

# Central Bank Purchases of Private Assets

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July 30, 2014

## Abstract

A model is constructed in which consumers and banks have incentives to fake the quality of collateral. Conventional monetary easing can exacerbate these problems, in that the misrepresentation of collateral becomes more profitable, thus increasing haircuts and interest rate differentials. Central bank purchases of private mortgages may not be feasible, due to misrepresentation of asset quality. If feasible, central bank asset purchase programs work by circumventing suboptimal fiscal policy, not by mitigating incentive problems in asset markets.

## 1 Introduction

During and since the global financial crisis, central banks have engaged in unconventional purchases of long-maturity assets, on a very large scale, particularly in the United States. For the Fed, such purchases, often referred to as quantitative easing (QE), have included purchases of long-maturity government debt in exchange for reserves, swaps of short-maturity government debt for long-maturity government debt, and purchases of mortgage-backed securities and agency securities. The focus of this paper is on the effects of the latter types of purchases. While asset purchases by the central bank involving only government debt act to change the composition of the outstanding consolidated government debt, central bank purchases of what are essentially private assets puts monetary policy in potentially different territory.

In purchasing private assets, the central bank needs to be concerned with the quality of the assets it purchases, and with the incentive effects of central bank actions. The fact that the central bank is a willing buyer of private assets may make it the victim of sellers of low-quality assets, and the changes in asset prices brought on by central bank actions may create incentives to cheat in the private sector. Further, if the central bank engages in private asset purchases, it needs to understand the relationship between its conventional monetary policy actions and its unconventional ones.

In this paper, we construct a model of asset exchange and monetary policy, in which economic agents have an incentive to cheat on the quality of collateral.

The basic structure comes from Lagos and Wright (2005) and Rocheteau and Wright (2005), and some details of the model are closely-related to models constructed in Williamson (2012, 2013), particularly in terms of the structure of financial intermediation and the relationship between fiscal and monetary policy.

In the model, the basic assets are currency, reserves, government debt, and housing. Assets are necessary for exchange to take place. Indeed, housing is a private asset which can be useful in exchange – though indirectly. Consumers own houses and take out mortgages with financial intermediaries using housing assets as collateral. Then, consumers use financial intermediary liabilities and currency in decentralized exchange. An equilibrium financial intermediation arrangement, in the spirit of Diamond-Dybvig (1983) (and as in Williamson 2012, 2013), is a type of insurance arrangement, and banks act to efficiently allocate liquid assets in exchange.

Limited commitment in the model requires that private debt be secured. Consumers secure mortgage debt with housing, and banks secure deposit liabilities with mortgage loans, government debt, and reserves. But, a key element in the model is that consumers, at a cost, can fake the quality of houses posted as collateral. Similarly, banks can fake the quality of mortgage debt at a cost. In this way, we capture elements that we think were important during the financial crisis, and in the period leading up to it. In the model, the incentive problems faced by consumers and banks are similar to the counterfeiting problem captured by Li, Rocheteau, and Weill (2012), though the technical features of how we deal with the incentive problems here are somewhat different.

We first study the behavior of the model under conventional monetary policy. A key feature of the equilibrium we study is that collateral is scarce in the aggregate. This scarcity creates inefficiency in exchange, and a liquidity premium on scarce collateral, which is reflected in a low real interest rate. Given the fiscal policy rule, treated as given, a Friedman rule allocation is not feasible.

We determine conditions under which the incentive problems in the model matter. Essentially, incentive constraints will bind in equilibrium if the costs of faking the quality of collateral are sufficiently low, and if the real interest rate on government debt is sufficiently low. An important feature of a secured debt contract when there is a binding incentive constraint, is that there is an endogenous haircut, as in Li, Rocheteau, and Weill (2012). That is, to convince lenders that the collateral is not faked, the borrower does not borrow up to the full value of the collateral.

When incentive constraints bind for banks or for consumers, there exist interest rate spreads and haircuts on collateral that can potentially be faked. As well, given binding incentive constraints, a decrease in the cost of faking collateral or a reduction in the real interest rate on government debt will in general lead to increases in both interest rate spreads and haircuts. As well, conventional monetary policy easing – a reduction in the nominal interest rate by the central bank – will reduce the real interest rate and thus exacerbate incentive problems.

If we determine optimal conventional monetary policy in the model, i.e. the

optimal choice of the nominal interest rate when there are no unconventional asset purchases by the central bank, then if some incentive constraints do not bind, a zero nominal interest rate is optimal, at least locally. However, if incentive constraints bind for banks and consumers, then the nominal interest rate is strictly positive at the optimum. This is quite different from what obtains in New Keynesian models. For example, in Werning (2012), a temporarily high discount factor (interpreted as a financial crisis shock) implies that the nominal interest rate should be zero for some time. In Eggertsson and Krugman (2012), a temporarily tighter borrowing constraint also implies that the nominal interest rate should be zero, and that the real interest rate is too high. In contrast, if incentive constraints bind for banks and consumers in our model, then the nominal interest rate and the real interest rate are too low at the zero lower bound. This is because a low real interest rate exacerbates incentive problems.

Can private asset purchases by the central bank improve matters? In our model, we assume that the central bank cannot lend directly to consumers, and must purchase mortgages outright from banks. Therefore, the central bank is faced with the same incentive problem as are bank depositors – banks can fake mortgage loans. As a result, there are circumstances in which asset purchase programs are not feasible because the central bank will only be supplied with fakes. Even if an asset purchase program is feasible, it may have no effect. For example, central bank asset purchases will be neutral if the central bank does not purchase the entire outstanding stock of mortgage debt, or if incentive constraints bind for consumers. The only case in which central bank asset purchases are not neutral and improve welfare is, surprisingly, in circumstances in which the incentive constraints for banks and households would not bind in the absence of the program. A successful central bank asset purchase program involves the purchase of the entire stock of mortgage debt. The program works essentially by circumventing suboptimal fiscal policy. The key credit market friction exists because collateral is scarce, and that scarcity can be eliminated if the stock of government debt is expanded. Short of that, monetary policy can mitigate the collateral scarcity by issuing reserves and purchasing mortgage debt at a high price. This acts to expand the value of the stock of eligible collateral and relax banks' collateral constraints, increasing welfare. Perhaps surprisingly, the increase in welfare coincides with an increase in the real interest rate on government debt (a reduction in the liquidity premium), though the real mortgage rate falls.

Related work on central bank intervention and collateral includes Kiyotaki and Moore (2012), Gertler and Kiyotaki (2012), Gertler and Karadi (2012), Rocheteau and Rodrigues-Lopez (2013), and Venkateswaran and Wright (2013). Geromichalos, Herrenbrueck and Salyer (2013) provides an explanation for the term premium on long-maturity assets.

In the second section the model is constructed, and an equilibrium is characterized and analyzed in Section 3. In the fourth and fifth sections, conventional monetary policy and unconventional central bank asset purchases are analyzed, respectively. The final section is a conclusion.

## 2 Model

The basic structure in the model is related to Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is indexed by  $t = 0, 1, 2, \dots$ , and in each period there are two sub-periods – the centralized market ( $CM$ ) followed by the decentralized market ( $DM$ ). There is a continuum of buyers and a continuum of sellers, each with unit mass. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + F_t + u(x_t)],$$

where  $H_t$  is labor supply in the  $CM$ ,  $F_t$  is consumption of housing services in the  $CM$ ,  $x_t$  is consumption in the  $DM$ , and  $0 < \beta < 1$ . Assume that  $u(\cdot)$  is strictly increasing, strictly concave, and twice continuously differentiable with  $u'(0) = \infty$ ,  $u'(\infty) = 0$ , and  $-x \frac{u''(x)}{u'(x)} < 1$ . Each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),$$

where  $X_t$  is consumption in the  $CM$ , and  $h_t$  is labor supply in the  $DM$ . Buyers can produce in the  $CM$ , but not in the  $DM$ , and sellers can produce in the  $DM$ , but not in the  $CM$ . One unit of labor input produces one unit of the perishable consumption good, in either the  $CM$  or the  $DM$ .

As well, there exists a continuum of banks. Each bank is an agent that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - H_t),$$

where  $X_t$  and  $H_t$  are consumption and labor supply, respectively, in the  $CM$ .

In the  $DM$ , there are random matches between buyers and sellers, and each buyer is matched with a seller. All  $DM$  matches have the property that there is no memory or recordkeeping, so that a matched buyer and seller have no knowledge of each others' histories. A key assumption is limited commitment – no one can be forced to work – and so lack of memory implies that there can be no unsecured credit. If any seller were to extend an unsecured loan to a buyer, the buyer would default.

Following Williamson (2012, 2013), assume limitations on the information technology that imply that currency will be the means of payment in some  $DM$  transactions, and some form of credit (here it will be financial intermediary credit) will be used in other  $DM$  transactions. Suppose that, in a fraction  $\rho$  of  $DM$  transactions – denoted *currency transactions* – there is no means for verifying that the buyer possesses any assets other than currency. Thus, in these meetings, the seller can only verify the buyer's currency holdings, and so means of payment other than currency are not accepted in exchange. However, in a fraction  $1 - \rho$  of  $DM$  meetings – denoted *non-currency transactions* – the seller can verify the entire portfolio held by the buyer. Assume that, in

any *DM* meeting, the buyer makes a take-it-or-leave-it offer to the seller. At the beginning of the *CM*, buyers do not know what type of match they will have in the subsequent *DM*, but they learn this at the end of the *CM*, after consumption and production have taken place. A buyer's type (i.e. whether they will need currency to trade in the *DM* or not) is private information, and at the end of the *CM*, a buyer can meet at most one bank of his or her choice.<sup>1</sup>

In addition to currency, there are three other assets in the model: nominal government bonds, reserves and housing. A government bond sells for  $z_t$  units of money in the *CM* of period  $t$ , and pays off one unit of money in the *CM* of period  $t + 1$ . One unit of reserves can be acquired in exchange for  $z_t$  units of money in the *CM* in period  $t$ , and pays off one unit of money in the *CM* of period  $t + 1$ . In principle, the prices of government bonds and reserves could be different in the *CM*, but if both assets are held in equilibrium, their prices are identical. Housing is in fixed supply, with a perfectly divisible stock of one unit of housing in existence forever. If an agent holds  $a_t$  units of housing at the beginning of the *CM* of period  $t$ , then that agent receives  $a_t y$  units of housing services, where  $y > 0$ . Only buyers receive utility from consuming housing services, and only the owner of a house can consume the services. Houses sell in the *CM* at the price  $\psi_t$ . Assume that there exists no rental market in housing.<sup>2</sup> Further, to guarantee that, in equilibrium, banks will never hold houses directly, assume that if a bank acquires a house in period  $t$ , that it immediately depreciates by 100%.<sup>3</sup>

### 3 Asset Exchange and Banking

In the spirit of Diamond-Dybvig (1983), banks play an insurance role. To illustrate this, suppose that banking is prohibited in this environment. Then, in the *CM*, a buyer would acquire a portfolio of currency, government bonds, reserves, and housing in the *CM*, anticipating that he or she may or may not need currency in the subsequent *DM*. In the *DM*, on the one hand, if the seller accepts only currency, then the buyer would exchange currency for goods. The government bonds, reserves, and housing in the buyer's portfolio would then be of no use in exchange, and the buyer would have to hold these assets until the next *CM*. On the other hand, if the buyer met a seller in the *DM* who could verify the existence of all assets in the buyer's portfolio, then government bonds, reserves, and housing could be used as collateral to obtain a loan from

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<sup>1</sup>Type is private information, and trading opportunities are limited at the end of the *CM* so as to prevent the unwinding of bank deposit contracts. See Jacklin (1987) and Wallace (1988).

<sup>2</sup>We could model the reasons for the lack of a rental market in the model, for example arising from a moral hazard problem - a renter has no private information about items needing repairs, but may have no incentive to make the repairs. But modeling these reasons the missing rental market need not add anything useful to the analysis.

<sup>3</sup>If we did not make this assumption, then when the real interest rate is zero or lower, and counterfeiting costs are sufficiently low, banks may hold houses directly in equilibrium. Allowing for this does not appear to admit any important insights, and only makes the analysis much more complicated.

the seller. Currency could also be traded in this circumstance, but ex post the buyer would have been better off by acquiring higher-yielding assets rather than currency in the preceding  $CM$ . A bank, as we will show, is able to insure buyers against the need for different types of liquid assets in different types of exchange. The bank's deposit contract will allow the depositor to withdraw currency as needed, and to trade bank deposits backed by assets when that is feasible in the  $DM$ .

As in Diamond-Dybvig models, the spatial separation assumptions we have made concerning meetings at the end of the  $CM$ , prevent the banking contracts, discussed in what follows, from being unwound.

### 3.1 Buyer's problem

Quasi-linear preferences for the buyer allows us to separate the buyer's contracting problem vis-a-vis the bank from his or her decisions about the remaining portfolio. In the  $CM$ , the buyer acquires housing  $a_t$ , at a price  $\psi_t$ , and holds this quantity of housing until the next  $CM$ , when the buyer receives the payoff  $\psi_{t+1} + y$  (market value of the housing, plus the payoff in terms of housing services). As well, the buyer can borrow in the form of a mortgage from a bank. A mortgage which is a promise to pay  $l_t^h$  units of consumption goods in the  $CM$  of period  $t + 1$  sells at the price  $q_t$ , in units of the  $CM$  good in period  $t$ . As well, a mortgage loan must be secured with housing assets, otherwise the borrower would abscond. But the buyer is able to produce "counterfeit housing," i.e. a buyer can produce assets that are indistinguishable to the bank from actual housing, at a cost of  $\gamma^h$  per unit of counterfeit housing. This stands in for incentive problems related to asset appraisals, or private information associated with the buyer's ability to service the mortgage debt.

In equilibrium, the buyer will not produce counterfeit housing (e.g. see Li, Rocheteau, and Weill 2012). For the buyer to keep himself or herself honest may require that the buyer not borrow up to the full value of the collateral, so let  $\theta_t^h \in [0, 1]$  denote the fraction of the housing assets of the buyer that lenders are permitted to seize if the buyer defaults.

The buyer's collateral constraint is

$$l_t^h \leq (\psi_{t+1} + y)a_t\theta_t^h, \quad (1)$$

i.e. the payoff required in the  $CM$  of period  $t + 1$  on the buyer's mortgage loan cannot exceed the payoff on the housing collateral, discounted by the "hair-cut"  $\theta_t^h$ . For now, we will assume that (1) binds, and we will later determine conditions that guarantee this. Then, given (1) with equality, the buyer solves

$$\max_{a_t, \theta_t^h} a_t \left[ -\psi_t + \beta(\psi_{t+1} + y) + (q_t - \beta)(\psi_{t+1} + y)\theta_t^h \right] \quad (2)$$

subject to

$$-\gamma^h + q_t(\psi_{t+1} + y)\theta_t^h \leq 0. \quad (3)$$

Here, (2) is the objective function for the buyer. The net payoff on one unit of housing assets, in the square parentheses in the objective function in (2), is minus the price of housing  $\psi_t$  plus the discounted direct payoff to the buyer from housing, plus the net discounted indirect payoff from using the housing as collateral to take out a mortgage. The constraint (3) is the incentive constraint for the buyer, which states that the net payoff to faking a house and borrowing against the fake house on the mortgage market must not be strictly positive. Off equilibrium, if the buyer were to fake a unit of housing and use the fake housing as collateral to borrow on the mortgage market, then he or she would default on the loan.

### 3.2 Bank's Problem

In the *CM*, when a bank writes deposit contracts with buyers, a buyer does not know his or her type, i.e. whether or not he or she will need currency to trade in the subsequent *DM*. Once the buyer learns his or her type, at the end of the *CM*, type remains private information to the buyer. The bank contract specifies that the buyer will deposit  $k_t$  units of goods with the bank in the *DM*, and gives the depositor one of two options. First, at the end of the period, the depositor can visit the bank and withdraw  $c_t$  in currency, in units of *CM* consumption goods, and have no other claims on the bank. Alternatively, if the depositor does not withdraw currency, he or she can have a claim to  $d_t$  units of consumption goods in the *CM* of period  $t + 1$ , and these claims can be traded in the intervening *DM*. In equilibrium, a bank maximizes the expected utility of its representative depositor, subject to the constraint that it earn a nonnegative net payoff, and satisfy a collateral constraint and an incentive constraint. If the bank did not solve this problem in equilibrium, then another bank could enter the industry, make depositors better off, and still earn a nonnegative expected payoff. As with a buyer, a bank must collateralize its deposit liabilities, though we assume that the bank can commit (say, by putting cash in the ATM) to meeting its promises to satisfy cash withdrawals. For the bank, collateral consists of mortgage loans, government bonds, and reserves. Further, the bank can create counterfeit loans in its asset portfolio, and in equilibrium the bank must have the incentive not to do that.

A depositing buyer receives expected utility from the bank's deposit contract,

$$EU = -k_t + \rho u \left( \beta \frac{\phi_{t+1}}{\phi_t} c_t \right) + (1 - \rho) u(\beta d_t), \quad (4)$$

i.e. the buyer deposits  $k_t$  with the bank in the *CM*, and with probability  $\rho$  exchanges currency worth  $\frac{\phi_{t+1}}{\phi_t} c_t$  in the *CM* of period  $t + 1$  with a seller, as the result of a take-it-or-leave-it offer by the buyer. With probability  $1 - \rho$  the buyer meets a seller who will accept claims on the bank, and the buyer makes a take-it-or-leave-it offer which nets  $\beta d_t$  in *DM* consumption goods from the seller. The bank's net payoff, given the deposit contract and the bank's asset

portfolio must be nonnegative in equilibrium, or

$$k_t - z_t(m_t + b_t) - \rho c_t - q_t l_t - \beta(1 - \rho)d_t + \beta \frac{\phi_{t+1}}{\phi_t} (m_t + b_t) + \beta l_t \geq 0, \quad (5)$$

where  $k_t - z_t(m_t + b_t) - \rho c_t - q_t l_t$  denotes the payoff in the *CM* of period  $t$  from acquiring deposits and purchasing reserves, government bonds, currency, loans, and housing. The quantity  $-\beta(1 - \rho)d_t + \beta \frac{\phi_{t+1}}{\phi_t} (m_t + b_t) + \beta l_t$  is the discounted net payoff to the bank in the *CM* in period  $t + 1$ , from making good on deposit claims and collecting the payoffs on reserves, government bonds and loans. As well, the bank is subject to limited commitment, just as other agents in the model are. The bank's asset portfolio serves as collateral that backs its deposit liabilities. Thus, the bank faces a collateral constraint

$$-(1 - \rho)d_t + \frac{\phi_{t+1}}{\phi_t} (m_t + b_t) + \theta_t l_t \geq 0, \quad (6)$$

which states that the bank's remaining deposit liabilities in the *CM* of period  $t + 1$  cannot exceed the value to the bank of the assets pledged as collateral against deposits. Here,  $\theta_t \in [0, 1]$  is the fraction of mortgage loans pledged as collateral, which is a choice variable for the bank that serves the same purpose as  $\theta_t^h$  for a buyer.

In equilibrium, the bank must choose the banking contract, subject to its constraints, to maximize the depositor's expected utility (4). As well, in equilibrium, the bank's net payoff must be zero, i.e. (5) holds with equality. Finally, we will assume that the bank's collateral constraint (6) binds in equilibrium, and we will later determine conditions that guarantee this. Essentially, we assume that collateral is scarce in the aggregate in equilibrium, in a well-defined sense. Banks, similar to buyers, face incentive constraints, but for banks this is due to the fact that banks can fake mortgages. Letting  $\gamma$  denote the cost of faking one unit of mortgage loans, the net payoff to faking a mortgage must be non-positive, or

$$-\gamma + \theta_t \beta u'(\beta d_t) \leq 0 \quad (7)$$

### 3.3 Government

We will make explicit assumptions about the powers of the monetary and fiscal authorities, and the policy rules they follow, but what is important in determining an equilibrium are the consolidated government budget constraints. The consolidated government issues currency, reserves, and nominal bonds, denoted by, respectively,  $C_t$ ,  $M_t$ , and  $B_t$ , in nominal terms, and issues liabilities and redeems them only in the *CM*. As well, the government makes a lump-sum transfer  $\tau_t$  to each buyer in the *CM* in period  $t$ .

Thus, the consolidated government budget constraints are given by

$$\phi_0 [C_0 + z_0 (M_0 + B_0)] - \tau_0 = 0 \quad (8)$$

$$\phi_t [C_t - C_{t-1} + z_t (M_t + B_t) - (M_{t-1} + B_{t-1})] - \tau_t = 0, \quad t = 1, 2, 3, \dots \quad (9)$$



## 4 Equilibrium

To solve for an equilibrium, we will first characterize the solutions to the buyer's and bank's problems. Then, we will make some assumptions about policy rules, and solve for a stationary equilibrium.

From the buyer's problem, (2) subject to (3), the solution for the buyer's "haircut" on housing collateral is

$$\theta_t^h = \min \left[ 1, \frac{\gamma^h}{q_t(\psi_{t+1} + y)} \right], \quad (10)$$

and asset prices must solve

$$-\psi_t + \beta(\psi_{t+1} + y) + \min \left[ (q_t - \beta)(\psi_{t+1} + y), \gamma^h \left( 1 - \frac{\beta}{q_t} \right) \right] = 0. \quad (11)$$

Equation (10) states that, if the cost of faking a house is sufficiently small, the price of a mortgage is sufficiently high, and the price of housing and the flow of housing services are sufficiently high, then the buyer will not borrow fully against his or her housing collateral. The buyer does this in order to demonstrate to the bank that it is not posting fake collateral. Equation (11) states that the net payoff to the buyer from acquiring one unit of housing is zero in equilibrium.

Recall that, in equilibrium, a bank chooses the bank's deposit contract  $(k_t, c_t, d_t)$ , its portfolio  $(m_t, b_t, l_t)$ , and a haircut on mortgage loan collateral  $\theta_t$ , to maximize the expected utility of depositors, subject to a zero net payoff constraint (5), the binding collateral constraint (6), and the incentive constraint (7). Then, the following must hold in equilibrium:

$$-z_t + \beta \frac{\phi_{t+1}}{\phi_t} u'(\beta d_t) = 0 \quad (12)$$

$$-q_t + \beta[\theta_t u'(\beta d_t) + 1 - \theta_t] = 0 \quad (13)$$

$$\beta \frac{\phi_{t+1}}{\phi_t} u' \left( \beta \frac{\phi_{t+1}}{\phi_t} c_t \right) - 1 = 0 \quad (14)$$

$$-(1 - \rho)d_t + \frac{\phi_{t+1}}{\phi_t} (m_t + b_t) + \theta_t l_t = 0 \quad (15)$$

$$\theta_t = \min \left( 1, \frac{\gamma}{\beta u'(\beta d_t)} \right). \quad (16)$$

Note that the quantities  $c_t$ ,  $d_t$ ,  $m_t$ ,  $b_t$ , and  $l_t$  in (14)-(15) denote, respectively, the quantities of currency and deposits promised to the representative depositor in the equilibrium banking contract, and the quantities of reserves, government bonds, and mortgage loans acquired by the representative bank in equilibrium.

In equilibrium, asset markets clear in the *CM*, so the representative bank's demands for currency, government bonds, and reserves are equal to the respective supplies coming from the government, i.e.

$$\rho c_t = \phi_t C_t, \quad (17)$$

$$b_t = \phi_t B_t, \quad (18)$$

$$m_t = \phi_t M_t. \quad (19)$$

As well, the demand for loans from banks equals the quantity supplied by buyers,

$$l_t = l_t^h, \quad (20)$$

and buyers' demand for housing is equal to the supply,

$$a_t = 1. \quad (21)$$

We will construct stationary equilibria, in which real quantities are constant forever, and all nominal quantities grow at the constant gross rate  $\mu$  forever, so that the gross rate of return on money,  $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$  for all  $t$ . Then, from the government's budget constraints (8) and (9), and (17)-(19),

$$\rho c + z(m + b) = \tau_0 \quad (22)$$

$$\rho c \left(1 - \frac{1}{\mu}\right) + \left(z - \frac{1}{\mu}\right)(m + b) - \tau = 0, \quad t = 1, 2, 3, \dots, \quad (23)$$

where  $\tau_t = \tau$  for  $t = 1, 2, 3, \dots$ , i.e. the transfer to buyers from the government may differ in period 0 from the transfer in each succeeding period. We will assume that the fiscal authority fixes the real value of the transfer in period 0,  $\tau_0 = V$ , i.e.  $V$  is exogenous. Then, from (23), we obtain

$$V \left(1 - \frac{1}{\mu}\right) + \left(\frac{m}{\mu} + \frac{b}{\mu}\right)(z - 1) = \tau, \quad (24)$$

where the tax on buyers  $\tau$  in each period  $t = 1, 2, 3, \dots$ , is endogenous. The fiscal policy rule is thus fixed in this sense, and the job of the central bank is to optimize treating the fiscal policy rule as given. So, in determining an equilibrium, all we need to take into account is equation (22) with  $\tau_0 = V$ , or

$$\rho c + z(m + b) = V. \quad (25)$$

In solving for a stationary equilibrium, it will prove convenient to express the equilibrium conditions in terms of the consumption allocation in the *DM*. This is helpful in part because, in this class of models, we can express aggregate welfare in terms of the *DM* consumption allocation. Let  $x_1$  and  $x_2$  denote, respectively, consumption in currency transactions and non-currency transactions in the *DM*, where  $x_1 = \frac{\beta c}{\mu}$  and  $x_2 = \beta d$ . Then, from (12)-(??) and (25), we obtain:

$$z = \frac{u'(x_2)}{u'(x_1)}, \quad (26)$$

$$q = \min \left[ \beta u'(x_2), \frac{u'(x_2)(\gamma + \beta) - \gamma}{u'(x_2)} \right], \quad (27)$$

$$-(1 - \rho)x_2u'(x_2) - \rho x_1u'(x_1) + V + l \min(\beta u'(x_2), \gamma) = 0, \quad (28)$$

$$\mu = \beta u'(x_1). \quad (29)$$

Similarly, from (11), (1) with equality, (20), and (21),

$$-\psi + \beta(\psi + y) + \min\left[(q - \beta)(\psi + y), \gamma^h \left(1 - \frac{\beta}{q}\right)\right] = 0, \quad (30)$$

$$l = \min\left[(\psi + y), \frac{\gamma^h}{q}\right]. \quad (31)$$

We will assume that conventional monetary policy consists of the choice of the price of short-term nominal government debt,  $z$ , which is then supported with the appropriate central bank balance sheet. Then, equations (26)-(31) can be used to solve for  $x_1$ ,  $x_2$ ,  $q$ ,  $\mu$ ,  $\psi$ , and  $l$ .

#### 4.1 Equilibrium with Sufficient Collateral

Our focus in this paper will be on the behavior of the model economy when collateral is sufficiently scarce. Collateral is not scarce in this economy if the value of collateral is large enough that an efficient allocation can be supported in equilibrium. Confining attention to stationary allocations, if we measure aggregate welfare as the sum of period utilities across economic agents, then efficiency is attained if and only if surplus is maximized in all *DM* exchanges, i.e. if  $x_1 = x_2 = x^*$ , where  $u'(x^*) = 1$ .

Then, from (26), a necessary condition for efficiency is  $z = 1$ , i.e. the nominal interest rate on government debt must be zero, so conventional monetary policy must conform to the Friedman rule. Also, in an efficient equilibrium, from (27),  $q = \beta$ , and from (11),  $\psi = \frac{\beta y}{1 - \beta}$ , so mortgages and houses are priced at their fundamental values, i.e. the sum of discounted payoffs on the respective assets. Of primary importance is that, for an efficient allocation to be feasible, the bank's collateral constraint (6) must be satisfied. From (28) and (31), the bank's collateral constraint holds if and only if

$$\underbrace{V}_{\text{public collateral}} + \underbrace{\min(\beta, \gamma) \min\left(\frac{y}{1 - \beta}, \frac{\gamma^h}{\beta}\right)}_{\text{private collateral}} \geq x^* \quad (32)$$

Inequality (32) states that the quantity of public collateral, given by  $V$  (the real value of the consolidated government debt), plus private collateral (the value of the stock of housing), must exceed the efficient quantity of consumption in the *DM*. If (32) holds, then an efficient allocation can be supported with conventional monetary policy, and the credit frictions – limited commitment and potential misrepresentation – in the model are irrelevant.

We will assume that (32) does not hold for any  $(\gamma, \gamma^h)$ , i.e.

$$V + \frac{\beta y}{1 - \beta} < x^*. \quad (33)$$

Inequality (33) states that the value of consolidated government debt plus the value of housing wealth to buyers is insufficient to support efficient exchange. Thus, (33) defines collateral scarcity. Note that, given the fiscal instruments available, the scarcity of collateral could be eliminated by fiscal policy, i.e. if the fiscal authority were to make  $V$  sufficiently large. Thus, a critical maintained assumption is that fiscal policy is suboptimal.

It will matter for the determination of equilibrium whether the incentive constraints of banks and buyers bind or not, so we will consider each of the four relevant cases in turn: both constraints bind; the buyer's constraint binds and the bank's does not; the bank's constraint binds and the buyers does not; and both constraints bind.

## 4.2 Non-binding Incentive Constraints for Buyers and Banks

A non-binding incentive constraint for the bank implies  $\theta = 1$ , which gives

$$\gamma \geq \beta u'(x_2), \quad (34)$$

and from (27),

$$q = \beta u'(x_2). \quad (35)$$

A non-binding incentive constraint for the buyer, using (35), gives

$$\gamma^h [1 - \beta u'(x_2)] \geq \beta u'(x_2) y, \quad (36)$$

and (30) gives

$$\psi = \frac{\beta u'(x_2) y}{1 - \beta u'(x_2)}, \quad (37)$$

so a necessary condition for (36) to hold and for positive asset prices is

$$\beta u'(x_2) < 1. \quad (38)$$

Finally, from (28) and (31), the bank's incentive constraint, after substitution, can be written

$$-(1 - \rho)x_2 u'(x_2) - \rho x_1 u'(x_1) + V + \frac{\beta u'(x_2) y}{1 - \beta u'(x_2)} = 0, \quad (39)$$

and then (26) and (39) solve for  $x_1$  and  $x_2$ , given  $z$ . For this to be an equilibrium requires  $z \leq 1$  (the zero lower bound on the nominal interest rate must be respected), and (34), (36), and (38) must be satisfied.

The real rates of return on assets are  $\frac{1}{z\beta u'(x_1)} - 1$  for government bonds,  $\frac{1}{q} - 1$  for mortgages, and  $\frac{\psi + y}{\psi} - 1$  for housing, so in this equilibrium in which incentive constraints do not bind,

$$\frac{1}{z\beta u'(x_1)} - 1 = \frac{1}{q} - 1 = \frac{\psi + y}{\psi} - 1 = \frac{1}{\beta u'(x_2)} - 1,$$

so real rates of return on assets (other than currency) are equalized. Note that collateral constraints for buyers and banks bind in this equilibrium if and only if  $u'(x_2) < 1$  in equilibrium, so that exchange is inefficient (surplus is not maximized) in *DM* non-currency trades. Thus,  $u'(x_2) < 1$  reflects a low supply of collateral in the aggregate, and this in turn shows up in the form of a low real rate of return on assets, i.e. a real return on assets less than the “fundamental” rate of return, which is  $\frac{1}{\beta} - 1$ .

### 4.3 Buyer’s Incentive Constraint Binds, Bank’s Incentive Constraint Does Not Bind

In this case, since  $\theta = 1$ , and  $\theta^h < 1$ , (34) and (35) hold, as in the previous case, but instead of (36) we have

$$\gamma^h [1 - \beta u'(x_2)] < \beta u'(x_2)y. \quad (40)$$

Then, (30) implies that

$$\psi = \frac{\beta u'(x_2)y + \gamma^h [u'(x_2) - 1]}{(1 - \beta) u'(x_2)}. \quad (41)$$

We can then write the bank’s collateral constraint in equilibrium as

$$-(1 - \rho)x_2 u'(x_2) - \rho x_1 u'(x_1) + V + \gamma^h = 0 \quad (42)$$

Inequality (40) implies that real rates of return on assets in this equilibrium satisfy

$$\frac{1}{\beta u'(x_2)} - 1 = \frac{1}{\beta u'(x_1)z} = \frac{1}{q} - 1 < \frac{\psi + y}{\psi} - 1,$$

so the real rate of return on government bonds is equal to the real rate of return on a mortgage loan, but those rates of return are less than the rate of return on housing. In this equilibrium, the incentive constraint for buyers binds, and the buyer must give the housing collateral a haircut, i.e. he or she does not borrow against the full value of the collateral, in order to demonstrate that the collateral is good. This then implies a rate of return differential between the mortgage rate and the rate of return on housing.

### 4.4 Buyer’s Incentive Constraint Does Not Bind, Bank’s Incentive Constraint Binds

In this case  $\theta < 1$ , so from (27),

$$q = \frac{u'(x_2)(\gamma + \beta) - \gamma}{u'(x_2)}, \quad (43)$$

and

$$\gamma < \beta u'(x_2). \quad (44)$$

From (30), we can then solve for the price of housing

$$\psi = \frac{y[u'(x_2)(\gamma + \beta) - \gamma]}{u'(x_2)[1 - \gamma - \beta] + \gamma}. \quad (45)$$

A necessary and sufficient condition for positive asset prices, from (45), is

$$\gamma < \frac{u'(x_2)(1 - \beta)}{u'(x_2) - 1}. \quad (46)$$

Then, given (46), for  $\theta^h = 1$ ,

$$\gamma^h \geq \frac{y[u'(x_2)(\gamma + \beta) - \gamma]}{u'(x_2)[1 - \gamma - \beta] + \gamma}. \quad (47)$$

Finally, we can write the bank's incentive constraint in equilibrium as

$$-(1 - \rho)x_2u'(x_2) - \rho x_1u'(x_1) + V + \frac{yu'(x_2)\gamma}{u'(x_2)(1 - \gamma - \beta) + \gamma} = 0 \quad (48)$$

In this equilibrium, the gross real rate of return on government debt is less than the gross rates of return on mortgages and houses, or,

$$\frac{1}{\mu z} - 1 < \frac{1}{q} - 1 = \frac{\psi + y}{\psi} - 1.$$

In this case, the bank's incentive constraint binds in equilibrium, so the bank does not borrow against the full value of its mortgage portfolio. This implies that there is an interest differential between government debt and mortgage debt, with the rate of return on housing equal to the rate of return on mortgage debt.

#### 4.5 Buyer's and Bank's Incentive Constraints Bind

In this case,  $\theta < 1$ , so (43) and (44) hold, and  $\theta^h < 1$  implies

$$\gamma^h \{u'(x_2)[1 - \gamma - \beta] + \gamma\} < y[u'(x_2)(\gamma + \beta) - \gamma] \quad (49)$$

From (30) the price of housing is given by

$$\psi = \frac{\beta y + \frac{\gamma \gamma^h [u'(x_2) - 1]}{u'(x_2)(\gamma + \beta) - \gamma}}{1 - \beta}. \quad (50)$$

Then, from (28), the bank's incentive constraint in equilibrium is

$$-(1 - \rho)x_2u'(x_2) - \rho x_1u'(x_1) + V + \frac{\gamma \gamma^h u'(x_2)}{u'(x_2)(\gamma + \beta) - \gamma} = 0 \quad (51)$$

In this equilibrium, the real rate of return on government debt is less than the rate of return on mortgages, which in turn is less than the rate of return on houses, i.e.

$$\frac{1}{\mu z^s} - 1 < \frac{1}{q} - 1 < \frac{y}{\psi},$$

Thus, in this case both incentive problems (for buyers and banks) present themselves. The incentive problem for the bank drives a wedge between the safe rate of interest and the mortgage rate, and the household's incentive problem gives an additional wedge between the mortgage rate and the rate of return on housing.

## 5 Conventional Monetary Policy

Our assumption (33), that fiscal policy is suboptimal, implies that an efficient allocation is unattainable in equilibrium. As a result, the credit frictions in the model matter, and monetary policy interacts with those frictions in interesting ways. The purpose of this section is to understand the role of conventional monetary policy in the context of these frictions.

From (26) and (29), the real interest rate on government debt in a stationary equilibrium can be written

$$r = \frac{1}{\beta u'(x_2)} - 1 = \underbrace{\frac{1}{\beta} - 1}_{\text{fundamental}} - \underbrace{\frac{[u'(x_2) - 1]}{\beta u'(x_2)}}_{\text{liquidity premium effect}}, \quad (52)$$

so the safe real rate of interest is lower than the fundamental, with the difference being due to a liquidity premium effect. The liquidity premium on government debt arises because of its role as collateral on bank balance sheets. Note that the liquidity premium depends on the inefficiency in transactions in the *DM* involving bank deposits, i.e. on the difference  $u'(x_2) - 1$ .

From our analysis in the previous four subsections, we can use inequalities (34), (36), (40), (44), (47), and (49) to construct Figures 1 and 2. These figures show, given the real interest rate on government debt,  $r$ , which incentive constraints bind given the costs of misrepresenting asset quality,  $\gamma$  and  $\gamma^h$ .

Figure 1 illustrates the case where  $r > 0$ , which implies that the parameter space is divided into four regions, one where neither incentive constraint binds, in the upper right, one where both constraints bind, in the lower left, one where the bank's incentive constraint binds and the buyer's does not, in the upper left, and the remaining one where the bank's incentive constraint does not bind and the buyer's does. An interesting feature here is that, if  $\frac{\beta y}{1-\beta} < \gamma^h < \frac{y}{r}$ , then the buyer's incentive constraint does not bind for low  $\gamma$ , but binds for high  $\gamma$ . This is because the buyer's incentive to fake housing assets depends on two things: the price of housing and the cost of faking a house. But the price of housing depends on the bank's incentive to fake a mortgage loan. When the cost of faking a mortgage loan is low, the bank effectively takes a haircut

on the mortgage loan in equilibrium, and this tends to reduce the demand for mortgages and their price, which in turn reduces the price of houses below what it would otherwise be. Since the price of houses is low when  $\gamma$  is low, this reduces the buyer's incentive to fake a house.

Next, in Figure 2, we show the configuration when  $r \leq 0$ . In this case, there are only three regions in the parameter space – the one in which neither incentive constraint binds has disappeared. When  $r \leq 0$ , from (??) and (??) the real rate of return on government debt is low, and all asset prices tend to be high, which gives banks and buyers a greater incentive to fake collateral. As a result, no matter how high the costs of faking assets are in this case, there cannot exist an equilibrium where neither incentive constraint binds.

As a step to understanding the effects of conventional monetary policy, it helps to consider the effects of changes in the real interest rate on government debt, given by (52), on other rates of return, and on interest rate differentials. As we showed in the previous four subsections, rates of return on government debt, mortgages, and housing, are equalized if neither incentive constraint binds. That is, if buyers and banks do not have the incentive to fake collateral in equilibrium, then all collateral is effectively equivalent, and bears the same liquidity premium, as given by (52). However, if either incentive constraint binds, this implies that some collateral is better than others, and this will be reflected in rates of return.

First, consider the case in which the bank's incentive constraint, (7), binds. This implies, from (43) and (52), that the rate of return on mortgages can be expressed as

$$\frac{1}{q} - 1 = r + \underbrace{\frac{[1 - \gamma(1+r)][1 - \beta(1+r)]}{\gamma[1 - \beta(1+r)] + \beta}}_{\text{interest rate spread}}. \quad (53)$$

Note that the interest rate spread – the second term on the right-hand side of (53) – is strictly positive if and only if  $\gamma > \frac{1}{1+r}$  and  $r < \frac{1}{\beta} - 1$ , i.e. if and only if the incentive constraint for the bank binds (see Figures 1 and 2) and there exists a liquidity premium on government debt (see (52)). That is, the interest rate spread on mortgages arises only when collateral is scarce in the aggregate, and mortgages are worse collateral than government debt, from the bank's point of view.

From (53), by inspection the interest rate spread is decreasing in the cost of misrepresentation  $\gamma$ . As we should expect, a lower cost of faking collateral for the bank implies a higher interest rate spread when the bank's incentive constraint binds. As well, by differentiating the right-hand side of (53) with respect to  $r$ , we can show that the interest rate spread is decreasing in  $r$ . Thus, with a lower real interest rate, which from (52) will occur if the liquidity premium on government debt rises, the interest rate differential rises. This occurs because a lower real interest rate and a higher liquidity premium increase the incentive for the bank to fake collateral.

As well, from (16) and (52), the fraction of the value of mortgages against which the bank can borrow, if the bank's incentive constraint binds, can be



expressed as

$$\theta = \gamma(1 + r). \quad (54)$$

Therefore, from (54), the haircut on mortgage debt held by the bank increases if the cost of faking mortgage collateral decreases, or if the real interest rate falls.

Next, if the buyer's collateral constraint binds, there will be an interest rate spread on housing, relative to government debt, but the size of this spread depends on whether or not the bank's incentive constraint binds. First, if the bank's incentive constraint does not bind, then from (41) and (52), the rate of return on housing from the buyer's viewpoint, is

$$\frac{y}{\psi} = r + \underbrace{\frac{[1 - \beta(1 + r)](y - \gamma^h r)}{\beta y + \gamma^h [1 - \beta(1 + r)]}}_{\text{interest rate spread}}. \quad (55)$$

Then, similar to the case for the interest rate spread on mortgages, the spread in the second term on the right-hand side of (55) is strictly positive if and only if the buyer's incentive constraint binds (see Figures 1 and 2) and there is a liquidity premium on government debt (see (52)). As well, the interest rate spread in (55) is decreasing in  $\gamma^h$  and in  $r$ . Therefore, a decrease in the cost of faking housing collateral for the buyer tightens the buyer's incentive constraint and increases the interest rate spread, and a decrease in the real interest rate on government debt has the same qualitative effects.

If the buyer's incentive constraint binds, and the bank's incentive constraint does not then, from (10) and (52), the fraction of housing wealth against which the buyer can borrow is given by

$$\theta^h = \frac{\gamma^h(1 - \beta)(1 + r)}{y + \gamma^h[1 - \beta(1 + r)]}. \quad (56)$$

Therefore, in this case the haircut applied to the value of housing wealth when a buyer borrows on the mortgage market increases when either the real interest rate on government debt falls, or when the cost of faking housing collateral falls.

Finally, consider the case in which the buyer's and bank's collateral constraints both bind. Then, the rate of return on housing is given by

$$\frac{y}{\psi} = r + \underbrace{\frac{[1 - \beta(1 + r)] \{y[\gamma + \beta - \gamma\beta(1 + r)] - \gamma\gamma^h r\}}{y[\gamma + \beta - \gamma\beta(1 + r)] + \gamma\gamma^h [1 - \beta(1 + r)]}}_{\text{interest rate spread}} \quad (57)$$

As in equations (53) and (55), the interest rate spread on the right-hand side of (57) is strictly positive if and only if there is a liquidity premium on government debt. As well, it is straightforward to show that the interest rate spread is decreasing in  $\gamma$ , in  $\gamma^h$ , and in  $r$ . Therefore, a decrease in the cost of collateral misrepresentation for buyers or for banks, either of which tightens incentive constraints, will increase the interest rate spread. As well, just as for the other

spreads, an increase in the liquidity premium on government debt, which reduces the real interest rate, will increase the interest rate spread in equation (57).

If the buyer's and bank's incentive constraints both bind then, from (10) and (52), the fraction of housing wealth against which the buyer can borrow on the mortgage market can be written as

$$\theta^h = \frac{\gamma^h}{y[\gamma + \beta - \gamma\beta(1+r)] + \gamma\gamma^h[1 - \beta(1+r)]}. \quad (58)$$

As in equation (56), from (58) a decrease in the cost of faking housing collateral, or a decrease in the real interest rate on government debt, will increase the haircut on housing collateral that buyers face in the mortgage market. An interesting effect in (58) is that a decrease in  $\gamma$  will reduce the haircut on housing collateral, in spite of the fact that lower  $\gamma$  implies a higher haircut for banks on mortgage collateral, from (54). This occurs because, from (50), a reduction in  $\gamma$  reduces the price of housing when both incentive constraints bind, and this reduces the incentive of buyers to fake housing collateral.

Figures 1 and 2 show what incentive constraints bind given  $r$ , which is endogenous, and the costs of misrepresenting collateral,  $\gamma$  and  $\gamma^h$ . For the purposes of learning more about how conventional monetary policy works in the model, it is useful to consider how changes in the real interest rate will affect the characteristics of an equilibrium – i.e. what incentive constraints bind – for given values of  $(\gamma, \gamma^h)$ . First, if we let  $R = 1 + r$  denote the gross real interest rate, from (52) we have

$$R = \frac{1}{\beta u'(x_2)} \quad (59)$$

in equilibrium. Since  $0 < x_2 \leq x^*$  in equilibrium, where  $u'(x^*) = 1$ , therefore  $0 < R \leq \frac{1}{\beta}$ . If we consider the extreme case in which  $R \rightarrow 0$ , then from our analysis summarized in Figure 2, if

$$\gamma^h(1 - \gamma - \beta) \geq (\gamma + \beta)y, \quad (60)$$

then the buyer's incentive constraint does not bind, and the bank's incentive constraint does. However, if

$$\gamma^h(1 - \gamma - \beta) < (\gamma + \beta)y, \quad (61)$$

then both incentive constraints bind as  $R \rightarrow 0$  (see Figure 3). Thus, if the real interest rate is sufficiently low, then for any finite misrepresentation costs, the bank's incentive constraint must bind, and the buyer's incentive constraint may bind.

At the other extreme, if  $R = \frac{1}{\beta}$ , then from our analysis summarized in Figure 1, the bank's incentive constraint binds if and only if

$$\gamma < \beta, \quad (62)$$

and the buyer's incentive constraint binds if and only if

$$\gamma^h < \frac{\beta y}{1 - \beta}. \quad (63)$$

See Figure 4, which summarizes the extreme case where  $R = \frac{1}{\beta}$ .

The extreme cases,  $R = 0$  and  $R = \frac{1}{\beta}$ , tell us something about the intervening possibilities, depicted in Figures 5 and 6, which show the cases  $\beta \geq \frac{1}{2}$  and  $\beta < \frac{1}{2}$ , respectively. In Figure 5, we can divide the parameter space into 6 regions as follows:

**Region 1:** This region is defined by (60). The bank's incentive constraint binds, and the buyer's incentive constraint does not bind, for all  $R \in (0, \frac{1}{\beta})$ .

**Region 2:** This region is defined by (61),

$$\gamma^h \geq \frac{\beta y}{1 - \beta}, \quad (64)$$

and (62). If  $R \in \left(0, \frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)}\right)$ , then both incentive constraints bind. If  $R \in \left[\frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)}, \frac{1}{\beta}\right]$ , then the bank's incentive constraint binds, and the buyer's incentive constraint does not bind.

**Region 3:** This region is defined by (62) and (63). Both incentive constraints bind for all  $R \in (0, \frac{1}{\beta})$ .

**Region 4:** This region is defined by

$$\gamma \geq \beta, \quad (65)$$

and

$$\gamma^h(1 - \gamma) \geq \gamma y. \quad (66)$$

For  $R \in \left(0, \frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)}\right)$ , both constraints bind; for  $R \in \left[\frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)}, \frac{1}{\gamma}\right)$ , the bank's incentive constraint binds and the buyer's constraint does not; for  $R \geq \frac{1}{\gamma}$ , neither incentive constraint binds.

**Region 5:** This region is defined by (65), (64), and

$$\gamma^h(1 - \gamma) < \gamma y. \quad (67)$$

For  $R \in \left(0, \frac{1}{\gamma}\right)$ , both incentive constraints bind; for  $R \in \left[\frac{1}{\gamma}, \frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)}\right)$ , the buyer's incentive constraint binds and the bank's does not; and for  $R \geq \frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)}$ , neither constraint binds.

**Region 6:** This region is defined by (65) and (63). For  $R \in \left(0, \frac{1}{\gamma}\right)$ , both incentive constraints bind; and for  $R \geq \frac{1}{\gamma}$ , the bank's incentive constraint does not bind, and the buyer's incentive constraint binds.

In the case where  $\beta < \frac{1}{2}$ , in Figure 6, we need to add an additional region of the parameter space:

**Region 7:** This region is defined by (65) and (60). For  $R \in \left(0, \frac{1}{\gamma}\right)$ , the bank's incentive constraint binds and the buyer's does not; for  $R \geq \frac{1}{\gamma}$ , neither incentive constraint binds.

As well, in Figure 6, for the case  $\beta < \frac{1}{2}$  we need to redefine Region 4 as:

**Region 4a:** This region is defined by (65), (66), and (61). For  $R \in \left(0, \frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)}\right)$ , both constraints bind; for  $R \in \left[\frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)}, \frac{1}{\gamma}\right)$ , the bank's incentive constraint binds and the buyer's constraint does not; for  $R \geq \frac{1}{\gamma}$ , neither incentive constraint binds.

Next, to understand the effects of conventional monetary policy, note from (39), (42), (48), (51), and (26), that a stationary equilibrium with scarce collateral is in general a solution  $(x_1, x_2)$  to

$$(1 - \rho)x_2u'(x_2) + \rho x_1u'(x_1) = V + K(x_2), \quad (68)$$

and

$$z = \frac{u'(x_2)}{u'(x_1)}. \quad (69)$$

Here,  $z \leq 1$  is the price of government debt, which is set exogenously by the central bank. In equation (68), the left-hand side can be interpreted as the aggregate demand for collateral as a function of consumption in the *DM*,  $(x_1, x_2)$ , while the right-hand side is the aggregate supply of collateral from the government,  $V$ , and the private sector,  $K(x_2)$ . From (39), (42), (48), and (51), the function  $K(x_2)$  depends on which incentive constraints bind. In particular, if neither constraint binds, then

$$K(x_2) = \frac{\beta u'(x_2)y}{1 - \beta u'(x_2)}. \quad (70)$$

If the buyer's incentive constraint binds, and the buyer's does not, then

$$K(x_2) = \gamma^h. \quad (71)$$

If the bank's incentive constraint binds, and the buyer's does not, then

$$K(x_2) = \frac{yu'(x_2)\gamma}{u'(x_2)(1 - \gamma - \beta) + \gamma}. \quad (72)$$

Finally, if both incentive constraints bind, then

$$K(x_2) = \frac{\gamma\gamma^h u'(x_2)}{u'(x_2)(\gamma + \beta) - \gamma}. \quad (73)$$

The most straightforward cases to consider are the ones in which neither constraint binds, or only one constraint binds so that  $K(x_2)$  is given by (70), (71), or (72). Then,  $K'(x_2) < 0$ , so (68) describes a downward-sloping locus in  $(x_1, x_2)$  space, and the solution to (68) and (69) is depicted in Figure 7. First, suppose that the central chooses to peg the price of government debt at  $z_1 < 1$ . In the figure,  $CC$  denotes the collateral constraint for the bank (68), and  $ZLB$  is the zero lower bound on the nominal interest rate on government debt, i.e. the locus along which  $z = 1$ . There is a unique equilibrium solution, and the initial equilibrium with  $z = z_1$  is at point  $A$ . If the central bank chooses a higher nominal interest rate, for example the central bank chooses  $z_2 < z_1$ , then the equilibrium will be at point  $B$ . Thus, at point  $B$ ,  $x_1$  is lower and  $x_2$  is higher. Lower  $x_1$  implies a higher inflation rate, from (29), and higher  $x_2$  implies a higher real interest rate on government debt, from (52).

Thus, in the cases in which neither incentive constraint binds, or one incentive constraint binds, what is conventionally considered a “tightening” in monetary policy, i.e. an increase in the nominal interest rate or a decrease in  $z$ , will increase the real interest rate  $r$ . From our analysis above, recall that an increase in  $r$  acts to relax collateral constraints and reduce haircuts when incentive constraints bind, and can induce incentive constraints not to bind if they already bind. Thus, if there is conventional “easing” in monetary policy, i.e. a reduction in the nominal interest rate on government debt, this will reduce the real interest rate, and act to aggravate incentive problems in credit markets, at least in these circumstances.

For the case where both incentive constraints bind, things are somewhat more complicated, as this opens up the possibility of multiple stationary equilibria. For clarity of exposition, consider the special case  $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$ , with our maintained assumption that  $\alpha < 1$ . Then, from (59), the gross real interest rate is  $R = \frac{x_2^\alpha}{\beta}$ , and from (68), (69), and (73) the following equation solves for  $R$  in an equilibrium in which both incentive constraints bind:

$$(R\beta)^{\frac{1}{\alpha}-1} \left( \rho z^{\frac{1}{\alpha}-1} + 1 - \rho \right) = V + \frac{\gamma\gamma^h}{\gamma + \beta - \gamma\beta R} \quad (74)$$

or

$$\Gamma(R) = \Upsilon(R). \quad (75)$$

Note, from (74) and (75) that  $\Gamma(R)$  is strictly increasing in  $R$ , concave if  $\alpha \geq \frac{1}{2}$ , and convex if  $\alpha \leq \frac{1}{2}$ , with  $\Gamma(0) = 0$ . The function  $\Upsilon(R)$  is strictly increasing and strictly convex, with  $\Upsilon(0) = V + \frac{\gamma\gamma^h}{\gamma+\beta} > 0$  and  $\Upsilon\left(\frac{1}{\beta}\right) = V + \frac{\gamma\gamma^h}{\beta}$ . Suppose that there is a stationary equilibrium in which both incentive constraints bind, and the gross real interest rate is  $R_1$ . Further, suppose that  $\Gamma'(R_1) < \Upsilon'(R_1)$ . Such an equilibrium is illustrated by point  $B$  in Figure 8, for the case  $\alpha = \frac{1}{2}$ . If  $R_1$  is an equilibrium gross interest rate and both incentive constraints bind, then from our analysis above,

$$R_1 \in \left( 0, \min \left[ \frac{1}{\gamma}, \frac{1}{\beta}, \frac{(\gamma^h + y)(\gamma + \beta) - \gamma^h}{\gamma\beta(\gamma^h + y)} \right] \right).$$

But,  $0 = \Gamma(0) < \Upsilon(0) = \frac{\gamma^h}{\gamma + \beta}$ . Therefore, there must exist another equilibrium gross real interest rate  $R_2 < R_1$  as illustrated by point *A* in Figure 8, with  $\Gamma'(R_2) > \Upsilon'(R_2)$ . Note that if the central bank increases  $z$ , i.e. if the nominal interest rate falls, then the equilibria are then at points *D* and *E*. Thus, in the high-real-interest-rate equilibrium, conventional monetary policy easing raises the real interest rate, in contrast to what happens in an equilibrium in which the incentive constraints do not bind or only one incentive constraint binds. However, if the high-real-interest-rate equilibrium exists, there exists a low-real-interest-rate equilibrium in which the comparative statics are the same as in equilibria in which no incentive constraints bind or there is only one binding incentive constraint – a decrease in the nominal interest rate reduces the real interest rate.

Thus, in general we can say that, given any parameter configuration and constant relative risk aversion, there exists an equilibrium in which conventional monetary policy easing lowers the real interest rate and tightens collateral constraints if both incentive constraints bind.

What can we say about optimality, under conventional monetary policy? First, if we add period utilities across agents in a stationary equilibrium, this gives us a welfare measure

$$W(x_1, x_2) = \rho[u(x_1) - x_1] + (1 - \rho)[u(x_2) - x_2], \quad (76)$$

which is the sum of surpluses in exchange in the *DM* in equilibrium. Then, the derivative of a level surface of the welfare function given by (76), evaluated along the zero lower bound locus where  $x_1 = x_2$  in Figure 7, is

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho}{1 - \rho}. \quad (77)$$

Similarly, the derivative of the locus defined by (68), also evaluated at the zero lower bound, is

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho}{(1 - \rho) - \frac{K'(x)}{u'(x) \left[1 + \frac{xu''(x)}{u'(x)}\right]}}, \quad (78)$$

where from (68) and (69) with  $z = 1$ ,  $x$  solves

$$xu'(x) = V + K(x),$$

Then, given our maintained assumption that  $-\frac{xu''(x)}{u'(x)} < 1$ , if  $K'(x) < 0$ , then

$$\frac{\rho}{1 - \rho} > \frac{\rho}{(1 - \rho) - \frac{K'(x)}{u'(x) \left[1 + \frac{xu''(x)}{u'(x)}\right]}},$$

so the zero lower bound is a locally optimal monetary policy if neither incentive constraint binds, or only one incentive constraint binds, from (70), (71), and (72). Thus, in spite of the fact that first-best efficiency – i.e.  $x_1 = x_2 = x^*$

– is not a feasible equilibrium allocation (because of scarce collateral) a zero nominal interest rate is optimal, at least locally.

But, when both incentive constraints bind, from (73)  $K'(x_2) > 0$ , which implies, from (77) and (78), that if both incentive constraints bind then the zero lower bound is suboptimal. This is an important result as it tells us that, in circumstances where the incentive problems in credit markets are most severe, conventional monetary policy should deviate from the zero lower bound.

This is very different from the policy conclusions that come from New Keynesian models. For example, in Werning (2012), prices are sticky and a temporarily high discount factor implies that it is optimal for the central bank to set the nominal interest rate at zero for some period of time. A high discount factor is intended, in this New Keynesian class of models, to stand in for a financial crisis shock. For example, in incomplete markets models, or New Keynesian models such as Eggertsson and Krugman (2012), tighter borrowing constraints (interpreted as arising from an event like a financial crisis) tend to lower the real rate of interest. But price stickiness then implies that the zero lower bound on the nominal interest rate can constrain monetary policy, i.e. the real interest rate is too high. In contrast, in our model we could think of the financial crisis as arising from lower costs of misrepresenting collateral, due to some combination of financial innovation and regulatory changes. In the model, all agents understand what is going on in the environment. Thus, the model cannot capture the runup to the financial crisis, which appears to have involved a misunderstanding by most financial market participants of the gravity of the incentive problems at work. However, once the nature of the incentive problems was well understood, the model captures the idea that decreases in  $\gamma$  and  $\gamma^h$  could lead to a tightening of collateral constraints and a decrease in the real interest rate. However, if both incentive constraints bind, then the zero lower bound is not optimal, which is quite different from the policy prescription coming from New Keynesian models.

## 6 Private Asset Purchases by the Central Bank

Our goal now is to try to understand the consequences of central bank intervention in the form of purchases of private mortgages. What we have in mind is related to the policy experiments conducted by the Federal Reserve System in the U.S. during and following the financial crisis, involving purchases of mortgage-backed securities (MBS). In our model, we assume that the central bank cannot purchase mortgages directly from buyers, but must purchase mortgages from private banks. Compared to reality, we have eliminated the step by which private mortgages are packaged by government-sponsored enterprises into MBS, but for our purposes this is a detail.

Purchases of mortgages by the central bank take the form of outright purchases from private banks at the price  $q_t$ , as private banks will make zero profits in lending to buyers and passing the mortgages on to the central bank. Let  $l_t^g$  denote the quantity of mortgage loans purchased in period  $t$ . Then, in a sta-

tionary equilibrium, with  $q_t = q$  and  $l_t^g = l^g$  for all  $t$ , we need to modify the government's budget constraint in a stationary equilibrium from (25) to

$$\rho c + z(m + b) = V + q^g l^g. \quad (79)$$

In equilibrium, a private bank must weakly prefer selling a mortgage to the central bank to holding the mortgage on its balance sheet. Therefore, modifying (13) and (16), in a stationary equilibrium in which  $x_2 = \beta d_t$  for all  $t$ ,

$$q \geq \min \left[ \beta u'(x_2), \frac{(\beta + \gamma)u'(x_2) - \gamma}{u'(x_2)} \right] \quad (80)$$

As well, the *purchase incentive constraint* must be satisfied, or

$$q \leq \gamma, \quad (81)$$

so that private banks do not have the incentive to sell fake mortgages to the central bank.

We need to consider two cases in turn. In the first, the central bank does not purchase the entire quantity of mortgage loans in the market each period, while in the second case the central bank holds the aggregate stock of mortgage debt, and therefore sets the market price.

## 6.1 Central Bank Asset Purchases Coexist with Private Mortgage Lending

First, suppose that, in a stationary equilibrium  $l^g < l$ , i.e. the central bank does not hold the entire stock of mortgage debt. Then, a private bank must be indifferent between selling a mortgage to the central bank and holding the mortgage on its own balance sheet, so (80) must hold with equality. Then, if  $\gamma \geq \beta u'(x_2)$  in equilibrium, from (80)  $q = \beta u'(x_2)$ , and so (81) is satisfied. However, if  $\gamma < \beta u'(x_2)$ , then from (80),  $q = \frac{(\beta + \gamma)u'(x_2) - \gamma}{u'(x_2)}$ , so if (81) holds, then  $\gamma \geq \beta u'(x_2)$ , a contradiction. Thus, if the central bank buys only a fraction of the supply of mortgages, the program is incentive compatible if and only if  $\gamma \geq \beta u'(x_2)$ , with  $q = \beta u'(x_2)$ . In other words, if the bank's incentive constraint binds in equilibrium, this is not compatible with central bank purchases of mortgages.

Then, in equilibrium, given (79), instead of (28) we get

$$-(1 - \rho)x_2 u'(x_2) - \rho x_1 u'(x_1) + V + q l^g + (l - l^g) \min(\beta u'(x_2), \gamma) = 0 \quad (82)$$

and then an equilibrium consists of quantities  $l$ ,  $x_1$ ,  $x_2$ , and a price for housing  $\psi$  solving (82), (26), (30), and (31). But, note that, since  $\gamma \geq \beta u'(x_2)$  and  $q = \beta u'(x_2)$ , (82) becomes

$$-(1 - \rho)x_2 u'(x_2) - \rho x_1 u'(x_1) + V + l \beta u'(x_2) = 0,$$

so the quantity of mortgages held on the central bank's balance sheet,  $l^g$ , is irrelevant. As the bank's incentive constraint does not bind in equilibrium, the



rates of return on mortgages and government bonds are identical, and there is no haircut on the mortgages on private banks' balance sheets. Therefore, from the bank's point of view, mortgages and government bonds have equivalent roles as collateral. Thus, if the central bank purchases mortgages, it simply swaps reserves, which are identical to government bonds, for mortgages, and so there is no effect of the effective stock of collateral held by banks. Thus, there is no effect of the central bank asset purchase program on quantities or prices. The asset purchase program simply displaces an equal quantity of private lending by banks.

## 6.2 Central Bank Purchases Account for All Mortgage Market Activity

From the previous subsection, we know that any central bank private asset purchase program that has any effect, let alone increasing welfare, must be a program in which all mortgages are held by the central bank. As (80) and (81) must hold, then

$$\gamma \geq \min \left[ \beta u'(x_2), \frac{(\beta + \gamma)u'(x_2) - \gamma}{u'(x_2)} \right]. \quad (83)$$

Therefore, if  $\gamma < \beta u'(x_2)$ , then (83) implies  $\gamma \geq \beta u'(x_2)$ , a contradiction. Therefore,  $\gamma \geq \beta u'(x_2)$  is necessary for the feasibility of the program, and  $q$  must satisfy

$$\beta u'(x_2) \leq q \leq \gamma \quad (84)$$

Then, given (79) instead of (28) we get

$$-(1 - \rho)x_2 u'(x_2) - \rho x_1 u'(x_1) + V + ql = 0 \quad (85)$$

Then, an equilibrium consists of quantities  $l$ ,  $x_1$ ,  $x_2$ , and a price for housing  $\psi$  solving (85), (26), (30), and (31) given a conventional monetary policy  $z \leq 1$ , and a mortgage price  $q$  satisfying (84).

Our goal now is to determine what policies are feasible and, then, what policies are optimal within the feasible set. Suppose first that the buyer's incentive constraint does not bind. Then, from (30) and (31),

$$\psi = \frac{qy}{1 - q}, \quad (86)$$

$$l = \frac{y}{1 - q}, \quad (87)$$

and the buyer's incentive constraint implies

$$\gamma^h(1 - q) \geq qy. \quad (88)$$

Then, substituting for  $l$  in (85) using (87), we get

$$-(1 - \rho)x_2 u'(x_2) - \rho x_1 u'(x_1) + V + \frac{qy}{1 - q} = 0 \quad (89)$$

An equilibrium is then  $(x_1, x_2)$  solving (26) and (89) given  $z \leq 1$ , and  $q$  satisfying (84) and (88). For any  $z$ , higher  $q$ , if that is feasible, will increase welfare, as this increases the last term on the left-hand side of (89), thus increasing the quantity of effective collateral in the aggregate and increasing  $x_1$  and  $x_2$ , which makes welfare increase, from (76).

But  $q$  cannot increase without bound, as ultimately either (84) or (88) must be violated. If

$$\gamma^h(1 - \gamma) \geq \gamma y, \quad (90)$$

then  $q = \gamma$  at the optimum. In this case, then, in the absence of the central bank's asset purchase program, neither incentive constraint would bind. Comparing equations (39) and (89), the optimal central bank asset purchase program must improve welfare if and only if

$$\frac{\gamma y}{1 - \gamma} > \frac{\beta u'(x_2)y}{1 - \beta u'(x_2)},$$

i.e. if and only if  $\gamma > \beta u'(x_2)$ , which holds in this region of the parameter space, except at the boundary. Thus the effect of the asset purchase program is as depicted in Figure 9, where the bank's collateral constraint shifts out from  $CC_1$  to  $CC_2$ . Along the locus  $z = z_1$  the nominal interest rate on government debt is a constant, so the asset purchase program serves to move the equilibrium allocation from  $A$  to  $B$ , and  $x_1$  and  $x_2$  increase, given conventional monetary policy. Since  $x_2$  rises, the real interest rate increases, and the inflation rate falls because  $x_1$  increases. As well, welfare must increase because  $x_1$  and  $x_2$  both rise. We get this effect, because the central bank asset purchase program bypasses suboptimal fiscal policy. Collateral is in short supply in equilibrium, and this scarcity could be relieved if the fiscal authority were to issue more government debt. Short of that, the central bank can issue reserves to purchase the entire stock of mortgages at a high price, and therefore increase the aggregate value of collateral in existence, which relaxes collateral constraints and increases welfare.

Next, if

$$\gamma^h(1 - \gamma) < \gamma y, \quad (91)$$

then  $q = \frac{\gamma^h}{\gamma^h + y}$  at the optimum. If (91) holds and  $\gamma \geq \beta u'(x_2)$ , then in the absence of the central bank asset purchase program either both incentive constraints are nonbinding, or the buyer's constraint binds and the bank's constraint does not. In the first case, from (39) and (89), welfare improves with the asset purchase program if and only if

$$\gamma^h > \frac{\beta u'(x_2)y}{1 - \beta u'(x_2)}, \quad (92)$$

which holds in this region of the parameter space. However, from (42) and (89), the equilibrium is identical with or without the optimal asset purchase program in the region in which the buyer's incentive constraint binds and the bank's does not in the absence of the program.

It remains to consider the case in which, under the central bank's asset purchase program, the buyer's incentive constraint binds. Then, from (30) and (31),

$$q \geq \frac{\gamma^h}{\gamma^h + y} \quad (93)$$

must hold. Then, from (84) and (93), this case is feasible if and only if (91) holds and  $\gamma \geq \beta u'(x_2)$ . Then, from (31) and (85), an equilibrium with the central bank's asset purchase program in place solves

$$-(1 - \rho)x_2 u'(x_2) - \rho x_1 u'(x_1) + V + \gamma^h = 0 \quad (94)$$

and (26), given  $z$  and  $q$ . Therefore, the choice of  $q$  by the central bank is irrelevant, so long as  $q$  satisfies (84) and (93). In this case, an increase in the price at which the central bank purchases mortgages increases the buyer's incentive to misrepresent the quality of collateral, and the haircut on housing increases, reducing the quantity of mortgage lending to just offset the increase in price, so that there is no effect on the total value of mortgages purchased by the central bank. The quantity of reserves issued by the central bank to purchase mortgages, which serve as collateral for banks, is thus invariant, in real terms, to the price  $q$ .

Does the central bank asset purchase program increase welfare if the buyer's incentive constraint binds? Given (90) and  $\gamma \geq \beta u'(x_2)$ , if the asset purchase program is feasible, then the bank's incentive constraint does not bind, and the buyer's incentive constraint may or may not bind, in the absence of the program. If neither incentive constraint binds in the absence of the central bank asset purchase program then, comparing (94) and (39), welfare increases with the asset purchase program if and only if (92) holds. Therefore, if neither incentive constraint binds in the absence of the program, then an optimal program will increase welfare. However, if the buyer's incentive constraint binds in the absence of the program then, since (94) and (42) are identical, the program is neutral.

### 6.3 Central bank Asset Purchase Programs: Summary and Implications for Monetary Policy

To summarize the previous two subsections:

1. If  $\gamma < \beta u'(x_2)$ , then central bank asset purchases are not feasible, as the central bank would receive only fake assets.
2. If  $\gamma \geq \beta u'(x_2)$ , and the central bank does not purchase the entire supply of mortgages, the purchase program is feasible, but it is neutral.
3. If  $\gamma \geq \beta u'(x_2)$  and  $\gamma^h \leq \frac{\beta u'(x_2)y}{1 - \beta u'(x_2)}$ , then it is feasible for the central bank to purchase the entire stock of mortgages, but the program will be neutral.

4. If  $\gamma > \beta u'(x_2)$  and  $\gamma^h > \frac{\beta u'(x_2)y}{1-\beta u'(x_2)}$ , then it is feasible and optimal for the central bank to purchase the entire stock of mortgages at a price  $q = \gamma$ . At that price, the purchase program is optimal.

We can then conclude that if, in equilibrium, in the absence of a central bank asset purchase program, either the bank's incentive constraint or the buyer's incentive constraint binds, then the purchase program is either not feasible, or it is neutral. Therefore, in such circumstances, the program should not be undertaken. Perhaps surprisingly, in circumstances in which incentive problems in credit markets matter, central bank intervention through this type of asset purchase program does no good.

Perhaps even more surprisingly, the circumstances under which central bank asset purchases work are when, given market prices, incentive constraints would not bind in the absence of the program. If, under circumstances in which asset purchases are effective, the central bank conducts such purchases optimally, this does not change the characterization of regions of the parameter space, as summarized in Figures 5 and 6. But, suppose that neither incentive constraint binds in the absence of the asset purchase program. Then, the real interest rate on government debt  $r > 0$ , and in Figure 1, costs of misrepresenting collateral and the real interest rate would place the economy in the upper right-hand region. Then, in Figure 10, in region *a*, an optimal asset purchase program would imply that neither incentive constraint would bind, and in region *b*, optimal asset purchases by the central bank imply a binding incentive constraint for the buyer. In the latter case, the incentive constraint binds because the asset purchase program increases the price of a mortgage (lowers the mortgage interest rate), and thus encourages misrepresentation, if the cost of misrepresentation is low enough, as is the case in region *b* in Figure 10.

If central bank asset purchases work, then the general equilibrium effect is depicted in Figure 11. In the figure, *ZLB* denotes the zero lower bound constraint on the nominal interest rate on government debt, and *CC*<sub>1</sub> is the bank's incentive constraint in equilibrium. The locus  $z = z_1$  represents  $(x_1, x_2)$  such that the nominal interest rate on government debt is a constant (with the price of government debt constant at  $z_1$ ). Then, in the figure, an equilibrium in the absence of the central bank asset purchase program is at point *A*, given a conventional monetary policy (a nominal interest rate on government debt). Holding conventional policy constant, suppose that the central bank introduces an optimal asset purchase program. This acts to shift the bank's incentive constraint to *CC*<sub>2</sub>, and  $x_1$  and  $x_2$  both increase, so welfare must go up. Note that, since  $x_2$  rises, the real interest rate on government debt must rise, from (52). This effect occurs because the asset purchase program relaxes collateral constraints and reduces the liquidity premium on government debt. But the mortgage interest rate drops as a result of the asset purchase program, as the real interest rate on mortgages is now  $\frac{1}{q} - 1 = \frac{1}{\gamma} - 1$ , which is necessarily lower than the initial interest rate on mortgages, as  $\gamma > \frac{1}{1+r}$  initially. As well, the inflation rate falls as a result of the asset purchase program, from (29), as  $x_1$  has increased. The inflation rate falls due to a Fisher effect. That is, the real

interest rate on government debt has risen, and the nominal interest rate is fixed by conventional monetary policy, so the inflation rate must decline.

Note that, as we showed previously, a zero nominal interest will be optimal, with or without the asset purchase program, if neither incentive constraint binds. Therefore, initially the optimum is at point  $D$  in the figure, and the optimum is then at point  $E$  with the optimal asset purchase program in place. Thus, just as in the general case with a positive nominal interest rate, at the zero lower bound optimal asset purchases by the central bank act to increase the real interest rate on government debt, reduce the real interest rate on mortgages, and reduce the inflation rate.

## 7 Conclusion

We have built a model where there are incentive problems in the mortgage market – banks can fake the quality of mortgage debt, and consumers can fake the quality of housing which is posted as collateral. These incentive problems, combined with a scarcity of collateralizable wealth, implies that conventional monetary policy easing can exacerbate credit market frictions by tightening incentive constraints. Perhaps surprisingly, when incentive constraints bind for both banks and consumers, a zero nominal interest rate is suboptimal because the real interest rate is too low at the zero lower bound.

Central bank purchases of mortgage debt may not be feasible, because of private banks' incentives to misrepresent the mortgages they pass on to the central bank. Even if central bank purchases are feasible, such programs will be neutral if the central bank does not purchase all mortgage debt, or the incentive constraints of consumers bind. If successful, a central bank asset purchase program works by circumventing suboptimal fiscal policy, increasing the value of the stock of collateralizable wealth, and relaxing the collateral constraints of banks. Welfare-improving central bank asset purchase programs reduce the real interest rate on mortgages, but increase the real interest rate on government debt, while reducing the inflation rate.

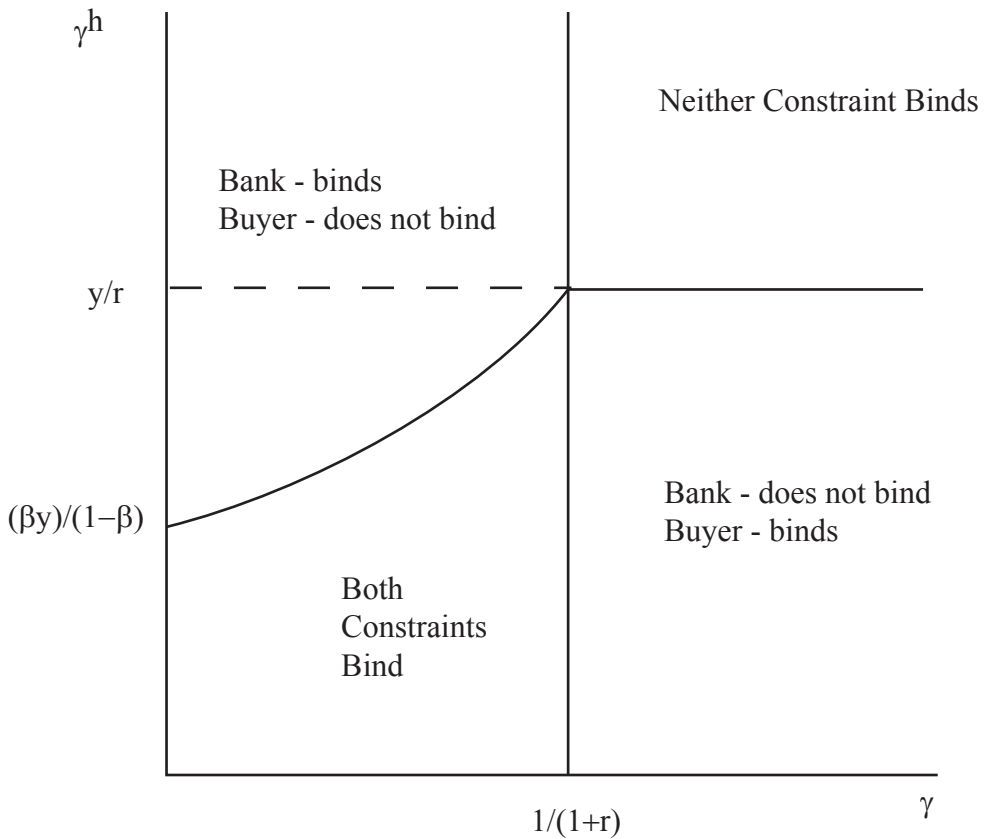
The paper does not address an important issue related to private asset purchases by the central bank. In general, such purchases will tend to favor some credit market participants relative to others. If private assets are purchased by the central bank, some choices must be made about which assets to purchase, and which assets not to purchase. If the purchase programs work as intended, this must have redistributive effects, and there are important issues that need to be addressed relating to political economy and central bank independence.

## 8 References

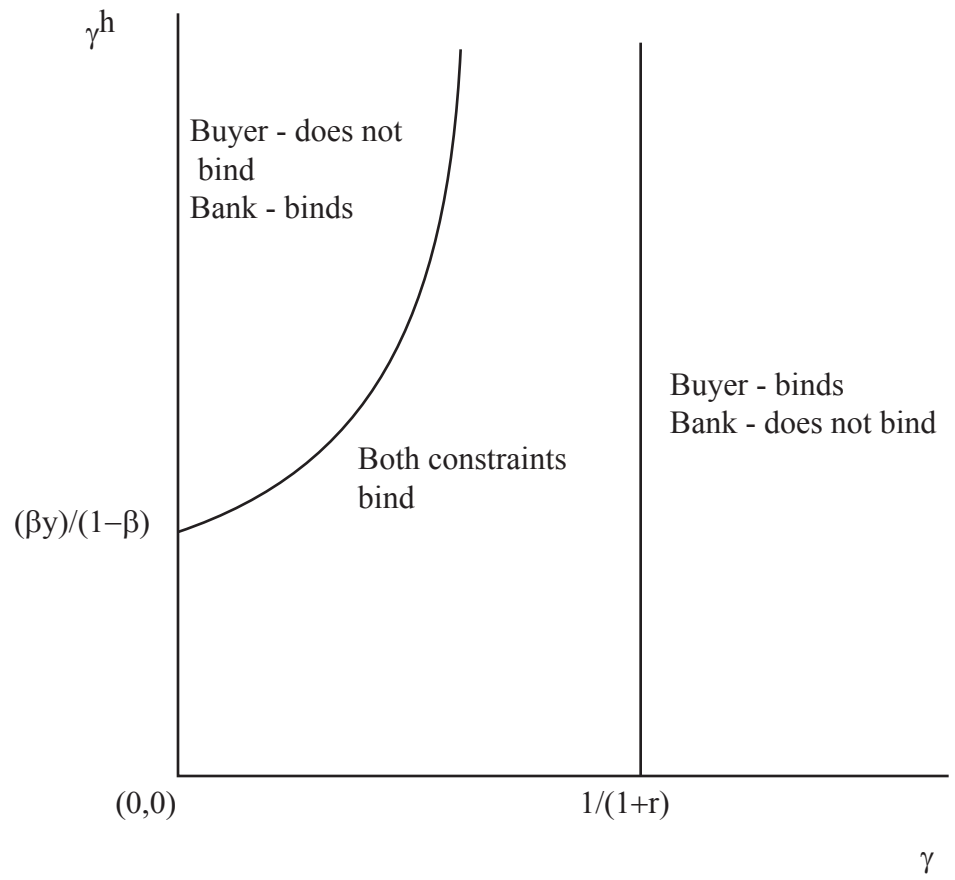
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**Figure 1: Equilibria when  $r > 0$**

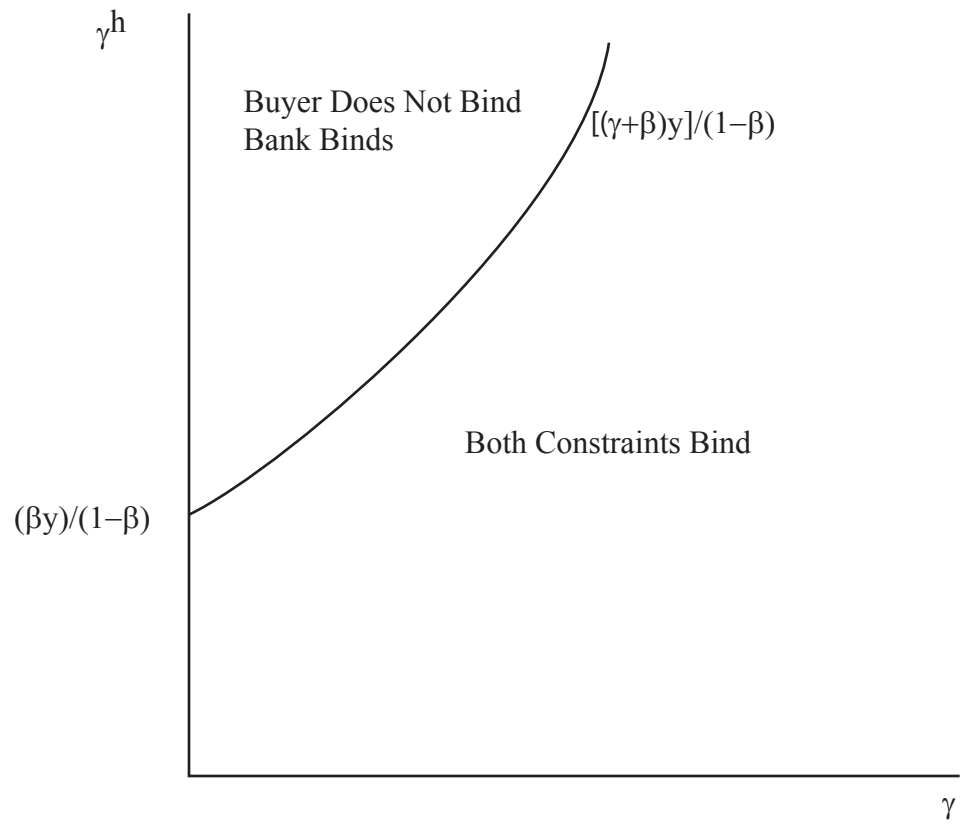


**Figure 2: Equilibrium When  $r < 0$**

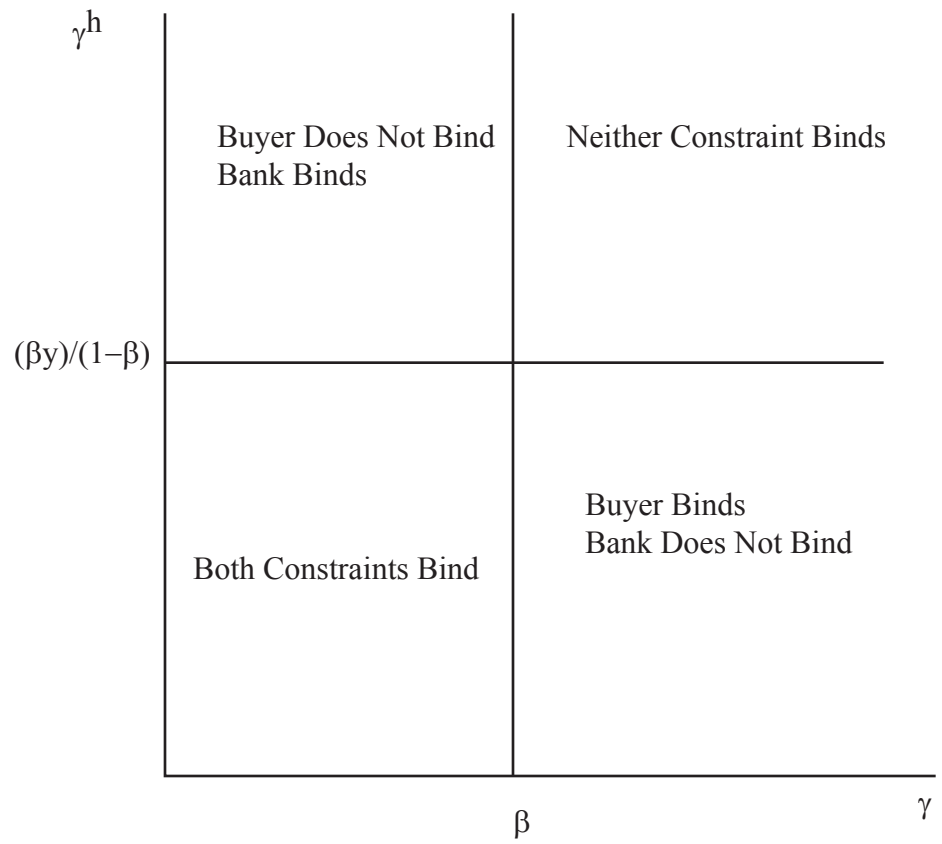




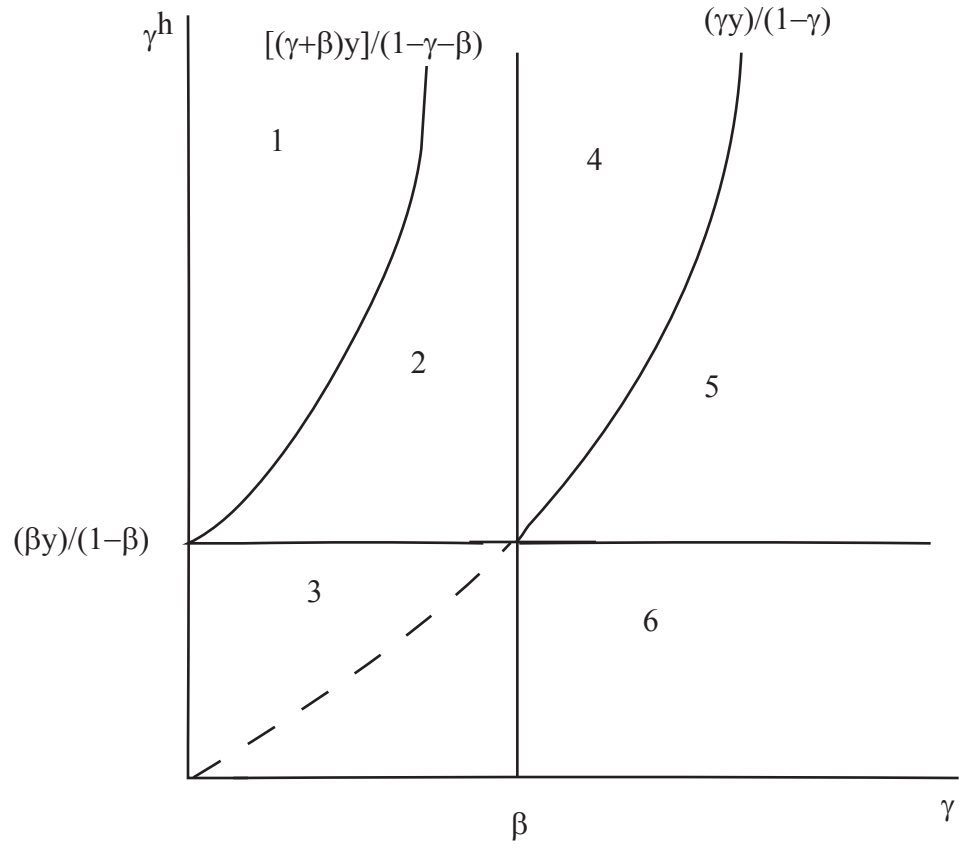
**Figure 3: Equilibria when  $R = 0$**



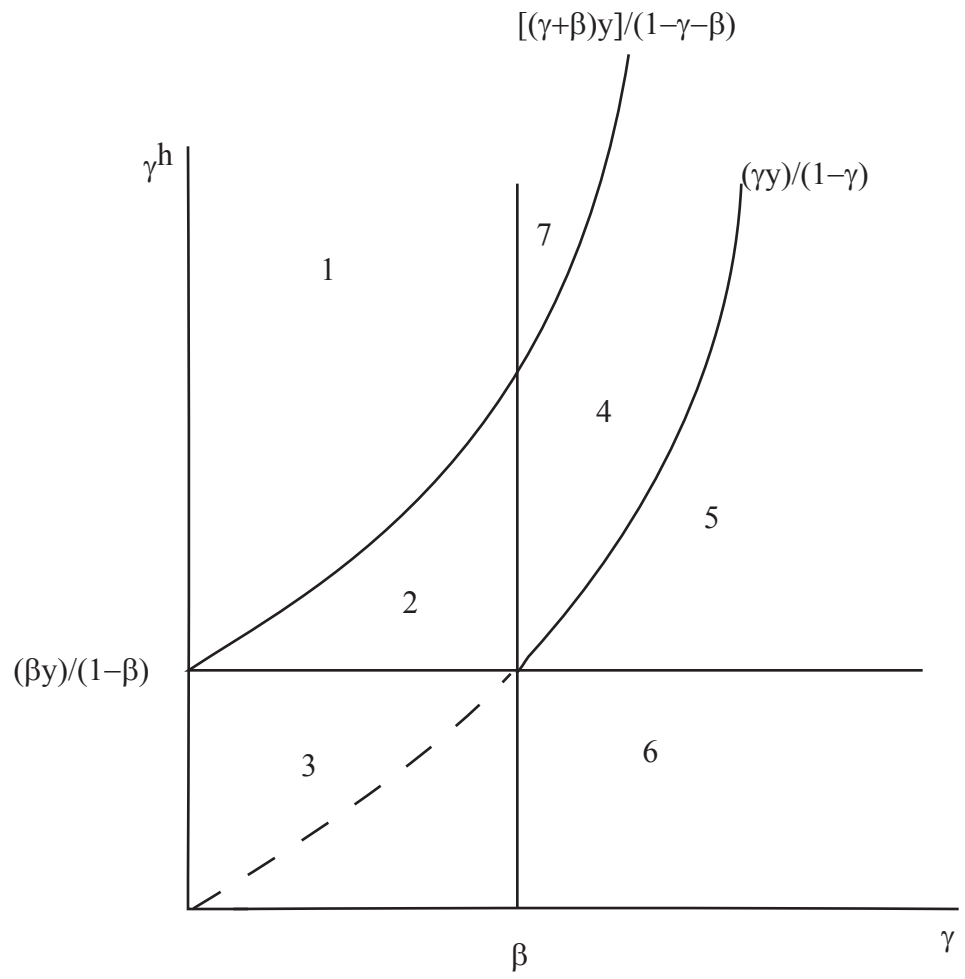
**Figure 4: Equilibria when  $R = 1/\beta$**



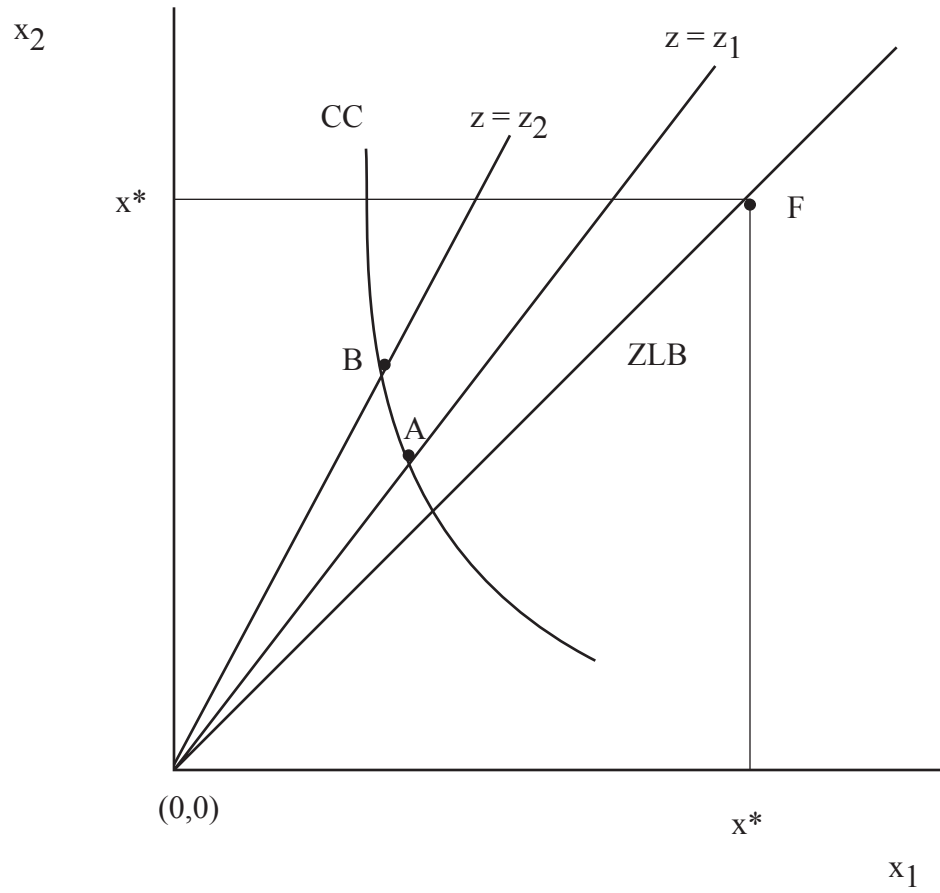
**Figure 5:**  $\beta \geq 1/2$



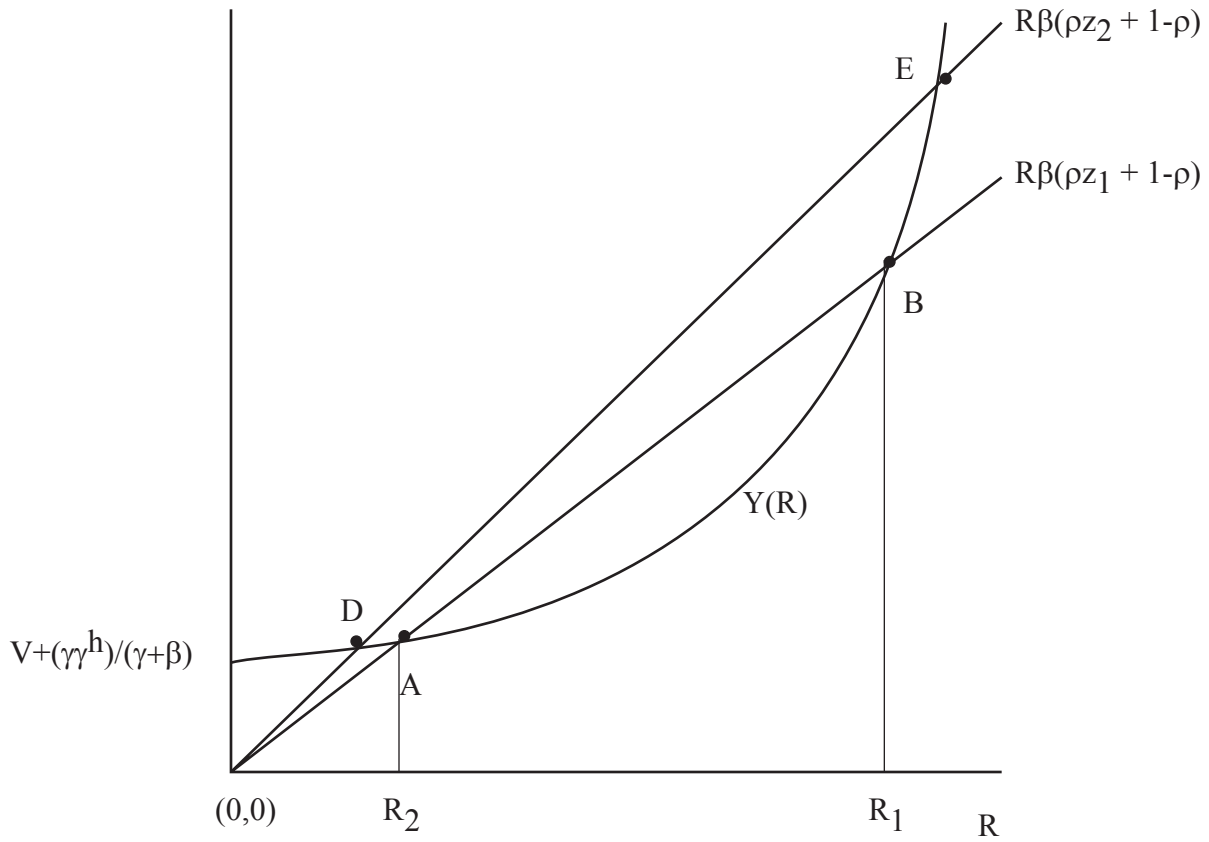
**Figure 6:**  $\beta < 1/2$



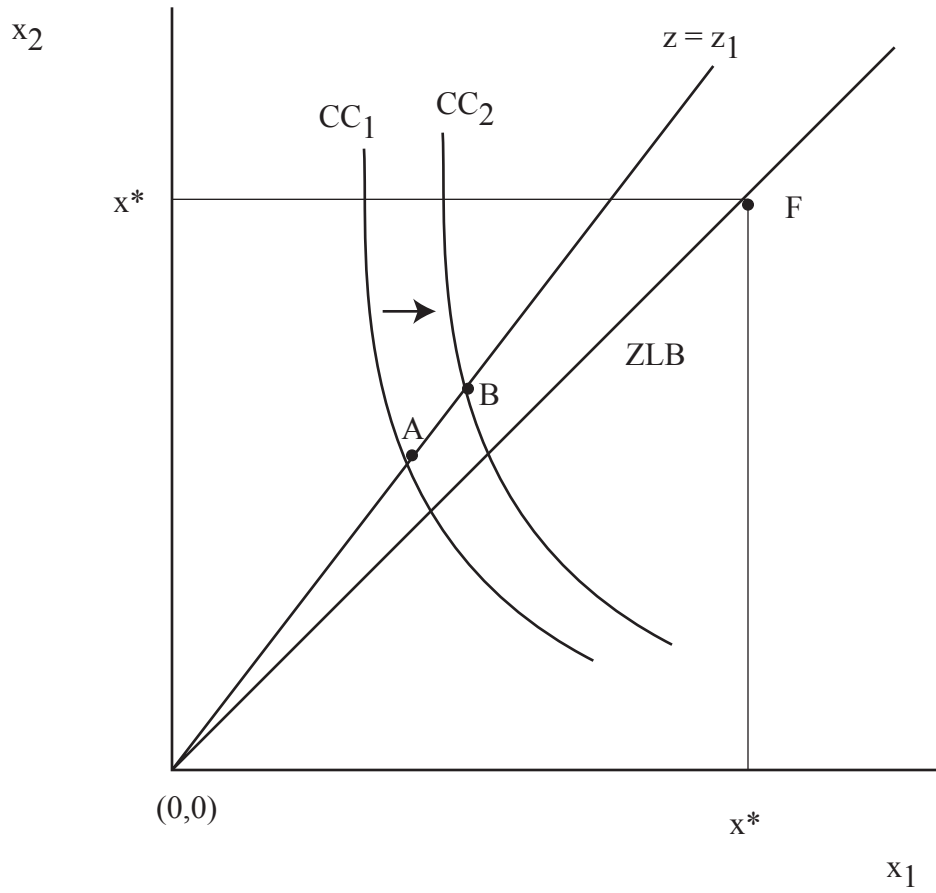
**Figure 7: Conventional Monetary Policy When Incentive Constraints Do Not Bind, or Only One Constraint Binds**



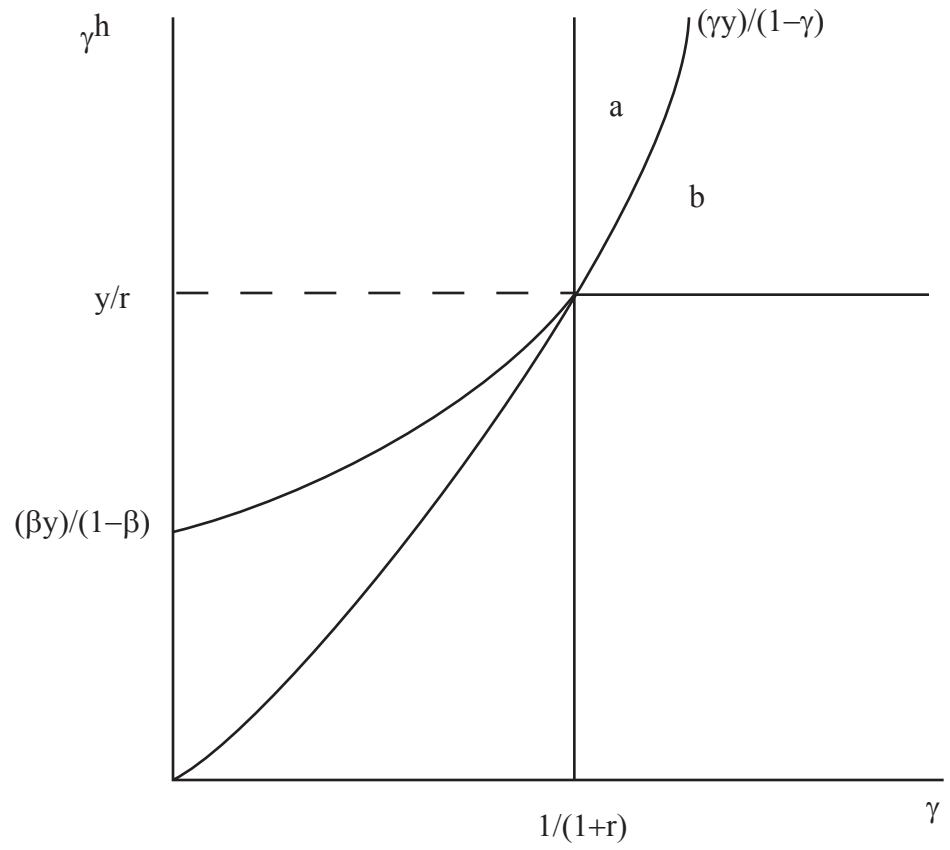
**Figure 8: Conventional Monetary Policy When Both Incentive Constraints Bind**



**Figure 9: Effects of Central Bank Asset Purchases  
When Purchases Are Not Neutral**



**Figure 10: Central Bank Asset Purchases are Effective in Regions a and b**





**Figure 11: Effects of Central Bank Asset Purchases With a Positive Nominal Interest Rate, and At the Zero Lower Bound**

