A Theory of Gross and Net Capital Flows over the Global Financial Cycle

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Abstract

We develop a theory to account for changes in gross and net capital flows over the global financial cycle (GFC). The theory relies critically on portfolio heterogeneity among investors within and across countries, related to risky portfolio shares and portfolio shares allocated to foreign assets. A global drop in risky asset prices during a downturn of the GFC changes relative wealth within and across countries due to portfolio heterogeneity. This leads to changes in gross and net capital flows that are consistent with the stylized facts: all countries experience a decline in gross capital flows (retrenchment), while countries that have a net debt of safe assets experience a rise in total net outflows and net outflows of safe assets and a drop in net outflows of risky assets. The model is applied to 20 advanced countries and calibrated to micro data related to within country portfolio heterogeneity, as well as cross country heterogeneity of net foreign asset positions of safe and risky assets. The implications of the calibrated model for gross and net capital flows are quantitatively consistent with the data.

Keywords: Global Financial Cycle; Capital Flows; Current Account; Portfolio Heterogeneity

JEL: F30; F40

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1 Introduction

Rey (2015) has characterized a global financial cycle (GFC) as exhibiting a strong global co-movement in asset prices, gross capital flows, leverage, credit and risk premia. In this paper we will focus on gross and net capital flows during the global financial cycle. We will show that heterogeneity in risky asset shares, both within and across countries, leads to changes in relative wealth following a change in global asset prices that can explain observed fluctuations in both gross and net flows. In response to a decline in global risky asset prices, the theory can account both for the observed global retrenchment and the observed fluctuations in global imbalances.

We will consistently define net capital flows as net capital outflows, i.e. capital outflows (purchases of foreign assets by domestic residents) minus capital inflows (purchases of domestic assets by foreign residents). One should keep in mind though that for a net borrower like the US, an increase in net capital outflows means that net borrowing declines. Net capital outflows are also equal to the current account, which is saving minus investment. We therefore also consider evidence related to saving and investment. We refer to gross capital flows as capital outflows, capital inflows or their sum. We also distinguish between capital flows of risky assets (portfolio equity and FDI) and safe assets (debt flows, banking flows and reserves).

We aim to understand the following set of facts related to capital flows during a downturn in the GFC (the opposite applies to an upturn):

1. There is a drop in gross capital inflows and outflows (global retrenchment).

2. Countries that are net borrowers of safe assets (negative net foreign asset position of safe assets) experience (i) a rise in overall net outflows, (ii) a rise in saving relative to other countries, (iii) a drop in investment relative to other countries, (iv) a drop in net outflows of risky assets and (v) a rise in net outflows of safe assets. The opposite is the case for countries with a positive net foreign asset position of safe assets.

3. The net foreign asset position of risky assets has no predictive power for the response of net capital flows, saving and investment to the GFC.

Miranda-Agrippino and Rey (2020), from hereon MAR, characterize the GFC as the common component of 858 asset price series around the world. Rey (2015) and MAR show that a drop in the GFC, defined as this common component in asset prices, is also associated with a drop in gross capital inflows and outflows (Fact 1). Related to this, Davis, Valente, and van Wincoop (2021b) find that a single global factor of capital inflows and outflows is highly correlated with the MAR factor that is based on asset prices. The same is found by
Large global declines in capital inflows and outflows at times of crises, with sharply falling asset prices, have been well documented.\textsuperscript{2} Facts 2 and 3 are related to findings by Davis et al. (2021b). They show that a global factor of gross capital inflows and outflows also accounts for 21 percent of the variance of net capital flows.\textsuperscript{3} Consistent with part (i) of Fact 2, and Fact 3, they show that total net capital outflows load positively on the product of the GFC and the net foreign asset position of safe assets, but do not depend on the product of the GFC and the net foreign asset position of risky assets. However, they do not separately consider capital flows of safe and risky assets.\textsuperscript{4}

In the next section we will provide further evidence for these three facts using data for 20 developed countries from 1996 to 2020. To provide a preview, consider two periods with significant global asset price changes: 2003-2007 (rise in global asset prices) and 2008-2009 (drop in global asset prices). Consistent with Fact 1, risky asset outflows rose from 3.2 percent of GDP in 2002 to 8.7 percent of GDP in 2007 and then fell back to 3.4 percent of GDP in 2009. Related to Fact 2, the charts in Figure 1 show saving, investment and net capital flows (percent of GDP) during these two periods minus their corresponding values prior to these periods (respectively 2002 and 2007). The net foreign asset position of safe assets is on the horizontal axis. The red dotted lines are the regression lines.

The charts on the right (2008-2009) show that net borrowers of safe assets experience a rise in overall net outflows. This is associated with an even larger rise in net outflows of safe assets and a fall in net outflows of risky assets. The charts for savings and investment show that saving and investment decline globally in response to a decline in the GFC, but net borrowers of safe assets experience a smaller decline in saving (a rise in saving relative to the rest of the world) and a larger drop in investment. The charts on the left show that the exact opposite happened when asset prices rose during the 2003-2007 period. If we instead put the net foreign asset position of risky assets on the horizontal axis, the regression lines are close to horizontal, with insignificant coefficients (Fact 3).

\textsuperscript{1}Habib and Venditti (2019) also find a close association between the global asset price factor and capital flows.

\textsuperscript{2}Milesi-Ferretti and Tille (2011) document the sharp fall in global capital flows during the 2008 crisis. Broner, Didier, Erce, and Schmukler (2013) emphasize the strong co-movement between capital inflows and outflows and show that both decline in the years after a crisis. Forbes and Warnock (2012) find that global factors are significantly associated with extreme capital flow episodes.

\textsuperscript{3}Related, Ghosh, Qureshi, Kima, and Zalduendo (2014) show that global factors are key determinants of net capital flow surges to emerging markets. Eichengreen and Gupta (2016) show that sudden stops (drop in net capital inflows) are increasingly driven by global factors. Lane and Milesi-Ferretti (2012) document a current account reversal in many countries as a result of the 2009 global financial crisis.

\textsuperscript{4}Also related to Fact 2 is Forbes and Warnock (2014). They find that most episodes of large capital flow changes are dominated by debt flows rather than equity flows. This is in line with Fact 2, which says that total net outflows and net outflows of safe assets fluctuate together along the GFC cycle, while net outflows of risky assets go in the opposite direction. Related, Shen (2022) provides evidence for 104 countries that net equity flows and net debt flows are negatively correlated.
To account for these facts, we develop a multi-country portfolio choice model where investors can hold risky assets of all countries as well as a global safe asset. There is a single good and therefore a single safe asset with a riskfree payoff in that good.\(^5\) Given significant evidence that changes in the global financial cycle are driven by shifts in global risk-aversion, we will consider a rise in global risk-aversion as the driver of the decline in the GFC. It leads to a portfolio shift from risky to safe assets.\(^6\) However, this is not important for the paper’s results. All that matters is that there is a global shock that lowers risky asset prices around the world.

Key to the model is portfolio heterogeneity within and across countries. A drop in risky asset prices then leads to changes in relative wealth within and across countries, which in turn leads to changes in gross and net capital flows. Without heterogeneity a rise in global risk-aversion has no effect on capital flows. In equilibrium the drop in demand for risky assets due to higher risk-aversion and lower wealth is exactly offset by the lower price of risky assets, which raises expected returns on risky assets. There will be no asset trade.

We introduce two types of portfolio heterogeneity within countries. First, portfolio shares invested in risky assets vary across investors. Second, the share of risky assets invested abroad also varies across investors and such that investors with a larger share of risky assets also tend to be more globally oriented (have less home bias). As we will discuss below, there is indeed micro evidence consistent with these assumptions. We model these two types of portfolio heterogeneity through variation across investors of risk-aversion and a parameter that affects home bias, but what matters is not the underlying assumptions that lead to portfolio heterogeneity but rather the portfolio heterogeneity itself.

This within-country portfolio heterogeneity is critical to understanding the drop in gross capital flows when global asset prices fall during a downturn in the GFC (Fact 1). Consider two groups of investors, those with a relatively high share invested in risky assets (group 1) and those with a relatively low share invested in risky assets (group 2). The wealth of investors in group 1 will then drop significantly more than those in group 2 when risky asset prices fall. With an unchanged supply of risky assets, in equilibrium investors in group 1 will then sell risky assets, while those in group 2 will buy risky assets. But when investors

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\(^5\)This means that we cannot consider gross flows of safe assets, which requires imperfect substitutability of safe assets such as through exchange rate risk. This will significantly complicate the model. We will discuss this further towards the end of the paper.

\(^6\)Forbes and Warnock (2012) find that an increase in global risk (proxied by the VIX) predicts a drop in capital outflows (“retrenchment”) and a drop in capital inflows (“sudden stops”). MAR and Habib and Venditti (2019) find that the GFC, as measured by a common component in global asset prices, is closely related to the VIX and the risk premium on corporate bonds. Jorda, Schularick, Taylor, and Ward (2019) find that the increase in stock price synchronization since the early 1990s is associated with a synchronization of risk-appetite across countries. Chari, Dilts-Stedman, and Lundblad (2020) document the significant impact of changes in global risk-aversion on capital flows to emerging markets and emerging market equity and bond prices.
in group 1 are also more globally diversified, they will end up selling foreign risky assets to
foreign investors as the domestic investors in group 2 do not hold a lot of foreign assets. This
implies a drop in capital outflows.

Although many papers have documented a drop in gross capital flows during a decline
in the GFC, there is no consensus on what accounts for this phenomenon. Our explanation
is consistent with micro evidence for Swedish household portfolio data reported in Calvet,
Campbell, and Sodini (2007, 2009a and 2009b). Sweden is unusual in that there is a wealth
tax that requires the government to collect detailed portfolio and wealth data for all Swedish
taxpayers. Risky shares, defined as the fraction invested directly in stock or in risky mutual
funds, are approximately uniformly distributed across households from 0 to 1. Calvet et al.
(2009a) finds, consistent with the theory, that when risky asset prices drop, households with
large risky shares sell risky assets and those with small risky shares buy risky assets. Calvet
et al. (2007 and 2009b) find that investors with larger risky shares also tend to be more
diversified.7

Cross-country portfolio heterogeneity takes the form of variation across countries in net
foreign asset positions of safe and risky assets. These are important to understand the
response of net capital flows to a global drop in asset prices (Facts 2 and 3). We model
these global imbalances through heterogeneity across countries in risk-aversion and expected
dividends of the risky assets. But what matters is the imbalances themselves, not the
particular parameters that give rise to them. The response of net capital flows (risky, safe
and total) depends on whether a country is a net borrower or lender of safe assets. The
intuition again operates through changes in relative wealth, but this time relative wealth
across countries rather than across investors within countries. A country that is a net
borrower of safe assets has a higher portfolio share of risky assets and therefore experiences
a larger drop in wealth. This leads to a larger drop in consumption (larger rise in saving)
and therefore an increase in the current account (larger net capital outflows).8

The response of net capital flows of risky assets also depends on changes in relative
wealth across countries. In analogy to the within-country intuition, consider two groups of
countries. Group 1 countries are net borrowers of safe assets (high portfolio share of risky

7While the Swedish portfolio data do not tell us directly what share of risky assets is invested abroad,
Calvet et al. (2007 and 2009b) report a Sharpe ratio loss measure that tells us how much lower the Sharpe
ratio of a household is than if it were globally diversified. They interpret this as related to global diversifica-
tion. A majority of Swedish households have a Sharpe ratio that is higher than that for the domestic stock
market, which they attribute to the large share of international securities in popular mutual funds. There is
a significant negative correlation of -0.49 between the risky asset share of households and their Sharpe ratio
loss, so that investors with a higher risky share are more diversified.

8When extending the model to include investment, there is a further increase in the current account. Due
to portfolio home bias and the larger drop in wealth of net borrowers of safe assets, their domestic risky asset
price drops more than average. Investment then drops more than average in a standard Tobin Q model of
investment.
assets), while group 2 countries are net lenders of safe assets (low portfolio share of risky assets). The relative wealth of group 1 countries falls when risky asset prices fall. Just as was the case for investors within a country, in equilibrium group 1 countries will then sell risky assets, while group 2 countries buy risky assets. For net borrowers of safe assets (group 1) this implies a drop in capital outflows of risky assets and a rise in capital inflows of risky assets.

The response of net flows of safe assets can be seen as a residual. The larger drop in wealth in a country that is a net borrower of safe assets implies that its relative saving increases and that it will sell risky assets. Both selling of risky assets and the rise in saving imply that the country will hold more safe assets. Net outflows of safe assets will therefore increase.

Introducing two types of cross-country heterogeneity allows us to address Fact 3 as we can independently vary the net foreign asset positions of safe and risky assets across countries. We find, both empirically and theoretically, that what matters for the net capital flow response to the decline in world risky asset prices is a country’s net foreign asset position of safe assets, not risky assets. Intuitively, what happens to the relative wealth of a country depends on its overall portfolio share of risky assets, which depends on whether it is a net borrower or lender of safe assets. Its wealth exposure to a decline in risky asset prices does not necessarily depend on its net foreign asset position of risky assets. Two countries may have the same exposure to risky assets overall, and therefore experience the same drop in wealth, even though one holds more domestic risky assets and the other more foreign risky assets. In addition, external risky liabilities do not affect the wealth of a country’s investors.

While the model is rich in the sense that it allows for a large number of countries and both within and cross-country heterogeneity, we keep the model otherwise sufficiently tractable that we can derive closed form analytical proofs for Facts 1-3. This helps in establishing the intuition behind the results. After discussing the theoretical results, we calibrate the parameters of a slightly extended model to evidence from 20 countries in order to compare the theoretical predictions regarding gross and net capital flows to those in the data. We use evidence from the Calvet et al. (2007, 2009a, and 2009b) papers to calibrate the parameters related to within-country portfolio heterogeneity. For the cross-country heterogeneity we

9Earlier papers, such as Maggiori (2017), Mendoza, Quadrini, and Rios-Rull (2009) and Gourinchas and Rey (2022), introduce just one type of cross-country heterogeneity to account for the fact that in the US the net foreign asset position of safe assets is negative and the net foreign asset position of risky assets is positive. But this is a very specific case. More generally, when plotting the net foreign asset positions of safe and risky assets, multiple countries of our 20 country sample occupy all 4 quadrants. With regards to the US, Atkeson, Heathcote, and Perri (2022) show that the net foreign asset position of risky assets has turned from positive to negative during the pre-Covid decade, while the net foreign asset position of safe assets has become even more negative. In the context of our model, higher expected domestic dividends (as during the pre-Covid decade in the U.S.) accounts for both the sign reversal of the U.S. net foreign asset position of risky assets and an even more negative net foreign asset position of safe assets.
match the observed net foreign asset positions of safe and risky assets for each of the 20 countries.

The remainder of the paper is organized as follows. Section 2 presents empirical evidence related to the three Facts. Section 3 describes the model. Section 4 discusses the impact of a global risk-aversion shock on risky asset prices and the interest rate on safe assets. Section 5 discusses the impact on gross capital flows when there is only within-country heterogeneity. Section 6 analyzes the impact on net capital flows when there is only cross-country heterogeneity. Section 7 introduces a couple of additional features and then calibrates the model to micro portfolio data and the net foreign asset positions of safe and risky assets of individual countries. It shows that a drop in global risky asset prices has implications for gross and net capital flows that is quantitatively consistent with the data discussed in Section 2. Section 8 concludes.

2 Empirical Results

In this section we will empirically establish the three stylized facts listed in the introduction, as well as provide a benchmark that we will use later to judge the quantitative predictions of the model.

We first describe the data used for the analysis. After that we estimate a static factor model of capital outflows and inflows of safe and risky assets in 20 developed countries over the sample 1996-2020. We show that the first factor is highly correlated with the asset price factor from MAR and refer to it as the GFC factor. We then regress equity prices, the real interest rate, gross and net capital flows, and savings and investment on this GFC factor and the factor interacted with global imbalances.

2.1 Data Description

We use capital flow data from twenty developed countries over the period 1996-2020.\(^\text{10}\) Here we present results using annual data, but for robustness we use data at the quarterly frequency in the Online Appendix.\(^\text{11}\)

Capital flows are obtained from the Balance of Payments data in the IMF’s International

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\(^{10}\)We do not include emerging markets. Davis, Fujiwara, Huang, and Wang (2021a) show that central bank foreign exchange reserve flows are one of the largest and most volatile components of the balance of payments in emerging market economies, while being only a minor part of the balance of payments in most advanced countries. Since we do not model foreign exchange reserve flows, it makes sense to focus on advanced countries in the empirical analysis. The countries included are United States, Singapore, Australia, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Iceland, Israel, Italy, Japan, Korea, Netherlands, Norway, Portugal and Sweden.

\(^{11}\)Quarterly capital flow data tend to be much nosier than annual capital flows data.
Financial Statistics (IFS). For country \( n \) in year \( t \), outflows and inflows of risky assets, \( OF_{n,t}^{risky} \) and \( IF_{n,t}^{risky} \), include FDI and portfolio equity flows. Outflows of safe assets, \( OF_{n,t}^{safe} \), includes portfolio debt flows, “other” outflows (bank lending and deposits), and central bank foreign exchange reserve accumulation. Inflows of safe assets, \( IF_{n,t}^{safe} \), includes portfolio debt flows and “other” inflows. Net flows are denoted \( NF_{n,t}^{risky} \) and \( NF_{n,t}^{safe} \) and are equal to outflows minus inflows. We use external assets and liabilities from the IMF International Investment Position data to obtain the net foreign asset positions of risky and safe assets, \( NFA_{n,t}^{risky} \) and \( NFA_{n,t}^{safe} \). We use the same classification of FDI and portfolio equity as risky assets and portfolio debt, “other”, and official reserves as safe assets. We normalize all flows and stock variables in country \( n \) and year \( t \) by that country’s prior year GDP. Normalized variables are written in lower case.

We believe that this separation between safe and risky assets is reasonable given the available data. Some of the portfolio debt and banking categories include high yield corporate debt, which would be better to add to the “risky” category. We only have a finer level of disaggregation available for inflows, not outflows. For portfolio debt and banking inflows, it is possible to observe the sector that issues the security: government, central bank, deposit taking corporations, and other sectors (other financial corporates, non-financial corporate, households, and non-profits). If we designate the last category (other sectors) as risky, and the rest as safe, we show in the Online Appendix that 80 to 90 percent of the variance of portfolio debt and banking inflows involves relatively safe assets. The inability to disaggregate portfolio debt and banking outflows along this line is therefore not likely to be a major concern.

In addition to safe and risky capital flows we we look at the response of saving and investment. This data is also from IFS. Investment is defined as gross fixed capital formation from the national accounts data. Saving is constructed from the national accounts and balance of payments data, as GDP plus net primary income plus net secondary income minus private consumption minus government consumption minus inventory investment. Like the capital flow data, we normalize savings and investment by the prior year’s GDP.

Risky asset prices are year-end equity prices from the OECD Main Economic Indicators. The interest rate on the safe asset is the one year U.S. Treasury rate from the Federal Reserve Board, minus the 1 year ahead U.S. inflation expectation from the Survey of Professional Forecasters.

### 2.2 Factor analysis

Consider the following static factor model with \( k \) factors:

\[
y_{n,t} = \bar{y}_n + F_t \lambda_n + \epsilon_{n,t} \tag{1}
\]
for $y_{n,t} = of_{n,t}^{\text{risky}}, if_{n,t}^{\text{risky}}, of_{n,t}^{\text{safe}}, if_{n,t}^{\text{safe}}$, where $\lambda_n$ is a $k \times 1$ vector of factor loadings and $F_t$ is a $1 \times k$ vector of global factors, and $\bar{y}_n$ is simply the average value of $y_{n,t}$ over the sample period.

The sample length is $T = 25$ years. Define $y_n$ as a $T \times 1$ vector that stacks the country-period scalars $y_{n,t} - \bar{y}_n$. We can then compactly write the factor model as

$$y = F\Lambda + \epsilon$$

where $y$ is a matrix that that stacks the series side by side. $\Lambda$ is a matrix that stacks $k \times 1$ vectors of factor loadings for each series side-by-side. $F$ is a $T \times k$ matrix that contains the factors $F_t$. The factor analysis gives us a matrix of loadings $\Lambda$. We regress the vector of capital inflows and outflows in each period $t$ on the matrix of loadings $\Lambda$ to estimate the values of the $k$ factors in period $t$.

Figure 2 plots the first factor from the factor model (blue line) together with the MAR asset price factor (red line). The latter is a monthly factor. We normalize their monthly factor to have a standard deviation and mean of respectively 1 and 0, and then annualize by taking the average over a year. Our factor based on capital flow data is clearly closely related to the MAR factor based on asset prices. The correlation between our first factor and the MAR factor is 0.80. Below we refer to the factor based on capital flow data as the GFC factor. While we will use this factor in the upcoming regression analysis, we show in the Online Appendix that very similar results are obtained if we run the same regressions but instead use the MAR asset price factor.

### 2.3 Effect of GFC factor on Asset Prices and Gross Capital Flows

We first consider the effect of the GFC factor on equity prices, the real interest rate and gross capital flows. The latter relates to Fact 1 in the introduction. Consider the following panel regression:

$$\Delta y_{n,t} = \alpha_n + \gamma \Delta y_{n,t-1} + \beta \Delta F_t + \varepsilon_{n,t}$$

where the dependent variable $\Delta y_{n,t}$ is the year-over-year change in either the log equity price $q_{n,t}$, risky capital outflows $of_{n,t}^{\text{risky}}$, risky capital inflows $if_{n,t}^{\text{risky}}$ or their sum $of_{n,t}^{\text{risky}} + if_{n,t}^{\text{risky}}$. The regressors are a country-fixed effect, the lag of the dependent variable and $\Delta F_t$, the the year-over-year change in our estimated GFC factor plotted in Figure 2. We also regress the year-over-year change in the 1-year U.S. real interest rate on a constant, its lag and $\Delta F_t$. The objective here is to determine the impact of an unexpected change in the global factor on the unexpected change in the endogenous variables. $\Delta F_t$ is approximately i.i.d., but changes in the endogenous variables are generally persistent, which is why we include
their lagged change in the regression.

The results from these regressions are presented in Table 1. A one standard deviation decrease in the GFC factor, $\Delta F_t = -1$, is associated with a 55 basis point fall in the real rate on one-year U.S. treasuries and a 16.2% fall in the average equity price. Gross risky asset flows, $o_{n,t}^{\text{risky}} + i_{n,t}^{\text{risky}}$, decrease by 5.5% of GDP. Both outflows and inflows of risky assets fall in response to the shock. These results are all statistically significant.

### 2.3.1 Effect of the GFC factor: Role of Global Imbalances

To establish Facts 2 and 3 in the introduction, we now ask to what extent global imbalances, in the form of net foreign asset positions of safe and risky assets, affect total net outflows (i.e. the current account), national saving and investment, as well as net outflows of risky and safe assets. Net capital outflows are zero at a global level, but we can ask whether the increase or decrease of net capital outflows in individual countries in response to a negative GFC shock is related to existing global imbalances.

It is useful to first take a look at the size of global imbalances. Figure 3 reports a scatterplot of $nfa_{n,t}^{\text{safe}}$ and $nfa_{n,t}^{\text{risky}}$ for each of the 20 countries, based on the average net foreign asset positions of safe and risky assets over the 25 year sample. While the data on net foreign asset positions are a panel, most of the variation is in the cross-section. Two points are worth making. First, there is less variation across countries in the net foreign asset position of risky assets than safe assets. Across the 20 countries, the standard deviation of the net foreign asset position of risky assets is 0.27, versus 0.79 for safe assets. Second, there is not a clear relationship between these two imbalances. Regressing $nfa_{n,t}^{\text{risky}}$ on $nfa_{n,t}^{\text{safe}}$ yields a coefficient of -0.05 with a standard error of 0.08. There are multiple countries in all four quadrants.

To determine the role of global imbalances, we expand the panel data regression (3) to include net foreign asset positions, both by themselves and interacted with the GFC factor:

$$\Delta y_{n,t} = \alpha_n + \gamma Z_{n,t-1} + \beta_1 \Delta F_t + \beta_2 \left( nfa_{n,t-1}^{\text{risky}} \times \Delta F_t \right) + \beta_3 \left( nfa_{n,t-1}^{\text{safe}} \times \Delta F_t \right) + \beta_4 \left( nfa_{n,t-1}^{\text{risky}} \right) + \beta_5 \left( nfa_{n,t-1}^{\text{safe}} \right) + \varepsilon_{i,t} \quad (4)$$

where for the dependent variable $\Delta y_{n,t}$ we consider the year-over-year changes in total net capital outflows, saving, investment, net risky capital outflows, and net safe capital outflows: $\Delta n_{f,t}$, $\Delta \text{save}_{n,t}$, $\Delta \text{invest}_{n,t}$, $\Delta n_{f,t}^{\text{risky}}$, and $\Delta n_{f,t}^{\text{safe}}$. The regressors include a country-fixed effect, one or both of the net foreign asset positions, $nfa_{n,t}^{\text{safe}}$ and $nfa_{n,t}^{\text{risky}}$, and their interaction with $\Delta F_t$. We also include lagged dependent variables. These are contained in $Z_{n,t-1}$, which includes $\Delta n_{f,t-1}$, $\Delta n_{f,t-1}^{\text{risky}}$ and $\Delta \text{save}_{n,t-1}$. We cannot include all lagged dependent variables in $Z_{n,t-1}$ as they are linearly related. Including $Z_{n,t-1}$ as opposed to
just $\Delta y_{n,t-1}$ has the advantage that the regression coefficients are related. For example, the regression for $\Delta n_{f,t}$ has coefficients that equal to sum of the coefficients for the regressions of $\Delta n_{f,sa}^{afe}$ and $\Delta n_{f,ri}^{risky}$.

The results from these regressions are shown in Table 2. For brevity we only present the coefficients on the GFC factor and the interactions between the GFC factor and the net foreign asset position. The coefficients of the non-interacted $n_{f,ri}^{risky}$ and $n_{f,sa}^{afe}$ are generally insignificant. It is first useful to note that for the net capital flow variables the coefficients on $\Delta F_t$ are generally insignificant. This makes sense as net capital flows cannot go up or down in all countries in the same direction in response to a change in the GFC since net capital flows aggregate to zero globally. For saving and investment the coefficients on $\Delta F_t$ are positive and highly significant, consistent with Figure 1. A downturn in the GFC therefore lowers both saving and investment. Globally the decline in saving and investment must be equal as world saving equals world investment.

The coefficient on $n_{f,sa}^{afe} \times \Delta F_t$ is always positive and significant for total net capital outflows. It is positive and significant for saving and negative for investment. It is negative and significant for net risky capital outflows, but positive and significant for net safe capital outflows. In response to a downturn in the GFC factor, a country with a negative foreign asset position of safe assets experiences a rise in total net outflows, which is the sum of an even larger rise in net outflows of safe assets and a drop in net outflows of risky assets. As discussed above, all countries experience a drop in saving and investment. For saving this drop is smaller for countries with a negative net foreign asset position of safe assets. In those countries saving therefore rises relative to other countries. These same countries experience a larger drop in investment. To summarize, during a downturn in the GFC, countries with a net debt of safe assets pay down their debt by selling foreign risky assets (negative net outflows of risky assets) and by increasing saving relative to investment (i.e. increase the current account).

In all of the regressions, the coefficient on the interaction term between the GFC factor and the country’s net foreign asset position of risky assets, $n_{f,ri}^{risky} \times \Delta F_t$, is insignificant. Moreover, when $n_{f,sa}^{afe} \times \Delta F_t$ and $n_{f,ri}^{risky} \times \Delta F_t$ are both included in the regression, the goodness of fit of the regression is little changed from the regression when $n_{f,sa}^{afe} \times \Delta F_t$ enters alone. We can conclude that the impact of the GFC factor on net capital flows, savings, and investment does not depend on the net foreign asset position of risky assets.

2.4 Robustness Analysis

The Online Appendix reports two types of robustness analysis. It presents results corresponding to Tables 1 and 2 for both quarterly data and when using the MAR factor as a measure of $F_t$. The results are robust to both alternatives.
3 Model Description

We now turn to a model that can replicate the stylized facts discussed above. There are $N+1$ countries with investors and households, and a single good. Although all agents have infinite horizons, we effectively collapse the future into a single period by assuming that all uncertainty is resolved in period 2. This simplification allows us to focus on portfolio heterogeneity within and across countries that is central to the main results. In each country there is a continuum of investors on the interval [0,1], who are heterogeneous with respect to risk-aversion and home bias. Investors can hold a safe asset and risky assets from each of the $N+1$ countries. Households only hold the safe asset and are all the same. Countries are heterogeneous with respect to risk-aversion and expected dividends of the risky assets. We take period 0 as given. The analysis focuses on the impact of a negative GFC shock in period 1 in the form of a rise in global risk-aversion.

3.1 Assets

The gross interest rate on the safe asset is $R_t$ from period $t$ to $t+1$. Over the same period, the return on the risky asset from country $n$ is

$$\frac{Q_{n,t+1} + D_{n,t+1}}{Q_{n,t}}$$

where $Q_{n,t}$ is the price of the asset and $D_{n,t}$ the dividend.

The period 1 dividend is set at 1 for all risky assets. There is uncertainty about future dividends, but this uncertainty is resolved at time 2. After that dividends will remain constant: $D_{n,t} = D_{n,2}$ for $t \geq 2$. In what follows it is useful to denote

$$D_n = \frac{D_{n,2}}{1 - \beta}$$

where $\beta$ is the time discount rate. $D_n$ is the present value of dividends at time 2, which is proportional to $D_{n,2}$. For simplicity of analysis, we assume that $D_n$ is uncorrelated across countries. This will be relaxed in the numerical analysis in Section 7.

3.2 Investors

3.2.1 Budget Constraint and Preferences

In period $t$ investor $i$ from country $n$ invests a fraction $z_{n,m,t}^i$ in the risky asset of country $m$. A fraction $1 - \sum_{m=1}^{N+1} z_{n,m,t}^i$ is invested in the safe asset. Wealth of investor $i$ in country
 evolves according to
\[ W_{n,t+1}^i = (W_{n,t}^i - C_{n,t}^i) R_{t+1}^{p,i,n} \]  
where \( C_{n,t}^i \) is consumption and \( R_{t+1}^{p,i,n} \) is the portfolio return from \( t \) to \( t+1 \):
\[ R_{t+1}^{p,i,n} = R_t + \sum_{m=1}^{N+1} z_{n,m,t}^i \left( \frac{Q_{m,t+1}^i + D_{m,t+1}^i}{Q_{m,t}} - R_t \right) \]  
The term in brackets is the excess return of the risky asset from country \( m \) over the safe asset.

Investors are assumed to have Rince preferences, which for investor \( i \) from country \( n \) we can write as
\[ \ln(V_{n,t}^i) = \max_{C_{n,t}^i, z_t} \left\{ (1-\beta) \ln(C_{n,t}^i) + \beta \ln \left( [E_t(V_{n,t+1}^i)^{1-\gamma_{i,n}}]^{1-\gamma_{i,n}} \right) \right\} \]  
where \( z_t = (z_{n,1,t},...,z_{n,1+N,t}) \) is the vector of portfolio shares chosen by the investor at time \( t \). The investor makes consumption and portfolio decisions. The rate of risk-aversion \( \gamma_{i,n} \) will generally vary across investors and countries. Risk-aversion only matters at time 1 as we take period 0 as given and uncertainty is resolved from time 2 onward.

### 3.2.2 Within-Country Portfolio Heterogeneity

We introduce two types of heterogeneity of investors within countries, which lead to heterogeneity of the risky share (portfolio share allocated to risky assets) and the foreign share (share of risky assets allocated to foreign assets). These two types of heterogeneity are associated with investor home bias and risk-aversion. Their main role in the model is to account for Fact 1 that gross outflows and inflows fall in response to the rise in global risk-aversion. Home bias heterogeneity is introduced by allowing perceived dividend risk of foreign assets to vary across investors. In period 1 all country \( n \) investors perceive the variance of \( D_n \) to be \( \sigma^2 \). For any foreign asset \( m \neq n \), investor \( i \) from country \( n \) perceives the variance of \( D_m \) to be \( \sigma^2 / \kappa_i \), with \( 0 < \kappa_i \leq 1 \). When \( \kappa_i = 1 \), all risky assets are perceived to be equally risky and there will be no home bias. When \( \kappa_i < 1 \) as a result of information asymmetries, foreign assets are perceived to be riskier, leading to a bias towards the domestic risky asset that varies across investors. The lower \( \kappa_i \), the stronger the home bias.

Risk-aversion heterogeneity is introduced as follows. Let
\[ \gamma_{i,n} = \frac{1}{\Gamma_i (1 + \epsilon_n^G) G} \]  
The higher \( \Gamma_i \), \( \epsilon_n^G \) or \( G \), the less risk-averse the investor is. The term \( G \) is common across
investors and countries. We will introduce a global risk-aversion shock through a drop in $G$. Variation of $\Gamma_i$ across investors leads to within-country risk-aversion heterogeneity. Variation in $\epsilon_n^G$ leads to cross-country heterogeneity that will be discussed below.

Both risk-aversion heterogeneity and home bias heterogeneity lead to heterogeneity of the risky share, while home bias heterogeneity also leads to variation in the foreign share. As we will see, to account for the decline in gross capital flows we also need a positive relationship across investors of their risky and foreign shares. In other words, investors that allocate a larger portfolio share to risky assets must (on average) also be more globally diversified. While for concreteness we model these portfolio share differences through risk-aversion and home bias parameters, the precise source of the portfolio heterogeneity does not matter for the results.

3.2.3 Cross-Country Portfolio Heterogeneity

We also introduce two types of cross-country heterogeneity: variation across countries in risk-aversion and the expected dividend of risky assets. Both again lead to portfolio heterogeneity, this time across countries. The parameter $\epsilon_n^G$ in (10) represents heterogeneity in risk-aversion across countries. For expected dividends, we denote the expectation at time 1 of $D_n$ as $\bar{D}_n$ and assume

$$\bar{D}_n = 1 + a + \bar{z} \frac{a^2}{a\bar{\psi}} (1 + \epsilon_n^D)$$

Here $a = \frac{\beta}{1-\beta}$ and $\bar{\psi}$ is the mean across investors of $\psi_i = \Gamma_i(1 + N\kappa_i)$. The parameter $\bar{z}$ will be equal to the mean risky share across all investors in all countries in the equilibrium of the model prior to the global risk-aversion shock.

Both $\epsilon_n^G$ and $\epsilon_n^D$ affect the country level risky shares, while $\epsilon_n^D$ also affects the foreign share. Both variation in the risky share and foreign share across countries lead to heterogeneity in the two net foreign assets positions $nfa_n^{safe}$ and $nfa_n^{risky}$ that are critical to Facts 2-3. The cross-sectional relationship between these two global imbalances is different for risk-aversion heterogeneity (negative relationship) and expected dividend heterogeneity (positive relationship). All that matters in the end is that the two imbalances are imperfectly correlated across countries. This will almost always be the case as long as there are at least two sources of heterogeneity. We chose risk-aversion and expected dividend heterogeneity for concreteness, though this is not important for the results.

3.2.4 Optimal Consumption and Portfolios

The value function will be proportional to the wealth of the agent: $V_{n,1}^i = \alpha_{1,i} W_{n,1}^i$ and $V_{n,t}^i = \alpha_2 W_{n,t}^i$ for $t \geq 2$. The coefficients $\alpha_{1,i}$ and $\alpha_2$ can be derived from the Bellman equation and depend on structural model parameters (see Appendix A), but are not important to the
analysis. Using (7), investors at time $t$ therefore maximize

$$
(1 - \beta) \ln(C^i_{n,t}) + \beta \ln(W^i_{n,t} - C^i_{n,t}) + \beta \ln \left( \left[ E_t \left( R^{p,i,n}_{t+1} \right) \right]^{1-\gamma_{i,n}} \right)
$$

(12)

Optimal consumption is then

$$
C^i_{n,t} = (1 - \beta)W^i_{n,t}
$$

(13)

All investors consume a fraction $1 - \beta$ of their wealth during each period. This leaves the investor with financial wealth $\beta W^i_{n,t}$ that is invested in safe and risky assets.

Since uncertainty is resolved at time 2, there is only a portfolio problem at time 1. Therefore the only portfolio return that matters is $R^{p,i,n}_2$, which for simplicity we will denote $R^{p,i,n}$. From (12) optimal portfolio shares are chosen to maximize the certainty equivalent of the portfolio return:

$$
[E(R^{p,i,n})]^{1-\gamma_{i,n}}
$$

(14)

Using a second order Taylor expansion of $(R^{p,i,n})^{1-\gamma_{i,n}}$ around the expected portfolio return, one can approximate this as maximizing

$$
E \left( R^{p,i,n} \right) - 0.5\gamma_{i,n} \text{var} \left( R^{p,i,n} \right)
$$

(15)

This leads to simple mean-variance portfolios.

As shown in the Online Appendix, risky asset prices at time 2 are $Q_{m,2} = [a/(1 + a)]D_m$. The period 2 asset payoffs are then $Q_{m,2} + D_{m,2} = D_m$. For ease of notation, from hereon we remove time subscripts from all time 1 variables. The portfolio return then becomes

$$
R^{p,i,n} = R + \sum_{m=1}^{N+1} z^{i}_{n,m} \left( \frac{D_m - RQ_m}{Q_m} \right)
$$

(16)

Maximizing (15) leads to the following optimal portfolios

$$
z^{i}_{n,n} = Q_n \Gamma_i \left( 1 + \epsilon^G_n \right) G \frac{\bar{D}_n - RQ_n}{\sigma^2}
$$

(17)

$$
z^{i}_{n,m} = Q_m \Gamma_i \kappa_i \left( 1 + \epsilon^G_n \right) G \frac{\bar{D}_m - RQ_m}{\sigma^2} \quad m \neq n
$$

(18)

For a given interest rate and risky asset prices, higher values of $\Gamma_i$, $\epsilon^G_n$ and $G$ (lower risk-aversion) imply a proportionally higher portfolio share allocated to all risky assets. A larger
implies a higher portfolio share allocated to foreign risky assets, without changing the portfolio share allocated to domestic risky assets (for given asset prices). Finally, a higher expected dividend $\bar{D}_m$ implies that all investors from all countries allocate a larger portfolio share to country $m$ risky assets.

3.3 Households

Saving by investors only depends on risky asset prices. Without any other agents, it is then impossible for risky asset prices to fluctuate as world saving is zero. We therefore introduce another set of agents, which we rather arbitrarily refer to as households, whose saving depends on the interest rate. These agents generate an interest rate elastic supply or demand of safe assets. Rather than households, it could be the government as well, or any other institutions that only demand or supply safe assets.

Since these agents only play the role of generating an interest rate elastic supply or demand of safe assets, we assume that they are identical across countries. We therefore omit the country subscript. Households in any one of the countries maximize

$$\sum_{s=0}^{\infty} \beta^s \left( \frac{C_{h_t+s}}{1 - \frac{1}{\rho}} \right)$$

They only hold the safe asset, $B_{h_t}$ in period $t$. Households inherit $B_{h_0}$ from time 0. They receive an endowment of $Y$ each period. The budget constraint is

$$B_{h_t} = R_{t-1} B_{h_{t-1}} + Y - C_{h_t}$$

The first-order condition is

$$C_{h_t} = (R_t \beta)^{-\rho} C_{h_{t+1}}$$

3.4 Asset Market Clearing

The period $t$ market clearing conditions for risky assets are

$$\beta \sum_{m=1}^{N+1} \int_0^1 z_{m,n,t}^i W_{m,n,t}^i di = Q_{n,t} K_n \quad n = 1, \ldots, N + 1$$

Here $K_n$ is the supply of risky asset $n$.

In addition there is a market clearing condition for safe assets. We can also use the aggregate market clearing condition for all assets that equates the demand to the supply of

\[13\] An alternative is to introduce investment that depends on the interest rate.
all assets:
\[(N + 1)B^h_t + \beta \sum_{n=1}^{N+1} \int_0^1 W^i_{n,t} di = \sum_n Q_{n,t}K_n \quad (23)\]

We can show that (23) corresponds to zero world saving. This needs to be the case as there is no investment in the model. In Section 7 we extend the model by introducing investment. In that case the aggregate asset market equilibrium condition corresponds to equality between world saving and investment.

### 3.5 Pre-Shock Equilibrium

We are interested in the impact of a global risk-aversion shock that lowers $G$. But we first describe the pre-shock equilibrium, for which we assume $G = 1$. We make a set of assumptions regarding initial conditions at time 0 that are intended to make sure that equilibrium values of endogenous variables are the same at time 1 as at time 0 before the shock. We can think of this as a type of pre-shock steady state.

**Assumption 1** Assume the following initial conditions for period 0: $W^i_{n,0} = (1 + a)/\bar{z}$ for all investors, $Q_{n,0} = a$, $R_0 = (1 + a)/a$, $B^h_0 = a \left( \frac{1}{N+1} \sum_{n=1}^{N+1} K_n - \frac{1}{\bar{z}} \right)$, and

\[
z_{n,n,0}^i = \Gamma_i \psi \bar{z} \left( 1 + \epsilon_G^G (1 + \epsilon_D^G) \right)
\]

\[
z_{m,n,0}^i = \Gamma_i \kappa_i \bar{z} \psi \left( 1 + \epsilon_G^G (1 + \epsilon_D^G) \right) \quad m \neq n
\]

Also assume $K_n = \left( E(\Gamma) (1 + \epsilon_G^G) + E(\Gamma \kappa) \sum_{m \neq n} (1 + \epsilon_G^G) \right) \frac{1}{\psi} (1 + \epsilon_D^G)$.

Here the expectation operator refers to the cross-sectional mean, so for example $E(\Gamma) = \int_0^1 \Gamma_i di$. The period 0 assumptions are such that the market clearing conditions (22)-(23) are satisfied for period 0. Allowing for within-country heterogeneity in $\Gamma_i$ and $\kappa_i$, as well as either cross-country heterogeneity in $\epsilon_G^G$ or $\epsilon_D^G$, it is easily checked that the risky share averages to $\bar{z}$ across across investors and countries.\(^\text{14}\)

Appendix A proves the following regarding the pre-shock equilibrium:

**Theorem 1** Under Assumption 1 and $G = 1$, there is an equilibrium where in period 1: $Q_n = a$, $W^i_n = (1 + a)/\bar{z}$, $z_{n,n}^i = z_{n,n,0}^i$ and $z_{n,m}^i = z_{n,m,0}^i$. In all periods $t \geq 1$: $R_t = (1 + a)/a$, $B^h_t = B^h_0$. In all periods $t \geq 2$: $Q_{n,t} = [a/(1 + a)]D_n$, $W^i_{n,t} = W^i_{n,2}$.

\(^{14}\)In the numerical Section 7, where we introduce both types of cross-country heterogeneity simultaneously, the average risky share is not exactly $\bar{z}$, but extremely close.
Therefore risky asset prices, the interest rate, wealth, portfolio allocation and household safe asset holdings are all the same in period 1 as in period 0. Since quantities of asset holdings are also the same in periods 0 and 1, there will be zero gross and net capital flows in the pre-shock equilibrium in period 1.

A couple of comments about the asset supplies $K_n$ are in order. Without cross-country heterogeneity ($\epsilon_n^G = \epsilon_n^D = 0$ for all $n$), Assumption 1 implies that $K_n = 1$ in all countries. When we introduce cross-country heterogeneity, equal supplies of risky assets in all countries will generally imply that the prices of risky assets vary across countries. We adjust the risky asset supplies such that the prices of risky assets are identical across countries in the pre-shock equilibrium. In a model with investment, a higher demand for an asset would eventually be accommodated through a higher supply. We think of the pre-shock equilibrium as capturing such an initial state.

### 3.6 Period 1 Capital Flows

After the risk-aversion shock, capital flows are generally no longer zero. Time 1 capital outflows $OF_{\text{risky}}^n$ are defined as purchases of foreign risky assets by country $n$ investors, while time 1 capital inflows $IF_{\text{risky}}^n$ are purchases of country $n$ risky assets by foreign investors. These are equal to\(^{15}\)

\[
OF_{\text{risky}}^n = \beta \int_0^1 \sum_{m \neq n} z_{i,n,m}^i W_{n}^i di - \int_0^1 \sum_{m \neq n} Q_m \frac{z_{i,m,n}^i}{z} di \tag{26}
\]

\[
IF_{\text{risky}}^n = \beta \int_0^1 \sum_{m \neq n} z_{i,m,n}^i W_{m}^i di - Q_n \int_0^1 \sum_{m \neq n} \frac{z_{i,m,n}^i}{z} di \tag{27}
\]

Define the fraction that investor $i$ from country $n$ invests in all risky assets (the “risky share”) as

\[
\frac{\sum_{m=1}^{N+1} z_{i,n,m}^i}{N+1}
\]

with its time zero value denoted $z_{i,0}^i$. Period 1 net capital outflows of safe assets are then

\[
NF_{\text{safe}}^n = B^h - B_0^h + \beta \int_0^1 \left(1 - z_{i,n}^i\right) W_{n}^i di - \frac{a}{z} \int_0^1 \left(1 - z_{i,n,0}^i\right) di \tag{29}
\]

Here $B^h - B_0^h$ are purchases of safe assets by households, while the rest captures the change\(^{15}\)The time 0 portfolio shares divided by $\bar{z}$ correspond to time 0 quantities of assets. By Assumption 1, investor $i$ in country $n$ has a financial wealth of $\beta W_{n,0}^i = a/\bar{z}$ at time 0, so that the value of the country $m$ risky asset held by the investor is $(a/\bar{z})z_{i,m,n,0}^i$. This corresponds to a quantity of $z_{i,m,n,0}/\bar{z}$ as $Q_{m,0} = a$ by Assumption 1.
in safe asset holdings from period 0 to 1 by investors.

4 Impact GFC Shock on Asset Prices

The key results of the paper relate to the impact of a negative GFC shock on gross and net capital flows, which depend critically on the wealth effects of a global decline in risky asset prices. But before addressing this, in this section we will first show how a rise in global risk-aversion gives rise to a global drop in risky asset prices, as well as a drop in the interest rate.

Since these asset price results do not depend on portfolio heterogeneity of the investors, we will consider here the simplest version of the model, where all investors have the same portfolios. Specifically, we assume $\Gamma_i = \Gamma$, $\kappa_i = \kappa$ and $\epsilon_n^C = \epsilon_n^D = 0$. In Section 5 we introduce within-country heterogeneity, which allows us to also account for Fact 1, a drop in gross capital outflows and inflows of risky assets. In Section 6 we consider cross-country heterogeneity, which allows us to account for Facts 2-3 related to net capital flows.

The negative GFC shock takes the form of a rise in global risk-aversion in period 1 (drop in $G$). All analytical results consider the derivatives of time 1 endogenous variables with respect to $G$ at $G = 1$. Without cross-country heterogeneity, all period 1 risky asset prices will be the same and denoted $Q$. The risky asset price $Q$ and interest rate $R$ can be jointly solved from the asset market clearing conditions (22)-(23) in period 1. After substituting optimal portfolio shares (17)-(18), investor wealth

$$W_n^i = 1 + a\bar{z} + Q - a$$

and the household budget constraint (20), these become

$$\left(\bar{D} - RQ\right)\left(\frac{1 + a}{\bar{z}} + Q - a\right) = \frac{1 + a}{\bar{z}}\frac{\sigma^2}{\bar{\psi}} G$$ (30)

$$\left(R_0 - 1\right)B_0^h + Y - C_1^h - \frac{Q - a}{1 + a} = 0$$ (31)

The first equation is the risky asset market equilibrium condition (RAE). The left hand side of (30) shows that demand for risky assets depends on the risky asset price $Q$ both negatively (first term) and positively (second term). On the one hand, a rise in $Q$ lowers the expected return on risky assets, lowering its demand. On the other hand, it raises wealth of investors, which raises demand for risky assets. We adopt the rather weak Assumption 2 to make sure that the first effect dominates.

Assumption 2 $\frac{\bar{\psi}^3}{E\bar{\psi}^2}(1 + a)^2 > \bar{z}^2\sigma^2$
This assumption assures that a higher asset price always reduces demand for risky assets. In the absence of within-country heterogeneity, it implies that in the pre-shock equilibrium \( \frac{\partial}{\partial Q} - R < R_0^2/\bar{z} \). With \( \bar{z} < 1 \), this condition says that the expected excess return on risky assets must be less than a number that is above 1, or 100 percent. This is evidently a very weak condition. A rise in \( Q \) then lowers demand for risky assets, while a drop in \( R \) raises it.

The aggregate asset market equilibrium condition (31) is equivalent to a zero saving condition \((S = 0)\). Since world saving is zero, and there is no cross-country heterogeneity, saving must be zero in each country. The term \((R_0 - 1)B_0^h + Y - C_{1h}^h\) is equal to household saving, while the term \(-Q - a)/(1 + a)\) is saving by investors. Household saving depends on the interest rate, while saving by investors depends on the risky asset price. The higher the risky asset price, the higher the wealth of the investors, which raises consumption and lowers saving.

Household saving depends on the interest rate as household consumption depends on the interest rate. Period 1 household consumption is

\[
C_{1h} = \frac{1}{1 + a(1 + a)^{1-\rho}R^\rho - 1} \left( Y + \frac{1 + a}{R}Y + \frac{1 + a}{a}B_0^h \right) \tag{32}
\]

Taking the derivative of (32) at the pre-shock equilibrium, we have

\[
\frac{\partial C_{1h}}{\partial R} = -\left( Y + (\rho - 1)C_{1h}^h \right) \frac{a^2}{(1 + a)^2} \equiv -\lambda \tag{33}
\]

where \( C_{1h} = Y + 1 - (1/\bar{z}) \) is constant household consumption in the pre-shock equilibrium. A higher interest rate lowers household consumption, which raises saving, and more so the higher \( \rho \). It follows that (31) implies a positive relationship between \( Q \) and \( R \).

The equilibrium is illustrated in Figure 4. The zero saving condition \( S = 0 \) is the upward sloping line and is not affected by a drop in \( G \). The risky asset market equilibrium schedule RAE is downward sloping. The pre-shock equilibrium is at point A. From (30) we can see that a rise in global risk-aversion (drop in \( G \)) shifts the RAE schedule to the left. The new equilibrium is at point C. The increase in global risk-aversion leads to a reallocation from risky to safe assets that lowers both \( Q \) and \( R \).

How much risky asset prices drop depends on how much the interest rate falls. The more the interest rate falls, the less risky asset prices drop. We can consider two extremes. One is a situation where household saving does not depend on the interest rate, which occurs when \( \lambda = 0 \). In that case the \( S = 0 \) schedule is vertical. The risky asset price \( Q \) is then unaffected by the global risk-aversion shock. Although higher risk-aversion lowers demand...
for risky assets, this is neutralized by a lower interest rate that equally raises demand for risky assets. Since household saving is unaffected by the interest rate, investor saving cannot change either in equilibrium. This can only be the case when the risky asset price does not change.

The other extreme is where household saving is infinitely interest rate elastic, so that \( \rho \to \infty \). The \( S = 0 \) schedule is then horizontal. In this case the interest rate does not change. This leads to an equilibrium at point B, where the drop in \( Q \) is largest as there is no reallocation back to risky assets due to a lower interest rate. In general, in order for the risky asset price to drop, there must be an interest rate elastic demand or supply for safe assets. This implies that either aggregate saving depends positively on the interest rate or aggregate investment (assumed zero here) depends negatively on the interest rate. When introducing investment, the \( S = 0 \) schedule becomes the \( S = I \) schedule.

Algebraically, the changes in \( Q \) and \( R \) in response to a change in \( G \) are:

\[
\frac{dQ}{dG} = \frac{\bar{z}(1 + a) \sigma^2}{(1 + a)^2 - \frac{\bar{z} \sigma^2}{\bar{\psi}} + \frac{\sigma^2}{\lambda}} (34)
\]
\[
\frac{dR}{dG} = \frac{1}{\lambda (1 + a)} \frac{dQ}{dG} (35)
\]

If Assumption 2 holds, the denominator of (34) is positive. The more sensitive household saving is to the interest rate (larger \( \lambda \)), the larger the drop in the risky asset price and the smaller the drop in the interest rate. Also note that both the risky asset price and interest rate drop more when \( \sigma^2/\bar{\psi} \) is higher, with \( \bar{\psi} = \Gamma(1 + N\kappa) \). This happens when risk or risk-aversion are higher. The RAE schedule in Figure 4 then shifts further to the left when \( G \) falls.

These findings are summarized as follows:

**Theorem 2** Assume that there is no heterogeneity of investors within or across countries, and Assumptions 1 and 2 hold. Then a rise in global risk-aversion lowers risky asset prices equally in all countries and also lowers the interest rate on the safe asset.

While a rise in global risk aversion naturally lowers risky asset prices, and is typically viewed as the driver behind a downturn of the global financial cycle, in what follows any other shock that lowers global risky asset prices will have similar effects. What matters for gross and net capital flows is the impact of such a global decline in risky asset prices on relative wealth as a result of portfolio heterogeneity.
5 Impact GFC Shock on Gross Capital Flows

We now allow $\Gamma_i$ and $\kappa_i$ to vary across investors within countries, while continuing to assume no cross-country heterogeneity.

It is useful to start with the portfolios of investors prior to the risk-aversion shock. Using (24)-(25), the risky share of investor $i$ in any country is

$$z^i = \alpha_i \bar{z}$$

where

$$\alpha_i = \frac{\psi_i}{\bar{\psi}} = \frac{\Gamma_i(1 + N\kappa_i)}{E(\Gamma(1 + N\kappa))}$$

has a cross-sectional mean of 1. The cross-sectional mean of portfolio shares is therefore $\bar{z}$. The cross-sectional variance is positive when $\text{var}(\alpha) > 0$.

In Appendix B we show that there is still an equal drop in all risky asset prices under a negative GFC shock when we allow for within-country heterogeneity. The key point of this section is to show that this global drop in risky asset prices leads to a drop in gross capital flows of risky assets. This operates through the impact of the lower asset prices on the relative wealth of individual investors. The period 1 wealth of investor $i$ from any country is

$$W^i = \frac{1}{\bar{z}} \left(1 + a + z^i(Q - a)\right)$$

Heterogeneity in risky shares across investors leads to heterogeneity of the wealth impact of the drop in risky asset prices. The larger the risky share of an investor, the larger the drop in wealth when risky asset prices fall.

This wealth drop in turn affects buying and selling of risky assets. Using the equilibrium portfolios (17)-(18) and the risky asset market clearing condition (22), we can show that the quantities of all risky assets that an investor holds go up or down in proportion to their holdings before the shock. The total quantity of risky assets held by investor $i$ is $\alpha_i$ before the shock and

$$W^i = \int_0^1 \alpha_j W^j d\alpha_j \frac{1}{\alpha_i}$$

after the shock. If the wealth of an investor drops relative to the weighted average wealth of all investors, it sells risky assets in equilibrium. Those investors whose wealth drops less than the weighted average will buy risky assets. The weights are determined by the quantity of risky assets that investors hold in their portfolio before the shock.
The change in the quantity of risky assets held by investor $i$ is then $Z^i dQ$, where

$$Z^i = \bar{z} \frac{\alpha_i (\alpha_i - 1 - \text{var}(\alpha))}{1 + a}$$  \hspace{1cm} (39)$$

Investor $i$ therefore sells risky assets when $\alpha_i > 1 + \text{var}(\alpha)$. The reason that $\alpha_i$ needs to be above $1 + \text{var}(\alpha)$ rather than the cross-sectional mean of $\alpha$ of 1 is that what matters is the wealth of investor $i$ relative to a weighted average of the wealth of all investors. More weight is given to investors that hold more risky assets. Since the cross-sectional mean of $\alpha_i$ is 1, less than half of investors sell risky assets, while more than half buy risky assets.

The impact of the shock on gross capital flows of risky assets depends on the extent of home bias of investors that are buying and selling risky assets. Specifically, let $z^i_F$ be the pre-shock “foreign share” of investor $i$, which is the fraction of risky assets allocated to foreign risky assets:

$$z^i_F = \frac{N \kappa_i}{1 + N \kappa_i}$$  \hspace{1cm} (40)$$

A higher $\kappa_i$ implies less home bias and therefore a larger $z^i_F$.

We now make the following assumption.

**Assumption 3** Assume that $\text{cov}(z_F, Z) > 0$.

Appendix B then proves the following result.

**Theorem 3** Assume that there is heterogeneity across investors within countries, but no cross-country heterogeneity, and Assumptions 1, 2, and 3 hold. Then a rise in global risk aversion leads to a drop in gross capital outflows and inflows of risky assets.

It also remains the case that all risky asset prices drop equally and net capital flows remain zero.

Assumption 3 is key to the drop in gross capital flows. It implies that investors with sufficiently large risky shares that sell risky assets ($Z^i > 0$) tend to invest a relatively large share of risky assets abroad (high $z^i_F$). To understand this, we can split investors into two groups: a group with high risky shares that is selling risky assets ($Z^i > 0$) and a group with lower risky shares that is buying risky assets ($Z^i < 0$). Overall the quantity of risky assets sold by the first group is equal to the quantity bought by the second group as $E(Z) = 0$ (the supply of risky assets is unchanged). However, it matters whether investors sell or buy domestic or foreign risky assets. When Assumption 3 holds, the first group is less home biased and therefore sells more foreign risky assets than the second group buys. These foreign risky assets are therefore sold at least partially to foreigners, which reduces gross capital outflows.
The decline in gross capital flows requires that risky shares and home bias vary across investors, and investors with a higher risky share also tend to have a higher foreign share (are less home biased). As we will discuss in Section 7, this is consistent with evidence from Swedish household portfolio data in Calvet et al. (2007, 2009a, and 2009b), which will be used to calibrate the model.

The proof in Appendix B for the decline in gross capital flows does not rely on the specifics for why the risky share and foreign share vary across investors. All that matters is that Assumption 3 holds, independent of the micro foundations that lead to cross-sectional variation of risky and foreign shares. But it is useful to discuss how Assumption 3 relates to the specific forms of within-country heterogeneity introduced in Section 3, related to the parameters $\Gamma_i$ and $\kappa_i$. We can ask under what assumptions about these structural parameters Assumption 3 holds.

Let $\bar{\kappa}$ and $\bar{\Gamma}$ be the cross-sectional mean of $\kappa$ and $\Gamma$. Write $\kappa_i = \bar{\kappa} + \epsilon_\kappa^i$ and $\Gamma_i = \bar{\Gamma} + \omega \epsilon_\kappa^i + \epsilon_\Gamma^i$. Assume that $\epsilon_\kappa^i$ and $\epsilon_\Gamma^i$ are uncorrelated and have symmetric distributions with mean zero. They are also such that $\Gamma_i$ and $\kappa_i$ are always positive. In Appendix B we then show that Assumption 3 is satisfied when $\omega \geq 0$ and $\text{var}(\epsilon_\kappa^i) > 0$. This means that we must have cross-sectional variation in home bias. We do not necessarily need cross-sectional variation in risk-aversion. To the extent that we do have cross-sectional variation in risk-aversion, Assumption 3 holds when the covariance between $\Gamma_i$ and $\kappa_i$ is non-negative.

To see why cross-sectional variation in $\kappa$ is sufficient for Assumption 3 to hold, note that both $\alpha_i$ and $z_F^i$ are larger the larger $\kappa_i$. A larger value of $\kappa_i$ leads to both a higher risky share and a higher foreign share. This is exactly what is needed to satisfy Assumption 3. When $\omega > 0$, so that the cross-sectional covariance between $\kappa$ and $\Gamma$ is positive, investors with low risk-aversion (and therefore a high risky share) also tend to have low home bias. This further contributes to a positive covariance between $Z^i$ and $z_F^i$ in Assumption 3.

However, risk-aversion heterogeneity alone is not sufficient as it does not lead to cross-sectional variation in home bias and therefore $z_F^i$. One can still distinguish between two groups of investors. Those with high enough risky shares will sell risky assets, while the others will buy. The overall quantities of risky assets bought and sold by the two groups are equal. But when all investors are equally home-biased, one group of domestic investors simply sells foreign risky assets to the other group of domestic investors, leaving gross capital flows unchanged.

6 Impact GFC Shock on Net Capital Flows

We now analyze what happens in response to the global risk-aversion shock when there is cross-country heterogeneity in either risk-aversion ($\epsilon_G^i$ varies across countries) or the expected
dividends of the risky assets ($r^D_n$ varies across countries). These asymmetries lead to pre-shock global imbalances in the form of non-zero net foreign asset positions of safe and risky assets. We analyze the role that these imbalances play in the impact of the global risk-aversion shock on net capital flows and risky asset prices. To keep the analysis tractable, we only consider one type of heterogeneity at a time and assume that there is no within-country heterogeneity ($\Gamma_i = \bar{\Gamma}$ and $\kappa_i = \bar{\kappa}$).

### 6.1 Net Foreign Asset Positions

The global imbalances are the pre-shock net foreign asset positions of safe and risky assets in period 1, which are equal to those in period 0. For country $n$ these are

$$
NFA^\text{safe}_n = B^h + (1 - z_n) \beta W_n \quad (41)
$$

$$
NFA^\text{risky}_n = \sum_{m \neq n} z_{n,m} \beta W_n - \sum_{m \neq n} z_{m,n} \beta W_m \quad (42)
$$

where $z_n = \sum_{m=1}^{N+1} z_{n,m}$ is the risky share of country $n$ investors. We have removed the investor superscripts as all investors within a country are identical.

First consider the net foreign asset position of safe assets. Since we have assumed that households are the same in all countries, this is determined by the portfolio allocation to safe assets by investors. Subtracting the average $NFA^\text{safe}_n$ across all countries, which is zero, and using that $\beta W_n = a/\bar{z}$ in the pre-shock equilibrium, we have

$$
NFA^\text{safe}_n = -\frac{a}{\bar{z}} \left( z_n - \frac{1}{N+1} \sum_{m=1}^{N+1} z_m \right) \quad (43)
$$

A country with a relatively large risky share is therefore a net debtor of safe assets.

Under risk-aversion heterogeneity, using pre-shock portfolio shares, we have

$$
z_n = \bar{z}(1 + \epsilon^G_n) \quad (44)
$$

When $\epsilon^G_n > 0$, investors in country $n$ are less risk averse than average. They therefore have a larger risky share and a smaller portfolio share of safe assets. The net foreign asset position of safe assets is then negative:

$$
NFA^\text{safe}_n = -a\epsilon^G_n \quad (45)
$$

Under expected dividend heterogeneity, using pre-shock portfolio shares, we have

$$
z_n = \bar{z} \left( 1 + \frac{1 - \kappa}{1 + N\kappa} \epsilon^D_n \right) \quad (46)
$$
If $\epsilon_n^D > 0$, the risky asset of country $n$ has a relatively high expected dividend. When there is home bias ($\kappa < 1$), this additional appeal of the country $n$ risky asset will raise the overall risky share of country $n$ and therefore lower the share allocated to safe assets. This again leads to a negative net foreign asset position of safe assets:

$$NFA_{n}^{safe} = -a \frac{1 - \kappa}{1 + N\kappa} \epsilon_n^D$$

(47)

Therefore both countries with lower risk-aversion and higher expected dividends will be net borrowers of safe assets.

Next consider the net foreign asset position of risky assets. Under risk-aversion heterogeneity we have

$$NFA_{n}^{risky} = a \frac{(N + 1)\kappa}{1 + N\kappa} \epsilon_n^G$$

(48)

Countries with low risk-aversion ($\epsilon_n^G > 0$) will have a positive net foreign asset position of risky assets. Country $n$ investors then allocate a larger portfolio share to foreign risky assets than foreign countries do in the country $n$ risky asset.

Under expected dividend heterogeneity we have

$$NFA_{n}^{risky} = -a \frac{(N + 1)\kappa}{1 + N\kappa} \epsilon_n^D$$

(49)

When $\epsilon_n^D > 0$, the high expected dividend of the country $n$ asset will reduce the holding of foreign risky assets by country $n$ investors and increase the holding of country $n$ risky assets by foreigners. This leads to a negative net foreign asset position of risky assets.

While both low risk-aversion and a high expected dividend of the domestic risky asset give rise to a negative net foreign asset position of safe assets, they have opposite effects on the net foreign asset position of risky assets. Low risk aversion leads to a positive net foreign asset position of risky assets, while a high expected domestic dividend leads to a negative net foreign asset position of risky assets. Any combination of $NFA_{n}^{safe}$ and $NFA_{n}^{risky}$ that we observe in the data can then be achieved in the model with a combination of $\epsilon_n^G$ and $\epsilon_n^D$. We will use this in the calibration of the model in the next section.

6.2 Net Capital Flows

We now consider how cross-country portfolio heterogeneity, and the associated global imbalances, impacts net capital flows in response to the global risk-aversion shock.

We assume either risk-aversion or expected dividend heterogeneity. Specifically, assume either $\epsilon_n^G = g_n \epsilon$ and $\epsilon_n^D = 0$ or $\epsilon_n^D = d_n \epsilon$ and $\epsilon_n^G = 0$, where $\sum_{n=1}^{N+1} g_n = \sum_{n=1}^{N+1} d_n = 0$. For example, when $\epsilon > 0$, countries for which $g_n > 0$ are less risk-averse and those for which
$g_n < 0$ are more risk-averse.

Now consider a country-specific variable $X_n$, which can be the risky asset price or net capital flows of safe or risky assets, or total net capital flows. Appendices C and D consider the impact of a global risk-aversion shock under respectively risk-aversion heterogeneity and expected dividend heterogeneity. To do so, we compute the second-order derivative

$$\frac{\partial^2 X_n}{\partial G \partial \epsilon}$$

at $\epsilon = 0$ and $G = 1$. We show that it is proportional to $g_n$ (risk-aversion heterogeneity) or $d_n$ (expected dividend heterogeneity), either positively or negatively. This tells us how the response to the global risk-aversion shock will vary across countries. Using the findings from Appendices C and D, Appendix E proves the following Theorem.

**Theorem 4** Assume that there is no within-country heterogeneity and there is either cross-country heterogeneity in risk-aversion or expected dividends. Assumptions 1 and 2 hold as well. Then, in response to a rise in global risk-aversion, countries with a negative net foreign asset position of safe assets experience (1) a positive overall net capital outflow, (2) a negative net outflow of risky assets, (3) a positive net outflow of safe assets, (4) a larger than average drop in the risky asset price. The opposite is the case for countries that have a positive net foreign asset position of safe assets. Moreover, the size of these changes is monotonically related to the size of the net foreign asset position of safe assets.

The intuition behind Theorem 4 is critically associated with the impact of asset price changes on relative wealth across the countries. Countries with a negative net foreign asset position of safe assets have a larger risky share. They experience a relatively large drop in wealth when risky asset prices fall in response to the rise in global risk aversion. As was the case for gross capital flows in the previous section, the impact of the shock on net capital flows also operates through this relative wealth mechanism.

First consider total net capital outflows, which is equal to the current account. We can write $CA_n = S_n$, where the latter is saving of country $n$ households and investors. Since the world current account is zero, we can also write this as $CA_n = S_n - \sum_{i=1}^{N+1} S_i / (N + 1)$. Net capital flows are therefore determined by relative saving. Countries that are net borrowers of safe assets experience a larger drop in their wealth. This leads to a larger drop in consumption and therefore a rise in saving relative to other countries. This implies overall net capital outflows.

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17For example, when $X_n$ is a net capital flow variable, we have $\partial X_n / \partial G = 0$ at $\epsilon = 0$. When the second-order derivative (50) depends positively on $g_n$, it means that countries that are less risk-averse experience a drop in $X_n$ (negative net outflows) in response to a rise in global risk-aversion ($dG < 0$).
Next consider net capital outflows of risky assets. Let $k_{n,m}$ be the quantity of the country $m$ risky asset held by country $n$ investors in period 1. $k_{n,m,0}$ is the analogous period 0 quantity. In Appendix F we show that

$$k_{n,m} = \frac{W_n}{\sum_{l=1}^{N+1} \omega_{l,m} W_l} k_{n,m,0} \quad (51)$$

Here $W_n$ is the wealth of country $n$ and $\omega_{l,m}$ is the share of the country $m$ risky asset supply that is held by investors from country $l$. These shares therefore add to 1. The quantity of risky assets held is therefore determined by relative wealth. This expression is analogous to (38). There we considered relative wealth across investors within a country, while now we now consider relative wealth across countries.

(51) implies that countries whose wealth drops relative to that of a weighted average of all countries will sell risky assets. Net borrowers of safe assets experience a larger drop in wealth and will therefore sell risky assets. Consider an example with just two countries, a Home country that is a net borrower of safe assets and a Foreign country that is a net lender of safe assets. The larger drop in wealth in the Home country implies that it will sell risky assets, including Foreign risky assets. The smaller wealth drop in Foreign implies that it will buy more risky assets, including Home risky assets. The Home country therefore experiences negative outflows and positive inflows of risky assets. The net outflow of risky assets will then be negative.

Next consider net capital flows of safe assets. In Appendix F we show that net capital flows of safe assets can be written as

$$NF_{n}^{safe} = S_n - \sum_{m=1}^{N+1} (k_{n,m} - k_{n,m,0}) Q_m \quad (52)$$

where $S_n$ is again country $n$ saving. Both increased saving and selling of risky assets lead to net capital outflows of safe assets. We have already discussed that a country that is a net borrower of safe assets will raise saving and sell risky assets, both as a result of a relatively large drop in wealth. Therefore countries with a negative net foreign asset position of safe assets will have net outflows of safe assets in response to the rise in global risk aversion.

Theorem 4 also says that the relative price of the risky asset of a country that is a net borrower of safe assets will drop in response to the negative GFC shock. We have seen that such countries will reduce their demand for risky assets as a result of their larger drop in wealth. Since they are biased towards their domestic asset ($\kappa < 1$), there will be a drop in relative demand of risky assets from countries with a negative net foreign asset positions of safe assets, which lowers their relative risky asset price. In terms of net capital flows, this has two implications. First, it reinforces the decline in relative wealth of net borrowers of safe
assets and therefore the net capital flow results discussed above. Second, in the next section we extend the model to include investment, which depends on the asset price (Tobin’s Q). This implies a larger drop of investment in countries that are net borrowers of safe assets, which reinforces the increase in the current account in these countries.

The following Corollary follows immediately from Theorem 4.

**Corollary 1** Assume that there is no within-country heterogeneity and there is either cross-country heterogeneity in risk-aversion or expected dividends. Assumptions 1 and 2 hold as well. Then, in response to a rise in global risk-aversion, knowing the sign of the net foreign asset position of risky assets is not informative about the sign of net capital outflows (total, safe and risky).

As we discussed above, what matters for the response to the global risk-aversion shock is whether a country is a net borrower or lender of safe assets. For a given sign of the net foreign asset position of safe assets, the net foreign asset position of risky assets can be either positive or negative, depending on the type of heterogeneity. Countries with a negative net foreign asset position of safe assets are either less risk-averse or have a risky asset whose expected dividend is relatively high. Low risk-aversion implies a positive net foreign asset position of risky assets, while high expected domestic dividends implies negative net foreign asset position of risky assets. Thus, knowing the sign of the net foreign asset position of risky assets is therefore not informative about the sign of the net foreign asset position of safe assets, which is what drives the results in Theorem 4.

### 7 Numerical Analysis

In the analytical results so far we have considered specific cases of either within-country or cross-country heterogeneity. We now turn to the numerical implications of the model, where we include both within-country and cross-country heterogeneity. We will compare the quantitative results from the model to the empirical results in Section 2.

In the analytical model presented thus far we make a few simplifying assumptions to ensure that the model has a closed form solution. For the purpose of the numerical exercise, we first relax two of these assumptions. After that we discuss the calibration of the model’s parameters. We finally consider the impact of the global risk-aversion shock and make comparisons to the empirical results in Section 2.

#### 7.1 Relaxing two model assumptions

We relax two assumptions in the model discussed so far. The first assumption is that the period 2 returns on risky assets are uncorrelated across countries. This simplifies the
portfolio expressions. The second assumption is that there is no investment and the capital stock therefore remains constant. Following the shock, there is trade in existing risky assets, but no creation of new risky assets.

Investment plays several roles. First, it affects net capital flows as the current account is saving minus investment. Second, as we have seen in Section 2, in the data both saving and investment drop in response to a decline in the GFC. In the model so far, saving cannot systematically drop across countries as global saving is zero. Third, investment affects gross capital flows. A drop in global investment after a negative GFC shock implies reduced risky asset supplies that in equilibrium imply reduced holdings of both domestic and foreign risky assets. This is an additional channel through which gross capital flows fall in response to a downturn of the GFC, on top of the channel discussed in Section 5 related to within-country heterogeneity.

### 7.1.1 Correlated returns risky assets

The period 2 return on the country \(m\) risky asset is \(D_m/Q_m\). We now allow for correlated dividends \(D_m\) that lead to correlated period 2 returns on the risky assets. This affects the period 1 portfolios of investors. Assume that \(D_m\) has a common component \(D\) and an idiosyncratic component \(F_m\):

\[
D_m = D + F_m
\]  

(53)

Assume that \(D\) and \(F_m\) are uncorrelated and that \(F_m\) is uncorrelated across countries. Assume that for investors in country \(n\) the variance of \(F_n\) is \(\sigma^2\), while the variance of \(F_m\) for investor \(i\) in country \(n\) with \(m \neq n\) is \(\sigma^2/\kappa_i\). Also let \(\sigma^2_d\) be the variance of \(D\).

Define

\[
\eta_i = \frac{\nu}{\nu - 1 + \nu(1 + N\kappa_i)}
\]

where \(\nu = \frac{\sigma^2_d}{\sigma^2 + \sigma^2} \) is the cross-country correlation of dividends. The Online Appendix then shows that for investor \(i\) in country \(n\), the optimal portfolio expressions are:

\[
z_{i,n,n} = \frac{Q_n\Gamma_i (1 + \epsilon^G_n) \sigma^2}{(1 - \eta_i)(\bar{D}_n - RQ_n) - \eta_i\kappa_i \sum_{m \neq n} (\bar{D}_m - RQ_m)} 
\]

(54)

and for \(m \neq n\)

\[
z_{i,n,m} = \frac{Q_m\Gamma_i (1 + \epsilon^G_n) \sigma^2}{(\bar{D}_m - RQ_m) - \eta_i\kappa_i (\bar{D}_n - RQ_n) + (\kappa_i - \eta_i\kappa_i^2)(\bar{D}_m - RQ_m) - \eta_i\kappa_i^2 \sum_{k \neq n,m} (\bar{D}_k - RQ_k)} 
\]

(55)

Notice that the previous case of uncorrelated returns corresponds to \(\eta_i = 0\) as in that case
\( \nu = \sigma_d^2 = 0 \). This yields the portfolio expressions that we presented earlier in equations (17) and (18).

### 7.1.2 Investment

We introduce installment firms that can produce \( I_n \) new capital goods in period 1 in country \( n \) at the price of \( Q_n \). These raise the capital stock at time 2. Production of capital goods requires a quadratic adjustment costs. Producing \( I_n \) units of the capital good requires

\[
aI_n + \frac{\xi (aI_n)^2}{2 K_n}
\]

units of the consumption good. Here \( K_n \) is the period 1 capital stock. The installment firms maximize the profit

\[
Q_n I_n - aI_n - \frac{\xi (aI_n)^2}{2 K_n}
\]

This implies

\[
\frac{I_n}{K_n} = \frac{1}{a^2 \xi} (Q_n - a)
\]

The period 2 capital stock is then

\[
K_{n,2} = K_n + I_n
\]

The period 1 asset market clearing conditions (22)-(23) remain the same, with the capital stock \( K_n \) replaced with \( K_{n,2} \). The \( S = 0 \) schedule discussed in Section 4 in the absence of heterogeneity now becomes an \( S = I \) schedule, where \( I \) is the value of investment, \( Q_n I_n \), which is \((Q^2 - aQ) / (a^2 \xi)\) in the absence of heterogeneity.

### 7.2 Heterogeneity Parameters

Heterogeneity within and across countries is key to the model. Before discussing the calibration of the other parameters, we therefore first consider the within-country heterogeneity parameters \( \Gamma_i \) and \( \kappa_i \) and the cross-country heterogeneity parameters \( \epsilon_n^G \) and \( \epsilon_n^D \).

To calibrate the within-country heterogeneity parameters, we rely on the Calvet et al. (2007, 2009a, and 2009b) papers mentioned earlier. These use Swedish administrative data from 1999 to 2002 on wealth, portfolio shares, and portfolio returns of individual households, and therefore provide a unique look at within-country heterogeneity in the desire to hold risky assets and diversified portfolios. The Online Appendix discusses in detail the calibration of \( \Gamma_i \) and \( \kappa_i \) from these Swedish data. Here we provide a brief summary. We first obtain information about the risky share \( z_i \) and the foreign share \( z_i^F \) of individual investors. After
that it is easy to back out the distributions of \( \Gamma_i \) and \( \kappa_i \).

Calvet et al. (2009a) provides information about the distribution of risky shares \( z_i \), defined as the portfolio fraction invested directly in stock or in risky mutual funds. The distribution is centered around 0.5. It is left-skewed during a boom in risky asset prices and right-skewed during a bust. But on average the risky shares are close to uniformly distributed between 0 and 1. We also use this to set \( \bar{z} = 0.5 \).

The foreign share \( z_i^F \) is not directly available from the Swedish administrative data. But it can be computed indirectly from information about the distribution of Sharpe ratios of the risky asset portfolios in Calvet et al. (2007). They report the distribution of the Sharpe ratio loss, which tells us for an individual investor how much lower the Sharpe ratio is in comparison to a diversified international benchmark portfolio. In the Online Appendix we derive the Sharpe ratio loss for individual investors in the model. There is a mapping in the model between the Sharpe ratio loss and the foreign share \( z_i^F \). The larger the Sharpe ratio loss, the less diversified the investor is, so that \( z_i^F \) is lower. Therefore the distribution of Sharpe ratio losses in Calvet et al. (2007) can be mapped into a distribution of \( z_i^F \) from the theoretical Sharpe ratios in the model.

While these results provide distributions of \( z_i \) and \( z_i^F \), we also need to know how they are correlated. Calvet et al. (2009b) show that the cross-sectional correlation between risky shares and Sharpe ratio losses is -0.49. In other words, investors that hold a higher risky share have a smaller Sharpe ratio loss relative to the international benchmark and are therefore more diversified. From the mapping between the Sharpe ratio loss and \( z_i^F \) discussed above, we use this to calibrate the correlation between \( z_i \) and \( z_i^F \) across investors. We use this to generate 100,000 pairs \((z_i, z_i^F)\).

We can map these to \( \kappa_i \) and \( \Gamma_i \) as follows. The risky share is \( z_i^F = N\kappa_i/(1 + N\kappa_i) \). This allows us to compute the \( \kappa_i \) from the \( z_i^F \). We have \( z_i = \bar{z} \frac{\bar{\psi}_i}{\bar{\psi}} \) where \( \bar{\psi}_i = \Gamma_i(1 + N\kappa_i)(1 - \eta_i(1 + N\kappa_i)) \), \( \bar{\psi} \) is mean of \( \psi_i \), and \( \eta_i = \frac{\nu}{1 - \nu + \nu(1 + N\kappa_i)} \). The calibration of the correlation \( \nu \) between risky asset returns is discussed below. Together with \( z_i \) and the calibrated \( \kappa_i \), we can use this to back out \( \Gamma_i / \bar{\Gamma} \). As discussed below, the cross-sectional mean of \( \Gamma_i \), \( \bar{\Gamma} \), plays no role for the results and we set it rather arbitrarily at 0.1.

Finally consider the cross-country heterogeneity parameters \( \epsilon_n^G \) and \( \epsilon_n^D \). They jointly determine \( nfa_{n}^{risky} \) and \( nfa_{n}^{safe} \) prior to the shock. Using a numerical solver, we set \( \epsilon_n^G \) and \( \epsilon_n^D \) to match the 1996-2020 sample averages of \( nfa_{n}^{risky} \) and \( nfa_{n}^{safe} \) in the data for the 20 countries in the empirical analysis of Section 2. These net foreign asset positions, as a share of GDP, are reported in Figure 3.\footnote{In the data the average across countries of the net foreign asset positions (as a share of GDP) is not exactly zero. To be consistent with the model, we recenter the net foreign assets positions in the data (as a share of GDP) by subtracting the cross-sectional mean.}
7.3 Other Parameters

The other model parameters and their calibrated values are shown in Table 3. The number of countries \( N + 1 \) is set at 20, corresponding to the empirical exercise in Section 2. We set \( a = \frac{\beta}{1-\beta} = 25 \), implying a 4 percent pre-shock interest rate. We set \( Y = 3 \), so that GDP is \( Y + 1 = 4 \). If we interpret \( Y \) as labor income, it is then equal to 75 percent of GDP.

We set the cross-country risky asset return correlation \( \nu \) based on data reported in Calvet et al. (2007). They report a Sharpe ratio for the international benchmark portfolio and the benchmark Swedish portfolio of respectively 45.2 and 27.4. These correspond to respectively \( \kappa_i = 1 \) and \( \kappa_i = 0 \) in the model. In the Online Appendix we use this to back out \( \nu = 0.33 \) based on the derivation of the Sharpe ratio loss. This value of the correlation is also plausible through direct observation of equity return correlations. For example, Quinn and Voth (2008) compute a century of global equity market correlations. Using correlations among 120 country pairs of monthly equity returns for non-overlapping 4-year intervals, they find an average correlation of 0.33, the same as what we calibrated based on Sharpe ratios in the Swedish data.

We cannot calibrate both \( \sigma^2 \) and \( \bar{\Gamma} \), only their ratio. The portfolio expressions depend on \( \Gamma_i / \sigma^2 \). As discussed, data from the Calvet et al. papers allow us to calibrate \( \Gamma_i / \bar{\Gamma} \). Given these values, the portfolio expressions depend on \( \sigma^2 / \bar{\Gamma} \). The pre-shock premium on risky assets is

\[
\frac{\bar{z} \sigma^2}{a^2 \psi} = \frac{\bar{z}}{a^2} \frac{\sigma^2}{\bar{\Gamma}} \frac{1}{E \left( \frac{\Gamma_i}{\bar{\Gamma}} (1 + N \kappa_i) \right)}
\]

The expectation in the last term refers to the mean across investors \( i \) and can be computed from the calibrated \( \Gamma_i / \bar{\Gamma} \) and \( \kappa_i \). We then set \( \sigma^2 / \bar{\Gamma} \) such that the risk premium is 4.6 percent. In Table 3 we set \( \bar{\Gamma} = 0.1 \) and \( \sigma = 2.4 \), but any other values with the same ratio of \( \sigma^2 / \bar{\Gamma} \) lead to the same results.

We set \( \rho \) as follows. In Section 2 we saw that a change in the GFC factor leads to a change in the log of risky asset prices relative to the interest rate of 16.2/0.55. We set \( \rho \) such that in the model the change in the average log risky asset price relative to the change in the interest rate is also 16.2/0.55 (see Online Appendix for further details). We do not take \( \rho \) as literally an intertemporal elasticity of substitution as we think of “households” in the model as representing all non-investors, including governments, non-profits, etc.

Finally, we calibrate the investment adjustment cost parameter \( \xi \) as follows. From Tables 1 and 2 we see that on average the investment/GDP ratio falls by about 0.5 percent and the savings/GDP ratio falls by about 0.7 percent for every 16.2 percent fall in the risky asset price. Since of course world savings has to equal world investment, we take the average of these two and calibrate the model to generate a 0.6 percent fall in the investment/GDP ratio for every 16.2 percent fall in the risky asset price. See again the Online Appendix for further
The parameters $\kappa_i$ affect the financial openness of countries. Since the Swedish data is from the period 1999-2002, which is towards the beginning of the sample 1996-2020 on which the results in Section 2 are based, it is useful to check the financial openness of countries implied by the calibrated parameters against two untargeted measures of financial openness. One is the sum of external assets plus liabilities of risky assets relative to GDP. The other is the mean foreign share. Both are averaged across countries and years of the sample. These are respectively 2.3 and 0.27 in the data. In the model they are respectively 2.9 and 0.24. These are both broadly in line with the data.

### 7.4 Results

In reporting the impact of a rise in global risk-aversion (drop in $G$), we report changes in endogenous variables per 1 percent drop in the average risky asset price. We compare these results to those in the data in Section 2. Table 1 shows that the change in the log risky asset price is 16.2 times $\Delta F_t$. We therefore set $\Delta F_t = -1/16.2$ in order to generate a one percent drop in risky asset prices. The results are reported in Table 4. The interest rate is in percent, while all other variables are as a percent of GDP. If one wishes to consider a really large shock that leads to a 50 percent drop in risky asset prices, all the numbers simply need to be multiplied by 50.

Table 4 reports the response of gross and net capital flows in the data to a 1% fall in the average risky asset price in the data, as well as in two versions of the model. The second column shows results from the benchmark version of the model described so far, which are discussed in this subsection. The third column shows results from an extension of the model with imperfect substitution between safe assets from different countries, which is discussed in the next subsection.

The top half of Table 4 reports the cross-country average change in a variable in response to a shock that leads to a 1% fall in global risky asset prices. The first three, indicated in italics, are targeted in the calibration of the model. These are the change in the risk-free interest rate and the average across countries of the change in savings/GDP and investment/GDP. The drop in the interest rate is by construction identical to that in the data. The same is also the case for saving an investment. In the model they change equally as

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19 For the first measure we use the data on external assets and liabilities of risky assets discussed in Section 2. For the second measure we calculate the foreign share as the ratio $A/B$, where $A$ is a country’s external portfolio equity assets and $B$ is its domestic equity market capitalization minus external portfolio equity liabilities plus external portfolio equity assets. The external portfolio equity assets and liabilities are from the same source as in Section 2, while the domestic market capitalization is from the World Bank’s World Development Indicators.

20 This linearity holds up approximately in the model as well.
global saving and investment are equal. In the data there is a small difference between the change in saving and investment due to measurement issues.

7.4.1 Gross Capital Flows

The average change across countries in gross capital flows as a percent of GDP (inflows plus outflows) is a non-targeted moment. The model closely matches the drop in gross capital flows in the data (-0.307 in model versus -0.342 in data). This decline in gross capital flows is associated with two modeling features, the within-country heterogeneity and the decline in global investment. It does not depend on cross-country heterogeneity. The drop in gross flows is virtually identical when we set all $\epsilon_n^G$ and $\epsilon_n^D$ equal to zero.

Figure 5 shows how the drop in gross capital flows depends on the drop in investment and on the correlation between the risky and foreign shares across investors.\textsuperscript{21} In the calibration this correlation is 0.43, which corresponds to the vertical dashed line. The line without investment corresponds to $\xi \to \infty$. It is immediate that the drop in gross capital flows is almost entirely the result of the within-country heterogeneity. The global drop in investment contributes very little. Figure 5 illustrates how the decline in gross capital flows depends critically on a positive correlation across investors of their risky and foreign shares. The decline in gross capital flows is virtually zero when this correlation is zero and depends almost linearly on the correlation between $z_i$ and $z_i^F$.

To further illustrate the importance of a positive relationship across investors of their risky and foreign shares, in Figure 6 we shut down investment ($\xi \to \infty$). It groups investors into 40 bins based on their pre-shock risky shares. The top panel shows purchases of risky assets as a percent of GDP. Without any investor heterogeneity, lower demand for risky assets due to higher risk-aversion and lower wealth is exactly offset by higher demand due to lower risky asset prices. In that case nobody would buy or sell risky assets. But we saw in Section 5 that with portfolio heterogeneity, in equilibrium investors with a high risky share, whose wealth drops the most, will sell risky assets, while those with a smaller risky share will buy risky assets. This is illustrated in the top panel of Figure 6. The integral of the risky asset purchase schedule is 0. Without investment, risky asset supplies are unchanged, so that purchases of risky assets (investors with low risky shares) are equal to sales of risky assets (investors with high risky shares).

This top panel of Figure 6 is very similar to Figure III.D of Calvet et al. (2009a), which shows the closely related change in the risky share due to active buying or selling of risky assets in 2002, a year during which the MSCI Sweden index fell by 48.6%. In those Swedish

\textsuperscript{21}We can change the correlation between the risky share and the foreign share in the model by changing the cross-sectional correlation between risky shares and Sharpe ratio losses. In the calibration we assume that this correlation is -0.49, based on Calvet et al. (2009b).
data the cutoff for the risky share above which investors sell risky assets is about 65 percent. This is very similar to the model in the top panel of Figure 6.

The bottom panel of Figure 6 reports purchases of foreign risky assets. These results are shown both for the calibrated model, where the correlation between the risky and foreign shares is 0.43, and when this correlation is zero. First consider the case of a zero correlation. With a large number of investors in each bin, purchases of foreign risky assets is then proportional to purchases of all risky assets in the top panel of Figure 6. Analogous to the top panel, purchases of foreign risky assets (investors with low risky shares) must then be equal to sales of foreign risky assets (investors with high risky shares). Therefore investors with high risky shares simply sell foreign risky assets to investors with low risky shares in the same country, so that there are no gross capital flows.

We can see that when the correlation between the risky and foreign shares is 0.43, as calibrated from the Swedish data, purchases of foreign risky assets as a function of the risky share (bottom panel) is no longer proportional to purchases of all risky assets as a function of the risky share (top panel). Investors with a high risky share now tend to have a high foreign share as well, which means that they sell a lot of foreign risky assets. Investors with a low risky share now tend to have a low foreign share, so that they mostly buy home risky assets and not a lot of foreign risky assets. The integral of the foreign risky asset purchase schedule is therefore negative. Overall the country is selling foreign risky assets. They sell these assets to foreigners, so that capital outflows decline.

### 7.4.2 Net Capital Flows

The results in the bottom section of Table 4 relate to net capital flows. Consider for example total net capital outflows as a share of GDP, $n_{f_n}$. In Section 2 we regressed $\Delta n_{f_n}$ on $n_{f_n}^{safe} \times \Delta F_t$ and found a coefficient on the interaction term of 0.01. This means that when $\Delta F_t = -1$, a country with $n_{f_n}^{safe} = -100$ (net foreign asset position of safe assets of -100 percent of GDP) experiences an increase in the total net outflows of 1 percent of GDP compared to an average country with $n_{f_n}^{safe} = 0$. In Table 4 we again set $\Delta F_t = -1/16.2$, associated with a 1% drop in risky asset prices. This 1% fall in risky asset prices then raises net capital outflows by 0.06 percent of GDP for a country with $n_{f_n}^{safe} = -100$ compared to the average country. In the model we first scale all $\Delta n_{f_n}$ to correspond to a 1% drop in the average risky asset price and then regress on $n_{f_n}^{safe}$. The same is done for saving and investment and for net outflows of risky and safe assets.

The results for saving, investment and net capital outflows in the model are all very close to those in the data. Recall that the changes in the saving and investment rates for an average country (with a zero net foreign asset position of safe assets) are reported in the top section of Table 4. In the average country both saving and investment will drop. The
bottom half of Table 4 tells us how much more or less they drop in a country with a net external debt of safe assets of 100% of GDP. We see that saving is higher and investment is lower in a country that has a net external debt of safe assets. For saving this is because of the larger drop in wealth of such a country, which leads to a larger drop in consumption and therefore higher saving. For investment it is because countries with a larger drop in wealth have a somewhat larger drop in their risky asset price (see Section 6) and therefore a larger drop in investment. As in the data, the net external debt of safe assets has a substantially larger impact on the saving response to the GFC shock than the investment response. Since total net capital outflows equals the current account, which is saving minus investment, a country with a net external debt of safe assets experiences an increase in net capital outflows, due both to the higher saving and lower investment, compared to a country with a zero net foreign asset position of safe assets.

Next consider the results for net capital outflows of risky and safe assets. As in the data, a country that has a net external debt of safe assets experiences an increase in net capital outflows of safe assets and a decrease in net outflows of risky assets in response to a negative GFC shock. However, in the data this response is substantially smaller. This is associated with a larger reallocation between safe and risky assets in the model. The sum of $nf^n_{sa}fe$ and $nf^n_{risky}$ is equal to total net capital outflows $nf^n$, where the model is close to the data. In the next subsection we present an extension of the model where each country has its own safe asset. When these safe assets are imperfect substitutes we can resolve this quantitative discrepancy between the model and the data.

Figure 7 presents scatter plots of changes in total net outflows, saving, investment, net risky outflows and net safe outflows in the model in response to a GFC shock that leads to a 1% drop in the average risky asset price. These changes are plotted against both $nf^n_{sa}fe$ (left column) and $nf^n_{risky}$ (right column). The slopes of the regression lines in the left column correspond to the model numbers reported in the second column of the bottom section of Table 4. There is clearly a strong link in the model between the changes in these variables in response to a negative GFC shock and the net foreign asset position of safe assets. By contrast, when plotting the changes against the net foreign asset position of risky assets in the right column, there is no clear relationship. This corresponds to Fact 3.

### 7.5 Imperfect Safe Asset Substitution

We now address the quantitative discrepancy between the model and the data regarding the response of net safe and risky capital outflows to the GFC shock. There is too much reallocation between safe and risky assets for a given net foreign asset position of safe assets. We need a speedbump to slow this down, without affecting any of the other results. A natural way to do so is to assume that each country has its own safe asset and these safe
assets are imperfect substitutes. We have seen that in response to a negative GFC shock, a country $n$ with a negative net foreign asset position of safe assets both saves more and swaps risky for safe assets. If there are $N+1$ safe assets and country $n$ investors are biased towards their domestic safe asset, there would be a relative increase in demand for the country $n$ safe asset. This leads to drop in the interest rate of the country $n$ safe asset, which weakens the reallocation in country $n$ from risky to safe assets.

It is beyond the scope of this paper to develop a full portfolio choice model with both $N+1$ risky and $N+1$ safe assets. Such a model would involve $N+1$ currencies as well. We consider here a much simpler framework that is similar to Gabaix and Maggiori (2015). It is assumed that investors only hold the domestic safe assets.\footnote{We continue to assume that households are the same in each country. They hold an equally weighted portfolio of all safe assets.} There are identical arbitrageurs in each country, which are like the financial intermediaries in Gabaix and Maggiori (2015). A full description of this extension is in the Online Appendix. Arbitrageurs maximize

$$\sum_{n=1}^{N+1} R_n B_n - \frac{1}{2} a_0 \sum_{n=1}^{N+1} (B_n - \bar{B}_n)^2$$

where $B_n$ are country $n$ safe asset holdings and $R_n$ is the interest rate on country $n$ safe assets. They therefore maximize the return on their safe asset portfolio minus quadratic costs of deviating from their pre-shock safe asset holdings $\bar{B}_n$. They enter with zero wealth, so that $\sum_{m=1}^{N+1} B_m = 0$. The optimal safe asset portfolio of arbitrageurs is then

$$B_n - \bar{B}_n = \frac{1}{a_0} \left( R_n - \frac{1}{N+1} \sum_{m=1}^{N+1} R_m \right)$$

(61)

The net foreign asset position of safe assets of country $n$ is $NFA_{n,safe} = -(N+1)B_n$. Dividing by GDP, using (61), we can derive

$$R_n - \frac{1}{N+1} \sum_{m=1}^{N+1} R_m = -\chi \left( nfa_{n,safe} - nfa_{n,0,safe} \right)$$

(62)
The interest rate then needs to fall.

Consider a country $n$ that is a net borrower of safe assets before the shock. We have seen that investors from this country reallocate from risky to safe assets as a result of the shock, leading to reduced net borrowing of safe assets. This lowers the interest rate, which reduces the equilibrium reallocation from risky to safe assets by country $n$ investors. Similarly, consider a country $n$ that is a net lender of safe assets. We have seen that this country reallocates from safe to risky assets as a result of the shock, leading to reduced net lending of safe assets. This raises the interest rate on the country $n$ safe asset, which reduces the equilibrium reallocation from safe to risky assets by investors from country $n$. The larger the parameter $\chi$, the bigger the speedbump that limits the equilibrium reallocation between safe and risky assets.

The impact of this imperfect substitution of safe assets depends critically on the value of $\chi$. When $\chi = 0$ the model reverts to the model with a single safe asset. Adrian, Erceg, Kolasa, Linde, and Zabczyk (2022) calibrate a model that incorporates this Gabaix and Maggiori (2015) friction. They calibrate the model with a value of $\chi = 0.02$ for advanced economies. That is the value we use in this extension.

The right hand column of Table 4 presents the results from introducing imperfect safe asset substitution into the model. First note that outside of net capital flows of safe and risky assets, imperfect substitution does not change the model results. The last two rows of Table 4 show that imperfect substitution reduces the magnitude of net flows of safe and risky assets in response to the GFC shock. They are now closely in line with the data.

8 Conclusion

We have developed a theory to account for changes in gross and net capital flows over the global financial cycle (GFC). The theory relies critically on portfolio heterogeneity among investors within and across countries. This portfolio heterogeneity affects relative wealth within and across countries in response to global asset prices changes during the global financial cycle. These relative wealth changes in turn affect gross and net capital flows in a way that is quantitatively consistent with the data for 20 advanced countries.

The model can be extended in several directions. One limitation is that we have only considered gross capital flows of risky assets. In the data there is a significant drop in gross flows of safe assets as well during a downturn of the GFC. The extension with imperfect substitution of safe assets discussed in Section 7.5 is not well equipped to address this. A less straightforward extension, where investors themselves hold safe assets of all countries, with returns on foreign safe assets dependent on exchange rate fluctuations, can address gross capital flows of safe assets. At least qualitatively the same mechanism explaining the
drop in gross flows of risky assets is likely to apply to safe assets as well. Foreign “safe” assets are risky from the perspective of domestic investors as a result of exchange rate uncertainty. When investors with a higher risky share tend to have a higher foreign share as well (of both risky and safe assets), they will sell not just foreign risky assets, but also foreign safe assets, reducing capital outflows of safe assets.

The model can be extended in numerous other directions to consider features of the GFC from which we have abstracted here. One direction is to consider the role of monetary policy and associated exchange rate fluctuations. Another is to allow for financial frictions, which would allow us to consider the need for macroprudential policies. Related to that, a third direction is to more explicitly model financial institutions and the constraints under which they operate. Finally, we have abstracted from the special role that the United States and the dollar play in the international financial system.
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Appendix

A Proof of Theorem 1

With period 1 dividends of 1, \( R_0 = (1 + a)/a \) and \( Q_{n,0} = Q_{n,1} = a \), (8) implies that \( R^{i,n}_t = (1 + a)/a \). We have \( W_{n,0}^i - C_{n,0}^i = \beta W_{n,0}^i = a/\bar{z} \), so that from (7) \( W_{n,1}^i = (1 + a)/\bar{z} \) for all investors. Substituting \( \bar{D}_t \) from Assumption 1, as well as \( Q_n = a \) and \( R = (1 + a)/a \), into the portfolio expressions (17)-(18) gives time 1 portfolio shares that are the same as the time zero portfolio shares (24)-(25). Substituting these portfolio expressions, as well as \( W_n^i = (1 + a)/\bar{z} \) and \( Q_n = a \), into the risky asset market clearing conditions (22), the markets clear in period 1 under Assumption 1 about \( K_n \). The aggregate asset market clearing condition (23) also holds in period 1, after substituting \( B_1^h = B_0^h \), \( W_n^i = (1 + a)/\bar{z} \), \( Q_n = a \) and the expression for \( B_0^h \) in Assumption 1.

Since \( R_t = 1/\beta \) for all \( t \geq 1 \), first-order condition (21) implies that household consumption is constant over time. Since income is constant, this implies \( C_t^h = Y + B_0^h/a \). The household budget constraint (20) then implies \( B_t^h = B_0^h \) for all \( t \geq 1 \). Since there is no uncertainty starting in period 2, we must have \( R_t = (Q_{n,t+1} + D_{n,2})/Q_{n,t} \) for \( t \geq 2 \). This is satisfied when \( R_t = (1 + a)/a \), \( Q_{n,t} = Q_{n,t+1} = (a/(1 + a))D_n = aD_{n,2} \). Investor wealth remains constant after period 2 since \( W_{n,t+1}^i = \beta R_t W_{n,t}^i \) for \( t \geq 2 \) and \( R_t = 1/\beta \).

We finally need to check the aggregate asset market clearing condition (23) for \( t \geq 2 \). Since household safe asset holdings, investor wealth and asset prices remain constant from period 2 onward, we only need to check it for \( t = 2 \). We have

\[
\sum_{n=1}^{N+1} \int_0^1 W_{n,2}^i di = \beta \frac{1 + a}{\bar{z}} \sum_{n=1}^{N+1} \int_0^1 R^{p,n}_t di = \frac{a}{\bar{z}} R(N + 1) + \frac{a}{\bar{z}} \sum_{n=1}^{N+1} \sum_{m=1}^{N+1} \int_0^1 z_{n,m}^i \frac{D_m - RQ_m}{Q_m} \int_0^1 W_{n,2}^i di = \frac{1 + a}{\bar{z}} (N + 1) + \sum_{m=1}^{N+1} K_m (D_m - (1 + a))
\]

Using \( B_2^h = B_0^h \), the period 2 aggregate asset market equilibrium can then be written as

\[
(N + 1)B_0^h + \frac{1}{\bar{z}} a(N + 1) + \frac{a}{1 + a} \sum_{n=1}^{N+1} D_n K_n - a \sum_{n=1}^{N+1} K_n = \sum_{n=1}^{N+1} Q_{n,2} K_n
\]

Using \( Q_{n,2} = (a/(1 + a))D_n \) and the expression for \( B_0^h \) in Assumption 1, it is immediate that this is satisfied.

We finally point out that the conjectured value functions are correct. We conjectured \( V_{n,1}^i = \alpha_1 W_{n,1}^i \) and \( V_{n,t}^i = \alpha_2 W_{n,t}^i \) for \( t \geq 2 \). First substituting the latter into the Bellman equation (9) for \( t \geq 2 \), together with \( C_{n,t}^i = (1 - \beta)W_{n,t}^i \) and \( W_{n,t+1}^i = W_{n,t}^i \), we have \( \alpha_2 = 1 - \beta \). Substituting \( V_{n,1}^i = \alpha_1 W_{n,1}^i \) into the Bellman equation (9) at time 1, together
with \( C_i^n = (1-\beta)W_i^n \) and \( W_{n,2}^i = \beta R_{p,i,n} W_i^n \), we have

\[
\ln(\alpha_{1,i}) = \ln(1-\beta) + \frac{\beta}{1-\beta} \ln(\beta) + \frac{\beta}{1-\beta} \ln\left(E(R_{p,i,n}^{1-\gamma_{i,n}})\right)
\]

Substituting the portfolio shares (24)-(25), \( Q_m = a \) and \( R = a/(1+a) \) into the portfolio return expression (16), \( \alpha_{1,i} \) becomes a function of structural model parameters.

**B Proof of Theorem 3**

The market equilibrium conditions (22)-(23) can be written as

\[
(\bar{D} - RQ) \left( \frac{1+a}{\bar{z}} + \frac{E\psi^2}{\bar{\psi}^2} (Q-a) \right) = \frac{1}{a} \frac{\sigma^2}{\bar{\psi}} \frac{dG}{dG}
\]

(B.1)

\[
(R_0 - 1) B_0^h + Y - C_1^h - \frac{Q-a}{1+a} = 0
\]

(B.2)

These remain very similar to (30)-(31). The only difference is that \( Q-a \) in the risky asset market equilibrium condition is now multiplied by \( E\psi^2/\bar{\psi}^2 \). Differentiating these equations at the pre-shock levels \( Q = a, R = (1+a)/a \) and \( G = 1 \), we have

\[
dQ = \bar{z}(1+a)(\frac{\sigma^2}{\bar{\psi}}) \frac{dG}{dG}
\]

(B.3)

\[
dR = \frac{\bar{z}(\sigma^2/\bar{\psi})}{\lambda \left((1+a)^2 - \bar{z}^2(\sigma^2 E\psi^2/\bar{\psi}^3)\right) + a^2} dG
\]

(B.4)

Assumption 2 implies that the denominator of both expressions is positive, so that a drop in \( G \) (rise in global risk-aversion) leads to a drop in both \( Q \) and \( R \).

Next consider capital flows. Use that \( z' = \alpha' \bar{z} \) with \( \alpha' = \psi'/\bar{\psi} \). (26)-(27) can then be written as

\[
OF_{n}^{\text{risky}} = IF_{n}^{\text{risky}} = \frac{a}{1+a} Q\bar{\psi}G \frac{\bar{D} - RQ}{\sigma^2} \int_0^1 z_F^i \alpha^i \left( \frac{1+a}{\bar{z}} + \alpha'(Q-a) \right) di - QE(z_F^i \alpha)
\]

(B.5)

with \( z \) and \( z_F \) defined in (36) and (40). First note that net flows of risky assets clearly remain zero. Since saving is also zero (see (B.2)), total net capital flows are zero and therefore net flows of safe assets are zero as well. Substituting (B.1), we have

\[
OF_{n}^{\text{risky}} = IF_{n}^{\text{risky}} = Q \frac{1+a}{\bar{z}} E(z_F^i \alpha) + (Q-a) E(z_F^i \alpha^2) \frac{1}{\bar{z}} + (1 + \text{var}(\alpha))(Q-a) - QE(z_F^i \alpha)
\]

(B.6)

This uses that \( 1 + \text{var}(\alpha) = E(\alpha^2) = E(\psi/\bar{\psi})^2 = (E\psi^2)/\bar{\psi}^2 \). Differentiating with respect to \( Q \) at \( Q = a \) gives

\[
dOF_{n}^{\text{risky}} = dIF_{n}^{\text{risky}} = \frac{a}{1+a} \bar{z} (Ez_F^i \alpha (\alpha - 1 - \text{var}(\alpha))) dQ
\]

(B.7)
It follows that both outflows and inflows of risky assets go down equally in response to an increase in global risk-aversion as long as $Ez_F\alpha(\alpha - 1 - \text{var}(\alpha)) > 0$. Using that the expectation of $\alpha(\alpha - 1 - \text{var}(\alpha))$ is equal to zero, this is the case when $\text{cov}(z_F, \alpha(\alpha - 1 - \text{var}(\alpha))) > 0$ or $\text{cov}(z_F, Z) > 0$, which is Assumption 3. This establishes that gross capital flows drop in response to the shock.

We finally show that $\text{cov}(z_F, Z) > 0$ is satisfied under a specific set of assumptions about the distributions of $\kappa$ and $\Gamma$. Assume that $\kappa_i = \bar{\kappa} + \epsilon_i^\kappa$ and $\Gamma_i = \bar{\Gamma} + \omega\epsilon_i^\Gamma + \epsilon_i^\Gamma$, where $\epsilon_i^\kappa$ and $\epsilon_i^\Gamma$ are uncorrelated, have symmetric distributions, and have mean zero. They are also such that $\Gamma_i$ and $\kappa_i$ are always positive. We then show that $\text{cov}(z_F, Z) > 0$ as long as $\omega \geq 0$ and $\text{var}(\epsilon^\kappa) > 0$. This means that we must have cross-sectional variation in home bias. We do not necessarily need cross-sectional variation in risk-aversion.

Define $\eta = N\Gamma\kappa$. Using that $\alpha = \psi/E(\psi)$ and $z_F\alpha = \eta/E(\psi)$, we can write the condition $\text{cov}(z_F, Z) > 0$ as

$$\text{cov}(\eta, \psi)E(\psi) - \text{var}(\psi)E\eta > 0 \quad (B.8)$$

Using that $\kappa = \bar{\kappa} + \epsilon^\kappa$ and $\Gamma = \bar{\Gamma} + \omega\epsilon^\Gamma + \epsilon^\Gamma$, we have

$$\psi = \bar{\Gamma}(1 + N\bar{\kappa}) + (\omega + N\omega\bar{\kappa} + N\bar{\Gamma})\epsilon^\kappa + (1 + N\bar{\kappa})\epsilon^\Gamma + N\epsilon^\kappa\epsilon^\Gamma + \omega N(\epsilon^\kappa)^2 \quad (B.9)$$

$$\eta = N\bar{\Gamma}\bar{\kappa} + N(\omega\bar{\kappa} + \bar{\Gamma})\epsilon^\kappa + N\bar{\kappa}\epsilon^\Gamma + N\epsilon^\kappa\epsilon^\Gamma + \omega N(\epsilon^\kappa)^2 \quad (B.10)$$

Using the assumed properties of $\epsilon^\kappa$ and $\epsilon^\Gamma$ (independent, symmetric distributions), it follows that

$$\text{var}(\psi) = (\omega + N\omega\bar{\kappa} + N\bar{\Gamma})^2\text{var}(\epsilon^\kappa) + (1 + N\bar{\kappa})^2\text{var}(\epsilon^\Gamma) + N^2\text{var}(\epsilon^\kappa)\text{var}(\epsilon^\Gamma) + \omega^2 N^2 E(\epsilon^\kappa)^4 - \omega^2 N^2 [\text{var}(\epsilon^\kappa)]^2$$

$$\text{cov}(\eta, \psi) = (\omega + N\omega\bar{\kappa} + N\bar{\Gamma})N(\omega\bar{\kappa} + \bar{\Gamma})\text{var}(\epsilon^\kappa) + (1 + N\bar{\kappa})N\bar{\kappa}\text{var}(\epsilon^\Gamma) + N^2\text{var}(\epsilon^\kappa)\text{var}(\epsilon^\Gamma) + \omega^2 N^2 E(\epsilon^\kappa)^4 - \omega^2 N^2 [\text{var}(\epsilon^\kappa)]^2$$

$$E(\psi) = \bar{\Gamma}(1 + N\bar{\kappa}) + \omega N\text{var}(\epsilon^\kappa)$$

$$E(\eta) = N\bar{\Gamma}\bar{\kappa} + \omega N\text{var}(\epsilon^\kappa)$$

Then collecting terms, we have

$$\text{cov}(\eta, \psi)E(\psi) - \text{var}(\psi)E\eta = (\omega + N\omega\bar{\kappa} + N\bar{\Gamma})N\bar{\Gamma}^2\text{var}(\epsilon^\kappa) + \bar{\Gamma}N^2\text{var}(\epsilon^\Gamma)\text{var}(\epsilon^\kappa) + \bar{\Gamma}\omega^2 N^2 E(\epsilon^\kappa)^4 - \omega^2 N(\omega + N\omega\bar{\kappa} + 2N\bar{\Gamma})[\text{var}(\epsilon^\kappa)]^2 - \omega N(1 + N\bar{\kappa})\text{var}(\epsilon^\Gamma)\text{var}(\epsilon^\kappa)$$

Rewrite this as

$$\text{cov}(\eta, \psi)E(\psi) - \text{var}(\psi)E\eta = \bar{\Gamma}N^2\text{var}(\epsilon^\Gamma)\text{var}(\epsilon^\kappa) + (\bar{\Gamma}^2 - \text{var}(\epsilon^\Gamma) - \omega^2 \text{var}(\epsilon^\kappa)) (1 + N\bar{\kappa})N\omega\text{var}(\epsilon^\kappa)$$

$$\left(\omega^2 E(\epsilon^\kappa)^4 - 2\omega^2 [\text{var}(\epsilon^\kappa)]^2 + \bar{\Gamma}^2 \text{var}(\epsilon^\kappa)\right) N^2\bar{\Gamma} \quad (B.11)$$

The first term of (B.11) is clearly greater than or equal to zero. Next consider the second term in the first line of (B.11). Since $\Gamma$ is assumed to be positive, we have $\omega\epsilon^\kappa + \epsilon^\Gamma > -\bar{\Gamma}$. Since $\epsilon^\kappa$ and $\epsilon^\Gamma$ are symmetrically distributed and have a mean of zero, it follows that
\[(\omega \epsilon + \epsilon \Gamma)^2 < \bar{\Gamma}^2\]. Taking the expectation, we have
\[
\omega^2 \text{var}(\epsilon^*) + \text{var}(\epsilon \Gamma) < \bar{\Gamma}^2
\] (B.12)
This implies that the second term of the first line of (B.11) is positive since \(\text{var}(\epsilon^*) > 0\).
Finally consider the last term of (B.11). We have
\[
E (\epsilon^*)^4 = \text{var}((\epsilon^*)^2) + [\text{var}(\epsilon^*)]^2 \geq [\text{var}(\epsilon^*)]^2
\]
Therefore the term in brackets in the last term of (B.11) is
\[
\omega^2 E (\epsilon^*)^4 - 2\omega^2 [\text{var}(\epsilon^*)]^2 + \bar{\Gamma}^2 \text{var}(\epsilon^*) \geq (\bar{\Gamma}^2 - \omega^2 \text{var}(\epsilon^*)) \text{var}(\epsilon^*)
\]
From (B.12) this is positive, which completes the proof that \(\text{cov}(\eta, \psi) E(\psi) - \text{var}(\psi) E\eta > 0\) and therefore \(\text{cov}(z_F, Z) > 0\).

C Cross Country Heterogeneity in Risk Aversion

Appendix E will proof Theorem 4. To do so, we first need to to derive the second order derivatives of risky asset prices and net capital flows (safe, risky, total) with respect to \(G\) and \(\epsilon\). In this section we do so for cross-country risk aversion heterogeneity. In Section D we do so for heterogeneity in expected dividends. We start by describing the market clearing conditions. After that we derive the second-order derivatives for risky asset prices, total net capital outflows and net outflows of risky assets as linear functions of \(g_n\). The last two also give us the second-order derivative for net flows of safe assets. We remove superscripts for individual agents as there is no within-country heterogeneity.

C.1 Market Clearing Conditions

The market clearing conditions are
\[
\frac{a}{1 + a} \sum_{m=1}^{N+1} \bar{z}_{m,n} \text{W}_m = \text{Q}_n \text{K}_n \quad n = 1, ..., N + 1 \quad (C.1)
\]
\[(N + 1) \text{B}_1^h + \frac{a}{1 + a} \sum_{n=1}^{N+1} \text{W}_n = \sum_n \text{Q}_n \text{K}_n \quad (C.2)
\]

First consider wealth. Using the expressions for portfolio shares (24)-(25) in the pre-shock equilibrium, we have
\[
\text{W}_n = \frac{1 + a}{\bar{\epsilon}} + (1 + \epsilon_n^G) \frac{1 - \kappa}{1 + N \kappa} (\text{Q}_n - a) + (1 + \epsilon_n^G) \frac{\kappa}{1 + N \kappa} \sum_m (\text{Q}_m - a) \quad (C.3)
\]
From Assumption 1 we have

\[
K_n = \frac{1 - \kappa}{1 + N\kappa} (1 + \epsilon_n^G) + \frac{(N + 1)\kappa}{1 + N\kappa} \tag{C.4}
\]

Since \(\sum_n (1 + \epsilon_n^G) = N + 1\), it follows that \(\sum_n K_n = N + 1\), so that from Assumption 1 \(B_0^h = a(1 - (1/\bar{z}))\). Therefore \(B^h = (1 + a)(1 - (1/\bar{z})) + Y - C_1^h\). Together with the expressions for \(K_n\) and \(W_n\), we can then write the aggregate asset market clearing condition (C.2) as

\[
1 - \kappa \sum_{m=1}^{N+1} (Q_m - a)\epsilon_m^G + \sum_{m=1}^{N+1} (Q_m - a) = (N + 1)(1 + a) \left( 1 - \frac{1}{\bar{z}} + Y - C_1^h \right) \tag{C.5}
\]

Taking the derivative with respect to \(G\), this implies:

\[
\frac{\partial R}{\partial G} = \frac{1}{(N + 1)(1 + a)\lambda} \frac{1 - \kappa}{1 + N\kappa} \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial G} \epsilon_m^G + \frac{1}{(N + 1)(1 + a)\lambda} \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial G} \tag{C.6}
\]

Next consider the market clearing conditions for risky assets (C.1). Substituting the portfolio shares (17)-(18) and wealth expressions (C.3) into (C.1), the market clearing conditions for risky assets are

\[
\frac{1 + a}{\bar{z}} \left( 1 + \epsilon_n^G \right) (1 - \kappa) + \kappa(N + 1) \frac{1 + a}{\bar{z}} + \frac{(1 - \kappa)^2}{1 + N\kappa} (1 + \epsilon_n^G)^2 (Q_n - a)
\]

\[
+ \frac{(1 - \kappa)\kappa}{1 + N\kappa} (1 + \epsilon_n^G)^2 \sum_m (Q_m - a) + \frac{(1 - \kappa)\kappa}{1 + N\kappa} \sum_m (1 + \epsilon_m^G)^2 (Q_m - a)
\]

\[
+ \frac{\kappa^2}{1 + N\kappa} \left( \sum_m (1 + \epsilon_m^G)^2 \right) \left( \sum_m (Q_m - a) \right) = \frac{1 + a}{a} K_n \frac{1}{\bar{G}} \frac{\sigma^2}{D - RQ_n} \tag{C.7}
\]

Differentiating (C.7) and substituting (C.6) gives

\[
\frac{(1 - \kappa)^2}{1 + N\kappa} (1 + \epsilon_n^G)^2 \frac{\partial Q_n}{\partial G} + \frac{(1 - \kappa)\kappa}{1 + N\kappa} (1 + \epsilon_n^G)^2 \sum_m \frac{\partial Q_m}{\partial G} + \frac{(1 - \kappa)\kappa}{1 + N\kappa} \sum_m (1 + \epsilon_m^G)^2 \frac{\partial Q_m}{\partial G}
\]

\[
+ \frac{\kappa^2}{1 + N\kappa} \left( \sum_m (1 + \epsilon_m^G)^2 \right) \left( \sum_m \frac{\partial Q_m}{\partial G} \right) = -\frac{1 + a}{a} K_n \frac{1}{\bar{G}} \frac{\sigma^2}{D - RQ_n}
\]

\[
+ \frac{1 + a}{a} RK_n \frac{1}{\bar{G}} \frac{\sigma^2}{(D - RQ_n)^2} \frac{\partial Q_n}{\partial G} + Q_n K_n \frac{1}{\bar{G}} \frac{\sigma^2}{(D - RQ_n)^2} \frac{1}{(N + 1)a\lambda} \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial G}
\]

\[
+ Q_n K_n \frac{1}{\bar{G}} \frac{\sigma^2}{(D - RQ_n)^2} (N + 1)a\lambda \frac{1 - \kappa}{1 + N\kappa} \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial G} \epsilon_m^G \tag{C.8}
\]
C.2 Impact on Relative Prices Risky Assets

We first consider the impact of the global risk-aversion shock on relative prices of risky assets. We set $\epsilon_m^n = g_n \epsilon$, with $\sum_n g_n = 0$. To show that the risky asset price $Q_n$ drops more the lower risk-aversion in country $n$, and therefore the higher $g_n$ when $\epsilon > 0$, we need to show that

$$\frac{\partial^2 Q_n}{\partial G \partial \epsilon}$$

depends positively on $g_n$.

To this end we need to differentiate (C.8) with respect to $\epsilon$ and evaluate at $\epsilon = 0$ and $G = 1$. At that point $\epsilon_m^n = 0$, $Q_m = a$, $R = (1 + a)/a$, $D - RQ = \sigma^2/(a \Gamma(1 + N \kappa))$ and $K_n = 1$. We also have from (C.4) that $\partial K_n/\partial \epsilon = (1 - \kappa)g_n/(1 + N \kappa)$. We use that the pre-shock equilibrium for risky asset prices and the interest rate do not depend on $\epsilon$, so that $\partial Q_m/\partial \epsilon = \partial R/\partial \epsilon = 0$. Since all first order derivatives of risky asset prices with respect to $G$ will be the same, we simply denote them $\partial Q/\partial G$ (see (34)). This gives

$$\frac{(1 - \kappa)^2}{1 + N \kappa} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{(2 - \kappa + N \kappa)\kappa}{1 + N \kappa} \sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon} + 2(1 - \kappa)g_n \frac{\partial Q}{\partial G} = -\frac{1}{\bar{z}}(1 + a)(1 - \kappa)g_n$$

$$+ \frac{1}{\bar{z}^2} (1 + a)^2 g_n (1 + N \kappa) \frac{1}{\sigma^2} \frac{\partial Q}{\partial G} + \frac{1}{\bar{z}^2} (1 + a)^2 (1 + N \kappa)^2 \Gamma \frac{1}{\sigma^2} \frac{\partial^2 Q_n}{\partial G \partial \epsilon}$$

$$+ \frac{1}{\bar{z}^2} a^2 (1 - \kappa) \Gamma(1 + N \kappa) \frac{\partial Q}{\partial G} + \frac{1}{\bar{z}^2} a \Gamma(1 + N \kappa)^2 \sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon}$$

Taking the sum over all $n$, using that $\sum_n g_n = 0$, it follows that $\sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon} = 0$. Therefore

$$\frac{\partial^2 Q_n}{\partial G \partial \epsilon} = \frac{(1 - \kappa)^2}{1 + N \kappa} g_n \frac{1 + a}{\bar{z}} + \frac{(2 - 1/\bar{z})(1 + a)^2 \bar{\psi} 1/\sigma^2 - \bar{\psi} a^2 \bar{\psi} \kappa \sigma^2}{(1 + a)^2 \bar{\psi} 1/\sigma^2 - (1 - \kappa)^2/(1 + N \kappa)^2} \frac{\partial Q}{\partial \epsilon} \quad (C.11)$$

Assumption 2 says $\bar{\psi} (1 + a)^2 > \sigma^2 \bar{z}^2$. It is immediate from this condition that the denominator of (C.11) is positive. To see that the numerator is positive, we can substitute the solution for $\partial Q/\partial G$ from (34). Multiplying through by the denominator of (34), which is positive, the numerator of the large ratio in (C.11) becomes $(1 + a)\bar{z} \sigma^2 / \bar{\psi}$, which is positive. It follows that (C.9) is a positive linear function of $g_n$, which implies that the risky asset price drops more in countries with lower risk-aversion, which are more leveraged.

C.3 Impact on Total Net Flows

We now consider the impact of the shock on total net capital flows (risky plus safe assets), which is equal to the current account, which is equal to saving. Therefore net flows of country $n$ are

$$NF_n = (R_0 - 1) B_0^h + Y - C_1^h + \frac{1}{\bar{z}} - \frac{1}{1 + a} W_n$$

(C.12)

Here $(R_0 - 1) B_0^h + Y - C_1^h$ is saving by households and $(1/\bar{z}) - W_n/(1 + a)$ is saving by investors. They earn dividend and interest income equal to $1/\bar{z}$ and consume $W_n/(1 + a)$. Using (C.3), $\sum_m CA_m = 0$, and that household consumption is the same in all countries, we
can write
\[
CA_n = \frac{1}{1+N} \sum_m (CA_n - CA_m) = -\frac{1}{1+N} \frac{1}{1+a} \sum_m (W_n - W_m) = \\
-\frac{1}{1+a} \frac{1-\kappa}{1+N \kappa} \sum_m ((1 + \epsilon_n^G)(Q_n - a) - (1 + \epsilon_m^G)(Q_m - a)) - \frac{1}{1+a} \frac{\kappa}{1+N \kappa} \epsilon_n^G \sum_m (Q_n - a)
\]  
(C.13)

The effect of a risk aversion shock is
\[
\frac{\partial CA_n}{\partial G} = -\frac{1}{1+a} \frac{1-\kappa}{1+N \kappa} \sum_m \left( (1 + \epsilon_n^G) \frac{\partial Q_n}{\partial G} - (1 + \epsilon_m^G) \frac{\partial Q_m}{\partial G} \right) - \frac{1}{1+a} \frac{\kappa}{1+N \kappa} \epsilon_n^G \sum_m \frac{\partial Q_m}{\partial G} 
\]  
(C.14)

Next take the derivative with respect to \(\epsilon\) and evaluate at \(\epsilon = 0\) and \(G = 1\). Using that \(\sum_{m=1}^{N+1} \frac{\partial^2 Q_m}{\partial G \partial \epsilon} = 0\), we have
\[
\frac{\partial^2 CA_n}{\partial G \partial \epsilon} = -\frac{1}{1+a} \frac{1-\kappa}{1+N \kappa} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} - \frac{1}{1+a} g_n \frac{\partial Q}{\partial G} 
\]  
(C.15)

Since \(\partial Q / \partial G > 0\), the last term is a negative linear function of \(g_n\). The same is the case for the first term as we have already established that \(\partial^2 Q_n / [\partial G \partial \epsilon]\) is a positive linear function of \(g_n\). It therefore follows that lower risk-aversion (higher \(g_n\) with positive \(\epsilon\)) implies a higher current account when \(G\) falls. Net capital outflows will therefore be higher in response to a global risk-aversion shock in countries that are less risk-averse.

C.4 Net Outflows Risky Assets

From (26) and (27), outflows and inflows of risky assets are
\[
OF_n^{\text{risky}} = \frac{a}{1+a} \sum_{m \neq n} z_{n,m} W_n - (1 + \epsilon_n^G) \frac{\kappa}{1+N \kappa} \sum_{m \neq n} Q_m 
\]  
(C.16)
\[
IF_n^{\text{risky}} = \frac{a}{1+a} \sum_{m \neq n} z_{m,n} W_m - Q_n \frac{\kappa}{1+N \kappa} \sum_{m \neq n} (1 + \epsilon_m^G) 
\]  
(C.17)

This uses (25) for \(z_{n,m,0}\) and \(z_{m,n,0}\). Substituting the time 1 portfolio shares in (18), net outflows of risky assets are
\[
NF_n^{\text{risky}} = \frac{a}{1+a} \left( 1 + \epsilon_n^G \right) W_n G \kappa \sum_m Q_m \frac{\bar{D} - RQ_m}{\sigma^2} 
\]  
(C.18)
\[
-\frac{a}{1+a} G \kappa Q_n \frac{\bar{D} - RQ_n}{\sigma^2} \sum_m (1 + \epsilon_m^G) W_m + (N + 1) Q_n \frac{\kappa}{1+N \kappa} - (1 + \epsilon_n^G) \frac{\kappa}{1+N \kappa} \sum_m Q_m 
\]
Taking the derivative with respect to $G$, we have

\[
\frac{\partial NF_{n}^{\text{risky}}}{\partial G} = \frac{a}{1 + a} \left(1 + \epsilon_{n}^{G}\right) \Gamma_{\kappa} \left(\sum_{m} Q_{m} \frac{\bar{D} - R Q_{m}}{\sigma^{2}}\right) \left(G \frac{\partial W_{n}}{\partial G} + W_{n}\right)
\]

\[+
\frac{a}{1 + a} \frac{(1 + \epsilon_{n}^{G}) W_{n} \Gamma_{\kappa}}{\sigma^{2}} \sum_{m} \left(\bar{D} - 2 R Q_{m}\right) \frac{\partial Q_{m}}{\partial G} - Q_{m}^{2} \frac{\partial R}{\partial G}\]

\[= \frac{a}{1 + a} \frac{\bar{D} Q_{n} - R Q_{n}^{2}}{\sigma^{2}} \sum_{m} \left(1 + \epsilon_{m}^{G}\right) W_{m} + G \sum_{m} \left(1 + \epsilon_{m}^{G}\right) \frac{\partial W_{m}}{\partial G}\]

\[+ \frac{\kappa(N + 1)}{1 + N_{\kappa}} \frac{\partial Q_{n}}{\partial G} - (1 + \epsilon_{n}^{G}) \frac{\kappa}{1 + N_{\kappa}} \sum_{m} \frac{\partial Q_{m}}{\partial G}\]  
(C.19)

Next we take the derivative with respect to $\epsilon$ at the starting point where $\epsilon = 0$ and $G = 1$. It is useful to first compute the derivatives involving wealth, using (C.3). Since the first-order derivatives of risky asset prices with respect to $\epsilon$ are zero, so is the first-order derivative of $W_{n}$ with respect to $\epsilon$. It is also useful to derive an expression for $\partial^{2} W_{n}/\partial G \partial \epsilon$.

We have

\[
\frac{\partial W_{n}}{\partial G} = (1 + \epsilon_{n}^{G}) \frac{1 - \kappa}{1 + N_{\kappa}} \frac{\partial Q_{n}}{\partial G} + (1 + \epsilon_{n}^{G}) \frac{\kappa}{1 + N_{\kappa}} \sum_{m} \frac{\partial Q_{m}}{\partial G}
\]  
(C.20)

Evaluated at the initial point, this is equal to $\partial Q/\partial G$. The second order derivative is

\[
\frac{\partial^{2} W_{n}}{\partial G \partial \epsilon} = g_{n} \frac{\partial Q}{\partial G} + \frac{1 - \kappa}{1 + N_{\kappa}} \frac{\partial^{2} Q_{n}}{\partial G \partial \epsilon} + \frac{\kappa}{1 + N_{\kappa}} \sum_{m} \frac{\partial^{2} Q_{m}}{\partial G \partial \epsilon}
\]  
(C.21)

We also use that $\partial R/\partial G = (\partial Q/\partial G)/((1 + a) \lambda)$ from (C.6).

Using this, taking the derivative of (C.19) with respect to $\epsilon$, and subtracting the same expression for country $k$, gives

\[
\frac{\partial^{2} \left(NF_{n}^{\text{risky}} - NF_{k}^{\text{risky}}\right)}{\partial G \partial \epsilon} = 2 \frac{a}{1 + a} \frac{\kappa(N + 1)}{1 + N_{\kappa}} (g_{n} - g_{k}) \frac{\partial Q}{\partial G} + a (g_{n} - g_{k}) \frac{\kappa(N + 1)}{1 + N_{\kappa}} \frac{\kappa(N + 1)}{1 + N_{\kappa}} \frac{\partial Q}{\partial G}
\]

\[+ \frac{a}{1 + a} \frac{(1 + \kappa)(N + 1)}{1 + N_{\kappa}} \frac{\partial^{2} (Q_{n} - Q_{k})}{\partial G \partial \epsilon} + (g_{n} - g_{k}) \frac{\kappa(N + 1)}{1 + N_{\kappa}} \left(1 - \frac{a(1 + a)(1 + N_{\kappa})}{\sigma^{2} \bar{z}}\right) \frac{\partial^{2} (Q_{n} - Q_{k})}{\partial G \partial \epsilon}\]

\[+ \frac{\kappa(N + 1)}{1 + N_{\kappa}} \frac{\partial^{2} (Q_{n} - Q_{k})}{\partial G \partial \epsilon} - \frac{\kappa(N + 1)}{1 + N_{\kappa}} (g_{n} - g_{k}) \frac{\partial Q}{\partial G}\]

\[= 2 \frac{a}{1 + a} \frac{\kappa(N + 1)}{1 + N_{\kappa}} (g_{n} - g_{k}) \frac{\partial Q}{\partial G} + a (g_{n} - g_{k}) \frac{\kappa(N + 1)}{1 + N_{\kappa}} \frac{\kappa(N + 1)}{1 + N_{\kappa}} \frac{\partial Q}{\partial G}
\]

\[+ \frac{a}{1 + a} \frac{(1 + \kappa)(N + 1)}{1 + N_{\kappa}} \frac{\partial^{2} (Q_{n} - Q_{k})}{\partial G \partial \epsilon} + (g_{n} - g_{k}) \frac{\kappa(N + 1)}{1 + N_{\kappa}} \left(1 - \frac{a(1 + a)(1 + N_{\kappa})}{\sigma^{2} \bar{z}}\right) \frac{\partial^{2} (Q_{n} - Q_{k})}{\partial G \partial \epsilon}\]

\[+ \frac{\kappa(N + 1)}{1 + N_{\kappa}} \frac{\partial^{2} (Q_{n} - Q_{k})}{\partial G \partial \epsilon} - \frac{\kappa(N + 1)}{1 + N_{\kappa}} (g_{n} - g_{k}) \frac{\partial Q}{\partial G}\]
Aggregating across $k$ and using that $\sum_k NF_{k}^{\text{risky}} = 0$, we have

$$\frac{\partial^2 NF_n^{\text{risky}}}{\partial G \partial \epsilon} = \frac{2a}{1 + a} \frac{\kappa(N + 1)}{1 + N\kappa} \frac{\partial Q}{\partial G} + a g_n \frac{\kappa(N + 1)}{1 + N\kappa}$$

$$+ \frac{a}{1 + a} \frac{\kappa(1 - \kappa)}{(1 + N\kappa)^2} \sum_k \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon} + \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon} + g_n \frac{\kappa(N + 1)}{1 + N\kappa} \left(1 - \frac{a(1 + a)(1 + N\kappa)\Gamma}{\sigma^2 \bar{z}}\right) \frac{\partial Q}{\partial G}$$

$$- \frac{a^3(N + 1)\Gamma \kappa}{\bar{z}(1 + a)\lambda\sigma^2} g_n \frac{\partial Q}{\partial G} - \frac{\kappa}{1 + N\kappa} \left(1 - \frac{a(1 + a)(1 + N\kappa)\Gamma}{\sigma^2 \bar{z}}\right) \sum_k \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon}$$

$$+ \frac{\kappa}{1 + N\kappa} \sum_k \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon} - \frac{\kappa(N + 1)}{1 + N\kappa} g_n \frac{\partial Q}{\partial G} \right) \] (C.22)

We need to show that (C.22) is a positive linear function of $g_n$. If so, it follows that countries with lower risk-aversion (higher $g_n$ when $\epsilon > 0$) have lower net outflows of risky assets when global risk aversion rises ($G$ falls). Using that $\sum_{k=1}^{N+1} \frac{\partial^2 Q_k}{\partial G \partial \epsilon} = 0$, collecting terms gives

$$\frac{\partial^2 NF_n^{\text{risky}}}{\partial G \partial \epsilon} = \frac{\kappa(N + 1)}{1 + N\kappa} \left(\frac{a}{1 + a} \frac{1 - \kappa}{1 + N\kappa} + \frac{a(1 + a)(1 + N\kappa)\Gamma}{\sigma^2 \bar{z}}\right) \frac{\partial^2 Q_n}{\partial G \partial \epsilon} +$$

$$\frac{\kappa(N + 1)}{1 + N\kappa} g_n \left[a + \left(2 \frac{a \bar{z}}{1 + a} - \frac{a(1 + a)(1 + N\kappa)\Gamma}{\sigma^2 \bar{z}}\right) - \frac{a^2 \Gamma(1 + N\kappa)}{(1 + a)\lambda\sigma^2 \bar{z}} \right] \frac{\partial Q}{\partial G} \ (C.23)$$

The first line is clearly a positive linear function of $g_n$ as we have already shown that $\frac{\partial^2 Q_n}{\partial G \partial \epsilon}$ is a positive linear function of $g_n$. Substituting the expression for $\frac{\partial Q}{\partial G}$ in (34), the second line becomes

$$\frac{\kappa(N + 1)}{1 + N\kappa} g_n \frac{a\sigma^2 \bar{z}^2}{(1 + a)^2 \Gamma(1 + N\kappa) - \sigma^2 \bar{z}^2 + \frac{a^2 \Gamma(1 + N\kappa)}{\lambda}}$$

This is also a positive linear function of $g_n$. The denominator is positive by Assumption 2 that $\sigma^2 \bar{z}^2 < \Gamma(1 + N\kappa)(1 + a)^2$.

Since the second-order derivative of total net outflows is a negative function of $g_n$, and the second-order derivative of net outflows of risky assets is a positive function of $g_n$, it follows that the second-order derivative of net outflows of safe assets is a negative function of $g_n$. Therefore a country with lower than average risk-aversion will have negative net outflows of risky assets due to the global risk-aversion shock, and positive total net outflows and net outflows of safe assets.

### D Cross Country Heterogeneity in Expected Dividends

Following the same steps as in Appendix C, we now consider the impact of heterogeneity across countries in expected dividends.
D.1 Market Clearing Conditions

The market clearing conditions remain the same as (22)-(23). Using the period 0 portfolio shares, which correspond to (24)-(25), wealth is

\[ W_n = \frac{1 + a}{\bar{z}} + \frac{1 - \kappa}{1 + N\kappa} (1 + d_n\epsilon)(Q_n - a) + \frac{\kappa}{1 + N\kappa} \sum_m (1 + d_m\epsilon)(Q_m - a) \quad (D.1) \]

From Assumption 1 we have \( K_n = 1 + d_n\epsilon \). Since \( \sum_n d_n = 0 \), it follows that \( \sum_n K_n = N + 1 \), so that from Assumption 1 \( B^0_h = a(1 - (1/\bar{z})) \). Therefore \( B^1_h = (1 + a)(1 - (1/\bar{z})) + Y - C^h_1 \).

Together with the expressions for \( K_n \) and \( W_n \), we can then write the aggregate asset market clearing condition (23) as

\[ \sum_{m=1}^{N+1} (1 + d_m\epsilon)(Q_m - a) = (N + 1)(1 + a) \left( 1 + \frac{1}{\bar{z}} + Y - C^h_1 \right) \quad (D.2) \]

Taking the derivative with respect to \( \Gamma \), we have

\[ \frac{\partial R}{\partial \Gamma} = \frac{1}{(N + 1)(1 + a)\lambda} \sum_{m=1}^{N+1} (1 + d_m\epsilon) \frac{\partial Q_m}{\partial \Gamma} \quad (D.3) \]

Next consider the market clearing conditions for risky assets (22). Substituting the portfolio shares (17)-(18) and wealth expressions (D.1) into (22), the market clearing conditions for risky assets are

\[ \frac{(1 + a)(1 + N\kappa)}{\bar{z}} + \frac{(1 - \kappa)^2}{1 + N\kappa} (1 + d_n\epsilon)(Q_n - a) + \kappa \left( 1 + \frac{1 - \kappa}{1 + N\kappa} \right) \sum_m (1 + d_m\epsilon)(Q_m - a) \]

\[ = \frac{1 + a}{a} (1 + d_n\epsilon) \frac{1}{\Gamma \sigma^2 D_n - RQ_n} \quad (D.4) \]

Differentiating with respect to \( \Gamma \) and substituting (D.3), we have

\[ \frac{(1 - \kappa)^2}{1 + N\kappa} (1 + d_n\epsilon) \frac{\partial Q_n}{\partial \Gamma} + \kappa \left( 1 + \frac{1 - \kappa}{1 + N\kappa} \right) \sum_m (1 + d_m\epsilon) \frac{\partial Q_m}{\partial \Gamma} \]

\[ = -\frac{1 + a}{a} (1 + d_n\epsilon) \frac{1}{\Gamma \sigma^2 D_n - RQ_n} + \frac{1 + a}{a} (1 + d_n\epsilon) \frac{1}{\Gamma \sigma^2} \frac{\partial Q_n}{\partial \Gamma} \]

\[ + \frac{1}{(N + 1)\alpha \lambda} (1 + d_n\epsilon) \frac{1}{\Gamma \sigma^2} Q_n \sum_{m=1}^{N+1} (1 + d_m\epsilon) \frac{\partial Q_m}{\partial \Gamma} \quad (D.5) \]

D.2 Impact on Relative Prices Risky Assets

We first consider the impact of the global risk-aversion shock on relative prices of risky assets. To show that the risky asset price \( Q_n \) drops more in countries with a higher expected dividend, and therefore a higher \( d_n \) when \( \epsilon > 0 \), we need to show that \( \partial^2 Q_n / [\partial \Gamma \partial \epsilon] \) depends
positively on $d_n$.

To this end we need to differentiate (D.5) with respect to $\epsilon$ and evaluate at $\epsilon = 0$ and $G = 1$. At that point $Q_m = a$, $R = (1 + a)/a$ and $\bar{D}_n - RQ = \sigma^2\bar{z}/(a\Gamma(1 + N\kappa))$. From the expression for $\bar{D}_n$ in Assumption 1 we have that $\partial \bar{D}_n/\partial \epsilon = d_n\sigma^2\bar{z}/[a\Gamma(1 + N\kappa)]$. We use that the pre-shock equilibrium for risky asset prices and the interest rate does not depend on $\epsilon$, so that $\partial Q_m/\partial \epsilon = \partial R/\partial \epsilon = 0$. Since all first order derivatives of risky asset prices with respect to $G$ will be the same, we simply denote them $\partial Q/\partial G$ (see (34)). This gives

$$
\frac{(1 - \kappa)^2}{1 + N\kappa} d_n \frac{\partial Q}{\partial G} + \frac{(1 - \kappa)^2}{1 + N\kappa} \frac{\partial^2 Q_m}{\partial G \partial \epsilon} + \kappa \left( 1 + \frac{1 - \kappa}{1 + N\kappa} \right) \sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon} = (D.6)
$$

$$(1 + a)^2 \frac{\Gamma(1 + N\kappa)^2}{\sigma^2 \bar{z}^2} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} - \frac{\Gamma(1 + N\kappa)^2}{\sigma^2 \bar{z}^2} d_n \left( 1 + a \right)^2 + \frac{a^2}{\lambda} \frac{\partial Q}{\partial G} + \frac{\Gamma(1 + N\kappa)^2 a^2}{(N + 1)\lambda \sigma^2 \bar{z}^2} \sum_{m=1}^{N+1} \frac{\partial^2 Q_m}{\partial G \partial \epsilon}
$$

Taking the sum across $n$, using that $\sum_{n=1}^{N+1} d_n = 0$, gives $\sum_{n=1}^{N+1} \partial^2 Q_n/[\partial G \partial \epsilon] = 0$. We then have

$$
\frac{\partial^2 Q_n}{\partial G \partial \epsilon} = d_n \frac{(1 - \kappa)^2}{1 + N\kappa} \frac{1}{\sigma^2 \bar{z}^2} \Gamma(1 + N\kappa)^2 \left( (1 + a)^2 + \frac{a^2}{\lambda} \right) \frac{\partial Q}{\partial G} + \frac{1}{(N + 1)\lambda \sigma^2 \bar{z}^2} \sum_{m=1}^{N+1} \frac{\partial^2 Q_m}{\partial G \partial \epsilon} (D.7)
$$

The numerator of this ratio is positive. The denominator is positive as well since from Assumption 2 we have $\bar{z}^2 \sigma^2 < (1 + a)^2 \Gamma(1 + N\kappa)$. Since $\partial Q/\partial G$ is positive, it follows that (D.7) is a positive linear function of $d_n$, which implies that the risky asset price drops more in countries with a higher expected dividend.

### D.3 Impact on Total Net Flows

We now consider the impact of the shock on total net capital flows, which is equal to the current account as in (C.13). As in (C.13), we have $CA_n = -\sum_m (W_n - W_m)/[(1 + a)(1 + N)]$. Using the wealth expression (D.1), we have

$$
CA_n = \frac{1}{1 + a} \frac{1 - \kappa}{1 + N\kappa} \left( -(1 + d_n \epsilon)(Q_n - a) + \frac{1}{1 + N} \sum_m (1 + d_m \epsilon)(Q_m - a) \right)
$$

The effect of a risk aversion shock is

$$
\frac{\partial CA_n}{\partial G} = \frac{1}{1 + a} \frac{1 - \kappa}{1 + N\kappa} \left( -(1 + d_n \epsilon) \frac{\partial Q_n}{\partial G} + \frac{1}{1 + N} \sum_m (1 + d_m \epsilon) \frac{\partial Q_m}{\partial G} \right)
$$

Next take the derivative with respect to $\epsilon$ and evaluate at $\epsilon = 0$ and $G = 1$. This gives

$$
\frac{\partial^2 CA_n}{\partial G \partial \epsilon} = -\frac{1}{1 + a} \frac{1 - \kappa}{1 + N\kappa} \left( d_n \frac{\partial Q}{\partial G} + \frac{\partial^2 Q_n}{\partial G \partial \epsilon} \right) (D.8)
$$

Since $\partial Q/\partial G > 0$ and we have already shown that $\partial^2 Q_n/[\partial G \partial \epsilon]$ is a positive linear function of $d_n$, it follows that this second derivative is a negative linear function of $d_n$. Therefore
countries with higher expected dividends (higher $d_n$ when $\epsilon > 0$) have larger net capital outflows when $G$ falls.

### D.4 Net Outflows Risky Assets

From (26) and (27), substituting the portfolio expressions (18) and (25), net outflows of risky assets are

\[
NF_{n}^{\text{risky}} = \frac{a}{1 + a} W_n \Gamma \kappa \sum_m Q_m \frac{\bar{D}_m - RQ_m}{\sigma^2} - \frac{a}{1 + a} \Gamma \kappa Q_n \frac{\bar{D}_n - RQ_n}{\sigma^2} W_m \\
+ \frac{\kappa (N + 1)}{1 + N \kappa} (1 + d_n \epsilon) Q_n - \frac{\kappa}{1 + N \kappa} \sum_m (1 + d_m \epsilon) Q_m
\]  

(D.9)

Taking the derivative with respect to $G$, we have

\[
\frac{1 + a}{a} \frac{\partial NF_{n}^{\text{risky}}}{\partial G} = W_n \frac{\Gamma \kappa}{\sigma^2} \sum_m (\bar{D}_m Q_m - RQ_m^2) \\
+ \frac{\Gamma \kappa}{\sigma^2} (\bar{D}_n Q_n - RQ_n^2) \sum_m W_m \\
- \frac{G \Gamma \kappa}{\sigma^2} (\bar{D}_n - 2RQ_n) \frac{\partial Q_n}{\partial G} \sum_m W_m \\
- \frac{G \Gamma \kappa}{\sigma^2} (\bar{D}_n - RQ_n^2) \sum_m \frac{\partial W_m}{\partial G} \\
+ \frac{G \Gamma \kappa}{\sigma^2} Q_n^2 \frac{\partial R}{\partial G} \sum_m W_m + \frac{1 + a}{1 + N \kappa} \kappa (N + 1) \frac{\partial Q_n}{\partial G} (1 + d_n \epsilon) - \frac{\kappa}{1 + N \kappa} \sum_m (1 + d_m \epsilon) \frac{\partial Q_m}{\partial G}
\]  

(D.10)

We will evaluate this derivative with respect to $\epsilon$ at the starting point where $\epsilon = 0$ and $G = 1$. Use that the derivative of any asset price $Q_m$ and wealth $W_m$ with respect to $\epsilon$ is zero at this point. It is also useful to derive an expression for $\partial^2 W_n/\partial G \partial \epsilon$. We have

\[
\frac{\partial W_n}{\partial G} = \frac{1 - \kappa}{1 + N \kappa} (1 + d_n \epsilon) \frac{\partial Q_n}{\partial G} + \frac{\kappa}{1 + N \kappa} \sum_m (1 + d_m \epsilon) \frac{\partial Q_m}{\partial G}
\]  

(D.11)

Evaluated at the initial point, this is equal to $\partial W_n/\partial G = \partial Q/\partial G$. The second order derivative is

\[
\frac{\partial^2 W_n}{\partial G \partial \epsilon} = \frac{1 - \kappa}{1 + N \kappa} d_n \frac{\partial Q}{\partial G} + \frac{1 - \kappa}{1 + N \kappa} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{\kappa}{1 + N \kappa} \sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon}
\]  

(D.12)
Taking the derivative of (D.10) with respect to $\epsilon$ then gives

$$
\frac{\partial^2 N_{F_{\text{risky}}}}{\partial G \partial \epsilon} = \frac{a}{1 + a} \frac{\kappa(1 - \kappa)(N + 1)}{(1 + N\kappa)^2} d_n \frac{\partial Q}{\partial G} + \frac{a}{1 + a} \frac{\kappa(1 - \kappa)(N + 1)\bar{z}}{(1 + N\kappa)^2} \frac{\partial^2 Q_n}{\partial G \partial \epsilon}
$$

$$
- a \frac{\kappa(N + 1)}{1 + N\kappa} d_n - \frac{\kappa(N + 1)}{1 + N\kappa} \frac{\partial Q}{\partial G} + \frac{\kappa(N + 1)}{1 + N\kappa} \left( \frac{(1 + a)(1 + N\kappa)\Gamma a}{\sigma^2 \bar{z}} - 1 \right) \frac{\partial^2 Q_n}{\partial G \partial \epsilon}
$$

$$
- \frac{a}{1 + a} \frac{\kappa(N + 1)\bar{z}}{1 + N\kappa} \frac{\partial Q}{\partial G} + \frac{\kappa(N + 1)}{1 + N\kappa} \frac{\partial^2 Q_n}{\partial G \partial \epsilon}
$$

Collecting terms, we have

$$
\frac{\partial^2 N_{F_{\text{risky}}}}{\partial G \partial \epsilon} = -a \frac{\kappa(N + 1)}{1 + N\kappa} d_n - \frac{a}{1 + a} \frac{\kappa^2(N + 1)^2 \bar{z}}{(1 + N\kappa)^2} \frac{\partial Q}{\partial G}
$$

$$
+ \frac{a}{1 + a} \frac{\kappa(N + 1)}{1 + N\kappa} \left( \frac{(1 - \kappa)^2}{1 + N\kappa} + \frac{1}{\sigma^2 \bar{z}} \right) (1 + a)^2 (1 + N\kappa) \Gamma \frac{\partial^2 Q_n}{\partial G \partial \epsilon}
$$

(D.13)

Using that $\sum_{k=1}^{N+1} \frac{\partial^2 Q_k}{\partial G \partial \epsilon} = 0$, substituting (34) and (D.7) and collecting terms, we have

$$
\frac{\partial^2 N_{F_{\text{risky}}}}{\partial G \partial \epsilon} = (D.14)
$$

This is clearly a positive linear function of $d_n$. The terms in the denominator are positive since $\sigma^2 \bar{z}^2 < (1 + a)^2 \Gamma(1 + N\kappa)$.

Since the second-order derivative of total net outflows is a negative function of $d_n$, and the second-order derivative of net outflows of risky assets is a positive function of $d_n$, it follows that the second-order derivative of net outflows of safe assets is a negative function of $d_n$. Therefore a country with a higher than average expected dividends will have negative net outflows of risky assets due to the global risk-aversion shock, and positive total net outflows and net outflows of safe assets.

### E Proof of Theorem 4

Given the results in Appendix C and D, Theorem 4 is now easy to proof. Let $X_n$ be either $Q_n$, $N_{F_n}$, $N_{F_{\text{risky}}}^n$ or $N_{F_{\text{safe}}}^n$. We have seen that for risk-aversion heterogeneity

$$
\frac{\partial^2 X_n}{\partial G \partial \epsilon}
$$

is a positive linear function of $g_n$ when $X_n$ is $Q_n$ or $N_{F_{\text{risky}}}^n$, while it is a negative linear function of $g_n$ when $X_n$ is $N_{F_n}$ or $N_{F_{\text{safe}}}^n$. Similarly, under expected dividend heterogeneity (E.1) is a positive linear function of $d_n$ when $X_n$ is $Q_n$ or $N_{F_{\text{risky}}}^n$, while it is a negative

55
linear function of $d_n$ when $X_n$ is $NF_n$ or $NF_n^{safe}$.

Assume without loss of generality that $\epsilon > 0$. First assume that there is risk-aversion heterogeneity. From Section 6.1, a country for which $g_n > 0$ then has a negative net foreign asset position in safe assets. The results then imply that as a result of a rise in global risk-aversion $Q_n$ and $NF_n^{risky}$ are lower than in the average country, while $NF_n$ and $NF_n^{safe}$ are higher than in the average country. This means that a country with a negative net foreign asset position of safe assets has a larger than average drop in the risky asset price, negative net outflows of risky assets and positive total net outflows and net outflows of safe assets. This uses that the first-order derivatives of all net outflow variables with respect to $G$ are zero. Since (E.1) is linear in $g_n$, the opposite will be the case for countries with a positive net foreign asset position of safe assets. It also follows that the size of these changes (in the relative risky asset price and net capital flow variables) is larger the larger the absolute size of the net foreign asset position of safe assets. The exact same results apply under expected dividend heterogeneity, using from Section 6.1 that a country for which $d_n > 0$ has a negative net foreign asset position of safe assets.

**F Section 6 Results**

Here we derive (51) and (52). The quantity that country $n$ holds at time 1 of risky assets from country $m$ is equal to

$$k_{n,m} = \beta z_{n,m} W_n Q_m$$

(F.1)

Using the portfolio expressions (17)-(18), this gives

$$k_{m,m} = \beta \Gamma (1 + \epsilon_m^G) G \frac{D_m - RQ_m}{\sigma^2} W_m$$

(F.2)

$$k_{n,m} = \beta \Gamma \kappa (1 + \epsilon_n^G) G \frac{D_m - RQ_m}{\sigma^2} W_n$$

(F.3)

In period 0 we have

$$k_{m,m,0} = \frac{1}{1 + N\kappa} (1 + \epsilon_m^G) (1 + \epsilon_m^D)$$

(F.4)

$$k_{n,m,0} = \frac{\kappa}{1 + N\kappa} (1 + \epsilon_n^G) (1 + \epsilon_m^D)$$

(F.5)

Therefore for all $n$

$$k_{n,m} \frac{1}{k_{n,m,0}} = a_m W_n$$

(F.6)

where

$$a_m = \beta (1 + N\kappa) \Gamma G \frac{\bar{D} - RQ_m}{\sigma^2} \frac{1}{1 + \epsilon_m^D}$$

(F.7)
The market equilibrium condition for country $m$ risky assets is

$$\sum_{l=1}^{N+1} k_{l,m} = K_m$$  \hspace{1cm} (F.8)

or

$$a_m \sum_{l=1}^{N+1} k_{l,m,0} W_l = K_m$$  \hspace{1cm} (F.9)

It follows that

$$a_m = \frac{K_m}{\sum_{l=1}^{N+1} k_{l,m,0} W_l} = \frac{1}{\sum_{l=1}^{n+1} \omega_{l,m} W_l}$$  \hspace{1cm} (F.10)

where

$$\omega_{l,m} = \frac{k_{l,m,0}}{K_m}$$  \hspace{1cm} (F.11)

is the fraction of the country $m$ quantity of risky assets that is held by country $l$ at time 0.

It then follows that

$$k_{n,m} = a_m k_{n,m,0} W_n = \frac{W_n}{\sum_{l=1}^{n+1} \omega_{l,m} W_l} k_{n,m,0}$$  \hspace{1cm} (F.12)

This is equation (51).

Next we derive (52). Start from (29):

$$NF_{saf}^n = B^h - B_0^h + \beta \left( 1 - \sum_{m=1}^{N+1} z_{n,m} \right) W_n - \beta \left( 1 - \sum_{m=1}^{N+1} z_{n,m,0} \right) W_{n,0}$$  \hspace{1cm} (F.13)

We have $\beta z_{n,m} W_n = k_{n,m} Q_m$. Therefore

$$NF_{saf}^n = B^h - B_0^h + \beta (W_n - W_{n,0}) - \sum_{m=1}^{N+1} k_{n,m} Q_m + \sum_{m=1}^{N+1} k_{n,m,0} Q_{m,0}$$  \hspace{1cm} (F.14)

We can further rewrite this as

$$NF_{saf}^n = B^h - B_0^h + \beta (W_n - W_{n,0}) - \sum_{m=1}^{N+1} (k_{n,m} - k_{n,m,0}) Q_m - \sum_{m=1}^{N+1} k_{n,m,0} (Q_m - Q_{m,0})$$  \hspace{1cm} (F.15)

The first term is saving of households. The second term is the change in financial wealth of investors, which is saving of investors plus valuation effects. The latter is $\sum_{m=1}^{N+1} k_{n,m,0} (Q_m - Q_{m,0})$. We then have

$$NF_{saf}^n = S_n - \sum_{m=1}^{N+1} (k_{n,m} - k_{n,m,0}) Q_m$$  \hspace{1cm} (F.16)

where $S_n$ is country $n$ saving. This corresponds to (52).
Table 1: Regressions on First Capital Flows Factor

<table>
<thead>
<tr>
<th></th>
<th>(\Delta R_t)</th>
<th>(\Delta q_{n,t})</th>
<th>(\Delta o_{n,t}^{\text{risky}})</th>
<th>(\Delta i_{n,t}^{\text{risky}})</th>
<th>(\Delta o_{n,t}^{\text{risky}} + \Delta i_{n,t}^{\text{risky}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta F_t)</td>
<td>0.559***</td>
<td>16.170***</td>
<td>3.272**</td>
<td>2.273**</td>
<td>5.531***</td>
</tr>
<tr>
<td></td>
<td>(0.190)</td>
<td>(0.982)</td>
<td>(1.292)</td>
<td>(0.942)</td>
<td>(2.161)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.363</td>
<td>0.376</td>
<td>0.313</td>
<td>0.176</td>
<td>0.237</td>
</tr>
</tbody>
</table>

Notes: \(\Delta R_t\) is the year-over-year change in the real 1-year U.S. treasury interest rate, \(\Delta q_{n,t}\) is the year-over-year log change in the stock market index in county \(n\), \(\Delta o_{n,t}^{\text{risky}}\) and \(\Delta i_{n,t}^{\text{risky}}\) are the year-over-year changes in risky capital outflows and inflows, and \(\Delta F_t\) is the year-over-year change in the GFC factor. All outflows and inflows are normalized by the prior year’s GDP. All regressions also include a country-fixed effect and a one year lag of the dependent variable. Robust standard errors are clustered by country. ***/***/** denotes significance at the 1/5/10% level.
<table>
<thead>
<tr>
<th></th>
<th>$\Delta n_{f_{t,n,t}}^{safe}$</th>
<th>$\Delta n_{f_{t,n,t}}^{safe}$</th>
<th>$\Delta n_{f_{t,n,t}}^{safe}$</th>
<th>$\Delta n_{f_{t,n,t}}^{risky}$</th>
<th>$\Delta n_{f_{t,n,t}}^{risky}$</th>
<th>$\Delta n_{f_{t,n,t}}^{risky}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta F_t$</td>
<td>-0.795</td>
<td>-0.436</td>
<td>-0.474</td>
<td>1.016*</td>
<td>0.795*</td>
<td>0.616*</td>
</tr>
<tr>
<td></td>
<td>(0.790)</td>
<td>(0.445)</td>
<td>(0.410)</td>
<td>(0.458)</td>
<td>(0.458)</td>
<td>(0.324)</td>
</tr>
<tr>
<td>$nfa_{n,t}^{safe} \times \Delta F_t$</td>
<td>0.025***</td>
<td>0.026***</td>
<td></td>
<td>-0.016**</td>
<td>-0.012**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.009)</td>
<td></td>
<td>(0.007)</td>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>$nfa_{n,t}^{risky} \times \Delta F_t$</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.084</td>
<td>0.205</td>
<td>0.205</td>
<td>0.519</td>
<td>0.526</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $\Delta n_{f_{t,n,t}}^{safe}$ is the year-over-year change in net safe capital outflows, $\Delta n_{f_{t,n,t}}^{risky}$ is the year-over-year change in net risky capital outflows, $\Delta n_{f_{t,n,t}}$ is the year-over-year change in total net capital outflows (safe plus risky), $\Delta save_{n,t}$ is the year-over-year change in saving, $\Delta invest_{n,t}$ is the year-over-year change in investment, $nfa_{n,t-1}^{safe}$ and $nfa_{n,t-1}^{risky}$ are a country’s net foreign asset positions of safe and risky assets. All variables are normalized by the prior years GDP. All regressions include a country-fixed effect and a one-year lag of the year-over-year change in net risky capital outflows, net safe capital outflows, and saving. Robust standard errors are clustered by country. ***/**/* denotes significance at the 1/5/10% level.
Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N + 1$</td>
<td>20</td>
<td>Number of countries</td>
</tr>
<tr>
<td>$a$</td>
<td>25</td>
<td>4 percent annual interest rate</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td>0.5</td>
<td>Median risky share of 50%</td>
</tr>
<tr>
<td>$Y$</td>
<td>3</td>
<td>Labor income share of 75%</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.1</td>
<td>Median risk aversion of 10</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.4</td>
<td>Equity risk premium 4.6 percent</td>
</tr>
<tr>
<td>$\rho$</td>
<td>17.2</td>
<td>Observed relative variation in risky and safe asset prices</td>
</tr>
<tr>
<td>$\xi$</td>
<td>6.75</td>
<td>Decline in investment following change in asset prices</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.33</td>
<td>Sharpe ratios from Calvet et al. (2007)</td>
</tr>
</tbody>
</table>

Table 4: Responses to 1 Percent Fall Risky Asset Prices

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model Benchmark</th>
<th>Model Imperfect Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average change after 1% fall in risky asset prices</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_t$</td>
<td>-0.035</td>
<td>-0.035</td>
<td>-0.035</td>
</tr>
<tr>
<td>$\text{invest}_{n,t}$</td>
<td>-0.031</td>
<td>-0.037</td>
<td>-0.037</td>
</tr>
<tr>
<td>$\text{save}_{n,t}$</td>
<td>-0.045</td>
<td>-0.037</td>
<td>-0.037</td>
</tr>
<tr>
<td>$of_{n,t}^{\text{risky}} + if_{n,t}^{\text{risky}}$</td>
<td>-0.342</td>
<td>-0.307</td>
<td>-0.288</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Change in a country with $nf_{n,0}^{\text{safe}} = -100%$ relative to a country with $nf_{n,0}^{\text{safe}} = 0%$</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$nf_{n,t}$</td>
<td>0.060</td>
<td>0.051</td>
<td>0.051</td>
</tr>
<tr>
<td>$\text{save}_{n,t}$</td>
<td>0.042</td>
<td>0.044</td>
<td>0.044</td>
</tr>
<tr>
<td>$\text{invest}_{n,t}$</td>
<td>-0.014</td>
<td>-0.007</td>
<td>-0.007</td>
</tr>
<tr>
<td>$nf_{n,t}^{\text{risky}}$</td>
<td>-0.097</td>
<td>-0.424</td>
<td>-0.131</td>
</tr>
<tr>
<td>$nf_{n,t}^{\text{safe}}$</td>
<td>0.157</td>
<td>0.475</td>
<td>0.182</td>
</tr>
</tbody>
</table>

Notes: Data moments are based on Tables 1 and 2, setting $\Delta F_t = -1/16.2$, leading to 1 percent drop in average risky asset price. Model moments are based on drop in $G$ that leads to an average drop in risky asset prices of 1 percent. Moments in italics are targeted. The last column is based on extension in Section 7.5 where there are $N + 1$ safe assets that are imperfect substitutes.
Figure 1: Flows during Rising (2003-2007) and Falling (2008-2009) Asset Prices (% GDP)

Notes: Scatter plots are for the 20 advanced countries in the sample. The left side column shows cumulative flows over 2003-2007 minus the corresponding flow in 2002. The right side column shows cumulative flows over 2008-2009 minus the corresponding flow in 2007. Both are as a percent of GDP. The net foreign asset position of safe assets, as a percentage of GDP, is on the horizontal axis. The regression line is in red.
Figure 2: First Factor from Capital Flow Factor Model and MAR factor
Figure 3: $NFA^{\text{risky}}$ and $NFA^{\text{safe}}$, all 20 Countries (% of GDP, average 1996-2015)
Figure 4: Asset Market Equilibrium following Global Risk Aversion Shock
Notes: The horizontal axis shows the correlation between the risky and foreign shares, which is varied in the model by changing the correlation between the risky share and the Sharpe ratio loss that is used in the calibration. The vertical dashed line marks the correlation between the risky share and the foreign share of 0.43 in the calibrated version of the model, which corresponds to a correlation between the risky share and the Sharpe ratio loss of -0.49. The model without investment refers to the case where $\xi \to \infty$. 
Notes: The 100,000 investors in a country are sorted by pre-shock risky shares and grouped into 40 bins of 2500 investors. Each dot represents the sum of purchases of risky assets or purchases of foreign risky assets across the 2500 investors in that bin. The reported results for each bin are averages across all $N + 1$ countries and shown as a percent of GDP. The bottom chart reports results both for the calibrated model, where the correlation between the risky and foreign share is 0.43, and the case where this correlation is zero.
Figure 7: Changes in Model after 1% Fall Risky Asset Prices

Notes: Scatter plots are generated from the model where $NFA_{Safe}$ and $NFA_{Risky}$ positions are calibrated to match the 20 countries in the empirical section.