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Can Trend Inflation Solve the Delayed Overshooting Puzzle?

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Abstract

We develop an open economy New Keynesian model with heterogeneity in price stickiness and positive trend inflation. The main insight of our analysis is that, in the presence of heterogeneity in price stickiness, there is a strong link between trend inflation and the timing of the peak response of the real exchange rate to a monetary policy shock. Without trend inflation, the real exchange rate peaks almost immediately. With trend inflation set at historical values, the peak occurs at around 2 years. Delayed overshooting is a consequence of the interaction between heterogeneity in price stickiness and trend inflation.

JEL codes: E52, F41, F44.

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1. Introduction

Open economy New Keynesian models have provided important insights into the design of optimal monetary and exchange rate policy.[1] It has been noted, however, that New Keynesian models cannot account for observed real exchange rate dynamics, calling their empirical relevance into question. Empirical evidence suggests that the real exchange rate is highly volatility and persistent. The difficulty of developing models that can reconcile the volatility of the real exchange rate with the fact that the effect of shocks on the real exchange rate die away slowly is known as the purchasing power parity puzzle (Rogoff, 1996).

To provide an explanation for the purchasing power parity puzzle, following Dornbusch (1976), the existing literature has focused primarily on the interaction between sticky goods prices and monetary shocks. Chari *et al.* (2002) demonstrate that while New Keynesian models can generate volatile real exchange rates, they fail to generate enough persistence to match that observed in the data. Moreover, as Eichenbaum and Evans (1995) have shown, empirically, the real exchange rate exhibits a hump-shaped path of adjustment in response to a monetary policy shock: a contractionary monetary shock leads to a gradual appreciation of the exchange rate, followed by a gradual depreciation. This is often referred to as the delayed overshooting puzzle.[2]

In this paper, we propose a solution to the delayed overshooting puzzle.[3] Our solution has two elements. First, we assume there is heterogeneity in price stickiness, as suggested by micro-evidence on prices. In our model, there are many sectors, each with a different

[1] See Corsetti *et al.* (2010) and the references contained therein.

[2] This result is confirmed by Grilli and Roubini (2006), Scholl and Uhlig (2008), and very recently, by Kim *et al.* (2017). Faust and Rogers (2003) and Bjornland (2008) find immediate overshooting. Cheung and Lai (2000) and Steinsson (2008) find that the real exchange rate displays hump-shaped dynamics.

[3] In proposing a solution to the delayed overshooting puzzle, our analysis also helps explain the purchasing power parity puzzle.

contract length. Within each sector, there is a Calvo style contract. We calibrate the share of each sector using micro-evidence on prices provided by Bils and Klenow (2004). Second, we allow for positive steady-state (trend) inflation. While the standard models make a simplifying assumption that trend inflation is zero, most central banks target an inflation rate of about 2 percent, and historically countries have had inflation rates considerably above this. For example, in 1979, when Paul Volcker became the chairman of the Federal Reserve Board, the annual average inflation rate in the United States was 11 percent.

We show that, when there is heterogeneity in price stickiness, trend inflation provides a strong internal propagation mechanism for the real exchange rate in response to monetary policy shocks. Heterogeneity is a necessary step in solving the delayed overshooting puzzle because it leads to inertia in price-setting. With heterogeneity in price stickiness, the peak response of the real exchange rate to a monetary policy shock is not immediate. Trend inflation acts to magnify the effects of heterogeneity in price stickiness. This magnification effect changes both the timing and size of the peak response of the real exchange rate. Without trend inflation, the peak response of the real exchange rate occurs 2 quarters after the shock. Allowing for positive trend inflation delays the peak response of the real exchange rate and, for example, when trend inflation is at 9 percent, the response of the real exchange rate to the monetary policy shock reaches its peak at 7 quarters.

The findings reported in Kim *et al.* (2017) provide empirical support for the main conclusion of our analysis. They reexamine delayed overshooting using a VAR with sign restrictions. They find that only during the Volcker period (from 1979 to 1987) did the exchange rate exhibit a hump-shaped pattern of adjustment to monetary policy shocks. The peak response of the real exchange rate is delayed by around 6 quarters for this period. During the rest of the sample period (specifically, 1974–79 and 1987-2007), the real exchange rate reaches its peak response within 2 months. This is important for our results because between 1979 and 1987 average inflation in the United States was over 6 percent. These empirical findings seem to reinforce the insight of our model that there is a strong link between trend inflation and delayed overshooting.

To develop some intuition for why trend inflation is important consider a two sector economy in which one sector has flexible prices and one sector has standard Calvo sticky-prices. When steady-state inflation is zero, a transitory, but persistent, increase in the interest rate generates a hump-shaped path of adjustment in the real exchange for the following reasons. In the sector with flexible prices, the relative price - that is, the price of a firm relative to the average sector-wide price - is more misaligned than in the sector with Calvo sticky-prices. Following the impact of the shock, firms in the flexible-price sector realize that prices are relatively misaligned, and to preserve relative prices across sectors, they reduce prices aggressively. In this case, aggregate inflation depends on lagged relative prices, and this introduces inertia into price-setting and the real exchange rate.

Positive trend inflation magnifies the inertial effect on relative prices because it makes sticky-price firms more forward-looking. With trend inflation, firms in the sticky-price sector set prices more aggressively, when given the opportunity to do so, because they realize that if they are unable to change prices in the future, the pace at which within-sector relative prices are misaligned will grow. As Coibion *et al.* (2012) show, such firms have an expenditure share which is decreasing with trend inflation, and consequently, the link between reset prices and the average price level is weaker. Due to complementarity between flexible and sticky-price firms, firms with flexible prices also respond more aggressively to monetary policy shocks. The backward-looking element of the real exchange rate therefore becomes more important with trend inflation and the hump-shaped dynamic of the real exchange rate is more pronounced.

In the context of the New Keynesian model, the importance of trend inflation has been emphasized by Ascari and Ropele (2009) and Ascari and Sbordone (2014). They show that allowing for positive trend inflation can significantly affect inflation dynamics and the conditions for determinacy. The empirical relevance of the New Keynesian Phillips curve under trend inflation is also discussed by Cogley and Sbordone (2008).

The introduction of heterogeneity in price stickiness has proved helpful in addressing criticisms directed at New Keynesian models. Kara (2015) shows that two problems associated with the Smets and Wouters (2007) model disappear when there is heterogeneity in price stickiness. First, that the model requires large price shocks to explain inflation dynamics (Chari *et al.*, 2008) and, second, that firm-level pricing in the model is inconsistent with that in reality (Bils *et al.*, 2012). As Taylor (2016) has recently commented, "... heterogeneity is not simply a nuisance; it has major implications for aggregate dynamics, and it has been offered as a response to criticism of staggered wage and price setting models. Often that criticism applies to a particular simple staggered contract model ... and that criticism disappears when heterogeneity is taken into account ...". The findings reported in this paper provide further support to such conclusions.

Finally, our paper is also closely related to the research of Carvalho and Nechio (2011), who formulate a multi-sector open economy New Keynesian model to study the purchasing power parity puzzle, and Carvalho *et al.* (2017), who focus on the source of interest rate persistence, building on the insight of Benigno (2004), that price-stickiness only matters for real exchange rate persistence when monetary policy features policy inertia. Kano (2016) argues that incomplete financial markets and shocks to trend inflation drive real exchange rate persistence.

The remainder of the paper is organized as follows. In section 2, we develop an open economy New Keynesian model with heterogeneity in price stickiness and positive steady-state (trend) inflation. In section 3, we provide a quantitative analysis of our model, which is calibrated to be consistent with micro evidence on price-stickiness. We also provide analytical results and develop intuition using a simplified two-sector model. Section 4 concludes.

2. Model Economy

In this section we develop the model economy. There are two identical countries - home and foreign - each populated by a continuum of households and firms with mass normalized to one. In each country, households supply labor to firms and consume a basket of home and foreign goods. Households have access to a complete set of internationally traded state-contingent securities. The only goods market frictions in our economy are monopolistic competition and Calvo local-currency price stickiness. Firms are divided into $i = 1, ..., N$ sectors and all firms serve the domestic and export market.

In what follows, we focus on the introduction sectoral heterogeneity, present pricing equations, and discuss the role of trend inflation and the specification of monetary policy in the home country, with the understanding that analogous expressions hold for the foreign country. Consumption, output, and the nominal price of the home/foreign output are denoted with h/f-subscripts. Asterisks denote foreign country variables. A complete description of the economy is provided in the Appendix.

2.1. CES Demands and Calvo Pricing Equations

The representative household consumes c_t units of home ($c_{h,t}$) and foreign ($c_{f,t}$) differentiated goods. We define the cumulative budget share of sectors $k = 1, ..., i$ in the economy as $\widehat{\alpha}_i = \sum_{k=1}^{i} \alpha_k$, with $\widehat{\alpha}_0 = 0$ and $\widehat{\alpha}_N = 1$. The unit interval for sector i is $[\widehat{\alpha}_{i-1}, \widehat{\alpha}_i]$ and the constant elasticity of substitution consumption aggregator over goods is,

$$c_{s,t} = \left\{ \sum_{i=1}^{N} \int_{\widehat{\alpha}_{i-1}}^{\widehat{\alpha}_i} \left[c_{i,s,t}(z)\right]^{(\varepsilon-1)/\varepsilon} dz \right\}^{\varepsilon/(\varepsilon-1)} \tag{1}$$

where z indexes the firm, $\varepsilon > 1$ is the elasticity of substitution, $\sum_{i=1}^{N} \alpha_i = 1$, and $s = h, f$. Given the form of preferences in equation (1), we express the sector $i = 1, ..., N$ demand curve as, $c_{i,s,t}(z) = [p_{i,s,t}(z) / p_{s,t}]^{-\varepsilon} c_{i,s,t}$, where the sector-level price index is given by, $p_{s,t}^{1-\varepsilon} = \sum_{i=1}^{N} \int_{\widehat{\alpha}_{i-1}}^{\widehat{\alpha}_i} [p_{i,s,t}(z)]^{1-\varepsilon} dz$.

Firm z in sector i has a Calvo hazard rate γ_i. Labor is the only factor of production. Firms have a linear technology and sell their product in the domestic and export market. The profit from domestic sales of firm z in sector i is, $\vartheta_{i,h,t}(z) = [p_{i,h,t}(z) - W_t] c_{i,h,t}(z)$, and in the export market, $\vartheta_{i,h,t}^{\star}(z) = \left[e_t p_{i,h,t}^{\star}(z) - W_t\right] c_{i,h,t}^{\star}(z)$, where the nominal exchange rate, denoted e_t, is defined as the price of one unit of foreign currency in units of home currency, and W_t is the nominal wage rate.

Firm z chooses the optimal reset price in the domestic and export market, $X_{i,h,t}(z)$ and $X_{i,h,t}^{\star}(z)$, respectively, to maximize expected profits,

$$\mathbb{E}_t \sum_{j=0}^{\infty} \gamma_i^j \left[\beta^j \left(\frac{c_{t+j}}{c_0}\right)^{-\sigma}\right] \left[\frac{\vartheta_{i,t+j}(z)}{p_{t+j}}\right] \quad \text{and} \quad \mathbb{E}_t \sum_{j=0}^{\infty} \gamma_i^j \left[\beta^j \left(\frac{c_{t+j}}{c_0}\right)^{-\sigma}\right] \left[\frac{\vartheta_{i,t+j}^{\star}(z)}{p_{t+j}^{\star}}\right] \tag{2}$$

subject to the demand for the good. In equations (2), the parameter σ is the inverse elasticity of intertemporal substitution of consumption, $\beta \in (0,1)$ is the subjective discount factor of the representative household, and p_{t+j} $(p_{t+j}^{\star})$ is the home (foreign) consumer price.

The home currency domestic and export market real reset prices (units of consumption) for firm z in sector i are given by the following expressions.

$$x_{i,h,t}(z) = \frac{\varepsilon}{\varepsilon - 1} \frac{\psi_{i,h,t}}{\phi_{i,h,t}} \quad \text{where,} \quad \begin{aligned} \psi_{i,h,t} &= w_t c_t^{1-\sigma} + \gamma_i \beta \mathbb{E}_t \left(\frac{\pi_{h,t+1}^{\varepsilon-\upsilon}}{\pi_{t+1}^{-\upsilon}} \psi_{i,h,t+1} \right) \\ \phi_{i,h,t} &= c_t^{1-\sigma} + \gamma_i \beta \mathbb{E}_t \left(\frac{\pi_{h,t+1}^{\varepsilon-\upsilon}}{\pi_{t+1}^{1-\upsilon}} \phi_{i,h,t+1} \right) \end{aligned} \tag{3}$$

and,

$$x^{\star}_{i,h,t}\left(z\right) = \frac{\varepsilon}{\varepsilon - 1}\frac{\psi^{\star}_{i,h,t}}{\phi^{\star}_{i,h,t}} \quad \text{where,} \quad \begin{aligned} \psi^{\star}_{i,h,t} &= w_t c_t^{-\sigma} c_t^{\star} + \gamma_i \beta \mathbb{E}_t \left[\frac{\left(\pi^{\star}_{h,t+1}\right)^{\varepsilon - \upsilon}}{\left(\pi^{\star}_{t+1}\right)^{-\upsilon}} \psi^{\star}_{i,h,t+1} \right] \\ \phi^{\star}_{i,h,t} &= \left(c_t^{\star}\right)^{1-\sigma} + \gamma_i \beta \mathbb{E}_t \left[\frac{\left(\pi^{\star}_{h,t+1}\right)^{\varepsilon - \upsilon}}{\left(\pi^{\star}_{t+1}\right)^{1-\upsilon}} \phi^{\star}_{i,h,t+1} \right] \end{aligned} \tag{4}$$

for $i = 1, ..., N$ and where $\pi_{t+1} \equiv p_{t+1}/p_t$ is the inflation rate for home consumer prices and analogous definitions are applied to home producer prices, $\pi_{h,t}$, and foreign consumer and producer prices, $\pi^{\star}_t$ and $\pi^{\star}_{h,t}$, respectively, and $w_t \equiv W_t/p_t$ is the real wage rate.

The optimal reset price in each market is expressed as the ratio of marginal cost (ψ) to marginal revenues (ϕ). Inflation terms appear because forward-looking firms know that the optimal price set in period t may remain fixed for a number of periods. Inflation erodes the markup over time; hence, firms use future expected inflation rates to discount future marginal costs. The higher the future expected rate of inflation, the higher the relative weight on expected future marginal costs (Ascari and Sbordone, 2014). In the open economy, inflation rates for both the price of home goods (in local currency terms) and the overall consumption basket affect the reset price, and the influence of these terms is determined by both the elasticity of substitution between home and foreign bundles of goods ($\upsilon > 0$) and the elasticity of substitution between differentiated goods ($\varepsilon > 1$).

Associated with optimal pricing equations are two relative price equations: one for relative producer price and consumer price indexes - specific to the open economy - and another for price dispersion - which explain the firm z price versus the producer price index at the sector level.

For example, for domestic sales in the home market - i.e., those conditions relevant for equations in (3), the evolution of the average sector price is,

$$\rho^{1-\varepsilon}_{i,h,t} = \gamma_i \left(\frac{\rho_{i,h,t-1}}{\pi_t}\right)^{1-\varepsilon} + \left(1 - \gamma_i\right) x^{1-\varepsilon}_{i,h,t} \tag{5}$$

where $\rho_{i,h,t} \equiv p_{i,h,t}/p_t$ is the price expressed in units of consumption. This condition links the average price to the reset price.

The dynamic equation for price dispersion is,

$$\Delta_{i,h,t} = (1-\gamma_i)\left[x_{i,h,t}\left(\frac{p_t}{p_{h,t}}\right)\right]^{-\varepsilon} + \gamma_i \pi_{h,t}^{\varepsilon}\Delta_{i,h,t-1} \tag{6}$$

where $\Delta_{i,h,t} \equiv \int_0^1 \left(p_{i,h,t}\left(z\right)/p_{h,t}\right)^{-\varepsilon} dz$. Whilst equations (5) and (6) are relatively standard, what matters for our analysis is that the γ_i parameters (the Calvo hazard rates) are used to match facts consistent with micro-data. Moreover, in the open economy, dynamic equations for the average price and price dispersion may differ across markets due to the assumption of local currency pricing.

2.2. International Risk Sharing and Monetary Policy

Home-currency, state-contingent securities are traded internationally and there is a non-traded domestic-currency riskless asset in each country. Since there are no impediments to trade in financial markets, the international risk sharing condition is,

$$q_t = \left(c_t^{\star}/c_t\right)^{-\sigma} \tag{7}$$

where $q_t = e_t\left(p_t^{\star}/p_t\right)$ is the real exchange rate.

We suppose monetary policy is conducted in the home economy using the following interest rate setting rule,

$$R_t/R = \left(\pi_t/\pi\right)^{\phi_\pi}\left(y_t/y\right)^{\phi_y}\exp\left(\upsilon_t\right) \tag{8}$$

where $R_t > 1$ is the short-term gross nominal interest rate, y_t is real GDP, and υ_t is the exogenous component of monetary policy. The foreign economy uses the same interest rate rule, targeting foreign consumer price inflation and real GDP, but has no exogenous component.

Interest rate persistence arises from serial correlation in the exogenous component of home monetary policy, which is an AR (1) process. This is an important point, in general, but has specific implications in the context of our analysis.[4] In a standard model, without trend inflation and sectoral heterogeneity, when υ_t is serial uncorrelated, Engel (2016) shows that endogenous real exchange rate persistence is bounded above by the interest rate smoothing parameter and by the probability of a firm not changing prices under Calvo pricing.[5] Carvalho *et al.* (2017) show that, when υ_t is serially correlated, increasing policy inertia may decrease real exchange rate persistence, in a multi-sector setting. We chose to specify the monetary policy rule in equation (8) without interest rate smoothing to highlight the role of trend inflation in generating delayed overshooting.

2.3. Calibration

Our calibration of the model proceeds in two steps. First, we assign standard values to the parameters of our model that are not directly related to heterogeneity in price-stickiness. We then assign values to hazard rates, sector shares, and the elasticity of substitution between differentiated goods (that is, γ_i and α_i and ε) using micro evidence on prices provided by Bils and Klenow (2004), hereafter BK.

In our calibration, we set steady-state inflation at 3.1 percent (which implies $\pi = 1.0077$) consistent with average post-1979 US inflation. Table 1 presents the values assigned to parameters not directly related to heterogeneity.

===== Table 1 =====

[4]The source of interest rate persistence observed in the data is also subject to ongoing debate. Rudebusch (2002) provides evidence that it arises mainly from persistent monetary shocks, whereas Coibion and Gorodnichenko (2012) point to policy inertia as the main source of interest rate persistence.

[5]This generalizes the result in Benigno (2004), who shows that, without interest rate smoothing, price stickiness may not matter for real exchange rate persistence.

The share of each sector in our economy is calibrated based on BK. BK report the frequency of price changes for around 300 product categories, which covers 70 percent of the US consumer price index. Following Kara (2015), we aggregate up from their 300 sectors so that we have 9 sectors with distinct reset probabilities. The aggregation is performed by forming probability focal points in increments of 0.1 percentage points $(0.2, ..., 1)$. The BK reset probabilities are rounded to 0.1 percentage point. Next, we allocate the BK reset probabilities to these 10 focal points. The sectors are scaled by the share in expenditure that is allocated to each focal point. The economy-wide mean frequency of price adjustment is around 0.4 with the share of flexible contacts at 34 percent.

Given the distribution of prices we then target a long-run average markup. Importantly, at the sectoral level, the markup is increasing in both trend inflation and the elasticity of substitution, such that the more sticky the sector, the stronger the effect of trend inflation on the markup. The long-run sector i markup is, $\mu_i = [\varepsilon / (\varepsilon - 1)] [(1 - \beta\gamma_i\pi^{\varepsilon-1}) / (1 - \beta\gamma_i\pi^{\varepsilon})]$, where $\varepsilon / (\varepsilon - 1) > 1$ is the standard markup arising from monopolistic competition. For the 34 percent of firms with flexible prices in our model we set $\varepsilon = 10$ such that the flexible-price sector markup is 11 percent. Given sectoral shares, and conditional on the sector-specific value of γ, we adjust ε such that the average markup across sectors is also 11 percent.

3. Real Exchange Rate Dynamics

In this section, we study the response of the real exchange rate to monetary policy shocks using a linearized version of our model. We start with a general version and focus the discussion on the role of trend inflation for delayed overshooting - the timing and size of the hump-shaped response of the real exchange rate to a monetary policy shock. We then present a two-sector version of the model in which we show how heterogeneity in price stickiness and trend inflation interact to generate our main result.

3.1. The Real Exchange Rate in a Calibrated Model

Figure 1 plots the impulse response functions (IRFs) of the real exchange rate with sectoral heterogeneity to a one-off shock that reduces the nominal interest rate by 1 percent, upon impact. Figure 2 plots the corresponding IRFs from a single-sector model with the same mean contract length. In both cases, trend inflation ranges between zero and 9 percent.

===== **Figure 1** =====

Figure 1 illustrates the role and importance of trend inflation in our analysis. Higher trend inflation raises both the peak response of the real exchange rate and position of the peak response. With zero trend inflation, delayed overshooting occurs at around 2 quarters. When trend inflation is at 9 percent, delayed overshooting occurs at around 7 quarters. This path of adjustment is consistent with the evidence presented in Kim *et al.* (2017).

===== **Figure 2** =====

Figure 2 demonstrates that the standard Calvo model cannot generate delayed overshooting. Moreover, without heterogeneity, since firms become more forward looking under trend inflation, the impact effect of the shock on the real exchange rate is diminished.

Overall, Figures 1 and 2 clarify the role played by trend inflation when there is heterogeneity in price stickiness. For example, focusing on the solid (black) line, which corresponds to the benchmark case of zero trend inflation, the half-life of the real exchange rate drops from over 10 quarters (with heterogeneity) to a little over 4 quarters. Although positive trend inflation, by itself, cannot lead to delayed overshooting, trend inflation has a significant impact on the path of adjustment of the real exchange rate.

These findings raise a natural question: how do heterogeneity in price stickiness and trend inflation work in tandem to generate delayed overshooting. The next subsection aims to provide an answer to this question. To examine how trend inflation affects the response of the real exchange rate to a monetary policy shock in the most transparent way possible we consider a simplified two-sector version of the economy presented above.

3.2. The Role of Flexible Prices in a Two-Sector Economy

In the two-sector economy, firms in sector 1 have flexible prices, and comprise a fraction α_1 of all firms, and firms in sector 2 (of which there are $\alpha_2 = 1 - \alpha_1$) have Calvo sticky-prices. For notational simplicity, we set $\alpha = \alpha_1$, $\gamma_1 = 0$ and $\gamma_2 = \gamma$. The aggregate amount of stickiness in each economy is then, $\overline{\gamma} \equiv \alpha + (1-\alpha)/(1-\gamma)$. We further assume labor is linear in utility, which eliminates the need to track price dispersion, and we assume no home-bias in consumption.

For domestic sales of home firms, optimal price setting in sectors 1 and 2 implies,

$$\widehat{\pi}_{1,h,t} = \widehat{c}_t - \widehat{c}_{t-1} + \widehat{\pi}_t \tag{9}$$

and,

$$\begin{aligned}\widehat{\pi}_{2,h,t} \quad = \quad & \beta \mathbb{E}_t \widehat{\pi}_{2,h,t+1} + \frac{\zeta(\pi)}{1-\alpha}\left[\widehat{c}_t - \left(\frac{\widehat{p}_{f,t} - \widehat{p}_{h,t}}{2}\right)\right] \\ & + \beta\tau(\pi)\left[\frac{\upsilon}{2}\left(\mathbb{E}_t\widehat{\pi}_{f,t+1} - \mathbb{E}_t\widehat{\pi}_{h,t+1}\right) + \varepsilon\mathbb{E}_t\widehat{\pi}_{h,t+1} + \mathbb{E}_t\widehat{\psi}_{2,h,t+1}\right]\end{aligned} \tag{10}$$

where $\zeta(\pi) \equiv (1-\gamma\pi^{\varepsilon-1})(1-\gamma\beta\pi^{\varepsilon})/\gamma\pi^{\varepsilon-1}$ and $\tau(\pi) \equiv (\pi-1)(1-\gamma\pi^{\varepsilon-1})$ are parameters and hatted variables denote log-deviations from steady state values. The inflation rate in sector 1 is $\widehat{\pi}_{1,h,t}$ and the inflation rate in sector 2 is $\widehat{\pi}_{2,h,t}$. The remaining inflation variables in equations (9) and (10) are economy-wide.

We start by considering equation (10). It is easiest to think about this expression without trend inflation first. In this case, $\widehat{\pi}_{h,t} = \beta\widehat{\pi}_{h,t+1} + \zeta(1)\left[\widehat{c}_t - (\widehat{p}_{f,t} - \widehat{p}_{h,t})/2\right]$, where $\zeta(1) \equiv (1-\gamma)(1-\gamma\beta)/\gamma$ and $\tau(1) = 0$. This is the standard representation of the New Keynesian

Phillips curve in the open economy. The term, $\widehat{c}_t - \left(\frac{\widehat{p}_{f,t} - \widehat{p}_{h,t}}{2}\right)$, represents marginal cost, or, $\widehat{w}_t - (\widehat{p}_{h,t} - \widehat{p}_t)$, where $\widehat{w}_t = \widehat{c}_t$ is the wage in units of consumption. Thus, in the open economy, the usual marginal cost term that appears in the New Keynesian Phillips curve depends on the relative price of imported and home goods in local currency.

Trend inflation has three implications for the (domestic sales) New Keynesian Phillips curve.[6] First, trend inflation alters the slope of the Phillips curve, via the term $[\zeta(\pi)]/(1-\alpha)$. With higher trend inflation, the slope of the Phillips curve is flatter; $\zeta'(\pi) < 0$. Because only a fraction of firms comprise the sticky-price sector, the term $\zeta(\pi)$ is multiplied by $\frac{1}{1-\alpha}$, such that as $\alpha \to 0$, there are no flexible price firms. Trend inflation also changes the weight on expected future inflation. In a single-sector closed economy, the coefficient on expected future inflation simplifies to $\beta[1 + \tau(\pi)\varepsilon]$. In the open economy, differentials in aggregate inflation rates for imported and home goods determine sector-level inflation, and this introduces a role for the trade elasticity, $\upsilon > 0$. Finally, there is a term specific to the presence of trend inflation, $\widehat{\psi}_{2,h,t+1}$, which accounts for dynamic changes in marginal costs.

Sector 1 does not have a Phillips curve-like representation because prices are flexible. However, it has the key mechanism, because inflation is backward looking with regard to consumption. To understand why, note that the relative price in sector 1, in period t, is $\widehat{\rho}_{1,h,t} = \widehat{\rho}_{1,h,t-1} + \widehat{\pi}_{1,h,t} - \widehat{\pi}_t$, where $\widehat{\pi}_{1,h,t+1} \equiv \widehat{p}_{1,h,t+1} - \widehat{p}_{1,h,t}$. The implication is that inflation in the flexible-price sector depends negatively on the relative price in that sector in the previous period. As such, if the relative price in the previous period (i.e., $\widehat{\rho}_{1,h,t-1}$) were high, flexible-price firms would reduce their prices, and thus reduce the within-sector relative price. This feature introduces inertia into pricing decisions. The optimal pricing decision of firms is, $\widehat{\rho}_{1,h,t} = \widehat{c}_t$, which leads immediately to equation (9).

[6] In sector 2, since the (domestic sales) Phillips curve is similar to that of a closed economy we draw on the discussion contained in Ascari and Sbordone (2014), pp. 693-4.

Summing across sectors using sector shares, domestic inflation is, $\widehat{\pi}_{h,t} = \alpha\widehat{\pi}_{1,h,t} + (1-\alpha)\,\widehat{\pi}_{2,h,t}$, which we re-express as,

$$\widehat{\pi}_{h,t} = \alpha\left(\widehat{c}_t - \widehat{c}_{t-1}\right) + \alpha\widehat{\pi}_t + (1-\alpha)\,\widehat{\pi}_{2,h,t} \tag{11}$$

This explains why the backward looking part of price setting (i.e., that in sector 1) affects inflation in domestic prices. With positive trend inflation, sticky-price (sector 2) firms that are able to reset their price do so more aggressively. Such firms realize that if they are unable to change prices in the future the pace at which within-sector relative prices are misaligned will grow. In turn, the optimal pricing decision of firms in sector 1 becomes more aggressive because there is a complementarity across all firms. Therefore, at the aggregate level, the backward-looking element of consumption becomes relatively more important.

3.3. Real Exchange Rate Dynamics in a Two-Sector Economy

We now consider the dynamics of the real exchange rate in our two-sector economy. Doing so requires accounting for optimal price setting in the export market - the export market versions of equations (9) and (10) - and the optimal price setting decisions of foreign firms. For simplicity, but without significant loss of generality, we assume strict inflation targeting, such that $\widehat{i}_t = \phi\widehat{\pi}_t + \widehat{\nu}_t$.

The real exchange rate in the two-sector economy is given by the following set of conditions.[7]

$$q_t = \frac{\alpha}{\lambda(\pi)}q_{t-1} + \beta\frac{1-\alpha}{\lambda(\pi)}\left\{\frac{\alpha}{1-\alpha}q_{t+1} + \frac{1}{\beta}\widehat{\pi}^R_t - \left[1+\varepsilon\tau(\pi)\right]\widehat{\pi}^R_{t+1} - \tau(\pi)\,\psi^R_{t+1}\right\} \tag{12}$$

where $\widehat{\pi}^R_t \equiv \widehat{\pi}_t - \widehat{\pi}^\star_t$ and $\lambda(\pi) \equiv \alpha(1+\beta) + \zeta(\pi)$ and,

$$\widehat{\psi}^R_t = \left(1-\gamma\beta\pi^\varepsilon\right)q_t + \gamma\beta\pi^\varepsilon\left(\varepsilon\mathbb{E}_t\widehat{\pi}^R_{t+1} + \mathbb{E}_t\widehat{\psi}^R_{t+1}\right) \tag{13}$$

and,

$$\mathbb{E}_t\Delta\widehat{q}_{t+1} = \phi\widehat{\pi}^R_t - \mathbb{E}_t\widehat{\pi}^R_{t+1} + \widehat{\nu}_t \tag{14}$$

[7] The derviation is contained in the Appendix.

with parameters $\zeta(\pi)$ and $\tau(\pi)$ defined above.

Equation (12) expresses the real exchange rate in terms of relative consumer prices inflation (that is; home versus foreign inflation; $\widehat{\pi}_t^R \equiv \widehat{\pi}_t - \widehat{\pi}_t^{\star}$), where, for example, $\widehat{\pi}_t = (1/2)(\widehat{\pi}_{h,t} + \widehat{\pi}_{f,t})$. Equation (13) is a dynamic equation in relative marginal costs, $\widehat{\psi}_t^R \equiv \widehat{\psi}_t - \widehat{\psi}_t^{\star}$. This additional variable is only relevant when there is trend inflation and is purely forward-looking. Finally, equation (14) links the real exchange rate and relative inflation to the home monetary policy shock, $\widehat{\nu}_t$. It is derived from international risk sharing and the relative consumption Euler equations.

The important implication of equation (12) is that the real exchange rate is backward looking, in the sense that it depends on its own lag, when $\alpha > 0$. When $\alpha \to 0$, the lagged term disappears and this condition simplifies considerably. This is the case in which there are only *sticky-price* firms. The point of our analysis is that the coefficient on the lag of the real exchange rate depends not only α, but also on $\lambda(\pi)$, where $\lambda'(\pi) < 0$. From inspection of equation (12), this means that, for a given α, as trend inflation rises, the weight on the lagged real exchange rate increases. This suggests the interaction of α and π - that is, the interaction of heterogeneity in price stickiness and trend inflation - generates delayed overshooting and strong endogenous persistence in the real exchange rate.

To understand how α and π lead to delayed overshooting, and to gain further insights about the role of trend inflation in our model, note that the general solution for the real exchange rate takes the form,

$$q_t = \chi_q q_{t-1} + \chi_\nu \widehat{\nu}_t \tag{15}$$

where χ_q and χ_ν are functions of underlying parameters of the model and are to be determined.

For the case of zero trend inflation we obtain the following expressions for χ_q and χ_ν,

$$\chi_q = \frac{\varpi - \left(\varpi^2 - 4\alpha/\beta\right)^{1/2}}{2} \quad \text{and} \quad \chi_\nu = -\frac{1-\alpha}{\varpi - (\chi_q + \rho)} \tag{16}$$

where $\varpi = 1 + (\zeta + \alpha)/\beta > 1$ and when $\phi = 1/\beta$. The parameter χ_q plays two distinct roles. First, it determines the weight attached to the lagged real exchange rate. Second, it influences the strength of the impact effect on the real exchange rate of a shock to monetary policy. The value of χ_q itself depends on the share of flexible-price firms in the economy. When there is a single sticky-price sector, and $\alpha \to 0$, we find that $\chi_q \to 0$. In this case, $\widehat{q}_t = -\left[1/(\varpi - \rho)\right]\widehat{\nu}_t$, and there is no endogenous persistence in the real exchange rate (Benigno, 2004). When the economy is mostly comprised of flexible price firms, the parameter χ_ν becomes smaller, and the real effects of monetary policy are muted. Thus, as the share of the flexible-price sector increases, the impact effect of the shock diminishes, but the lag term in equation (15) becomes more important. Once this is combined with serial correlation in $\widehat{\nu}_t$, the multi-sector structure of our model can give rise to a hump-shaped path of adjustment in the real exchange rate.

Having established the role of heterogeneity in price stickiness we now turn to study the role of trend inflation. When we extend the model to allow for trend inflation it is no longer possible to characterize χ_q in a simple manner. Therefore, we determine the value of χ_q numerically. In Figure 3, we plot χ_q for different values of trend inflation and for the range $\alpha \in (0, 0.34)$, the upper bound of which corresponds to the 34 percent of flexible-price firms reported in the BK data. The remaining parameter values we use are the same as in the benchmark calibration with the mean contract length constant across the cases considered.

===== Figure 3 =====

Figure 3 shows that whilst χ_q increases with the share of flexible-price firms in the economy it is also increasing with trend inflation. Moreover, the impact of trend inflation on χ_q

is more pronounced the larger the share of firms with flexible prices. This explains why the hump-shaped response of the real exchange rate in our calibrated model (Figure 2) is considerably more pronounced once trend inflation reaches 9 percent.

How does increased backward-lookingness - both from trend inflation and an increase in the proportion of firms with flexible prices - help generate a more pronounced hump-shaped response in the real exchange? Consider equation (15) again. To generate a peak response *one period* after the shock we require, $q_t < q_{t+1}$, which is satisfied only if,

$$\chi_q > 1 - \rho \tag{17}$$

Without heterogeneity, $\chi_q = 0$, and since $0 < \rho < 1$, equation (17) is not satisfied. With heterogeneity, since $0 < \chi_q < 1$, the model can potentially generate a hump-shaped real exchange rate. However, a relatively high value of χ_q can be replaced with a relative high value for ρ. That is, so long as some firms set prices flexibly, serially correlated errors in the Taylor rule can also lead to a hump-shaped path of adjustment for the real exchange rate. This possibility is discussed in Engel (2016) and Carvalho *et al.* (2017). They both argue that the source of the persistence of nominal interest rates is key to understanding persistence in the real exchange rate when there are monetary policy shocks. An implication of our findings, is that long-run (trend) inflation also plays a role in generating real exchange rate persistence because it leads to delayed overshooting.

As we discuss above, positive trend inflation magnifies inertia in price setting because it makes sticky-price firms more forward-looking. Firms in the sticky price sector that reset their price make larger adjustments because, if they are unable to change prices for sometime, inflation will erode their markup and expenditure share. A reduced expenditure share acts to weaken the link between reset prices and inflation. Because there is complementarity between resetting firms in the sticky-price sector and firms in the flexible-price sector this mechanism effectively makes flexible-price firms more backward-looking. It is this channel, magnified through trend inflation, that leads to our main result.

4. Conclusions

This paper studies the delayed overshooting puzzle using an open economy New Keynesian model with positive trend inflation and heterogeneity in price stickiness. We show that trend inflation plays an important role in determining the timing of the hump-shaped response of the real exchange rate to monetary policy shocks. Without trend inflation, the peak effect of the change in the real exchange rate is around 2 quarters after the shock. When long-run inflation is closer to historical values, the peak effect can be nearer 2 years. Our results are supported by the evidence of delayed overshooting in periods of relatively high inflation, as recently reported in Kim *et al.* (2017). The importance of heterogeneity in price stickiness and trend inflation have been noted separately. We show that the interaction between these two factors has implications for real exchange rate dynamics and can also provide answers to important puzzles in macroeconomics.

Appendix

In this Appendix we present details and derivations omitted in the main text.

A.1. Household Optimization and Resource Constraints

The home household's intertemporal utility function is,

$$\mathbb{U}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \delta \frac{L_t^{1+\eta}}{1+\eta} \right) \tag{18}$$

where $\beta \in (0,1)$ is a subjective discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, $1/\eta$ is the Frisch elasticity of labor supply (L_t, where $\delta > 0$), and,

$$c_t = \left[\omega^{1/\upsilon} c_{h,t}^{(\upsilon-1)/\upsilon} + (1-\omega)^{1/\upsilon} c_{f,t}^{(\upsilon-1)/\upsilon} \right]^{\upsilon/(\upsilon-1)} \tag{19}$$

where $\omega \in (0,1)$ is a measure of openness to trade in goods (home-bias parameter) and $\upsilon > 0$ is the elasticity of substitution between home and foreign goods. Utility is maximized choosing consumption bundles of the home and foreign good, $c_{h,t}$ and $c_{f,t}$, internationally traded home currency state-contingent securities, B_{t+1}, a non-traded riskless asset, A_t, and labor supply, subject to the flow budget constraint,

$$p_{h,t} c_{h,t} + p_{f,t} c_{f,t} + \mathbb{E}_t \left[Q_{t,t+1} B_{t+1} \right] + A_t R_t = W_t L_t + B_t + A_{t-1} + \int \vartheta_t(z)\, dz + T_t \tag{20}$$

where $Q_{t,t+1}$ is the period t price of securities normalized by the probability of the occurrence of the state, W_t is the nominal wage rate, $\int \vartheta_t(z)\, dz$ is aggregate profit, and T_t is a lump-sum transfer. The first-order conditions can be expressed as,

$$Q_{t,t+1} = \frac{\beta c_{t+1}^{-\sigma} / c_t^{-\sigma}}{p_{t+1}/p_t} \quad \text{and} \quad \mathbb{E}_t \left[Q_{t,t+1} \right] = \frac{1}{R_t} \tag{21}$$

and,

$$\frac{c_{f,t}}{c_{h,t}} = \frac{1-\omega}{\omega} \left(\frac{p_{h,t}}{p_{f,t}} \right)^{\upsilon} \quad \text{and} \quad w_t = \delta L_t^{\eta} c_t^{\sigma} \tag{22}$$

where $p_t = \left[\omega p_{h,t}^{1-\upsilon} + (1-\omega) p_{f,t}^{1-\upsilon}\right]^{1/(1-\upsilon)}$ is the consumer price index, $R_t > 1$ is the short-term gross nominal interest rate (with $Q_{t,t} \equiv 1$) and $w_t \equiv W_t/p_t$ is the wage rate in units of consumption.

Home-currency state-contingent securities are traded internationally. Since there are no impediments to trade in financial markets, $Q_{t,t+1} = Q^{\star}_{t,t+1}$, where $Q^{\star}_{t,t+1} \equiv \beta \left(c^{\star}_{t+1}/c^{\star}_t\right)^{-\sigma} \left(e_t p^{\star}_t / e_{t+1} p^{\star}_{t+1}\right)$. Defining the real exchange rate as, $q_t = e_t \left(p^{\star}_t/p_t\right)$, we can write the international risk sharing condition as, $q_t = \left(c^{\star}_t/c_t\right)^{-\sigma}$, where $q_0 \left(c_0/c^{\star}_0\right)^{-\sigma} = 1$. Total resources in the domestic economy are,

$$\begin{aligned} L_t &= \sum_{i=1}^{N} \int_{\widehat{\alpha}_{i-1}}^{\widehat{\alpha}_i} c_{i,h,t}(z) + \sum_{i=1}^{N} \int_{\widehat{\alpha}_{i-1}}^{\widehat{\alpha}_i} c^{\star}_{i,h,t}(z) \\ &= \left[\omega \Delta_{h,t} + (1-\omega) \Delta^{\star}_{h,t} \left(\frac{e_t p^{\star}_{h,t}}{p_{h,t}}\right)^{-\upsilon} q_t^{\upsilon - 1/\sigma}\right] \left(\frac{p_{h,t}}{p_t}\right)^{-\upsilon} c_t \end{aligned} \tag{23}$$

where we defined $\Delta_{h,t} \equiv \sum_{i=1}^{N} \alpha_i \Delta_{i,h,t}$ and $\Delta^{\star}_{h,t} \equiv \sum_{i=1}^{N} \alpha_i \Delta^{\star}_{i,h,t}$. In the foreign economy,

$$L^{\star}_t = \left[\omega \Delta^{\star}_{f,t} + (1-\omega) \Delta_{f,t} \left(\frac{p_{f,t}}{e_t p^{\star}_{f,t}}\right)^{-\upsilon} q_t^{-\upsilon + 1/\sigma}\right] \left(\frac{p^{\star}_{f,t}}{p^{\star}_t}\right)^{-\upsilon} c^{\star}_t$$

where $\Delta^{\star}_{f,t} \equiv \sum_{i=1}^{N} \alpha^{\star}_i \Delta^{\star}_{i,f,t}$ and $\Delta_{f,t} \equiv \sum_{i=1}^{N} \alpha^{\star}_i \Delta_{i,f,t}$. The foreign economy is also characterized by demand for goods and labor supply conditions equivalent to those in (22).

A.2. Firm Optimization

Consider the domestic sales of the home firm z in sector i with nominal period profits, $\vartheta_{i,h,t}(z) = p_{i,h,t}(z)\, y_{i,h,t}(z) - W_t l_{i,h,t}(z)$. Define the stochastic discount factor as, $\Phi_{t,t+j} = \beta^j u'(c_{t+j})/u'(c_0)$. The problem for the firm is to choose the reset price, $X_{i,h,t}(z)$, to solve the following unconstrained problem,

$$\max \mathbb{E}_t \sum_{j=0}^{\infty} \Phi_{t,t+j} \gamma^j \left\{ \frac{X_{i,h,t}(z)}{p_{t+j}} \left[X_{i,h,t}(z)/p_{h,t+j}\right]^{-\varepsilon} - \frac{W_{t+j}}{p_{t+j}} \left[X_{i,h,t}(z)/p_{h,t+j}\right]^{-\varepsilon} \right\} c_{h,t+j} \tag{24}$$

where $\varepsilon > 1$. This solution to this problem is,

$$\mathbb{E}_t \sum_{j=0}^{\infty} \Phi_{t,t+j} \gamma_i^j \left(p_{i,h,t+j}\right)^{\varepsilon} \left[X_{i,h,t}\left(z\right)/p_{t+j} - \frac{1}{\theta} w_{t+j} \right] c_{h,t+j} = 0 \tag{25}$$

We introduce trend inflation the demand curve, $c_{h,t+j} = \omega \left[\left(\pi_{h,t,t+j}/\pi_{t,t+j}\right)\left(p_{h,t}/p_t\right)\right]^{-\upsilon} c_{t+j}$ where $p_t = p_{t+j}/\pi_{t,t+j}$. We eliminate wages using the labor-leisure condition, $w_{t+j} = \delta c_{t+j}^{\sigma} L_{t+j}^{\eta}$. Since all firms that have the opportunity to reset their price are identical, we drop the z index and define $x_{i,h,t} \equiv X_{i,h,t}/p_t$ as the real reset price (units of consumption). This leads to,

$$x_{i,h,t} = \frac{1}{\theta} \frac{\psi_{i,h,t}}{\phi_{i,h,t}} \tag{26}$$

where,

$$\begin{aligned} \psi_{i,h,t} &\equiv \mathbb{E}_t \sum_{j=0}^{\infty} \left(\gamma_i \beta\right)^j \left(\pi_{h,t,t+j}^{\varepsilon-\upsilon}/\pi_{t,t+j}^{-\upsilon}\right) \delta L_{t+j}^{\eta} c_{t+j} \\ \phi_{i,h,t} &\equiv \mathbb{E}_t \sum_{j=0}^{\infty} \left(\gamma_i \beta\right)^j \left(\pi_{h,t,t+j}^{\varepsilon-\upsilon}/\pi_{t,t+j}^{1-\upsilon}\right) c_{t+j}^{1-\sigma} \end{aligned}$$

In the main text we express $\psi_{i,h,t}$ and $\phi_{i,h,t}$ recursively following Ascari and Sbordone (2014).

To derive the export pricing equation we follow the same steps but also use of the international risk-sharing condition. The unconstrained optimization problem is to choose $X_{i,h,t}^{\star}\left(z\right)$ to maximize,

$$\mathbb{E}_t \sum_{j=0}^{\infty} \gamma_i^j \Delta_{t,t+j} \left\{ \frac{q_{t+j} X_{i,h,t}^{\star}\left(z\right)}{p_{t+j}^{\star}} \left[X_{i,h,t}^{\star}\left(z\right)/p_{h,t+j}^{\star}\right]^{-\varepsilon} - w_{t+j} \left[X_{i,h,t}^{\star}\left(z\right)/p_{h,t+j}^{\star}\right]^{-\varepsilon} \right\} c_{h,t+j}^{\star} \tag{27}$$

which leads to, $\mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \Phi_{t,t+j} \left(p_{h,t+j}^{\star}\right)^{\varepsilon} \left[\left(q_{t+j}/p_{t+j}^{\star}\right) X_{i,h,t}^{\star}\left(z\right) - \frac{1}{\theta} w_{t+j} \right] c_{h,t+j}^{\star} = 0$. Again, we substitute in the discount factor, the demand curve, and the labor-leisure equation, and we also drop the z index, with the real reset price defined in units of consumption in the destination market; i.e., $x_{h,t}^{\star} \equiv X_{h,t}^{\star}/p_t^{\star}$. This generates,

$$x_{h,t}^{\star} = \frac{1}{\theta} \frac{\psi_{h,t}^{\star}}{\phi_{h,t}^{\star}} \tag{28}$$

where,

$$\psi^\star_{h,t} \equiv \mathbb{E}_t \sum_{j=0}^{\infty} (\gamma_i \beta)^j \left[\left(\pi^\star_{h,t,t+j}\right)^{\varepsilon-\upsilon} / \left(\pi^\star_{t,t+j}\right)^{-\upsilon} \right] \delta L^\eta_{t+j} c^\star_{t+j}$$

$$\phi^\star_{h,t} \equiv \mathbb{E}_t \sum_{j=0}^{\infty} (\gamma_i \beta)^j \left[\left(\pi^\star_{h,t,t+j}\right)^{\varepsilon-\upsilon} / \left(\pi^\star_{t,t+j}\right)^{1-\upsilon} \right] \left(c^\star_{t+j}\right)^{1-\sigma}$$

using $p^\star_t = p^\star_{t+j}/\pi^\star_{t,t+j}$ and $q_{t+j} = e_{t+j}p^\star_{t+j}/p_{t+j} = \left(c^\star_{t+j}/c_{t+j}\right)^{-\sigma}$. Foreign conditions are derived analogously. For reference, we note, $\vartheta^\star_{i,f,t}(z) = p^\star_{i,f,t}(z)\, y^\star_{i,f,t}(z) - W^\star_t l^\star_{i,f,t}(z)$ and $\vartheta_{i,f,t}(z) = \frac{p_{i,f,t}(z)}{e_t} y_{i,f,t}(z) - W^\star_t l_{i,f,t}(z)$ are profits of foreign firms (domestic and export, respectively).

A.3. Equation for Real Exchange Rate Dynamics (Section 3.2)

Period utility is such that optimal household conditions are $W_t/p_t = \delta c_t$ and $W^\star_t/p^\star_t = \delta c^\star_t$ and $q_t = c_t/c^\star_t$. Sectoral demand functions for goods are

$$c_{h,t} = \frac{1}{2}\left(p_{h,t}/p_t\right)^{-\upsilon} c_t \quad \text{and} \quad c_{f,t} = \frac{1}{2}\left(p_{f,t}/p_t\right)^{-\upsilon} c_t \tag{29}$$

and,

$$c^\star_{f,t} = \frac{1}{2}\left(p^\star_{f,t}/p^\star_t\right)^{-\upsilon} c^\star_t \quad \text{and} \quad c^\star_{h,t} = \frac{1}{2}\left(p^\star_{h,t}/p^\star_t\right)^{-\upsilon} c^\star_t$$

where $p_t = \left[(1/2)\, p^{1-\upsilon}_{h,t} + (1/2)\, p^{1-\upsilon}_{f,t}\right]^{1/(1-\upsilon)}$ and $p^\star_t = \left[(1/2)\, p^{\star 1-\upsilon}_{h,t} + (1/2)\, p^{\star 1-\upsilon}_{f,t}\right]^{1/(1-\upsilon)}$. Firms are divided into sectors: a fraction $1-\alpha$ of firms have a Calvo technology and a fraction α of firms are free to change their price each period. In the home economy,

$$p^{1-\varepsilon}_{h,t} = \underbrace{\int_0^\alpha \left[p_{1,h,t}(z)\right]^{1-\varepsilon} dz}_{\text{flexible prices}} + \underbrace{\int_\alpha^1 \left[p_{2,h,t}(z)\right]^{1-\varepsilon} dz}_{\text{Calvo sticky-price}} \tag{30}$$

and,

$$\left(p^\star_{h,t}\right)^{1-\varepsilon} = \int_0^\alpha \left[p^\star_{1,h,t}(z)\right]^{1-\varepsilon} dz + \int_\alpha^1 \left[p^\star_{2,h,t}(z)\right]^{1-\varepsilon} dz \tag{31}$$

for domestic sales and export sales, respectively. Similar equations hold for $p^{\star}_{f,t}$ and $p_{f,t}$.

Consider equation (3) in the text. With $\sigma = 1$ and $\gamma = 0$, we have; $x_{i,h,t} \equiv X_{i,h,t}/P_t = \psi_{i,h,t}/\theta\phi_{i,h,t}$, where $\psi_{i,h,t} = c_t + \gamma_i\beta\mathbb{E}_t\left(\frac{\pi_{h,t+1}^{\varepsilon-\upsilon}}{\pi_{t+1}^{-\upsilon}}\psi_{i,h,t+1}\right)$ and $\phi_{i,h,t} = 1 + \gamma_i\beta\mathbb{E}_t\left(\frac{\pi_{h,t+1}^{\varepsilon-\upsilon}}{\pi_{t+1}^{1-\upsilon}}\phi_{i,h,t+1}\right)$ and $c_t = (1/\delta)\,w_t$. In linearized form, $\widehat{x}_{i,h,t} = \widehat{\psi}_{i,h,t} - \widehat{\phi}_{i,h,t}$, where,

$$\widehat{\psi}_{i,h,t} = (1-\gamma_i\beta\pi^{\varepsilon})\,\widehat{c}_t + \theta_i\beta\pi^{\varepsilon}\left[(\varepsilon-\upsilon)\,\widehat{\pi}_{h,t+1} + \upsilon\widehat{\pi}_{t+1} + \widehat{\psi}_{i,h,t+1}\right] \tag{32}$$

and,

$$\widehat{\phi}_{i,h,t} = \beta\gamma_i\pi^{\varepsilon-1}\left[\widehat{\phi}_{i,h,t+1} + (\varepsilon-\upsilon)\,\widehat{\pi}_{h,t+1} - (1-\upsilon)\,\widehat{\pi}_{t+1}\right] \tag{33}$$

The sector average price is connected to the reset price by,

$$\widehat{x}_{i,h,t} = \frac{1}{(1-\gamma_i)\left(\frac{x_{i,h}}{\rho_{i,h}}\right)^{1-\varepsilon}}\left(\widehat{\rho}_{i,h,t} - \gamma_i\pi^{\varepsilon-1}\widehat{\rho}_{i,h,t-1}\right) + \frac{\gamma_i\pi^{\varepsilon-1}}{(1-\gamma_i)\,(x_{i,h}/\rho_{i,h})^{1-\varepsilon}}\widehat{\pi}_t \tag{34}$$

where $(x_{i,h}/\rho_{i,h})^{1-\varepsilon} = (1-\gamma_i\pi^{\varepsilon-1})\,/\,(1-\gamma_i)$. Noting that $\widehat{\rho}_{i,h,t+1} = \widehat{\rho}_{i,h,t} + (\widehat{\pi}_{1,h,t+1} - \widehat{\pi}_{t+1})$ and eliminating $\widehat{x}_{i,h,t}$ and collecting terms leads to,

$$\widehat{\pi}_{i,h,t} = \beta\widehat{\pi}_{i,h,t+1} + \zeta_i\,(\widehat{c}_t - \widehat{\rho}_{i,h,t}) + \beta\,(\pi-1)\,\Theta_i\left[\upsilon\widehat{\pi}_{t+1} + (\varepsilon-\upsilon)\,\widehat{\pi}_{h,t+1} + \widehat{\psi}_{i,h,t+1}\right] \tag{35}$$

where $\rho_{i,h,t} \equiv p_{i,h,t} - p_t$ and where $\Theta_i = (1-\gamma_i\pi^{\varepsilon-1})$ and $\zeta_i = \Theta_i\,(1-\gamma_i\beta\pi^{\varepsilon})\,/\gamma_i\pi^{\varepsilon-1}$ for sector $i = 1, ..., N$.

Making similar steps,

$$\widehat{\pi}^{\star}_{i,h,t} = \beta\widehat{\pi}^{\star}_{i,h,t+1} + \zeta_i\left(\widehat{c}_t - q_t - \widehat{\rho}^{\star}_{i,h,t}\right) + \beta\,(\pi-1)\,\Theta_i\left[\upsilon\widehat{\pi}^{\star}_{t+1} + (\varepsilon-\upsilon)\,\widehat{\pi}^{\star}_{h,t+1} + \widehat{\psi}^{\star}_{i,h,t+1}\right] \tag{36}$$

and,

$$\widehat{\pi}^{\star}_{i,f,t} = \beta\widehat{\pi}^{\star}_{i,f,t+1} + \zeta_i\left(\widehat{c}^{\star}_t - \widehat{\rho}^{\star}_{i,f,t}\right) + \beta\,(\pi-1)\,\Theta_i\left[\upsilon\widehat{\pi}^{\star}_{t+1} + (\varepsilon-\upsilon)\,\widehat{\pi}^{\star}_{f,t+1} + \widehat{\psi}^{\star}_{i,f,t+1}\right] \tag{37}$$

and,

$$\widehat{\pi}_{i,f,t} = \beta\widehat{\pi}_{i,f,t+1} + \zeta_i\,(\widehat{c}^{\star}_t - \widehat{\rho}_{i,f,t} + q_t) + \beta\,(\pi-1)\,\Theta_i\left[\upsilon\widehat{\pi}_{t+1} + (\varepsilon-\upsilon)\,\widehat{\pi}_{f,t+1} + \widehat{\psi}_{i,f,t+1}\right] \tag{38}$$

which we refer to as sectoral Phillips curves with trend inflation.

We now consider the case in which $i = 1, 2$ and $\gamma_1 = 0$ and $\gamma_2 = \gamma$. For $i = 1$, in real terms,

$$\widehat{\rho}_{1,h,t} = \widehat{c}_t \;\; ; \;\; \widehat{\rho}_{1,f,t} = \widehat{\rho}^{\star}_{1,f,t} + \widehat{q}_t \;\; ; \;\; \widehat{\rho}^{\star}_{1,f,t} = \widehat{c}^{\star}_t \;\; ; \;\; \widehat{\rho}^{\star}_{1,h,t} = \widehat{\rho}_{1,h,t} - \widehat{q}_t \tag{39}$$

which implies the law of one price holds in sector 1. These equations determine sector-2 prices via price indexes - for example, see equations (30) and (31) for home good. We have the following sector-2 prices (which we also express in real terms),

$$\begin{aligned}
\widehat{\rho}_{2,h,t} &= \left[(\widehat{p}_{h,t} - \widehat{p}_t) - \alpha \widehat{c}_t\right] / (1-\alpha) \\
\widehat{\rho}_{2,f,t} &= \left[(\widehat{p}_{f,t} - \widehat{p}_t) - \alpha \left(\widehat{c}^{\star}_t + q_t\right)\right] / (1-\alpha)
\end{aligned}$$

and,

$$\begin{aligned}
\widehat{\rho}^{\star}_{2,f,t} &= \left[\left(\widehat{p}^{\star}_{f,t} - \widehat{p}^{\star}_t\right) - \alpha \widehat{c}^{\star}_t\right] / (1-\alpha) \\
\widehat{\rho}^{\star}_{2,h,t} &= \left[\left(\widehat{p}^{\star}_{h,t} - \widehat{p}^{\star}_t\right) - \alpha \left(\widehat{c}_t - q_t\right)\right] / (1-\alpha)
\end{aligned}$$

We substitute these four conditions into $i = 1$ equations (35)-(38) and eliminate sector-1 price levels. For example, we re-write the sector-2 Phillips curve for domestic sales in the home economy as,

$$\widehat{\pi}_{2,h,t} = \beta \widehat{\pi}_{2,h,t+1} + \beta (\pi - 1) \Theta \left[\upsilon \widehat{\pi}_{t+1} + (\varepsilon - \upsilon) \widehat{\pi}_{h,t+1} + \widehat{\psi}_{2,h,t+1}\right] + \left(\frac{\zeta}{1-\alpha}\right) \left[\widehat{c}_t + (\widehat{p}_t - \widehat{p}_{h,t})\right] \tag{40}$$

where $\widehat{\pi}_{2,h,t} = \left(\frac{1}{1-\alpha}\right) \widehat{\pi}_{h,t} + \left(\frac{\alpha}{\alpha-1}\right) (\Delta \widehat{c}_t + \widehat{\pi}_t)$. We make similar steps for expressions determining $\left\{\widehat{\pi}_{2,f,t}, \widehat{\pi}^{\star}_{2,f,t}, \widehat{\pi}^{\star}_{2,h,t}\right\}$.

We then use these four equations to construct country-specific Phillips curves using $\widehat{\pi}_t = (1/2)\left(\widehat{\pi}_{h,t} + \widehat{\pi}_{f,t}\right)$ and $\widehat{\pi}^{\star}_t = (1/2)\left(\widehat{\pi}^{\star}_{h,t} + \widehat{\pi}^{\star}_{f,t}\right)$. For the home economy,

$$\begin{aligned}
\frac{\widehat{\pi}_t}{\alpha} - \Delta G_t &= \beta \left(\frac{\widehat{\pi}_{t+1}}{\alpha} - \Delta G_{t+1}\right) + \left(\frac{1-\alpha}{\alpha}\right) \beta (\pi - 1) \Theta \left(\varepsilon \widehat{\pi}_{t+1} + \widehat{\psi}_{1,t+1}\right) \\
&\quad + \frac{\zeta_i}{\alpha} \left(\widehat{c}^{W}_t + \frac{1}{2} q_t\right)
\end{aligned}$$

where $G_t \equiv \widehat{c}_t^W + \widehat{p}_t^W + \frac{1}{2}\widehat{e}_t$ and $\widehat{\psi}_{1,t+1} \equiv (1/2)\left(\widehat{\psi}_{1,h,t+1} + \widehat{\psi}_{1,f,t+1}\right)$. Likewise, in the foreign economy,

$$\begin{aligned}\frac{\widehat{\pi}_t^\star}{\alpha} - (\Delta G_t - \Delta\widehat{e}_t) &= \beta\left[\frac{\widehat{\pi}_{t+1}^\star}{\alpha} - (\Delta G_{t+1} - \Delta e_{t+1})\right] + \left(\frac{1-\alpha}{\alpha}\right)\beta(\pi-1)\Theta\left(\varepsilon\widehat{\pi}_{t+1}^\star + \widehat{\psi}_{1,t+1}^\star\right) \\ &\quad + \frac{\zeta_i}{\alpha}\left(\widehat{c}_t^W - \frac{1}{2}q_t\right)\end{aligned}$$

where $\widehat{\psi}_{1,t+1}^\star \equiv (1/2)\left(\widehat{\psi}_{1,f,t+1}^\star + \widehat{\psi}_{1,h,t+1}^\star\right)$. Finally, defining $\widehat{\pi}_t^R \equiv \widehat{\pi}_t - \widehat{\pi}_t^\star$,

$$\frac{\widehat{\pi}_t^R}{\alpha} - \Delta\widehat{e}_t = \beta\left(\frac{\widehat{\pi}_{t+1}^R}{\alpha} - \Delta e_{t+1}\right) + \left(\frac{1-\alpha}{\alpha}\right)\beta(\pi-1)\Theta\left(\varepsilon\widehat{\pi}_{t+1}^R + \widehat{\psi}_{1,t+1}^R\right) + \frac{\zeta_i}{\alpha}q_t \tag{41}$$

where $\widehat{\psi}_{t+1}^R \equiv (1/2)\left(\widehat{\psi}_{1,t+1} - \widehat{\psi}_{1,t+1}^\star\right)$. Now all we need to note is that $\Delta e_t = \Delta q_t + \widehat{\pi}_t^R$ to generate the first equation reported in the text.

Equation (41) contains three endogenous variables: $\left\{\widehat{\pi}_t^R, q_t, \psi_t^R\right\}$. The real exchange rate is explained by interest parity condition, $\Delta\widehat{e}_{t+1} = \widehat{i}_t - \widehat{i}_t^\star$, and the monetary policy stance of each economy, which we specify as $\widehat{i}_t = \phi\widehat{\pi}_t + \nu_t$ and $\widehat{i}_t^\star = \phi\widehat{\pi}_t^\star$. In this case, $\Delta q_{t+1} = \phi\widehat{\pi}_t^R - \widehat{\pi}_{t+1}^R + \nu_t$, which is the third equation in the text. The final term is $\psi_{t+1}^R = (1/2)\left(\widehat{\psi}_{1,t+1} - \widehat{\psi}_{1,t+1}^\star\right)$, which accounts for the dynamic changes in real marginal costs (wages in consumption units) across countries. Since $\widehat{w}_t = \widehat{c}_t$, in the home economy, we have, $\widehat{\psi}_{1,t} = (1-\theta\beta\pi^\varepsilon)\,\widehat{c}_t + \theta\beta\pi^\varepsilon\left(\varepsilon\widehat{\pi}_{t+1} + \widehat{\psi}_{1,t+1}\right)$, using the definition of $\widehat{\psi}_{1,t+1}$ given above. A similar expression holds for $\widehat{\psi}_{1,t}^\star$. Finally, taking the difference of these two expressions, using the definition $\widehat{\psi}_{t+1}^R$, and applying international risk sharing, $\widehat{q}_t = \widehat{c}_t - \widehat{c}_t^\star$, we generate the second equation in the text.

A.4. Solution for the Real Exchange Rate in a Special Case (Section 3.2)

In the text, we present a solution for the two-sector model when set $\pi = 1$ and $\phi\beta = 1$. Eliminate $\phi\widehat{\pi}_t^R - \widehat{\pi}_{t+1}^R$ using equation (14). This generates,

$$\beta\Delta\widehat{q}_{t+1} - \frac{\alpha}{1-\alpha}\Delta\widehat{q}_t = -\beta\frac{\alpha}{1-\alpha}\Delta\widehat{q}_{t+1} + \frac{\zeta}{1-\alpha}\widehat{q}_t + \beta\widehat{\nu}_t \tag{42}$$

In levels, this is a second order difference equation,

$$\widehat{q}_t = \lambda_1 \widehat{q}_{t-1} + \lambda_2 \widehat{q}_{t+1} + \lambda_3 \widehat{\nu}_t$$

where,

$$\lambda_1 \equiv \frac{\phi\alpha}{\varpi} \;\; ; \;\; \lambda_2 \equiv \frac{1}{\varpi} \;\; ; \;\; \lambda_3 \equiv -\frac{1-\alpha}{\varpi} \;\; ; \;\; \varpi = 1 + \phi\left(\zeta + \alpha\right)$$

and $\widehat{\nu}_t = \rho \widehat{\nu}_{t-1} + \widehat{\varepsilon}_t$. We guess the solution is of the form $\widehat{q}_t = \kappa_1 \widehat{q}_{t-1} + \kappa_2 \widehat{\nu}_t$, such that we can write,

$$\widehat{q}_t = \left(\frac{\lambda_1}{1-\lambda_2\kappa_1}\right)\widehat{q}_{t-1} + \left(\frac{\lambda_2\rho\kappa_2 + \lambda_3}{1-\lambda_2\kappa_1}\right)\widehat{\nu}_t$$

This implies,

$$0 = \lambda_2\kappa_1^2 - \kappa_1 + \lambda_1 \;\text{ and }\; \kappa_2 = \frac{\lambda_3}{1-\lambda_2\left(\kappa_1+\rho\right)} \quad \rightarrow \quad \widehat{q}_t = \kappa_1 \widehat{q}_{t-1} + \frac{\lambda_3}{1-\lambda_2\left(\kappa_1+\rho\right)}\widehat{\nu}_t$$

where the solution to κ_1 is $\left[1 \pm \left(1-4\lambda_1\lambda_2\right)^{0.5}\right]/2\lambda_2$. When we re-insert the definitions of $\lambda_1, \lambda_2, \lambda_3$, we have,

$$\begin{aligned} q_t &= \chi_q q_{t-1} + \chi_\nu \widehat{\nu}_t \\ \chi_q &= \frac{\varpi \pm \left(\varpi^2 - 4\phi\alpha\right)^{1/2}}{2} \;\text{ and }\; \chi_\nu = -\frac{1-\alpha}{\varpi - \left(\chi_q + \rho\right)} \end{aligned} \tag{43}$$

Consider the case when there is no flexible price sector and $\alpha \to 0$. We must have $\chi_q \to 0$. This case only obtains when $\chi_q = \left[\varpi - \left(\varpi^2 - 4\phi\alpha\right)^{1/2}\right]/2$, which is reported in the text.

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Table 1: Assigned Parameters Values

Assigned Parameters Values			
Description	Parameter	Value	Target/Source
Discount Factor	β	0.99	$(\beta^{-4} - 1) \times 100 = 4.102\%$
Elasticity of intertemporal substitution	η	1/5	Gali (2015)
Frisch elasticity of labor supply	σ	1/3	Carvalho and Nechio (2011)
Home-bias parameter	ω	0.9	Carvalho and Nechio (2011)
Armington parameter	ν	1.5	Carvalho and Nechio (2011)
Taylor rule parameter (inflation)	ϕ_π	1.5	standard
Taylor rule parameter (output)	ϕ_y	0.5/4	standard
Serial correlation in υ_t	ρ	0.85	standard

Figure 1: Dynamic Path of the Real Exchange Rate (Multi-Sector)

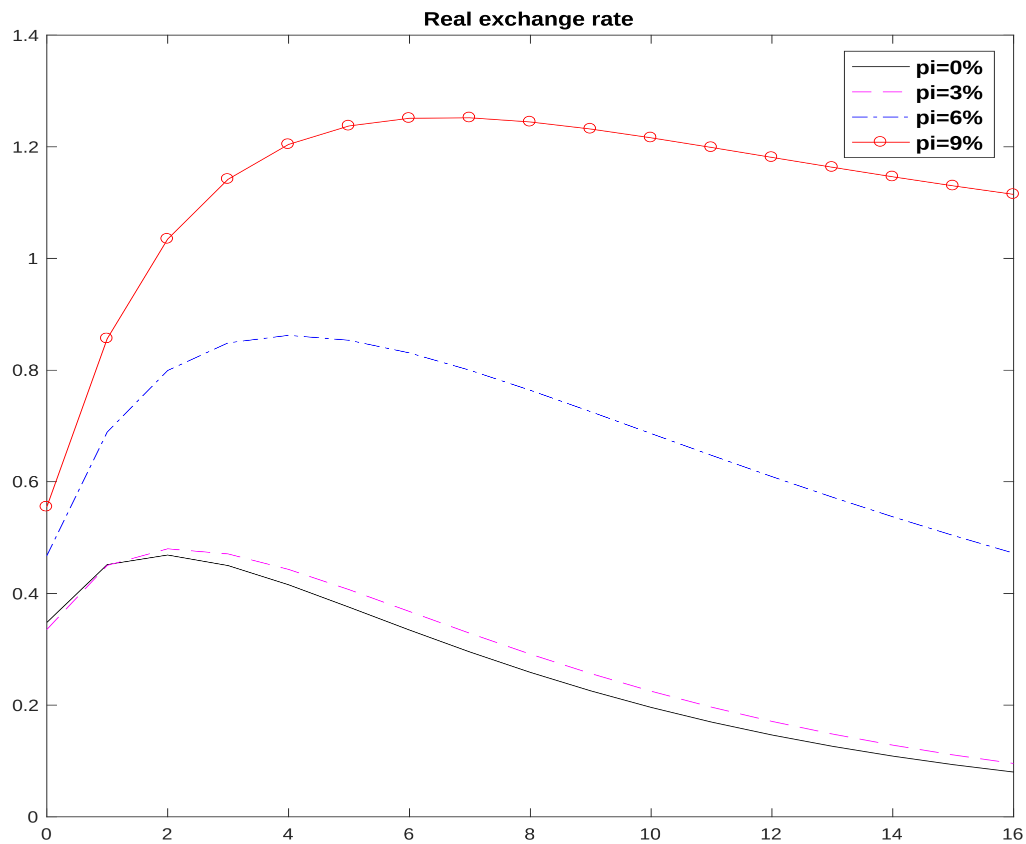

Figure 2: Dynamic Path of the Real Exchange Rate (Single-Sector)

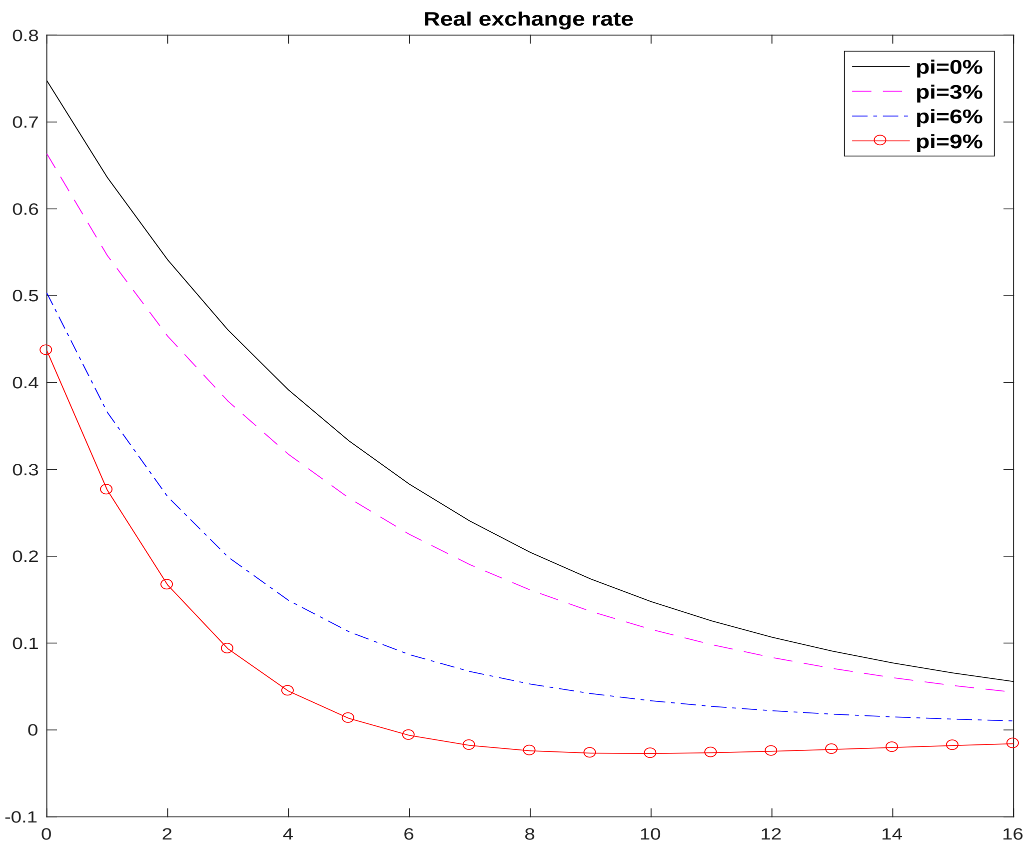

Figure 3: Plot of the χ_q Parameter