Sticky Leverage

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Abstract

We develop a tractable general equilibrium model that captures the interplay between nominal long-term corporate debt, inflation, and real aggregates. We show that unanticipated inflation changes the real burden of debt and, more significantly, leads to a debt overhang that distorts future investment and production decisions. For these effects to be both large and very persistent it is essential that debt maturity exceeds one period. Finally we also show that interest rates rules can help stabilize our economy. (Key words: Debt deflation, debt overhang, monetary non-neutrality)

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1 Introduction

The onset of the financial crisis in 2008 triggered the most aggressive monetary policy response in developed countries in at least 30 years. At the same time, financial markets now occupy a much more prominent role in modern macroeconomic theory. Typical models of financial frictions focus on debt and identify leverage as both a source, and an important mechanism of transmission, of economic fluctuations.[1] Surprisingly, the fact that debt contracts are almost always denominated in nominal terms is usually ignored in the literature.[2,3] Yet, nominal debt creates an obvious link between inflation and the real economy, a potentially important source of monetary nonneutrality even with fully flexible prices.

The goal of this paper is to develop a tractable general equilibrium model that captures the interplay between nominal debt, inflation, and real aggregates, and explore some of its main implications. In our model, as in reality, firms fund themselves by choosing the appropriate mix of nominal defaultable debt and equity securities to issue in every period. Debt is priced fairly by bondholders, who take into account default and inflation risk, but is attractive to issue because of the tax-deductibility of interest payments. Macroeconomic quantities are obtained by aggregating across the optimal decisions of each firm and by ensuring consistency with the consumption, savings and labor choices of representative households.

[1] Some examples include Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Cooley, Marimon and Quadrini (2004), Gourio (2012), Jermann and Quadrini (2012), and Gomes and Schmid (2013).

[2] Among the very rare exceptions are Dopke and Schneider (2006), Christiano, Motto, and Rostagno (2009), Fernandez-Vilaverde (2010), Bhamra et al (2011), and De Fiore et al. (2011).

[3] At the end of 2012 U.S. non-financial businesses alone had nearly 12.5 trillion dollars in outstanding credit market debt - about 75% of GDP. Nearly all of these instruments are in the form of nominal liabilities, often issued at fixed rates of interest.

We have two main results. First, because debt contracts are written in nominal terms, unanticipated changes in inflation, regardless of their source, always have real effects, even if prices and wages are fully flexible. In particular, lower than expected inflation increases the real value of debt, worsens firms' balance sheets, and makes them more likely to default. If defaults and bankruptcies have resource costs, this immediately and adversely impacts output and consumption. More importantly however, when debt is long-lived, low inflation *endogenously* creates a debt overhang that persists for many periods - even though debt is freely adjustable.

As a result, even non-defaulting firms must begin to cut future investment and production plans, as the increased (real) debt lowers the expected rewards to their equity owners. It is this debt overhang phenomenon - emphasized in empirical studies of financial crises and almost entirely missing from standard models with only short term debt - that accounts for most of the effects of changes in inflation on the economy.[4]

Second, by adding a monetary policy rule linking short term nominal interest rates to inflation and output, our setting offers a different insight into the ongoing monetary stimulus around the world. In particular, a standard Taylor rule parameterization implies that central banks should try to raise the rate of inflation in response to adverse real shocks, such as declines in productivity and in wealth.

The friction we emphasize is probably not suited to understand the response of the economy to all shocks. Nevertheless, we believe an environment with long-term nominal debt contracts offers a clearer understanding of financially driven recessions than traditional models emphasizing sticky prices and wages. While those models often imply that adverse shocks would be miti-

[4]Recent examples include Reinhart and Rogoff (2011) and Mian and Sufi (2011).

gated if prices were allowed to fall, our setting suggests the exact opposite. In particular, as Fisher (1933) suggests, deflation would only magnify the real burden of debt and further worsen economic activity. The monetary policy implications are also subtly different. In our model, central banks should respond to episodes of excessive leverage not necessarily by lowering the effective real interest rate in the economy but by actively and immediately pushing up the rate of inflation.

While the notion that a debt deflation may have significant macroeconomic consequences goes back at least to Fisher (1933), it has not been incorporated into the modern quantitative macroeconomic literature until quite recently. Our paper contributes to the literature by introducing nominal long-term debt in an aggregate business cycle model and studying its role as a nominal transmission channel.

Another important novelty of our paper is that we solve for firms' time-consistent optimal policies for long-term debt when firms can adjust debt freely every period. We present a numerical approach that allows the analysis of model dynamics with perturbation techniques, and as such alleviates the curse of dimensionality of fully nonlinear global methods.[5]

Other macro economic analyses with long-term debt and default include Gomes and Schmid (2013) and Miao and Wang (2010). In both cases the debt is real. In Gomes and Schmid (2013) firms pick their debt at the time of birth and face costly adjustment thereafter. Their focus is on the role of asset prices to capture firm heterogeneity and forecast business cycles. The setting in Miao and Wang (2010) is closer to ours but in their model firms act myopically and fail to take into account that their current leverage choice

[5] A similar time consistency issue arises in dynamic public policy problems, as studied, for instance, by Klein, Krusell and Rios-Rull (2008).

influences future leverage, and through that, the current value of debt. As a result their approach is not really suitable to fully understand the effects of debt overhang.

The asset pricing implications of allowing for nominal corporate debt in a model driven by productivity and inflation shocks is studied by Kang and Pflueger (2012). Their empirical analysis supports the view that inflation uncertainty raises corporate default rates and bond risk premiums. Their model assumes constant labor and considers only two-period debt.

Fernandez-Villaverde (2010) considers nominal government debt, and Christiano, Motto, and Rostagno (2009) embed nominal entrepreneurial loans into a medium-scale DSGE model. Both use short-term debt and emphasize the role of sticky prices. De Fiore et al. (2011) examine optimal monetary policy when firms have nominal debt with default risk. Eggertson and Krugmann (2012) study leverage and debt overhang with short term debt, but their focus is on sticky prices and the role of fiscal policy, and investment plays no role in the analysis. Neither offers a quantitative analysis and both rely on short-term debt. Occhino and Pescatori (2012a,b) examine debt overhang with one-period debt. Jermann and Yue (2013) study interest rate swaps in conjunction with short-term nominal debt and default.

Some studies on sovereign default have also considered long-term debt in equilibrium models, in particular, Arellano and Ramanarayanan (2012), Hatchondo, Martinez and Sosa Padilla (2014) and Aguiar and Amador (2014). In these studies, debt is real and the problem of a sovereign differs along several dimensions from the problem of a firm in our model. For instance, firms in our model have the ability to issue equity to reduce debt, while sovereigns do not have that choice.

Debt overhang has been studied in the corporate finance literature, but

usually in static (real) models that focus solely on optimal firm decisions and where debt overhang arises exogenously. An early example is Myers (1977), and recent dynamic models are provided in Hennessy (2004), Moyen (2005), and Chen and Manso (2010).

More broadly, our paper also expands on the growing literature on the macroeconomic effects of financial frictions. This includes Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Cooley, Marimon and Quadrini (2004), Gertler and Karadi (2011) and Jermann and Quadrini (2012).

The next section describes our model. Section 3 shows some key properties of the model regarding the real effects of inflation and outlines our solution strategy. Section 4 discusses the calibration of our baseline model. Section 5 shows our quantitative findings and is followed by some concluding remarks.

2 Model

To focus on the novel mechanisms associated with long-term nominal debt financing and investment, the model is kept as parsimonious as possible. Firms own the productive technology and the capital stock in this economy. They are operated by equity holders but partially financed by defaultable debt claims. The firms' optimal choices are distorted by taxes and default costs. Households consume the firms' output and invest any savings in the securities issued by firms. The government plays a minimal role: it collects taxes on corporate income and rebates the revenues to the households in lump-sum fashion.

2.1 Firms

We start by describing the behavior of firms and its investors in detail. At any point in time production and investment take place in a continuum of measure one of firms, indexed by j. Some of these firms will default on their debt obligations, in which case they are restructured before resuming operations again. This means that firms remain on-going concerns at all times, so that their measure remains unchanged through time. Although this is not an essential assumption, it greatly enhances tractability to use an environment where all firms make identical choices.[6]

2.1.1 Technology

Each firm produces according to the function:

$$y_t^j = A_t F\left(k_t^j, n_t^j\right) = A_t \left(k\right)^{\alpha} n^{1-\alpha}, \tag{1}$$

where A_t is aggregate productivity. Solving for the static labor choice we get the firms' operating profit:

$$R_t k_t^j = \max_{n_t^j} A_t F\left(k_t^j, n_t^j\right) - w_t n_t^j \tag{2}$$

where $R_t = \alpha y_t / k_t$ is the implicit equilibrium "rental rate" on capital. Given constant returns to scale, all firms chose identical ratios k^j/n^j, so R_t is identical across firms.

Firm level profits are also subject to additive idiosyncratic shocks, $z_t^j k_t^j$, so

[6] Gilchrist, Sim and Zakrajsek (2010) and Gomes and Schmid (2013) present models where the cross-section of firms moves over time with entry and default events.

that operating profits are equal to:

$$\left(R_t - z_t^j\right) k_t^j. \tag{3}$$

We assume that z_t^j is i.i.d. across firms and time, has mean zero, and cumulative distribution $\Phi(z)$ over the interval $[\underline{z}, \bar{z}]$, with $\int_{\underline{z}}^{\bar{z}} \phi(z)\, dz = \int d\Phi(z)$. We think of these as direct shocks to firms' operating income and not necessarily output. They summarize the overall firm specific component of their business risk. Although they average to zero in the cross section, they can potentially be very large for any individual firm.

Finally, firm level capital accumulation is given by the identity:

$$k_{t+1}^j = \left(1 - \delta + i_t^j\right) k_t^j \equiv g(i_t^j) k_t^j \tag{4}$$

where i_t^j denotes the investment to capital ratio.

2.1.2 Financing

Firms fund themselves by issuing both equity and defaultable nominal debt. Let B_t^j denote the stock of outstanding defaultable nominal debt at the beginning of period t.

To capture the fact that outstanding debt is of finite maturity, we assume that in every period t a fraction λ of the principal is paid back, while the remaining $(1 - \lambda)$ remains outstanding. This means that the debt has an expected life of $1/\lambda$. In addition to principal amortization, the firm is also required to pay a periodic coupon c per unit of outstanding debt.

Letting q_t^j denote the market price of one unit of debt in terms of consumption goods during period t, it follows that the (real) market value of new debt

issues during period t is given by:

$$q_t^j(B_{t+1}^j - (1-\lambda)B_t^j)/P_t = q_t^j(b_{t+1}^j - (1-\lambda)b_t^j/\mu_t) \tag{5}$$

where $b_t^j = B_t^j/P_{t-1}$, P_t is the overall price level in period t, and we define $\mu_t = P_t/P_{t-1}$ as the economy wide rate of inflation between period $t-1$ and t. We will work with the real value of these outstanding liabilities throughout the remainder of the paper.

2.1.3 Dividends and equity value

In the absence of new debt issues, (real) distributions to shareholders are equal to:

$$(1-\tau)\left(R_t - z_t^j\right)k_t^j - ((1-\tau)c + \lambda)\frac{b_t^j}{\mu_t} - i_t^j k_t^j + \tau\delta k_t^j.$$

where τ is the firm's effective tax rate. The first term captures the firm's operating profits, from which we deduct the required debt repayments and investment expenses and add the tax shields accrued through depreciation expenditures. This expression for equity distributions is consistent with the fact that interest payments are tax deductible.

It follows that the value of the firm to its shareholders, denoted $J(\cdot)$, is the present value of these distributions plus the value of any new debt issues. It is useful to write this value function in two parts, as follows:

$$J\left(k_t^j, b_t^j, z_t^j, \mu_t\right) = \max\left[0, (1-\tau)\left(R_t - z_t^j\right)k_t^j - ((1-\tau)c + \lambda)\frac{b_t^j}{\mu_t} + V\left(k_t^j, b_t^j, \mu_t\right)\right] \tag{6}$$

where the continuation value $V(\cdot)$ obeys the following Bellman equation:

$$V\left(k_t^j, b_t^j, \mu_t\right) = \max_{b_{t+1}^j, k_{t+1}^j} \left\{ \begin{array}{c} q_t^j \left(b_{t+1}^j - (1-\lambda)\frac{b_t^j}{\mu_t}\right) - (i_t^j - \tau\delta)k_t^j \\ +\mathrm{E}_t M_{t,t+1} \int_{\underline{z}}^{\bar{z}} J\left(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j, \mu_{t+1}\right) d\Phi\left(z_{t+1}\right) \end{array} \right\} \tag{7}$$

where the conditional expectation $\mathrm{E_t}$ is taken only over the distribution of aggregate shocks. This value function $V(\cdot)$ thus summarizes the effects of the decisions about future investment and financing on equity values.

Several observations about the value of equity (6) will be useful later. First, limited liability implies that equity value, $J(\cdot)$, is bounded and will never fall below zero. This implies that equity holders will default on their credit obligations whenever their idiosyncratic profit shock z_t^j is above a cutoff level $z_t^* \leq \bar{z}$, defined by the expression:

$$(1-\tau)\left(R_t - z_t^{j*}\right)k_t^j - \left((1-\tau)\,c + \lambda\right)\frac{b_t^j}{\mu_t} + V\left(k_t^j, b_t^j, \mu_t\right) = 0. \tag{8}$$

It is this value z_{t+1}^{j*} that truncates the integral in the continuation value of (7).

Second, the stochastic discount factor $M_{t,t+1}$ is exogenous to the firm and must be determined in equilibrium, in a manner consistent with the behavior of households/investors. Finally, the value function is homogenous of degree one in capital k_t^j and debt b_t^j and so is the default cutoff z_t^{j*}.

2.1.4 Default and credit risk

The firm's creditors buy corporate debt, at price q_t^j, and collect coupon and principal payments, $(c+\lambda)\frac{b_{t+1}^j}{\mu_{t+1}}$, until the firm defaults.

In default, shareholders walk away from the firm, while creditors take over and restructure the firm. Creditors become the sole owners and investors

of the firm and are entitled to collect the current after-tax operating income $(1-\tau)\left(R_{t+1}-z_{t+1}^{j}\right)k_{t+1}^{j}$. After this restructuring, creditors resume their customary role by selling off the equity portion to new owners while continuing to hold the remaining debt. This means that in addition to the current cash flows, the creditors have a claim that equals the total enterprise, or asset, value, $V\left(k_{t+1}^{j},b_{t+1}^{j}\right)+q_{t+1}^{j}\left(1-\lambda\right)b_{t+1}^{j}$.[7]

Restructuring entails a separate loss, in the amount ξk_{t+1}^{j}, with $\xi\in[0,1]$.[8]

With these assumptions, the creditors' valuation of their holdings of corporate debt at the end of period t is:[9]

$$b_{t+1}^{j}q_{t}^{j}=\mathrm{E}_{t}M_{t,t+1}\left\{\begin{array}{c}\Phi(z_{t+1}^{j*})\left[c+\lambda+\left(1-\lambda\right)q_{t+1}^{j}\right]\frac{b_{t+1}^{j}}{\mu_{t+1}}+\int_{z_{t+1}^{j*}}^{\bar{z}}\left[\left(1-\tau\right)\left(R_{t+1}-z_{t+1}^{j}\right)k_{t+1}^{j}\right.\\ \left.+V\left(k_{t+1}^{j},b_{t+1}^{j},\mu_{t+1}\right)+\left(1-\lambda\right)\frac{q_{t+1}^{j}b_{t+1}^{j}}{\mu_{t+1}}-\xi k_{t+1}^{j}\right]d\Phi(z_{t+1})\end{array}\right\}. \tag{9}$$

The right hand side of this expression can be divided in two parts. The first term reflects the cash flows received if no default takes place, while the integral contains the payments in default, net of the restructuring charges.

It is immediate to establish that this market value of corporate debt is decreasing in restructuring losses, ξ, and the default probability, implied by the cutoff z^{j*}. It can also be shown that debt prices are declining in the expected rate of inflation - since equity values increase in μ_{t+1}. Finally, note that q_{t}^{j} is also homogeneous of degree zero in k_{t+1}^{j} and b_{t+1}^{j}.

[7] This is only one of several equivalent ways of describing the bankruptcy procedures that yields the same payoffs for shareholders and creditors upon default. Equivalently we could assume that they sell debt and continue to run the firm as the new equity holders.

[8] We can think of these costs as including legal fees, but also other efficiency losses and frictions associated with the bankruptcy and restructuring processes. These costs represent a collective loss for bond and equity holders, and may also imply a loss of resources for the economy as a whole.

[9] Note that creditors discount the future using the same discount factor as shareholders, $M_{t,t+1}$. This is consistent with our assumption that they belong to the same risk-sharing household.

All together, our assumptions ensure that when the restructuring process is complete a defaulting firm is indistinguishable from a non-defaulting firm. All losses take place in the current period and are absorbed by the creditors. Since all idiosyncratic shocks are i.i.d. and there are no adjustment costs, default has no further consequences. As a result, both defaulting and non-defaulting firms adopt the same optimal policies and look identical at the beginning of the next period.

2.2 Households

The general equilibrium is completed with the household sector. This is made of a single representative family that owns all securities and collects all income in the economy, including a rebate on corporate income tax revenues. Preferences are time-separable over consumption C and hours worked, N:

$$U = \mathrm{E}\left\{\sum_{t=0}^{\infty} \beta^t \frac{\left[u\left(C_t, N_t\right)\right]^{1-\sigma} - 1}{1-\sigma}\right\} \tag{10}$$

where the parameters $\beta \in (0,1)$ and $\sigma > 0$ are tied to the rate of inter temporal preference and household risk aversion. We further assume that momentary utility is described by the Cobb-Douglas function:

$$u\left(C_t, N_t\right) = C_t^{1-\theta}\left(3 - N_t\right)^{\theta} \tag{11}$$

where the value of θ will be linked to the elasticity of labor supply.

As is common in the literature, we find it useful to assume that each member of the family works or invests independently in equities and debt, and all household income is then shared when making consumption and savings decisions.

2.3 Equilibrium and aggregation

Given the optimal decisions of firms and households implied by the problems above we can now characterize the dynamic competitive equilibrium in this economy.

We focus on Markov perfect equilibria where the aggregate state vector is $s = (B, K, \mu, A)$, where B and K denote the aggregate levels of debt and capital in the economy. The nature of the problem means that, outside default, this equilibrium is symmetric, in the sense that all firms make identical decisions at all times. The only meaningful cross-sectional difference concerns the realization of the shocks z_t^j which induce default for a subgroup of firms with mass $1 - \Phi(z^*)$. Default implies a one-time restructuring charge for firms, but these temporary losses have no further impact on the choices concerning future capital and debt. Thus all firms remain ex-ante identical in all periods so that we can drop all subscripts j for firm specific variables so that in equilibrium $B_t = b_t$ and $K_t = k_t$.

Aggregate output in the economy, Y_t, can be expressed as:

$$Y_t = y_t \ - \left[1 - \Phi\left(z^*\right)\right] \xi^r \xi k_t. \tag{12}$$

As discussed above, ξk_t captures the loss that creditors suffer in bankruptcy. Some of these losses may be in the form of legal fees and might be recouped by other members of the representative family. But some may represent a genuine destruction of resources. The relative balance between these two alternatives is governed by the parameter $\xi^r \in [0, 1]$. In the special case where $\xi^r = 0$, default entails no loss of resources at the aggregate level.

The aggregate capital stock, $K_t = k_t$, obeys the law of motion:

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{13}$$

where aggregate investment is $I_t = i_t k_t$.

To complete the description of the economy we require that both goods and labor markets clear. This is accomplished by imposing the aggregate resource constraint:

$$Y_t = C_t + I_t \tag{14}$$

and the labor market consistency condition:

$$N_t = n_t. \tag{15}$$

3 Characterization

To highlight the economic mechanisms at the heart of the model, we now provide a detailed characterization of the firms' leverage choice.

We establish two sets of results in this section. First, we show analytically that, with long-term debt, real leverage responds persistently to i.i.d. inflation shocks. That is, nominal leverage is effectively sticky. A debt overhang channel then transmits changes in real leverage to changes in real investment. Second, we show that the solution of the firm's problem is characterized by a generalized Euler equation in which the policy function enters jointly with its derivative. We present a numerical algorithm that allows us to apply first-order perturbation techniques to study this class of models.

Under constant returns to scale, the firm's problem is linearly homogenous in capital and therefore it has leverage, $\omega = b/k$, as a single endogenous state

variable. Conditionally on not defaulting the value of a firm per unit of capital, $v(\omega) = V/k$, can be written as:

$$v(\omega) = \max_{\omega', i} \left\{ \begin{array}{c} q\left(\omega' g(i) - (1-\lambda)\frac{\omega}{\mu}\right) - i + \tau\delta \\ +g(i)\mathrm{E}M' \int_{\underline{z}}^{z^{*\prime}} \left[(1-\tau)(R'-z') - ((1-\tau)c+\lambda)\frac{\omega'}{\mu'} + v(\omega')\right] d\Phi(z') \end{array} \right\} \tag{16}$$

where we use primes to denote future values, and the definition $g(i) = (1-\delta+i)k$. For ease of notation we omit the dependence on the aggregate state variables for the functions $v(\omega)$, $q(\omega')$, as well as for prices M and R.

The market value of the outstanding debt (9) can be expressed as:

$$\omega' q(\omega') = \mathrm{E}M' \left\{ \begin{array}{c} \Phi(z^{*\prime})[c+\lambda]\frac{\omega'}{\mu'} + (1-\lambda)\frac{q'(h(\omega'))\omega'}{\mu'} \\ +(1-\Phi(z^{*\prime}))[(1-\tau)R' - \xi + v(\omega')] - (1-\tau)\int_{z^{*\prime}}^{\bar{z}} z' d\Phi(z) \end{array} \right\}. \tag{17}$$

Explicitly writing next period's debt price $q(h(\omega'))$ as a function of the optimal policy, $\omega' = h(\omega)$, on the right hand side of equation (17), highlights the potential time-inconsistency problem faced by the firm. With long-lived debt, the price of debt $q(\omega')$ depends on future debt prices and thus on next period's leverage choice ω''. As no commitment technology is available, time-consistency requires that next period's leverage be a function of the current policy choice, so that $\omega'' = h(\omega')$.

Finally, the optimal default cutoff level, z^*, can be expressed as a function of the leverage ratio, as:

$$z^*(\omega) = R - c\frac{\omega}{\mu} - \frac{\lambda}{(1-\tau)}\frac{\omega}{\mu} + \frac{1}{(1-\tau)}v(\omega). \tag{18}$$

Differentiating this expression with respect to outstanding leverage ω we get:

$$\frac{\partial z^{*}(\omega)}{\partial \omega} = -\left(c + \frac{\lambda}{1-\tau}\right)\frac{1}{\mu} + \frac{1}{(1-\tau)}\frac{\partial v(\omega)}{\partial \omega} < 0. \tag{19}$$

Intuitively, an increase in outstanding debt increases the required principal and coupon payments, and by reducing the cutoff $z^{*}(\omega)$, makes default more likely.[10]

3.1 Debt overhang and the impact of inflation

We now characterize firm behavior in response to a change in the inflation rate, μ. To isolate how a shock to μ is propagated in the model, it is assumed in this section that inflation follows an exogenous i.i.d process. For the quantitative analysis in the next section, the inflation rate is persistent and endogenously driven by real and monetary shocks.

The necessary first-order conditions with respect to investment and leverage are given by:[11],

$$1 - q(\omega')\omega' = \mathrm{E}M' \int_{\underline{z}}^{z^{*}(\omega')} \left[(1-\tau)(R' - z') - ((1-\tau)c + \lambda)\frac{\omega'}{\mu'} + v(\omega')\right] d\Phi(z') \tag{21}$$

[10]The envelope condition implies that:

$$\frac{\partial v(\omega)}{\partial \omega} = -q\frac{1-\lambda}{\mu} \leq 0. \tag{20}$$

When debt maturity exceeds one period ($\lambda < 1$) an increase in outstanding debt decreases the (expected) future payments to equity holders further encouraging default.

[11]We assume throughout this section that first order conditions are also sufficient. It is straightforward to derive conditions on the distribution for the idiosyncratic shock $\Phi(z)$ to guarantee that this is true when $\lambda = 1$.

and

$$q(\omega')\,g(i)+\frac{\partial q(\omega')}{\partial \omega'}\left(\omega' g(i)-(1-\lambda)\frac{\omega}{\mu}\right)=-(1-\tau)\,g(i)\,\mathrm{E}M'\Phi(z^{*\prime})\frac{\partial z^{*}(\omega')}{\partial \omega'}. \tag{22}$$

The first order condition for investment, (21), equates the marginal reduction in equity cash flows today, on the left hand side, to the expected increase in (after tax) dividend and capital gains tomorrow, after netting out any debt payments.

The optimal condition for leverage, (22), recognizes that the debt price, $q(\omega')$, falls when new debt is issued, $\frac{\partial q(\omega')}{\partial \omega'} < 0$, thus reducing the marginal benefits of more debt today. The marginal costs of new debt, on the right hand side, reflects the impact of new debt on the probability of future default, $\frac{\partial z^{*}(\omega')}{\partial \omega'}$.

We now show that the effect of unanticipated inflation on leverage depends crucially on the maturity of debt. Proposition 1 establishes this result under general conditions.

Proposition 1. *Consider an economy where optimal choices are described by the optimality conditions (21)-(22), and where:*

- *there are no aggregate resource costs associated with bankruptcy, i.e.,* $\xi^{r}=0$*; and*
- *all realizations of exogenous shocks have been zero for a long time so that at time* $t-1$ $\mu_{t-1}=\mu$ *and* $\omega_t=\omega$*, are at their steady state values.*

Suppose that at time t *the economy experiences a temporary decline in the inflation rate so that* $\mu_t<\mu_{t-1}$*. Then* $\omega_{t+1}\geq\omega_t$*, with equality if and only if* $\lambda=1$*.*

Proof. With $\lambda = 1$ the current inflation rate, μ_t, has no direct effect on the choice of $\omega' = \omega_{t+1}$ in (22). Moreover, since

- μ_t is i.i.d., there is no effect on the expected default cutoff (18) and the equilibrium price of debt, $q(\omega')$;
- $\xi^r = 0$, there are no resource costs and neither aggregate consumption, C, nor the stochastic discount factor, M', are affected;

It follows that there are no indirect general equilibrium effects either, and the optimal choice of leverage, ω_{t+1} is unaffected by the shock.

However, when $\lambda < 1$ a decline in μ raises the marginal benefit of issuing new debt (since $\frac{\partial q(\omega')}{\partial \omega'} \leq 0$) and ω' will increase accordingly. ∎

Proposition 1 establishes that even i.i.d movements in inflation will be propagated when debt maturity is not instantaneous. This is because a change in the existing leverage has a direct impact on the marginal benefits of leverage.

Intuitively an unanticipated decline in the rate of inflation μ increases the (real) value of currently outstanding liabilities, $(1-\lambda)b/\mu$, which is not automatically retired. Given the (increased) probability of default in future periods, it is not optimal for equity holders to undo the increase in the real value of outstanding debt all at once. As a result, the firm will keep an elevated level of ω for a prolonged period. Thus, real leverage responds persistently to shocks and nominal leverage is effectively sticky.

This less than instantaneous response of optimal leverage reflects the fact that the *marginal* debt price effect in equation (22), $\frac{\partial q(\omega')}{\partial \omega'}\left(\omega' g\left(i\right) - \left(1-\lambda\right)\frac{\omega}{\mu}\right)$, acts like an (endogenous) convex adjustment cost that slows down the response to shocks. Indeed, the marginal price effect is an increasing function of the

amount of debt issued, $\left(\omega' g(i) - (1-\lambda)\frac{\omega}{\mu}\right)$ which discourages the firm from making rapid adjustments to its leverage.[12]

Finally, note that by relaxing the extreme assumptions in Proposition 1, persistent movements in leverage can occur even when $\lambda = 1$. This can happen when the underlying inflation movements are persistent or if there are some resource costs associated with default ($\xi^r \neq 0$). However, in our quantitative analysis, these indirect effects are typically not very strong.

We now turn to discuss the effects of inflation on real investment decisions. Since the firm's problem is linear in investment, equation (21) pins down the required equilibrium value of the rental rate, R'. Optimal investment is only determined in general equilibrium.

Proposition 2. *Consider the economy of Proposition 1. Suppose that at time t the economy experiences a temporary decline in the inflation rate so that $\mu_t < \mu_{t-1}$. Then $R_{t+1} = R_t$, if and only if $\lambda = 1$.*

Proof. Proposition 1 implies that when $\lambda \neq 1$ optimal leverage ω' changes with inflation. It follows that $z^*(\omega')$ and R' must also change. ∎

Proposition 2 shows that inflation is generally non-neutral in our model, even though we abstract from other common frictions like sticky prices and wages. Inflation impacts the required rate of return on capital, an effect that is essentially equivalent to that of changing the tightness in financing constraints. Under the assumptions made in the propositions, inflation shocks are transmitted to real investment through debt alone, and thus act through

[12] Models with collateral constraints, such as Kiyotaki and Moore (1997) or Jermann and Quadrini (2012), require a separate friction on equity or debt issuance to produce real effects from shocks to the collateral constraint. Here, with defaultable long-term debt, no additional friction is needed.

a debt overhang channel.[13]

To summarize these two propositions, Figure 1 displays an example of impulse responses to a one time decrease in the price level - alternatively an i.i.d realization of unexpectedly low inflation. As shown in the third row, the permanent decline in the price level induced by the shock eventually produces an identical decline in nominal debt and nominal leverage, so that in the long run there are no real effects. Initially, however, nominal debt only partially offsets the inflation shock, that is, nominal debt and nominal leverage are effectively sticky. As a result, we can see in the second row that (real) leverage, ω, and the default rate, both stay elevated for a prolonged period while investment declines persistently.

3.2 Solution strategy

The solution of the model is significantly complicated by the presence of the derivative $\frac{\partial q(\omega')}{\partial \omega'}$ in (22). This subsection presents the rest of the equations that describe the firm's problem and outlines our numerical solution strategy.

The competitive equilibrium is characterized by (12)-(22), plus the first-order conditions for consumption and labor supply implied by the household's preferences (11).

To understand the role of the derivative $\frac{\partial q(\omega')}{\partial \omega'}$ it is useful to differentiate

[13] Although we have no formal proof, in our simulations, the effects of high inflation always decrease R'.

the debt price function to obtain:

$$
\begin{aligned}
& q\left(\omega'\right)+\omega'\frac{\partial q\left(\omega'\right)}{\partial\omega'} \qquad (23)\\
= \quad & \mathrm{E}M'\left\{\begin{array}{c}
\Phi(z^{*}\left(\omega'\right))\left(c+\lambda\right)\frac{1}{\mu'}+\left(1-\Phi(z^{*\prime})\right)v_{\omega}\left(\omega'\right)\\
+\left(1-\lambda\right)\frac{q(h(\omega'))}{\mu'}+\left(1-\lambda\right)\frac{\omega'}{\mu'}\frac{\partial q}{\partial\omega'}\left(h\left(\omega'\right)\right)\cdot h_{\omega}\left(\omega'\right)\\
+\frac{\partial z^{*}}{\partial\omega}\left(\omega'\right)\phi\left(z^{*}\left(\omega'\right)\right)\left(\begin{array}{c}
-\left(1-\tau\right)\left(R'-z^{*}\left(\omega'\right)\right)\\
+\left(c+\lambda\right)\frac{\omega'}{\mu'}+\xi-v\left(\omega'\right)
\end{array}\right)
\end{array}\right\}.
\end{aligned}
$$

This shows that the derivative of the debt price is linked to the marginal impact of the current leverage choice, ω', on the future choice, ω'', which is captured by the derivative of the policy function $h_{\omega}\left(\omega'\right)$. The presence of this term complicates the solution by standard local approximation methods because $h_{\omega}\left(\omega\right)$ is not known and has to be solved for jointly with the policy function $h\left(\omega\right)$ itself. Essentially, there is one additional variable to solve for, namely $h_{\omega}\left(\omega\right)$, but no additional equation.

Our strategy is to differentiate the derivative of the debt price function, equation (23), and the first-order condition for leverage, equation (22). The two resulting equations include second-order derivatives for the debt price and the policy function, $\frac{\partial^{2}q(\omega')}{\partial\omega'^{2}}$ and $h_{\omega\omega}\left(\omega\right)$. Although we continue to need a boundary condition, this can now be imposed on a higher order term. Specifically, we assume that the derivative of the policy function exhibits constant elasticity in ω, but is otherwise unrestricted, so that

$$\ln h_{\omega}=A_{1}\left(s\right)+h_{1}\ln\omega$$

which implies a restriction that can be used as an additional equation to

characterize local dynamics,

$$h_{\omega\omega} = \frac{h_\omega}{\omega} h_1. \tag{24}$$

$A_1(s)$ is allowed to be any arbitrary function of the state vector, s, and h_1 is a constant that can be determined from the deterministic steady state. With this approach, we do not constrain the first-order dynamics for h_ω, and we have a system of equations that can be fully characterized using first-order perturbation methods. As is well documented, first-order solutions around the deterministic steady state provide very accurate approximations for typical business cycle models.[14]

To implement this strategy however we need to solve for the deterministic steady state to evaluate the constant h_1. As before, solving for the deterministic steady state is more involved than for standard models, because the presence of $h_\omega(\omega)$ leaves the system of nonlinear equations that characterize the deterministic steady state short by one equation.

To address this problem, we instead compute the deterministic steady state using value function iteration over a grid for ω. Computing time is relatively short because the model is deterministic and there is a univariate grid. Also, this global solution only needs to produce the steady state value for ω, and not all the derivatives of the policy function. This is because the nonlinear system of equations for the deterministic steady state is only short one equation. Effectively, knowing the the steady state value for ω provides us with the missing equation. Our appendix provides additional details.

As an alternative, Miao and Wang (2010) solve a model with real long

[14] Assuming instead that $h_{\omega\omega}$ equals its constant steady state value, produces moderately different local dynamics.

term debt by taking the extreme approach of setting $h_\omega(\omega') = 0$, ignoring any of these effects. Thus, in their model firms act myopically not realizing that their current leverage choice influences future leverage and through that the current value of debt.

The presence of derivatives of unknown functions that characterizes the solution of our model is also a feature of time-consistent solutions for problems of dynamic public policy, as studied, for instance, by Klein, Krusell and Rios-Rull (2008). In their model, the government anticipates how future policy will depend on current policy via the state of the economy. Our solution method shares some of the features of the approach described in their paper.

Our solution approach allows us to overcome the particular challenges implied by time-consistent firm behavior with long-term debt without all the limitations of fully nonlinear global methods. Indeed, once the deterministic steady state for the firms' problem is found, our approach can take advantage of the scalability of perturbation methods to easily handle additional state variables, such as a nominal interest included in a monetary policy rule.

4 Parameterization

The model is calibrated at quarterly frequency. While we choose parameters to match steady state targets whenever feasible, we use model simulations to pin down parameters that determine the stochastic properties of the model economy. Whenever possible, the steady state targets correspond to empirical moments computed over the $1955.I - 2012.IV$ period. In the first part of our quantitative analysis we work with an exogenous inflation process and assess the dynamic effects and the relative importance of changes in inflation.

4.1 Inflation and productivity processes

We start by constructing estimates of the joint behavior of innovations to productivity and inflation. Initially, we assume the following general VAR(1) representation for the stationary component of productivity and inflation:

$$\begin{bmatrix} a_t \\ \pi_t \end{bmatrix} = \Gamma \begin{bmatrix} a_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^a \\ \varepsilon_t^\pi \end{bmatrix}.$$

where $a_t = \ln A_t$, $\pi_t = \ln \mu_t - \ln \bar{\mu}$, and $\bar{\mu}$ is the average (gross) rate of inflation during this period.[15] To do this, we first construct series for Solow residuals and inflation using data on GDP, hours, capital stock and the GDP deflator from the BEA and the BLS. Estimating this autoregressive system yields empirical measures of the standard deviations σ_a and σ_π of the productivity and inflation shocks, as well as their cross-correlation, $\rho_{\pi a}$.

For our sample period, we find that:

$$\Gamma = \begin{bmatrix} 0.98 & -0.094 \\ 0.012 & 0.85 \end{bmatrix}$$

and $\sigma_a = 0.0074$ and $\sigma_\pi = 0.0045$, and $\rho_{\pi a} = -0.19$. We call this the VAR specification of our shocks. We also consider a more restrictive AR(1) specification where $\rho_{\pi a} = 0$ and $\Gamma_{12} = \Gamma_{21} = 0$. In this case, the diagonal elements of Γ are 0.97 and 0.85, respectively, with $\sigma_a = 0.007$ and $\sigma_\pi = 0.0040$. This version of the model allows us to examine the case where the exogenous inflation shocks have no real effects, other than those on firm leverage and offers more reliable variance decomposition results.

[15] We normalize $\ln(\bar{A}) = 0$.

4.2 Idiosyncratic profit shocks

Instead of adopting a specific distribution for the p.d.f. $\phi(z)$ we use a general quadratic approximation of the form:

$$\phi(z) = \eta_1 + \eta_2 z + \eta_3 z^2. \tag{25}$$

The distribution is assumed symmetric with $\bar{z} = -\underline{z} = 1$. Our other assumptions about this distribution's mean imply that $\eta_2 = 0$, and η_3 is tied to the only free parameter η_1. The value for η_1 is selected to target the unconditional volatility of the leverage ratio, a key variable in our model. While η_1 is closely linked to the volatility of leverage, σ_ω, the model cannot fully match the empirical counterpart for σ_ω.

Together with the average leverage ratio, ω, and expected debt life, $1/\lambda$, the value for η_1 is an important determinant of the persistence of inflation shocks. Intuitively, this is because when the mass of firms accumulated around the default, $\phi(z^*)$, is sizable, any shock will have a large impact on the default probability, $\Phi(z^*)$, and on bond prices, $q(\omega)$. This matters because the sensitivity of debt prices, $\partial q/\partial \omega$, effectively determines the magnitude of the implicit costs to adjusting the stock of debt in equation (22). Thus, when $\phi(z^*)$ - governed by the choice of η_1 - is large, debt will be more persistent and the effects on the real economy will last longer.

4.3 Technology and preferences

Our choices for the capital share α, depreciation rate δ, and the subjective discount factor β correspond to fairly common values and pin down the capital-output, investment-output, and the average rate of return on capital in our

economy. As long as they remain in a plausible range, the quantitative properties of the model are not very sensitive to these parameter values. The preference parameter θ is chosen so that in steady state working hours make up one third of the total time endowment, and σ is set to deliver a plausible level of risk aversion.

4.4 Institutional parameters

The parameter λ pins down average debt maturity. This is an important parameter for determining the propagation of inflation shocks. Our benchmark calibration implies an average maturity of 5 years, and an actual duration of about 4 years. These values are similar to initial maturities of industrial and commercial loans, but significantly shorter than those for corporate bonds. Given the importance of this parameter, we prefer to err on the safe side and focus on the results when debt maturity is conservatively calibrated. We will document how the results change with alternative average maturities.

The tax wedge τ is chosen so that average firm leverage matches the observed leverage ratio for the U.S. non-financial business sector. Average leverage is 0.42 in the period since 1955. The (statutory) wedge implied by corporate income rates and the tax treatment of individual interest and equity income during this period which is about 25%. As a result we should think of τ as capturing other relative benefits of using debt rather than equity (e.g. issuance costs).

The coupon rate is set to $c = \exp(\mu)/\beta - 1$, so that the price of default free debt is equal to one. The default loss parameter, ξ, is chosen to match the average default rate per quarter of 0.25% for the postwar period. The chosen default cost parameters implies average steady-state recovery rates at

default of about 30%. The magnitude of ξ is obviously important but its choice cannot be arbitrary. Specifically, choosing a higher default cost implies that firms are less likely to default in equilibrium so that overall expected default costs remain unchanged. Finally, we set the aggregate loss parameter $\xi^r = 1$, so that all restructuring charges involve a deadweight resource loss.

Table 1 summarizes our parameter choices for the benchmark calibration. The model is quite parsimonious and requires only 10 structural parameters, in addition to the stochastic process for the shocks. Table 2 shows the implications of these choices for the first and second (unconditional) moments of a number of key variables.

The second panel in Table 2 shows that our quantitative model shares many of the properties of other variations of the stochastic growth model. All the main aggregates, have plausible volatilities, except for labor, as is typical in standard Real Business Cycle models.

Table 1: **Calibration**

Parameter	Description	Value
β	Subjective Discount Factor	0.99
γ	Risk Aversion	1
θ	Elasticity of Labor	0.63
α	Capital Share	0.36
δ	Depreciation Rate	0.025
λ	Debt Amortization Rate	0.05
ξ^r	Fraction of Resource Cost	1
ξ	Default Loss	0.29
τ	Tax Wedge	0.40
η_1	Distribution Parameter	0.6815

Table 2: **Aggregate Moments**

	Data	Model AR(1)	Model VAR(1)
First Moments			
Investment/Output, I/Y	0.21	0.24	0.24
Leverage	0.42	0.42	0.42
Default Rate, $1-\Phi(z^*)$	0.25%	0.24%	0.24%
Second Moments			
σ_Y	1.66%	1.45%	1.59%
σ_I/σ_Y	4.12	3.48	3.67
σ_C/σ_Y	0.54	0.37	0.39
σ_N/σ_Y	1.07	0.38	0.44
σ_ω	1.7%	0.72%	0.91%
$\sigma_{q\omega}$	1.7%	1.67%	1.76%

5 Quantitative analysis

We now investigate the quantitative importance of the frictions induced by nominal long-term debt. First, we are interested in determining how the model responds to inflation shocks and how much endogenous propagation can plausibly be generated by the combination of our sticky leverage and debt overhang mechanisms. Next, we show how the model can be modified so that the inflation rate is determined endogenously and investigate how a standard monetary policy rule can help stabilize the economy following different shocks.

5.1 Model with exogenous inflation

The model predicts that under very general conditions, even exogenous i.i.d. movements in inflation can induce prolonged movements in corporate leverage and investment, and, in output and consumption. Figure 2 shows how these

responses look in a plausible quantitative version of the model, when inflation follows the exogenous AR(1) process specified above, which is assumed to be uncorrelated with productivity.

We can see that following lower than expected inflation, the default rate increases as the real value of outstanding corporate liabilities increases. This increase in the default rate immediately produces output losses since restructuring costs are not rebated to households and represent real deadweight losses.

As Proposition 1 implies, leverage - we report its market value, $q\omega'$ - rises and remains elevated for a long time even though inflation quickly returns to its long run mean. This persistence in leverage contributes to a prolonged, and significant contraction in investment spending - the debt overhang result.

Initially at least, there is an important change in the intertemporal allocation of resources, as consumption actually rises, reflecting the fact that the required rental rate on capital has increased. Soon however, the lower capital stock further contributes to lowering output, and consumption declines as well. Labor initially mirrors the behavior of consumption, as households seek to smooth their leisure decisions as well. Over time, reduced capital contributes to lowering the marginal product of labor.[16]

5.2 Variance Decomposition

Table 3 shows the contribution of inflation shocks to the total variance in the key macro and financial aggregates for the benchmark model as well as the sensitivity of this measure with respect to some key assumptions. Specifically, we now assume that our model is driven by *independent* AR(1) shocks to inflation and productivity to ensure that the measured contributions are entirely

[16]With GHH preferences, consumption and labor do not move at all on impact and then immediately decline.

due to their effects on the endogenous variables. [17]

Regardless of our calibration, inflation shocks are always the dominant source of variations in leverage and default rates. In addition, in our benchmark model inflation is also responsible for 44% of the variance of investment and 22% of the variance of output. Given that our model does not include many features found in medium-scale business cycle models and therefore omits possible sources of shocks, we interpret the significant contribution of inflation shocks to business cycles primarily as a sign that the inflation nonneutrality in our model can be quantitatively important.

In the baseline case leverage matches the data for the period since 1955. When our economy is calibrated to an average leverage ratio of only 32%, inflation still accounts for about a quarter of the movements in investment. If, instead, the leverage ratio matches the value observed in the period since 2005, which is 52%, inflation shocks account for two thirds of the investment variance and almost half of the variance of output.

Finally, Table 3 shows that with one period debt, real quantities are almost unaffected by movements in the inflation rate, and we essentially recover monetarily neutrality. There are two reasons for this. First, for a given inflation process, percentage gains and losses on bonds produced by inflation are smaller the shorter the maturity. Second, the debt overhang channel is entirely absent with one-period debt.

Overall, these findings are striking. After all, this is a model with exogenous inflation shocks that by assumption have no direct impact on real quantities. However, despite the fact that the price level is entirely flexible, inflation still has potentially very large real effects.

[17] Since movements in TFP are the only other source of fluctuations here, the fraction of the variance coming from TFP shocks is 1 minus the number reported in the table.

Table 3: **Variance Decomposition (due to inflation)**

	Y	Inv	Cons	Hrs	Lev	Default
Benchmark	.23	.44	.17	.13	.89	.99
Low Leverage (.32)	.11	.24	.05	.03	.74	.99
High Leverage (.52)	.42	.67	.36	.38	.92	.99
Shorter Maturity, $\lambda = .06$	.21	.41	.13	.10	.84	.99
One-period Debt, $\lambda = 1$	0	0	0	0	.98	.99

5.3 Model with endogenous inflation

We now consider the case of endogenous inflation changes. We follow the popular practice of using a monetary policy rule of the form:[18]

$$\ln\left(r_t/\bar{r}\right) = \rho_r \ln\left(r_{t-1}/\bar{r}\right) + (1-\rho_r)v_\mu \ln\left(\mu_t/\bar{\mu}\right) + (1-\rho_r)v_y \ln\left(Y_t/\bar{Y}\right) + \zeta_t, \quad (26)$$

where ζ_t is an exogenous monetary policy shock, r_t is the nominal (gross) one period interest rate which obeys the Euler equation:

$$r_t = \frac{1}{\mathrm{E_t} M_{t,t+1}/\mu_{t+1}}, \quad (27)$$

and the bars denoting the steady-state values of the relevant variables.

We follow the literature and set the monetary policy responses $\rho_y = 0.5$ and $\rho_\mu = 1.5$. The smoothing parameter is $\rho_r = 0.6$.

Thus, in this version of the model, inflation is driven either by exogenous shocks to monetary policy itself or by endogenous monetary policy responses

[18] Another common alternative is to assume a money demand equation of the form $M_t/P_t = L(Y_t, r_t)$, arising from a cash in advance constraint or money in the utility function and where money growth, ΔM_t, is controlled by the monetary authority and possibly stochastic.

to movements in output and real interest rates. We investigate these two possibilities in turn.

5.3.1 Monetary policy and endogenous inflation

Figure 3 shows how the exogenous behavior of the inflation rate in Figure 2 can simply be thought as the endogenous response to discretionary changes in monetary policy. Specifically, we now consider the effects of an exogenous shock ζ_t to the policy rule. The shock is set so as to produce an inflation response in the second panel that closely resembles the exogenous inflation shock considered above. Specifically, this is accomplished by assuming that ζ_t follows an AR(1) with a persistence parameter of 0.94.[19]

As can be seen, the responses of the key variables are similar regardless of whether the inflation movements are exogenous or induced by the monetary policy rule. What matters for the response of the real economy is the actual behavior of the inflation rate itself.

5.3.2 Productivity shocks

With endogenous monetary policy, the inflation rate changes when the economy is hit by real shocks. Put another way, the monetary policy rule changes the response of the real economy to the underlying shocks by generating more or less inflation.

Figure 4 documents the impact of a shock to (the logarithm of) total factor productivity, a_t, with and without a monetary policy response. The green lines show the baseline responses to the productivity shock without the monetary policy rule, so that inflation is unaffected by the shock. In this case, we observe

[19]For this example only we set the interest smoothing parameter $\rho_r = 0.3$.

the patterns common to other quantitative equilibrium models.

The blue lines represent the responses when the nominal interest rate follows the monetary police rule (26). Here, output, investment, labor, and to some extent consumption, move much less. This is because the monetary policy rule increases the inflation rate and reduces the real burden of outstanding debt. As discussed above (in reverse), this increase in inflation then lowers corporate defaults, and positively impacts investment and the other real variables.[20]

This experiment emphasizes that our debt overhang result is entirely driven by the long-lived nature of our debt contracts and does not rely on nominal frictions. However, when debt is nominal, inflation significantly helps to eliminate this overhang problem.

5.3.3 Wealth shocks

Figure 5 examines the case of a shock to the stock of capital in the economy. This experiment can be seen as capturing some aspects of the contraction seen since 2007/08, with sharply declining real estate values.

We think of this as a rare one off event. Formally, this is implemented by an unexpected decrease in the value of the capital stock k of 5% through a one time increase in the depreciation rate, δ. On impact, this destruction of the capital stock lowers both overall firm and equity values. This leads to an immediate spike in corporate defaults, and an increase in the leverage ratio.

When the inflation rate remains constant (green lines), persistence in lever-

[20]Interestingly, the lower output in the policy rule, everything else equal, would lead to a lower interest rate. However, in this model, the nominal interest rate actually increases because inflation increases too. See Cochrane (2011) for a detailed analysis of how inflation is determined by Taylor rules. In particular, the unique stable solution of the difference equation for inflation implied by the Taylor rule typically produces opposing movements in current inflation and output.

age and debt overhang produce long lasting real declines in investment and output. However, when monetary policy responds according to the rule (26) (blue lines), the inflation rate increases immediately. This reduces the burden of outstanding liabilities. The effects of this shock on the real aggregates are significantly mitigated, and the economy recovers a lot faster. With a 5% reduction in the capital stock, the model's implied inflation increases by a total of 2.7% above its steady state level over the first year after the shock.

A priori, this policy is consistent with a popular policy prescription from several macroeconomists in the immediate aftermath of the crisis and summarized in the following quote:

"I'm advocating 6 percent inflation for at least a couple of years," says Rogoff, 56, who's now a professor at Harvard University. "It would ameliorate the debt bomb and help us work through the deleveraging process." (Bloomberg, May 19, 2009).

6 Conclusion

In this paper we have presented a general equilibrium model with nominal long-term debt that can help us better understand the monetary non-neutralities associated with Irving Fisher's (1933) debt deflation. The model also sheds some light on the ongoing financial crisis and possible monetary policy responses. Unlike other popular models of monetary non-neutralities, we eschewed price rigidities. Yet our model is capable of generating very large and persistent movements in output and investment.

Almost unavoidably, our attempt to write a parsimonious and tractable model leaves out many important features. In particular, we ignore nominal debt contracts other than those held by firms, even though household debt is

roughly equal in magnitude and subject to similarly large restructuring costs.

Our analysis also abstracts from the role of movements in credit risk premia and the behavior of asset prices in general. In addition, while convenient, the assumption of constant returns to scale, which nearly eliminates firm heterogeneity and renders the model so tractable, also limits our ability to study firm behavior more comprehensively.

These and other simplifying assumptions can and should be further explored in future work. Nevertheless we believe none is essential to the main ideas in the paper.

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7 Appendix on solution method

This section provides additional details on our solution method. The model is solved with a first-order perturbation approximation around the deterministic steady state.

To solve for the deterministic steady state, R is initially taken as given. Three equations, (16), (17), and (18), are used to solve for the steady state level of leverage by value function iteration. The first-order condition for investment is then used to find an updated value for R, and this is repeated until convergence. The full model's steady state is then obtained by combining the steady state value for leverage with the system of equations characterizing equilibrium, that is, equations (12)-(22), the first-order conditions for consumption and labor supply implied by the household's preferences (11), (23) and the derivatives of (23) and (22) with respect to ω. For model dynamics, this system of equations is augmented with one equation determining the behavior of the second derivative of the policy function, $h_{\omega\omega} = \frac{h_\omega}{\omega} h_1$, which is based on the assumption that the first derivative of the policy function exhibits constant elasticity in ω. The coefficient h_1 is set equal to $h_{\omega\omega}\frac{\omega}{h_\omega}$ evaluated at the deterministic steady state.

Figure 1: A decline in the price level. This figure shows the effect of an unanticipated one time decline in the inflation rate μ_t on the key variables of the model.

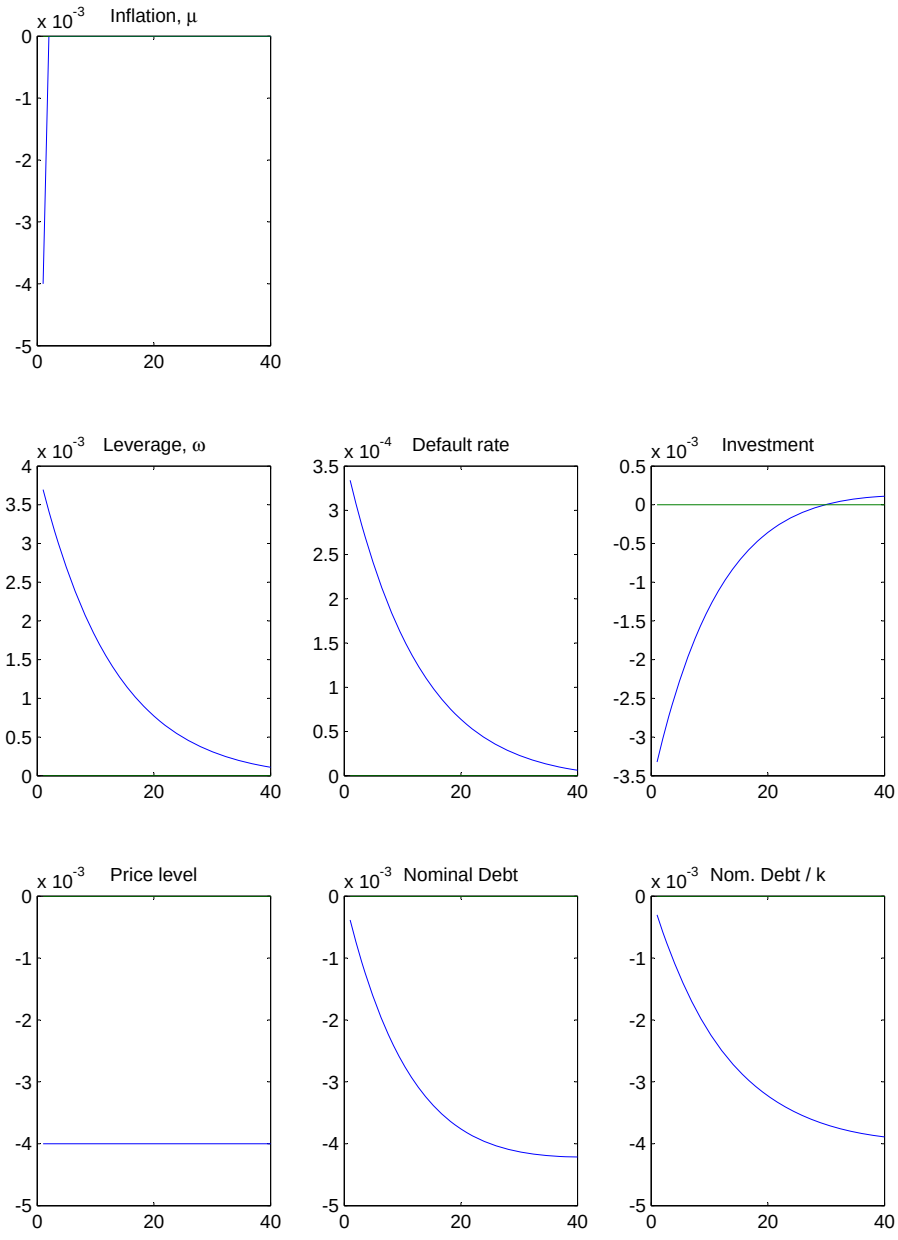

Figure 2: An exogenous inflation shock. This figure shows the effect of an unanticipated decline in the inflation rate μ_t on the key variables of the model.

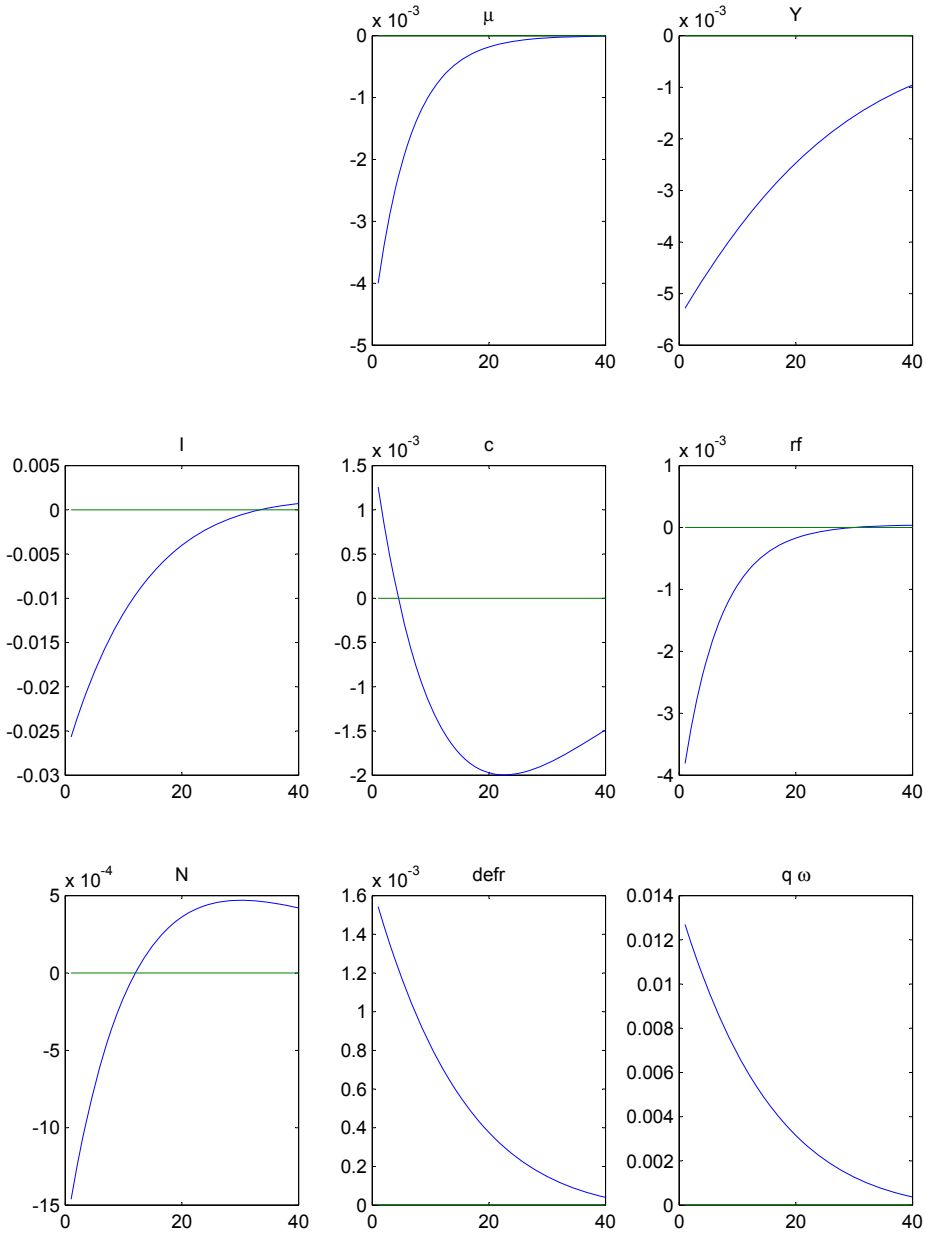

Figure 3: An endogenous inflation shock. This figure shows the effect of an exogenous shock to the monetary policy rule on the key variables of the model. The blue line illustrates the response to an exogenous change in monetary policy, ζ, while the green line shows the response in the baseline case with exogenous inflation.

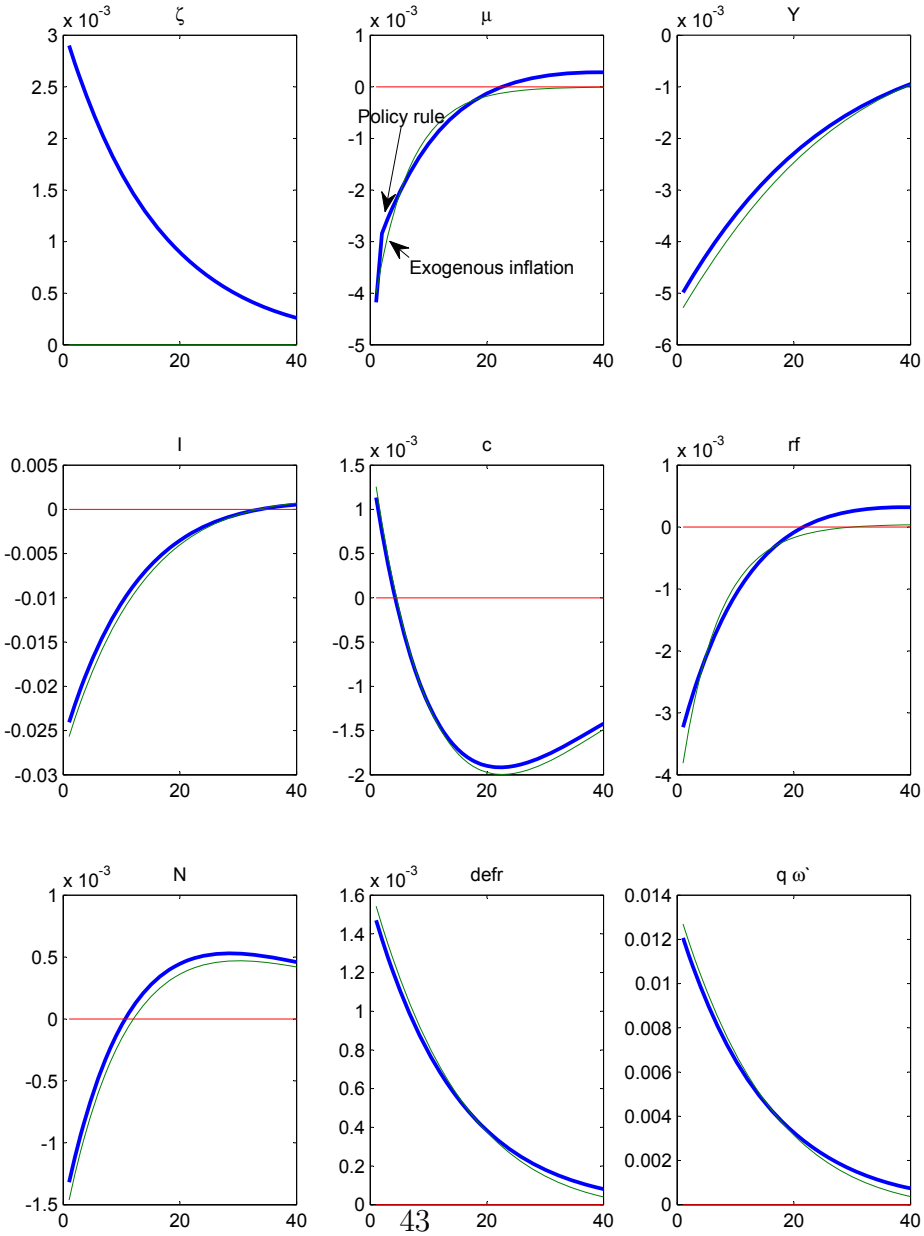

Figure 4: A productivity shock. This figure shows the effect of an exogenous shock to productivity, A. It compares the effects when inflation is exogenous (green line) and when it adjusts endogenous as a consequence of a monetary policy rule (blue line).

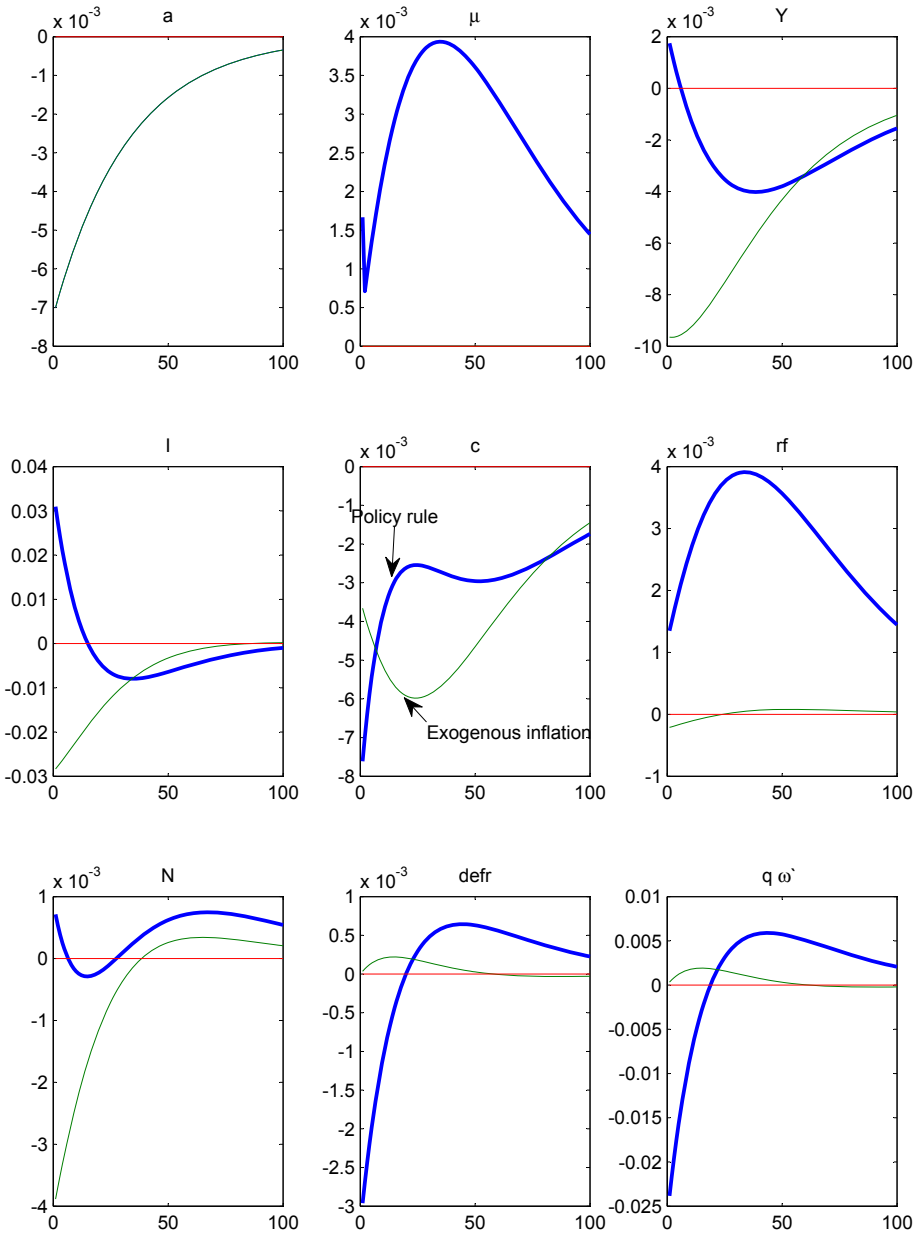

Figure 5: A wealth shock. This figure shows the effect of an exogenous destruction of the capital stock, k, through a temporary increase in the depreciation rate, δ. It compares the effects when inflation is exogenous (green line) and when it adjusts endogenous as a consequence of a monetary policy rule (blue line).