock directly, from the change in present value of future surpluses. Ideally, the present value should contain interest rates and risk premiums as well as surpluses. Monetary policy may affect the present value of surpluse by changing discount rates, even if it cannot change surpluses. We should also have a serious analysis of monetary and fiscal policy coordination. For my illustrative calculation, I will simply choose to pair monetary policy with no change in present value of surpluses, as I have done in the other illustrative calculations, $\pi_{t+1} - E_t\pi_{t+1} = 0$. This choice generally does not mean no change in surpluses, but a change in surpluses that matches the change in discount rate effects on their present values.

Since we pick one innovation $\delta_{\pi_{t+1}} = 0$, we only need one eigvenvalue greater than one. Hence, following the usual rules, we need a "passive" monetary policy $\phi < 1$. This choice implies $\lambda_+ > 1$ but $\lambda_- < 1$. Hence, the model dynamics keep an additional eigenvector,

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1-\rho_d\beta & \sigma(\rho_\pi - \phi_\pi) & -\sigma(1-\rho_i\beta) & k \\ \gamma & 1-\rho_\pi & -\sigma\gamma & 2\gamma \\ \phi_\pi\gamma & \phi_\pi(1-\rho_\pi) & (1-\rho_i)(1-\rho_i\beta) - \sigma\gamma\rho_i & 2\gamma\phi_\pi \end{bmatrix} \begin{bmatrix} z_{dt} \\ z_{\pi t} \\ z_{it} \\ z_{\lambda t} \end{bmatrix}$$

$$k = 1 - \beta - \sigma\gamma + \sqrt{(1+\beta+\sigma\gamma)^2 - 4\beta(1+\sigma\gamma\phi_\pi)}$$

and we add

$$z_{\lambda t+1} = \lambda_- z_{\lambda t} + \delta_{\lambda t+1}$$

Now we can compute responses to fiscal shocks, identified by the innovation in π_{t+1}, and to other shocks orthgonalized, i.e. holdng fiscal shocks and thus the innovation in inflation constant.

To impose no shock to inflation, we must have

$$\begin{bmatrix} \gamma & 1-\rho_\pi & -\sigma\gamma & 2\gamma \end{bmatrix} \begin{bmatrix} v_{dt} \\ v_{\pi t} \\ v_{it} \\ v_{\lambda t} \end{bmatrix} = 0$$

In my calculations, when there is only a monetary policy shock v_i, this means

$$\begin{bmatrix} -\sigma\gamma & 2\gamma \end{bmatrix} \begin{bmatrix} v_{it} \\ v_{\lambda t} \end{bmatrix} = 0,$$

i.e. we pair the v_i shock with a contemporaneous

$$v_{\lambda t} = \frac{\sigma}{2} v_{it}.$$

In sum, then, to find the response to a monetary policy shock, we simulate

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} -\sigma(1-\rho_i\beta) & 1-\beta-\sigma\gamma+\sqrt{(1+\beta+\sigma\gamma)^2 - 4\beta(1+\sigma\gamma\phi_\pi)} \\ -\sigma\gamma & 2\gamma \\ (1-\rho_i)(1-\rho_i\beta) - \sigma\gamma\rho_i & 2\gamma\phi_\pi \end{bmatrix} \begin{bmatrix} z_{it} \\ z_{\lambda t} \end{bmatrix}$$

$$z_{it+1} = \rho_i z_{it}$$

$$z_{\lambda t+1} = \lambda_- z_{\lambda t}$$

$$z_{i1} = v_{i1} = 1/\left[(1-\rho_i)(1-\rho_i\beta) + \sigma\gamma(\phi_\pi - \rho_i)\right]$$

$$v_{\lambda 1} = z_{\lambda 1} = \sigma/2 \; v_{i1}$$