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Get the Lowdown: The International Side of the Fall in the U.S. Natural Rate of Interest*

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Abstract

I investigate the downward drift of U.S. interest rates from 1984:Q1 to 2019:Q4. For this, I bring the workhorse two-country New Keynesian model to data on the U.S. and an aggregate of its major trading partners using Bayesian techniques. I show that the U.S. natural (or equilibrium) interest rate recovered from the model has fallen more gradually than the long-run U.S. real rate, cushioned by productivity shocks. Since inflation expectations became well-anchored in the '90s, this implies that the continued interest rate decline is largely explained by the real rate tracking the natural rate downward. Foreign productivity spillovers have had significant effects on the U.S. natural rate and on U.S. output potential. However, foreign shock propagation contributed little to the upswing in U.S. output relative to potential or to sustaining inflation close to target, both of which are attributed almost entirely to mark-up compression (cost-push shocks) and an accommodative monetary policy in the U.S.

JEL Classification: F41, F42, E12, E52, C11.

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1 Introduction

At the 2020 economic symposium at Jackson Hole, Federal Reserve Chair Jerome H. Powell discussed the main takeaways of the U.S. central bank's first-ever public review of its monetary policy framework (strategy, tools, and communication practices) conducted during $2019-20$. He also announced a major shift in the Fed's strategy from the flexible "inflation targeting" approach adopted in 2012 to a flexible "average inflation targeting" strategy going forward.[1] To make the Fed's case, Chair Powell cited prominently the decline in the U.S. natural rate of interest and the related slowdown in U.S. output potential since the mid-2000s stating that:

> "Estimates of the neutral federal funds rate, which is the rate consistent with the economy operating at full strength and with stable inflation, have fallen substantially, in large part reflecting a fall in the equilibrium real interest rate, or "r-star" [*a.k.a.* the natural rate of interest]. This rate is not affected by monetary policy but instead is driven by fundamental factors in the economy, including demographics and productivity growth—the same factors that drive potential economic growth. [...] This decline in assessments of the neutral federal funds rate has profound implications for monetary policy. With interest rates generally running closer to their effective lower bound even in good times, the Fed has less scope to support the economy during an economic downturn by simply cutting the federal funds rate." Chair Jerome H. Powell speech at the 2020 Jackson Hole Symposium, August 27, 2020 (Powell (2020)).

The natural rate concept so central to the argument laid out by Chair Powell here is, in fact, not a novel one. Its origins are often traced back to the work of Wicksell (1898) if not to earlier contributions (see, e.g., Niehans (1987)). However, it is the case that the natural rate concept has gained much of its prominence in the debates about monetary policy praxis and strategy since the 1990s as it became an integral part of the workhorse New Keynesian model—the theoretical framework that lies at the core of much of mainstream macro and central bank modeling nowadays.[2]

[1] The interested reader can explore the Fed's $2019-20$ Monetary Policy Framework Review and the resulting changes announced on August 27, 2020 to the Fed's Statement on Longer-Run Goals and Monetary Policy Strategy (adopted effective January 24, 2012; amended effective January 29, 2019) here: Board of Governors (2020). On the concepts of "inflation targeting" and "average inflation targeting", see e.g. Svensson (1999) and Nessen and Vestin (2005).

[2] From a New Keynesian perspective, monetary policy is judged to be expansionary (contractionary) when inflation expectations are well-anchored and the real interest rate is below (above) the natural rate. To the extent that monetary policy influences the deviation of the short-run real rate from the short-run natural rate, it also exploits the trade-off between inflation and slack arising through the Phillips curve relationship. See on this point, e.g., Woodford (2003), Barsky et al. (2014), Bernanke (2015a), and Galí (2018).

In the context of the New Keynesian model, a fall in the natural rate of interest has major implications for the conduct and efficacy of monetary policy and increases the likelihood of zero-lower bound episodes where the monetary policy space can become constrained during economic downturns (Ball et al. (2016), Clarida (2020)).[3] Furthermore, Caldara et al. (2020) argue that the decline in the natural rate is not exclusive to the U.S. and show through simulations that a simultaneously binding zero-lower bound constraint abroad can prolong the duration of a zero-lower bound episode in the U.S. and worsen its downturn. The challenge for monetary policymakers arises, at least in part, because the natural rate of interest is inherently unobservable and must be estimated.

Figure 1 illustrates the empirical evidence available on the U.S. natural rate of interest over the 1984:Q1-2019:Q4 period using the most prominent estimates in the literature obtained with a variety of empirical (time series) and semi-structural estimation techniques (Laubach and Williams (2003), Kiley (2015), Lubik and Matthes (2015), Holston et al. (2017), Johannsen and Mertens (2018), and Del Negro et al. (2019)).[4] There is some disagreement across estimates, but Figure 1 clearly illustrates the broad consensus that exists regarding the downward shift of the U.S. natural rate around the $2007-09$ global financial crisis (a fall from above 2 percent before to less than 1 percent afterwards) to which Chair Powell alluded in his 2020 Jackson Hole speech.[5]

Most of the U.S. estimates summarized in Figure 1 do not consider explicitly the role of international spillovers, except Holston et al. (2017). In their paper, Holston et al. (2017) extend the Laubach and Williams (2003) semi-structural estimation strategy from the closed-economy to the open-economy setting. Their work and related contributions have been instrumental in documenting the substantial comovement of the natural rate across countries and over time (on this point, see also Wynne and Zhang (2018a) and Wynne and Zhang (2018b)). This body of evidence suggests international spillovers or global factors can play an important role in shaping the natural rate not just in the U.S.[6]

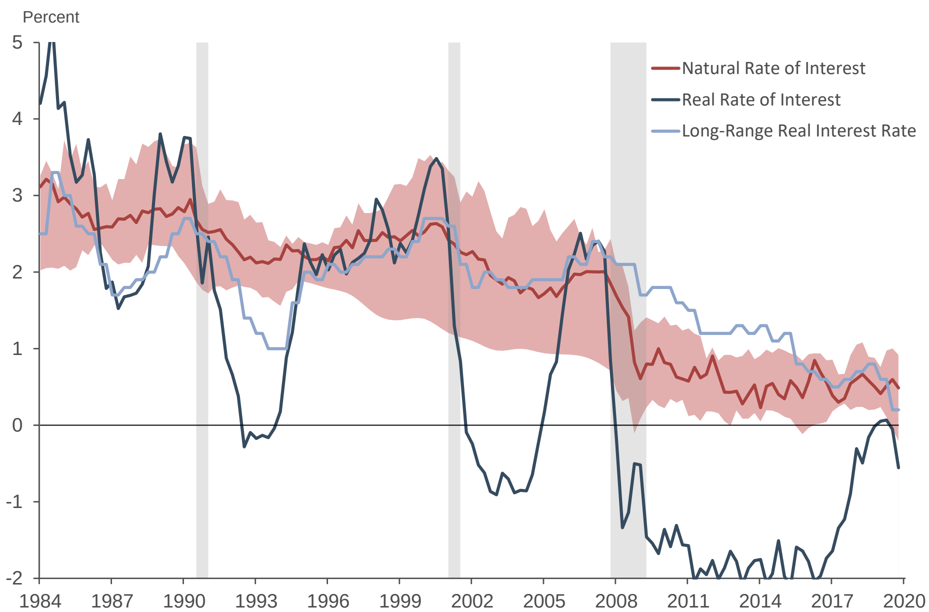
[3] The uncertainty surrounding the estimates of the natural rate leads also to a number of additional risk management concerns for central banks when conducting monetary policy, as noted by Evans et al. (2016). For an exploration of the Fed's monetary policy credibility and some of the challenges it poses limiting the efficacy of tools like forward guidance, see Cole and Martínez-García (2020).

[4] Other related estimates include Clark and Kozicki (2005), Trehan and Wu (2007), Pescatori and Turunen (2015), and Wynne and Zhang (2018a).

[5] Figure 1 also includes, as a reference, the survey-based measures of the short-run and of the long-range real interest rate based on forecasts from Blue Chip Economic Indicators (Aspen Publishers (2020)).

[6] Other empirical work on the natural rate in other countries (in many cases, using open-economy or small-open economy specifications) includes, e.g., Mesonnier and Renne (2007), Horváth (2009), Leu and Sheen (2011), Berger and Kempa (2014), Goyal and Arora (2016), Fries et al. (2018), Armelius et al. (2018),

Figure 1. Estimates of the U.S. Natural Rate of Interest vs.
Short-Run and Long-Run Survey-Based Measures of the Real Interest Rate



Note: The shaded bars indicate the NBER chronology of U.S. recessions. The U.S. real rate is the 3-month nominal Treasury bill minus the one quarter ahead CPI inflation expectations from Blue Chips Economic Indicators (dark green line). The U.S. long-range real interest rate is the 5-year average, 5-year forward forecast of the 3-month nominal Treasury bill minus the 5-year average, 5-year forward forecast of the annual CPI inflation rate also from Blue Chips Economic Indicators (light blue line). The six estimates of the natural interest rate from Laubach and Williams (2003), Kiley (2015), Lubik and Matthes (2015), Holston et al. (2017), Johannsen and Mertens (2018), and Del Negro et al. (2019) are summarized using their median (dark red line) and max-min range (light red shaded area). Observations for the estimated natural rate are not available for all six estimates of the natural rate since 2016.
Shources: NBER; Aspen Publishers (2020); Laubach and Williams (2003); Kiley (2015); Lubik and Matthes (2015); Holston et al. (2017); Johannsen and Mertens (2018); Del Negro et al. (2019); and author's calculations.

However, even in those studies that explicitly take into account the possibility of international spillovers, the semi-structural estimates of the U.S. natural rate that the literature recovers do not lend themselves to monetary policy analysis. They are obtained under a flexible reduced-form representation consistent with two key New Keynesian equilibrium conditions, the IS curve and the Phillips curve relationships, but assuming the natural rate behaves as a purely exogenous process. Instead, in this paper I estimate a fully structural specification building on the theoretical two-country work of Martínez-García (2019) and Clarida (2019), among others.[7] What that entails is that, unlike the empirical and semi-structural estimates found elsewhere in the literature, the U.S. natural rate estimate I obtain is endogenously determined and tied to the driving shocks of the economy.

Structural estimates of the U.S. natural rate of interest motivated by the New Keynesian theory do exist such as the closed-economy estimate of Cúrdia et al. (2015). Small-open economy specifications have also been estimated as well such as in Grossman et al. (2019) where the authors show the natural rate of six small (but advanced) economies to be strongly correlated with the U.S. natural rate.[8] However, to my knowledge, this is the first paper to provide a structural estimate of the U.S. natural rate in a two-country setting. And, as a result, this paper not only provides evidence of international comovement, but also a novel empirical measure of the contribution of foreign shock spillovers to the U.S. natural rate.

I argue that the two-country setup adopted here is better suited to explore the domestic and foreign sources of fluctuations on the U.S. natural rate (and on U.S. output potential) than the small-open economy model. This is because the two-country model recognizes the influence that domestic conditions can have on terms of trade for large economies like the U.S. (the U.S. is therefore not treated as a price-taker in international markets) and because the model also explicitly incorporates trade and technological linkages between the U.S. and the rest of the world. However, recovering the unobservable natural rate from the data is not without its challenges in a structural framework like this one. To handle those challenges, I propose an estimation methodology underpinned by three main considerations:

First, the structural open-economy model describes the cyclical dynamics of the economy. Hence, the observed data is detrended using survey-based long-range forecasts from Blue

Neto and Candido (2018), Brand et al. (2018), and Belke and Klose (2020), among others. For further evidence on interest rates in the U.S. and abroad see also Hamilton et al. (2016) and Borio et al. (2017).

[7] For a related theoretical exploration of the small open-economy structural framework, see e.g. Galí and Monacelli (2005), Divino (2009), and Goyal (2011).

[8] Models of the natural rate of interest that rely on a closed-economy specification include Andrés et al. (2009), Hristov (2016), Del Negro et al. (2017), Hirose and Sunakawa (2017), Neri and Gerali (2018), and Andrade et al. (2018). Among the small open-economy estimates of the natural rate of interest, I should count Justiniano and Preston (2010), Funke et al. (2011), Çebi (2012), and Gómez et al. (2019).

Chip Economic Indicators (Aspen Publishers (2020)) as proxies for trends before the data is mapped to the endogenous inflation and interest rate generated by the model. That allows me to define the cyclical patterns of the data from the perspective of professional forecasters themselves. For the model productivity, I use measured labor productivity as the corresponding observable. I find evidence of a unit root and detrend the data accordingly through a Beveridge-Nelson decomposition.

Second, I propose a selection of priors and observables based on their broad information content with which to pin down the cyclical dynamics of the U.S. natural rate of interest (and U.S. output potential) while simultaneously recognizing the consequences of the zero-lower bound. In regards to the latter, the key insight is to exploit survey-based forecast data from Blue Chip Economic Indicators (Aspen Publishers (2020)) as those forecasts are formed externally by private agents who are aware of the effects of the zero-lower constraint and incorporate those in their predictions about the future. Survey-based forecasts, therefore, help condition the endogenous expectations and the expected path of the model solution to be consistent with the zero-lower bound.

Third, the open-economy model is estimated with data for the U.S. and a trade-weighted aggregate of advanced and emerging economies that includes 33 of the major trading partners of the U.S. over the period from the onset of the Great Moderation in 1984:Q1 till 2019:Q4. All data is collected from the Congressional Budget Office (CBO (2020)), the Federal Reserve Bank of Dallas' Database of Global Economic Indicators (Grossman et al. (2014)), and the Conference Board Total Economy Database™ (Conference Board (2020)), with survey-based forecasts obtained from Blue Chip Economic Indicators (Aspen Publishers (2020)).

With this three-pronged empirical strategy, I show that the downward drift in the U.S. natural rate observed since at least the $2007-09$ global financial crisis can be attributed partly to the gradual decline in the U.S. long-run real rate shown in Figure 1. I show that international spillovers explain some of the cyclical fluctuations of the U.S. natural rate. Moreover, I also find that the cyclical component of U.S. output potential has fallen below trend while the U.S. short-term natural rate has stayed above the U.S. long-run real rate since $2007-09$ to a large extent as a consequence of the below-trend path of U.S. labor productivity over the past decade.[9]

In analyzing the U.S. experience, Caldara et al. (2020) argue that another important concern for U.S. policymakers is how inflation appears to have become less sensitive to

[9] Related to this, see the thematic discussion on some of the most talked about possible explanations of the decline of the real interest rates and the role that productivity plays on it in Bernanke (2015b), Bernanke (2015c), and Bernanke (2015d).

domestic slack also during the 2000s. I find through the lens of the two-country model that the cyclical comovement between inflation and slack has indeed changed over the past $10-15$ years in the U.S. However, I argue that this can be attributed, at least to a certain extent, to the more prominent role that (negative) cost-push shocks have played during this time. Although cost-push shocks can arise in different ways, the model interprets those as markups and explains the recovered sequence of negative cost-push shock realizations as a prolonged period of markup compression. More research on these cost-push shocks should help shed further light on their nature.

I also show in the paper that the post-$2007-09$ global financial crisis period has been characterized by a more robust U.S. cyclical output path than what could have been expected given the concurrent decline in U.S. cyclical output potential. This appears to be largely supported by a sequence of negative cost-push shocks. In turn, inflation has been fairly stable and close to the Fed's 2 percent target sustained by monetary policy shocks that mostly compensated for the drag on inflation resulting from those same negative cost-push shocks that provided a boost to economic activity. In fact, albeit to a lesser extent, monetary shocks also contributed to lift U.S. cyclical output up.

Finally, I argue that the estimation biases that arise when one ignores the open-economy dimension of the U.S. economy (either working with a closed-economy specification or mischaracterizing the comovement and exogenous spillovers of the productivity shock process) can distort the estimates of the monetary policy shocks and, therefore, result in erroneous empirical inferences about the conduct and efficacy of monetary policy in the U.S.

The remainder of the paper proceeds as follows: Section 2 describes the workhorse open-economy New Keynesian model. Section 3 discusses the Bayesian estimation methodology I use in my empirical analysis including the choice of the relevant priors. Section 4 reports my main findings regarding the open-economy estimates of the U.S. natural rate of interest and its determinants. In this section, I also explore the macro performance of the U.S. economy and of monetary policy after the $2007-09$ global financial crisis through the lens of the model. Section 5 concludes.

2 The Open-Economy Model

I take the workhorse two-country New Keynesian model with nominal rigidities à la Calvo (1983) as my baseline model. This model highlights two key international transmission mechanisms. First, exogenous international propagation occurs because the shock innovations can be correlated across countries and due to productivity cross-country spillovers (an exogenous form of technological diffusion). Second, endogenous international propagation through trade also occurs. The building blocks of the model are laid out in Martínez-García and Wynne (2010) and Martínez-García (2019).

The main equilibrium conditions of the workhorse two-country New Keynesian model are log-linearized around a deterministic, zero-inflation steady state. This framework provides a natural extension of the three-equation closed-economy New Keynesian setup (see, e.g., Woodford (2003)). In fact, each country can be fully described with a variant of the same three equations—that is, with an open-economy Phillips curve, an open-economy dynamic Investment-Saving (IS) equation, and a Taylor (1993)-type monetary policy rule. In the limiting case where households have preferences defined exclusively over domestically-produced varieties of goods, the model reduces to the standard three-equation closed-economy New Keynesian specification for each of the two countries.

The log-linearized equilibrium conditions do not suffice by themselves to constrain the solution at the zero-lower bound, but those equilibrium conditions still have to be satisfied by the constrained solution. However, as I explain in a bit more detail later (and more extensively in Martínez-García (2020)), estimation does not necessitate the inclusion of the zero-lower bound constraint. Adding survey data on expectations to the observable set, as I do here, suffices to obtain estimates consistent with the zero-lower bound constraint given that the observed macro variables and survey-based forecasts already internalize the constrained path of an economy where policy rates can occasionally become stuck at the zero-lower bound.

2.1 Model Specification

I denote $\widehat{z}_t \equiv \ln\left(\frac{Z_t}{\overline{Z}}\right)$ the deviation of a given variable in logs from its steady-state and, similarly, $\widehat{\overline{z}}_t \equiv \ln\left(\frac{\overline{Z}_t}{\overline{Z}}\right)$ is the deviation of a variable in logs from its steady-state in the counterfactual scenario where all frictions are removed. I use the superscript $*$ to distinguish the foreign country from the home country. Given this notation, the open-economy Phillips curve relationship for each country can be written down as follows:

$$\widehat{\pi}_t \approx \beta \mathbb{E}_t\left(\widehat{\pi}_{t+1}\right) + \Phi\left(\varphi+\gamma\right)\left[\kappa \widehat{x}_t + \left(1-\kappa\right)\widehat{x}_t^* + \left(1-\xi\right)\widehat{u}_t + \xi \widehat{u}_t^*\right], \tag{1}$$

$$\widehat{\pi}_t^* \approx \beta \mathbb{E}_t\left(\widehat{\pi}_{t+1}^*\right) + \Phi\left(\varphi+\gamma\right)\left[\left(1-\kappa\right)\widehat{x}_t + \kappa \widehat{x}_t^* + \xi \widehat{u}_t + \left(1-\xi\right)\widehat{u}_t^*\right], \tag{2}$$

where $\mathbb{E}_t(\cdot)$ are expectations formed conditional on information up to time t. Moreover, $\widehat{\pi}_t$ and $\widehat{\pi}_t^*$ denote home and foreign inflation (quarter-over-quarter changes in the consumption price index), $\widehat{y}_t$ and $\widehat{y}_t^*$ are the home and foreign output, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$ are the home and foreign output potential (the output achievable absent all frictions), and $\widehat{x}_t \equiv \left(\widehat{y}_t - \widehat{\overline{y}}_t\right)$ and $\widehat{x}_t^* \equiv \left(\widehat{y}_t^* - \widehat{\overline{y}}_t^*\right)$ refer to the home and foreign output gaps expressed as the difference between output and output potential.

The slope of the open-economy Phillips curve in (1)–(2), $\Phi(\varphi+\gamma) \equiv \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)(\varphi+\gamma) > 0$, is a function of the inverse of the intertemporal elasticity of substitution $\gamma > 0$, the inverse of the Frisch elasticity of labor supply $\varphi > 0$, the intertemporal discount factor $0 < \beta < 1$, and the Calvo (1983) price stickiness parameter $0 < \alpha < 1$. The composite coefficient $\kappa \equiv (1-\xi)\left[1-(\sigma\gamma-1)\left(\frac{\gamma}{\varphi+\gamma}\right)\left(\frac{(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right] > 0$ determines how the home and foreign output gaps are weighted to determine the firms' marginal costs and their impact on inflation and depends on two additional parameters: the steady state import share parameter (the degree of openness) $0 < \xi < 1$ and the elasticity of intratemporal substitution between home and foreign goods (or trade elasticity) $\sigma > 0$.[10]

Home and foreign exogenous cost-push shocks, $\widehat{u}_t$ and $\widehat{u}_t^*$ respectively, also appear in the open-economy Phillips curves (1) and (2). These shocks are described with a bivariate VAR(1) process of the following form:

$$\begin{aligned}\begin{pmatrix}\widehat{u}_t\\ \widehat{u}_t^*\end{pmatrix} &\approx \begin{pmatrix}\delta_u & 0\\ 0 & \delta_u\end{pmatrix}\begin{pmatrix}\widehat{u}_{t-1}\\ \widehat{u}_{t-1}^*\end{pmatrix} + \begin{pmatrix}\widehat{\varepsilon}_t^u\\ \widehat{\varepsilon}_t^{u*}\end{pmatrix},\\ \begin{pmatrix}\widehat{\varepsilon}_t^u\\ \widehat{\varepsilon}_t^{u*}\end{pmatrix} &\sim N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\sigma_u^2 & \rho_{u,u^*}\sigma_u^2\\ \rho_{u,u^*}\sigma_u^2 & \sigma_u^2\end{pmatrix}\right),\end{aligned} \tag{3}$$

where $0 < \delta_u < 1$ is the persistence parameter, $\sigma_u > 0$ is the volatility parameter, and $0 < \rho_{u,u^*} < 1$ determines the correlation of the cost-push shock innovations across countries. Cost-push shocks act as exogenous (country-specific) marginal cost shifters on the open-economy Phillips curve. These cost-push shocks are motivated as exogenous price markups, as shown in Martínez-García (2020).

The open-economy dynamics IS equation ties the path of the output gap of each country

[10] The composite coefficient κ is equal to $(1-\xi)$ only in the knife-edge case where $\sigma\gamma = 1$.

to both home and foreign aggregate demand as:

$$\widehat{x}_t \approx \mathbb{E}_t\left[\widehat{x}_{t+1}\right] + \gamma^{-1}\left[\Omega\left(\widehat{\overline{r}}_t - \widehat{r}_t\right) + (1-\Omega)\left(\widehat{\overline{r}}_t^* - \widehat{r}_t^*\right)\right], \tag{4}$$

$$\widehat{x}_t^* \approx \mathbb{E}_t\left[\widehat{x}_{t+1}^*\right] + \gamma^{-1}\left[(1-\Omega)\left(\widehat{\overline{r}}_t - \widehat{r}_t\right) + \Omega\left(\widehat{\overline{r}}_t^* - \widehat{r}_t^*\right)\right], \tag{5}$$

where the real rates in the home and foreign country are defined by Fisher's equation as $\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$ and $\widehat{r}_t^* \equiv \widehat{i}_t^* - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]$ respectively, with $\widehat{i}_t$ and $\widehat{i}_t^*$ being the home and foreign one-period nominal interest rates. The home and foreign natural rates of interest (the real rates achievable absent all frictions) are denoted $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$ respectively.

The open-economy dynamic IS equilibrium conditions given by (4) and (5) show that the local output gap moves with the home and foreign real interest rate gaps $\widehat{r}_t^{gap} \equiv \left(\widehat{\overline{r}}_t - \widehat{r}_t\right)$ and $\widehat{r}_t^{*gap} \equiv \left(\widehat{\overline{r}}_t^* - \widehat{r}_t^*\right)$ (that is, interest rate gaps are the difference between the natural rate and the real rate) for the home and foreign countries weighted by the composite coefficient $\Omega \equiv (1-\xi)\left(\frac{1-2\xi(1-\sigma\gamma)}{1-2\xi}\right) > 0$. The real interest rate gaps reflect the strength of the aggregate demand that could be sustained absent all frictions relative to the global aggregate demand with nominal rigidities. The composite Ω determines how home and foreign aggregate demand forces are weighted.[11].

In the limiting case where $\xi = 0$, equations (1) – (5) simply reduce to the closed-economy Phillips curve and dynamic IS equations for each country.

Monetary policy rule. The home and foreign Taylor (1993)-type monetary policy rules that complete the model track the local natural rate of interest while responding to local inflation in deviations from target and also to fluctuations of the local output gap, i.e.,

$$\widehat{r}_t \approx \widehat{\overline{r}}_t + \psi_\pi \widehat{\pi}_t + \psi_x \widehat{x}_t + \widehat{m}_t, \tag{6}$$

$$\widehat{r}_t^* \approx \widehat{\overline{r}}_t^* + \psi_\pi \widehat{\pi}_t^* + \psi_x \widehat{x}_t^* + \widehat{m}_t^*, \tag{7}$$

where the parameters $\psi_\pi > 1$ and $\psi_x \geq 0$ determine the strength of the central banks' responses to inflation deviations and the output gap, respectively. The monetary policy rules in (6) and (7) recognize that the effects of monetary policy operate through short-run fluctuations of the real interest rate and, therefore, allow me to remain agnostic about the set of policy tools that the central bank uses to influence the real rate at the zero-lower bound and away from it.

[11] The composite coefficient Ω equals $(1-\xi)$ only in the knife-edge case where $\sigma\gamma = 1$.

The monetary policy rules in (6) and (7) incorporate home and foreign monetary policy shocks, $\widehat{m}_t$ and $\widehat{m}_t^*$, with the following bivariate VAR(1) stochastic process:

$$\begin{gathered}
\begin{pmatrix} \widehat{m}_t \\ \widehat{m}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_m & 0 \\ 0 & \delta_m \end{pmatrix} \begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix}, \\
\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_{m,m^*}\sigma_m^2 \\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2 \end{pmatrix} \right),
\end{gathered} \tag{8}$$

which introduces exogenous persistence through the parameter $0 < \delta_m < 1$, volatility through the parameter $\sigma_m > 0$, and correlation of the shock innovations through the parameter $0 < \rho_{m,m^*} < 1$.

The frictionless equilibrium. The natural rate of interest and the output potential for each country correspond to the real rate and output of the frictionless equilibrium. Without nominal rigidities, the frictionless allocation is not affected by monetary policy in any way and, therefore, is not affected by the zero-lower bound on nominal interest rates either. The natural (real) rates of interest of each country, $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$ respectively, can be expressed as a function of the expected changes in home and foreign output potential, i.e.,

$$\widehat{\overline{r}}_t \approx \gamma \left[\Theta \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1} \right] - \widehat{\overline{y}}_t \right) + (1-\Theta) \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1}^* \right] - \widehat{\overline{y}}_t^* \right) \right], \tag{9}$$

$$\widehat{\overline{r}}_t^* \approx \gamma \left[(1-\Theta) \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1} \right] - \widehat{\overline{y}}_t \right) + \Theta \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1}^* \right] - \widehat{\overline{y}}_t^* \right) \right]. \tag{10}$$

Hence, equations (9) and (10) show that the natural rates respond to expected *changes* in potential economic activity measured by home and foreign output potential growth. The composite coefficient $\Theta \equiv (1-\xi) \left[\frac{1+(\sigma\gamma-1)(2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))} \right] > 0$ determines how home and foreign output potential growth are weighted.[12] The key takeaway from these equations is that the local natural rate of interest does not depend on the expected local potential growth alone as it also varies with the expected output potential growth abroad.

The home and foreign output potential in the frictionless equilibrium, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$, can be written as a function of the home and foreign productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$, in the following

[12] The composite coefficient Θ equals $(1-\xi)$ only in the knife-edge case where $\sigma\gamma = 1$.

terms:

$$\widehat{\overline{y}}_t \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left[\Lambda \widehat{a}_t + (1-\Lambda)\,\widehat{a}^*_t\right], \tag{11}$$

$$\widehat{\overline{y}}^*_t \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left[(1-\Lambda)\,\widehat{a}_t + \Lambda \widehat{a}^*_t\right], \tag{12}$$

where the composite coefficient $\Lambda \equiv 1+\frac{1}{2}\left[\frac{\left(\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right] > 0$ weights the impact of domestic and foreign productivity on the output potential of each country. Similar to Cole and Obstfeld (1991), local output potential is insulated from productivity shocks originating abroad (i.e., $\Lambda = 1$) in the knife-edge case where $\sigma\gamma = 1$. That entails that only local productivity enters into the function that determines the output potential of each country. This is because in this special case perfect risk-sharing across countries can be achieved solely through movements in international relative prices (or terms of trade).

The exogenous productivity shocks, $\widehat{a}_t$ and $\widehat{a}^*_t$, are described with a bivariate VAR(1) process of the following form:

$$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}^*_t \end{pmatrix} \approx \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}^*_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^a_t \\ \widehat{\varepsilon}^{a*}_t \end{pmatrix},$$
$$\begin{pmatrix} \widehat{\varepsilon}^a_t \\ \widehat{\varepsilon}^{a*}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_a & \rho_{a,a^*}\sigma^2_a \\ \rho_{a,a^*}\sigma^2_a & \sigma^2_a \end{pmatrix}\right), \tag{13}$$

where $0 < \delta_a < 1$ is the persistence parameter, $\sigma_a > 0$ is the volatility parameter, and $0 < \rho_{a,a^*} < 1$ introduces correlation of the productivity shock innovations across countries. The specification also permits cross-country spillovers in the stochastic process through the parameter $0 < \delta_{a,a^*} < 1$ which I interpret as an exogenous form of cross-country technological diffusion.

2.2 A Closer Inspection

The Taylor (1993) rules in (6) and (7) reflect the home and foreign central banks' (*de iure* or *de facto*) mandate to respond to local economic conditions only. This implies that monetary policy reacts to developments abroad only to the extent that those shocks impact local conditions. Moreover, monetary policy tracks the domestic natural rate of interest implying that a neutral monetary policy stance where the real and natural rate equate would require inflation to be at its target and output at its potential. One important implication for the propagation of shocks of such a monetary policy can be summarized with the following lemma:

Lemma 1 *The class of monetary policy rules described by (6) − (7) fully insulates inflation and the output gap from the productivity shocks which, therefore, only respond to domestic and foreign cost push-shocks and monetary policy shocks.*

Replacing the monetary policy equations given by (6) − (7) into the open-economy dynamic IS equations in (4) − (5), it follows that:

$$\widehat{x}_t \approx \mathbb{E}_t\left[\widehat{x}_{t+1}\right] - \gamma^{-1}\left[\begin{array}{c}\psi_\pi\left(\Omega\widehat{\pi}_t + (1-\Omega)\,\widehat{\pi}^*_t\right) + \psi_x\left(\Omega\widehat{x}_t + (1-\Omega)\,\widehat{x}^*_t\right) + ... \\ \Omega\widehat{m}_t + (1-\Omega)\,\widehat{m}^*_t\end{array}\right], \quad (14)$$

$$\widehat{x}^*_t \approx \mathbb{E}_t\left[\widehat{x}^*_{t+1}\right] - \gamma^{-1}\left[\begin{array}{c}\psi_\pi\left((1-\Omega)\,\widehat{\pi}_t + \Omega\widehat{\pi}^*_t\right) + \psi_x\left((1-\Omega)\,\widehat{x}_t + \Omega\widehat{x}^*_t\right) + ... \\ (1-\Omega)\,\widehat{m}_t + \Omega\widehat{m}^*_t\end{array}\right]. \quad (15)$$

Let me collect the 18 structural parameters of the model as well as the parameters of the shock processes in the vector:

$$\lambda = \left(\beta, \gamma, \varphi, \sigma, \xi, \alpha, \psi_\pi, \psi_x; \delta_a, \delta_{a,a^*}, \sigma_a, \rho_{a,a^*}, \delta_u, \sigma_u, \rho_{u,u^*}, \delta_m, \sigma_m, \rho_{m,m^*}\right)^T. \quad (16)$$

Combining (14) − (15) with the open-economy Phillips curve equations in (1) − (2), the purely forward-looking expectational difference system for the vector of endogenous variables $\left(\widehat{\pi}_t, \widehat{\pi}^*_t, \widehat{x}_t, \widehat{x}^*_t\right)^T$ takes the following form:

$$\begin{pmatrix}\widehat{\pi}_t \\ \widehat{\pi}^*_t \\ \widehat{x}_t \\ \widehat{x}^*_t\end{pmatrix} = C\left(\lambda\right)\mathbb{E}_t\begin{pmatrix}\widehat{\pi}_{t+1} \\ \widehat{\pi}^*_{t+1} \\ \widehat{x}_{t+1} \\ \widehat{x}^*_{t+1}\end{pmatrix} + D\left(\lambda\right)\begin{pmatrix}\widehat{u}_t \\ \widehat{u}^*_t \\ \widehat{m}_t \\ \widehat{m}^*_t\end{pmatrix}. \quad (17)$$

If a unique stable solution for $\left(\widehat{\pi}_t, \widehat{\pi}^*_t, \widehat{x}_t, \widehat{x}^*_t\right)^T$ indeed exists, then it is explained solely by the exogenous vector of cost-push shocks and monetary policy shocks $\left(\widehat{u}_t, \widehat{u}^*_t, \widehat{m}_t, \widehat{m}^*_t\right)^T$.[13] This result follows because the home and foreign natural rates of interest, $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}^*_t$ respectively, drop out of the dynamic IS equations in (14) − (15) under the monetary policy rules in (6) − (7). Hence, the productivity shocks, $\widehat{a}_t$ and $\widehat{a}^*_t$, which enter into the model solely

[13] Blanchard and Kahn (1980) and King and Watson (1998), among others, discuss general conditions that ensure local existence and uniqueness. The determinacy region of this open-economy equilibrium conditions is studied in Martínez-García (2019).

through the output potential, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$, in equations (11) – (12) and through the natural rate of interest, $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$, in equations (9) – (10) drop out as well.

The mapping of the home and foreign natural rates into productivity shocks is given by:

Lemma 2 *The natural rate of interest in each country* (9) – (10) *is a linear combination of the home and foreign productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$, similar in structure to the output potential for each country given by* (11) – (12)*.*

Putting together the equations that describe output potential in both countries given by (11) – (12) and the stationary VAR(1) process for the productivity shocks posited in (13) with the equations that describe the natural rate of interest in (9) – (10), the natural rates can be expressed in terms of the home and foreign productivity as follows:

$$\widehat{\overline{r}}_t \approx \gamma \left(\frac{1+\varphi}{\gamma+\varphi} \right) \left[\begin{array}{c} \left(\Theta^{\Lambda} (\delta_a - 1) + \left(1 - \Theta^{\Lambda}\right) \delta_{a,a^*} \right) \widehat{a}_t + ... \\ \left(\Theta^{\Lambda} \delta_{a,a^*} + \left(1 - \Theta^{\Lambda}\right) (\delta_a - 1) \right) \widehat{a}_t^* \end{array} \right], \tag{18}$$

$$\widehat{\overline{r}}_t^* \approx \gamma \left(\frac{1+\varphi}{\gamma+\varphi} \right) \left[\begin{array}{c} \left(\Theta^{\Lambda} \delta_{a,a^*} + \left(1 - \Theta^{\Lambda}\right) (\delta_a - 1) \right) \widehat{a}_t + ... \\ \left(\Theta^{\Lambda} (\delta_a - 1) + \left(1 - \Theta^{\Lambda}\right) \delta_{a,a^*} \right) \widehat{a}_t^* \end{array} \right], \tag{19}$$

where I define the composite coefficient Θ^{Λ} to be $\Theta^{\Lambda} \equiv (\Theta\Lambda + (1-\Theta)(1-\Lambda)) > 0$. Hence, the impact of productivity shocks on the natural rate depends not just on the preference parameters but also on the persistence of the productivity shocks, δ_a, and on the technological diffusion parameter, δ_{a,a^*}.

The equations in (18) – (19) are a linear mapping of the productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$, into the natural rates, $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$. Together with the VAR specification for the productivity shocks in (13), these equations are fundamental to recover the natural interest rates and to analyze their determinants. Accordingly, the selection of observable variables with which to pin down the productivity process and disentangle exogenous productivity spillovers will be critical for identification purposes in my subsequent estimation. For that, some more consideration must be given to the vector of endogenous variables $\widehat{Y}_t = (\widehat{x}_t, \widehat{x}_t^*, \widehat{\pi}_t, \widehat{\pi}_t^*, \widehat{r}_t, \widehat{r}_t^*)^T$ corresponding to the main log-linearized equilibrium conditions of the model (the Phillips curve, the dynamic IS equation, and the monetary policy rule of both countries).

First, I recall the home and foreign output definitions:

$$\widehat{y}_t \equiv \widehat{\overline{y}}_t + \widehat{x}_t, \tag{20}$$

$$\widehat{y}_t^* \equiv \widehat{\overline{y}}_t^* + \widehat{x}_t^*, \tag{21}$$

from which home and foreign output, $\widehat{y}_t$ and $\widehat{y}_t^*$, can be related to the corresponding output potential and output gap and, therefore, mapped to observable data. However, given the linear-in-labor production technology assumed in the model, I find more persuasive to incorporate information more directly tied to the productivity shocks using instead the observable measured labor productivity of each country as it holds that:

$$\widehat{y}_t - \widehat{l}_t \approx \widehat{a}_t, \tag{22}$$

$$\widehat{y}_t^* - \widehat{l}_t^* \approx \widehat{a}_t^*, \tag{23}$$

where $\widehat{l}_t$ and $\widehat{l}_t^*$ denote home and foreign employment. This helps strengthen the identification of my estimates around the natural rates of interest which is, after all, the main concern in this paper. Another practical advantage of using measured labor productivity as an observable is that it implicitly takes into account population growth and demographic trends present in the data, but about which the structural model itself is silent.

Second, the monetary policy rule equations in (6) − (7) expressed in terms of the home and foreign real rates are needed to complete the model. While the endogenous real rate can be mapped to survey-based measures of the U.S. real interest rate that are readily available, the same cannot be said for the rest of the world real rate. Given that, I opt for practical reasons to approximate the solution constrained by the zero-lower bound for the rest of the world with the unconstrained one. The short-term nominal rate for the rest of the world is well above zero within my estimation sample, so I do not expect that abstracting from the rest of the world zero-lower bound constraint will introduce significant distortions in the estimation. In doing so, I then use the Fisher equation to relate the foreign real rate of interest, $\widehat{r}_t^*$, to the observable nominal interest rate, $\widehat{i}_t^*$, as follows:

$$\widehat{r}_t^* \approx \widehat{i}_t^* - \mathbb{E}_t\left(\widehat{\pi}_{t+1}^*\right). \tag{24}$$

Accordingly, I take the home real rate and the foreign nominal rate, $\widehat{r}_t$ and $\widehat{i}_t^*$, as my two observable policy variables.

Third, I can re-write the open-economy Phillips curves in (1) − (2) and the dynamic IS equilibrium equations in (4) − (5) together with the output definitions in (20) − (21) as follows:

$$\begin{pmatrix} \widehat{\pi}_t \\ \widehat{\pi}^*_t \\ \widehat{y}_t \\ \widehat{y}^*_t \end{pmatrix} = \Phi_1(\lambda) \begin{pmatrix} \mathbb{E}_t(\widehat{\pi}_{t+1}) \\ \mathbb{E}_t(\widehat{\pi}^*_{t+1}) \\ \mathbb{E}_t(\widehat{x}_{t+1}) \\ \mathbb{E}_t(\widehat{x}^*_{t+1}) \end{pmatrix} + \Phi_2(\lambda) \begin{pmatrix} \widehat{r}_t \\ \widehat{r}^*_t \end{pmatrix} + \Phi_3(\lambda) \begin{pmatrix} \widehat{a}_t \\ \widehat{a}^*_t \\ \widehat{u}_t \\ \widehat{u}^*_t \end{pmatrix}, \tag{25}$$

where

$$\Phi_1(\lambda) \equiv \begin{pmatrix} \beta & 0 & \Phi(\varphi+\gamma)\kappa & \Phi(\varphi+\gamma)(1-\kappa) \\ 0 & \beta & \Phi(\varphi+\gamma)(1-\kappa) & \Phi(\varphi+\gamma)\kappa \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\Phi_2(\lambda) \equiv \begin{pmatrix} -\Phi(\varphi+\gamma)\frac{1}{\gamma}(\kappa\Omega+(1-\kappa)(1-\Omega)) & -\Phi(\varphi+\gamma)\frac{1}{\gamma}(\kappa(1-\Omega)+(1-\kappa)\Omega) \\ -\Phi(\varphi+\gamma)\frac{1}{\gamma}(\kappa(1-\Omega)+(1-\kappa)\Omega) & -\Phi(\varphi+\gamma)\frac{1}{\gamma}(\kappa\Omega+(1-\kappa)(1-\Omega)) \\ -\frac{\Omega}{\gamma} & -\frac{1}{\gamma}(1-\Omega) \\ -\frac{1}{\gamma}(1-\Omega) & -\frac{\Omega}{\gamma} \end{pmatrix},$$

$$\Phi_3(\lambda) \equiv \begin{pmatrix} \Phi(1+\varphi)\Delta_1^\kappa & \Phi(1+\varphi)\Delta_2^\kappa & \Phi(1+\varphi)(1-\xi) & \Phi(1+\varphi)\xi \\ \Phi(1+\varphi)\Delta_2^\kappa & \Phi(1+\varphi)\Delta_1^\kappa & \Phi(1+\varphi)\xi & \Phi(1+\varphi)(1-\xi) \\ \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left(\Lambda+\Delta_1^{a,a^*}\right) & \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left((1-\Lambda)+\Delta_2^{a,a^*}\right) & 0 & 0 \\ \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left((1-\Lambda)+\Delta_2^{a,a^*}\right) & \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left(\Lambda+\Delta_1^{a,a^*}\right) & 0 & 0 \end{pmatrix}$$

with $\Delta_1^\kappa \equiv \left[\kappa\Delta_1^{a,a^*} + (1-\kappa)\Delta_2^{a,a^*}\right]$ and $\Delta_2^\kappa \equiv \left[(1-\kappa)\Delta_1^{a,a^*} + \kappa\Delta_2^{a,a^*}\right]$.

Then, the present-value form of the equilibrium conditions in (25) can be written as:

$$\begin{pmatrix} \widehat{\pi}_t \\ \widehat{\pi}^*_t \\ \widehat{y}_t \\ \widehat{y}^*_t \end{pmatrix} = \Phi_2(\lambda) \begin{pmatrix} \widehat{i}_t \\ \widehat{i}^*_t \end{pmatrix} + \sum\nolimits_{\tau=1}^{T} \Phi_1(\lambda)^\tau \Phi_2(\lambda) \begin{pmatrix} \mathbb{E}_t(\widehat{r}_{t+\tau}) \\ \mathbb{E}_t(\widehat{r}^*_{t+\tau}) \end{pmatrix} + \ldots$$
$$\left[\sum\nolimits_{\tau=0}^{T} \Phi_1(\lambda)^\tau \Phi_2(\lambda) A_1^\tau\right] \begin{pmatrix} \widehat{a}_t \\ \widehat{a}^*_t \\ \widehat{u}_t \\ \widehat{u}^*_t \end{pmatrix} + \Phi_1(\lambda)^{T+1} \begin{pmatrix} \mathbb{E}_t(\widehat{\pi}_{t+T+1}) \\ \mathbb{E}_t(\widehat{\pi}^*_{t+T+1}) \\ \mathbb{E}_t(\widehat{x}_{t+T+1}) \\ \mathbb{E}_t(\widehat{x}^*_{t+T+1}) \end{pmatrix}, \tag{26}$$

where $A_1 \equiv \begin{pmatrix} \delta_a & \delta_{a,a^*} & 0 & 0 \\ \delta_{a,a^*} & \delta_a & 0 & 0 \\ 0 & 0 & \delta_u & 0 \\ 0 & 0 & 0 & \delta_u \end{pmatrix}$.

Ruling out bubbles in the solution with $\lim_{T \to +\infty} \Phi_1(\lambda)^{T+1} \begin{pmatrix} \mathbb{E}_t(\widehat{\pi}_{t+T+1}) \\ \mathbb{E}_t(\widehat{\pi}^*_{t+T+1}) \\ \mathbb{E}_t(\widehat{x}_{t+T+1}) \\ \mathbb{E}_t(\widehat{x}^*_{t+T+1}) \end{pmatrix} = 0$, it follows from (26) that the dynamics of output and inflation for the home and foreign countries are related to the contemporaneous realization of the vector of productivity shocks and cost-push shocks $(\widehat{a}_t, \widehat{a}^*_t, \widehat{u}_t, \widehat{u}^*_t)^T$ as well as to the path of current and expected future real interest rates, $\widehat{r}_t$ and $\widehat{r}^*_t$, i.e.,

$$\begin{pmatrix} \widehat{\pi}_t \\ \widehat{\pi}^*_t \\ \widehat{y}_t \\ \widehat{y}^*_t \end{pmatrix} = \Phi_2(\lambda) \begin{pmatrix} \widehat{r}_t \\ \widehat{r}^*_t \end{pmatrix} + \sum_{\tau=1}^{+\infty} \Phi_1(\lambda)^{\tau} \Phi_2(\lambda) \begin{pmatrix} \mathbb{E}_t(\widehat{r}_{t+\tau}) \\ \mathbb{E}_t(\widehat{r}^*_{t+\tau}) \end{pmatrix} + \left[\sum_{\tau=0}^{+\infty} \Phi_1(\lambda)^{\tau} \Phi_2(\lambda) A_1^{\tau}\right] \begin{pmatrix} \widehat{a}_t \\ \widehat{a}^*_t \\ \widehat{u}_t \\ \widehat{u}^*_t \end{pmatrix}. \tag{27}$$

Equation (27) can be re-written similarly replacing the foreign real interest rate with the foreign nominal interest rate using the Fisher equation in (24). However, that is of no practical consequence because neither the endogenous expectations of the nominal nor the real foreign interest rate can be constrained with observed data.

Equation (27) suggests that expectations about future inflation and output *per se* are not required if I include expectations about the current and future real interest rate with which to discipline the solution of the model in my estimation. I appeal to this in order to augment the set of observables with survey-based measures of the contemporaneous and expected future real interest rate for the U.S. Those forecasts are formed by private agents that recognize the significance of the zero-lower bound on monetary policy and, therefore, help bring some consistency with the implications of the zero-lower bound to my empirical inferences and estimates based on the set of log-linearized equilibrium conditions described in Subsection 2.1.

Fourth, the current short-term home nominal interest rate, $\widehat{i}_t$, is observable and there is survey data on expected home inflation $(h+1)-$quarters ahead, $\mathbb{E}^{survey}_t(\widehat{\pi}_{t+h})$, and on the expected home short-term nominal interest rate $h-$quarters ahead, $\mathbb{E}^{survey}_t\left(\widehat{i}_{t+h}\right)$. Here I use the superscript *survey* to refer to the observed forecast data. Given the home country counterpart of (24), i.e., the home country Fisher equation:

$$\widehat{r}_t \approx \widehat{i}_t - \mathbb{E}_t\left(\widehat{\pi}_{t+1}\right), \tag{28}$$

it follows that the expectations for the home real interest rate at different horizons $h > 0$ can be constructed as:

$$\mathbb{E}_t\left(\widehat{r}_{t+h}\right) \approx \mathbb{E}_t\left(\widehat{i}_{t+h}\right) - \mathbb{E}_t\left(\widehat{\pi}_{t+h+1}\right). \tag{29}$$

By analogy, I posit the following auxiliary equations allowing for some measurement error on the expected future home real interest rate path:

$$\begin{aligned} &\widehat{r}_t \approx \widehat{r}_t^{survey}, \\ &\widehat{r}_t^{survey} \approx \widehat{i}_t - \mathbb{E}_t^{survey}\left(\widehat{\pi}_{t+1}\right), \end{aligned} \tag{30}$$

$$\begin{aligned} &\mathbb{E}_t^{survey}\left(\widehat{r}_{t+h}\right) \approx \mathbb{E}_t^{survey}\left(\widehat{i}_{t+h}\right) - \mathbb{E}_t^{survey}\left(\widehat{\pi}_{t+h+1}\right), \ \forall h > 0, \\ &\mathbb{E}_t\left(\widehat{r}_{t+h}\right) \approx \mathbb{E}_t^{survey}\left(\widehat{r}_{t+h}\right) + \widehat{o}_t^h, \ \forall h > 0. \end{aligned} \tag{31}$$

The measurement error term $\widehat{o}_t^h$ is modeled as i.i.d., uncorrelated Gaussian white noise, i.e.,

$$\widehat{o}_t^h \sim N\left(0, \sigma_h^2\right), \ \forall h > 0. \tag{32}$$

In my estimation, I include survey-based measures of the current U.S. real interest rate and forecasts of the U.S. real rate for $h = 1, ..., 4$ quarters ahead which is all that is available from the Blue Chip Economic Indicators survey dataset (Aspen Publishers (2020)).

Augmenting the first-order approximation of the equilibrium conditions of the two-country workhorse New Keynesian model (i.e., equations (1) − (13)) with the Fisher equation on foreign interest rates in (24) and the auxiliary measurement equations on expectations given by (31) − (32) in order to internalize the impact of the zero-lower bound on the estimated model is a novel methodological contribution of this paper. This system of equilibrium conditions and the expectations-augmented set of observables $\widehat{\widetilde{Y}}_t^o \equiv \left(\widehat{\pi}_t, \widehat{\pi}_t^*, \widehat{a}_t, \widehat{a}_t^*, \widehat{r}_t^{survey}, \widehat{i}_t^*, \mathbb{E}_t^{survey}\left(\widehat{r}_{t+1}\right), ..., \mathbb{E}_t^{survey}\left(\widehat{r}_{t+4}\right)\right)^T$ suffice to estimate the key structural parameters of the model and, in particular, to recover a consistent estimate of the U.S. natural rate of interest even in periods when U.S. policy rates were stuck at the zero-lower bound.

3 Estimation Approach

3.1 Data

I use the log-linear equilibrium conditions of the workhorse open-economy New Keynesian model and the auxiliary measurement equations described in Section 2 as my structural equations for estimation. All my data is collected from the Congressional Budget Office (CBO (2020)), the Federal Reserve Bank of Dallas' Database of Global Economic Indicators (Grossman et al. (2014)), and the Conference Board Total Economy Database™ (Conference Board (2020)). The survey data is from Blue Chip Economic Indicators (Aspen Publishers (2020)). This dataset includes time series for the U.S. and an aggregate of its 33 major trading partners from the onset of the Great Moderation period in 1984:Q1 as dated by McConnell and Pérez-Quirós (2000) until 2019:Q4.[14] The sample period, therefore, covers the entire Great Moderation period as well as the $2007-09$ global financial crisis and its aftermath. The rest of the world aggregate is trade-weighted as explained in Grossman et al. (2014).

The U.S. macro data is all from the Congressional Budget Office (CBO (2020)) and includes: (1) the quarter-over-quarter annualized inflation rate of the Consumer Price Index For All Urban Consumers (CPI-U): All Items (SA, $1982-84=1$) ($\Delta \ln CPI_t^{U.S.}$); (2) measured labor productivity calculated as the ratio between Real Gross Domestic Product (SAAR, Mil.Chn.2012.$) and the civilian employment calculated as the Civilian Labor Force: 16 Years and Over (SA, Mil.) multiplied by one minus the Civilian Unemployment Rate: 16 Years and Over (SA, in units) ($\ln LP_t^{U.S.}$); and (3) the nominal 3−Month Treasury Bill Yield (%, per annum) ($i_t^{U.S.}$). The U.S. survey data is from Blue Chip Economic Indicators (Aspen Publishers (2020)) and includes: (1) quarterly averages of the monthly reports of the Consumer Price Index Consensus Forecasts one- to five-quarters ahead in quarter-over-quarter (annualized) percent change ($\mathbb{E}_t^{survey}\left(\Delta \ln CPI_{t+1}^{U.S.}\right), ..., \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{t+5}^{U.S.}\right)$); and (2) quarterly averages of the monthly reports of the 3−Month Treasury Bill Yield Consensus Forecasts one- to four-quarters ahead in percent (per annum) ($\mathbb{E}_t^{survey}\left(i_{t+1}^{U.S.}\right), ..., \mathbb{E}_t^{survey}\left(i_{t+4}^{U.S.}\right)$). From Aspen Publishers (2020), I also obtain: (1) the 5-year expected average, 5-year forward of the annual CPI inflation rate ($\mathbb{E}_t^{survey}\left(\Delta^{ann} \ln CPI_{y+5\rightsquigarrow y+10}^{U.S.}\right)$ where the subscript y refers to the current year and the superscript ann denotes annual

[14] The countries other than the U.S. included in my sample are: Australia, Austria, Belgium, Bulgaria, Canada, Chile, China, Colombia, Costa Rica, Czech Republic, France, Germany, Greece, Hungary, India, Indonesia, Italy, Japan, Malaysia, Netherlands, Nigeria, Philippines, Poland, Portugal, Russia, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, and the U.K.

rate); and (2) the 5-year expected average, 5-year forward of the annual 3-Month Treasury Bill Yield ($\mathbb{E}_t^{survey}\left(i_{y+5\rightsquigarrow y+10}^{ann,U.S.}\right)$).[15]

The data that I collect from Grossman et al. (2014) and Conference Board (2020) for the 33 largest trading partners of the U.S. are: (1) the quarter-over-quarter annualized inflation rate on headline CPI ($\Delta \ln CPI_t^{RoW}$); (2) the measured labor productivity ($\ln LP_t^{RoW}$); and (3) the short-term nominal interest rate in percent (per annum) (i_t^{RoW}). All of the foreign country macro data is from Grossman et al. (2014) except the employment series needed to compute measured labor productivity for which I rely on the Conference Board (2020)'s Persons Employed (thousands) annual series interpolated at quarterly frequency with the Denton-Chollette interpolation method (as in Dagum and Cholette (2006)).

Mapping the endogenous variables of the workhorse open-economy New Keynesian model to the observed data requires that I filter out the trend before the estimation. Most business cycle models like the one I explore in this paper are agnostic about trends and are most pertinent for investigating business cycle frequencies, so it is customary to rely on filtered data. What's different in this case is that here I exploit the long-range survey-based forecasts available $\left(\mathbb{E}_t^{survey}\left(\Delta^{ann} \ln CPI_{y+5\rightsquigarrow y+10}^{U.S.}\right), \mathbb{E}_t^{survey}\left(i_{y+5\rightsquigarrow y+10}^{ann,U.S.}\right)\right)^T$ as a proxy for the trends on the inflation and interest rate data. For measured labor productivity, I simply assume that the permanent component of the series follows a random walk with drift. Hence, I postulate the following set of observation equations for the U.S.:

$$\Delta \ln CPI_t^{U.S.} = \overline{\pi}_t^{\text{long-run}} + \widehat{\pi}_t, \tag{33}$$

$$\ln LP_t^{U.S.} = \widehat{a}_t^T + \left(\widehat{y}_t - \widehat{l}_t\right),$$

$$\widehat{a}_t^T = \widehat{a}^T + \widehat{a}_{t-1}^T + \eta_t^{a^T}, \ \eta_t^{a^T} \sim i.i.d.N\left(0, \sigma_{a^T}^2\right), \tag{34}$$

$$i_t^{U.S.} - \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{t+1}^{U.S.}\right) = r_t^{U.S.} = \overline{i}_t^{\text{long-run}} - \overline{\pi}_t^{\text{long-run}} + \widehat{r}_t, \tag{35}$$

$$\begin{aligned}\mathbb{E}_t^{survey}\left(r_{t+h}^{U.S.}\right) &= \mathbb{E}_t^{survey}\left(i_{t+h}^{U.S.}\right) - \mathbb{E}_t^{survey}\left(i_{t+1+h}^{U.S.}\right) \\ &= \overline{i}_t^{\text{long-run}} - \overline{\pi}_t^{\text{long-run}} + \mathbb{E}_t^{survey}\left(\widehat{r}_{t+h}\right), \text{ for } h = 1, ..., 4,\end{aligned} \tag{36}$$

$$\mathbb{E}_t^{survey}\left(\Delta \ln CPI_{t+h}^{U.S.}\right) = \overline{\pi}_t^{\text{long-run}} + \mathbb{E}_t^{survey}\left(\widehat{\pi}_{t+h}\right), \text{ for } h = 1, ..., 5, \tag{37}$$

$$\mathbb{E}_t^{survey}\left(i_{t+h}^{U.S.}\right) = \overline{i}_t^{\text{long-run}} + \mathbb{E}_t^{survey}\left(\widehat{i}_{t+h}\right), \text{ for } h = 1, ..., 4, \tag{38}$$

[15] I match the long-range forecasts from the March report with Q1 and Q2 of the given year and, similarly, those of the October report with Q3 and Q4.

and, similarly, the following set of observation equations for the rest of the world aggregate:

$$\Delta \ln CPI_t^{RoW} = \overline{\pi}_t^{\text{long-run}*} + \widehat{\pi}_t^*, \tag{39}$$

$$\ln LP_t^{RoW} = \widehat{a}_t^{T*} + \left(\widehat{y}_t^* - \widehat{l}_t^*\right),$$

$$\widehat{a}_t^{T*} = \overline{a}^{T*} + \widehat{a}_{t-1}^{T*} + \eta_t^{a^{T*}}, \ \eta_t^{a^{T*}} \sim i.i.d.N\left(0, \sigma_{a^{T*}}^2\right), \tag{40}$$

$$i_t^{RoW} = \overline{i}_t^{\text{long-run}*} + \widehat{i}_t^*. \tag{41}$$

The equations in (33) − (41) map the observable series to the endogenous variables characterized by the model.[16]

Detrending Inflation and Interest Rates. I take the trend components for the expected U.S. inflation and the expected U.S. nominal short-term interest rates to be their corresponding observable survey-based long-range forecasts, i.e.,

$$\mathbb{E}_t^{survey}\left(\Delta \ln CPI_{y+5 \rightsquigarrow y+10}^{U.S.}\right) \approx \overline{\pi}_t^{\text{long-run}} \approx \overline{\pi}_t^{\text{long-run}*}, \tag{42}$$

$$\mathbb{E}_t^{survey}\left(i_{y+5 \rightsquigarrow y+10}^{ann,U.S.}\right) \approx \overline{i}_t^{\text{long-run}} \approx \overline{i}_t^{\text{long-run}*}. \tag{43}$$

The added assumption that long-run trends are approximately equal across countries is consistent with the theory, but has the practical advantage that allows me to proxy for the unobserved rest of the world long-range inflation and nominal interest rate trends with the observed survey-based long-range forecasts of U.S. inflation and the U.S. nominal short-term interest rate.

Accordingly, I use the quarter-over-quarter growth rate (in logs) for U.S. headline CPI in deviations from the U.S. long-range 5-year average, 5-year forward forecast, that is, $\Delta^{ann} \ln CPI_t^{U.S.} - \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{y+5 \rightsquigarrow y+10}^{U.S.}\right)$, as my counterpart for the cyclical home inflation, $\widehat{\pi}_t$. For the rest of the world inflation aggregate, I simply use its quarter-over-quarter growth rate (in logs) for headline CPI in deviations from the U.S. long-range 5-year average, 5-year forward forecast such that $\Delta \ln CPI_t^{RoW} - \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{y+5 \rightsquigarrow y+10}^{U.S.}\right)$ is my empirical counterpart for the cyclical foreign inflation, $\widehat{\pi}_t^*$.

I remove the trend on current and expected home real interest rates and foreign nominal

[16] The long-run inflation and nominal interest rate are defined as $\overline{\pi}_{t+h}^{\text{long-run}} \equiv \mathbb{E}_{t+h}\left(\overline{\pi}_{i \rightsquigarrow +\infty}\right)$ and $\overline{i}_{t+h}^{\text{long-run}} \equiv \mathbb{E}_{t+h}\left(\overline{i}_{i \rightsquigarrow +\infty}\right)$ for all h. Giving the properties of the expectations operator, it follows that $\mathbb{E}_t\left(\overline{\pi}_{t+h}^{\text{long-run}}\right) = \mathbb{E}_t\left(\mathbb{E}_{t+h}\left(\overline{\pi}_{i \rightsquigarrow +\infty}\right)\right) = \mathbb{E}_t\left(\overline{\pi}_{i \rightsquigarrow +\infty}\right) = \overline{\pi}_t^{\text{long-run}}$ which is a result implicit reflected in equation (37). Similar reasoning implies that $\mathbb{E}_t\left(\overline{i}_{t+h}^{\text{long-run}}\right) = \mathbb{E}_t\left(\mathbb{E}_{t+h}\left(\overline{i}_{i \rightsquigarrow +\infty}\right)\right) = \mathbb{E}_t\left(\overline{i}_{i \rightsquigarrow +\infty}\right) = \overline{i}_t^{\text{long-run}}$ as reflected in (38).

interest rates analogously. I start by removing the trend on expected U.S. inflation with the U.S. long-range inflation forecast such that $\mathbb{E}_t^{survey}\left(\Delta \ln CPI_{t+h}^{U.S.}\right) - \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{y+5 \rightsquigarrow y+10}^{U.S.}\right)$ is the empirical counterpart of $\mathbb{E}_t\left(\widehat{\pi}_{t+h}\right)$, for $h = 0, ..., 4$. I also detrend the expected U.S. short-term nominal interest rate with the U.S. long-range interest rate forecast such that $\mathbb{E}_t^{survey}\left(i_{t+h}^{U.S.}\right) - \mathbb{E}_t^{survey}\left(i_{y+5 \rightsquigarrow y+10}^{ann,U.S.}\right)$ is my empirical counterpart for $\mathbb{E}_t\left(\widehat{i}_{t+h}\right)$. From here, I conclude that the U.S. real interest rate detrended as $i_t^{U.S.} - \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{t+1}^{U.S.}\right) - \left(\mathbb{E}_t^{survey}\left(i_{y+5 \rightsquigarrow y+10}^{ann,U.S.}\right) - \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{y+5 \rightsquigarrow y+10}^{U.S.}\right)\right)$ is the natural empirical counterpart for $\widehat{r}_t$ and that the expected path of the U.S. real interest rate can be approximated as:

$$\mathbb{E}_t^{survey}\left(i_{t+h}^{U.S.}\right) - \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{t+h+1}^{U.S.}\right) - \left(\mathbb{E}_t^{survey}\left(i_{y+5 \rightsquigarrow y+10}^{ann,U.S.}\right) - \mathbb{E}_t^{survey}\left(\Delta \ln CPI_{y+5 \rightsquigarrow y+10}^{U.S.}\right)\right), \tag{44}$$

which constitutes the empirical counterpart for the expectations $\mathbb{E}_t\left(\widehat{r}_{t+h}\right)$, for $h = 1, ..., 4$. In regards to the rest of the world, I use the short-term nominal interest rate aggregate in deviations from the U.S. long-range 5-year average, 5-year forward nominal interest rate forecast implying that $i_t^{RoW} - \mathbb{E}_t^{survey}\left(i_{y+5 \rightsquigarrow y+10}^{ann,U.S.}\right)$ is my empirical counterpart for the cyclical foreign nominal interest rate, $\widehat{i}_t^*$.

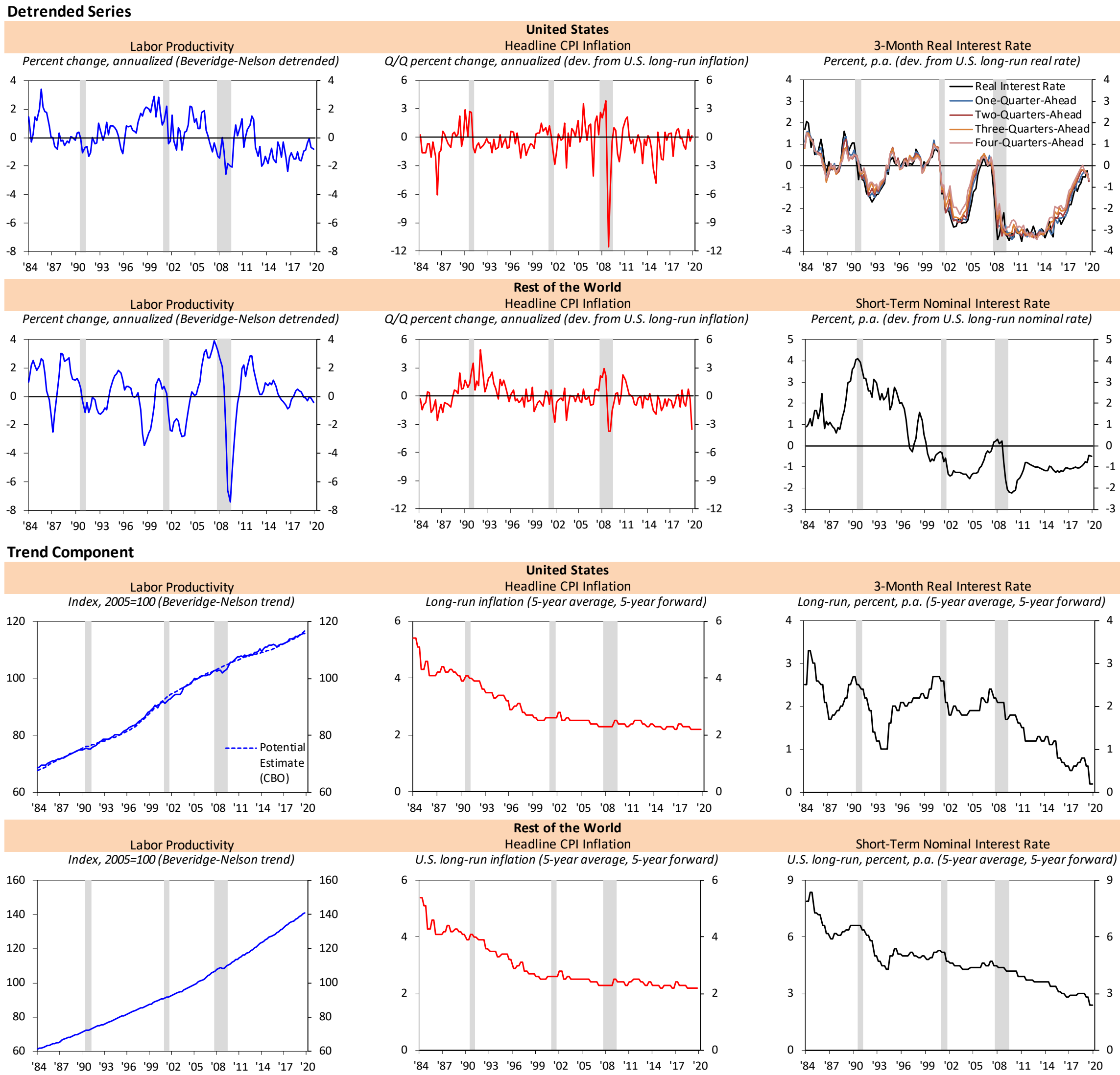
Detrending Labor Productivity. I assume the permanent component on measured labor productivity to be a random walk with drift. I convert the observed data on labor productivity into 400 times the natural logarithm for the 1980:Q1-2019:Q4 period. I test for the presence of stochastic trends in the transformed data series using the augmented Dickey-Fuller unit root test with lag selection based on the Schwarz information criterion (BIC). I fail to reject the unit root hypothesis against the alternative of stationarity at all conventional significance levels for U.S. labor productivity (t-statistic $= -1.017411$ for no lags of difference terms, with a p-value of 0.7465) as well as for the rest of the world labor productivity aggregate (t-statistic $= -0.145750$ for one lag of difference terms, with a p-value of 0.9413).

Following the footsteps of Beveridge and Nelson (1981), Morley et al. (2003), and Morley (2011), I apply the Beveridge-Nelson decomposition to detrend each labor productivity series using an AR(15) specification to approximate its cyclical component as this flexible structure works well with the measured labor productivity data. I extract the cyclical component in this way and use it as my empirical counterpart for the productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$.[17]

[17] The implementation of the Beveridge-Nelson decomposition uses the E-views add-in BNDecom with $s = 100$ steps ahead prediction implemented with E-views 10.

Figure 2. U.S. and Rest-of-the-World Dataset for the Estimation

(In Deviations from Trend)



Note: Shaded bars indicate NBER recessions in the U.S. All detrended data is expressed in percent, annualized. The long-run data used to detrend U.S. and rest of the world inflation is the 5-year average, 5-year forward of U.S. CPI inflation from Blue Chip Economic Indicators. The long-run data used to detrend U.S. and rest of the world short-term interest rates is the 5-year average, 5-year forward forecast of the U.S. 3-month Treasury bill also from. The labor productivity variables are detrended using a Beveridge-Nelson decomposition with an AR(15) on the cyclical component and s=100 forecast periods ahead. For the U.S., the trend is the inferred labor productivity trend is compared with the potential output series from the Congressional Budget Office.
Sources: Aspen Publishers (2020); CBO (2020); Conference Board (2020); Grossman et al. (2014); NBER; and author's calculations.

All the cyclical series and their corresponding trends are plotted in Figure 2. The cyclical data is reported at quarterly frequency, expressed in percentage terms, and annualized.

Notable Data Patterns. The U.S. labor productivity trend based on the Beveridge-Nelson decomposition approach described earlier tracks fairly well the estimates of labor productivity potential from CBO (2020). The Beveridge-Nelson estimates clearly imply that measured labor productivity in the U.S. has been significantly below trend since the mid-2000s, even before the $2007-09$ global financial crisis hit the world economy, at a time when most estimates of the U.S. natural rate of interest also began to fall (as can be seen in Figure 1).

Inflation targeting became quite popular during the 90s around the world as the preferred monetary policy strategy with which to stabilize inflation and inflation expectations (Bernanke and Mishkin (1997)). In the U.S. case, the disinflation engineered by Chair Paul Volcker and continued through the efforts of Chair Alan Greenspan led to a stabilization of inflation and inflation expectations around 2 percent as can be seen in the long-range U.S. inflation forecasts in Figure 2. However, it was not until the Chairmanship of Ben Bernanke that the Federal Reserve adopted many of the features of a "flexible inflation targeter." Then-Vice Chair Janet Yellen facilitated the efforts that would codify the FOMC's approach to "flexible inflation targeting" with an explicit commitment to keep long-run inflation at 2 percent in the Fed's 2012 Statement on Longer-Run Goals and Monetary Policy Strategy. The Longer-Run Statement reflected the lessons learned from fighting inflation during the previous decades and the experience of inflation-targeting central banks around the world.

Figure 1 and Figure 2 showcase that the decline in the U.S. long-range nominal interest rate during the past $10-15$ years has come from a gradual fall in the long-range U.S. real interest rate while long-range inflation expectations have remained well-anchored at about 2 percent. This decline of the long-run U.S. real and natural rates underlies many of the concerns raised during the Fed's $2019-20$ Monetary Policy Framework review (Caldara et al. (2020)). And, it also motivated the subsequent revision of the Longer-Run Statement and the adoption of flexible "average inflation targeting" announced by Chair Powell at the 2020 Jackson Hole Symposium (Powell (2020) and Clarida (2020)).

3.2 Eliciting Priors

Table 1 lists the 18 parameters of the model (8 structural parameters and 10 parameters for the exogenous shock processes) and the 4 measurement error parameters. Not all of them affect the frictionless allocation and influence the natural rate, though. In fact, only the 4 parameters that describe the exogenous productivity shock VAR(1) process and 4 of the preference parameters (the trade elasticity σ, the import share ξ, the intertemporal elasticity of substitution γ, and the inverse of the Frisch elasticity of labor supply φ) affect the frictionless allocation.

Of the 22 parameters of the estimated model, I parameterize only the intertemporal discount factor β set to 0.995012479 in order to attain an annualized real interest rate of about $-400 \ln(\beta) = 2$ percent. All other 21 parameters are estimated and, therefore, require that I take a stand on the priors for each. For that, I follow closely the approach to prior selection for the workhorse open-economy New Keynesian model laid out in Martínez-García et al. (2012), Martínez-García and Wynne (2014), and Martínez-García (2015).

Structural parameters. I center the prior of the inverse of the Frisch elasticity of labor supply φ and the prior of the intertemporal elasticity of substitution γ both at 5. I adopt a Gamma prior distribution for these two parameters where I impose a loose prior standard deviation of 0.25 in order to let the data speak for itself. The frequency of price adjustments is tied to the Calvo (1983) parameter, α, and for this I adopt a Beta prior centered at 0.75 with a tight prior standard deviation of 0.02. The prior mean for α implies that prices remain unchanged for an average of four quarters. I adopt the Gamma distribution centered around 1.5 for the intratemporal elasticity of substitution between home and foreign goods, σ, based on the trade elasticity values of Backus et al. (1994). For this prior, I adopt a standard deviation of 0.15. For the share of imported goods in the consumption basket ξ, I choose a tight Beta distribution centered around 0.18 with a small standard deviation of 0.01. This prior is centered around an average U.S. import share of 18% to be consistent with the evidence reported in Martínez-García (2018). All of these choices are largely consistent with those found elsewhere in the international macro literature (see, e.g., Chari et al. (2002), Martínez-García et al. (2012), Martínez-García and Søndergaard (2013), and Martínez-García and Wynne (2014), among others).

I estimate the policy parameter that determines the response to inflation, $\psi_\pi - 1$, with a Gamma prior centered at 0.5 and a prior standard deviation of 0.01. Similarly, I select a Gamma distribution for the parameter that defines the policy response to fluctuations of the output gap, ψ_x, with a prior mean of 0.5 and a standard deviation of 0.01. These priors are centered around the conventional values advocated by Taylor (1993) in his seminal exploration of the conduct of monetary policy in the U.S. The domain of the Gamma prior for $\psi_\pi - 1$ ensures that the Taylor principle is satisfied (i.e., $\psi_\pi > 1$) and, accordingly, that only a very small probability is placed on parameter values for which a solution does not exist or is not unique. In any event, all draws that fall outside the determinacy region are discarded in the estimation.

Shock process and measurement error parameters. I estimate the restricted specification of the VAR(1) process for productivity shocks in (13) using the detrended labor productivity series described in Subsection 3.1 (and plotted in Figure 2). The estimates I get are fairly similar to those found in the literature (e.g., in Heathcote and Perri (2002)), albeit perhaps a bit less persistent. Based on that evidence, I set the prior means of the productivity shock parameters to match those estimates: the prior mean of δ_a (the persistence parameter) is set to 0.87, the prior mean of δ_{a,a^*} (the cross-country spillover parameter) to -0.008 although I should note here that in my estimation this coefficient is not statistically different from zero, and the prior mean of ρ_{a,a^*} (the correlation between domestic and foreign innovations) to 0.15. The prior mean for the volatility of both series σ_a is set to 0.79 which equals the standard deviation of the U.S. labor productivity residuals.[18] I select Beta priors for δ_a, δ_{a,a^*}, and ρ_{a,a^*} with very tight standard deviations of 0.001, 0.001, and 0.01 respectively. The prior distribution for the volatility σ_a is the Inverse Gamma with again a very tight prior of 0.001. This aims to keep the parameters for the VAR(1) process that describes the productivity shocks close to their estimated values.

I choose a Beta distribution for the first-order autocorrelation of the monetary shock, δ_m, as well as for the persistence of the cost-push shock, δ_u. The priors are centered around 0.90 and 0.50, respectively, with a fairly tight prior standard deviation equal to 0.01. The prior volatilities of the monetary shock and the cost-push shock, σ_m and σ_u, are centered at 0.50, and 0.10, respectively. I select an Inverse Gamma distribution to represent the prior of each of these volatility parameters, with a standard deviation of 0.01 for both. I choose Beta priors for the cross-country correlation of the monetary policy innovations and the cost-push shock innovations, ρ_{m,m^*} and ρ_{u,u^*}. I center both at 0 with a standard deviation of 0.01. Finally, I adopt an Inverse Gamma prior distribution for the measurement error volatilities, $\sigma_1, ..., \sigma_4$, all of which are centered at 0.15 with a very tight standard deviation of 0.005.

[18] The restricted bivariate VAR(1) estimates are omitted in the paper, but can be found in Martínez-García (2020).

Table 1 - Structural and Shock Parameters: Parameterization and Prior Distributions

Structural Parameters		Domain	Density	Mean	Std. Dev.
		Prior Distributions			
Non-policy parameters					
Intertemporal discount factor	β	$(0,1)$	Fixed	0.995	–
Inv. Intert. elasticity of substitution	γ	$\mathbb{R}^+$	Gamma	5	0.25
Inv. Frisch elasticity of labor supply	φ	$\mathbb{R}^+$	Gamma	5	0.25
Elast. of subst. Home & Foreign	σ	$\mathbb{R}^+$	Gamma	1.50	0.15
Cons. share of foreign goods	ξ	$(0,1)$	Beta	0.18	0.01
Calvo (1983) price stickiness	α	$(0,1)$	Beta	0.75	0.02
Policy parameters					
Response to inflation	$\psi_\pi - 1$	$\mathbb{R}^+$	Gamma	0.50	0.01
Response to output gap	ψ_x	$\mathbb{R}^+$	Gamma	0.50	0.01
Shock Parameters					
Productivity shock persistence	δ_a	$(-1,1)$	Beta	0.87	0.001
Productivity shock spillover	δ_{a,a^*}	$(-1,1)$	Beta	-0.008	0.001
Productivity shock volatility	σ_a	$\mathbb{R}^+$	InvGamma	0.79	0.001
Productivity shock corr. innovations	ρ_{a,a^*}	$(-1,1)$	Beta	0.15	0.01
Cost-push shock persistence	δ_u	$(-1,1)$	Beta	0.50	0.01
Cost-push shock volatility	σ_u	$\mathbb{R}^+$	InvGamma	0.10	0.01
Cost-push shock corr. innovations	ρ_{u,u^*}	$(-1,1)$	Beta	0.00	0.01
Monetary shock persistence	δ_m	$(-1,1)$	Beta	0.90	0.01
Monetary shock volatility	σ_m	$\mathbb{R}^+$	InvGamma	0.50	0.01
Monetary shock corr. innovations	ρ_{m,m^*}	$(-1,1)$	Beta	0.00	0.01
Measurement error exp. real rate (one-quarter-ahead)	σ_1	$\mathbb{R}^+$	InvGamma	0.15	0.005
Measurement error exp. real rate (two-quarters-ahead)	σ_2	$\mathbb{R}^+$	InvGamma	0.15	0.005
Measurement error exp. real rate (three-quarters-ahead)	σ_3	$\mathbb{R}^+$	InvGamma	0.15	0.005
Measurement error exp. real rate (four-quarters-ahead)	σ_4	$\mathbb{R}^+$	InvGamma	0.15	0.005

Note: In order to reduce the indeterminacy region consistent with the Taylor principle, I transform the sensitivity of monetary policy to deviations from the inflation target ψ_π to adjust its range (set above one) to the domain of the Gamma distribution. Restricting the policy parameter ψ_π to satisfy the Taylor principle in this open-economy model is neither necessary nor sufficient to ensure determinacy but is a conventional practice to truncate the parameter space to lie mostly on the determinacy region.

Note: Para (v) and Para (s) list the pair of parameters that characterize each prior distribution. For the Normal distribution, the mean is μ=v and the variance is σ^2=s^2. For the Beta distribution, the mean is μ=$v/(v+s)$ and the variance is σ^2=$vs/((v+s)^2(v+s+1))$. For the Gamma distribution, the mean is μ=vs and the variance is σ^2=vs^2. For the Uniform distribution, the upper and lower bound of the support are v and s respectively, while the mean is μ=$(v+s)/2$ and the variance is σ^2=$(v-s)^2/12$. For the Inverse Gamma distribution, the mean is μ=s/(v-1) and the variance is σ^2=$s^2/((v-1)^2(v-2))$.

3.3 Methodology

I set the number of observables used for the estimation to be equal to the number of structural and measurement error shocks in the model to avoid the well-known stochastic singularity problem. I take as given that the observable endogenous variables vector is given by $\widehat{\widetilde{Y}}_t^o \equiv \left(\widehat{\pi}_t, \widehat{\pi}_t^*, \widehat{a}_t, \widehat{a}_t^*, \widehat{r}_t^{survey}, \widehat{i}_t^*, \mathbb{E}_t^{survey}\left(\widehat{r}_{t+1}\right), ..., \mathbb{E}_t^{survey}\left(\widehat{r}_{t+h}\right)\right)^T$. The selection of this vector of observables, albeit conditioned partly by data availability, has been motivated through theory in Subsection 2.2 as a plausible way to bring the workhorse open-economy New Keynesian model to the data in a way that is consistent with the implications of the zero-lower bound constraint.[19] I estimate the equilibrium conditions and auxiliary measurement equations of the model with Bayesian methods, as surveyed by Martínez-García et al. (2012) and Martínez-García and Wynne (2014), among others.

The vector $\widetilde{\lambda} = \left(\gamma, \varphi, \sigma, \xi, \alpha, \psi_\pi, \psi_x; \delta_a, \delta_{a,a^*}, \sigma_a, \rho_{a,a^*}, \delta_u, \sigma_u, \rho_{u,u^*}, \delta_m, \sigma_m, \rho_{m,m^*}, \sigma_1, ..., \sigma_4\right)^T$ includes the 21 parameters to be estimated. With the software package Dynare (see, e.g., Villemot (2011)), the Bayesian estimation proceeds as follows: for a given draw of $\widetilde{\lambda}$, the model is solved to obtain its state-space representation. If a unique stable solution exists, then the Kalman filter evaluates the likelihood function $\mathcal{L}\left(\widetilde{\lambda} \mid \widehat{\widetilde{Y}}_t^o\right)$ in order to infer the posterior as $p\left(\widetilde{\lambda} \mid \widehat{\widetilde{Y}}^o\right) \propto \mathcal{L}\left(\widetilde{\lambda} \mid \widehat{\widetilde{Y}}^o\right) p\left(\widetilde{\lambda}\right)$ where $p\left(\widetilde{\lambda}\right)$ is the prior density. Otherwise, $\mathcal{L}\left(\widetilde{\lambda} \mid \widehat{\widetilde{Y}}^o\right) p\left(\widetilde{\lambda}\right)$ is set to zero. The Monte Carlo-based Metropolis-Hastings (MH) algorithm generates two Markov chains with a stationary distribution on the basis of $1,000,000$ draws per chain. That approximates the posterior distribution of the vector $\widetilde{\lambda}$ which, under general regularity conditions, is asymptotically normal around the mode. The algorithm implemented then goes on to maximize the posterior density kernel with a Newton-type optimization routine.

4 Main Empirical Findings

Why should I be concerned with the open-economy model? Because ignoring endogenous and exogenous cross-country spillovers can bias the recovered path of the natural rate of interest and of output potential for the U.S. It can also result in misleading empirical inferences and

[19] Incidentally, another practical use of the model is that it can shape one's views about the preferred estimation strategy (priors, observable variables, etc.) given that it provides the testing grounds where to "safely" detect estimation problems such as those that arise from weak identification. Estimating the model with simulated data has been quite useful to inform my own work in this paper.

an erroneous characterization of the driving forces (shocks) that account for the observed data through the lens of the model. Furthermore, in the presence of weak structural identification it may be possible to specify the *wrong* model (say, the closed-economy variant) and select priors and observables in such a way as to approximate reasonably well the empirical fit of the *true* model (the open-economy model) by muting the invalid or incorrect cross-equation restrictions of the *wrong* specification. For policy analysis purposes, the errors that arise from selecting a misspecified model instead of the *true* one can be very significant indeed.

All these issues are extensively discussed in Martínez-García et al. (2012) and Martínez-García and Wynne (2014). Being aware of that, I consider the baseline open-economy model as my benchmark but I also investigate two alternative specifications with which to tease out the potential biases that could arise from a misspecified or weakly identified estimated model.

∘ One alternative explores the extent to which priors on the exogenous technological propagation across countries affect the performance of the open-economy model. To be more precise, I consider a scenario where productivity shock innovations are thought to be largely uncorrelated across countries with ρ_{a,a^*} centered around 0 while exogenous technological diffusion through the parameter δ_{a,a^*} centered around 0.12 becomes the dominant mechanism for the international propagation of the productivity shocks (unlike what I observe with the detrended labor productivity data). In this case, I retain a degree of uncertainty keeping the same prior distribution and standard deviation used in the benchmark for both parameters.

∘ The other alternative considers the implications of ignoring the impact of cross-country endogenous and exogenous interconnectedness by assuming a closed-economy framework that sets a degenerate prior distribution on the steady state import share ξ with probability mass one on the value zero. Another additional preference parameters is also dropped from the closed-economy specification as a consequence of this: that is the trade elasticity σ which together with the degree of openness ξ are the two key parameters that determine the endogenous cross-country propagation of shocks in this framework. Furthermore, the closed-economy case excludes also four parameters related to the exogenous international shock propagation (the cross-country productivity spillovers δ_{a,a^*} and the three parameters that describe the comovement of shock innovations ρ_{a,a^*}, ρ_{u,u^*}, and ρ_{m,m^*}) whose prior distribution is also degenerate with probability mass one set on the value zero. In other words, this closed-economy estimate comes closest to the structural model of the U.S. natural rate proposed by Cúrdia et al. (2015) which abstracts entirely from endogenous as well as exogenous spillovers from the rest of the world.

With all of this in mind, I now proceed to explore empirically the evidence on the U.S.

natural rate of interest and the U.S. output potential through the lens of the workhorse open-economy New Keynesian model. I concurrently also assess the significance of the cross-country endogenous and exogenous spillovers in the data.

4.1 Estimates

In Table 2, I report the estimated parameters of the open-economy New Keynesian model under the baseline priors summarized in Table 1. I also consider the estimation when the prior means on the cross-country productivity spillovers and the productivity shock innovation correlation parameters are chosen to emphasize technological diffusion instead of the mechanism found in the data which depends much more on the correlation of the productivity shock innovations (i.e., setting the prior means at $\delta_{a,a^*} = 0.12$, $\rho_{a,a^*} = 0$ instead of at their data-consistent values $\delta_{a,a^*} = -0.008$, $\rho_{a,a^*} = 0.15$). These alternative prior means have the potential to bias the estimates of the model if the key exogenous propagation parameters, δ_{a,a^*} and ρ_{a,a^*}, turn out to be weakly identified.

I find that, with the given tight priors, the estimates of δ_{a,a^*} and ρ_{a,a^*} end up dominated by one's prior mean choices as can be seen from the reported Bayesian estimates in Table 2. While this suggests that both parameters are indeed weakly identified, the results in Table 2 indicate that elsewhere the estimation bias introduced by the choice of priors on those two productivity shock process parameters has only limited effects. In fact, Table 2 shows that all parameter estimates except δ_{a,a^*} and ρ_{a,a^*} themselves are almost identical irrespective of what prior beliefs I hold about them.

Adding the cyclical path recovered from the estimated model to the trend in the data given by the long-range U.S. real interest rate, Plot A in Figure 3 illustrates the estimates of the U.S. natural rate of interest in levels. Plot A in Figure 3 shows that the evolution over time of the U.S. natural rate of interest estimated with the baseline open-economy model is similar to that seen in most conventional (semi-structural or time series) estimates of the U.S. natural rate reported in the literature (as shown earlier in Figure 1). Moreover, the long-range U.S. natural rate inferred from the benchmark model as the 5-year average, 5-year forward rate overlaps almost exactly with the long-range real interest rate obtained from Aspen Publishers (2020) as theory would suggest.

Table 2 - Structural and Shock Parameters: Posterior Distributions

	Structural Parameters							
	Log-density: -4935.043161				Log-density: -4960.411632			
	Baseline				Alternative Prior δ_{a,a^*}, ρ_{a,a^*}			
	Prior μ	Post. μ	95%-CI		Prior μ	Post. μ	95%-CI	
γ	5	6.07	5.63	6.49	5	5.94	5.51	6.36
φ	5	4.43	4.04	4.80	5	4.39	4.01	4.78
σ	1.5	0.42	0.34	0.49	1.5	0.49	0.41	0.57
ξ	0.18	0.13	0.12	0.14	0.18	0.13	0.12	0.15
α	0.75	0.78	0.75	0.81	0.75	0.77	0.74	0.79
$\psi_\pi - 1$	0.5	0.45	0.44	0.47	0.5	0.45	0.44	0.47
ψ_x	0.5	0.47	0.46	0.49	0.5	0.47	0.46	0.49
	Exogenous Shock Parameters							
δ_a	0.87	0.86	0.86	0.87	0.87	0.87	0.87	0.87
δ_{a,a^*}	-0.008	-0.009	-0.010	-0.007	0.12	0.12	0.12	0.12
σ_a	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79
ρ_{a,a^*}	0.15	0.15	0.14	0.17	0.00	0.00	-0.02	0.02
δ_u	0.50	0.82	0.81	0.83	0.50	0.79	0.78	0.80
σ_u	0.10	0.91	0.80	1.02	0.10	0.88	0.77	0.98
ρ_{u,u^*}	0.00	-0.01	-0.02	0.01	0.00	-0.01	-0.03	0.01
δ_m	0.90	0.81	0.80	0.83	0.90	0.78	0.77	0.80
σ_m	0.50	1.40	1.36	1.45	0.50	1.40	1.35	1.45
ρ_{m,m^*}	0.00	0.03	0.01	0.04	0.00	0.03	0.01	0.04
	Measurement Error							
σ_1	0.15	0.23	0.22	0.24	0.15	0.24	0.23	0.25
σ_2	0.15	0.32	0.30	0.34	0.15	0.35	0.33	0.36
σ_3	0.15	0.38	0.36	0.40	0.15	0.42	0.39	0.44
σ_4	0.15	0.41	0.39	0.43	0.15	0.46	0.42	0.47

This table reports the point estimates and 95 percent confidence intervals (CI) for all the model parameters. I estimate the model with the observed data over the full sample starting in 1984:Q1 and ending in 2019:Q4. Estimates are reported also for the scenario where the productivity shock parameters δ_{a,a^*} and ρ_{a,a^*} are centered around a different prior mean. The prior means recorded for the baseline are those summarized in Table 1. I use Matlab 7.13.0.564 and Dynare v4.2.4 for the stochastic simulation and estimation.

Plot B of Figure 3 allows me to compare the baseline open-economy estimate with the U.S. natural rate inferred under the alternative prior specification considered in Table 2 on two key parameters of the bivariate productivity shock process. The estimated exogenous productivity spillovers are significantly different and that can potentially impact the U.S. natural rate recovered as those parameters enter directly into the natural rate formula given in equation (18). However, as can be seen in Figure 3, the different prior choices considered here turn out to have only modest effects on the recovered U.S. natural rate of interest in levels. That suggests that, while in theory the form in which productivity shocks exogenously propagates across countries is an important consideration, in practice the U.S. data suggests that it has had only a limited effect on the U.S. natural rate of interest.

The international transmission of shocks depends on the endogenous trade mechanism as well. In Table 3, I report the results comparing the estimates that I obtain from the baseline open-economy model against those of the closed-economy alternative. I find that the structural parameter estimates show more substantive differences in this case than for the exercise reported in Table 2 that explores instead the exogenous cross-country propagation of productivity shocks. Interestingly, the estimation of the inverse of the elasticity of intertemporal substitution γ is one of the parameters most affected—with estimates significantly lower in the closed-economy case than in the open-economy case. The fact that open-economy models tend to require larger values of γ is something that was already recognized by Martínez-García et al. (2012). This, in principle, could impact the U.S. natural rate estimates recovered as the preference parameter γ features prominently in the natural interest rate formula in (18).

Most importantly, perhaps, is that notable differences between the open-economy and closed-economy estimates appear on the estimated volatilities of the monetary policy shock and the cost-push shock, σ_m and σ_u respectively. The closed-economy model favors less volatility of the monetary policy shocks (about 20% less) and less volatility of the cost-push shocks (about 30% less) than the open-economy specification does. While these shocks do not have a direct effect on the frictionless equilibrium (that is, they do not affect the natural rate equation in (18) directly), they do matter a great deal for assessing the macro performance of the economy in relation to other important macro variables—output, the output gap, and inflation—at business cycle frequencies. From my findings it, therefore, follows that ignoring the open-economy dimension is likely to bias one's understanding and inferences about the impact that monetary policy shocks have on the economy as well as their contribution to business cycle fluctuations. The closed-economy specification also confounds domestic and foreign shocks and their impacts on the economy.

Table 3 - Structural and Shock Parameters: Posterior Distributions								
Structural Parameters								
	Log-density: -4935.043161				Log-density: -3939.472782			
	baseline				closed-economy			
	Prior μ	Post. μ	95%-CI		Prior μ	Post. μ	95%-CI	
γ	5	6.07	5.63	6.49	5	5.51	5.10	5.93
φ	5	4.43	4.04	4.80	5	4.62	4.22	5.00
σ	1.5	0.42	0.34	0.49	–	–	–	–
ξ	0.18	0.13	0.12	0.14	–	–	–	–
α	0.75	0.78	0.75	0.81	0.75	0.74	0.72	0.77
$\psi_\pi - 1$	0.5	0.45	0.44	0.47	0.5	0.44	0.43	0.46
ψ_x	0.5	0.47	0.46	0.49	0.5	0.49	0.48	0.51
Exogenous Shock Parameters								
δ_a	0.87	0.86	0.86	0.87	0.87	0.87	0.86	0.87
δ_{a,a^*}	-0.008	-0.009	-0.010	-0.007	–	–	–	–
σ_a	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79
ρ_{a,a^*}	0.15	0.15	0.14	0.17	–	–	–	–
δ_u	0.50	0.82	0.81	0.83	0.50	0.84	0.83	0.84
σ_u	0.10	0.91	0.80	1.02	0.10	0.64	0.56	0.71
ρ_{u,u^*}	0.00	-0.01	-0.02	0.01	–	–	–	–
δ_m	0.90	0.81	0.80	0.83	0.90	0.87	0.86	0.88
σ_m	0.50	1.40	1.36	1.45	0.50	1.13	1.09	1.16
ρ_{m,m^*}	0.00	0.03	0.01	0.04	–	–	–	–
Measurement Error								
σ_1	0.15	0.23	0.22	0.24	0.15	0.22	0.21	0.23
σ_2	0.15	0.32	0.30	0.34	0.15	0.29	0.28	0.31
σ_3	0.15	0.38	0.36	0.40	0.15	0.35	0.33	0.36
σ_4	0.15	0.41	0.39	0.43	0.15	0.37	0.35	0.39

This table reports the point estimates and 95 percent confidence intervals (CI) for all model parameters. I estimate the model with the observed data over the full sample from 1984:Q1 to 2019:Q4. The estimates are reported also for the closed-economy specification estimated for the U.S. alone. The prior means recorded for the baseline are those summarized in Table 1. The baseline results are also the same ones reported in Table 2. I use Matlab 7.13.0.564 and Dynare v4.2.4 for the stochastic simulation and estimation.

Nonetheless, when I look at Plot B of Figure 3 which includes the recovered estimates of the U.S. natural rate in levels from the closed-economy specification and compare them with those of the open-economy model, I conclude that this form of modeling misspecification produces only a modest discrepancy in the estimated path of the U.S. natural rate. In other words, the erroneous empirical inferences occur not so much because the estimates of the U.S. natural rate are radically different but because of the conceptual interpretation that one would give to the shocks driving the cyclical fluctuations of the U.S. natural rate (only U.S. productivity shocks tied to expected U.S. output potential growth in the closed-economy case vs. a combination of U.S. and foreign productivity shocks in the benchmark model).

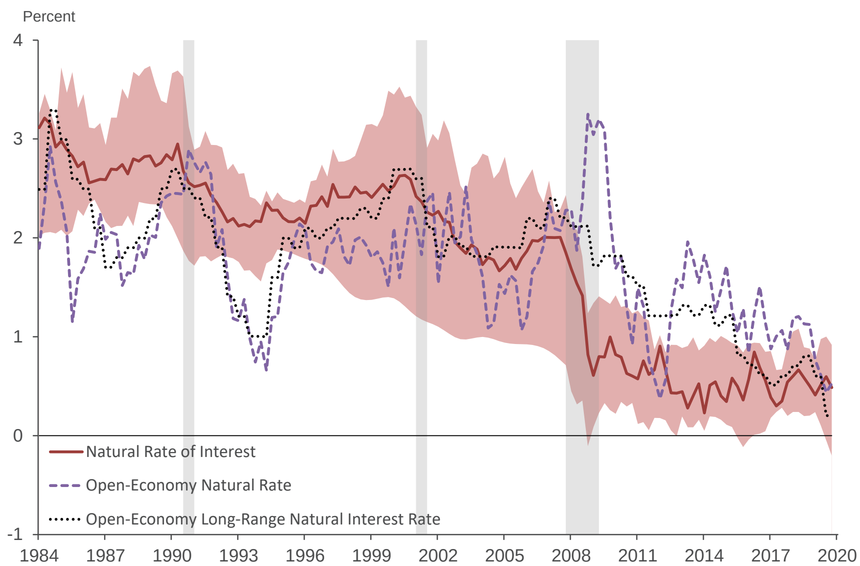
The closed-economy model is nested as a special case of the workhorse open-economy New Keynesian model and, therefore, offers a more parsimonious representation of the data-generating process underlying the observed data. This more parsimonious parameterization can become the preferred one because, as shown in Martínez-García and Wynne (2014), Bayesian model comparison techniques (posterior odds tests) tend to favor more parsimonious specifications like the closed-economy one in data samples comparable in size to the one I study here even when the *true* data-generating process is the open-economy one. That outcome hinges to a large extent on how informative the prior distributions and even the observed data are about the open-economy features of the model, but can be difficult to overcome even when more informative data is brought to bear when some of the key parameters of the open-economy model remain weakly identified.

The cautionary tale for policymakers here is that, even when the degree of openness is not particularly large, abstracting from the open-economy features of the model can fit the data well but lead researchers to favor a misspecified model (the closed-economy one). What my findings show is that weak identification and in particular misspecification—erroneously abstracting from exogenous and endogenous propagation across countries—can have consequences as illustrated in Table 3. In practice, however, a judicious choice of observables and the exploration of alternative priors (potentially even using simulated data as testing grounds first) can be useful in guiding the estimation and can mitigate those concerns to some extent. In fact, in the comparison between the benchmark and the alternative model specifications, I find that the selection of observables is key to obtain estimates of the path of the U.S. natural rate that are robust to some of those forms of misspecification/weak identification. Yet, as noted before, this will not prevent inferential errors on the nature of the shocks driving the U.S. natural rate and possibly about other implications of the model (such as those related to monetary policy and cost-push shocks) as well.

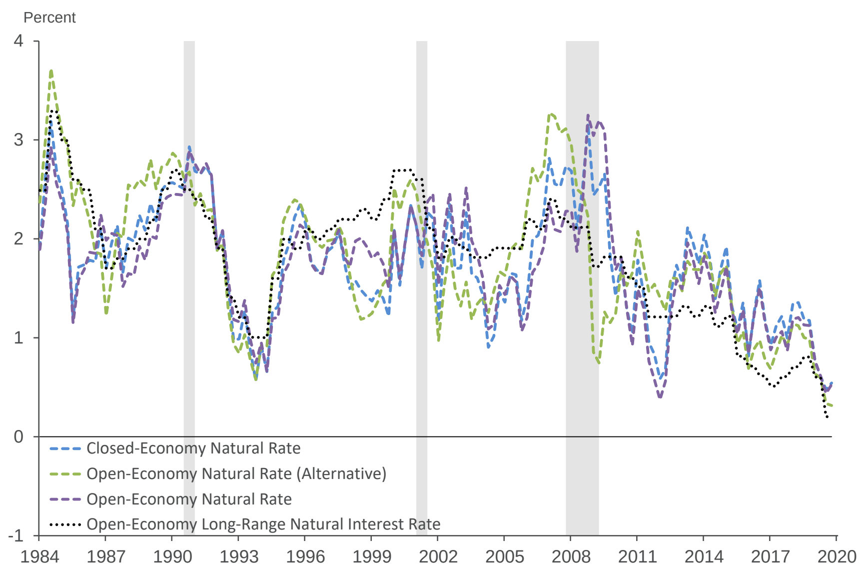
Figure 3. Estimates of the U.S. Natural Rate

(Scenarios: Open-Economy Baseline, Closed-Economy, Alternative Priors on Technology Diffusion)

A. U.S. Natural Rate: Comparison Across Estimates



B. U.S. Natural Rate: Comparison Based on Different Model Specifications



Note: The shaded bars indicate the NBER chronology of U.S. recessions. The natural rate estimates from the workhorse open-economy model are reported in levels under the different scenarios for the estimation by adding together the real rate based on forecasts from Blue Chip Economic Indicators from Aspen Publishers (2020) and the gap between the natural rate and the real rate estimated in the model. Similarly, the long-range combines the 5-year average, 5-year forward real rate based on forecasts from Blue Chip Economic Indicators from Aspen Publishers (2020) with the 5-year average, 5-year forward gap between the natural and the real rate inferred with the model. I use Matlab 7.13.0.564 and Dynare v4.2.4 for the stochastic simulation and estimation.

Sources: NBER; CBO (2020); Conference Board (2020); Grossman et al. (2014); Aspen Publishers (2020); Laubach and Williams (2003); Kiley (2015); Lubik and Matthes (2015); Holston et al. (2017); Johannsen and Mertens (2018); Del Negro et al. (2019); and author's calculations.

Finally, I should point out that all the structural estimates of the U.S. natural rate of interest shown in Figure 3 tend to accord with the prevailing narrative of a decline in the U.S. natural rate since at least the beginning of the $2007-09$ global financial crisis. One needs to be mindful that my structural benchmark model ties such behavior to the long-range decline in the U.S. real interest rate observed in the survey data and to cyclical productivity shocks. Surely other factors as of yet unmodeled—for instance, the safety and liquidity features of bonds discussed in the work of Del Negro et al. (2017)—could be contributing to the decline in the natural rate as well. Hence, the results presented here should be interpreted with caution due to possibly omitted features. However, in my view the evidence nonetheless suggests that domestic and foreign productivity shocks are a central part of any explanation of the cyclical behavior of the U.S. natural rate.

4.2 Historical Decomposition

Given the similarity between the closed-economy and the open-economy estimates of the U.S. natural rate seen in Figure 3, one could naturally conjecture that the U.S. natural rate must be explained mostly by U.S. productivity shocks. If that was the case, then abstracting from the open-economy linkages altogether would be without much loss of generality. Figure 4 provides evidence to the contrary plotting the historical decomposition of the contribution to the cyclical part of the U.S. natural rate (Panel A) and the cyclical component of U.S. output potential (Panel B) from U.S. and rest of the world productivity shocks recovered from the benchmark open-economy model. Figure 5 complements those results plotting the corresponding historical decomposition of the U.S. output gap in Panel A and of the U.S. cyclical inflation in Panel B.[20]

The salient findings that emerge from the historical decomposition analysis of the cyclical components of the U.S. natural rate and of the U.S. output potential in Figure 4 are:

First, domestic productivity shocks explain a large part of the cyclical fluctuations in the U.S. natural rate and U.S. output potential, but foreign productivity shocks have a sizeable contribution accounting for those cyclical movements (particularly for the U.S. natural rate

[20] In deriving the historical decomposition in Figure 4 and Figure 5, the parameter set is based on the calibrated parameter values and the posterior mean for all estimated parameters. Apart from the contribution of the smoothed shocks, there is also a contribution from the initial values in the Kalman filter which refers to the part of the smoothed endogenous variable fluctuations explained by the unknown initial values of the state variables. The influence of the initial values, however, decays pretty quickly in all my estimations.

of interest). This illustrates why important information would be lost if I were to rely exclusively on the more parsimonious closed-economy specification.

Second, the decline in the long-range natural rate of interest in the U.S. shown in Figure 3 has been partly cushioned in the recovery phase after the $2007-09$ global financial crisis by the positive contributions of the cyclical U.S. natural rate plotted in Panel A of Figure 4. The counterpart to that is the negative cyclical contributions recovered for the estimate of the U.S. output potential seen in Panel B of Figure 4. Moreover, if anything, the negative cyclical contributions to U.S. output potential appear to have preceded to some extent the $2007-09$ global financial crisis itself.

As I discussed in Subsection 2.2 while analyzing the implications of the benchmark model for the international transmission of shocks, setting monetary policy to track the natural rate of interest has the immediate consequence of isolating entirely the output gap and inflation from fluctuations arising from both home and foreign productivity shocks. Not surprisingly then, assuming that policymakers have aimed to track the natural rate of interest in setting polity as I do in the benchmark, I find that only the contributions of monetary policy shocks and most notably cost-push shocks feature in the historical decompositions of the U.S. output gap and U.S. cyclical inflation in Figure 5. Interestingly, the estimated contribution of foreign shocks to U.S. output gap and U.S. cyclical inflation shown in Figure 5 is rather small unlike what I observe with the cyclical fluctuations of the U.S. natural rate and U.S. output potential in Figure 4.

This suggests, for starters, that the contribution of foreign spillovers on the conventionally observable set of U.S. macro variables that includes the cyclical output, inflation, and short-term nominal interest rate is rather small or even negligible. In contrast, the foreign spillovers—to be precise the foreign productivity spillovers—have a more substantive contribution on the unobservable cyclical part of the U.S. natural rate and of the U.S. output potential. Therefore, a closed-economy model not only helps me recover a close estimate of the U.S. natural rate but provides me with a close approximation of sources of business cycle fluctuation for the set of observable variables (particularly so for the U.S. cyclical inflation component and the U.S. output gap). From a monetary policy analysis perspective, the major issue that arises in the data is that a misspecified closed-economy variant of the model would imply that the U.S. natural rate and the expected U.S. output potential move in tandem while the evidence in Figure 4 indicates that foreign shocks can indeed drive a wedge between both at least in regards to their cyclical component.

Furthermore, through the lens of the New Keynesian model, I observe that cost-push shocks (overwhelmingly domestic) have contributed negatively to U.S. cyclical inflation and positively to the U.S. output gap. This has had the effect of keeping the U.S. cyclical output component well above its potential counterpart, notably since the $2007-09$ global financial crisis. Cost-push shocks can be interpreted in different ways, but in my benchmark model they arise from stochastic shocks to the price markups. Congruent with that interpretation, I would argue that the period since the mid-2000s has been characterized by a sequence of negative cost-push shocks which can be thought of as a sequence of negative price markup shocks or, alternatively, as a prolonged period of compression in price markups.

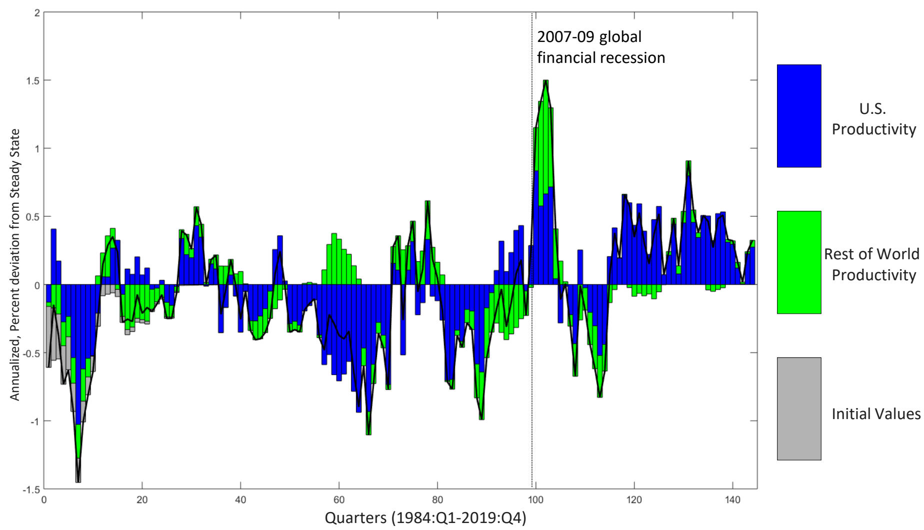
Given that U.S. output potential has been below trend for much of the period since the $2007-09$ (Panel B of Figure 4), it follows that this period of markup compression has been key to support U.S. cyclical output in the face of a concurrent fall in the U.S. output potential. The added twist to this story is that at the same time monetary policy has been quite robust, as seen in Figure 5. This has contributed to largely make up for the drag on U.S. cyclical inflation caused by this period of markup compression. And, accordingly, it has kept U.S. inflation close to its long-run trend (and close to the Fed's own inflation target) while, simultaneously, it has resulted in an additional boost to cyclical output for the U.S.

4.3 International Propagation

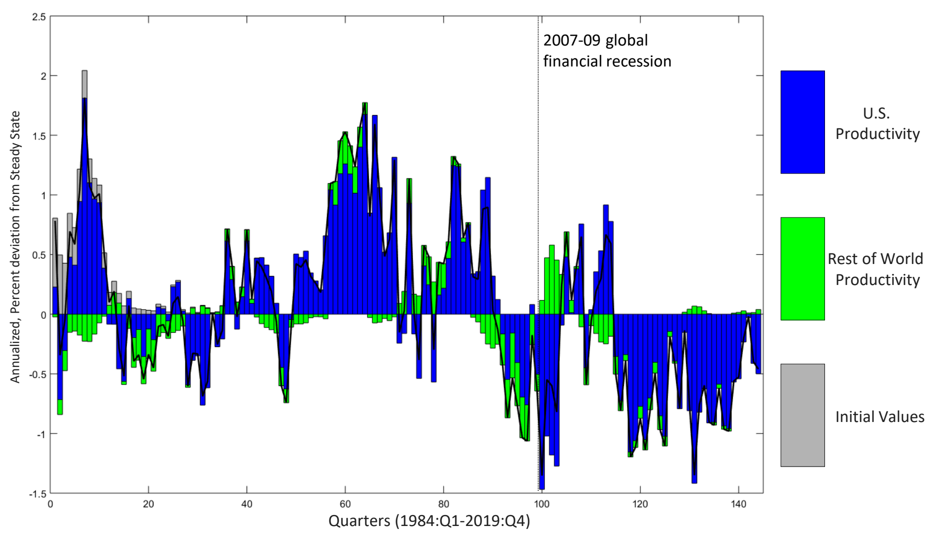
Figure 6 illustrates the Bayesian impulse response functions of the different shock innovations (a positive one-standard deviation) on the cyclical part of the U.S. natural rate and of the U.S. output potential (Panel A) as well as on the U.S. output gap and U.S. cyclical inflation (Panel B). The responses of U.S. output potential and the U.S. natural rate to a productivity shock innovation whether domestic or foreign are sizeable but of opposing sign, as seen in Panel A of Figure 6. Mechanically, the dynamic responses are related to equations (18) and (11) and to the stationary bivariate VAR(1) process for the productivity shocks found in (13). The intuition is that a fall in labor productivity below trend such as that which resulted in the negative labor productivity period observed in the U.S. since the mid-2000s (see Figure 2) would lead to a fall of U.S. output potential below its trend in the frictionless equilibrium. In this case, however, private agents hit by lower productivity today still expect higher output potential in the future due to the mean-reversion property of the stationary productivity shock process. Hence, they seek to smooth out their consumption path by anticipating today some of that future higher output and, because of that, the natural rate goes above its long-run level as a counterbalance that helps clear the bond markets.

Figure 4. Structural Estimates of the U.S. Natural Rate
and of U.S. Potential Output
(Open-Economy Model)

A. U.S. Natural Rate (Cyclical Component)



B. U.S. Potential Output (Cyclical)

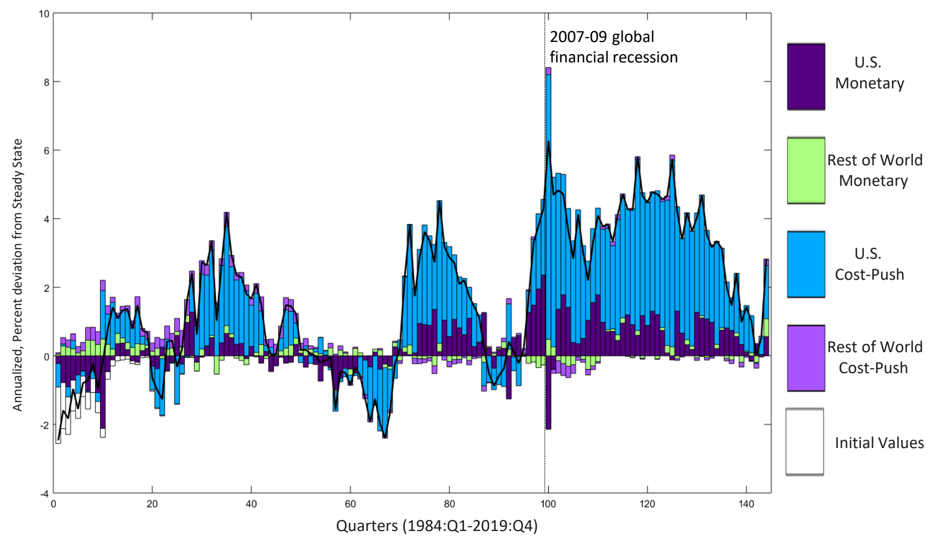


Note: The figure plots the historical decomposition of the U.S. natural rate and the U.S. output potential from the estimated workhorse open-economy New Keynesian model. The contribution of U.S. productivity shocks and of rest-of-the-world productivity shocks is colored in blue and green, respectively. I use Matlab 7.13.0.564 and Dynare v4.2.4 for the estimation.

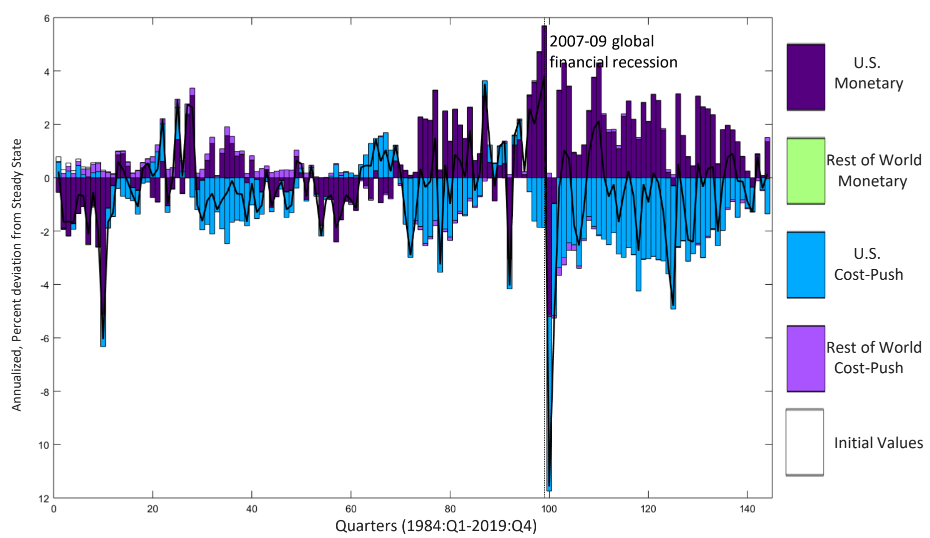
Sources: CBO (2020); Conference Board (2020); Grossman et al. (2014); Aspen Publishers (2020); and author's calculations.

Figure 5. Structural Estimates of the U.S. Output Gap
and of U.S. Cyclical Inflation
(Open-Economy Model)

A. U.S. Output Gap



B. U.S. Cyclical Inflation



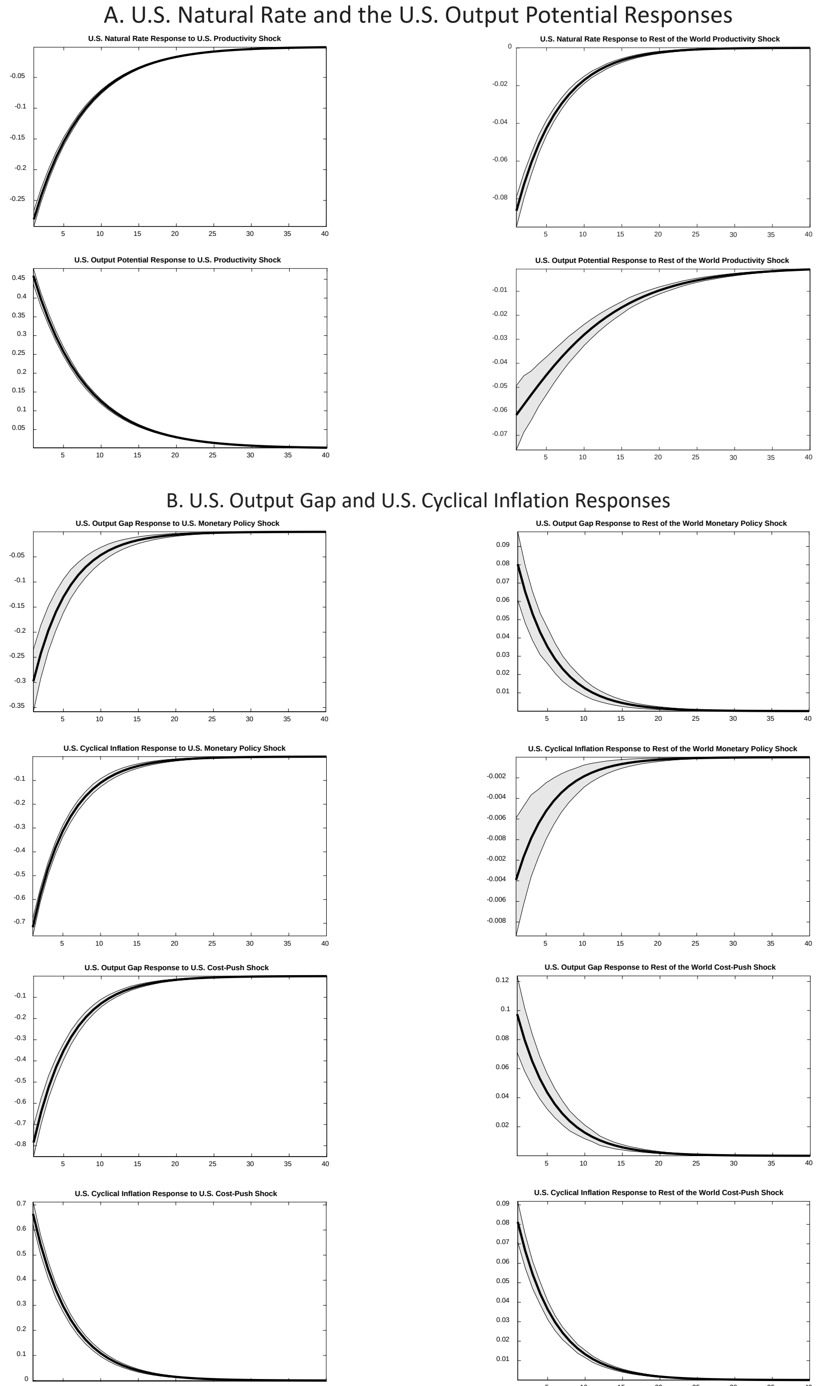
Note: The figure plots the historical decomposition of the U.S. output gap and of the cyclical component of U.S. inflation from the estimated workhorse open-economy New Keynesian model. The contribution of U.S. monetary policy shocks and of rest-of-the-world monetary policy shocks is colored in violet (grape) and light green, respectively. The contribution of U.S. "cost-push" shocks and of rest-of-the-world "cost-push" shocks is colored in light blue and lavender, respectively. I use Matlab 7.13.0.564 and Dynare v4.2.4 for the estimation.
Sources: CBO (2020); Conference Board (2020); Grossman et al. (2014); Aspen Publishers (2020); and author's calculations.

Simultaneously, a decline in U.S. productivity—or an increase in the rest of the world productivity for that matter—makes U.S. goods relatively more scarce and, accordingly, makes it relatively cheaper for U.S. households to purchase the more abundant foreign goods. This has a substitution effect, but also a real income effect, in the frictionless allocation. In the estimated benchmark model, the result is that U.S. households work somewhat less but also would want to consume less overall. Accordingly, the natural rate goes below its long-run level to clear the markets. These income and substitution effects working out through trade do not qualitatively reverse the dominant effect of consumption-smoothing that I described earlier, but are a robust and powerful force in the international propagation of shocks. Notice that, in fact, the effect of a foreign productivity shock on the U.S. natural rate which arises solely through trade is about a third that of a U.S. productivity shock.

In contrast, panel B of Figure 6 reveals only modest impacts in magnitude on the U.S. output gap and more so on the U.S. cyclical inflation arising from foreign monetary and cost-push shocks. Those Bayesian impulse response functions in themselves provide a mechanical rationale for the small contribution of foreign shocks that I have noted earlier when discussing the historical decomposition of U.S. cyclical inflation and U.S. output gap shown in Figure 5. Hence, once again, if I were only concerned about key macro variables such as U.S. cyclical inflation or the U.S. output gap, I would not miss much if I were to rely on a misspecified closed-economy model. Although, to be fair, Table 3 shows that the estimates of the volatility of monetary policy shock innovation and even of the cost-push shock innovations are smaller in the closed-economy case; hence, a bias still appears here due to the underestimation of the size of these shock innovations. The main bias in regard to the propagation of shocks arises, however, from ignoring the notable effects of foreign productivity shocks on the cyclical U.S. output potential and, particularly, on the cyclical part of the U.S. natural interest rate.

Another interesting fact to point out in regards to Figure 6 is that, as discussed theoretically by Martínez-García (2019), U.S. monetary policy shocks move the U.S. output gap and U.S. cyclical inflation in the same direction while U.S. cost-push shocks move them in opposite directions. That is in a nutshell why, through the lens of the New Keynesian model, a sequence of negative cost-push shocks (the prolonged period of price markup compression to which I alluded earlier) is what accounts for most of the U.S. macro performance in the aftermath of the $2007-09$ global financial crisis as seen in panel B of Figure 5.

Figure 6. Estimated Impulse Response Functions to a
Positive One-Standard Deviation Shock Innovation
(Open-Economy Model)



Note: This figure plots the impulse response functions to a one standard deviation together with the 90 percent confidence intervals. I use Matlab 7.13.0.564 and Dynare v4.2.4 for the estimation.
Sources: CBO (2020); Conference Board (2020); Grossman et al. (2014); Aspen Publishers (2020); and author's calculations.

4.4 Has the Phillips Curve Broken Down?

Much has been made about the apparent weakness of the Phillips curve relationship and what that entails for monetary policy. This concern was duly noted during the Fed's $2019-20$ Monetary Policy Framework Review as seen in Caldara et al. (2020) and was even cited by Chair Powell in announcing the Fed's subsequent policy strategy change in his 2020 Jackson Hole speech.[21] Martínez-García (2019) shows that, for equilibrium outcomes, the comovement between output gap and cyclical inflation depends on the nature of the shocks and the contribution that different shocks make during the sample period under consideration.

Hence, when investigating the Phillips curve relationship, the comovement between unconditional measures of cyclical inflation and the output gap may be instead confounded by the varying contribution of different shocks over time. In fact, the challenges are even more severe when, on top of that, one does not have a theoretically-consistent measure of the output gap (i.e., when the researcher cannot either recover or observe the *true* output gap measure). Furthermore, Martínez-García (2019) also indicates that even for equilibrium outcomes, one ought to expect a wedge between the domestic output gap and domestic cyclical inflation that is itself related to the foreign output gap.

To explore the consequences all of this on conventional reduced-form estimates of the Phillips curve relationship, Table 4 runs simple OLS regressions of the observed U.S. cyclical inflation on the measures of the U.S. output gap and the rest of the world output gap recovered from the benchmark model for a subsample that ends in 2007:Q4 as well as for the full sample. In this exercise, the OLS estimates show an statistically-insignificant negative coefficient on the U.S. output gap before the $2007-09$ global financial crisis which turns larger and more statistically-significant for the full sample. Whenever the rest of the world output gap is included in the regressions, the coefficient on the foreign output gap is statistically significant and positive for the pre-$2007-09$ global financial crisis subsample and larger for the full sample.

This is, in a way, to be expected given how the estimated cost-push shocks propagate as can be seen in the impulse response functions in Panel B of Figure 6 and how cost-push shock account for a large part of the fluctuations of the U.S. output gap and U.S. cyclical inflation in Panel B of Figure 5. These OLS estimates, therefore, do not negate the structural Phillips curve relationship nor do they suggest that the relationship has broken

[21] To be precise, Powell (2020) states that: "(t)he muted responsiveness of inflation to labor market tightness [an often-used measure of resource utilization slack thought to be related to the output gap], which we refer to as the flattening of the Phillips curve, also contributed to low inflation outcomes."

down and monetary policy has no efficacy to stimulate economic activity even though they display some significant variation over time when I estimate the reduced-form relationship between inflation and output gaps. In fact, the structural model estimates and the historical decomposition provided in this paper clearly assume otherwise—that is, the structural model clearly assumes the structural Phillips curve relationship exists and has been stable over the entire sample period.

Table 4 - U.S. Cyclical Inflation vs. Estimated Output Gap

	OLS Regression 1	OLS Regression 2	OLS Regression 3	OLS Regression 4
	$\widehat{\pi}_t$	$\widehat{\pi}_t$	$\widehat{\pi}_t$	$\widehat{\pi}_t$
	1984:Q1-2007:Q4	1984:Q1-2019:Q4	1984:Q1-2007:Q4	1984:Q1-2019:Q4
	-0.13	-0.21	-0.06	-0.15
$\widehat{x}_t$	(0.09)	(0.06)	(0.09)	(0.06)
	[−1.50]	[−3.67]	[−0.68]	[−2.75]
			0.19	0.27
$\widehat{x}_t^*$	–	–	(0.06)	(0.07)
			[2.99]	[3.93]
R^2	0.02	0.09	0.11	0.18
F-stat	2.23	13.44	5.67	15.13

Standard errors in () and t-statistics in [].

What this means is that the open-economy New Keynesian benchmark model estimated in this paper accounts for the performance of the U.S. economy without a structural break in the Phillips curves given by equations (1) and (2). However, to be fair, this does not necessarily imply that no structural break has occurred. What the exercise in Table 4 does in this setting is simply to show that addressing questions such as the much-discussed flattening of the Phillips curve is not an easy question to settle and is one that requires a different (and presumably more sophisticated) econometric strategy beyond the scope of this paper. Furthermore, the exercise also suggests that—to some extent—the variation in the reduced-form estimates of the Phillips curve relationship can be attributed to varying contributions of the different shocks over time rather than to a structural break in the relationship.

5 Concluding Remarks

In this paper, I investigate the determinants of the U.S. natural rate of interest decline through the lens of the workhorse open-economy New Keynesian model over the period from 1984:Q1 until 2019:Q4. Apart from explicitly modeling key endogenous (trade) and exogenous (technological diffusion and shock innovation correlations) linkages, I also adopt a specification of monetary policy in terms of its objectives (to affect the deviations of the real interest rate from the natural rate) rather than in terms of a specific instrument (such as the Federal Funds Rate) in order to respond to inflation and the output gap. Moreover, I exploit survey-based forecasts of current and expected future U.S. real interest rates to discipline the endogenous expectations and the path the model solution takes in order to be congruent with the zero-lower bound constraint on the Federal Funds Rate. This strategy allows me to estimate a log-linearized subset of the equilibrium conditions of the model without explicitly including the occasionally-binding zero-lower bound constraint and still recover consistent estimates of the model and, in particular, of the U.S. natural rate of interest.

I collect data for the U.S. and an aggregate of 33 of its largest trading partners. Long-range survey-based forecasts on U.S. inflation and the U.S. nominal interest rate are employed to detrend the corresponding observables for the U.S. and the rest of the world aggregate, while I rely on the Beveridge-Nelson decomposition to extract the cyclical component from measured labor productivity. Then I use Bayesian estimation techniques to recover a novel open-economy, structural estimate of the unobserved U.S. natural rate. Such an structural estimate is useful for monetary policy analysis as well as to gauge the exogenous and endogenous cross-country spillovers that influence the U.S. natural rate and the macro performance of the U.S. economy during my sample period.

I argue that weak identification problems persist to some degree even after appropriately tailoring the selection of observables and priors to better grasp the significance of international spillovers, so the possibility that model comparison techniques would favor the more parsimonious closed-economy representation should not be discounted neither the associated costs (costs resulting from missing out the open-economy dimension) be minimized. While estimates of the U.S. natural rate of interest are not too dissimilar even when recovered from a closed-economy variant of the model, misspecifying the cross-country linkages can result in significant errors of judgement about the drivers of the U.S. natural rate, about the way shocks propagate through the economy, and even about the contribution of monetary policy itself to business cycles. Indeed, focusing on the more parsimonious closed-economy model can bias one's understanding of U.S. business cycles in significant ways.

Furthermore, I document several findings of significance for the U.S. natural rate and for U.S. monetary policy:

First, I observe a significant decline in the U.S. natural rate of interest in the aftermath of the $2007-09$ global financial crisis. The decline in the long-run natural rate of interest in the U.S. follows the downward slide in the long-run real interest rate forecasts inferred from Blue Chip Economics Indicators survey (Aspen Publishers (2020)) data. Through the lens of the workhorse open-economy New Keynesian model, I find that the decline has been partly cushioned during the recovery phase after the $2007-09$ global financial crisis keeping the U.S. natural rate above its long-range path with positive contributions from its cyclical component. The counterpart to the above-trend natural rate estimate is the fall below-trend of the U.S. output potential recovered from the data. The evidence suggests that while U.S. productivity shocks play a large role on the cyclical fluctuations of the U.S. natural rate (and the U.S. output potential), foreign productivity shocks have contributed significantly as well.

Second, the estimation also showcases the importance of international productivity spillovers. The findings suggest that the technological diffusion and the correlation of shock innovations across countries incorporated in the specification of the bivariate shock process for detrended labor productivity has modest effects. However, the endogenous transmission mechanism that operates through the trade channel is a potent force in the U.S. economy. Fluctuations of the U.S. output gap and of U.S. cyclical inflation are isolated from the productivity shocks, but very much depend on the realizations of (predominantly domestic in origin) cost-push shocks and monetary policy shocks.

Third, I observe that the period since the mid-2000s has been characterized by a sequence of negative cost-push shocks which can be thought of as a prolonged period of compression in price markups. Through the lens of the open-economy benchmark model, it follows that this period of compressed markups has contributed positively to boost U.S. cyclical output in the face of a concurrent dip below trend in the U.S. output potential (and in U.S. measured labor productivity) since at least the beginning of the $2007-09$ global financial crisis, if not before.

Fourth, the findings provide evidence that monetary policy has been quite robust during the entire sample period under consideration. In the aftermath of the $2007-09$ global financial crisis, in particular, monetary policy has contributed to make up for much of the drag on U.S. cyclical inflation that the model attributes to the concurrent period of negative domestic cost-push shocks (or price markup compression) in the U.S. This has kept U.S. inflation close to its long-range level (and close to the Fed's own inflation target) over the past decade, but it has also resulted in a non-trivial boost to cyclical output in the U.S.

Finally, much discussion remains about the perceived flattening of the Phillips curve relationship during the same time period, but the findings in this paper suggest that some of the instability in the reduced-form estimates of the Phillips curve could simply reflect a variation in the mix of shocks over time. Simply put, cost-push shock realizations appear to have contributed more to the variation in the U.S. output gap and U.S. cyclical inflation after the $2007-09$ global financial crisis than before.

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