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Risk Management for Sovereign Debt Financing with Sustainability Conditions[*]

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Abstract

We develop a model of debt sustainability analysis with optimal financing decisions in the presence of macroeconomic, financial and fiscal uncertainty. We define a coherent measure of refinancing risk, and trade off the risks of debt stock and flow dynamics, subject to debt sustainability constraints and endogenous risk and term premia. We optimize both static and dynamic financing strategies, compare them with several simple rules and consol financing to demonstrate economically significant effects of optimal financing, and show that the stock-flow tradeoff can be critical for sustainability. We quantify the minimum refinancing risk and the maximum rate of debt reduction that a sovereign can achieve given its economic fundamentals, and extend the model to identify optimal timing for debt flow adjustments that allow the sovereign to go beyond these limits. We put the model to the data on three real-world cases: a representative euro zone crisis country, a low-debt country (Netherlands) and a high-debt country (Italy). These applications illustrate the use of the model in informing diverse policy decisions on sustainable public finance. The model is part of the European Stability Mechanism toolkit to assess debt sustainability and repayment capacity of member states in the context of financial assistance.

JEL Classification: C61, C63, D61, E3, E47, E62, F34, G38, H63

Keywords: sovereign debt, sustainability, debt financing, optimization, stochastic programming, scenario analysis, conditional Value-at-Risk, risk measures

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1 Introduction

In the aftermath of the 2008 global financial crisis, sovereign debt increased sharply in most advanced economies. Average public debt, as a percentage of gross domestic product, increased by 30% from trough to peak (Figure 1), with almost all countries experiencing a significant increase. In the euro area, public debt rose to about 84% in 2010, decisively contributing to a sovereign debt crisis, with five countries requiring financial assistance. As the crisis countries exited their fiscal adjustment programs by 2018, Italy emerged as a new challenge. These episodes, and the fact that public debt remains high globally, have prompted a renewed interest in debt sustainability analysis (DSA) and in policy discussions concerning the most appropriate DSA variables (debt stock level, or debt flow measured by the annual gross financing needs), and the thresholds on these variables beyond which debt dynamics are likely unsustainable. Such thresholds are estimated by the international institutions —the International Monetary Fund (IMF), the European Stability Mechanism (ESM) or the European Central Bank— and can be used to specify transparent rules for accessing official lending. Amid this debate, it has become clear that the standard DSA models need to be strengthened to serve as early warning tools monitoring the sustainability of public accounts. In the words of Klaus Regling, managing director of ESM,

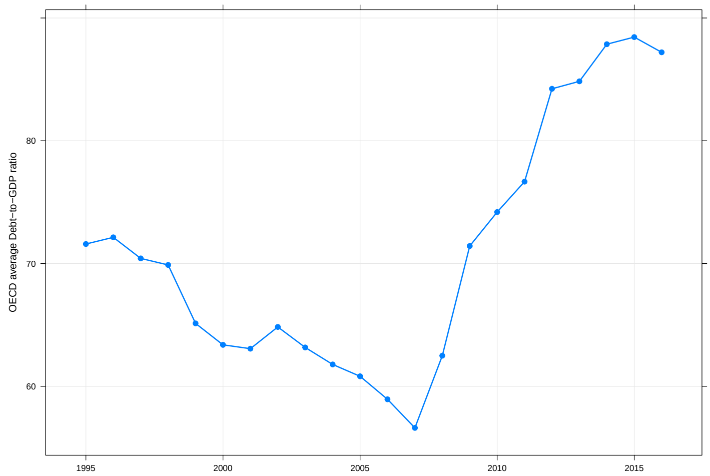
> "Partly in reaction to the European experience, the traditional approach to debt sustainability assessment has evolved. A fresh view on things is one way to further improve the analysis of debt sustainability, and develop best practices."[1]

We provide such a fresh view by developing a novel normative model optimizing debt financing decisions with the sustainability conditions embedded within official sector DSA, under macroeconomic, financial, and fiscal uncertainty. The model addresses questions like: How do issuance strategies trade off the main sustainability variables of debt stock and debt flow? How do the inter-related stock and flow dynamics depend on the sensitivity of interest rates to debt? Can the debt dynamics of a country with given fundamentals satisfy acceptable sustainability conditions or are they unsustainable? If debt dynamics are unsustainable, how and when should adjustments be made to restore sustainability?

The model optimizes the maturity of debt instruments to minimize expected interest costs, constrained by exogenous sustainability thresholds on both debt flow and stock. The constraint on gross financing needs limits the refinancing risks due to funding requirements spikes. The stock dynamics constraint sets the minimum pace of debt reduction that will avoid unsustainable dynamics (or, the maximum sustainable rate of debt increase for low debt countries).

We follow the risk management literature to define a coherent measure of refinancing risk, and optimize the debt stock-flow risks tradeoff, bounded by sustainability thresholds on the *conditional Value-at-Risk* (CVaR) measure of tail risks. This approach —engrained in the risk management of financial institutions— is particularly appropriate for DSA given the need to assess sustainable debt dynamics in a probabilistic way under multi-dimensional uncertainty. We represent uncertainty by a discrete time- and state-space scenario tree, and use stochastic programming to optimize dynamic debt financing strategies. An important feedback effect,

[1]Opening address at the ESM conference on debt sustainability, 11 December 2018, Luxembourg.



Source: OECD, doi 10.1787/a0528cc2-en, accessed March 2018.

Figure 1 – Government debt growth of OECD countries

which is especially strong for highly indebted countries, is between a sovereign's debt and the level and slope of the yield curve. We allow for endogenous *risk* and *term premia* to drive issuance decisions and, consequently, the debt level, to capture this feedback loop.

Our model breaks new ground in DSA with the introduction of a risk measure, the optimization of debt financing decisions, the simultaneous treatment of debt stock and flow, and dual —stock and flow— sustainability constraints. The model generates sustainability-constrained frontiers of the stock-flow tradeoff embedded in the financing decisions. This tradeoff is a significant determinant of debt dynamics and can be critical for sustainability. We optimize both static (state independent) and dynamic (state dependent) financing strategies, and show that optimal financing can have economically significant non-linear effect on sustainability.

We test the model on a calibrated representative eurozone crisis economy, compare the optimized strategies with several simple rules and consol financing, and show that debt financing optimization matters. The dynamic strategies dominate, but we settle on an intermediate adaptive strategy (time dependent but state independent) which is less volatile in the issuance patterns and, hence, applicable in practice. We use the adaptive strategy to document the stock-flow tradeoff and show that optimal financing can reduce the likelihood of unsustainable debt dynamics. We also demonstrate the effect of endogenous risk and term premia.

We then apply the model to the debt data of the Dutch State Treasury Agency (DSTA) and the Italian Parliamentary Budget Office (PBO) representing, respectively, a low- and a high-debt non-crisis economy.[2] We find that the current DSTA practices are near optimal, we document a premium for the DSTA policy of maintaining a minimum level of short term reserves, and identify possible improvements. Using the Italian data we assess the sustainability of debt dynamics under no policy changes, under the plans of the newly elected Italian government, and under the negotiated 2019 budget agreement with the European Commission (EC), and answer questions of relevance to an independent fiscal council. We show that the 2019 budget

[2]We acknowledge collaboration with the European Stability Mechanism in calibrating a representative eurozone crisis country, SEO Amsterdam Economics as part of a comprehensive assessment of the DSTA funding policies (Hers et al., 2019, pp. 30–36), and the Italian Parliamentary Budget Office for the Italian application.

improves the country's debt dynamics, although the probability of sustainability is a low 0.55.

We also quantify the minimum level of refinancing risks and the maximum debt reduction pace that a sovereign could reach, given its economic fundamentals, and extend the model to identify the optimal timing of adjustments to gross financing needs so that a sovereign can go beyond these thresholds to restore sustainability. In particular, we identify *hot spots* of unsustainable gross financing needs, and compute the minimum size and optimal timing of the required adjustments. We modify the crisis economy calibration to create non sustainable debt dynamics and test the model extension. Results show the benefits from timely implementing additional adjustments, limit the adjustments to avoid potential adverse effects on the economy, and illustrate the cost of procrastination. This feature of the model is helpful in designing official sector financial assistance programmes. Applying this extension to the Italian data we estimate that additional fiscal effort of at least 3.5% of GDP is required over the next decade to achieve sustainable dynamics with a high probability of 0.85.

1.1 Related literature

We draw from literature on the economics of sovereign financing and sustainability, risk management, and planning under uncertainty.

Economics of sovereign debt

The literature on what determines sustainable debt levels is extensive (D'Erasmo et al., 2016). Our paper adds to recent contributions which model an active fiscal policy maker, but instead of allowing for endogenous default,[3] we follow a different path. We assume an exogenous process of primary surplus and focus on the decisions of an active public debt management office to minimize net interest payments while avoiding, if possible, the default risk from breaching some thresholds. This approach has a clear practical advantage in that, by abstracting from the government's utility function for preferences over deficit, we can solve the operational problem of a treasury facing sustainability rules like those imposed by official lenders, accounting for the term structure of legacy debt and using a realistic set of financing instruments.

Several contributions focus on the role of the maturity structure on the tradeoff between borrowing costs and refinancing risks. Cole and Kehoe (2000) find that sovereigns can avoid self-fulfilling refinancing crisis by lengthening debt maturity, whereas countries in recession gamble for redemption by shortening the maturity (Conesa and Kehoe, 2015), and, likewise, Arellano and Ramanarayanan (2012); Broner et al. (2013) show that maturity shortens with rising interest rates. On the other hand, Barro (2003) shows that a tax-smoothing objective leads to contingent consols, or, in the absence of contingent debt, "maturity structure that has no holes", i.e., the same amount to be paid in each future period. Angeletos (2002) characterizes the maturity structure that insulates public debt from interest rate risk and smooths a government's refinancing needs by issuing a perpetuity and investing in short-term reserves, thereby doing away with contingent debt; see also Bianchi et al. (2018) for the international setting. The

[3]See Aguiar et al. (2016); Arellano and Ramanarayanan (2012); Ayres et al. (2018); Bai et al. (2015); Bocola and Dovis (2019); Chatterjee and Eyigungor (2012); Conesa and Kehoe (2014); Hatchondo et al. (2016); Hatchondo and Martinez (2009); Mihalache (2017).

earlier papers issue single-period debt and characterize its duration (long vs short), while the recent multi-period works use consols with exponentially decaying coupons (Angeletos, 2002; Arellano and Ramanarayanan, 2012; Bocola and Dovis, 2019; Hatchondo and Martinez, 2009; Mihalache, 2017) or contingent consols (Barro, 2003). However, consol financing finds very limited use in practical applications.[4] We will see that, in practice, the maturity structure of legacy debt does have "holes", and by adding to this literature the salient tradeoff between debt stock level and smooth financing needs, and using a multi-period financing strategy with a richer set of realistic instruments, we can deal with such important practical considerations. Importantly, we add the dual sustainability constraints.

The above papers differ on the debt maturity recommendations, due to differences in their respective models of default dynamics. For instance, Cole and Kehoe (2000); Conesa and Kehoe (2015) relate the debt maturity structure to rollover risk, Bocola and Dovis (2019) relate maturity to fundamental and non-fundamental (self-fulfilling) risks, and Ayres et al. (2018) relate the maturity structure to good or bad equilibria depending on the distributional characteristics of the feedback between debt and interest rates. (Barro, 2003, footnote 5) emphasizes the significance of maturity structures, in the absence of contingent consols. Also, with the exception of Angeletos (2002); Bocola and Dovis (2019), they treat interest rates as exogenous. Our model is the first, to the best of our knowledge, *normative* model to optimize over a range of maturities with a realistic set of instruments to limit both refinancing risk and explosive debt dynamics, accounting for the term structure of legacy debt with potential maturity gaps, and endogenous interest rates.

For the sustainability constraints, we are informed by the use of thresholds in official sector DSA. Recent empirical studies (Dias et al., 2014; IMF, 2013; Irwin, 2015; Schumaker and Weder di Mauro, 2015), raise the question on the right definition of public debt to be used for assessing market access. Gabriele et al. (2017) show that borrowing costs are determined by jointly considering debt stock and refinancing needs. We supplement this literature by modeling the tension between stock and flow within DSA, with sustainability constraints on both.

Risk management

The risk management of financial institutions hinges on trading off risks with expected rewards under uncertainty, and regulated industries adopt standards to measure and limit risks, e.g., Basel III and Solvency II. Dating back to the seminal work of Markowitz (1952) on portfolio selection, the tradeoffs are quantified through efficient frontiers. Missale (1997, 2000) recognizes these tradeoffs in the context of sovereign debt management, pointing out that tax or deficit smoothing follows from a tradeoff between minimization of the expected cost of debt servicing and of budgetary risk, and the optimal smoothing of Barro (2003) follows from a risk minimization specification. Bolder (2003) uses simulations to study the tradeoffs of a given financing strategy, and Velandia (2018) describes a sovereign asset and liability framework. Our work advances these descriptive approaches to a prescriptive model with sustainability constraints.

[4]There are presently very few outstanding consol bonds (Meyer et al., 2019), and we are unaware of sovereigns issuing exponentially decaying perpetuities (Arellano and Ramanarayanan, 2012; Hatchondo and Martinez, 2009) or debt with probabilistic maturities (Chatterjee and Eyigungor, 2012).

The quantification of uncertainty in DSA has only recently received attention by international organizations and academia (Barnhill and Kopits, 2003; Celasun et al., 2006; Consiglio and Zenios, 2016; Guzman and Lombardi, 2018). We represent stochastic variables on a discrete time- and state-space *scenario tree* to model endogenous risk and term premia, contribute a measure of tail risk of the two key DSA variables, and trace the tradeoffs in a parametric optimization model. We adapt the CVaR tail risk measure, grounded in the coherence properties of Artzner et al. (1999), and used in Basel III. This approach is important for DSA, since un-sustainability are rare extreme events, and tail risk allows a sustainability assessment with an acceptable probability level.

Planning under uncertainty

We formulate the model using multi-period stochastic programming with recourse, which dates back to Dantzig (1963), and received renewed attention in the 1980s with the development of solution algorithms (Birge and Louveaux, 2011; Kall and Wallace, 1993) and advances in parallel computing methods (Censor and Zenios, 1997) to solve large-scale problems. Stochastic programming financial applications proliferated, see, e.g., Mulvey and Ziemba (1998); Zenios (2007), and models were developed for the Turkish Ministry of Finance (Balibek and Köksalan, 2010), the Italian Treasury (Consiglio and Staino, 2012), and sensitivity analysis for sustainability (Consiglio and Zenios, 2016). These works deal with the short-term problem of public debt stock management to optimize the cost of debt issuance given an exogenous stochastic yield curve, but do not consider the flow dynamics, economic and fiscal shocks, or sustainability constraints. These are the precursors to our model.

1.2 Contribution

Our model, uniquely among existing works, looks at both stock and flow dynamics in a multi-period setting. It optimizes *dynamic* financing strategies for the full term structure of legacy debt payments, and not just aggregate debt stock, and with a realistic set of instruments beyond the binary choice between consols and short-term treasuries. We utilise a rich representation of uncertain economic, financial, and fiscal variables, using scenario trees calibrated to expectations, volatilities, and correlations. As a result of these innovations we quantify new tradeoffs and obtain qualitative policy insights for real world applications.

The model is part the ESM toolkit to assess debt sustainability and repayment capacity of member states in the context of financial assistance —mandated in ESM Treaty Article 13 1.b—, including the crisis countries under financial assistance programs totalling €295bn. Two of us are involved in the use of the model, but DSA for program countries is done with proprietary data and is not reported here. Two of us collaborate with SEO Amsterdam Economics and the Italian PBO using publicly available data, and we report our findings without implicating these organizations.

The paper proceeds as follows. We lay out the economic problem and describe scenario trees (Section 2), develop the optimization model (Section 3, with details for a tree implementation in Appendix A), and discuss model calibration (Section 4). We then put the model to the data (Section 5). In Section 6 we extend the model to identify hot spots and compute the minimum size and optimal timing of adjustments to improve the debt sustainability outlook, and apply it to the crisis country and Italy. In section 7 we carry additional tests to evaluate the economic significance of the results and understand when optimization matters most. Section 8 concludes.

2 Layout of the model

2.1 The economic problem

We consider a sovereign that at period t is endowed with output Y_t, runs a primary balance PB_t, and owes a stock of debt D_{t-1}. The sovereign's *gross financing needs* are given by the *flow dynamics*

$$GFN_t = i_{t-1}D_{t-1} + A_t - PB_t, \tag{1}$$

where i_{t-1} is the *effective nominal interest rate* on debt at $t-1$, and A_t is the amortization schedule corresponding to the amount of D_{t-1} maturing at t.[5] The *debt stock dynamics* are given by the recursive equation

$$D_t = (1 + i_{t-1})D_{t-1} - PB_t. \tag{2}$$

To finance its needs the sovereign choses from J debt instruments of different maturities. The sovereign makes *financing decisions* to issue $X_t(j)$ nominal amount of instrument j at t. The *debt financing equation* satisfies

$$\sum_{j=1}^{J} X_t(j) = GFN_t. \tag{3}$$

The interest on issued debt is determined by the market risk-free rate plus endogenous premia idiosyncratic to the sovereign, including term premia for debt of different maturities (Broner et al., 2013; Engen and Hubbard, 2004; Paesani et al., 2006; Qiang and Phillippon, 2005). These premia depend on debt dynamics (Bassanetti et al., 2016; Gabriele et al., 2017; Paesani et al., 2006), and we integrate into our model a calibrated functional response of interest rates to debt. Following Engen and Hubbard (2004); Paesani et al. (2006) we model premia as a nonlinear function of debt-to-GDP ratio, $d_t = \frac{D_t}{Y_t}$, and of the maturity of the issued instrument, so that the interest rate for instrument j issued at t is given by

$$r_t(j) = r_{ft} + \rho(d_t, j). \tag{4}$$

r_{ft} is the term structure of the base maturity (which we take to be the 5-year maturity) forward risk-free rate, and $\rho(d, j)$ endogenizes the *risk premium* and *term premia* for different maturities j.

The vector X of issuance of all debt types j at each time t, determines the *effective interest*

[5] Eqn. (1) can incorporate one off flow adjustments, e.g., due to sales of state-owned assets or debt restructuring.

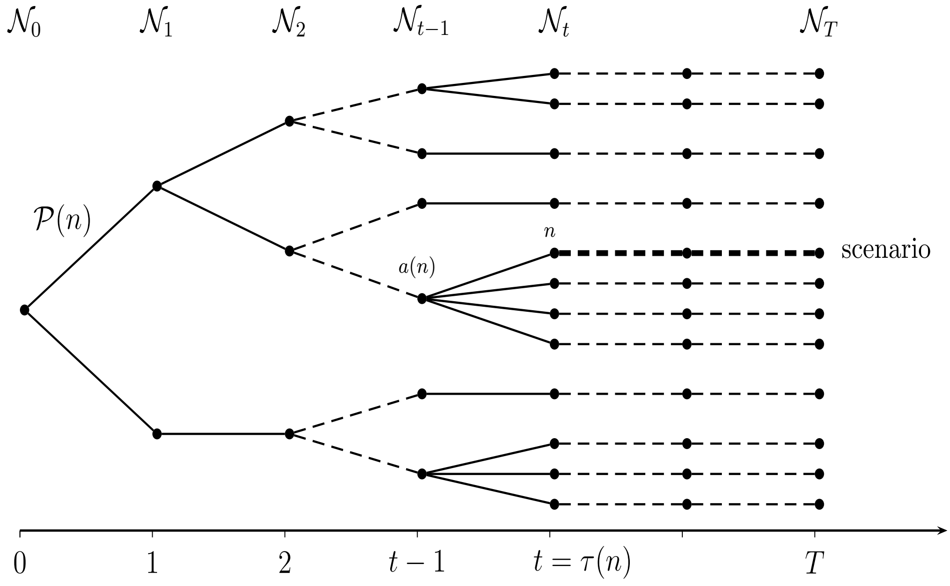


Figure 2 – A scenario tree

rate $i \doteq i(X)$, and we have[6]

$$i_t = \frac{i_{t-1}(D_{t-1} - A_t) + \sum_{j=1}^{J} r_t(j) X_t(j)}{D_t}. \tag{5}$$

In this paper we model the optimal choice of financing variables X. These variables consequently determine the debt dynamics, that endogenously determine risk and term premia (eqn. 4), which, in turn, influence the maturities to be issued. This *feedback loop* $X \to D \to r \to X$ links stock and flow not only through quantities, but also through prices. The feedback from debt stock into interest rates gives raise to vicious and/or virtuous cycles, and critically influences financing decisions as we demonstrate in subsection 5.1.3.

2.2 Modelling uncertainty

We model uncertainty using a discrete multi-period *scenario tree*, see Figure 2. Time steps are indexed by $t = 0, 1, 2, \dots T$, where 0 is here-and-now, and T is the risk horizon. Data are indexed on the tree by a set of *states* $\mathcal{N}_t$, with each $n \in \mathcal{N}_t$ representing a possible state of the economy at t. $\mathcal{N}$ denotes all possible states during the risk horizon. The number of states at t is N_t, and the total number of states is N. Not all states at t can be reached from every state at $t-1$, and $\mathcal{P}(n)$ denotes the set of states on the unique *path* from the *root state* 0 up to n. Each path that leads to a terminal state $n \in \mathcal{N}_T$ is a *scenario*. The unique ancestor of n is denoted by $a(n)$, with $a(0) = 0$. A function $\tau(n)$ identifies the time period of states on a path, i.e., $\tau(n) = t, \tau(a(n)) = t-1$, and so on. For any state n at t, all information at states m on the path $\mathcal{P}(n)$ is known since $\tau(m) < t$.

Problem data and model variables are indexed by states n. The values of exogenous variables

[6]For simplicity, this recursive equation assumes that the effective interest rate for D_{t-1} and A_t is the same. In practice, this does not need to be the case and differences are fully accounted for in our tests.

are known at each state, whereas endogenous variables take state-dependent values, determined by the optimization model. The conditional probabilities of states $n \in \mathcal{N}_t$ are denoted by π_t^n, and p^n denote unconditional probabilities of states $n \in \mathcal{N}$ except the root. Scenario probabilities are the unconditional probabilities of states $n \in \mathcal{N}_T$.

The multi-period setup allows us to represent the term structure of legacy debt payments, thus capturing refinancing risk due to service payment spikes. This feature of the model is consistent with recent contributions with endogenous defaults (Aguiar et al., 2016; Bocola and Dovis, 2019), but it is absent from most models that use a lump sum of debt stock, or debt with binary short-long maturity structures (Arellano and Ramanarayanan, 2012; Ayres et al., 2018; Ghosh et al., 2011; Mihalache, 2017). Likewise, it permits finer granularity in the choice of financing instruments, beyond short-term treasuries and consols. On the tree we define debt financing decisions, model the stochastic dynamics of debt stock and flow, and define risk measures of the distributions of both dynamics. At each state of the tree we also compute the calibrated non-linear function for the risk and term premia to endogenize interest rates.

We pose the economic problem of the sovereign on the scenario tree to optimize financing strategies that can be time-dependent or time- and state-dependent. As a special case we obtain optimal time invariant strategies, mimicking simple financing rules.

3 The optimization model

We develop the parametric optimization model to trade off cost with the risk of violating sustainability conditions on debt stock or flow. We discuss the objective function, define the financing decision variables, and set the constraints for a model of a sovereign facing uncertain exogenous economic output and primary balance, and endogenous refinancing rates. The model captures the salient factors influencing the decisions of a sovereign issuer, namely, the tradeoff between short-term costs and long-term risks, and the assessment of future refinancing and insolvency risks arising from shocks in the economy, the fiscal position, and the capital markets.

The model constrains both debt flow and stock dynamics by exogenous thresholds. Debt flows give a vulnerability signal at any risk horizon, whereas debt stock dynamics reflect long-term insolvency risks. These variables are linked through the endogenous interest rates. For debt flow risks we constrain a tail measure of the distribution. For debt stock, we constrain that it remains on a non-increasing path to limit insolvency risks. Naturally, relaxing the constraint on one variable improves the optimal value for the other.

The model also accommodates market demand and other policy considerations, such as inter-temporal smoothing or boundary conditions on the financing strategy.

3.1 Objective function

We consider a sovereign issuer minimizing the expected interest payments on its debt, subject to a constraint on the level of refinancing risks. We measure gross financing needs, like debt stock, as a ratio to GDP, to account for economic output uncertainty, and denote this random variable by *gfn*. Letting $\Psi(\cdot)$ denote the risk measure, we write the optimization problem as:

$$\underset{X}{\text{Minimize}} \qquad \sum_{\substack{n\in\mathcal{N}_t,\\ \text{for all } t=0,1,2,\ldots T.}} p^n NIP_t^n \tag{6}$$

s.t.

$$\Psi(gfn) \leq \omega. \tag{7}$$

The objective function minimizes the expected *net interest payments* (NIP) faced by the sovereign, which is the primary relevant variable for the treasury. For instance, the primary goal of the US Treasury Department debt management function is to finance government borrowing needs at the lowest cost over time, against acceptable risks to the budget, and, likewise for the DSTA and the Italian Treasury, with a medium and long term view.[7] Interest payments consist of interest service payments on legacy debt I_t^n, plus service payments on debt created by the financing decisions on the path leading to n. To trace service payments on endogenously created debt requires some ingenuity to exploit the tree structure. Let $CF_t^n(j,m)$ denote the nominal amount of interest payments due at state n of period t, per unit $X_{\tau(m)}^m(j)$ issued at state m of period $\tau(m)$ on the path $\mathcal{P}(n)$. This amount is computed from scenarios of the term structure of interest rates and the terms of the issued instrument, and since the yield curve depends on debt dynamics (cf. eqn. 4) it is endogenous. The net interest payments are given by

$$NIP_t^n = I_t^n + \sum_{m\in\mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j)\, CF_t^n(j,m). \tag{8}$$

Net interest payments minus interest on legacy debt is what the sovereign controls through financing decisions. NIP/D is the effective interest rate of debt (eqn. 5).

The gross financing needs-to-GDP ratio in (7), *gfn*, takes scenario values $gfn_t^n = \frac{GFN_t^n}{Y_t^n}$, where Y_t^n is the state-dependent economic output. We denote by *gfn* the random variable over all states $\mathcal{N}$, and by gfn_t over states $\mathcal{N}_t$ at t. Inequality (7) bounds the risk on gross financing needs (as percentage of GDP) by ω, reflecting the sovereign's risk tolerance.

3.2 Decision variables

The financing decisions on the tree are denoted by $X_t^n(j)$ and the debt financing equation (3), for all states $n \in \mathcal{N}_t$, and times periods $t = 0, 1, 2, \ldots T$, becomes

$$\sum_{j=1}^{J} X_t^n(j) = GFN_t^n. \tag{9}$$

[7]For the US Treasury policy see `https://www.treasury.gov/resource-center/data-chart-center/quarterly-refunding/Pages/overview.aspx`, for DSTA see the preface to Outlook 2019 at `https://www.dsta.nl/documenten/publicaties/2018/12/14/outlook-2019`, and for the Italian Treasury see Section I of Outlook 2017 at `http://www.dt.tesoro.it/export/sites/sitodt/modules/documenti_en/debito_pubblico/presentazioni_studi_relazioni/Public_Debt_Report_2017.pdf`. All sites accessed in March 2019.

Using proportional weights $w_t^n(j)$ for financing with instrument j in state n at time t, we write the debt financing equation as

$$\sum_{j=1}^{J} w_t^n(j) = 1, \qquad (10)$$

$$w_t^n(j) = \frac{X_t^n(j)}{GFN_t^n}. \qquad (11)$$

We envisage three possible strategies in optimizing the financing decisions. In the simplest case, the issuing Treasury restricts weights $w(j)$ to be time- and state-invariant, thereby searching for optimal weights of financing instruments that are kept fixed for all periods and all states. This is a *fixed-mixed strategy* and results in simple *rules* for debt financing.

The second possibility is more flexible, specifying state-invariant but time-dependent weights $w_t(j)$, so that debt is financed using weighted allocations in the available instruments that adapt with time, but are identical for all states at each period. This is an *adaptive fixed-mix strategy.*

The third possibility is the most flexible with time- and state-dependent optimal weights $w_t^n(j)$. This allows the issuer to implement a decision, wait to observe the state at the next time period, implement the optimal decision for that state, and wait again. This is a *dynamic strategy.* Decisions are made at $t = 0$ based on all available information at the root state, including conditional expectations about future uncertain information, and, as new information arrives at subsequent time periods, the model makes *recourse* decisions. The decision $w_t^n(j)$ is *adapted* to state n from the information at the ancestor state $a(n)$. The tree structure precludes decisions from being adapted to states that have not yet been observed, satisfying the *non-anticipativity* property of stochastic programming. Adaptation and non-anticipativity imply that stochastic programming prescribes implementable policies without clairvoyance.

A dynamic strategy has more degrees of freedom than the alternatives, allowing the Treasury to achieve better results, and our tests will demonstrate significant Pareto improvements when going from simple rules to adaptive fixed-mix and to dynamic strategies. However, we favor the use of adaptive fixed-mix since dynamic strategies pose a practical problem. In particular, while Treasuries enjoy flexibility in setting their financing strategy, they also face both demand and supply constraints that may preclude a dynamic strategy. Treasuries tend to pre-commit to annual issuance plans, and they avoid surprising the markets over the course of the year. For instance, the Dutch agency considers "predictability —no undue surprises— an important underlying objective in our funding policies." (op. cit., p. 4), and the Italian and U.S. Treasury use almost identical language in their policy statements. Dynamic strategies can surprise the markets by changing the issuance strategy depending on the state of the economy, and adaptive fixed-mix is a more appropriate strategy. Financing with simple rules is sub-optimal, but it is prevalent in standard DSA, and we use it for comparison purposes in some experiments.

3.3 Constraints

We now specify the model constraints. For simplicity, we give state-invariant equations when possible, and we introduce the states n for implementation on a tree in Appendix A.

Risk measure and the debt flow constraint

We use the coherent CVaR risk measure, see, e.g., (Zenios, 2007, pp. 58–63), to consider the expected value of the tail of the distribution of *gfn* for confidence level α. This is the aggregate conditional Value-at-Risk of debt flow (CFaR) over the tree, to be distinguished from a risk measure of flow at each time period, and the risk function in eqn. (7) is defined as

$$\Psi(gfn) \doteq \mathbb{E}\left(gfn \mid gfn \geq gfn^{\diamond}\right). \tag{12}$$

$gfn^{\diamond}$ is the right α-percentile of the aggregate gross financing needs, i.e., the lowest value $gfn^{\diamond}$ such that the probability of gross financing needs less or equal to $gfn^{\diamond}$ is greater or equal to α. $gfn^{\diamond}$ is the Value-at-Risk of aggregate debt flow, and we use $gfn^{\diamond\diamond}$ to denote aggregate CFaR.

In a seminal contribution Rockafellar and Uryasev (2000, 2002) showed that CVaR for discrete random variables can be minimized using linear programming, and we use this key property for a tractable formulation of the model on a tree. The flow constraints on the scenario tree are given in Appendix A.1.

Debt stock constraint

For sustainability, the debt dynamics should not be exploding, and, for highly indebted countries, access to official lending comes with strict conditionality that debt stock should be reduced. We constrain the debt stock dynamics by adding to model (6)–(7) the condition

$$\frac{\partial d}{\partial t} \leq \delta. \tag{13}$$

For highly indebted countries we set $\delta < 0$, targeting a minimum debt reduction pace, in line with official lending rules, but for low debt countries we can allow debt increase with $\delta \geq 0$. Since d is a stochastic variable this constraint can be imposed on the worst case (i.e., always), or on the average, or with high probability. Consistently with the flow constraint, we treat debt stock with a probabilistic risk constraint, so that the target pace of debt reduction is achieved with high probability. The stock constraints on the tree are given in Appendix A.2.

Changing δ parametrically, together with ω, we trace a three-dimensional frontier trading-off refinancing and debt stock risks with cost. In the model we optimize the tradeoff between gross financing needs and costs, and bound debt stock with a (probabilistic) constraint.

Debt dynamics

To complete the model we specify the accounting identities for debt flow and stock dynamics. There are two sets of linear equations, linking the flow (respectively, stock) at period t to those at $t-1$. The model funds legacy debt and any debt created endogenously with maturity within the risk horizon, and we introduce an indicator function $\mathbb{1}^t(j,t')$, with $t' < t$, that takes the value 1 if instrument j issued at time t' matures at t, and 0 otherwise. This function tracks maturing endogenous debt, and at each t the amount of previously issued debt that matures, is given by $\sum_{t'=0}^{t}\sum_{j=1}^{J} X_{t'}(j)\mathbb{1}^t(j,t')$,. This amount is financed together with maturing legacy debt and due interest. The debt dynamics on the tree are given in Appendix A.3.

Maturity smoothing and boundary conditions

Inter-temporal smoothness of debt financing produces strategies satisfying the predictability ("no undue surprises") criterion. If M_j denotes the maturity of the jth instrument, the *weighted average maturity of issued debt* at t under the adaptive fixed-mix strategy is given by

$$WAMI_t = \sum_{j=1}^{J} w_t(j) M_j, \tag{14}$$

and inter-temporal changes are limited with a user specified parameter λ,

$$\left| WAMI_t - WAMI_{t-1} \right| \leq \lambda. \tag{15}$$

Smoothing conditions apply to adaptive fixed-mix and dynamic strategies, since fixed-mix is by definition smooth. The scenario tree formulation is given in Appendix A.4.

We can also specify boundary conditions. For instance, the sovereign may want to start with a WAMI close to the weighted average maturity of its legacy debt k_0 or the historical WAMI, and end at the risk horizon with a target k_T, and we set

$$WAMI_0 = k_0, \;\; WAMI_T = k_T. \tag{16}$$

Model specification

The model consists of the objective function, the flow risk constraint, and the decision variable definitions (eqns. (6)–(11) and Appendix A.1), the stock risk constraint (eqn. 13 and Appendix A.2), the stock and flow dynamics (Appendix A.3), smoothing (15), and boundary conditions (16). Interest rates are endogenously given by the scenario-dependent function (Appendix A.5). The stochastic programming model on the scenario tree is given in Appendix A.6.[8]

4 Model calibration

We calibrate the model on three real-world cases: a representative highly vulnerable euro-area country facing a liquidity crisis and entering an adjustment program, Netherlands with low debt levels and sound fiscal policies, and Italy with high debt levels but not (yet) facing a liquidity crisis. Using the crisis country we draw conclusions about the significance of optimizing debt financing and of the stock-flow tradeoffs for debt sustainability. For the Netherlands we conduct a retrospective analysis of the debt management practices of DSTA, and for Italy we assess the 2019 budget agreement and contribute to the ongoing policy debate between Italy and the European Commission. These diverse applications highlight the use of the model in informing policy decisions on sustainable public finance.

The legacy debt profile of the first case is typical of the eurozone crisis countries, and the macroeconomic variables projections reflect euro-area averages estimated by the European

[8]The model can incorporate additional practical considerations, such as foreign currency debt (Bohn, 1990), upper and lower bounds, or targets, on the issued maturities (Perold, 1984) (Zenios, 2007, sec. 3.2.2), inter-temporal smoothing of gross financing needs, and "principles-based constraints" (Guzman and Stiglitz, 2016).

institutions and are, presently, the eurozone policy targets, but this example should not be construed as representing any one country in particular. The debt-to-GDP ratio is high, signalling a potential solvency crisis, and the large amounts of legacy debt maturing early on create an imminent liquidity crisis. The 5-year forward risk-free ratet is modelled according to the Euribor forward rate. Macroeconomic variables converge to the euro-area long-term average after a 6 to 7-year business cycle with a deep recession followed by strong recovery. Standard deviations and correlation coefficients for the stochastic variables are computed from eurozone historical data. Our baseline calibration implies (ex post) sustainable long run debt dynamics, as per the current consensus for eurozone crisis countries, but we also test a *non sustainable* variant —by keeping all other country characteristics fixed and lowering long term expected growth. The model is calibrated for the period 2019–2059, extending past 2049 when all legacy debt matures.

We then calibrate the model to the data of Netherlands and Italy. In the former case we use macroeconomic, financial, and fiscal projections from the DSTA. In the later we use the Italian legacy debt and the eurozone forward rates, and for macroeconomic and fiscal variables we evaluate the projections from the IMF, the Italian Minsitry of Finance, and the EC.

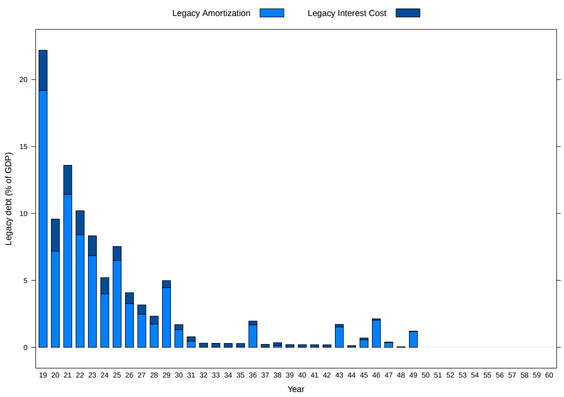
For the crisis country and Italy we follow the international institutions and optimize over a set of 3-year (short-term), 5-year (medium-term) and 10-year (long-term) bonds. For the Netherlands we follow DSTA, and optimize over a 1-year treasuries proxy for the money markets, and 3-, 5-, 10-, 20-, and 30-year bonds.

We use GAMS for data management and model setup, with solver BARON to fit the trees, and CONOPT to solve the model (GAMS Development Corporation, 2016). Runs are carried on a Dell Precision, running Linux, Intel Xeon QuadCore and 32Gb of memory. The model for adaptive fixed-mix and for simple rules is non-linearly constrained. From the special structure of the model constraints we obtain a starting solution solving a linear system of equations, and the non-linearly constrained problem is then solved with relative efficiency. The linear models on a 1024-scenario tree are solved within a minute of computer time. The nonlinear models are solved within minutes for 64-scenario trees but beyond that the computational time may take a few hours. We develop and test the model on 64 scenarios, but the applications solved in this paper use 1024 scenarios. Appendix Table B.1 summarizes solution times for different models.

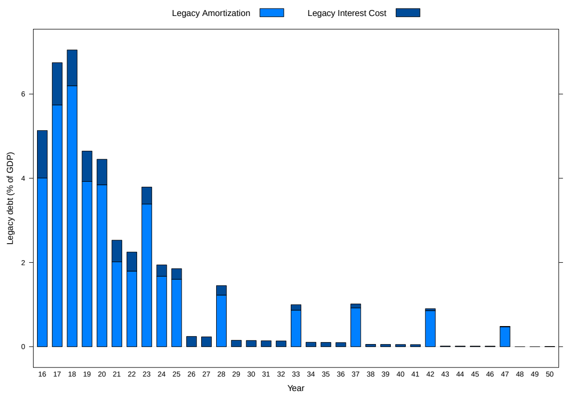
4.1 Legacy debt

We display in Figure 3 the legacy debt service profile for the crisis country (Panel A), Netherlands (Panel B), and Italy (Panel C). For the crisis country we use the 2019 legacy debt amortization, for Netherlands we use 2016 debt data to conduct a three-year retrospective analysis, and for Italy we use the 2018 data on which the 2019 budget agreement was based.[9] The initial

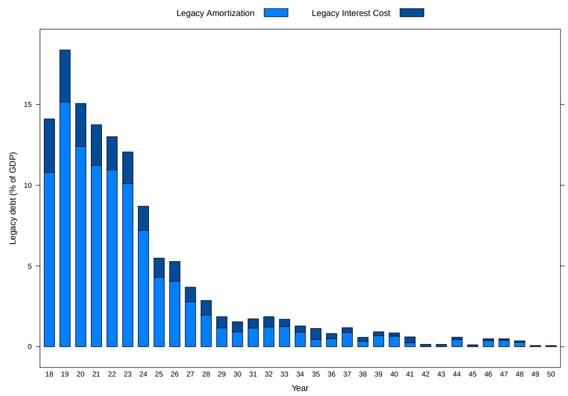
[9]For Italy we compile the data from publicly available databases, `http://en.upbilancio.it/dts-06/`, accessed on May 31, 2018. For fixed income bonds we compute the principal by summing the nominal value of bonds maturing in each year after May 31, 2018, and, likewise, for the interest rate payment of fixed rate bonds. For floating rate and inflation linked bonds, we make an assumption about future EURIBOR rate of 1% and inflation rate 2% over the risk horizon. The remaining part of the debt concerns internal liabilities which are accounted in Table 4 of the report "The Public Finances: Borrowing Requirement and Debt" issued by Bank of Italy and available at `https://www.bancaditalia.it/pubblicazioni/finanza-pubblica/index.html?com.dotmarketing.htmlpage.language=1`.



(a) Highly indebted crisis country



(b) Netherlands



(c) Italy

Figure 3 – Legacy debt service profile

debt stock for the crisis country is 100% of GDP, and is front loaded with about 20% maturing within a year. Italy has legacy debt of about 130% of GDP, with 15% maturing within a year, and Netherlands has initial legacy debt slightly below 50% of GDP, with about 6% maturing annually during the first three years, and dropping to 2% within ten years, with some payments well into the future. We point out that the legacy debt of the crisis country and Netherlands has "maturity holes", and the promised maturity payments are not approximated by the hyperbolic distribution documented, for instance, with the US debt (Hilscher et al., 2018).

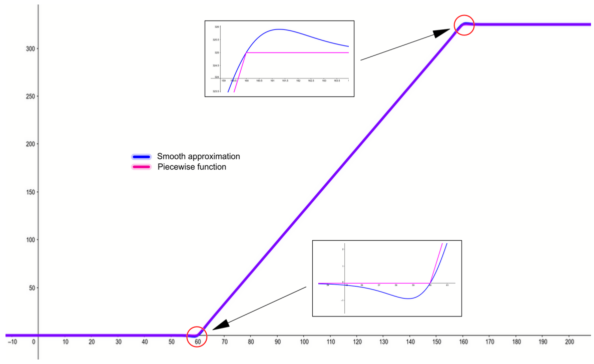
4.2 Yield curve

Risk and term premia (eqn. 4) take the functional form

$$\rho(d_t, j) = a_j + (1 + b_j)\hat{\rho}(d_t), \tag{17}$$

where a_j and b_j are maturity-specific constants, and $\hat{\rho}(d)$ is a non-linear function that captures the endogenous feedback of debt stock into interest rates,

$$\hat{\rho}(d) \doteq \hat{\rho}\left[\frac{d_{max} - d}{1 + \exp\left(d_{max} - d\right)} - \frac{d_{min} - d}{1 + \exp\left(d_{min} - d\right)}\right]. \tag{18}$$



(a) Endogenous premium

Equation (17)				Equation (18)	
Coefficient	Bond maturity (years)			Coefficient	Value
	3	5	10		
a_j	-35	0	72	$\hat{\rho}$	3.25
b_j	-0.13	0	0.13	d_{min}	60
				d_{max}	160

(b) **Parameters** for the calibration of endogenous yield curves

Figure 4 – Endogenous risk premium as a smooth approximation of a piece-wise debt function

The scenario-dependent yield curve is given in Appendix A.5.

Eqns. (17)–(18) generate yield curves that shift and twist with debt changes. If $\beta_j = 0$ for all maturities, changes in debt level cause parallel shifts. When b_j is higher (lower) for long-term than short-term debt, a debt increase causes curve steepening (flattening).

Equation (18) is a smooth nonlinear approximation of the simpler, but non-continuously differentiable, piece-wise step function illustrated in Figure 4 (Panel A). The piece-wise function specifies zero premium for debt ratios below the 60% level prescribed in the Stability and Growth Pact, which grows linearly with slope 3.25 for higher debt values up to a peak of 325 basis points when the debt ratio is greater or equal than 160%. The implicit assumption is that the sovereign is cut off the market when the spread exceeds 325bp —as has been observed with the eurozone crisis countries— and funding costs are stabilized by official sector support under strict conditionality.

The parameters estimated from the yield curve dynamics in the European periphery during the last euro area debt crisis are reported in Figure 4 (Panel B).

4.3 Scenarios

There have been significant advances in the calibration of scenario trees to match market observed moments for multiple risk factors. Notably, Høyland and Wallace (2001); Klaassen (2002) develop calibration methods for use in stochastic programming models, and Consiglio et al. (2016) develop a parsimonious model to generate risk-neutral and objective probabilities that match an arbitrary number of moments. We use the later to fit trees of forward rates,

	Growth	5-year rates	Primary balance
	Correlations		
Growth	1.00		
5-year rates	-0.20	1.00	
Primary balance	0.25	-0.03	1.00
St. Dev	0.75	0.85	0.15
Long-term mean (Crisis country)	3.5	3.25	1.00
Long-term mean (Netherlands)	3.8	3.40	0.0
Long-term mean (Italy-IMF)	2.3	3.25	1.5

Table 1: Data to calibrate the scenario trees

GDP growth, and primary balance.

We calibrate the tree to converge to the long-term expected values, matching the means, standard deviations, and correlations of Table 1. For computational tractability we calibrate a tree for five years, for a total of 256 scenarios. The long-term growth, primary balance, and 5-year forward risk-free rates for the crisis country are provided by the ESM, and each scenario is extended past the fifth year until the risk horizon using cyclical dynamics with normally distributed random shocks. For Netherlands we use statistics of GDP growth rate, primary balance, and risk-free rate based on the CPB Netherlands Bureau for Economic Policy Analysis,[10] and the tree converges to long term growth rate and primary balance projections from the same source (Hers et al., 2019, p. 31). For Italy we use baseline projections over the next five years from the 2018 IMF World Economic Outlook report, converging to their long-term averages thereafter. Standard deviations and correlations are common for the three cases, and reflect historical patterns in the euro-area. Figure 5 gives the macroeconomic, financial, and fiscal scenarios for the crisis country (Panel A), Netherlands (Panel B), and the baseline for Italy (Panel C).

[10] See https://www.cpb.nl/en/about-cpb

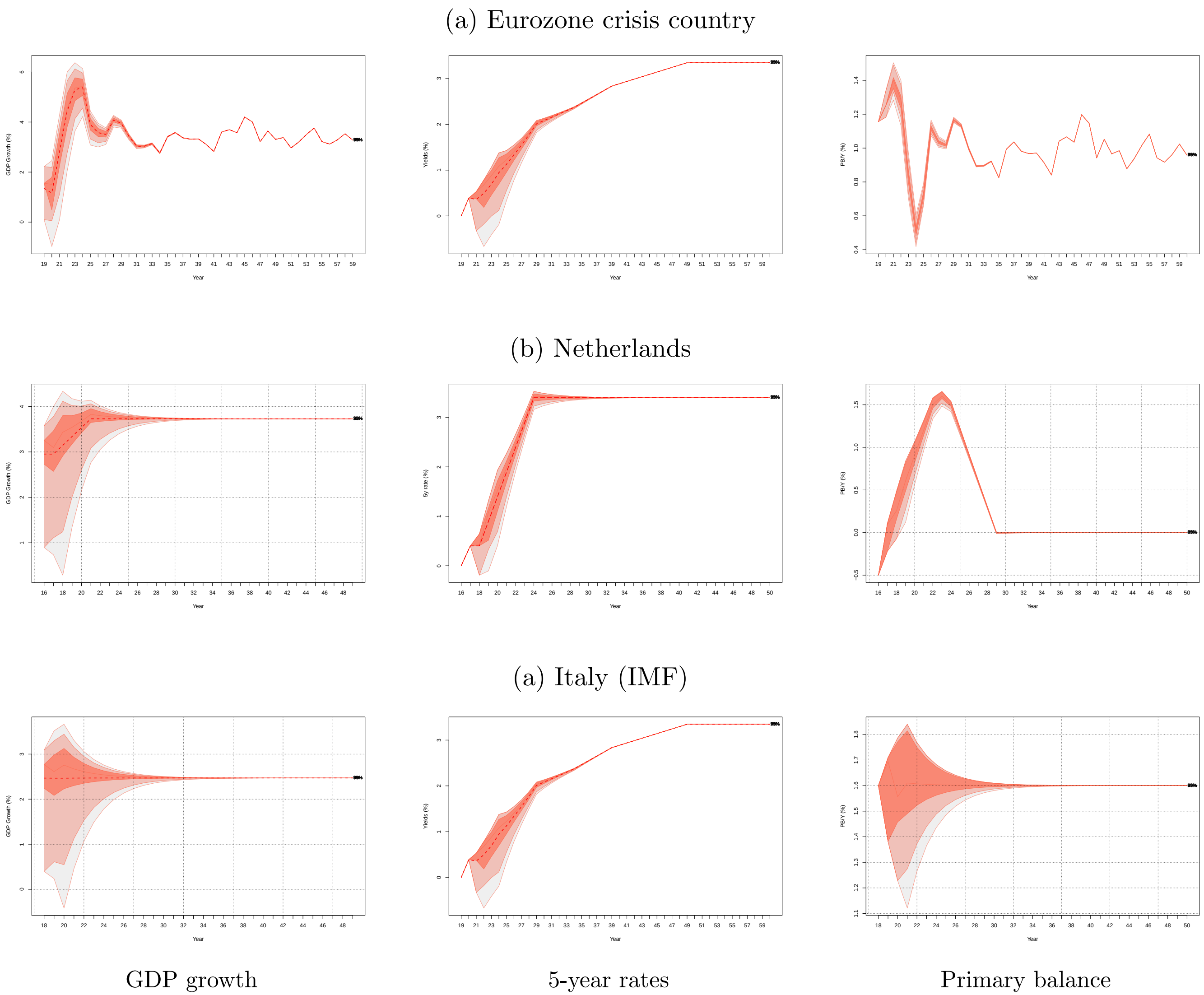


Figure 5 – Scenarios for macroeconomic (GDP growth), financial (5-year forward risk-free rates), and fiscal (primary balance) variables.

5 Model at work

We first test the model on the crisis economy. We compare dynamic, adaptive fixed-mix, and fixed-mix strategies, with several fixed-mix rules used by public debt management offices and in official sector DSA, including also consol financing (Cochrane, 2015). We highlight two findings of practical significance. First, optimized debt financing can have an impact on sustainability, and, second, significant tradeoffs between debt stock and flow are embedded in a financing strategy. We also illustrate the implications of endogenous interest rates for debt sustainability. After comparing the different strategies with the benchmarks, we settle on the adaptive fixed-mix strategy for all experiments, although, quantitatively, there is greater scope for tradeoffs with dynamic strategies.

We then test the model on the Dutch data, to assess, retroactively, the DSTA policies and financing practices. Finally, using Italian data we contribute to the debate between the newly elected Italian government and the EC on whether the 2019 budget proposal risks deteriorating Italy's public debt outlook and raise sustainability concerns.

We set the CVaR parameter $\alpha = 0.05$, and trace the cost vs risk tradeoffs as ω varies from a very large value to the smallest possible value for which the model has a feasible solution. For the eurozone crisis country we do not always impose the stock risk constraint (13) in order to better understand unsustainable debt dynamics. For Netherlands, this constraint is never binding. For Italy this is a critical constraint in all runs. We report on the effective cost of issued debt (NIP) for increasing risk tolerance ω, and provide fan charts for the stochastic debt stock and flow dynamics, showing the median and selected percentiles. We examine, in particular, whether the flow dynamics stay within the range 15% to 20% of GDP,[11] and that stock dynamics are non-exploding or are decreasing with a target pace from an initial high value.

5.1 Eurozone crisis economy

5.1.1 The significance of optimized debt financing

We compare the optimized strategies with various simple rules. Optimization theory tells us that results are always better with dynamic strategies, compared to fixed-mix or adaptive fixed-mix, and in this section we look at the economic significance of this outcome for crisis countries, and especially whether optimized strategies do materially better than the rules used in practice. We also compare with consol financing and shed some light on the practical performance of this instrument.

Figure 6 shows the cost-risk tradeoff with our optimized strategies. On the same figure we show results with three simple rules —issuing always long-term (i.e., using 0-0-100 weights for the 3y-5y-10y funding instruments), issuing always medium- (0-100-0) or short-term (100-0-0) instruments—, a *benchmark* issuing in all tenors with a fixed weighted average maturity of about 5 years (40-40-20), which is the financing strategy assumed by international institutions. We also show a consol financing, with coupon payments decaying exponentially at the rate of 2.5%.

[11] These are the currently used thresholds for official lending, drawn from empirically observed market capacity to refinance a sovereign's debt. Typically, 15% is the threshold for emerging and 20% for developed economies.

Dynamic strategies are the best performing for any risk tolerance, and the 40-40-20 benchmark, all simple rules, and the consol, underperform the adaptive and dynamic optimal ones. The cost savings from more flexible optimization increase as risk tolerance declines, with savings as large as an economically significant 1% GDP.

The performance of the simple rules is is in agreement with existing literature, given the upward sloping yield curve in the calibration. When risk tolerance is high and the sovereign focuses on cost minimization, it issues at the shortest available tenor (Arellano and Ramanarayanan, 2012; Broner et al., 2013; Conesa and Kehoe, 2015) and the optimal financing strategy coincides with the 100-0-0 rule. When the service profile of legacy debt is decreasing and risk tolerance is low, issuing at the longest available maturity is the optimal strategy (Cole and Kehoe, 2000; Conesa and Kehoe, 2014), but we see that for the non-monotonically decreasing legacy debt of the crisis country, with "holes" in the maturity structure, the 0-0-100 rule is sub-optimal. This discrepancy highlights the value-added of a normative model with fine granularity of the term structure of legacy and issued debt. Our model speaks when stylized assumptions fail to hold in practice, and for the non-boundary cases.

Interestingly, the 40-40-20 benchmark lies on the fixed-mix efficient frontier. This empirical rule, obtained through a non-analytical institutional decision making process, is the optimal fixed-mix strategy for crisis economies under current eurozone growth and primary balance targets. It also reveals an intermediate risk preference by the institutions using this strategy. We consider this an important finding: it validates that the existing strategy is prudent and optimal, in a static context, but also shows that significant improvements are possible with adaptive (and dynamic) strategies.

We find, consistently with existing literature, that consol financing is a low risk strategy, with lower risk than the simple rules, the benchmark, or the optimized fixed-mix (see "Consol bond" in Figure 6). However, it is more costly.[12] Adaptive fixed-mix and dynamic strategies can achieve the same level of risk like the consol, at lower cost by 0.5–1% of GDP. An adaptive fixed-mix frontier of optimal portfolios composed of a consol and the 3-year bond (not shown), starts from the lowest risk level achieved with the consol and converges to the 100-0-0 low cost portfolio. However, for the crisis country legacy debt with gaps in the maturity structure, not approximated well by a hyperbolic distribution (Hilscher et al., 2018), these portfolios are more expensive by 0.5–0.75% GDP than the adaptive fixed-mix portfolio of the model. They are of course more expensive than the dynamic portfolios, and slightly more expensive than the optimal fixed-mix portfolios. Overall, the model generates financing strategies that dominate, in the cost-risk space, simple rules and consol financing.

We compare now the debt dynamics implied by the benchmark rule and the adaptive fixed-mix strategy. We make a Pareto move from the point 40-40-20 on Figure 6 to the adaptive fixed-mix frontier, by moving to the left and reducing ω from 18.18% to 14.74% of GDP, and show the debt dynamics with the benchmark and the optimized strategy in Figure 7.[13] Panel A shows that this move reduces gross financing needs volatility, smoothing over gaps in the

[12]We assume that the consol is priced at zero spread over the benchmark 5-year rate, and this biases the comparison in its favor since, in general, consols will command a spread due to their very long effective maturity.

[13]Alternatively, we could move down and lower expected NIP by about 0.4% of GDP and the effective cost of debt by 40bp, but this will keep refinancing risks unchanged.

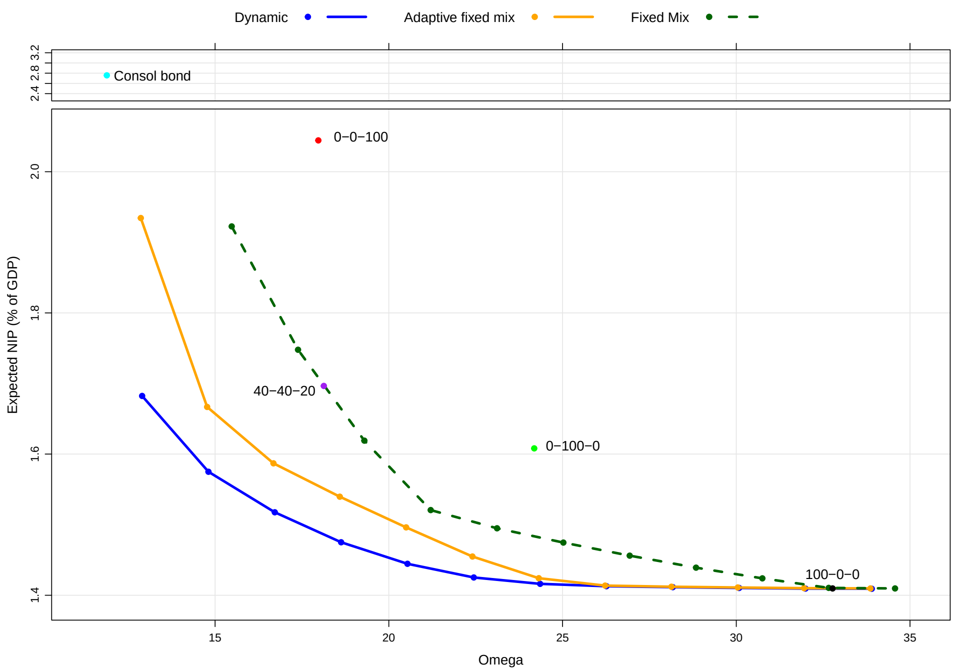


Figure 6 – Expected net interest payment and risk for different financing strategies

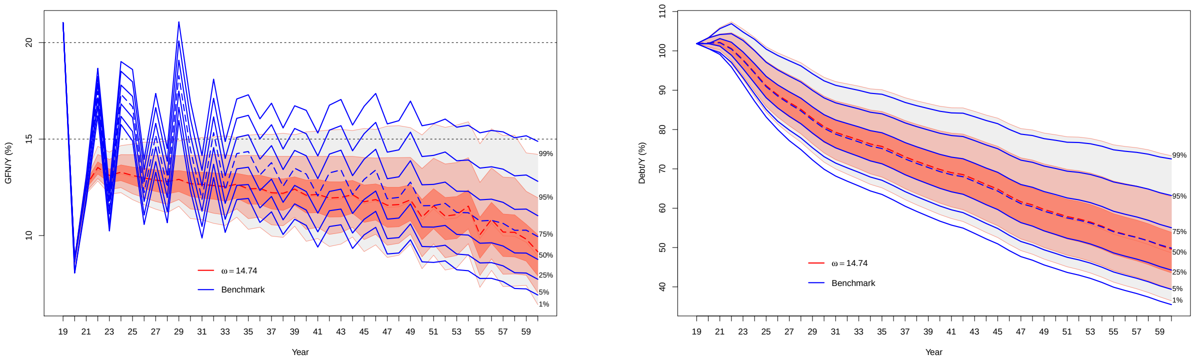
maturity structure, and the fan chart stays below the 15% threshold throughout the horizon with probability 0.95. This comes at the expense of slightly slower pace of debt reduction (Panel B).

These two panels demonstrate the efficacy of the model in addressing sustainability concerns. Large spikes in gross financing needs may derail sovereign debt management, for instance, because temporary liquidity problems may weigh on long-term solvency, and an optimal financing strategy can smooth out these spikes, making the optimized strategies especially relevant for sovereigns with debt sustainability concerns. We observe from Panel A that the optimal strategy keeps financing needs below the 15% threshold after 2019 with probability 0.99, whereas with the benchmark rule the threshold is violated with probability 0.99 six times during the first decade, and there is a 0.05 probability of more violations until 2049 when legacy debt expires.

Of course, debt-financing decisions may not always restore by themselves the sustainability of explosive debt dynamics, but optimizing certainly renders significant improvements. We address the issue of restoring sustainability with adjustments of gross financing needs in Section 6.

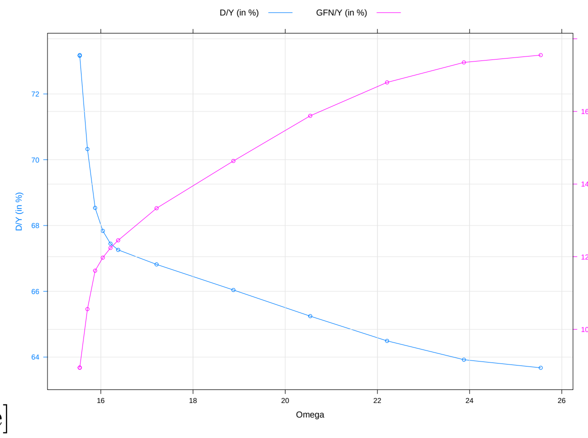
5.1.2 Tradeoff between debt stock and flow dynamics

The tension between stock and flow is a byproduct of the tradeoff between borrowing costs and refinancing risks (Bolder, 2003; Missale, 1997, 2000). In our setting, the choice of an optimal financing strategy impacts significantly the stock and flow dynamics. Going from cost minimization to the minimization of refinancing risk, we obtain more diversified financing strategies with less volatile gross financing needs, but stock declines more slowly. By varying ω from the lowest possible value for which the model has a feasible solution, to high values such that the risk constraint (7) is not binding, we quantify the tradeoff between stock and flow.

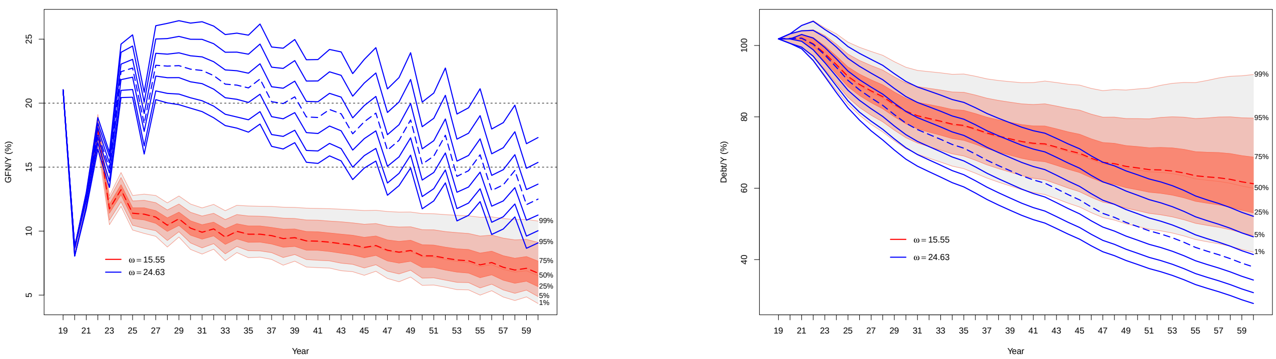


(a) Gross financing needs (% GDP) **(b)** Debt stock (% GDP)

Figure 7 – Debt dynamics with the optimal adaptive fixed-mix strategy and the benchmark



[Debt stock and gross financing needs, averaged over the tree]



(b) Gross financing needs (% GDP) **(c)** Debt stock (% GDP)

Figure 8 – Trading off stock and flow dynamics for different levels of risk tolerance

We illustrate this tradeoff in Figure 8. Panel A shows the average debt stock and average gross financing needs over the tree, for different risk tolerance values. Panels B–C convey more information with the same message showing the inter-temporal stochasticity of stock and flow dynamics, for two values of ω. Smoother gross financing needs when risk tolerance is low (Panel

B) imply smoother demands on the tax base.[14] The stock-flow tradeoff operates through the maturity and cost channel of different financing instruments, and improving gross financing dynamics using longer-dated bonds, with an upward sloping yield curve, increases the effective interest rate of debt thereby worsening the debt stock dynamics (Panel C).

5.1.3 Risk management implications of interest rate endogeneity

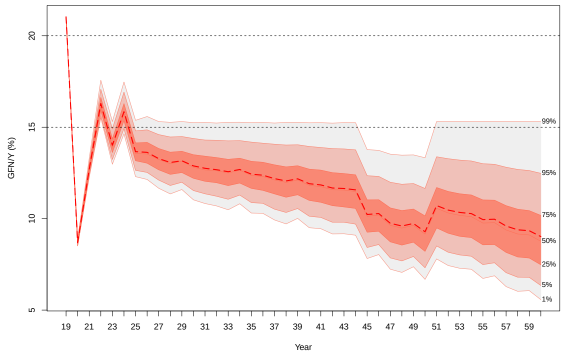
Endogenous rates can create both vicious and virtuous debt-cost cycles. Higher sensitivity of interest rates to debt creates worse conditions for a treasury, and an increase in the parameter $\hat{\rho}$ (eqn. 18) shifts the efficient frontiers of Figure 6 to the right and up (see Appendix Figure C.1). The effect is more severe on countries verging on unsustainability, and to illustrate we lower the long term growth rate of the crisis country to 3% and run the model with and without endogenous premia. The effective cost of debt is higher by 81bp with endogenous interest rates due to the premia, and to make the two runs comparable we add a fixed premium 81bp to the 5-year forward rate scenarios. In Figure 9 we show the flow and stock dynamics. Comparing Panels A and B we observe significantly higher gross financing needs with endogenous interest rates. With exogenous rates, gross financing needs stay below the threshold after the first three years, whereas with endogenous premia the sustainability threshold is breached with probability 0.5 during the first five years. Endogenous premia are more impactful on stock dynamics, with significant qualitative differences between Panels C and D. With endogenous premia, d decreases with low probability 0.25, and is in the range 60–130% GDP, whereas without endogeneity it remains in the range 70–110% GDP and it is non-increasing at the 0.5-0.75 level. The higher upside is a manifestation of the vicious cycle, and the lower downside of the virtuous cycle.

These results provide supporting evidence for a conjecture by (Bohn, 1990, p. 1218) that "if debt management affected interest rates, the qualitative nature of the government's optimization problem would change significantly". Ignoring interest rate endogeneity biases the results downwards, and DSA may lead to erroneous conclusions. Furthermore, and abstracting from moral hazard considerations, our results suggest that mitigating interest rate endogeneity would be positive both from a cost and risk management perspective, and can keep debt flow dynamics within acceptable thresholds and mitigates exploding stock dynamics. This is one of the aims of financial assistance programmes to countries facing debt sustainability problems, and our model establishes whether this aim can be achieved.

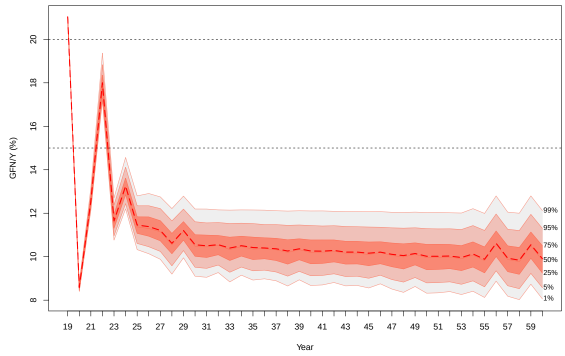
5.2 Netherlands

We evaluate the DSTA strategies of issuing debt in all tenors with maturity anchored at a "7-year benchmark and deviations", the policy of raising a minimum amount from the money market, and the policy of no undue surprises. Netherland's low legacy debt is spread over a long horizon, and under the assumed scenarios, debt-to-GDP ratio declines to a long term average of about 30% and gross financing needs are below 11% (see Table 2), so that debt dynamics are sustainable. Instead, we use the model to assess the different DSTA policies, and show the corresponding risk frontiers in Figure 10 (Frontiers A to F).

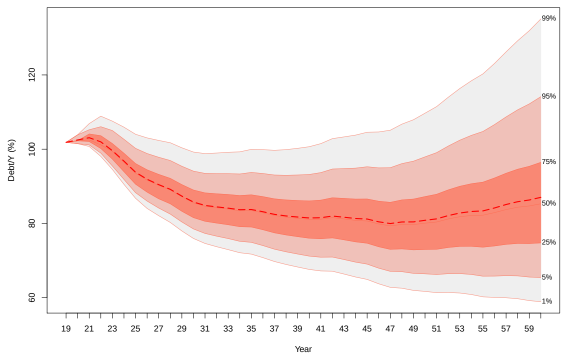
[14] This finding is consistent with Missale (1997) that "the optimal taxation approach follows from a specification of [the] tradeoff which gives all the weight to risk minimization".



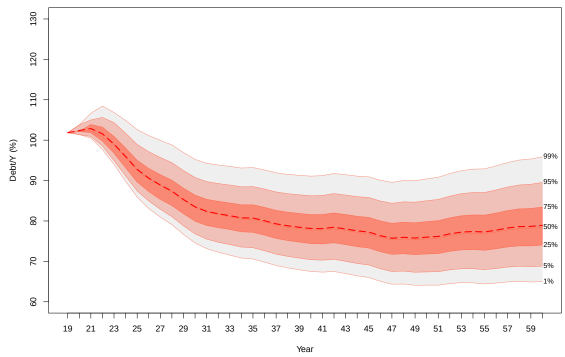
(a) *gfn* with endogenous yield curve



(b) *gfn* without endogenous yield curve



(c) *d* with endogenous yield curve



(d) *d* without endogenous yield curve

Figure 9 – Effect of endogenous yield curve on costs, risks, and debt dynamics.

To evaluate whether the current debt issuance strategy is optimal, we run the fixed-mix model imposing two additional constraints to comply with current policies: anchor the WAMI (eqn. 14) at 8.2 years,[15] and impose a lower bound of 5% of GDP issued in the money markets. With a WAMI anchor the model optimizes the combination of long maturity bonds to counterbalance the short money-market issuance and achieve the target WAMI. Frontier A shows that portfolios satisfying the benchmark policies have low NIP in the range 0.83–0.86 % GDP, with low associated risk in the range 7.1–8.1% GDP. The frontier exhibits marginal cost savings for increasing risk. The current DSTA strategy, indicated by the point marked “DSTA benchmark”, achieves minimum risk. With its low cost, and given the shape of Frontier A, there is not much room for improvement. One interesting result of the model is that it delivers the required target maturity by combining the available instruments, whereas DSTA typically issues 20- or 30-year bonds and then uses maturity swaps to achieve the target WAMI. The granularity of the model with instruments of different maturities eliminates the maturity swap.

We remove the WAMI anchor to obtain Frontier B, showing that it is possible to achieve lower costs at similar levels of risk, except for the most risk averse solutions. Cost savings increase with risk tolerance. The left most part of Frontier B intersects with Frontier A, leading

[15]This is the 7-year benchmark plus deviations of +1.2 years, based on the DSTA current portfolio composition.

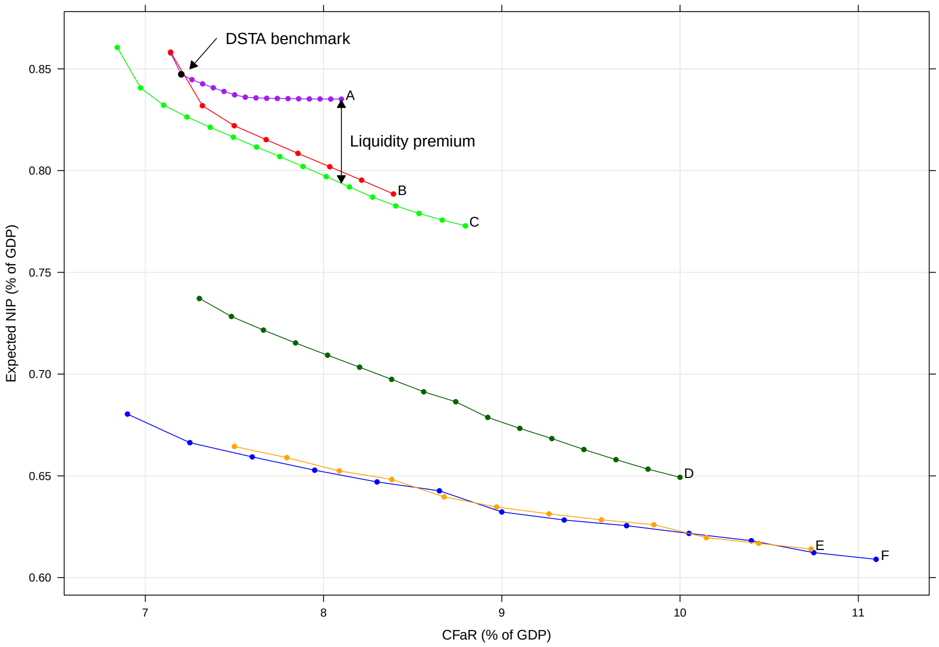


Figure 10 – Cost-risk tradeoffs for the policies of the Dutch State Treasury Agency. A- optimal fixed-mix strategy with anchored weighted average maturity at 8.2 years and 5% GDP minimum in money markets; B- optimal fixed-mix without maturity anchor; C- optimal fixed-mix without anchor or money markets minimum; D- adaptive fixed-mix with anchored weighted average maturity and money markets minimum; E- adaptive fixed-mix without maturity anchoring; F- adaptive fixed-mix without anchor or money markets minimum.

to the conclusion that the DSTA policy of deviations from the 7-year maturity anchor is ex ante cost reducing, as long as the DSTA is willing to accept some small risks.

Next, we drop the minimum money markets policy and obtain Frontier C with further slight cost reduction at any given risk level. Issuing in the money markets increases the model refinancing risks, and the model counterbalances the money markets minimum target with more expensive long term financing to achieve a given risk level. Hence, removing the money markets target we achieve the same risk with a less expensive portfolio of medium maturities. DSTA is expected to have some presence in the money markets, and it typically uses the money market to bridge funding needs, intervene in financial sector and the like, so they view a minimum size of the money market as a stabilizing influence on their policy, and the current policy of extending maturity to counter-balance the short term money market issuance has been optimal. However, the model recommends less reliance on the money markets and it documents that the DSTA money markets policy target comes with a "liquidity premium", shown in the figure as the distance from Frontier A to C. The minimum risk portfolio it the left-most point on Frontier C, with WAMI 8.6. The benchmark WAMI 8.2 corresponds to the third from the left point and deviating from the minimum risk or benchmark portfolio can reduce costs slightly.

Overall, within the current policies of issuing a fixed mix of tenors with a target WAMI and money markets presence, the DSTA benchmark is near optimal. However, the target WAMI can be achieved without maturity swaps.

We now use adaptive fixed-mix to obtain Frontier D (with anchored WAMI and minimum

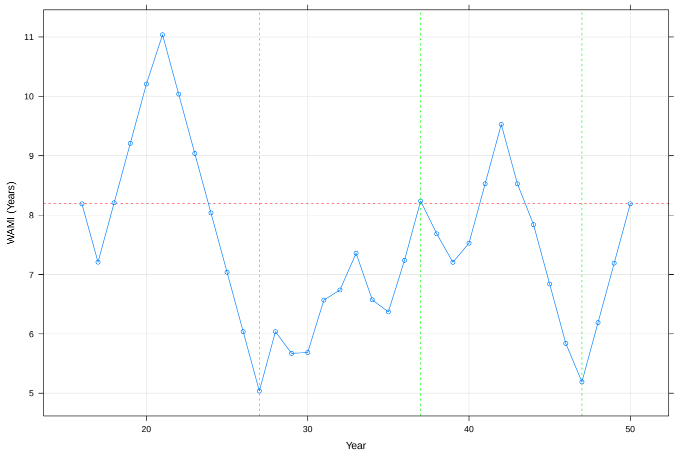


Figure 11 – Adaptive fixed-mix weighted average maturity of issued debt for the Netherlands

money markets requirement), E (without anchored WAMI but a money markets minimum) and F (without anchoring or minimum money markets). We notice cost reductions of up to 0.2% GDP without worsening risks. Interestingly, the adaptive fixed-mix strategy maintains an active money markets exposure, as evidenced by the coincidence of Frontier E (with minimum liquidity target) and F (without minimum liquidity target). The average WAMI of the least risky portfolio (left-most point of Frontier E) is about 9.6 years which is 1.4 years longer than the current DSTA strategy. Of course the adaptive fixed-mix strategy changes the portfolio WAMI. The model can limit these changes, if they are deemed to violate the "no undue surprise" policy. Importantly, however, the model computes ex ante the optimal debt issuance strategy providing *forward guidance* so that there are no surprises. Recall that the anchor of 8.2 was obtained as upward deviations from the 7-year target, following past DSTA tri-annual evaluations of its interest rate risk framework, so the agency is already engaged in limited adaptation of its issuance strategy. Frontiers E and F show that such adaptations are a good practice for risk management, but the position of these frontiers compared to the benchmark Frontier A suggests that further improvements are possible. We run the adaptive fixed-mix model, using 8.2 years as boundary conditions (eqn. 16), and obtain the WAMI of Figure 11 for forward guidance: it suggests an increase of the long-term issuance early on, to avoid rollover risks as large payments are due the first few years, and subsequent reductions when debt service payments are low, before converging to the boundary value of 8.2. The forward guidance is consistent with interest rate expectations, going long early one to take advantage of the low rates and reducing the issued maturity as rates increase.

5.3 Italy

Parliamentary elections in March 2018 brought to power a center-right coalition government of Lega Nord and the anti-establishment M5S. The coalition partners had campaigned on the promise of large deficits to boost growth, and spreads on the Italian treasury securities more than doubled from the low (pre-crisis) levels of mid-2016.[16] The proposed deficit, and unrealistic expectations on economic growth, risked triggering an excessive deficit procedure by the EC. With its high debt level, concerns were expressed that Italy was facing unsustainable debt dynamics with a potential massive debt restructuring task.[17] Following six-month negotiations with the EC, a compromise was reached on December 19th. In the words of Commissioner Vice-President Dombrovskis the solution is not ideal, but[18]

> "it avoids opening the excessive deficit procedure at this stage. And it corrects the situation of serious non-compliance with the Stability and Growth Pact. One important positive element is that the new budget is based on a plausible economic scenario."

The Commissioner pointed at the "urgent need to put public debt on a clear downwards path".

We use the model to evaluate the compromise agreement by examining if a downward debt path is possible (i.e., eqn. (13) can be satisfied with $\delta < 0$) and gross financing needs stay within the 20% threshold. The Italian case reaches the limits of our model, namely that growth is not endogenously linked to primary balance. These are exogenous inputs, and it is beyond the scope of our current work to endogenize them, because, as we have seen in the case of Italy, they are determined from auxiliary models following protracted negotiations. Instead, we use the model to assess the sustainability of debt dynamics under (i) no policy change, (ii) the growth and surplus projections of the Italian government, and (iii) the negotiated agreement.

We conduct a parametric analysis on the long-term projections of growth and primary balance. We run the model for combinations of these variables, and evaluate the probability that the tail of gross financing needs will not violate the threshold, and the debt stock will be non-increasing. In Figure 12 we show a color-coded heat map of combinations of growth and balance, with dark green denoting extremely low probability (0.01) of unsustainable dynamics, and red denoting high probability (0.85). On this map we place different combinations of long-term growth and primary balance projections. "IMF" is obtained from the IMF World Economic Outlook report for 2018, and shows the results with no policy change, with a probability 0.99 of un-sustainable debt dynamics. "Pre EC-agreement" corresponds to the government scenarios and is an improvement over the no policy change. However, this point is considered implausible by the EC, reflecting a "surplus of ambition" (Eichengreen and Panizza, 2016), and still has a high probability 0.85 of unsustainable dynamics. "Post EC-Agreement" presents further improvements, with probability 0.45 for unsustainable dynamics, but the sagacity of an agreement with about 50% chance of achieving its objectives is questionable. With additional fiscal effort the probability of sustainable dynamics can be increased to 0.85 (light green), and we analyze this issue next.

[16]For a discussion of the Italian situation see Papadia and Gonclaves-Reposo, "The higher yield on Italian government securities is becoming a burden for the real economy", Bruegel Blog Post, Feb. 5, 2019, available at http://bruegel.org/2019/02/the-higher-yield-on-italian-government-securities-is-becoming-a-burden-for-the-real-economy/, accessed March 2019.

[17]Reuters, Oct. 4, 2018, "'Complete insanity' of Italy debt plans may lead to huge restructuring - euro officials", Oct. 4., 2018, available at https://www.reuters.com/article/italy-budget-eurozone/complete-insanity-of-italy-debt-plans-may-lead-to-huge-restructuring-euro-officials-idUSL8N1WK2R6, accessed March 2019.

[18]Available at http://europa.eu/rapid/press-release_SPEECH-18-6886_en.htm, accessed March 2019.

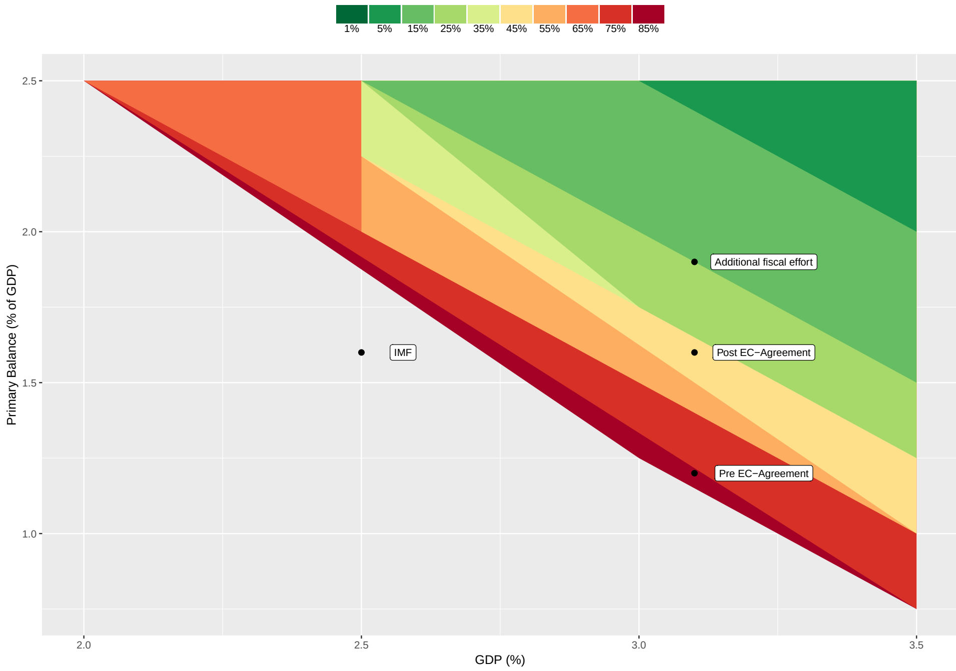


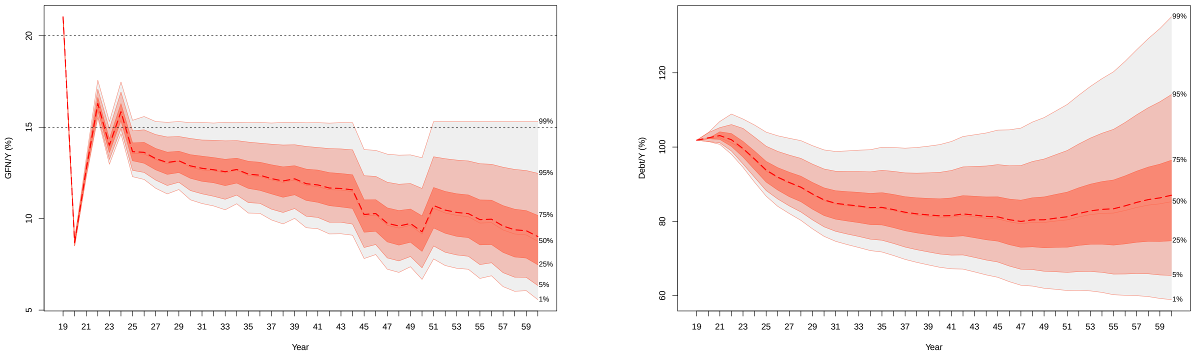
Figure 12 – Probability of the Italian debt violating debt stock or flow thresholds
Color coded are the probabilities of gross financing needs or debt stock violating the thresholds, for different combinations of primary balance and long term growth. Also shown are the results with the 2018 IMF projections for the Italian economy, the projections of the Italian government (Pre EC-agreement), and the projections agreed between the Italian government and the EC (Post EC-agreement).

6 Adjusting gross financing needs for sustainability

We extend the model to identify additional efforts that may be needed if the exogenously given primary balance process fails to deliver sustainable debt dynamics with high probability. We compute optimal adjustments to restore sustainability, but since we abstract from the government's utility function for deficit preferences, the model is silent as to whether the government should want, or be able, to pursue such efforts. This is left to negotiations between the sovereign and its lenders.

Depending on the assumed economic fundamentals of the country, there is always a threshold for $\hat{\omega}$ (eqn. 7) below which refinancing risks can not be reduced further, even with the smartest financing strategies. Likewise estimating the lowest $\hat{\delta}$ (eqn. 13), identifies policies to reduce debt at an ambitious pace that may be beyond the potential of the economy.[19] Quantifying these thresholds is an important contribution of our model, which is especially useful for debt sustainability negotiations involving the official sector and sovereigns in distress. Knowing this minimum (unavoidable) level of exposure to refinancing risks can preclude the quest for unrealistic policy targets. We extend the model to answer the question: What can be done to ensure that the desired, but seemingly unattainable, targets of refinancing risks and pace of debt reduction are reached? We identify the *hot spots* where adjustments may be required. Adjustments can mobilize a combination of domestic resources, such as higher revenues (e.g., tax proceeds, privatisations) or expenditure reductions, and external resources, such as official

[19]The model is always feasible if we allow debt to grow and ignore refinancing risks.



(a) *gfn* **(b)** *d*

Figure 13 – Debt dynamics for an economy calibrated under non sustainable conditions

sector financing or debt restructuring.

We introduce a variable u_t to denote adjustments as a proportion of GDP. If the sovereign manages to save (or to raise) an additional amount $u_t Y_t^n$ at state n of period t, then the debt financing eqn. (9) becomes

$$\sum_{j=1}^{J} X_t^n(j) + u_t Y_t^n \geq GFN_t^n. \tag{19}$$

$u_t Y_t^n$ represents the part of gross financing needs that is not financed by issuing new debt. If u_t is unbounded and carries no cost, the model will always meet financing needs through u_t, and to subordinate such adjustments we add a penalty term $\mathcal{M} \sum_{t=0}^{T} u_t$ to the objective function (6), where a large constant $\mathcal{M}$ ensures that we compute the *minimum* adjustment required to meet the desired targets on refinancing risks and stock reduction. The timing of these adjustments are the hot spots. We apply the model to the eurozone crisis country to illustrate the use of adjustments, and we estimate additional fiscal effort for Italy to attain sustainable dynamics with probability 0.85, instead of the low 0.55 with the current EC agreement.

6.1 Eurozone crisis economy

We revisit the crisis economy, with lowered long-term expected growth of 3% that creates non sustainable debt dynamics. The optimal adaptive fixed-mix strategy is now unable to reduce refinancing risks below 15% of GDP with high probability. Figure 13 shows the dynamics of gross financing needs and debt stock for this risk level. Comparing with the optimized dynamics of the baseline calibration —Figure 8 with $\omega = 15.55$— we observe a significant increase of gross financing needs and explosive growth of debt stock. There are repeated liquidity crises within the first five years and more than 0.5 probability of long term insolvency.

We compute the minimum adjustments that reduce financing risks below the threshold of 15% and show the results in Figure 14. The minimum achievable risk level is 13.7% using the adjustments shown in Panel A. Panel B shows that the new gross financing needs stay within the threshold, after the first period, with probability 0.99. The model suggests adjustments of more than 5% of GDP in the early periods for a total adjustment of 10.14% of GDP. If these were, for example, additional fiscal efforts, they may be overly ambitious (Eichengreen and Panizza, 2016), and difficult to implement due to technical and/or political reasons. Likewise, such adjustments through privatization proceeds can cause fire sales. To address such concerns we cap the adjustments at 3% of GDP per period, and show the results in Panels C-D. Although the adjustment per year is smaller, it carries on for two more years, and the total adjustment is slightly higher at 10.52% of GDP. Gross financing needs dynamics slightly worsen, but remain within the threshold with very high probability. Debt stock dynamics (not shown) are decreasing with both adjustments.

To study the effect of delays (Blanchard et al., 1990), we also run the model disallowing adjustments in the first year. Panels E-F show that the adjustments following this delay last for an additional two years, and the total 12.55% of GDP is about 2% GDP higher than those required if the country does not procrastinate. Gross financing needs worsen. This result is in agreement with Blanchard et al. (1990) that "delaying adjustment substantially affects the size of the needed policy action". These findings can guide policy decisions for public finance and operational decisions for official sector support.

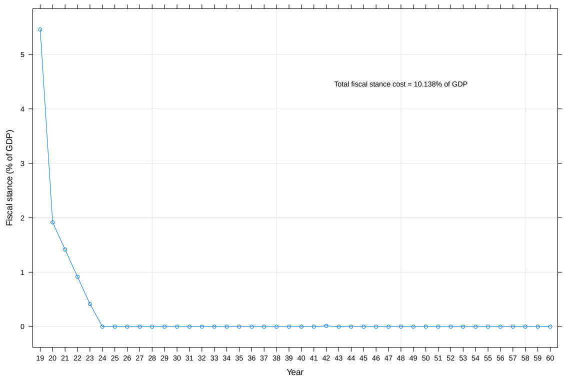
6.2 Italy

We estimate additional fiscal effort to reach debt dynamics that are sustainable with probability 0.85, i.e., see "Additional fiscal effort" in Figure 12. There are many ways to reach this point. For instance, an annualized increase of primary balance by about 0.5% GDP, for a total of 5% GDP over the next decade, will shift "Post EC-agreement" vertically up towards the dark green area. We run the model extension and find that the 0.85 confidence level can be reached with much lower extra effort of 2.5% GDP, concentrated during the first year. This may be politically untenable and it ignores the shock from such a strong fiscal stance on the Italian economy, so we cap the additional effort by a politically feasible 0.3% GDP, and find that a total fiscal effort of about 3.5% over the next twelve years —0.3% for eleven years, 0.1% in year twelve, and zero thereafter— moves the Italian debt dynamics to the target sustainability level.

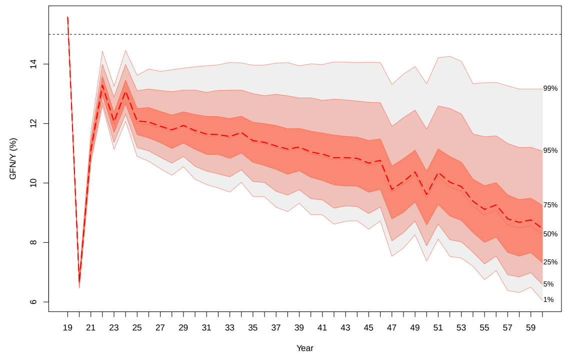
The above estimates are lower bounds. Accounting for fiscal multipliers, the fiscal effort will not shift "Post EC-agreement" up, but leftwards at an angle equal to minus the fiscal multiplier. If the multiplier is one, the country will shift from the "Post EC-agreement" at the -45 degree angle and can not escape the yellow shaded area. For a reasonable fiscal multiplier of 0.6 (Blanchard and Leigh, 2013) the country shifts towards the green shaded area, and using elementary geometry we can obtain a more accurate (higher) estimate for the extra fiscal effort. Using the model iteratively with exogenously generated combinations of primary balance and GDP growth scenarios accounting for the fiscal multipliers, we can obtain even more accurate estimates. The model can inform these difficult policy questions.

7 Further testing

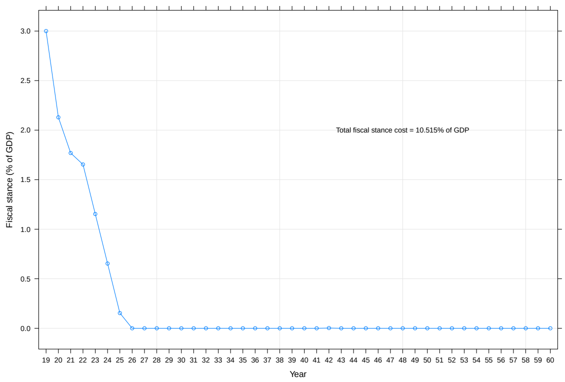
We perform three additional tests to (i) highlight the economic significance of the debt stock and flow tradeoff, (ii) shed light on the conditions under which optimization matters the most, and, (iii) illustrate the quantifiable tradeoffs between cost and risk.



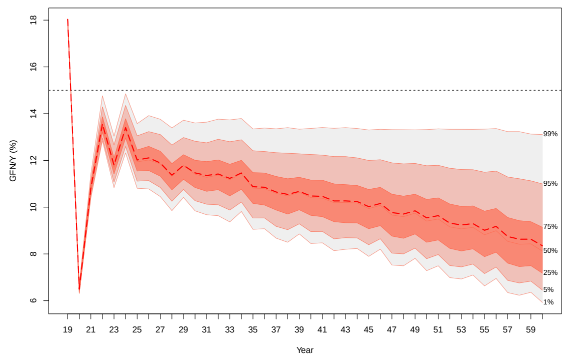
(a) Unrestricted adjustments



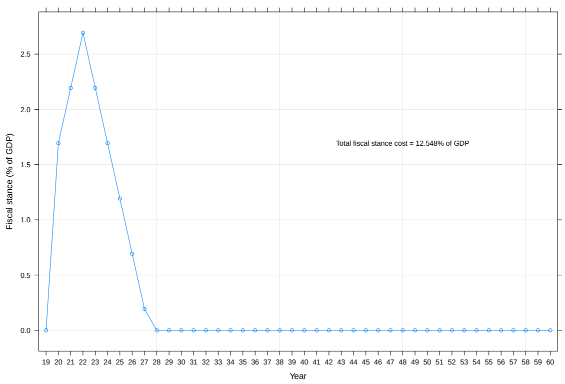
(b) *gfn* with unrestricted adjustments



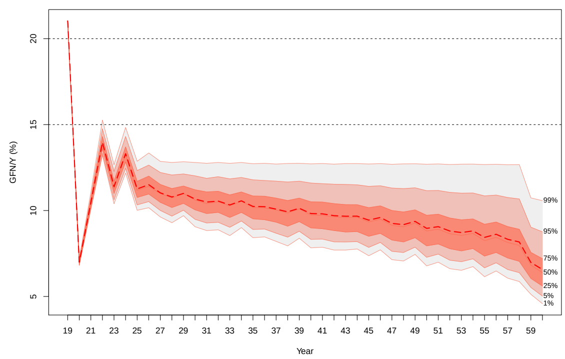
(c) Restricted adjustments



(d) *gfn* with restricted adjustments



(e) Delayed adjustments



(f) *gfn* with delayed adjustments

Figure 14 – Adjusting gross financing needs for sustainability
Hot spots and adjustments of gross financing needs required to reach an acceptable (and sustainable) refinancing risk for the eurozone crisis country calibrated under non sustainable conditions.

7.1 The economic relevance of tradeoffs

Staying well within sustainability thresholds is desirable, as it is always desirable to reduce refinancing risks, but when can this be too costly? How much should a Treasury increase the effective interest rate on its debt to reduce tail refinancing risks by 1%? Is the relationship between these variables linear? Conceptual tradeoffs are only pertinent for policy makers as long as they have material quantitative effects, and addressing these questions without a rich and realistic quantitative tool can generate misleading policy advice.

In Table 2 we report results for the three countries, and observe economically significant flow-stock tradeoffs. Reducing risk tolerance for the crisis country (Panel A) from a high level $\omega = 26$ to the lowest attainable $\omega = 15.5$ implies an increase in the debt's effective interest rate of 0.8 percentage points (pp) on average over the tree, with debt stock-to-GDP ratio increasing by 9.3pp and gross financing needs dropping by 8.4pp. To reduce the refinancing risk, WAMI (not shown) increases by about 5 years. For Netherlands (Panel B), the most significant savings from risk minimization accrue to gross refinancing needs that are reduced by about 7pp with a slight increase of effective interest rates and debt stock. For Italy we run the model with the three projections (Panels C–E). Leaving aside the cross-panel differences, that were discussed in subsection 5.3, we notice the economic relevance of the tradeoffs in each panel. Gross financing needs improve by about 8pp and debt ratios by 4-5pp, in all panels. Effective interest rate changes only by 0.3pp since Italy has high debt level and its risk premium can not be significantly reduced simply by optimizing the financing strategy.

Our analysis unveils some marked non-linearities in the risk management of financing strategies. Intuitively, when risk tolerance is very high, the flow risk constraint is barely binding (if at all). In those cases, a given reduction in ω will have relatively small effects on issuance, cost, flow, and stock dynamics, which will be mostly driven by cost minimization. In contrast, for low values of risk tolerance, the same reduction in tail risks implies larger changes in the relevant variables of the problem. These non-linear effects are particularly evident for WAMI and debt stocks. In our tests for the crisis country (Panel A), reducing risk tolerance from 26 to 19 increases effective rates marginally (0.2pp) and requires a maturity extension (not shown) of 0.8 years. However, reducing risk tolerance from 17 to the lowest possible 15.5 has a larger impact on costs (0.6pp) and almost doubles the maturities. The magnitude of these impacts depends on the calibration of the economy. The most significant effects are, naturally, for the crisis country, with an almost perfect inverse relationship between debt stock and flow.

(a) Crisis country

ω	Effective rate (%)	GFN (% GDP)	Debt (% GDP)
15.5	3.4	9.2	73.9
16	2.9	12.2	68.6
17	2.8	13.5	67.7
19	2.8	14.8	66.9
22	2.7	16.9	65.4
24	2.6	17.4	64.8
26	2.6	17.6	64.6

(b) Netherlands

ω	Effective rate (%)	GFN (% GDP)	Debt (% GDP)
7.5	2.7	4.5	33.4
9.2	2.6	6.0	33.1
10.8	2.5	7.4	33.0
12.5	2.5	8.4	32.8
14.2	2.5	9.4	32.7
15.8	2.5	10.3	32.7
17.5	2.4	11.2	32.6

(c) Italy (IMF projections)

ω	Effective rate (%)	GFN (% GDP)	Debt (% GDP)
23.6	4.4	18.1	137.2
25.2	4.3	20.2	135.7
26.9	4.3	21.5	135.0
28.6	4.3	22.7	134.3
30.2	4.2	24.0	133.8
31.9	4.2	25.2	133.4
33.6	4.2	26.4	133.0

(d) Italy (Pre EC-agreement)

ω	Effective rate (%)	GFN (% GDP)	Debt (% GDP)
20.9	4.3	16.6	129.1
22.5	4.2	18.3	127.7
24.2	4.2	19.6	126.9
25.9	4.1	20.9	126.2
27.5	4.1	22.1	125.5
29.2	4.0	23.5	125.0
30.9	4.0	24.7	124.6

(e) Italy (Post EC-agreement)

ω	Effective rate (%)	GFN (% GDP)	Debt (% GDP)
17.8	4.1	14.1	119.0
19.4	4.0	15.7	117.6
21.1	4.0	17.0	116.6
22.8	3.9	18.3	115.8
24.4	3.9	19.5	115.0
26.1	3.8	20.9	114.4
27.8	3.8	22.2	114.0

Table 2: The economic significance of risk tolerance ω on effective interest rate, gross financing needs, and debt stock

7.2 When does optimisation matter most

Optimizing the financing strategy is more significant for worse debt dynamics, as we have seen from the results with the eurozone crisis country compared to the Netherlands. We run a controlled experiment and find that optimization helps more when the stock of legacy debt is larger or its maturity shorter. Ceteris paribus, more legacy debt means worse initial conditions, and this implies both higher average gross financing needs with the associated refinancing risks, and higher funding costs due to interest rate endogeneity. The same is true when legacy debt amortizes on a shorter horizon, since this means higher refinancing pressures. In either case the efficient frontier for adaptive fixed-mix strategies shifts up and to the right (Appendix Figure C.2), and adaptive or dynamic strategies become more beneficial (Appendix Table C.1).

7.3 The cost of risk management

The model quantifies the tradeoff between borrowing costs and refinancing risks, embedded in the financing decisions, giving rise to the stock and flow dynamics used to assess sustainability. Under an upward sloping yield curve with upward sloping endogenous term premia, reduced financing risks imply higher expected interest payments as risk averse sovereigns issue long-term instruments which are more expensive (Appendix Figure C.3.) The shift from long- to short-term issuance with higher risk tolerance is in agreement with Cole and Kehoe (2000); Conesa and Kehoe (2014), leading to even higher risks when a country is in trouble, called *gambling for redemption* by Conesa and Kehoe (2015). Our results are consistent with their argument as risk tolerance increases, and our model quantifies these tradeoffs for any shape of the yield curve.

8 Conclusions

We developed a granular and flexible normative model to optimize debt financing strategies and trace the tradeoffs embedded in the choice of financing strategies. We provide new qualitative and quantitative insights on sovereign debt dynamics in the presence of sustainability constraints. The model is part the European Stability Mechanism toolkit to assess debt sustainability and repayment capacity of member states in the context of financial assistance. We apply the model to a representative eurozone crisis country, Netherlands, and Italy. From the eurozone crisis country we draw several lessons about the efficacy of the model in addressing sustainability issues, and from the Netherlands and Italy we draw policy implications for the respective countries.

We find that optimization of the issued debt maturities can have significant economic effects. For countries with unsustainable dynamics, optimizing the financing strategy may significantly improve debt dynamics, and even satisfy sustainability thresholds by smoothing excessive gross financing needs. We document the tradeoffs between debt stock and flow, and demonstrate a significant effect of endogenous interest rates. We find, as expected, that dynamic strategies dominate adaptive strategies that in turn dominate fixed-mix strategies. Interestingly the prevailing fixed-mix benchmark is near optimal in a static setting, but significant improvements are possible with adaptive and dynamic strategies. Debt financing with consol bonds is a low risk strategy but can be the most expensive one, financing with short term debt is low cost but the most risky, and the optimized dynamic and adaptive fixed-mix strategies dominate both. We rule out dynamic strategies whose volatile maturity recommendations may cause undue surprises to the markets, but the adaptive fixed-mix strategy can change smoothly and it is practical. Optimized adaptive fixed-mix can provide forward guidance to the market about future debt issuance by a sovereign. The benefits of optimizing are relatively larger when risk tolerance is lower, the stock of legacy debt is larger or its maturity shorter, and funding costs are more sensitive to debt dynamics.

A model extension identifies hot spots where adjustments may be needed so that risks can be reduced below what is possible with the assumed country fundamentals. This analysis can inform the terms of official sector lending.

We find that the Dutch State Treasury Agency is following a near-optimal financing strategy, within their policy constraints. The shift towards longer maturities has been in the right direction, although it came ex post and falls somewhat short from the optimum maturity. We quantify a liquidity premium from the policy of issuing some debt in the money markets, and show that the model can issue debt of a targeted weighted average maturity without resorting to maturity swaps. We also show how the DSTA can ex ante adjust the maturity structure using forward guidance to avoid undue surprises. For Italy we show that without policy changes the debt dynamics are unsustainable at the 0.99 confidence level, and that the 2019 budget agreement with the EC improves the debt dynamics but the probability of sustainability is only 0.55. We estimate that additional fiscal effort of about 3.5% of GDP over the next decade can render debt dynamics sustainable with a reasonably high probability 0.85.

We see at least two avenues for further work:

1. The ability of a country to create a primary surplus may be limited by "fiscal fatigue" (Ghosh et al., 2011). To address this issue we need to link primary balance scenarios with debt-to-GDP ratios. This is not simply a question of calibrating appropriate economic simulations using fiscal multipliers. We would also need to link primary balance scenarios with the endogenous debt-to-GDP dynamics within the model, internalizing the loop $X \rightarrow D \rightarrow r \rightarrow Y \rightarrow PB \rightarrow X$. This extension links growth to debt.

2. Following the ongoing debate on the merits of sovereign contingent debt, with a special focus on GDP-linked bonds and sovereign-contingent convertible debt instruments (CoCo) (IMF, 2017), it is worth incorporating such instruments into our model. This would require two extensions. First, to link the contingent payments to the appropriate risk factor from a calibrated scenario tree, such as the payments of GDP-linked bonds to GDP scenarios, or of sovereign-CoCo to the factor triggering a standstill. Second, to link the new scenario tree to the financing decisions. To do so, we must incorporate additional factors relating to contingent contracts, and model the cashflow payments of discrete contingent debt in the case of sovereign-CoCos. This requires integer programming.

The model extension for adjusting gross financing needs also raises interesting questions. If the required adjustments imply additional fiscal effort, we need to model the feedback from fiscal effort to growth. If the adjustments imply debt restructuring, we must consider its impact on the sovereign's yield curve, and take into account lenders' considerations through "principles-based constraints" (Guzman and Stiglitz, 2016). We must also strike a balance between the use of domestic and external resources. Sensitivity analysis on the model can address these questions, but endogenizing them calls for further work. Finally, the scenario tree permits model implementation with additional uncertain variables, such as inflation, exchange rates or CDS spreads, thus allowing for instruments denominated in foreign currencies or contingent debt. Our stochastic programming model on scenario trees is a versatile and effective tool for debt sustainability analysis, and provides a fertile ground for further work on relevant research and policy questions.

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Disclaimer

The views herein are those of the authors, and are not to be reported as those of the European Stability Mechanism or of Bank of Spain.

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Online Companion

A The scenario optimization model

A.1 Debt flow constraint

Following Rockafellar and Uryasev (2000, 2002), we compute aggregate CFaR (cf. eqn. 12) on the tree using the following linear system, for all states $n \in \mathcal{N}$,

$$gfn^{\diamond\diamond} = gfn^{\diamond} + \frac{1}{1-\alpha} \sum_{n \in \mathcal{N}} p^n z^n \tag{20}$$

$$z^n \geq gfn_t^n - gfn^{\diamond} \tag{21}$$

$$z^n \geq 0. \tag{22}$$

The flow risk constraint (eqn 7) becomes

$$gfn^{\diamond\diamond} \leq \omega. \tag{23}$$

Since $n \in \mathcal{N}$ is equivalent to $n \in \mathcal{N}_t$ for all $t = 0, 1, 2, \ldots T$, it follows that eqn. (21) with time indexed gfn_t^n but time independent z^n, is well defined.

Bounding the aggregate CFaR by a threshold does not guarantee that CFaR will be below the threshold at each time period. It may exceed the threshold at some time t' at the α confidence level of the distribution $gfn_{t'}$, and t' will be a *hot spot*. In several tests of the model on multiple countries we consistently found that the aggregated formulation also limits the risk at every time period. However, an unusually large spike of legacy debt could create a hot spot where the dis-aggregated measure exceeds the threshold. In this eventuality we can impose CFaR constraints at the hot spot to shape risk, by computing the CVaR for gross financing needs at t',

$$\Psi(gfn_{t'}) \doteq \mathbb{E}\left(gfn_{t'} \mid gfn_{t'} \geq gfn_{t'}^{\diamond}\right), \tag{24}$$

where $gfn_{t'}$ is the random variable of gross financing needs at t', $gfn_{t'}^{\diamond}$ is the right α-percentile, and CFaR is denoted by $gfn_{t'}^{\diamond\diamond}$. The disaggregated risk measure can also be formulated using linear inequalities, based on the work of Rockafellar and Uryasev (2002), and shape the risk profile at the hot spot as shown by Jobst et al. (2006) for credit portfolios.

A.2 Debt stock constraint

We compute changes of d on the tree by

$$\Delta_t^n = d_t^n - d_{t-1}^{a(n)}. \tag{25}$$

Δ_t^n is a random variable and to impose the debt stock constraint (eqn 13) we use again a tail risk measure, consistent with our flow risk measure, so that debt stock is non-increasing at the α confidence level. Similarly to the flow constraint, we model (13) on the tree using the linear system for all states $n \in \mathcal{N}$,

$$\begin{aligned}
\Delta_t^n &= d_t^n - d_{t-1}^{a(n)} && (26)\\
\Delta^{\diamond\diamond} &= \Delta^{\diamond} + \frac{1}{1-\alpha}\sum_{n\in\mathcal{N}} p^n y^n && (27)\\
y^n &\geq \Delta_t^n - \Delta^{\diamond} && (28)\\
y^n &\geq 0 && (29)\\
\Delta^{\diamond\diamond} &\leq \delta. && (30)
\end{aligned}$$

$\Delta^{\diamond}$ is the Value-at-Risk of debt stock changes, and $\Delta^{\diamond\diamond}$ is the conditional Value-at-Risk. With this formulation, the changes are bounded by δ at the α confidence level.

A.3 Debt dynamics

We give the accounting identities for debt dynamics, for states $n \in \mathcal{N}_t$ at time period $t = 0, 1, 2, \ldots T$. To exploit the tree structure we use the state dependent indicator function $\mathbb{1}^{\tau(n)}(j, \tau(m))$ to keep track of maturing endogenous debt,

$$\mathbb{1}^t(j,\tau(m)) = \begin{cases} 1, \text{ if instrument } j \text{ issued at } \tau(m) \text{ matures at } t = \tau(n), \text{ where } m \in \mathcal{P}(n), \\ 0, \text{ otherwise.} \end{cases} \quad (31)$$

The flow dynamics (eqn. 1) are written as

$$\begin{aligned}
GFN_t^n \quad = \quad & \underbrace{I_t^n + A_t^n}_{\substack{\text{Legacy service}\\ \text{payments}}} \quad - \quad \underbrace{PB_t^n}_{\substack{\text{Primary}\\ \text{balance}}} && (32a)\\
& + \underbrace{\sum_{m\in\mathcal{P}(n)}\sum_{j=1}^{J} X_{\tau(m)}^m(j)\, CF_t^n(j,m)}_{\substack{\text{Interest payment}\\ \text{of debt financing decisions}}} && (32b)\\
& + \underbrace{\sum_{m\in\mathcal{P}(n)}\sum_{j=1}^{J} X_{\tau(m)}^m(j)\, \mathbb{1}^t(j,\tau(m))\,.}_{\substack{\text{Principal amortization}\\ \text{of debt financing decisions}}} && (32c)
\end{aligned}$$

I_t^n is the part of $i_{t-1}D_{t-1}$ of eqn. (1) due to legacy debt and (32b) is the part due to endogenously created debt by our financing decisions. Similarly, A_t^n is the part of A_t of eqn. (1) due to legacy debt and (32c) is the part due to financing decisions.

The debt stock dynamics eqn. (2) can be expressed in terms of flows on the tree,

$$D_t^n = D_{t-1}^{a(n)} + GFN_t^n - \sum_{m\in\mathcal{P}(n)}\sum_{j=1}^{J} X_{\tau(m)}^m(j)\,\mathbb{1}^t(j,\tau(m)) - A_t^n. \quad (33)$$

Substituting (32) into (33) we link financing decisions to the effective interest rate on debt, which was the point of departure for our model,

$$D_t^n = D_{t-1}^{a(n)} + I_t^n - PB_t^n + \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j)\, CF_t^n(j,m). \tag{34}$$

Comparing this with (2) we get the *effective cost of debt* i_t at state n as

$$i_t^n = \frac{I_t^n + \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j)\, CF_t^n(j,m)}{D_t^n}. \tag{35}$$

The numerator is the net interest payment optimised in the objective function (6).

A.4 Smoothing

The absolute value function (15) is not continuously differentiable but in the context of our model we can stay in the realm of linearly constrained optimization by introducing variables v_t^+ and v_t^- to denote, respectively, increase and decrease of weighted average maturity at t,

$$v_t^+ \geq WAMI_t - WAMI_{t-1}, \text{ and } v_t^- \geq WAMI_{t-1} - WAMI_t. \tag{36}$$

Constraining these variables to be non-negative, we get their value equal to the absolute value of WAMI changes and we bound it by $0 \leq v_t^+, v_t^- \leq \lambda$.

A.5 Yield curve

Equation (4) on the tree reads

$$r_t^n(j) = r_{ft}^n + \rho(d_t^n, j). \tag{37}$$

The risk and term premia take state dependent values

$$\rho(d_t^n, j) = a_j + (1 + b_j)\hat{\rho}(d_t^n), \tag{38}$$

where the state-dependent eqn. (18) is written as

$$\hat{\rho}(d_t^n) \doteq \hat{\rho}\left[\frac{d_{max} - d_t^n}{1 + \exp\left(d_{max} - d_t^n\right)} - \frac{d_{min} - d_t^n}{1 + \exp\left(d_{min} - d_t^n\right)}\right]. \tag{39}$$

d_t^n is obtained dividing eqn. (34) by the state-dependent economic output Y_t^n.

A.6 Model specification

The complete model consists of the objective function, flow risk constraints, and decision variable definitions (6)–(11) and (20)–(23), stock risk constraints (26)–(30), flow (32) and stock (34) dynamics, smoothing constraints (14)–(15) and (36), boundary conditions (16), and the endogenous yield curve (37)–(39). Non-negativity constraints $w_t^n(j) \geq 0$ exclude short sales.

To optimize dynamic strategies we use variables $w_t^n(j)$, for adaptive fixed-mix strategies replace the time- and state-dependent variables by time-dependent and state-invariant $w_t(j)$, and for fixed-mix strategies use time- and state-invariant $w(j)$.

A.7 Adjusting gross financing needs

Equation (19) holds with inequality because the adjustment u_t is time-dependent but state-invariant, whereas the total amount raised from a given adjustment is state-dependent due to GDP. This means that, under some states of the economy, a surplus could be created. We assume that this state-dependent surplus will be used to pay down debt, and eqn. (33) is modified accordingly,

$$D_t^n = D_{t-1}^{a(n)} + GFN_t^n - u_t Y_t^n - \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j) \mathbb{1}^t(j, \tau(m)) - A_t^n. \tag{40}$$

A smoothing constraint can be imposed on u_t, similar to the WAMI smoothing. We can also allow for state- and time-dependent adjustments, u_t^n, to structure *contingent contracts* for a country under an assistance program.

B Model size and solution times

Model (No. of scenarios)	Non-linearities	Problem size Variables	Constraints	Model setup min:sec	Solution time hrs:min:sec
Stochastic program (64)	No	26,451	37,030	00:01	00:00:05
Stochastic program (1024)	No	402,771	563,878	00:20	02:25:12
Adaptive fixed-mix (64)	Yes	24,068	24,065	00:01	00:00:35
Adaptive fixed-mix (1024)	Yes	362,756	362,753	00:18	03:17:45
Adaptive fixed-mix (64)	Endogeneity	29,272	29,183	00:05	00:01:16
Adaptive fixed-mix (1024)	Endogeneity	443,224	443,135	18:59	05:52:07

Table B.1: Problem size and solution times for different models

C Supplementary results

This appendix refers to sections 5.1.3, 7.2 and 7.3. Figure C.1 shows the effect of the parameter $\hat{\rho}$ of the sensitivity of funding costs to debt stock (eqn. 18), Figure C.2 and Table C.1 show when optimisation matters more, Figure C.3 illustrates the cost of risk management.

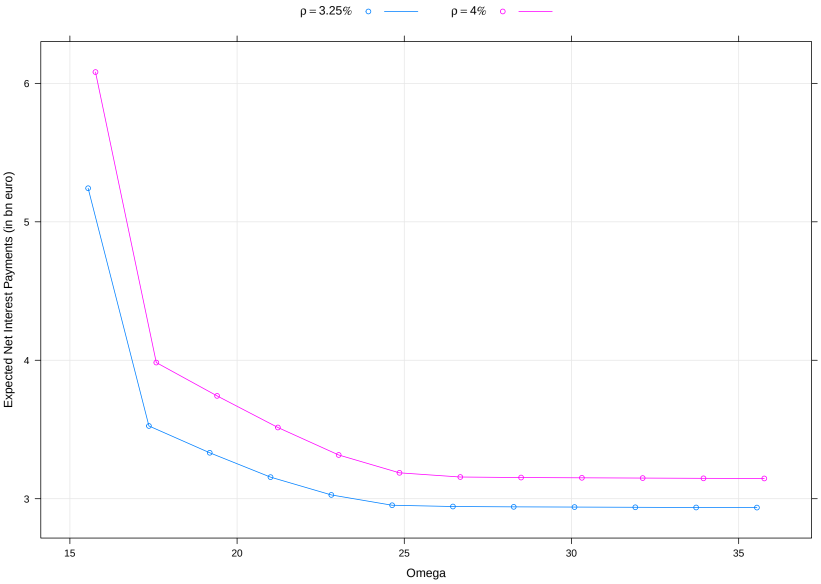
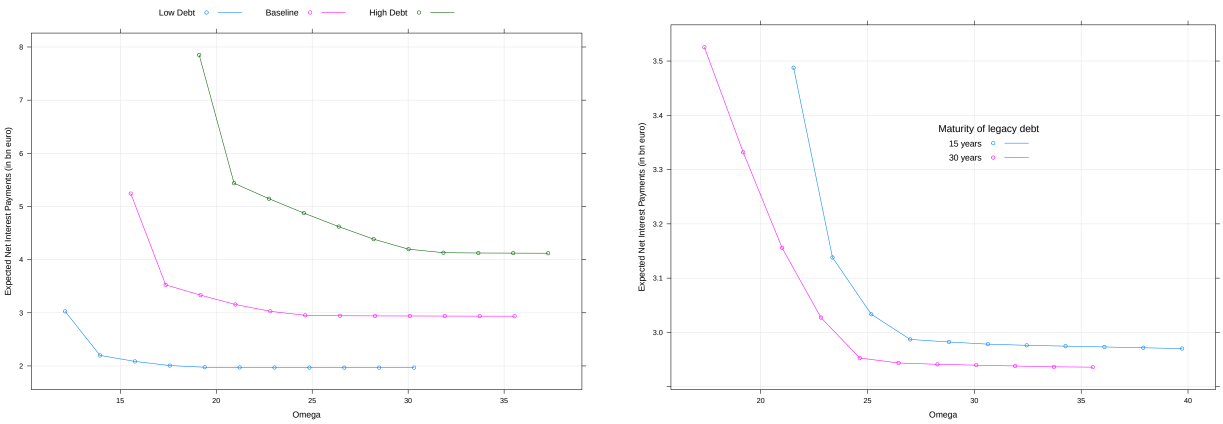


Figure C.1 – Effect of increasing sensitivity $\hat{\rho}$ of funding costs to debt stock on costs and risks.



(a) Expected net interest payment averaged over the tree, and ω for different initial stocks of debt

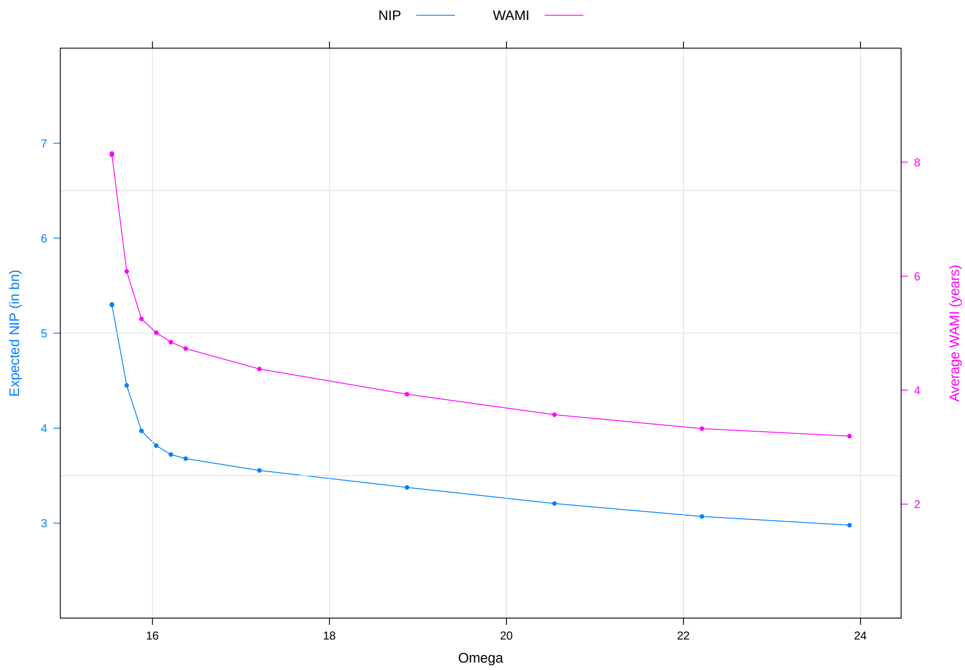
(b) Expected net interest payment averaged over the tree, and ω for different maturities of legacy debt

Figure C.2 – The relative benefits of optimization for varying initial debt stock and maturity

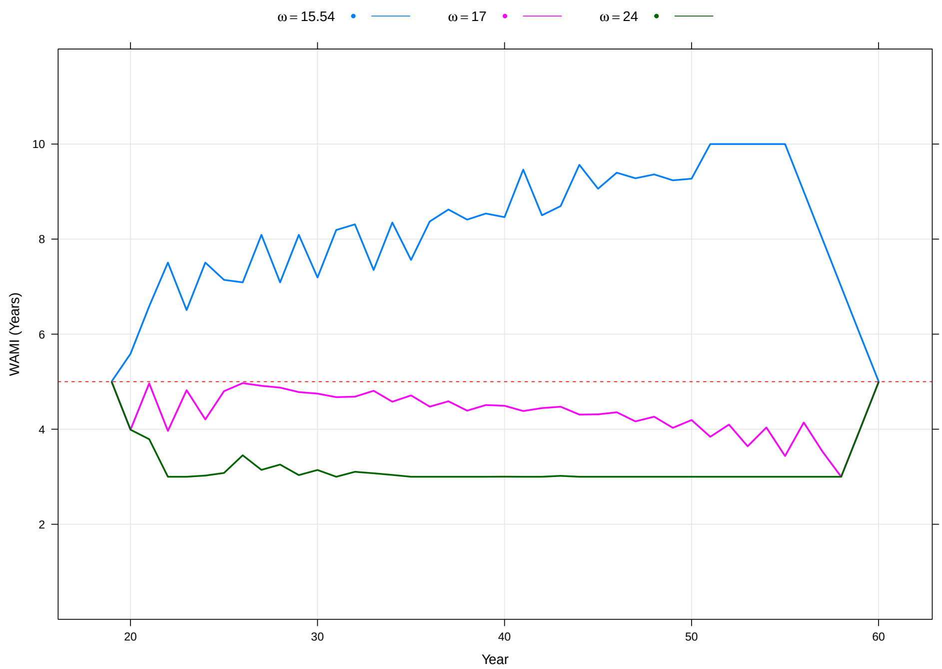
	Baseline	High debt	Short maturity
$\omega = 20$			
Adaptive fixed-mix (AFM)	3.22	5.79	3.86
Fixed-mix (FM)	3.57	8.17	5.21
Difference AFM-FM	-0.34	-2.38	-1.51
ω^*	18.19	22.82	22.74
Adaptive fixed-mix	3.41	5.09	3.16
Benchmark 40-40-20	4.12	6.15	4.20
Difference AFM-benchmark	-0.71	-1.06	-1.05

Table C.1: Expected net interest payments, averaged over the tree, for different financing strategies and calibrated economies

ω^* is the risk tolerance of the optimal adaptive fixed-mix strategies when set equal to the level of risk of the benchmark rule in each of the calibrated economies, to make for a fair comparison. The benefits from using more flexible optimal strategies are relatively larger in the presence of worse initial conditions, and the table reports the lowest expected interest costs that can be achieved with fixed-mix and adaptive fixed-mix strategies for an arbitrary risk tolerance and different initial conditions of legacy debt and its maturity. The outperformance (in terms of lower interest costs) of the more flexible strategies is more evident in the worst scenarios. The same is true when comparing adaptive fixed-mix strategies and the 40-40-20 rule for the same level of risk tolerance, ω^*.



(a) Expected net interest payments and weighted average maturity at issuance, averaged over the tree



(b) Dynamics of the weighted average maturity at issuance

Figure C.3 – The cost of risk management
Panel A shows that reduced refinancing risk implies higher expected interest payments and longer maturities under an upward-sloping yield curve. Panel B compares the evolution of WAMI over time for low, medium, and high risk tolerance.