Technology Choice and the Long- and Short-Run Armington Elasticity

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Abstract

This paper studies the international transmission of productivity shocks when the Armington elasticity is endogenized through firms' technology choice. With costly adjustment, technology choice allows for a low short-run elasticity and a high long-run elasticity. I provide analytical results which demonstrate how technology choice provides a solution to the Backus-Smith puzzle - the observed negative correlation between relative consumption and the real exchange rate. I then embed technology choice in a quantitative model of international trade with heterogeneous firms and endogenous producer entry. When the cost of adjustment is parameterized to match the correlation between relative consumption and the real exchange rate, the cross-correlation of GDP is higher than the cross-correlation of consumption, thereby providing a solution to the quantity anomaly.

JEL Classification: F41.

Keywords: Armington Elasticity, Backus-Smith Puzzle, Quantity Anomaly, Technology Choice.

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1. Introduction

This paper studies the international transmission of productivity shocks. The key innovation is that the long- and short-run Armington elasticities differ because firms are subject to costly technology choice. The Armington elasticity - which in simple settings also determines the trade elasticity - is the key parameter for the majority of open economy models of the macroeconomy.[1] In the model I develop, firms, which produce a final non-traded good for consumption, not only choose the quantities of domestic and imported inputs, but also make a technology choice - how intensively they want imported goods to be used in the production process.[2] Technology choice alters the elasticity of substitution across inputs because technology is a factor of production which cannot be fully varied in the short-run (a quasi-fixed factor). Shocks to productivity generate deviations from the long-run choice of technology and allow a low short-run Armington elasticity alongside a higher long-run elasticity.

To understand why differences between the long- and short-run Armington elasticities play an important role in the international business cycle, I use a two-country, two-good endowment economy with frictions in international financial markets (financial autarky).[3] A sufficiently

[1]The Armington elasticity is not only important for the transmission of productivity shocks. For example, it affects the international diversification of portfolio holdings (Heathcote and Perri, 2013) and the extent to which trade integration matters for monetary policy (De Paoli, 2009). Estimates of short-run elasticities are regularly below 0.5 (Hooper *et al.*, 2000), whereas long-run elasticities tend to be over 5 (Anderson and van Wincoop, 2004). See Imbs and Mejean (2015) and Feenstra *et al.* (2018) for further discussion.

[2]A natural interpretation of this structure is that it represents trade in intermediates, which accounts for over 60 percent of all international trade (Johnson, 2014).

[3]I also allow for home-bias in imports. Since home-bias can be modeled as a melting iceberg trade cost, arguably, there are two frictions in this setting (affecting international trade in financial assets and goods). Throughout the paper I abstract from iceberg trade costs and assume home-bias is part of the production technology. I address the role of international financial markets for the transmission of shocks in the quantitative analysis.

low static Armington elasticity and a realistic degree of home-bias - which allows for strong wealth effects - can resolve the Backus-Smith puzzle.[4] In response to a positive home endowment shock, consumption becomes relatively higher in the home economy, while its price, converted into a common currency, also rises.[5] Although a low value of the static Armington elasticity can resolve the Backus-Smith puzzle, it generates negative transmission - whereby the relative price of home's output (terms of trade) also improves, despite an increase in supply of the home good - and the possibility of negative cross-correlation of consumption across countries, even in the presence of positive cross-correlation of endowments.[6]

I then add a second friction - technology choice, with costs of adjustment - to the endowment setting. In the short-run, firms choose inputs with a given technology. In the long-run, firms also choose technology on a given frontier. Now consider a positive shock to the home endowment.[7] Upon impact, the home terms of trade deteriorate, and the demand for inputs is relatively insensitive to the change in relative prices. The reason for this insensitivity is the slow adjustment in technology. If, for example, technology was costless to adjust, firms would place themselves immediately on the technology frontier. Technology would then track the change in the terms of trade and the strength of this change would depend on the

[4] Efficient risk-sharing implies a positive relationship between relative consumption and the real exchange rate. However, as Backus and Smith (1993) illustrate, for OECD countries, the empirical correlation between bilateral real exchange rates and relative consumption tends to be negative.

[5] A very clear exposition of this idea is provided by Corestti *et al.* (2008). One implication of this result is that market-incompleteness alone is not sufficient to break the tight link between consumption and real exchange rates. A restriction on trade intensity and the trade elasticity is also required.

[6] At higher elasticities the cross-correlation of consumption rises above that of endowments (the quantity anomaly). There is a very small parameter range in which the cross-correlation of consumption remains positive and the quantity anomaly does not appear. However, with a static elasticity, it is no longer possible to solve the Backus-Smith puzzle.

[7] Whilst temporary changes in endowments alters the mix of inputs, at a given frontier, permanent shocks, such as episodes of trade liberalization, can also affect the technology frontier.

gap between the long- and short-run elasticities and the trade intensity. In this case, import demand would be as sensitive to a change in prices as when the static Armington elasticity is set at its (high) long-run value.

Partial adjustment in technology leads to considerably less sensitivity of demand to changes in the terms of trade and costs of adjustment act to break the tight link between relative prices and the demand for inputs. Moreover, in equilibrium, the fact that technology adjusts slowly, places a restriction on the change in the terms of trade, insofar as they always deteriorate in response to a positive home shock.[8] Since partial adjustment in technology alters the terms of trade it also changes the path of the real exchange rate and relative consumption. In particular, whilst there is an immediate fall in the real value of the home currency (consistent with a deterioration in the home terms of trade), a transitory change in the endowment can have long-lasting effects, which act to raise the value of the home currency over time. Whilst home and foreign consumption both rise in response to the shock, the strength of the change in the former (latter) becomes relatively weaker (stronger). Technology choice therefore alters the correlation between relative consumption and the real exchange rate, and the correlation of consumption across countries, relative to that of endowments.

I provide explicit analytical expressions for the correlation of consumption across countries and the correlation between relative consumption and the real exchange rate. I link both statistics to the speed of adjustment - the cost of changing technology - and the gap between the long- and short-run Armington elasticities - the elasticity when technology is given and when it is free to adjust. I first show that, with technology choice, it is possible to resolve the quantity anomaly, despite a high long-run Armington elasticity. This is because, with partial adjustment, the cross-correlation of consumption is less sensitive to the cross-correlation of

[8] In effect, with technology choice, negative transmission is ruled out, because the impact Armington elasticity needs to be sufficiently large for the path of technology to be stationary.

endowments. At the same time, the Backus-Smith puzzle is resolved, because the correlation between relative consumption and the real exchange rate is falling (towards negative one) in the cost of adjustment - the persistence in technology - and the gap between the long- and short-run elasticities.

In a next step, I embed technology choice in a quantitative model of international trade with heterogeneous firms and endogenous producer entry (Ghironi and Melitz, 2005). This framework successfully captures important features of US trade dynamics, but for my purposes, there are additional reasons to endogenize movements in output across countries in this way. First, the presence of a non-traded sector is often assumed in analysis that provides candidate solutions to the Backus-Smith puzzle.[9] With fixed costs of exporting, the mass of firms that do not trade in the export market is determined endogenously.[10] Second, with endogenous produce entry - as Liao and Santacreu (2015) show - there is a potential for strong endogenous international productivity spillovers, through returns to variety, which drive co-movement in GDP.[11] As I show analytically, strong co-movement in final outputs (endowments) is important for solving the quantity anomaly, even without technology choice.

I focus primarily on the possibility of jointly solving the consumption-correlation puzzles

[9]For example, Benigno and Theonissen (2008) consider a model with non-traded sectors and a single traded bond. In Lambrias (2019), there are sectoral news shocks, and complete markets. In effect, non-traded goods are an exteme version of trade frictions. Thus, one can alternatively ask, how effect are trade costs at solving such puzzles (Obstfeld and Rogoff, 2001).

[10]Ghironi and Melitz (2005) focus on fixed (and melting iceberg) costs of trade. Also see Alessandria and Choi (2008), who analyse the implications of sunk costs of entering foreign markets, without domestic producer entry.

[11]This channel is not present in the standard real business cycle model. With endogenous firm entry, a positive productivity shock in the home economy increases entry into the domestic market because the value of creating a firm rises. The introduction of new goods raises producers' efficiency. Since goods developed in the home economy can be exported the foreign economy it can also benefit from a greater number of goods.

discussed above. To do so, I begin by showing that once the cost of adjustment is parameterized such that the correlation between relative consumption and the real exchange rate matches the data, the cross-correlation of GDP is above that of consumption and also of exogenous productivity. In general, whilst it is always possible to generate a relatively low cross-correlation of consumption, this only occurs with unrealistically high costs of adjustment. Furthermore, when I compare the benchmark calibration of partial adjustment with full adjustment I find the response of the real exchange rate is considerably more sensitive to a change in productivity. This leads to large responses of home export participation and also foreign consumption and foreign GDP. Finally, if I shut-off producer entry and the extensive margin of exports, it is no longer possible to generate high cross-correlation of GDP.

There is other research that demonstrates why it is important to allow the short-run trade elasticity to differ from the long-run elasticity at business cycle frequencies. Drozd and Nosal (2012) focus on international relative prices. They develop a model with search costs, where sales require sluggish marketing capital. Crucini and Davis (2016) develop a model with frictions in local distribution services. They reconcile the low (high) import elasticity assumed in the international macro (trade) literatures. More recently, Drozd *et al.* (2019) model a dynamic trade elasticity, via adjustment costs in the trade share, to account for the trade co-movement puzzle.[12] In my analysis, temporary changes in productivity cause firms to deviate from their desired long-run level of technology. Combined with short-run adjustment costs, the optimal mix of domestic and foreign imports is relatively insensitive to short-run changes in relative prices. I use the model to address international consumption-correlation puzzles.

It is also important to understand the rational for modeling the difference between the

[12]A similar approach is taken in Erceg *et al.* (2006) where such a cost enters the consumption aggregator.

long- and short-run Armington elasticities with technology choice. My formulation of the Armington elasticity is similar in spirit to the analysis of León-Ledesma and Satchi (2019).[13] Their analysis considers the standard production function used in models of the real business cycle and is motivated by observed cyclical fluctuations in factor shares and evidence of a low elasticity of substitution between capital and labor at business cycle frequencies. In my case, technology choice alters the elasticity of substitution of elasticity between domestic and imported goods in terms of an otherwise-standard CES aggregator. Whilst Armington elasticities are notoriously hard to estimate, there is considerable evidence that the short-run (long-run) elasticities are lower (greater) than one.

Finally, my work relates to models of the extensive margin of exports with producer dynamics - i.e., research that builds on the original work of Ghironi and Melitz (2005). For example, Jaef and Lopez (2014) consider the role of firm entry and the extensive margin of exports in the propagation of productivity shocks across countries. They suggest that entry and exit considerations add relatively little over the standard representative firm model and that the Backus-Smith puzzle remains unresolved. Liao and Santacreu (2015) hypothesize that fluctuations in the number of goods embedded in trade flows may be one of the forces driving productivity co-movement and thereby output co-movement. Cavallari (2013) shows that the presence of imported investment goods matters for replicating these the high co-movement of output in the data when there are nominal price rigidities. Finally, Cacciatore (2014) shows how labor market rigidities affect the impact of trade integration on business cycle synchronization. Embedding technology choice within this class of model allows the elasticity of substitution to differ in the long- and short-run and addresses important puzzles in international macroeconomics.

The remainder of the paper is organized as follows. In sections 2 and 3 I describe and

[13] León-Ledesma and Satchi (2019) focus on balanced growth considerations and dynamics in the labor share.

analyze an endowment economy with technology choice. In section 4, I embed technology choice in a model of international trade with heterogeneous firms and endogenous producer entry. I then undertake a quantitative analysis. Section 5 concludes.

2. An Endowment Economy

In this section and the section that follows I develop and analyze an endowment economy.[14] There are two identical countries - home and foreign - each populated by a continuum of households with mass normalized to one. Countries trade in a country-specific good and the law of one price holds. I focus primarily on characterizing the dynamics of technology for a given change in endowments and the correlation between consumption across countries and relative consumption with the real exchange rate.

In what follows, I focus the exposition of the model on the home country, with the understanding that analogous expressions hold for the foreign country. Consumption, output, and the nominal price of the home/foreign output are denoted with h/f-subscripts. Asterisks denote foreign country variables.

2.1. Households

Households have the following intertemporal utility function over consumption, $\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$, where period utility is increasing and strictly concave and the parameter $\beta \in (0,1)$ is the discount factor. The representative household enters period t with bond holdings, b_t, and share holdings, x_t. It receives gross interest income on bond holdings, r_t, dividend income on share holdings, and the value of selling its initial share position, $d_t + v_t$. The household maximizes expected lifetime utility subject to the following budget constraint,

$$c_t + b_{t+1} + v_t x_{t+1} = y_t + \left(d_t + v_t\right) x_t + \pi_t + \left(1 + r_t\right) b_t \tag{1}$$

[14]This modelling choice is based on parsimony. The same results can be generated from a production economy with endogenous labor supply, once the Frisch elasticity is assumed to be zero.

where y_t is the home endowment. The choice of bonds and shares yields,

$$u'(c_t) = \beta E_t\left[(1+r_{t+1})\, u'(c_{t+1})\right] \quad \text{and} \quad v_t u'(c_t) = E_t\left(d_{t+1}+v_{t+1}\right)\beta u'(c_{t+1}) \tag{2}$$

Both equations in (2) are standard. They are Euler equations for bonds and share holdings.

2.1. Dynamic Import Demand

Production of non-traded goods in the home economy is subject to the following short-run production function,

$$G_t = \left[e^{\xi d(\theta_t)} y_{h,t}^{\xi} + \theta_t^{\xi} e^{\xi d(\theta_t)} y_{f,t}^{\xi}\right]^{1/\xi} \tag{3}$$

where $a_h + a_f = 1$ and $y_{h,t}$ $(y_{f,t})$ is the domestic (imported) input. In what follows, I refer to the parameter $v \equiv 1/(1-\xi) < 1$ as the short-run Armington elasticity because, in my model, it characterizes the elasticity of substitution between the domestic and imported good, for a given level of home technology, $\theta_t > 0$. Given the specification of equation (3), the short-run trade (import) elasticity is $1 - v$, and so, in this endowment setting, there is a direct mapping between elasticities.[15] At this point, it is worth noting a common objection to assuming a low Armington elasticity. A low elasticity implies that a reduction in trade costs reduces trade volumes, which contradicts the evidence on the effect of trade liberalization episodes. With technology choice, the Armington elasticity is only low in the short-run, and this critique does not apply.

Following León-Ledesma and Satchi (2019), I refer to the term $d(\theta_t)$ in equation (3) as the (log) technology frontier. I posit that the choice of θ_t is costly in units of the final good. The period profit function of the firm is therefore, $d_t = G_t - p_{h,t}y_{h,t} - p_{f,t}y_{f,t} - \chi\left(\frac{\theta_t}{\theta_{t-1}}\right)G_t$, where $\chi(\cdot) = \chi'(\cdot) = 0$ and $\chi''(\cdot) > 0$, and the variable $p_{h,t}$ $(p_{f,t})$ is the real price of the

[15] Once I allow for differences across firms, there is a distinction between these two terms which depends on the shape of the underlying productivity distribution.

home (imported) good. Although the steady state of the model does not depend on the value of $\chi''(\cdot)$, the dynamics do. However, given the solution procedure, I do not need to specify any other features of the function $\chi(\cdot)$.

Firms maximize expected discounted profits, which leads to the following first-order conditions,

$$\frac{p_{f,t}}{p_{h,t}} = \frac{G_f(t)}{G_h(t)} \tag{4}$$

and,

$$\frac{p_{f,t} y_{f,t}}{G_f(t)} = \frac{\chi(t) - 1}{\theta_t d'(t)} + \frac{\theta_t}{\theta_{t-1}} \chi'(t) - E_t m_{t+1} \frac{\theta_{t+1}}{\theta_t} \chi'(t+1) \tag{5}$$

where,

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{G_{t+1}}{G_t} \tag{6}$$

is a stochastic discount factor, $G_h(t)$, for example, is the period t marginal productivity with respect to good h, and $d'(t) < 0$. Equation (4) is standard and states that the relative price of inputs equals the ratio of the marginal productivity. Equation (5) determines the dynamics of technology. I interpret this condition as being a dynamic import demand equation, since, without adjustment costs, $\frac{y_{f,t}}{G_t} p_{f,t} = \frac{\chi(t)-1}{\theta_t d'(t)}$. In this sense, technology choice appears similar to Leibovici and Waugh's (2019) model of dynamic import demand. In their case, however, a time-to-ship friction implies that the stochastic discount factor enters the demand equation, which affects the price elasticity, and also acts as a time-varying trade wedge.

2.2. Resources and Market Clearing

The resource equation of the home economy is,

$$y_t = y_{h,t} + y^{\star}_{h,t} \tag{7}$$

where $y^{\star}_{h,t}$ is home (foreign) exports (imports). In equilibrium, $x_t = 1$ and $b_t = 0$, for all t, such that goods market clearing is,

$$c_t + \varphi\left(\frac{\theta_t}{\theta_{t-1}}\right) G_t = G_t \tag{8}$$

Finally, with financial autarky, net exports are zero,

$$0 = p_{f,t} y_{f,t} - q_t p^{\star}_{h,t} y^{\star}_{h,t} \tag{9}$$

where q_t is the real exchange rate. The world economy is characterized by 17 equations (including foreign equivalents to the conditions described above). Given home and foreign endowments, $y_t, y^{\star}_t$, equations (2)-(9), and their foreign equivalents, solve for input demands, $y_{h,t}, y_{f,t}$, domestic and import prices, $p_{h,t}, p_{f,t}$, consumption and production, c_t, G_t, technology, d_t, θ_t, foreign equivalents, and the real exchange rate, q_t.

3. Analytical Results

In this section I do two things. I show how the path of technology evolves, for a given endowment, how technology depends on the costs of adjustment, and why endogenous changes in technology alter the path of the real exchange rate. I then provide explicit expressions for the cross-country correlation of consumption and the correlation of relative consumption and the real exchange rate.

At this point I specify the following technology frontier,

$$d\left(\theta_t\right) = \frac{1}{\gamma\lambda} \ln\left[a_h^{(1-\gamma)\lambda} + a_f^{(1-\gamma)\lambda} \theta_t^{-\gamma\lambda}\right] \tag{10}$$

where $\lambda \equiv \xi / \left(\xi - \gamma\right)$ is a composite parameter and $\omega \equiv 1/\left(1-\gamma\right)$. In what follows, I refer to ω as the long-run Armington elasticity, in the sense that, absent adjustment costs, this frontier implies the following production function, $G_t = \left(a_h^{1-\gamma} y_{h,t}^{\gamma} + a_f^{1-\gamma} y_{f,t}^{\gamma}\right)^{1/\gamma}$.

3.1. The Path of Technology

In this section I discuss the how changes in the endowment affect technology choice and how technology choice affects the real exchange rate.[16]

Proposition 1 The path of the relative technology is,

$$\widehat{b}_t - \widehat{b}_{t-1} = \frac{(1-\alpha)(1-\nu)}{\chi/\alpha}\left[\left(\frac{2}{2\alpha-1}\right)\widehat{q}_t - \left(\frac{1-\nu}{\omega-\nu}\right)\widehat{b}_t\right] + \beta E_t\left(\widehat{b}_{t+1} - \widehat{b}_t\right) \tag{11}$$

where $\widehat{q}_t$ is the real exchange rate and $\widehat{b}_t \equiv \widehat{\theta}_t - \widehat{\theta}^\star_t$ is relative technology. Parameters $\alpha \equiv a_h < 1$, $\omega > \nu$, and $\chi \geq 0$ measure the (inverse) openness to trade, short and the long-run Armington elasticities, and the cost of adjusting technology, respectively.

Proof See Appendix. ∎

To understand the implications of equation (11), I use the resource constraint and demand equations to express the real exchange rate as a function of the endowment, conditional on technology,

$$\widehat{q}_t = a_\nu\left[\left(\widehat{y}_t - \widehat{y}^\star_t\right) - \alpha(1-\nu)\widehat{b}_t\right] \quad ; \quad a_\nu \equiv (2\alpha-1)/[1-2\alpha(1-\nu)] \tag{12}$$

Suppose that there is full adjustment and $\chi \to 0$. Proposition 1 implies that the change in technology is, $\widehat{b}_t = \left(\frac{\omega-\nu}{1-\nu}\right)\left(\frac{2}{2\alpha-1}\right)\widehat{q}_t$.[17] With $\omega > \nu$ and $\nu < 1$, full adjustment means a rise in the home endowment is associated with a higher real exchange rate (a deterioration in the home terms of trade). If we return to the static demand equations, as in equation (4), we find, $\widehat{y}_{f,t} - \widehat{y}_{h,t} = -\left(\frac{\nu}{2\alpha-1}\right)\widehat{q}_t - (1-\nu)\widehat{b}_t$. Technology choice therefore reflects a shift in demand, which works in the same direction as the change in the real exchange rate. In the special case of full adjustment, when period t technology and the real exchange rate are proportional, relative demand is, $\widehat{y}_{f,t} - \widehat{y}_{h,t} = -\left(\frac{\omega}{2\alpha-1}\right)\widehat{q}_t$; where ω is the long-run elasticity.

[16] In all cases below, variables with a circumflex denote deviations from steady-state values.

[17] The relationship between the terms of trade, $\widehat{\tau}_t \equiv \widehat{p}_{f,t} - \widehat{p}_{h,t}$, and the real exchange rate, $\widehat{q}_t = (2\alpha-1)\widehat{\tau}_t$, is unaffected by technology choice and the structure of international asset markets. Thus, in what follows, the results I discuss concern both variables.

Further eliminating the change in technology in equation (12), under full adjustment, the response of the real exchange rate to a change in the endowment is simply, $\widehat{q}_t = a_\omega \left(\widehat{y}_t - \widehat{y}^\star_t\right)$.

It is worth focusing temporarily on full adjustment because this is consistent with the representative firm being on the long-run technology frontier. The parameter a_ω is of interest, theoretically, because, as ω falls, and a_ω rises, there is a discontinuity in the response of the real exchange rate to a change in the endowment at $\omega = 1 - \frac{1}{2\alpha} \in \left(0, \frac{1}{2}\right)$. For ω less than (greater than) $1 - \frac{1}{2\alpha}$, the real exchange rate falls (rises) in response to a positive home endowment shock.[18] It is this former case which Corsetti *et al.* (2008) refer to as negative transmission, by which a positive endowment shock leads to an improvement in the home terms of trade, despite an increase in the supply of the home good.[19]

Empirically, it is the relative demand condition, i.e., $\widehat{y}_{f,t} - \widehat{y}_{h,t} = -\frac{\omega}{2\alpha-1}\widehat{q}_t$, that is used to generate estimates of trade elasticities. At business frequencies this elasticity can be very low. For example, Drozd and Nosal (2012) construct a volatility ratio of the demand for domestic and imported goods to the relative price, which suggest a value as low as 0.44. Leibovici and Waugh (2019) estimate a value of 0.3 using US time series data.[20] On the contrary, in the long-run, this elasticity can be very high. For example, Anderson and van Wincoop (2004) report that the import demand elasticity is generally found to lie between 5 and 10 and Romalis (2007) estimates values between 6 and 11 for Canada and the US.

[18] This parameter is also declining in ω, such that, for high Armington elasticities (i.e., high values of ω), the real exchange rate is relatively insensitive to movements in the endowment. This implies there are two values of ω which generate the same volatility in the real exchange rate and that volatility can rise without bound ω approaches $1 - \frac{1}{2\alpha}$.

[19] See Enders and Müller (2009) for evidence in favor of this possibility. It is this feature that leads to the possibility multiple equilibria (Bodenstein, 2010).

[20] Using quarterly data, Blonigen and Wilson (1999) suggest a value of 0.81 for Canada and the US, with considerable variation across sectors.

Proposition 2 A unique stationary solution for technology requires,

$$\omega > \nu \qquad \text{and} \qquad \nu > 1 - \frac{1}{2\alpha} \tag{13}$$

where $\alpha > 1/2$.

Proof See Appendix. ■

Proposition 2 shows that, with technology choice, there is a lower bound on the short-run Armington elasticity.[21] The immediate economic implication of this restriction is clear from equation (12). For a given path of technology, negative transmission is ruled out; i.e., $\nu > 1 - \frac{1}{2\alpha} \Leftrightarrow a_\nu > 0$. Put differently, given technology, a positive shock to the home endowment must be matched by higher world demand at lower prices.

The path of technology is given by the following expression,

$$\widehat{b}_t = \psi_b \widehat{b}_{t-1} + \psi_y \left(\widehat{y}_t - \widehat{y}_t^\star\right) \tag{14}$$

where,

$$\psi_b \equiv \frac{1 - \sqrt{1 - 4\left(\delta_b\right)^2 \beta}}{2\beta\delta_b} \qquad \text{and} \qquad \psi_y \equiv \frac{\delta_y}{1 - \beta\left(\delta_b \psi_b\right)} \tag{15}$$

and $\delta_b = 1/\left[(1+\beta) - (\alpha/\chi)(1-\nu)(1-\psi_\nu)\frac{(1-\nu)/(\nu-\omega)}{(1-\psi_\omega)/(1-\alpha)}\right]$, $\delta_y \equiv (\alpha/\chi)(1-\nu)(1-\psi_\nu)\delta_b$, and $\psi_j \equiv (2j\alpha - 1)/[1 - 2\alpha(1-j)]$, for $j = \nu, \omega$. Equation (14) is quite straightforward to interpret. When the cost of adjustment is high, and $\chi \to \infty$, then $\delta_b \to \frac{1}{1+\beta}$ (hence $\psi_b \to 1$) and adjustment is very long-lived. When $\chi = 0$, adjustment is immediate, and $\delta_b = 0$ (hence $\psi_b = 0$). Similarly, we can also consider how long adjustment takes for different values of the long-run elasticity, ω. For $\nu > 1 - \frac{1}{2\alpha}$, I find that $\delta_b > 0$ ($\psi_b < 1$) is decreasing

[21] In the Appendix, I show that no such restriction applies to the case with complete international financial markets. However, with complete markets, it is not possible to resolve the Backus-Smith puzzle.

(increasing) in ω, which means the higher is the long-run elasticity, the greater is the period of adjustment.

With partial adjustment in technology, the period t real exchange rate is,

$$\widehat{q}_t = a_\nu \left[\left(\widehat{y}_t - \widehat{y}_t^\star\right) - \alpha \left(1 - \nu\right) \psi_y \sum_{k=0}^{\infty} \psi_b^k \left(\widehat{y}_{t-k} - \widehat{y}_{t-k}^\star\right) \right] \tag{16}$$

where I impose $a_\nu > 0$. Since the composite parameter, ψ_y, in equation (16), is positive, there are competing effects on the real exchange rate from a one-off change in the endowment. The first term, i.e., that associated with the period t endowment, reflects positive transmission. The second term occurs because a rise in the endowment raises technology, i.e., $\widehat{b}_t > 0$, and this has a countervailing effect. The second effect is also potentially long-lived. As I discuss above, if adjustment is immediate, then $\widehat{q}_t = a_\omega \left(\widehat{y}_t - \widehat{y}_t^\star\right)$. However, if the change in technology persists beyond the effect of the change in the endowment, the initial positive change in the real exchange rate can turn negative (i.e., the real exchange rate falls below it's long-run level) and then begins to rise. This difference is a result in the shift in demand - and hence the change in relative prices - induced by technology choice.

3.2. International Correlations

In this section, I solve for the cross-country correlation of consumption in terms of endowments (the quantity anomaly) and the correlation of relative consumption and the real exchange rate (the Backus-Smith puzzle). For simplicity, I assume the endowments, $\widehat{y}_t$ and $\widehat{y}_t^\star$, are mean zero iid random variables, with unit variance, and correlation coefficient $\rho_{y,y^\star}$.

Since the endowment processes are iid, using equation (14), the variance of technology is,

$$\sigma_b^2 = 2 \left(\frac{\psi_y^2}{1 - \psi_b^2} \right) \left(1 - \rho_{y,y^\star}\right) \tag{17}$$

Higher international correlation of endowments acts to reduce the variability of technology because, if home and foreign output co-move, there can be only limited change in the relative position of technology across countries. Since we already know ψ_b and ψ_y rise and fall with the cost of adjustment, $\chi \geq 0$, as might be expected, a higher cost of adjustment also reduces volatility. Finally, since the countries are symmetric, $\rho_{b,y} = -\rho_{b,y^\star} = \left[\frac{1-\psi_b^2}{2}\left(1-\rho_{y,y^\star}\right)\right]^{0.5}$, such that technology is positively (negatively) correlated with the a shock to the home (foreign) endowment and the correlation between the home endowment and technology is falling in $\chi \geq 0$.[22]

Given a solution for the variance of technology I now discuss the main analytical results of this section.

Proposition 3 The cross-country correlation of consumption is,

$$\rho_{c,c^\star} = \frac{\sigma_{c,c^\star}}{\sigma_c^2} \tag{18}$$

where,

$$\sigma_{c,c^\star} = \frac{1-\tau+(1+\tau)\,\rho_{y,y^\star}}{2} \tag{19}$$

and,

$$\sigma_c^2 = \frac{1+\tau+(1-\tau)\,\rho_{y,y^\star}}{2} \tag{20}$$

and,

$$\tau \equiv \psi_\nu^2 + \tau_1\left[\left(\frac{\tau_1}{1-\psi_b^2}\right)+2\psi_\nu\right] \quad ; \quad \tau_1 \equiv (1-\psi_\nu)(1-\nu)\,\alpha\psi_y \tag{21}$$

Parameters $\psi_b < 1$ and $\psi_y > 0$ are defined in equation (15) and $\psi_\nu \equiv (2\nu\alpha - 1)\,/\,[1-2\alpha(1-\nu)]$.

Proof See Appendix. ■

[22]This final result occurs because, when the costs of adjustment are relatively high, so is the persistence of technology, and it is this process that allows movement from the short-run elasticity, ν, to the long-run, ω.

Proposition 3 can be best understood by appealing to full adjustment. In this case, it is easy to verify that,

$$\tau = \psi_\omega^2 \text{ where } \psi_\omega = \frac{2\omega\alpha - 1}{1 - 2\alpha(1-\omega)} \tag{22}$$

Under full adjustment, equation (18) has very specific implications for the cross-correlation of consumption, and the quantity anomaly, which requires $\rho_{c,c^\star} < \rho_{y,y^\star}$. First, for $\omega = 1 - \frac{1}{2\alpha}$ ($\tau \to \infty$), there is perfect negative correlation of consumption across countries, and $\rho_{c,c^\star} = -1$. Second, for $\omega = \frac{1}{2\alpha}$ ($\tau = 0$), there is perfect correlation of consumption across countries, and $\rho_{c,c^\star} = 1$. Finally, as ω rises above $\omega = \frac{1}{2\alpha}$, the cross-correlation of consumption falls, such that, as $\omega \to \infty$, then $\rho_{c,c^\star} \to \rho_{y,y^\star}$. This means the parameter range in which the quantity anomaly can be resolved requires a low Armington elasticity, and in particular, $\omega < \frac{1}{2}$ ($\tau < 1$). Such a parameterization also implies the possibility of negative transmission - that the real exchange rate will fall in response to a rise in the home endowment - depending on the extent of home-bias.

Although it is possible to solve the quantity anomaly by appealing to a low static Armington elasticity there is a problem with this approach. In particular, despite the fact that $\rho_{c,c^\star}$ is increasing in $\rho_{y,y^\star}$, the relationship is only linear when $\omega = \frac{1}{2}$. In general, the cross correlation of consumption changes with the cross-correlation of output in the following way: $sign\left(\frac{\partial^2}{\partial \rho_{y,y^\star}} \rho_{c,c^\star}\right) = sign\left(\psi_\omega^2 - 1\right)$. Thus, when $\omega < \frac{1}{2}$, the cross-correlation of consumption is highly sensitive to a change in the cross-correlation of output, and a small reduction in $\rho_{y,y^\star}$ leads to a large fall in $\rho_{c,c^\star}$. Since, when $\omega < \frac{1}{2}$, we already know $\rho_{c,c^\star} < \rho_{y,y^\star}$, it means, even for relatively high levels of $\rho_{y,y^\star}$, it is possible that $\rho_{c,c^\star} < 0$. This makes it clear why the quantity anomaly is hard to solve: it requires a low Armington elasticity and a relatively high correlation of output across countries.

Now consider technology choice. In this case, it helps to temporarily suppose the cross-correlation of endowments is zero, because then $\rho_{c,c^\star} = \frac{1-\tau}{1+\tau}$. Given the definitions of the composite parameters τ and τ_1, it is straightforward to verify that, since ψ_b and τ_1 are increasing in $\chi \geq 0$, the correlation of consumption across countries is decreasing in the costs of adjustment (i.e., $\partial\rho_{c,c^\star}/\partial\tau < 0$). Intuitively, this makes sense. If the cost of adjustment is zero ($\chi = 0$), firms are at the long-run technology frontier, and the Armington elasticity is such that, $\rho_{c,c^\star} = (1-\psi_\omega^2)\,/\,(1+\psi_\omega^2)$. If we then assume $\omega > \frac{1}{2}$, we find $\rho_{c,c^\star} > \rho_{y,y^\star}$. When changes in technology are costly ($\chi > 0$), the effective short-run elasticity falls. Insofar as the short-run elasticity, $\nu > 1 - \frac{1}{2\alpha} \in \left(0, \frac{1}{2}\right)$, is relatively low, the cross-correlation of consumption will lie below that of endowments.

Since it is possible to determine that $\rho_{c,c^\star} < \rho_{y,y^\star}$, for some $\chi > 0$, it remains to determine how the cross-correlation of consumption reacts to changes in the cross-correlation of endowments. This only requires knowledge of the composite parameter τ, which is determined by ψ_ν^2, which is larger than unity, and $\tau_1\left[\left(\frac{\tau_1}{1-\psi_b^2}\right) + 2\psi_\nu\right]$, where $\tau_1 > 0$ and $\psi_b \in (0,1)$. With costly adjustment, therefore, there are offsetting effects, since $\frac{\tau_1}{1-\psi_b^2}$ has a positive effect on τ, and ψ_ν has a negative effect. It is this second term that matters because, whilst $\tau > 1$, it must lie below ψ_v^2, unless there is a very high cost of adjustment, in which case the outcomes converge. What this means, at a practical level, is that it is possible to resolve the quantity anomaly at lower levels of $\rho_{y,y^\star}$. Moreover, this can also be achieved with a high long-run Armington elasticity.

Proposition 4 The correlation between relative consumption and the real exchange rate is determined by,

$$\rho_{c^R,q} = \frac{\sigma_{c^R,q}}{\sigma_{c^R} \times \sigma_q} \tag{23}$$

where,

$$\sigma_{c^R,q} = \left(\frac{2\alpha\nu - 1}{2\alpha - 1}\right)\sigma_q^2 + 2a_\nu\left\{\left[1 - \frac{\tau_1/\left(1-\psi_b^2\right)}{1-\psi_\nu}\right]\frac{\tau_1}{1-\psi_\nu}\right\} \tag{24}$$

and,

$$\sigma_{c^R}^2 = 2\tau \quad \text{and} \quad \sigma_q^2 = 2a_\nu^2 \left\{ 1 - \left[2 - \frac{\tau_1 / \left(1 - \psi_b^2\right)}{1 - \psi_\nu} \right] \frac{\tau_1}{1 - \psi_\nu} \right\} \tag{25}$$

and τ and τ_1 are defined in Proposition 3 and $\rho_{y,y^\star} = 0$ is assumed.

Proof See Appendix. ■

The relationship between relative consumption and the real exchange rate is given by $\widehat{c}_t^R = \left(\frac{2\alpha\nu-1}{2\alpha-1}\right) \widehat{q}_t + \alpha(1-\nu)\widehat{b}_t$. Under full adjustment, since $\widehat{b}_t = \left(\frac{\omega-\nu}{1-\nu}\right) \frac{2}{2\alpha-1} \widehat{q}_t$ (obtained by setting $\psi_b = 0$ in equation (14)), I find, $\widehat{c}_t^R = \left(\frac{2\alpha\omega-1}{2\alpha-1}\right) \widehat{q}_t \Leftrightarrow \sigma_{q,c^R} = \left(\frac{2\alpha\omega-1}{2\alpha-1}\right) \sigma_q^2$. The first condition is discussed in Corsetti *et al.* (2008). As they show, when $\omega < \frac{1}{2\alpha}$ (which lies between one-half and unity, since α also lies between one-half and unity), it is possible to resolve the Backus-Smith puzzle, such that, $\sigma_{q,c^R} < 0$.[23] The explanation for this possibility is related to the transmission of shocks under financial autarky. In the context of the discussion above, however, when $\frac{1}{2} < \omega < \frac{1}{2\alpha}$, although the the cross-correlation of consumption is positive, it is less than the cross-correlation in output. Thus, whilst it is possible to solve the Backus-Smith puzzle together with the quantity anomaly (i.e., impose $\omega < \frac{1}{2}$) this requires a very high cross-correlation in endowments.

With technology choice, the extent to which the Backus-Smith puzzle is resolved depends on the speed of adjustment. Since $\nu > 1 - \frac{1}{2\alpha} \in \left(0, \frac{1}{2}\right)$, the covariance term, $\sigma_{c^R,q}$, can be negative when $\nu < \frac{1}{2\alpha}$. However, there is an additional reason for a negative covariance. This effect is driven by the term in braces in (24). What matters is the extent to which this term is negative, which is the case when $(1-\nu)\alpha\left(\frac{\psi_y}{1-\psi_b^2}\right) > 1$. Since $\nu < 1$ and $\alpha < 1$, this condition amounts to requiring ψ_b be relatively close to one, which only occurs when the cost of adjustment is high, and when the effective short-run elasticity is low. There is,

[23] Because the correlation coefficient in equation (23) is, in general, a complicated expression, I focus discussion on the covariance term - equation (24) - and the extent to which this is negative.

however, a second point to this. The backward-lookingness of technology is higher when the long-run elasticity is high (i.e., $\psi_b < 1$ is increasing in ω). Thus, the greater the gap between the long- and short-run elasticities, the stronger will be the impact of technology choice on the covariance of relative consumption and the real exchange rate.

4. Quantitative Model with Costly Trade

In sections 2 and 3, I demonstrated that it is possible to have high a long-run Armington elasticity (consistent with the implications from trade liberalization episodes) alongside a low short-run Armington elasticity (which is consistent with the negative co-movement between consumption and the real exchange rate at business cycle frequencies).

In this section, I endogenize output movements across countries by embedding technology choice in a production economy. I focus on exogenous changes in aggregate productivity as the source of business cycle fluctuations. I do so using the framework proposed by Ghironi and Melitz (2005). In this model, the entry and exit of firms in the export market (the extensive margin of exports) creates an endogenous mass of non-traded firms and a wedge between the average domestic price for a good, and it's export price, when evaluated in the same currency. In addition, allowing for endogenous producer entry generates endogenous productivity spillovers, via returns to variety.

4.1. Model Overview

In the home economy there is representative household which supplies labor, L_t, to domestic firms, and consumes a final, non-traded good, c_t, according the the function $\sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$, which has standard properties.[24]

The final good is produced from intermediate goods according to a production function

[24] Again, I focus on the home economy for the purposes of exposition. The foreign economy is symmetric.

defined over a continuum of differentiated goods:

$$G_t = \left[e^{\xi d(\theta_t)} \left(\int_{\omega \in \Omega_t} y_{h,t}(\omega)^\sigma \, d\omega \right)^{\xi/\sigma} + \theta_t^\xi e^{\xi d(\theta_t)} \left(\int_{\omega^\star \in \Omega_t^\star} y_{f,t}(\omega^\star)^\sigma \, d\omega^\star \right)^{\xi/\sigma} \right]^{1/\xi} \tag{26}$$

where $y_{h,t}(\omega)$ is variety $\omega \in \Omega_t$ of the home good, $y_{f,t}(\omega^\star)$ is variety $\omega^\star \in \Omega_t^\star$ of the the foreign good. In this setting, the parameter $0 < \sigma < 1$ determines the substitutability between varieties (within-country substitutability) and determines the markup of price over marginal cost. As in the endowment case, $d(\theta_t)$ is defined in equation (10), and it determines a time-varying cross-country substitutability.

There are a continuum of firms in the home country, each producing a differentiated variety. Labor is the only factor of production and there is an aggregate productivity shifter, denoted a_t. Prior to entry, firms are identical, and face a labor-intensive entry cost, denoted $f_e > 0$. Upon entry, each firm draws its productivity level, $z \geq 1$, from a common distribution $G(z)$. Exporting incurs an additional, labor-intensive, per-period fixed cost, denoted $f_x > 0$. Thus, once a firm knows its productivity level, it may produce only for the domestic market, or produce for the both domestic and export markets. Each period firms face a constant probability of exit, $\delta < 1$.

There is a mass n_t of firms, with average productivity $\overline{z} \equiv \left[\int_1^\infty z^{\sigma-1} dG(z)\right]^{1/(\sigma-1)}$, and a mass of $n_{x,t}$ exporting firms, with average productivity $\overline{z}_{x,t} \equiv \left[\frac{1}{1-G(z_{x,t})} \int_{z_{x,t}}^\infty z^{\sigma-1} dG(z)\right]^{1/(\sigma-1)}$. The mass of exporters is such that, $n_{x,t} = [1 - G(z_{x,t})]\, n_t$, where $1 - G(z_{x,t})$ is the ex-ante probability that a firm exports and $z_{x,t}$ is the minimum (cut-off) level of productivity for export participation. Finally, there is an unbounded mass of prospective entrants. The decision to enter is based on the present discounted value of the expected future profits with entrants at time t only starting to produce at time $t+1$. Since all firms face a constant probability of exit, a proportion of new entrants will never produce, and in period t, the mass of home firms is, $n_t = (1-\delta)(n_{t-1} + n_{e,t-1})$, where $n_{e,t}$ is the mass of entrants.[25]

4.2. Parameterization and Calibration of the Steady-State

The steady-state of the model is close to Ghironi and Melitz (2005) and I following much of their parameterization. There are two main differences. First, period utility is assumed to be, $u(c_t, L_t) = \ln c_t - \frac{\eta}{1+1/\varsigma} L_t^{1+1/\varsigma}$, where ς is the Frisch elasticity. I choose η such that $L = 1$. Second, I do not include iceberg trade costs, and this impacts the expenditure share of domestic goods in final output. Nevertheless, there is a simple mapping between trade costs and home-bias in final production; in particular, $\tau^{1-\nu} = \frac{1-\alpha}{\alpha}$, where $\nu = \sigma$. The value of τ is set at 1.3 in Ghironi and Melitz (2005), and since $\sigma = 3.8$, this implies setting $\alpha = 0.6758$.

Table 2 presents the parameters used to determine the steady-state of the model and their respective targets.

===== **Table 2 here** =====

The time period for the model is a quarter. The Frisch elasticity is set at 0.74, well-within the range of standard estimates. With a long-run elasticity of substitution set at 3.8 the implied steady-state price markup is 35.7 percent.

Firms draw their productivity from a Pareto distribution, where $G(z) = 1 - z^{-\kappa}$, where κ is the curvature parameter. The Pareto assumption implies average productivity is, $\overline{z} = \{\kappa/[\kappa - (\sigma-1)]\}^{1/(\sigma-1)}$, and average exporter productivity is, $\overline{z}_{x,t} = \overline{z} \times z_{x,t}$. It also means I can express the share of exporters as, $n_{x,t}/n_t = z_{x,t}^{-\kappa}$, where $\kappa > (\sigma - 1)$. Firm heterogeneity is measured by the standard deviation of log plant sales, which, in the model,

[25] A description of the non-linear equations of the model is provided in Appendix B.1.

is given by $1/\left[\kappa-(\sigma-1)\right]$. I match the value of 1.67 reported in Bernard *et al.* (2003) by using the shape parameter of the productivity distribution set to $\kappa = 3.4$.

The fraction of firms that produce is determined by the cost of exporting, f_x. I set this parameter such that 21 percent of firms export, as reported in Bernard *et al.* (2003), which implies a productivity premium for exporting firms of 58.2%. I normalize f_e to unity. Given these values, $\alpha = 0.6758$ implies an expenditure share of 73.3 percent and an import-GDP ration of 22.5 percent.

Aggregate productivity is assumed to follow an autoregressive process,

$$\lambda_{t+1} = A\lambda_t + \varepsilon_{t+1} \quad ; \quad \varepsilon_{t+1} \sim N(0, V) \tag{27}$$

where $\lambda_t = \left[\widehat{a}_t, \widehat{a}_t^{\star}\right]^T$ and $\varepsilon_{t+1} = \left[\varepsilon_{t+1}^{a}, \varepsilon_{t+1}^{a^{\star}}\right]^T$ is the vector of shocks. The parameter values assigned to the matrix of autoregressive terms, A, and the covariance terms, V, are taken from Backus *et al.* (1994), and represent a well-known benchmark. In this case, the cross-correlation of exogenous productivity is given by 0.31.[26]

4.3. International Correlations

In this section I report international correlations.[27] First, I consider those discussed above; namely, the international cross-correlation of consumption and GDP and the correlation between relative consumption and the real exchange rate. To do so, requires two more parameters; the short-run elasticity, $\nu < \omega$, and the cost of adjustment parameter, $\chi > 0$. From Proposition 2, the more open the economy, the lower the admissible value of the short-run elasticity. Based on this restriction, and the parameterization of home-bias, I consider a lower value of 0.35. Solving the Backus-Smith Puzzle, under full adjustment, however, also

[26]This correlation is with the HP filter set at 1,600. All moments reported below are HP filtered.

[27]In this section, I focus primarily on model-based correlations. Impulse response functions (for partial and full adjustment) to a one-off home productivity shock are reported in Appendix B.2.

requires a not-to-high elasticity. Based on this, I consider an upper value for the short-run elasticity of 0.5.[28]

Figure 1 plots the correlations of interest against a range of values for $\chi \in [0, 5]$.

===== **Figure 1 here** =====

In both panels of Figure 1, the left-hand side (when $\chi = 0$) corresponds to the model of Ghironi and Melitz (2005), in that, the elasticity of substitution is 3.8; that is, both economies are at their long-run technology frontier.[29] The left (right)-hand side panel considers the lower (upper) value for the short-run elasticity of substitution.

In both cases, we can see that it is possible to solve the Backus-Smith puzzle, in that, given the structure of shocks to productivity, the correlation between relative consumption and the real exchange rate is negative. For the lower value of the short-run elasticity, only a small value of χ (less than 0.5) is required. With a higher value for the short-run elasticity a larger cost of adjustment is required to generate negative correlation. This suggests that relatively small changes in the short-run elasticity can have large effects on the potential resolution of the Backus-Smith puzzle, which is consistent with using a static Armington elasticity. Despite incorporating technology choice, the basic structure of the model, and transmission mechanism for shocks, is unchanged. What matters here is that it is possible to solve the Backus-Smith puzzle whilst allowing for a high long-run elasticity.

[28]Empirical estimates, as discussed above, are at, or below, 0.5 (Hooper *et al.*, 2000). In terms of calibrated or estimated DSGE models, Lubik and Schorfheide (2005) find a mean estimate of 0.43 and Enders and Müller suggest 0.32. Mandelman *et al.* (2011) and Rabanal *et al.* (2011) both assume 0.62 in their analysis. In all of these papers there is no distinction between long- and short-run elasticities.

[29]Figure 1 plots variables measured in terms of welfare. That is, with the variety effect present. For the cross-correlation of GDP and consumption, this point has only limited bearing on the results.

The second clear result from Figure 1 is that it is also possible to solve the quantity anomaly. This also requires a lower value for the short-run elasticity of substitution. I addition, it requires relatively slow adjustment. From the left panel, the cross-correlations of consumption is lower than GDP at values of $\sigma_{c^R,q} < 0.7$, which is consistent with the data. For example Corsetti *et al.* (2012), report $\rho_{c^R,q} = -0.71$ alongside $\rho_{c,c^\star} = 0.6$ and $\rho_{y,y^\star} = 0.68$, for the US *vis-à-vis* remaining OECD countries. In the right-hand side panel, the cross-correlation of consumption and GDP both rise with the cost of adjustment and it is not possible to jointly solve both the Backus-Smith puzzle and the quantity anomaly, unless the cost of adjustment is unreasonably high.

In Figure 2, I calculate the correlations for the extensive margin of exports ($n_{x,t}$) for both values of the short-run elasticity.

===== **Figure 2 here** =====

Figure 2 shows that the extensive margin of exports has little correlation with the real exchange rate when the cost of adjustment is zero (long-run frontier). At levels of adjustment that solve the Backus-Smith puzzle, however, this correlation rises to around 0.4. Alessandria and Choi (2019) suggest there is very little correlation between exporters and the real exchange rate and Fitzgerald and Haller (2019) suggest that although export participation does rise in response to (favorable) changes in the real exchange rate, the implied elasticity is considerable less than for changes in tariffs. A second feature of these results is that the correlation between export participation and overall economic activity is falling in the cost of adjustment. The correlation between the extensive margin of exports and GDP is around 0.8 when the cost of adjustment are zero, which is potentially problematic. For the short-run low elasticity case, and at costs of adjustment which resolve the Backus-Smith puzzle, this correlation is somewhat lower. Finally, with adjustment costs, there is very low cross-correlation in extensive margins.

One final point is worth considering. Why not simply lower the elasticity of substitution to a value that is consistent with the lower of the two short-run values considered in this section. The answer is quite simple. Doing so produces extreme results, consistent with the right-hand side of the panels in Figures 1 and 2. For example, in Figure 1, the international correlation of consumption is around -0.5, and in Figure 2, the correlation of the extensive margin of exports and the real exchange rate is 1.

4.4. Robustness

In this section, I consider the robustness of the results discussed above. First, I change the structure of international financial markets. In particular, I suppose there is either a complete set of internationally traded claims or there is a single foreign-currency traded bond. Second, I change the parameterization of the model under financial autarky. In this case, I first raise the long-run elasticity of substitution to 7.9, the value reported in Drozd and Nosal (2012). I then allow for persistent, internationally correlated shocks to the labor-wedge (or, taste shocks, similar to that considered in Stockman and Tesar (1995)). Finally, I alter preferences, along the lines suggested by Greenwood *et al.* (1988), and eliminate the wealth effect in labor supply.[30]

The benchmark case, reported in Table 2, below, refers to financial autarky, with $\nu = 0.35$ and $\chi = 1.032$, such that the correlation between the real exchange rate and relative consumption matches that in the data.

===== **Table 2 here** =====

[30] Details of these changes are discussed in the Appendix B.3. In Appendix B.4. I also present results for the model without producer entry and export participation decisions.

Table 2 shows that the impact of changing financial markets is considerable. Relative to financial autarky (*FA*), economies with either complete markets (*CM*) or a traded bond (*Bond*) generate strong (weak) international cross-correlation of consumption (GDP) and the correlation between relative consumption and the real exchange rate is close to unity. Baxter and Crucini (1995), among others, show that differences between bond economies and those with complete markets depend crucially on the persistence of exogenous shocks. Under the parameterization of shocks I consider, cross-correlations of consumption and GDP should be similar. However, correlations for exporters do change across specifications, with the correlation between the real exchange rate and the mass of exporters falling when there is a single traded bond.

In a second step, I change the parameterization of the benchmark model. I find that raising the long-run elasticity of substitution from 3.8 to 7.9 has little effect on the macro-correlations and slightly reduces the correlation between the real exchange rate and the mass of exporters. This is not too surprising, since the value of 3.8 is already considerably above values often assumed in open economy models of the business cycle (of around 1.5). Eliminating the wealth effect in labor supply has stronger effects; whilst it lowers the correlation between relative consumption and the real exchange rate the cross-correlation of consumption rises. Finally, when I add shocks to the labor wedge, I find there is negative cross correlation of consumption, which is partly explained by the strong persistence of the process. In this, and the former case, it is also possible to offset these results, by lowering the costs of adjustment, such that the correlation between relative consumption and the real exchange rate remains at the benchmark value of -0.71.

5. Conclusion

This paper studies the international transmission of productivity shocks when the Armington elasticity is endogenized through firms' technology choice. With technology choice, the

Armington elasticity is low in the short-run and high in the long-run. I show that it is possible to resolve the Backus-Smith puzzle - the observed negative correlation between the real exchange rate and cross-country consumption - and the quantity anomaly - the observation that the cross-correlation of GDP across countries is higher than the cross-correlation of consumption.

Appendices A

Appendix A.1. (Derivation of Firms Dynamic First-Order Condition)

The standard problem, $\max\limits_{\{y_{h,t},y_{f,t},y_t\}} G_t - p_{h,t}y_{h,t} - p_{f,t}y_{f,t} + \lambda_t \left\{ \left[a_h^{1-\gamma} y_{h,t}^{\gamma} + a_f^{1-\gamma} y_{f,t}^{\gamma} \right]^{1/\gamma} - G_t \right\}$, implies, $p_{h,t} = a_h^{1-\gamma}\left(\frac{y_{h,t}}{y_t}\right)^{\gamma-1}$ and $p_{f,t} = a_f^{1-\gamma}\left(\frac{y_{f,t}}{y_t}\right)^{\gamma-1}$, such that, $\frac{p_{f,t}}{p_{h,t}} = \left(\frac{a_f}{a_h}\frac{y_{f,t}}{y_{h,t}}\right)^{\xi-1}$. Now consider the problem with technology choice program but no adjustment cost. It is, $\max\limits_{\{y_{h,t},y_{f,t},G_t,\theta_t\}} G_t - p_{h,t}y_{h,t} - p_{f,t}y_{f,t} + \lambda_t \left\{ \left[e^{\xi d(\theta_t)} y_{h,t}^{\xi} + \theta_t^{\xi} e^{\xi d(\theta_t)} y_{f,t}^{\xi} \right]^{1/\xi} - G_t \right\}$. This implies, $\theta_t^{\xi}\left(\frac{y_{f,t}}{y_{h,t}}\right)^{\xi-1} = \frac{p_{f,t}}{p_{h,t}}$ and $1 = -s(\theta_t)\,\theta_t^{\xi}\left(\frac{y_{f,t}}{y_{h,t}}\right)^{\xi}$, where $s(\theta_t) = 1 + \frac{1}{\theta_t d'(\theta_t)}$ and $d'(\theta_t)$ is assumed. The problem with technology choice will replicate the standard problem if I assume, $d(\theta_t) = \frac{1}{\rho\lambda}\ln\left[a_h^{(1-\rho)\lambda} + a_f^{(1-\rho)\lambda}\theta_t^{-\rho\lambda}\right]$.

For the problem with technology choice and adjustment costs, there is an additional first-order condition,

$$\frac{\partial \ln G_t}{\partial \ln \theta_t}\left[1 - \varphi\left(\frac{\theta_t}{\theta_{t-1}}\right)\right] = \frac{\theta_t}{\theta_{t-1}}\varphi'(t) - E_t m_{t+1}\frac{\theta_{t+1}}{\theta_t}\varphi'(t+1) \tag{28}$$

I use equation (28) to generate the condition reported in the main text as equation (5) by noting that the production has the following properties: $G_t = \frac{\partial G_t}{\partial y_{h,t}}y_{h,t} + \frac{\partial G_t}{\partial y_{f,t}}y_{f,t}$ and $\frac{\partial G_t}{\partial \theta_t} = d'(\theta_t)\frac{\partial G_t}{\partial y_{h,t}}y_{h,t} + \left[\frac{1}{\theta_t} + d'(\theta_t)\right]\frac{\partial G_t}{\partial y_{f,t}}y_{f,t}$.

Appendix A.2. (Proof of Proposition 1)

In what follows a circumflex denotes the deviation of a variable from its steady-state value. The home dynamic import equation is equation (5) in the main text. There is foreign equivalent. The difference between the home and foreign technology (the relative position) is given,

$$\begin{aligned} \Delta\widehat{b}_t \;=\;& \frac{1/\chi}{1-s}\left[\left(\widehat{p}_{f,t} + \widehat{y}_{f,t} - \widehat{G}_t\right) - \left(\widehat{p}^{\star}_{h,t} + \widehat{y}^{\star}_{h,t} - \widehat{G}^{\star}_t\right) + \frac{(1-\nu)(1-\omega)}{\omega-\nu}\alpha\widehat{\theta}^R_t\right] \\ & + \beta\Delta E_t\widehat{b}_{t+1} \end{aligned} \tag{29}$$

where $\widehat{b}_t \equiv \widehat{\theta}^R_t$ and, in the steady-state, $p_f = p_h$, such that, $s_t = s = -\alpha/(1-\alpha)$.

Short-run import demand curves are,

$$\widehat{p}_{f,t} + \widehat{y}_{f,t} - \widehat{G}_t = (1-\nu)\left[\widehat{p}_{f,t} - \left(\widehat{\theta}_t + \widehat{D}_t\right)\right] \tag{30}$$

and,

$$\widehat{p}^\star_{h,t} + \widehat{y}^\star_{h,t} - \widehat{G}^\star_t = (1-\nu)\left[\widehat{p}^\star_{h,t} - \left(\widehat{\theta}^\star_t + \widehat{D}^\star_t\right)\right] \tag{31}$$

where $\widehat{D}_t = -\left(1/\left(1-s\right)\right)\widehat{\theta}_t$. Finally, the home production function is,

$$\begin{aligned}
\widehat{G}_t &= \left(\frac{1}{1+\theta^{\xi/(1-\xi)}}\right)\left(\widehat{y}_{h,t} + \widehat{D}_t\right) + \left(\frac{\theta^{\xi/(1-\xi)}}{1+\theta^{\xi/(1-\xi)}}\right)\left[\widehat{y}_{f,t} + \left(\widehat{\theta}_t + \widehat{D}_t\right)\right] \\
&= \alpha\widehat{y}_{h,t} + (1-\alpha)\,\widehat{y}_{f,t}
\end{aligned}$$

where the second condition follows from the result that $\theta^{\xi/(1-\xi)} = (1-\alpha)/\alpha$, when $p_f = p_h$. Since this equation is the same as when technology choice is absent, we have a standard result in that the difference between the production functions, i.e., $\widehat{G}_t - \widehat{G}^\star_t$, implies the real exchange rate and the terms of trade are related in the following way: $\widehat{q}_t = (2\alpha - 1)\,\widehat{\tau}_t$. Using this final result, along with equations (29)-(31), generates the equation for relative technology reported in Proposition 1.

Appendix A.3. (Proof of Proposition 2)

I proceed in two stages. First, I solve the model conditional on the path of technology. Second, I derive equation (14), which solves for the path of technology. Under financial autarky, I solve for variables $\left\{\widehat{b}_t, \widehat{\tau}_t, \widehat{q}_t, \widehat{c}^R_t\right\}$ using the following conditions,

$$\Delta\widehat{b}_t = \left(\frac{1-\alpha}{\chi}\right)(1-\nu)\,\alpha\left[2\widehat{\tau}_t - \left(\frac{1-\nu}{\omega-\nu}\right)\widehat{b}_t\right] + \beta\Delta E_t\widehat{b}_{t+1} \tag{32}$$

$$\widehat{y}_t - \widehat{y}^\star_t = \left[1 - 2\alpha\,(1-\nu)\right]\widehat{\tau}_t + \alpha\,(1-\nu)\,\widehat{b}_t \tag{33}$$

$$\widehat{q}_t = (2\alpha - 1)\,\widehat{\tau}_t \tag{34}$$

$$\widehat{c}_t^R = (2\alpha\nu - 1)\,\widehat{\tau}_t + \alpha(1-\nu)\,\widehat{b}_t \tag{35}$$

where equation (35) is the financial autarky condition and $\widehat{c}_t^R \equiv \widehat{c}_t - \widehat{c}_t^\star$ is relative consumption. Equations (33)-(35) can be used to produce equation (12) in the main text, which is a solution for the real exchange rate, conditional on technology. Eliminating the terms of trade from the equation for the path of technology implies,

$$\widehat{b}_t = \delta_b \widehat{b}_{t-1} + \beta\delta_b E_t \widehat{b}_{t+1} + \delta_y\left(\widehat{y}_t - \widehat{y}_t^\star\right) \tag{36}$$

where,

$$\begin{aligned}
\delta_b &= \left\{(1+\beta) - (1-\nu)\left[\frac{\alpha\left(\frac{1-\alpha}{\chi}\right)(1-\nu)}{1-2\alpha(1-\nu)}\right]\left[\frac{1-2\alpha(1-\omega)}{\nu-\omega}\right]\right\}^{-1} \\
\delta_y &\equiv \left[2\frac{\alpha\left(\frac{1-\alpha}{\chi}\right)(1-\nu)}{1-2\alpha(1-\nu)}\right]\delta_b
\end{aligned}$$

and $\widehat{y}_t$ and $\widehat{y}_t^\star$ are both mean zero iid random variables. I solve for technology, in equation (36), by using the method of undetermined coefficients. I guess the solution takes the form, $\widehat{b}_t = \psi_b \widehat{b}_{t-1} + \psi_y\left(\widehat{y}_t - \widehat{y}_t^\star\right)$. Applying expectations at period $t+1$ implies, $E_t\widehat{b}_{t+1} = \psi_b\widehat{b}_t$ and $E_t\widehat{b}_{t+1} = \psi_b^2\widehat{b}_{t-1} + \psi_b\psi_y\left(\widehat{y}_t - \widehat{y}_t^\star\right)$. As such,

$$\widehat{b}_t = \psi_b \widehat{b}_{t-1} + \psi_y\left(\widehat{y}_t - \widehat{y}_t^\star\right) = \delta_b\widehat{b}_{t-1} + \beta\delta_b\left[\psi_b^2\widehat{b}_{t-1} + \psi_b\psi_y\left(\widehat{y}_t - \widehat{y}_t^\star\right)\right] + \delta_y\left(\widehat{y}_t - \widehat{y}_t^\star\right)$$

which provides the following solutions for ψ_b and ψ_y,

$$\psi_b = \delta_b + \beta\delta_b\psi_b^2 \quad \text{and} \quad \psi_y = \beta\delta_b\psi_b\psi_y + \delta_y$$

These conditions are reported in (15) in the main text. For $\psi_b < 1$, we require, $\delta_b < \frac{1}{1+\beta}$, or,

$$\frac{1-2\alpha(1-\omega)}{1-2\alpha(1-\nu)}\frac{1}{\nu-\omega} < 0$$

since $\alpha\left(\frac{1-\alpha}{\chi}\right)(1-\nu)^2 > 0$. Imposing $\nu < \omega$ requires the numerator and denominator to be positive, which is reported in Proposition 2.

I now compare financial autarky to complete markets. In this case, I make two replacements to the conditions above. Relative demand is,

$$\widehat{y}_t - \widehat{y}_t^\star = \nu \widehat{\tau}_t + (1-2\alpha)\left(\nu \widehat{q}_t - \widehat{c}_t^R\right) + (1-\nu)\left[2\alpha(1-\alpha)\right]\widehat{b}_t$$

and,

$$\widehat{c}_t^R = \widehat{q}_t$$

is the standard risk-sharing condition. Note that, under full adjustment, $\widehat{b}_t = \left(\frac{\omega-\nu}{1-\nu}\right) 2\widehat{\tau}_t$, and so, $\widehat{q}_t = \frac{2\alpha-1}{\omega+(1-\omega)(2\alpha-1)^2}\left(\widehat{y}_t - \widehat{y}_t^\star\right)$. In this case, the real exchange rate can never fall when the home endowment rises. Following the same steps as before, I find,

$$\widehat{b}_t = \eta_b \widehat{b}_{t-1} + \beta\eta_b E_t \widehat{b}_{t+1} + \eta_y\left(\widehat{y}_t - \widehat{y}_t^\star\right) \tag{37}$$

where,

$$\begin{aligned}
\eta_b &= \left\{(1+\beta) - (1-\nu)\left[\frac{\alpha\left(\frac{1-\alpha}{\chi}\right)(1-\nu)}{1-4\alpha(1-\nu)(1-\alpha)}\right]\left[\frac{1-4\alpha(1-\omega)(1-\alpha)}{\nu-\omega}\right]\right\}^{-1} \\
\eta_y &= 2\frac{\alpha\left(\frac{1-\alpha}{\chi}\right)(1-\nu)}{\nu+(1-\nu)(2\alpha-1)^2}\eta_b
\end{aligned}$$

and the stability criteria is now $\frac{1}{1+\beta} > \eta_b$. This holds when $\nu < \omega$ and $\omega > 1 - \frac{1}{4\alpha(1-\alpha)}$ and this implies I only require $\omega > 0$ such that there is no lower bound on the long-run Armington elasticity.

Appendix A.4. (Proof of Proposition 3)

First, consider $\widehat{b}_t = \psi_b \widehat{b}_{t-1} + \psi_y\left(\widehat{y}_t - \widehat{y}_t^\star\right)$, which is a standard first-order autoregressive process. As such, the variances of relative technology is,

$$\begin{aligned}
E_t \widehat{b}_t^2 &= \psi_b^2 E_t \widehat{b}_{t-1}^2 + 2\psi_b\psi_y E_t\left(\widehat{b}_{t-1}\widehat{y}_t^R\right) + \psi_y^2 E_t\left(\widehat{y}_t - \widehat{y}_t^\star\right)^2 \\
&\Rightarrow \sigma_b^2 = \left(\frac{\psi_y^2}{1-\psi_b^2}\right)\sigma_{(y-y^\star)}^2
\end{aligned} \tag{38}$$

where $\sigma^2_{(y-y^\star)} = \sigma^2_y + \sigma^2_{y^\star} - 2\sigma^2_{y,y^\star} = 2\sigma^2_y (1 - \rho_{y,y^\star})$. Similarly, the covariance between relative technology and the home endowment is,

$$\sigma_{b,y} = \psi_b \sigma_{b_{-1},y} + \psi_y \sigma_{y,y} - \psi_y \sigma_{y,y^\star} = \psi_y \left(\sigma^2_y - \sigma_{y,y^\star}\right) = -\sigma_{b,y^\star} \tag{39}$$

Second, note that it is possible to solve for country-level consumption using, $\widehat{c}^W_t = \widehat{y}^W_t$. This allows me to write:

$$\widehat{c}_t = \xi_1 \widehat{y}_t + \xi_2 \widehat{y}^\star_t + \xi_3 \widehat{b}_t \quad \text{and} \quad \widehat{c}^\star_t = \xi_1 \widehat{y}^\star_t + \xi_2 \widehat{y}_t - \xi_3 \widehat{b}_t$$

where,

$$\xi_1 \equiv \frac{1+\psi_\nu}{2} \quad ; \quad \xi_2 \equiv \frac{1-\psi_\nu}{2} \quad ; \quad \xi_3 \equiv (1-\psi_\nu) \frac{\alpha (1-\nu)}{2}$$

and $\psi_\nu \equiv (2\nu\alpha - 1) / [1 - 2\alpha (1-\nu)]$.

I then express the covariances of consumption across countries in terms of the composite parameters, ξ_1, ξ_2, ξ_3, and the variance of technology, σ^2_b and $\sigma_{b,y}$, which implies,

$$\sigma_{c,c^\star} = 2\xi_1 \xi_2 \sigma^2_y - \xi^2_3 \sigma^2_b - \xi_3 (\xi_1 - \xi_2)(\sigma_{b,y} - \sigma_{b,y^\star}) + \left(\xi^2_1 + \xi^2_2\right) \sigma_{y,y^\star} \tag{40}$$

Applying (38) and (39) to the preceding condition and then applying definitions ξ_1, ξ_2, ξ_3 and simplifying generates equation (19) in the main text.

The variance of home consumption is,

$$\sigma^2_c = \left(\xi^2_2 + \xi^2\right) \sigma^2_y + \xi^2_3 \sigma^2_b + 2\xi_3 (\xi_1 - \xi_2) \sigma_{b,y} + 2\xi_1 \xi_2 \sigma_{y,y^\star} \tag{41}$$

Applying definitions ξ_1, ξ_2, ξ_3 generates equation (20) in the main text. The proposition is completed by noting $\rho_{c,c^\star} = \sigma_{c,c^\star}/\sigma^2_c$ and $\rho_{y,y^\star} = \sigma_{y,y^\star}/\sigma^2_y$, setting $\sigma^2_y = 1$, and releasing, since the economies are symmetric, that $\sigma_c = \sigma_{c^\star}$.

The final stage is to prove we can recover the cross-correlation of consumption under full adjustment, which is such that, $\tau = \psi^2_\omega$, where $\psi_\omega = (2\alpha\omega - 1) / [1 - 2\alpha (1-\omega)]$. Note that under full adjustment, when $\chi \to 0$, then $\tau = \psi_\nu^2 + \tau_1^2 + 2\tau_1\psi_\nu$ and $\tau_1 = (1-\psi_\nu)(1-\nu)\alpha\psi_y(0)$, where $\psi_y(0) = \delta_y$ is such that,

$$\tau_1 = \frac{2(1-\alpha)}{1-2\alpha(1-\nu)} \frac{2\alpha(\omega-\nu)}{1-2\alpha(1-\omega)} = \frac{(1-\psi_\omega)/(1-\alpha)}{(1-\nu)/(\omega-\nu)} \tag{42}$$

Substituting this value for τ_1 into the expression for τ, I generate equation (22) reported in the main text.

Appendix A.5. (Proof of Proposition 4)

The solution for the real exchange rate, conditional on technology, is given by equation (12). The covariance between the real exchange rate and technology is,

$$\sigma_{q,b} = a_\nu \left[2\psi_y - \alpha(1-\nu)\, 2\left(\frac{\psi_y^2}{1-\psi_b^2}\right)\right]\left(\sigma_y^2 - \sigma_{y,y^\star}\right) \tag{43}$$

The relationship between relative consumption and the real exchange rate, conditional on technology, is $\widehat{c}_t^R = \left(\frac{2\alpha\nu-1}{2\alpha-1}\right)\widehat{q}_t + \alpha(1-\nu)\widehat{b}_t$, and this allow me to write, $\sigma_{q,c^R} = \left(\frac{2\alpha\nu-1}{2\alpha-1}\right)\sigma_q^2 + \alpha(1-\nu)\sigma_{q,b}$. Together with equation (43), this generates equation (24) in the main text.

Next, I derive expressions for the variances of the real exchange rate and relative consumption, denoted σ_q^2 and $\sigma_{c^R}^2$. To determine the former, I again use the expression for the real exchange rate, which implies,

$$\sigma_q^2 = 2a_\nu^2\left[1 + [\alpha(1-\nu)]^2\left(\frac{\psi_y^2}{1-\psi_b^2}\right) - 2\alpha(1-\nu)\psi_y\right]\left(\sigma_y^2 - \sigma_{y,y^\star}\right)$$

This is the second condition in equation (25). Using the expression for relative consumption, I find,

$$\sigma_{c^R}^2 = \left(\frac{2\alpha\nu-1}{2\alpha-1}\right)^2\sigma_q^2 + [\alpha(1-\nu)]^2\sigma_b^2 + 2\left(\frac{2\alpha\nu-1}{2\alpha-1}\right)\alpha(1-\nu)\sigma_{q,b}$$

Substituting σ_q^2 and $\sigma_{q,b}$, and simplifying, delivers,

$$\sigma_{c^R}^2 = 2\psi_\nu^2\left(\sigma_y^2 - \sigma_{y,y^\star}\right) + \left(\psi_\nu^2 + 1 - 2\psi_\nu\right)[\alpha(1-\nu)]^2\sigma_b^2 + 4\psi_\nu(1-\psi_\nu)\alpha(1-\nu)\sigma_{b,y}$$

where I have also introduced $\psi_\nu = (2\alpha\nu - 1) / [1 - 2\alpha(1-\nu)]$. Finally, eliminating σ_b^2 and $\sigma_{b,y}$ delivers the first condition in equations (25).

Again, it is useful to recover these relationships under full adjustment. First, applying conditions for τ and τ_1 under full adjustment to the variance of relative consumption, $\sigma_{c^R}^2$, the result is immediate (see the first expression in equations (25) in the main text). Next, consider the real exchange rate, σ_q^2. Again applying τ and τ_1 to the second expression in equations (25) in the main text implies, $a_\nu^2\left[1 - \frac{\psi_\nu^2 - \tau + 2\tau_1}{(1-\psi_\nu)^2}\right] = a_\omega^2$, which means, $\sigma_q^2 = 2a_\omega^2\left(\sigma_y^2 - \sigma_{y,y^\star}\right)$. Finally, the same procedure implies $\sigma_{c^R,q} = 2\left(\frac{2\alpha\omega-1}{2\alpha-1}\right) a_\omega^2\left(\sigma_y^2 - \sigma_{y,y^\star}\right)$. Note that this implies, $\sigma_{q,c^R} = \left(\frac{2\alpha\omega-1}{2\alpha-1}\right)\sigma_q^2$, which is consistent with the expression reported in Corsetti *et al.* (2008).

Appendices B

Appendix B.1. (Summary of Quantitative Model with Technology Choice)

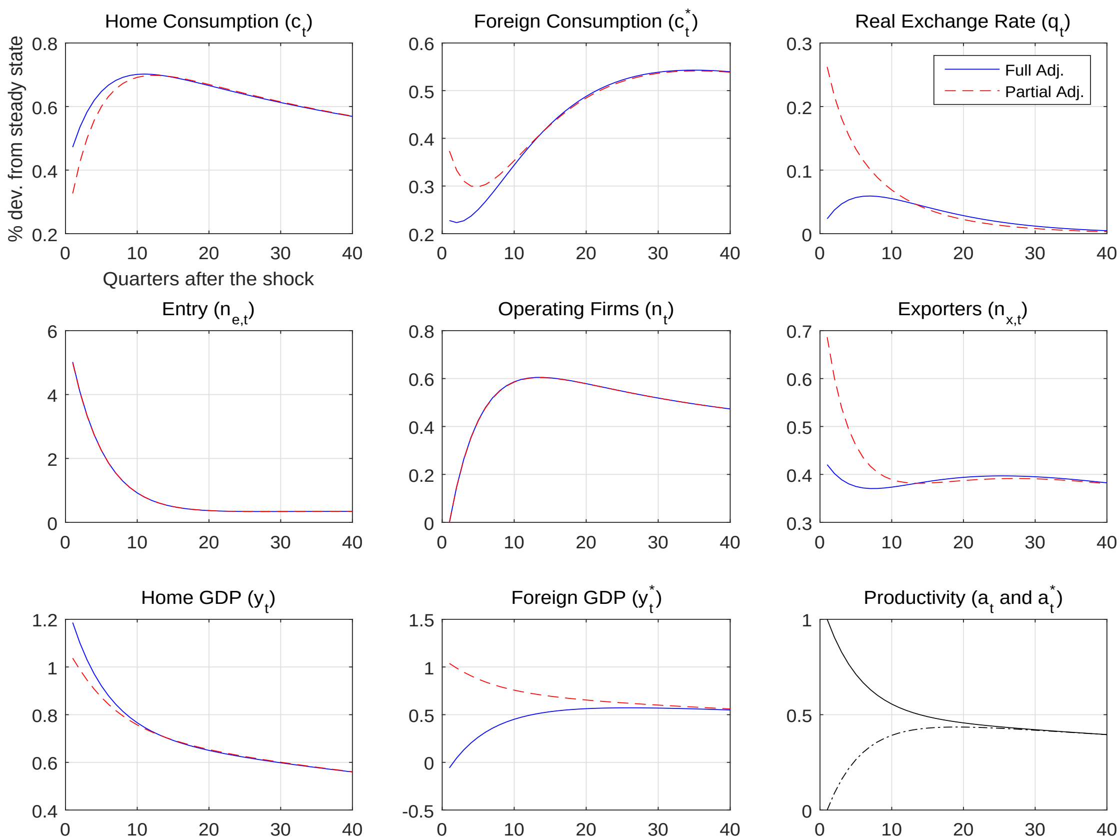
The home economy is characterized by:

Description	Equation
Goods market clearing	$G_t = \left[\left(e^{d(\theta_t)}Y_{h,t}\right)^{\xi} + \left(\theta_t e^{d(\theta_t)}Y_{f,t}\right)^{\xi}\right]^{1/\xi}$
Income-expenditure	$w_t L_t + \pi_t n_t = c_t + \varphi\left(\theta_t/\theta_{t-1}\right) + v_t n_{e,t}$ where $G_t = c_t + \varphi\left(\frac{\theta_t}{\theta_{t-1}}\right)$
Domestic and Export pricing	$p_{h,t} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_t}{a_t z}$ and $p^{\star}_{h,t} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_t/q_t}{a_t \bar{z}_{x,t}}$
Labor supply	$w_t = -u_c\left(t\right)/u_L\left(t\right)$
Domestic demand	$Y_{h,t} = a_h\left[n_t^{1/(1-\sigma)}p_{h,t}\right]^{-\nu} G_t$ where $y_{h,t} = n_t^{\sigma/(1-\sigma)}Y_{h,t}$
Import demand	$Y_{f,t} = a_f\left[\left(n^{\star}_{x,t}\right)^{1/(1-\sigma)} p_{f,t}\right]^{-\nu} G_t$ where $y_{f,t} = \left(n^{\star}_{x,t}\right)^{\sigma/(1-\sigma)} Y_{f,t}$
Exporter cut-off	$f_x\left(\frac{w_t}{a_t}\right)\int_{z_{x,t}}^{\infty}\left[\left(z_{x,t}/z\right)^{1-\sigma}\right] dG\left(z\right) = q_t n_{x,t}^{\sigma/(1-\sigma)} p^{\star}_{h,t} Y^{\star}_{h,t}/\sigma$
Share of exporters	$n_{x,t}/n_t = 1 - G\left(z_{x,t}\right)$
Profit	$\pi_t = \pi_{h,t} + \frac{n_{x,t}}{n_t}\pi^{\star}_{h,t}$ where $\pi_{h,t} = n_t^{\sigma/(1-\sigma)}p_{h,t}Y_{h,t}/\sigma$, $\pi^{\star}_{h,t} = f_x\left(\frac{w_t}{a_t}\right)\int_{z_{x,t}}^{\infty}\left[\left(z_{x,t}/z\right)^{1-\sigma} - 1\right] dG\left(z\right)$
Shares Euler	$f_e\left(w_t/a_t\right) = \left(1-\delta\right)E_t\beta\left[u_c\left(t+1\right)/u_c\left(t\right)\right]\left[f_e\left(w_{t+1}/a_{t+1}\right) + \pi_{t+1}\right]$
Mass of entrants	$n_t = \left(1-\delta\right)\left(n_{t-1} + n_{e,t-1}\right)$
Technology choice	$\frac{1-\chi(t)}{\theta_t d'(t)} + \frac{y_{f,t}}{G_t}p_{f,t}n^{\star}_{x,t} = \left(\frac{\theta_t}{\theta_{t-1}}\right)\chi'\left(t\right) - E_t m_{t+1}\left(\frac{\theta_{t+1}}{\theta_t}\right)\chi'\left(t+1\right)$
Financial autarky	$q_t p^{\star}_{h,t} n_{x,t}^{1/(1-\sigma)} Y^{\star}_{h,t} = p_{f,t}\left(n^{\star}_{x,t}\right)^{1/(1-\sigma)} Y_{f,t}$

The home and foreign economy are symmetric.

Appendix B.2. (Impulse Responses to a Home Productivity Shock)

The figure below presents impulse responses to a one-time home productivity shock of 1 percent for selected variables under two scenarios. The first, *Full Adj.*, refers to the case in which $\chi = 0$. The second, *Partial Adj.*, refers to the case in which $\nu = 0.35$ and $\chi = 1.032$.



Appendix B.3. (Alternative Specifications)

Under complete markets, I replace "Financial autarky", which is given by, $q_t p^{\star}_{h,t} n_{x,t}^{1/(1-\sigma)} Y^{\star}_{h,t} = p_{f,t} \left(n^{\star}_{x,t}\right)^{1/(1-\sigma)} Y_{f,t}$, as in section B.1, with $q_t = c_t/c^{\star}_t$.

To allow for a bond I allow the home economy to trade in a home and foreign currency bond and the foreign economy to trade only in the foreign currency bond. The national budget constraint for the home economy is given by,

$$\frac{q_t b_{f,t}}{\left(1+r^{\star}_t\right)\Theta\left(q_t b_{f,t}\right)} = \left[w_t L_t + \pi_t n_t - v_t n_{e,t} - c_t\right] + q_t b_{f,t-1} \tag{44}$$

where $\Theta\left(q_t b_{f,t}\right)$ is cost of bond holding that renders the model stationary. Bond market clearing implies $b^{\star}_{f,t} + b_{f,t} = 0$ and the national budget constraint for the foreign is redundant. These two conditions replace, "Financial autarky". I then use labor market clearing,

$$w_t L_t = (\sigma - 1)\left(n_t \pi_{h,t} + n_{x,t}\pi^{\star}_{h,t}\right) + w_t\left(\sigma\frac{f_x}{a_t} n_{x,t} + \frac{f_e}{a_t} n_{e,t}\right) \tag{45}$$

to replace "Income-expenditure" (and the foreign equivalent). I also include two (domestic) consumption Euler equations and an uncovered interest parity condition to account for trade in the foreign currency bond. Overall, I have added three equations to explain $\{b_{f,t}, r_t, r^{\star}_t\}$. As in Ghironi and Melitz, if $b_{f,t} = 0$ for all t, then equations (44) and (45) imply "Financial autarky". In the steady-state I assume a zero net foreign asset position and so linearize the national budget constraint around long-run consumption; i.e., $\widehat{b}_{f,t} \equiv db_{f,t}/c$. I adopt the cost specification in Benigno and Thoenissen (2006) with a cost of adjustment specified such that, in the steady state, the premium for home households holding foreign bonds is 10 basis points (annualized).

I change the shock structure to that used in Alessandria (2009). Based on equation (27) in the main text, where, $\lambda_{t+1} = A_0 + A\lambda_t + \varepsilon_{t+1}$, I have $\lambda_t = \left[\ln\left(a_t\right), \ln\left(\varsigma_t\right), \ln\left(a^{\star}_t\right), \ln\left(\varsigma^{\star}_t\right)\right]^T$, $A_0 = \left[a, \varsigma, a^{\star}, \varsigma^{\star}\right]^T$, and $\varepsilon = \left[\varepsilon^a, \varepsilon^\theta, \varepsilon^{a^{\star}}, \varepsilon^{\theta^{\star}}\right]^T$. The coefficient matrix for the autoregressive terms is,

$$A = \begin{bmatrix} 0.88 & 0 & 0.06 & 0 \\ -0.45 & 0.96 & -0.18 & -0.02 \\ 0.06 & 0 & 0.88 & 0 \\ -0.18 & -0.02 & -0.45 & 0.96 \end{bmatrix} \tag{46}$$

Innovations are such that $\text{corr}\left(\varepsilon^{a}, \varepsilon^{a^\star}\right) = 0.385$, $\text{corr}\left(\varepsilon^{\theta}, \varepsilon^{\theta^\star}\right) = 0.48$, $\text{corr}\left(\varepsilon^{a}, \varepsilon^{\theta}\right) = \text{corr}\left(\varepsilon^{a^\star}, \varepsilon^{\theta^\star}\right) = -0.54$, and $\text{corr}\left(\varepsilon^{a}, \varepsilon^{\theta^\star}\right) = \text{corr}\left(\varepsilon^{a^\star}, \varepsilon^{\theta}\right) = -0.34$. Finally, $\text{var}(\varepsilon^{a}) = \text{var}\left(\varepsilon^{a^\star}\right) = 0.0061^2$ and $\text{var}\left(\varepsilon^{\theta}\right) = var\left(\varepsilon^{\theta^\star}\right) = 0.03^2$.

GHH preferences (Greenwood *et al.*, 1988) imply, $u\left(c_t, L_t\right) = \ln\left(c_t - \frac{\eta}{1+1/\varsigma} L_t^{1+1/\varsigma}\right)$. This affects the labor-leisure condition and the share Euler equation, as reported in section B.1. Specifically,

$$u_c\left(c_t, L_t; \eta\right) = \frac{1}{c_t - \frac{\eta L_t^{1+1/\varsigma}}{1+1/\varsigma}} \quad \text{and} \quad u_L\left(c_t, L_t; \eta\right) = \frac{-\eta L^{1/\varsigma}}{c_t - \frac{\eta L_t^{1+1/\varsigma}}{1+1/\varsigma}} \tag{47}$$

where the steady-state Frisch elasticity is $\left[u_L\left(\cdot\right)\right] / \left[L u_{LL}\left(\cdot\right) - L \frac{u_{cL}(\cdot)}{u_{cc}(\cdot)}\right]$. In the baseline case, we used ς, since the period utility function was separable.

Appendix B.4. (International Correlations with a Fixed Mass of Varieties and No Export Decision)

The figure below presents correlations with productivity shocks; with preferences that are separable and those as in Greenwood *et al.* (1988); *GHH*. All statistics have been HP-filtered with the smoothing parameter set at $1,600$.

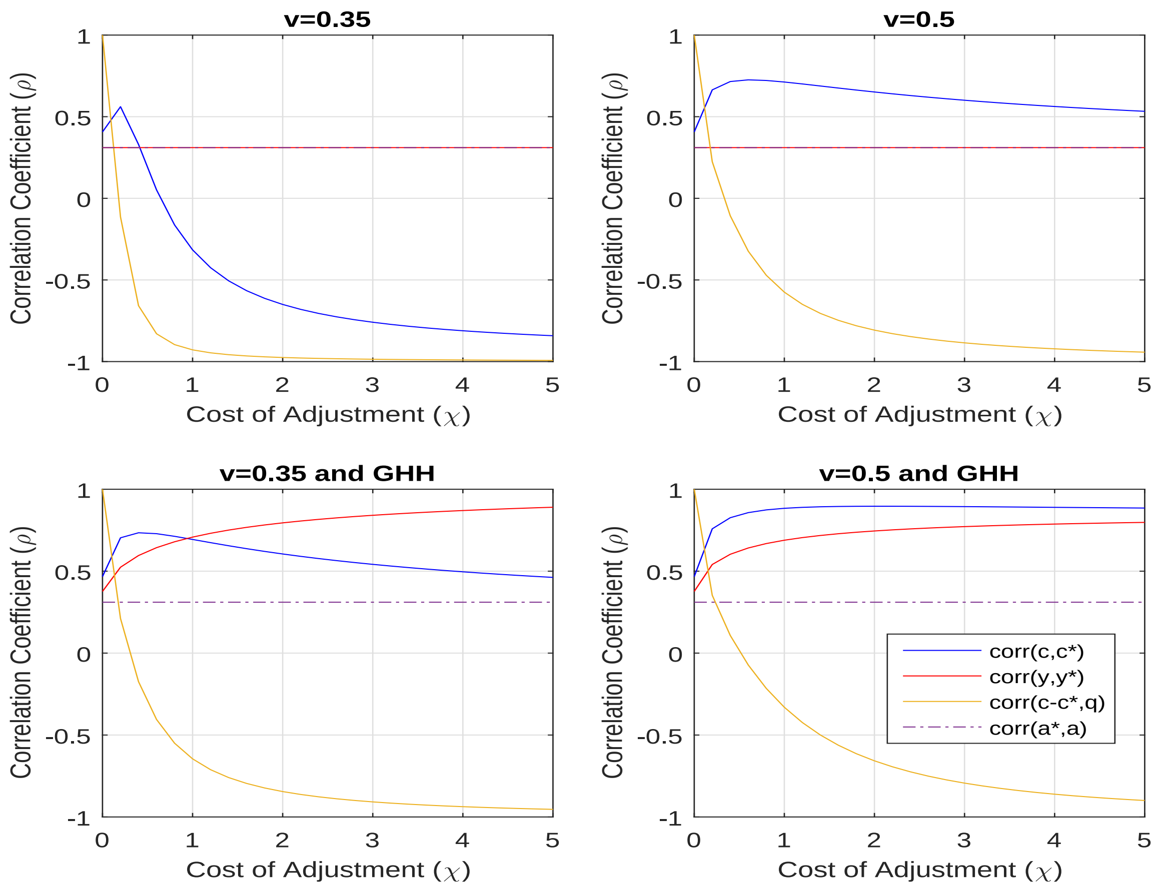


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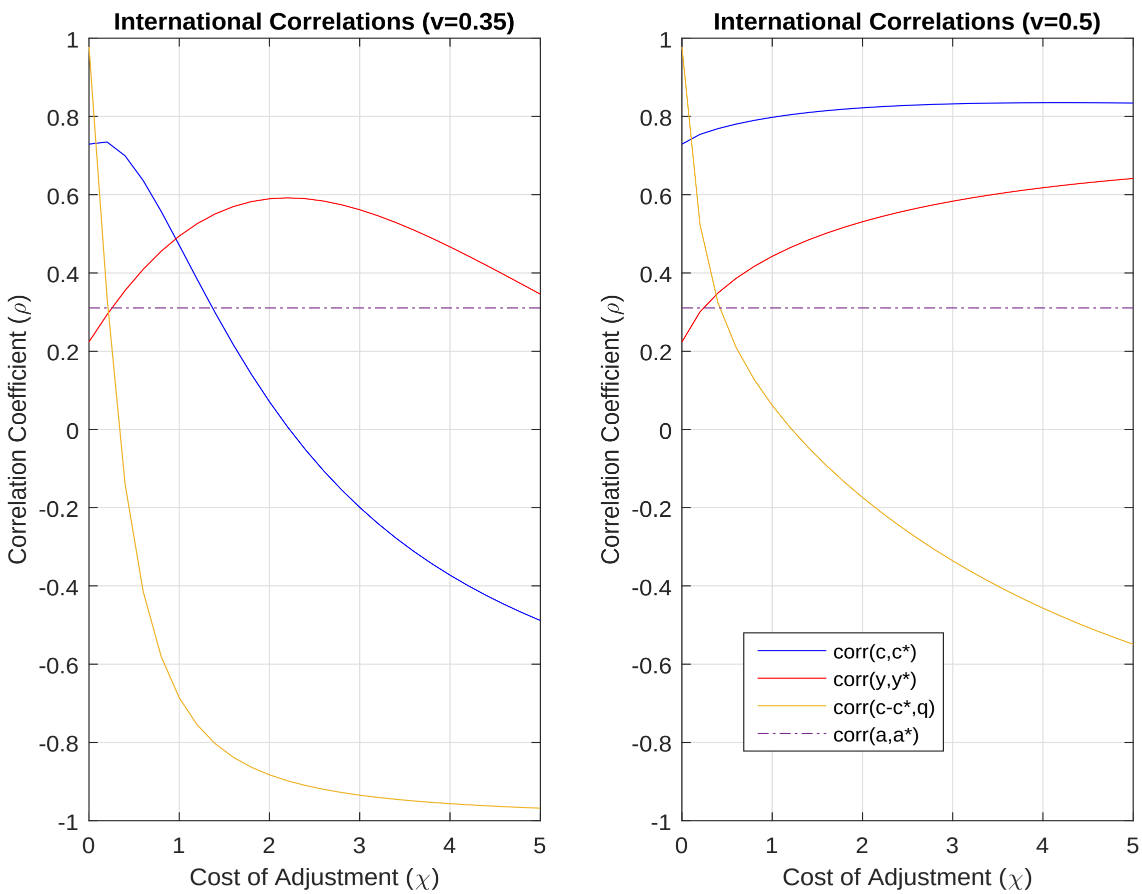
Table 1: Steady-State Parameter Values

Parameters Set Exogenously			
	Parameter	Value	Target/Source
Discount factor	β	0.99	$(\beta^{-4}-1)\times 100 = 4.01\%$
Frisch elasticity	ς	0.74	Heathcote *et al.* (2010)
LR elasticity of sub'n	ω	3.8	Bernard *et al.* (2003)
Sunk cost	f_e	1	Normalization

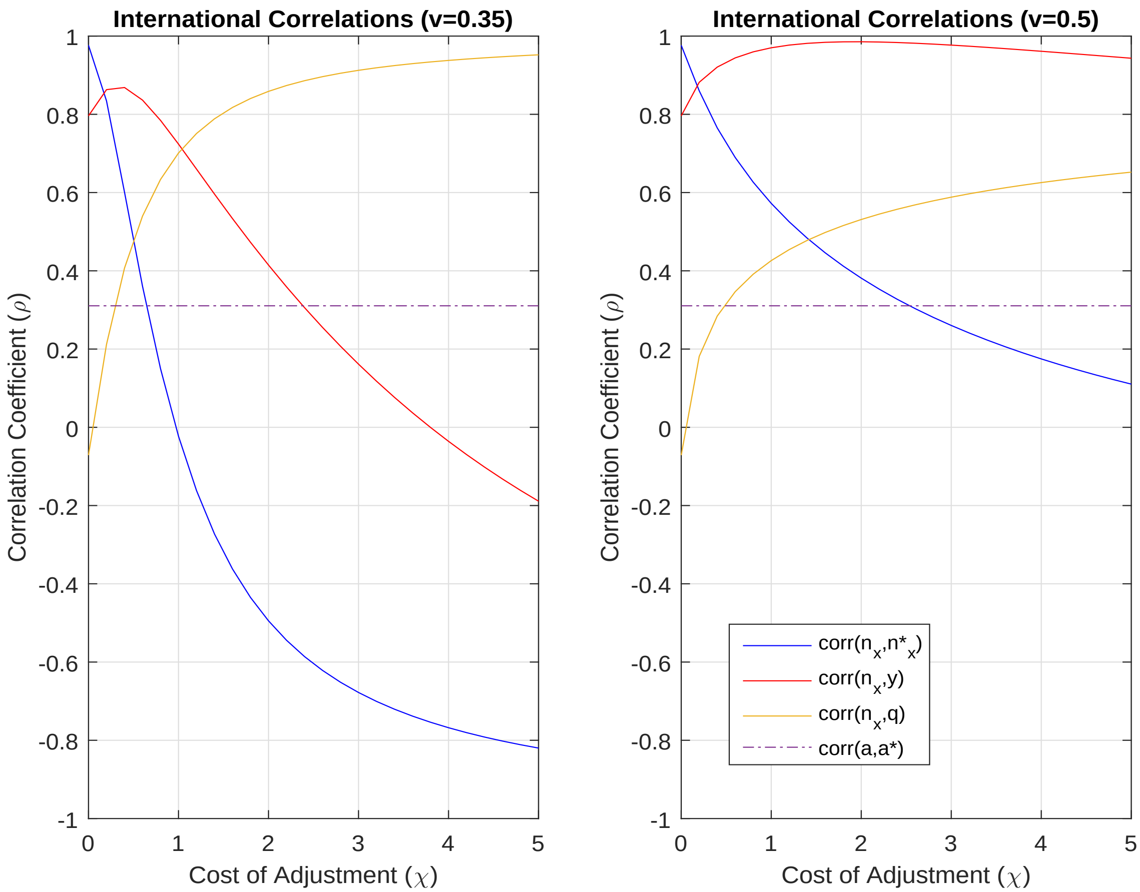
Calibrated Parameters				
	Parameter	Value	Target	Source
Exit rate (annual)	δ	0.029	11.78%	BLS
Fixed export cost	f_x	0.001	21%	Bernard *et al.* (2003)
s.d. of sales	κ	3.400	1.67%	Bernard *et al.* (2003)
Labor supplied	η	0.936	1	Normalization
Expenditure share	α	0.676	73.3%	Ghironi and Melitz (2005)

Figure 1: International Correlations (Consumption, GDP, and Real Exchange Rate)[‡]



[‡]Notes: All statistics have been HP-filtered with the smoothing parameter set at $1,600$.

Figure 2: Correlations (with Extensive Margin of Exports)[§]



[§]Notes: All statistics have been HP-filtered with the smoothing parameter set at $1,600$.

Table 2: International Correlations under Alternative Specifications[¶]

		Specifications					
		Financial Markets			*Parameterization*		
Statistic	Data	FA	CM	Bond	Elas.	Shocks	GHH
$\rho_{c,c^\star}$	0.60	0.45	0.96	0.88	0.45	-0.32	0.59
$\rho_{y,y^\star}$	0.68	0.50	0.26	0.19	0.48	0.36	0.57
$\rho_{c^R,q}$	-0·71	-	1	0.96	-0.70	-0.88	-0.82
$\rho_{n_x,n_x^\star}$	–	-0.04	0.39	0.79	0.20	-0.78	0.15
$\rho_{n_x,y}$	–	0.71	0.44	0.70	0.76	-0.10	0.54
$\rho_{n_x,q}$	–	0.71	0.69	0.22	0.62	0.94	0.65

[¶]Notes: Column *Financial Markets-FA* reproduces the specifications from Figures 1 and 2, with $\nu = 0.35$ and $\chi = 1.032$. This is the benchmark. Columns *CM* and *Bond* refer to economies with complete markets and a single bond. Columns *Elas.*, *Shocks*, and *GHH* differ from the benchmark in that their is a high long-run Armington elasticity, there is a different specification for shocks, and preferences have been changed, as outlined in the text. All statistics have been HP-filtered with a smoothing parameter of $1,600$. Column *Data* reports figures from Corsetti *et al.* (2008) for U.S. vis-à-vis remaining OECD countries.