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# A Theory of Capital Flow Retrenchment

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**Abstract**

The empirical literature shows that gross capital inflows and outflows both decline following a negative global shock. However, to generate a positive co-movement between gross inflows and outflows, the theoretical literature relies on asymmetric shocks across countries. We present a model where there is heterogeneity across investors within countries, but there are no asymmetries across countries. We show that a negative global shock (rise in global risk-aversion) generates an identical drop in gross inflows and outflows. The within-country heterogeneity relates to the willingness of investors to hold risky assets and foreign assets.

**Keywords:** Capital Flows; Retrenchment; Portfolio Heterogeneity

**JEL**: F30; F40

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# 1 Introduction

It is well known that gross capital inflows (purchases of domestic assets by foreigners) and gross capital outflows (purchases of foreign assets by domestic investors) both fall during a negative global shock. Milesi-Ferretti and Tille (2011) document the sharp fall in gross capital outflows and inflows during the 2008 crisis. Broner et al. (2013) emphasize the strong co-movement between capital inflows and outflows and show that both decline in the years after a crisis. Forbes and Warnock (2012) find that global factors are significantly associated with extreme capital flow episodes, like sudden stops in gross capital inflows and retrenchment in gross capital outflows. Davis, Valente and van Wincoop (2021) and Miranda-Agrippino and Rey (2022) find that a common global factor explains the movements of both gross capital inflows and outflows and is highly correlated with a global factor of risky asset prices.

While the empirical literature finds evidence of capital flow retrenchment in response to a negative global shock, Broner et al. (2013) emphasize that asymmetries across countries are needed to generate a positive correlation between gross inflows and outflows. If investors in different countries have the same perception of risk and expected returns, portfolios would shift in the same direction. For example, a higher expected return on US assets would imply a portfolio shift to US assets by both US and foreign investors, which leads to higher capital inflows and lower capital outflows for the US. A variety of asymmetries across countries have been suggested in the literature to generate a positive co-movement between capital inflows and outflows.

Caballero and Simsek (2020) introduce asymmetry in the form of investor fickleness, where foreign investors are fickle and sell home country assets in a crisis. Home investors do not suffer from this fickleness and buy home assets at fire sale prices. This leads to a fall in capital inflows (fickleness) and a fall in outflows (retrenchment due to lower home asset prices). Brennan and Cao (1997) and Tille and van Wincoop (2014) focus on information asymmetries between home and foreign investors. Davis and van Wincoop (2018) introduce exogenous portfolio demand shocks in opposite directions for home and foreign investors. Tille and van Wincoop (2010) show that inflows and outflows become positively correlated due to various types of time-varying risk that impact foreign and domestic investors differently. Gourio, Siemer and Verdelhan (2015) introduce expropriation risk. Foreigners face potential expropriation of assets, while domestic investors do not.

In this paper we propose a model with symmetric countries where a common global shock leads to a joint decline in gross capital inflows and outflows. Investors in each country choose a portfolio of risky assets from each country and a global safe asset. Key to the model is within-country portfolio heterogeneity. Individual investors within each country differ in their risky shares (share of portfolio invested in risky assets) and foreign shares (share of risky asset portfolio invested in foreign risky assets). Investors with a higher risky share also tend to have a higher foreign share. But the distribution of these portfolio shares is identical across countries. There is no cross-country heterogeneity.

The intuition for the decline in gross capital inflows and outflows is as follows. A negative global shock reduces global risky asset prices. This reduces the relative wealth of investors with higher risky shares. These investors will then sell risky assets, while the investors with lower risky shares will buy. If investors with higher risky shares on average also have higher foreign shares, they will sell both home and foreign assets. Investors with low risky shares tend to have small foreign shares and therefore buy mostly home assets. This means that investors with high risky shares sell foreign assets mostly to foreigners, reducing gross capital outflows. In the aggregate, investors sell foreign assets and buy home assets. The wealth weighted average of a country's home bias increases, not because of an asymmetric shock that leads to a retrenchment to domestic assets, but because of the endogenous shift in the wealth distribution following the global shock.

The nature of the global shock and the parameters driving the portfolio heterogeneity are not critical to these results. For concreteness we will consider a rise in global risk-aversion, which reduces all risky asset prices equally. Such a shock is commonly considered as a key driver of the global financial cycle (GFC). Heterogeneity of risky and foreign shares is a result of investor heterogeneity of risk-aversion and a home bias parameter. While the model is rich in the sense that it allows for multiple countries and heterogeneity of investors within countries, we keep the model otherwise sufficiently tractable that we can derive a closed form analytical results regarding retrenchment.

Our explanation for the joint decline in capital inflows and outflows is consistent with micro evidence from Swedish household portfolio data reported by Calvet, Campbell and Sodini (2007, 2009*a* and 2009*b*). Sweden is unusual in that there is a wealth tax that requires the government to collect detailed portfolio and wealth data for all Swedish taxpayers. Risky shares, defined as the fraction of financial wealth not held in bank accounts or money market funds, are approximately uniformly distributed across households from 0 to 1. Calvet, Campbell and Sodini (2009*a*) find, consistent with the theory, that when risky asset prices drop, households with large risky shares sell risky assets and those with small risky shares buy risky assets.

In addition, Calvet, Campbell and Sodini (2007 and 2009*b*) find that investors with larger risky shares also tend to be more diversified (implying larger foreign shares). While the Swedish portfolio data do not tell us directly what share of risky assets is invested abroad, Calvet, Campbell and Sodini (2007 and 2009*b*) report a Sharpe ratio loss measure that tells us how much lower the Sharpe ratio of a household is than if it were globally diversified. A majority of Swedish households have a Sharpe ratio that is higher than that for the domestic stock market, suggesting that the Sharpe ratio loss is related to global diversification. There is a significant positive correlation of 0.49 between the risky share of households and their Sharpe ratio, so that investors with a higher risky share are more diversified.[1]

Consistent with the Swedish household portfolio data, our analysis of gross flows will relate to portfolio decisions made by households. We will not consider gross banking flows. Banks are of course ultimately owned by households, but banking sector capital flows are unrelated to decisions made directly by households.[2] When taking the model to the data, we will therefore consider gross non-banking flows. We will calibrate investor portfolio heterogeneity using the Swedish household portfolio data. We find that the model is consistent with the empirical change in gross capital flows relative to the global change in risky asset prices.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the impact of a global risk-aversion shock on gross capital flows. Section 4 presents the results from the calibrated model. Section 5 concludes.

# 2 Model Description

The model consists of $N+1$ symmetric countries, with one risky asset per country. There is a single consumption good, and thus one world-wide risk-free asset. Risky assets are shares in a country-specific capital that pay a stochastic dividend. The safe asset is supplied by a representative borrower in each country. These borrowers hold an initial stock of debt of the safe asset. They adjust their consumption/savings decisions, and thus the supply of the safe asset, in response to changes in the risk-free interest rate.

In each country there is a continuum of investors who solve a portfolio choice problem to allocate their wealth across the different assets. These investors are heterogeneous with respect to risk-aversion and home bias.

Although all agents have infinite horizons, we effectively collapse the future into a single period by assuming that all uncertainty is resolved in period 2. This simplification allows us to focus on the portfolio heterogeneity within countries that is central to the main results. We take period 0 as given. The analysis focuses on the impact of a shock to investor risk aversion in period 1.

[1] Using the Survey of Consumer Finances, the Online Appendix of Calvet, Campbell and Sodini (2007) reports that the distribution of Sharpe ratio losses is similar in the United States as in Sweden, suggesting that the Swedish data are not atypical.

[2] Acalin (2023) develops of theory of gross banking flows in a model with regional banks and global banks. Some of the regional banks wish to borrow, while others wish to lend. It is assumed that all this borrowing and lending goes through global banks. Deleveraging by global banks reduces the flows between regional and global banks, reducing both gross inflows and outflows.

## 2.1 Assets

### 2.1.1 Safe Assets

Safe assets are produced by a representative borrower in each country. The gross interest rate on the safe asset is $R_t$ from period $t$ to $t+1$. Borrowers have an initial debt of $B_0$ and receive an endowment of $Y$ each period.

Borrowers are identical across countries, so that we can omit country-specific subscripts. Borrowers in any one of the countries maximize

$$\sum_{s=0}^{\infty} \beta^s \frac{\left(C_{t+s}^b\right)^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}} \tag{1}$$

subject to the budget constraint

$$B_t = R_{t-1}B_{t-1} - Y + C_t^b \tag{2}$$

The first-order condition is

$$C_t^b = (R_t\beta)^{-\rho} C_{t+1}^b \tag{3}$$

This first order condition leads to a safe asset supply, $B_t$, that depends on the risk free interest rate and the previous period's stock of borrower debt, $B_{t-1}$.

### 2.1.2 Risky Assets

In each country there is a country-specific risky asset, $K_n$, that has a price $Q_{n,t}$ and pays a dividend $D_{n,t}$ in period $t$.

The return on the risky asset from country $n$ is

$$\frac{Q_{n,t+1} + D_{n,t+1}}{Q_{n,t}} \tag{4}$$

where $Q_{n,t}$ is the price of the asset and $D_{n,t}$ the dividend.

The period 1 dividend is set at 1 for all risky assets. There is uncertainty about future dividends, but this uncertainty is resolved at time 2. After that dividends will remain constant: $D_{n,t} = D_{n,2}$ for $t \geq 2$. In what follows it is useful to denote

$$D_n = \frac{D_{n,2}}{1-\beta} \tag{5}$$

where $\beta$ is the time discount rate. $D_n$ is the present value of dividends at time 2, which is proportional to $D_{n,2}$. We assume that $D_n$ is uncorrelated across countries. We relax this assumption in Section 4 when computing numerical results.

## 2.2 Investors

In each country there is a continuum of investors indexed $i$ on the interval [0,1].

### 2.2.1 Budget Constraint and Preferences

In period $t$ investor $i$ from country $n$ invests a fraction $z^i_{n,m,t}$ in the risky asset of country $m$. A fraction $1 - \sum_{m=1}^{N+1} z^i_{n,m,t}$ is invested in the safe asset. Wealth of investor $i$ in country $n$ evolves according to

$$W^i_{n,t+1} = \left(W^i_{n,t} - C^i_{n,t}\right) R^{p,i,n}_{t+1} \tag{6}$$

where $C^i_{n,t}$ is consumption and $R^{p,i,n}_{t+1}$ is the portfolio return from $t$ to $t+1$:

$$R^{p,i,n}_{t+1} = R_t + \sum_{m=1}^{N+1} z^i_{n,m,t} \left( \frac{Q_{m,t+1} + D_{m,t+1}}{Q_{m,t}} - R_t \right) \tag{7}$$

The term in brackets is the excess return of the risky asset from country $m$ over the safe asset.

Investors are assumed to have Rince preferences, which for investor $i$ from country $n$ we can write as

$$\ln(V^i_{n,t}) = \underset{C^i_{n,t}, \mathbf{z}_t}{max} \left\{ (1-\beta) \ln(C^i_{n,t}) + \beta \ln \left( \left[ E_t (V^i_{n,t+1})^{1-\gamma_i} \right]^{\frac{1}{1-\gamma_i}} \right) \right\} \tag{8}$$

where $\mathbf{z}_t = \left( z^i_{n,1,t}, .., z^i_{n,1+N,t} \right)$ is the vector of portfolio shares chosen by the investor at time $t$. The investor makes consumption and portfolio decisions. The rate of risk-aversion $\gamma_i$ will generally vary across investors. Risk-aversion only matters at time 1 as we take period 0 as given and uncertainty is resolved from time 2 onward.

### 2.2.2 Portfolio Heterogeneity

We introduce two types of heterogeneity of investors within countries, which lead to heterogeneity of the risky share (portfolio share allocated to risky assets) and the foreign share (share of risky assets allocated to foreign assets). These two types of heterogeneity are associated with investor home bias and risk-aversion. Home bias heterogeneity is introduced by allowing perceived dividend risk of foreign assets to vary across investors. In period 1 all country $n$ investors perceive the variance of $D_n$ to be $\sigma^2$. For any foreign asset $m \neq n$, investor $i$ from country $n$ perceives the variance of $D_m$ to be $\sigma^2/\kappa_i$, with $0 < \kappa_i \leq 1$. When $\kappa_i = 1$, all risky assets are perceived to be equally risky and there will be no home bias. When $\kappa_i < 1$ as a result of information asymmetries, foreign assets are perceived to be riskier, leading to a bias towards the domestic risky asset that varies across investors. The lower $\kappa_i$, the stronger the home bias.

Risk-aversion heterogeneity is introduced as follows. Let

$$\gamma_i = \frac{1}{\Gamma_i G} \tag{9}$$

The higher $\Gamma_i$ or $G$, the less risk-averse the investor is. Variation of $\Gamma_i$ across investors leads to within-country risk-aversion heterogeneity. A fall in $G$ is the common global shock to risk aversion in period 1. It affects all investors in all countries equally.

### 2.2.3 Optimal Consumption and Portfolios

The value function will be proportional to the wealth of the agent: $V^i_{n,1} = \alpha_{1,i} W^i_{n,1}$ and $V^i_{n,t} = \alpha_2 W^i_{n,t}$ for $t \geq 2$. The coefficients $\alpha_{1,i}$ and $\alpha_2$ can be derived from the Bellman equation and depend on structural model parameters (see Online Appendix), but are not important to the analysis. Using (6), investors at time $t$ therefore maximize

$$(1-\beta)\ln(C^i_{n,t}) + \beta ln(W^i_{n,t} - C^i_{n,t}) + \beta \ln\left(\left[E_t\left(R^{p,i,n}_{t+1}\right)^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}}\right) \tag{10}$$

Optimal consumption is then

$$C^i_{n,t} = (1-\beta)W^i_{n,t} \tag{11}$$

All investors consume a fraction $1-\beta$ of their wealth during each period. This leaves the investor with financial wealth $\beta W^i_{n,t}$ that is invested in safe and risky assets.

Since uncertainty is resolved at time 2, there is only a portfolio problem at time 1. Therefore the only portfolio return that matters is $R^{p,i,n}_2$, which for simplicity we will denote $R^{p,i,n}$. From (10) optimal portfolio shares are chosen to maximize the certainty equivalent of the portfolio return:

$$[E(R^{p,i,n})^{1-\gamma_i}]^{\frac{1}{1-\gamma_i}} \tag{12}$$

Using a second order Taylor expansion of $(R^{p,i,n})^{1-\gamma_i}$ around the expected portfolio return, one can approximate this as maximizing[3]

$$E\left(R^{p,i,n}\right) - 0.5\gamma_i var\left(R^{p,i,n}\right) \tag{13}$$

This leads to simple mean-variance portfolios.

[3]A second-order Taylor expansion gives $(R^{p,i,n})^{1-\gamma_i} = (ER^{p,i,n})^{1-\gamma_i} + (1-\gamma_i)(ER^{p,i,n})^{-\gamma_i}(R^{p,i,n} - ER^{p,i,n}) - 0.5\gamma_i(1-\gamma_i)(ER^{p,i,n})^{-\gamma_i-1}(R^{p,i,n} - ER^{p,i,n})^2$. Taking the expectation, we have $E(R^{p,i,n})^{1-\gamma_i} = (ER^{p,i,n})^{1-\gamma_i} - 0.5\gamma_i(1-\gamma_i)(ER^{p,i,n})^{-\gamma_i-1}var(R^{p,i,n})$. Taking this to the power $1/(1-\gamma_i)$, and linearly expanding around $ER^{p,i,n} = 1$ and $var(R^{p,i,n}) = 0$, gives (13).

For expected dividends, we denote the expectation at time 1 of $D_n$ as $\bar{D}$ and assume

$$\bar{D} = 1 + a + \bar{z}\frac{\sigma^2}{a\bar{\psi}} \tag{14}$$

Here $a = \frac{\beta}{1-\beta}$ and $\bar{\psi}$ is the mean across investors of $\psi_i = \Gamma_i(1 + N\kappa_i)$. The parameter $\bar{z}$ will be equal to the mean risky share across all investors in the equilibrium of the model prior to the global risk-aversion shock.

As shown in the Online Appendix, risky asset prices at time 2 are $Q_{m,2} = [a/(1+a)]D_m$. The period 2 asset payoffs are then $Q_{m,2} + D_{m,2} = D_m$. For ease of notation, from hereon we remove time subscripts from all time 1 variables. The portfolio return then becomes

$$R^{p,i,n} = R + \sum_{m=1}^{N+1} z^i_{n,m}\left(\frac{D_m - RQ_m}{Q_m}\right) \tag{15}$$

Maximizing (13) leads to the following optimal portfolios

$$z^i_{n,n} = Q_n\Gamma_i G\frac{\bar{D} - RQ_n}{\sigma^2} \tag{16}$$

$$z^i_{n,m} = Q_m\Gamma_i\kappa_i G\frac{\bar{D} - RQ_m}{\sigma^2} \qquad m \neq n \tag{17}$$

Define the fraction that investor $i$ from country $n$ invests in all risky assets (the "risky share") as

$$z^i_n = \sum_{m=1}^{N+1} z^i_{n,m} \tag{18}$$

For a given interest rate and risky asset prices, a higher value of $\Gamma_i$ (lower risk-aversion) implies a proportionally higher portfolio share allocated to all risky assets. A larger $\kappa_i$ implies a higher portfolio share allocated to foreign risky assets, without changing the portfolio share allocated to domestic risky assets (for given asset prices). The negative shock to $G$ lowers the portfolio share allocated to all risky assets equally across all investors.

## 2.3 Asset Market Clearing

The period $t$ market clearing conditions for risky assets are

$$\beta\sum_{m=1}^{N+1}\int_0^1 z^i_{m,n,t}W^i_{m,t}di = Q_{n,t}K_n \qquad n = 1, ..., N+1 \tag{19}$$

Here $K_n$ is the supply of risky asset $n$.

In addition there is a market clearing condition for safe assets. We can also use the aggregate market clearing condition for all assets that equates the demand to the supply of all assets:

$$\beta \sum_{n=1}^{N+1} \int_0^1 W_{n,t}^i di = \sum_n Q_{n,t} K_n + (N+1) B_t \tag{20}$$

## 2.4 Pre-Shock Equilibrium

We are interested in the impact of a global risk-aversion shock that lowers $G$. But we first describe the pre-shock equilibrium, for which we assume $G = 1$. We make a set of assumptions regarding initial conditions at time 0 that are intended to make sure that equilibrium values of endogenous variables are the same at time 1 as at time 0 before the shock. We can think of this as a type of pre-shock steady state.

**Assumption 1** *Assume the following initial conditions for period 0:* $W_{n,0}^i = (1+a)/\bar{z}$ *for all investors,* $Q_{n,0} = a$, $R_0 = (1+a)/a$, $K_n = 1$, $B_0 = a\left(\frac{1}{\bar{z}} - 1\right)$, *and*

$$z_{n,n,0}^i = \Gamma_i \frac{1}{\bar{\psi}} \bar{z} \tag{21}$$

$$z_{m,n,0}^i = \Gamma_i \kappa_i \frac{1}{\bar{\psi}} \bar{z} \quad m \neq n \tag{22}$$

It is straight forward to show that given the initial conditions in Assumption 1, when $G = 1$, all period 1 first-order conditions and market clearing conditions are satisfied with the same levels of asset prices, interest rates, wealth, portfolio allocation, and debt in period 1 as in period 0. See the Online Appendix for a formal proof.

**Theorem 1** *Under Assumption 1 and* $G = 1$, *there is an equilibrium where in period 1:* $Q_n = a$, $W_n^i = (1+a)/\bar{z}$, $z_{n,n}^i = z_{n,n,0}^i$ *and* $z_{n,m}^i = z_{n,m,0}^i$. *In all periods* $t \geq 1$: $R_t = (1+a)/a$, $B_t = B_0$. *In all periods* $t \geq 2$: $Q_{n,t} = [a/(1+a)]D_n$, $W_{n,t}^i = W_{n,2}^i$.

Since quantities of asset holdings are also the same in periods 0 and 1, there will be no capital flows in the pre-shock equilibrium in period 1.

## 2.5 Period 1 Capital Flows

After the risk-aversion shock, capital flows are generally no longer zero. Time 1 capital outflows $OF_n^{risky}$ are defined as purchases of foreign risky assets by country $n$ investors, while time 1 capital inflows $IF_n^{risky}$ are purchases of country $n$ risky assets by foreign investors. These are equal to[4]

$$OF_n^{risky} = \beta \int_0^1 \sum_{m \neq n} z_{n,m}^i W_n^i di - \int_0^1 \sum_{m \neq n} Q_m \frac{z_{n,m,0}^i}{\bar{z}} di \tag{23}$$

$$IF_n^{risky} = \beta \int_0^1 \sum_{m \neq n} z_{m,n}^i W_m^i di - Q_n \int_0^1 \sum_{m \neq n} \frac{z_{m,n,0}^i}{\bar{z}} di \tag{24}$$

In a model with symmetric countries net flows must of course be zero. We therefore only consider gross outflows and inflows of the risky assets.

# 3 Impact Global Risk-Aversion Shock on Gross Capital Flows

We now consider the derivative of gross capital flows with respect to $G$ at $G = 1$. In the next section we will consider large changes in $G$. The Online Appendix first derives the derivatives of asset prices with respect to $G$. We have

$$\frac{dQ}{dG} = \frac{\bar{z}(1+a)\frac{\sigma^2}{\bar{\psi}}}{(1+a)^2 - \frac{E\psi^2}{\bar{\psi}^2}\frac{\bar{z}^2\sigma^2}{\bar{\psi}} + \frac{a^2}{\lambda}} \tag{25}$$

$$\frac{dR}{dG} = \frac{1}{\lambda(1+a)}\frac{dQ}{dG} \tag{26}$$

Since risky asset prices from all countries are the same in this symmetric framework, we denote the risky asset price as $Q$, without a country subscript. $E\psi^2$ denotes the mean across investors of $\psi_i^2$. We define $\lambda = -\partial C_1^b/\partial R$. The Online Appendix shows that $\lambda > 0$ since a higher interest rate lowers consumption. We make a very weak assumption in the Online Appendix to make sure that higher risky asset prices lower demand for risky assets. It corresponds to the sum of the first two terms in the denominator of (25) being positive. It follows that a drop in $G$ (rise in global risk-aversion) lowers risky asset prices as well as the interest rate.

We now discuss under what conditions this drop in global risky asset prices leads to a drop in gross inflows and outflows of risky assets. Since all countries in the model are symmetric, from hereon we drop country subscripts.

It is useful to start with the portfolios of investors prior to the risk-aversion shock. Using

[4]The time 0 portfolio shares divided by $\bar{z}$ correspond to time 0 quantities of assets.

(21)-(22), the risky share of investor $i$ is

$$z^i = \alpha_i \bar{z} \tag{27}$$

where

$$\alpha_i = \frac{\psi_i}{\bar{\psi}} = \frac{\Gamma_i(1+N\kappa_i)}{E\left(\Gamma(1+N\kappa)\right)}$$

has a cross-sectional mean of 1. The cross-sectional mean of risky shares is therefore $\bar{z}$. The cross-sectional variance is positive when $var(\alpha) > 0$.

The period 1 wealth of investor $i$ from any country is

$$W^i = \frac{1}{\bar{z}}\left(1 + a + z^i(Q-a)\right) \tag{28}$$

Heterogeneity in risky shares across investors leads to heterogeneity of the wealth impact of the drop in risky asset prices following an increase in risk aversion. The larger the risky share of an investor, the larger the drop in wealth when risky asset prices fall.

This wealth drop in turn affects buying and selling of risky assets. Using the equilibrium portfolios (16)-(17) and the risky asset market clearing condition (19), we can show that the quantities of all risky assets that an investor holds go up or down in proportion to their holdings before the shock. The total quantity of risky assets held by investor $i$ is $\alpha_i$ before the shock and

$$\frac{W^i}{\int_0^1 \alpha_j W^j dj}\alpha_i \tag{29}$$

after the shock. If the wealth of an investor drops relative to the weighted average wealth of all investors, it sells risky assets in equilibrium. Those investors whose wealth drops less than the weighted average will buy risky assets. The weights are determined by the quantity of risky assets that investors hold in their portfolio before the shock.

The change in the quantity of risky assets held by investor $i$ is then $Z^i dQ$, where

$$Z^i = \bar{z}\frac{\alpha_i(\alpha_i - 1 - var(\alpha))}{1+a} \tag{30}$$

Investor $i$ therefore sells risky assets when $\alpha_i > 1 + var(\alpha)$. The reason that $\alpha_i$ needs to be above $1 + var(\alpha)$ rather than the cross-sectional mean of $\alpha$ of 1 is that what matters is the wealth of investor $i$ relative to a weighted average of the wealth of all investors. More weight is given to investors that hold more risky assets. Since the cross-sectional mean of $\alpha_i$ is 1, less than half of investors sell risky assets, while more than half buy risky assets.

The impact of the shock on gross capital flows of risky assets depends on the extent of home bias of investors that are buying and selling risky assets. Specifically, let $z_F^i$ be the pre-shock foreign share of investor $i$, which is the fraction of risky assets allocated to foreign risky assets:

$$z_F^i = \frac{N\kappa_i}{1 + N\kappa_i} \tag{31}$$

A higher $\kappa_i$ implies less home bias and therefore a larger $z_F^i$.

We now make the following assumption.

**Assumption 2** *Assume that $cov(z_F, Z) > 0$.*

We can then derive the following result.

**Theorem 2** *Assume that there is heterogeneity across investors within countries and Assumptions 1 and 2 hold. Then a rise in global risk aversion leads to a drop in gross capital outflows and inflows.*

This result follows from the expressions of gross flows (23)-(24), after substituting the expressions for portfolio shares, wealth (28) and the risky asset market equilibrium condition (19). A formal proof is in the Online Appendix.

Assumption 2 is key to the drop in gross capital flows. It implies that investors with sufficiently large risky shares that sell risky assets ($Z^i > 0$) tend to invest a relatively large share of risky assets abroad (high $z_F^i$). To understand this, we can split investors into two groups: a group with high risky shares that is selling risky assets ($Z^i > 0$) and a group with lower risky shares that is buying risky assets ($Z^i < 0$). Overall the quantity of risky assets sold by the first group is equal to the quantity bought by the second group as $E(Z) = 0$ (the supply of risky assets is unchanged). However, it matters whether investors sell or buy domestic or foreign risky assets. When Assumption 2 holds, the first group is less home biased and therefore sells more foreign risky assets than the second group buys. These foreign risky assets are therefore sold at least partially to foreigners, which reduces gross capital outflows.

The decline in gross capital flows requires that risky shares and home bias vary across investors, and investors with a higher risky share also tend to have a higher foreign share (are less home biased). As we will discuss in Section 4, this is consistent with evidence from Swedish household portfolio data in Calvet, Campbell and Sodini (2007, 2009*a*, and 2009*b*), which will be used to calibrate the model.

It is useful to discuss how Assumption 2 relates to the specific forms of within-country heterogeneity introduced in Section 2, related to the parameters $\Gamma_i$ and $\kappa_i$. Let $\bar{\kappa}$ and $\bar{\Gamma}$ be the cross-sectional mean of $\kappa$ and $\Gamma$. Write $\kappa_i = \bar{\kappa} + \epsilon_i^{\kappa}$ and $\Gamma_i = \bar{\Gamma} + \omega\epsilon_i^{\kappa} + \epsilon_i^{\Gamma}$. Assume that $\epsilon_i^{\kappa}$ and $\epsilon_i^{\Gamma}$ are uncorrelated and have symmetric distributions with mean zero. In the Online Appendix we then show that Assumption 2 is satisfied when $\omega \geq 0$ and $var(\epsilon^{\kappa}) > 0$. This means that we must have cross-sectional variation in home bias. We do not necessarily need cross-sectional variation in risk-aversion. To the extent that we do have cross-sectional variation in risk-aversion, Assumption 2 holds when the covariance between $\Gamma_i$ and $\kappa_i$ is non-negative.

A larger value of $\kappa_i$ leads to both a higher risky share and a higher foreign share. This is exactly what is needed to satisfy Assumption 2. By contrast, variation in risk-aversion only affects the risky share and not the foreign share. By itself it is therefore not sufficient to satisfy Assumption 2.

# 4 Numerical Analysis

We make one change to the model for the numerical analysis. We allow the correlation of period 2 returns of risky assets to be non-zero. We set it at $\nu$. This leads to more complex portfolio expressions as asset prices affect portfolio shares of all other assets. These portfolio expressions are derived in the Online Appendix. For what follows we just mention that the expression of $\psi_i$ changes. It remains the case that the pre-shock risky share is $z_i = \bar{z}\frac{\psi_i}{\bar{\psi}}$, but now $\psi_i = \Gamma_i(1 + N\kappa_i)(1 - \eta_i(1 + N\kappa_i))$, where $\eta_i = \frac{\nu}{1-\nu+\nu(1+N\kappa_i)}$.

## 4.1 Heterogeneity Parameters

Heterogeneity across investors is key to the model. Before discussing the calibration of the other parameters, we therefore first consider the heterogeneity parameters $\Gamma_i$ and $\kappa_i$.

To calibrate the investor heterogeneity parameters, we rely on the Calvet, Campbell and Sodini (2007, 2009*a*, and 2009*b*) papers mentioned earlier. These use Swedish administrative data from 1999 to 2002 on wealth, portfolio shares, and portfolio returns of individual households, and therefore provide a unique look at investor heterogeneity in the desire to hold risky assets and diversified portfolios. The Online Appendix discusses in detail the calibration of $\Gamma_i$ and $\kappa_i$ from these Swedish data. Here we provide a brief summary.

Calvet, Campbell and Sodini (2009*a*) provides information about the distribution of risky shares $z_i$, which they define as the share of the investor's wealth not in bank accounts or money market mutual funds. The distribution is centered around 0.5. It is left-skewed during a boom in risky asset prices and right-skewed during during a bust. But on average the risky shares are close to uniformly distributed between 0 and 1. We also use this to set $\bar{z} = 0.5$.

The foreign share $z_i^F$ is not directly available from the Swedish administrative data. But it can be computed indirectly from information about the distribution of Sharpe ratios of the risky asset portfolios in Calvet, Campbell and Sodini (2007). Of course, lack of international diversification is not the only reason for a low Sharpe ratio. Holding a non-diversified domestic portfolio will also reduce the Sharpe ratio. But Calvet, Campbell and Sodini (2007) show that around two-thirds of Swedish households have a Sharpe ratio that exceeds that for the domestic Swedish index. From this we conclude that the majority of investors hold a diversified domestic portfolio and heterogeneity in international diversification drives the heterogeneity in Sharpe ratios.

Calvet, Campbell and Sodini (2007) report the distribution of the Sharpe ratio loss, which tells us for an individual investor how much lower the Sharpe ratio is in comparison to a diversified international benchmark portfolio. In the Online Appendix we derive the Sharpe ratio loss for individual investors in the model. There is a mapping in the model between the Sharpe ratio loss and the foreign share $z_i^F$. The larger the Sharpe ratio loss, the less diversified the investor is, so that $z_i^F$ is lower.

While these results provide distributions of $z_i$ and $z_i^F$, we also need to know how they are correlated. Calvet, Campbell and Sodini (2009*b*) show that the cross-sectional correlation between risky shares and Sharpe ratio losses is -0.49. In other words, investors that hold a higher risky share have a smaller Sharpe ratio loss relative to the international benchmark and are therefore more diversified. From the mapping between the Sharpe ratio loss and $z_i^F$ discussed above, we use this to calibrate the correlation between $z_i$ and $z_i^F$ across investors. The mapping between the Sharpe ratio loss and $z_i^F$ is not linear, so calibrating the model to match the -0.49 correlation between $z_i$ and the relative Sharpe ratio loss results in a correlation of about 0.44 between $z_i$ and $z_i^F$.

We use this calibration strategy to generate 100,000 pairs $(z_i, z_i^F)$. We can map these to $\kappa_i$ and $\Gamma_i$ as follows. The risky share is $z_i^F = N\kappa_i/(1 + N\kappa_i)$. This allows us to compute the $\kappa_i$ from the $z_i^F$. We have $z_i = \bar{z}\frac{\psi_i}{\bar{\psi}}$ where $\psi_i = \Gamma_i(1 + N\kappa_i)(1 - \eta_i(1 + N\kappa_i))$, $\bar{\psi}$ is mean of $\psi_i$, and $\eta_i = \frac{\nu}{1-\nu+\nu(1+N\kappa_i)}$. The calibration of the correlation $\nu$ between risky asset returns is discussed below. Together with $z_i$ and the calibrated $\kappa_i$, we can use this to back out $\Gamma_i/\bar{\Gamma}$. As discussed below, the cross-sectional mean of $\Gamma_i$, $\bar{\Gamma}$, plays no role for the results and we set it rather arbitrarily at 0.1.

## 4.2 Other Parameters

The number of countries $N + 1$ is set at 20. For the theory the number of countries doesn't matter, as long as it is at least two. However, the number of countries affects portfolio diversification and therefore affects the calibration of $\kappa_i$ and $\Gamma_i$, as discussed above. We set the number of countries in the model at 20 because later, when evaluating the quantitative predictions of the model, we compare the model results to those from a simple panel data regression with 20 advanced countries.[5]

[5]The countries included are the United States, Singapore, Australia, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Iceland, Israel, Italy, Japan, Korea, the Netherlands, Norway, Portugal and Sweden.

We set $a = \frac{\beta}{1-\beta} = 25$, implying a 4 percent pre-shock interest rate. We set the borrower intertemporal elasticity of substitution $\rho = 0.5$, in line with the evidence from Beaudry and van Wincoop (1996). We set the period one borrower endowment $Y = 3$. This combined with a dividend of 1 means that GDP is $Y + 1 = 4$. If we interpret the dividend as capital income, then capital income is one-quarter of GDP.

We set the cross-country risky asset return correlation $\nu$ based on data reported in Calvet, Campbell and Sodini (2007). They report a Sharpe ratio for the international benchmark portfolio and the benchmark Swedish portfolio of respectively 45.2 and 27.4. These correspond to respectively $\kappa_i = 1$ and $\kappa_i = 0$ in the model. In the Online Appendix we use this to back out $\nu = 0.33$ based on the derivation of the Sharpe ratio loss. This value of the correlation is also plausible through direct observation of equity return correlations. For example, Quinn and Voth (2008) compute a century of global equity market correlations. Using correlations among 120 country pairs of monthly equity returns for non-overlapping 4-year intervals, they find an average correlation of 0.33.

We cannot calibrate both $\sigma^2$ and $\bar{\Gamma}$, only their ratio. The portfolio expressions depend on $\Gamma_i/\sigma^2$. As discussed, data from the Calvet et al. papers allow us to calibrate $\Gamma_i/\bar{\Gamma}$. Given these values, the portfolio expressions depend on $\sigma^2/\bar{\Gamma}$. The pre-shock premium on risky assets is $\bar{z}\sigma^2/(a^2\bar{\psi})$, which depends on $\sigma^2/\bar{\Gamma}$. We set it such that the risk premium is 5.8 percent. This is equal to the average difference over the period 1996-2018 between the gross annual return of the MSCI World Index and the 1 year Treasury bill rate. We set $\bar{\Gamma} = 0.1$ and $\sigma = 2.7$, but any other values with the same ratio of $\sigma^2/\bar{\Gamma}$ lead to the same results. While the risk-premium affects the change in risky asset prices and gross capital flows in the model in response to a given risk-aversion shock, it does not affect their ratio, which will be the focus of the analysis below.

The parameters $\kappa_i$ affect the financial openness of countries. It is useful to check the financial openness of countries implied by the calibrated parameters against an untargeted measure of financial openness. We can calculate the ratio of gross external non-bank assets and liabilities to GDP (non-bank external asset and liabilities include FDI, portfolio equity, and portfolio debt). For the 20 advanced countries mentioned earlier, the average ratio over the 1996-2018 sample is 3.16. This is close to the model, where the ratio is 3.0.

## 4.3 Results

In this subsection we report the response of gross flows following a shock to $G$. To evaluate the model's results we first need to establish an empirical benchmark. We will look at the change in gross flows relative to the change in risky asset prices following a global shock. A good candidate for that shock is the Global Financial Cycle Factor from Miranda-Agrippino and Rey (2020) (henceforth MAR).

In a panel data regression with annual data from 20 advanced economies over the years 1996-2018, we regress a measure of risky asset prices or gross capital flows on the global financial cycle factor. Specifically, we consider the following panel regression:

$$\Delta y_{n,t} = \alpha_n + \gamma \Delta y_{n,t-1} + \beta \Delta F_t + \varepsilon_{n,t} \tag{32}$$

where the dependent variable $\Delta y_{n,t}$ is year-over-year log change in the equity price index or the year-over-year change in gross non-bank capital inflows and outflows as a share of GDP. The equity price index is taken from national stock market data from the OECD, and non-bank capital inflows and outflows are the sum of FDI, portfolio equity, and portfolio debt inflows and outflows from the IMF's International Financial Statistics Database. The regressors are a country-fixed effect, the lag of the dependent variable and $\Delta F_t$, the year-over-year change in the global financial cycle factor.[6]

The only coefficient we are interested in is $\beta$, the coefficient on the global financial cycle factor. In the regression for the equity price index we find a coefficient of 18.8 (t-stat 12.3). In the regression for gross capital flows we find a coefficient of 5.3 (t-stat 2.7). A global shock that leads to a 10% fall in risky asset prices then leads to a $10 \times 5.3/18.8 = 2.8$ percentage point fall in gross non-bank capital inflows and outflows as a share of GDP. This is our empirical benchmark.

Next we simulate the model with a shock to $G$ large enough to cause a 10% fall in the risky asset price. We find that in our baseline calibration, gross capital inflows and outflows as a share of GDP fall by 2.7%. This closely matches the empirical counterpart of 2.8%.

Figure 1 shows how the drop in gross capital flows in the model depends on the correlation between the risky and foreign shares across investors. In the calibration this correlation is 0.44. Figure 1 illustrates how the decline in gross capital flows depends critically on a positive correlation across investors of their risky and foreign shares. The decline in gross capital flows is zero when this correlation is zero and depends almost linearly on the correlation between $z_i$ and $z_i^F$.

In Figure 2 we group investors into 40 bins based on their pre-shock risky shares. The top panel shows purchases of all risky assets, both home and foreign, as a percent of GDP. Without any investor heterogeneity, nobody would buy or sell risky assets. Lower demand for risky assets due to higher risk-aversion and lower wealth is exactly offset by higher demand due to lower asset prices (higher expected returns). But we saw in Section 3 that with portfolio heterogeneity, in equilibrium investors with a high risky share, whose wealth drops the most, will sell risky assets, while those with a smaller risky share will buy risky assets. This is illustrated in the top panel of Figure 2. The integral of the risky asset

[6]The monthly factor from MAR has been converted to annual by simply averaging over the months in a calendar year.

purchase schedule is 0. Risky asset supplies are unchanged, so that purchases of risky assets (investors with low risky shares) are equal to sales of risky assets (investors with high risky shares).

This top panel of Figure 2 is very similar to Figure III.D of Calvet, Campbell and Sodini (2009*a*), which shows the closely related change in the risky share due to active buying or selling of risky assets in 2002, a year during which the MSCI Sweden index fell by 48.6%. In those Swedish data the cutoff for the risky share above which investors sell risky assets is about 65 percent. This is very similar to the model in the top panel of Figure 2.

The bottom panel of Figure 2 reports purchases of foreign risky assets. These results are shown both for the calibrated model, where the correlation between the risky and foreign shares is 0.44, and when this correlation is zero. First consider the case of a zero correlation. With a large number of investors in each bin, purchases of foreign risky assets is then proportional to purchases of all risky assets in the top panel of Figure 2. Analogous to the top panel, purchases of foreign risky assets (investors with low risky shares) must then be equal to sales of foreign risky assets (investors with high risky shares). Therefore investors with high risky shares simply sell foreign risky assets to investors with low risky shares in the same country, so that there are no gross capital flows.

We can see that when the correlation between the risky and foreign shares is 0.44, as calibrated from the Swedish data, purchases of foreign risky assets as a function of the risky share (bottom panel) are no longer proportional to purchases of all risky assets as a function of the risky share (top panel). Investors with a high risky share now tend to have a high foreign share as well, which means that they sell a lot of foreign risky assets. Investors with a low risky share now tend to have a low foreign share, so that they mostly buy home risky assets and not a lot of foreign risky assets. The integral of the foreign risky asset purchase schedule is therefore negative. Overall the country is selling foreign risky assets. They sell these assets to foreigners, so that capital outflows decline.

# 5 Conclusion

We construct a model where, consistent with lots of empirical evidence, there is a retrenchment in gross capital flows in response to a negative global shock. The model relies on within-country portfolio heterogeneity with respect to risky and foreign shares. There are no cross-country asymmetries. Countries are ex-ante and ex-post identical, with the same portfolio distribution across investors.

The portfolio heterogeneity within countries leads to an endogenous shift in relative wealth from internationally diversified agents to home biased agents following the negative global shock that lowers global asset prices. This endogenous shift in the wealth distribution within countries raises the wealth-weighted home bias, which generates an aggregate retrenchment.

When we calibrate the model to household portfolio data, we find that it can account for the observed decline in gross non-bank capital flows relative to the decline in global asset prices following a negative global shock.

# References

**Acalin, Julien.** 2023. "The Global Financial Cycle Meets Global Imbalances." *mimeo.*

**Beaudry, Paul, and Eric van Wincoop.** 1996. "The intertemporal elasticity of substitution: An exploration using a U.S. panel of state data." *Economica*, 495–512.

**Brennan, Michael J, and H Henry Cao.** 1997. "International portfolio investment flows." *Journal of Finance*, 52(5): 1851–1880.

**Broner, Fernando, Tatiana Didier, Aitor Erce, and Sergio L Schmukler.** 2013. "Gross capital flows: Dynamics and crises." *Journal of Monetary Economics*, 60(1): 113–133.

**Caballero, Ricardo J, and Alp Simsek.** 2020. "A model of fickle capital flows and retrenchment." *Journal of Political Economy*, 128(6): 2288–2328.

**Calvet, Laurent E, John Y Campbell, and Paolo Sodini.** 2007. "Down or out: Assessing the welfare costs of household investment mistakes." *Journal of Political Economy*, 115(5): 707–747.

**Calvet, Laurent E, John Y Campbell, and Paolo Sodini.** 2009*a*. "Fight or flight? Portfolio rebalancing by individual investors." *The Quarterly journal of economics*, 124(1): 301–348.

**Calvet, Laurent E, John Y Campbell, and Paolo Sodini.** 2009*b*. "Measuring the financial sophistication of households." *American Economic Review*, 99(2): 393–98.

**Davis, J. Scott, and Eric van Wincoop.** 2018. "Globalization and the increasing correlation between capital inflows and outflows." *Journal of Monetary Economics*, 100: 83–100.

**Davis, J. Scott, Giorgio Valente, and Eric van Wincoop.** 2021. "Global drivers of gross and net capital flows." *Journal of International Economics*, 128: 103397.

**Forbes, Kristin, and Francis E. Warnock.** 2012. "Capital flow waves: Surges, stops, flight, and retrenchment." *Journal of International Economics*, 88: 235–251.

**Gourio, Francois, Michael Siemer, and Adrien Verdelhan.** 2015. "Uncertainty and international capital flows." *Working Paper, MIT Sloan.*

**Milesi-Ferretti, Gian Maria, and Cedric Tille.** 2011. "The Great Retrenchment: International Capital Flows During the Global Financial Crisis." *Economic Policy*, 26(66): 285–342.

**Miranda-Agrippino, Silvia, and Hélene Rey.** 2020. "US monetary policy and the global financial cycle." *The Review of Economic Studies*, 87(6): 2754–2776.

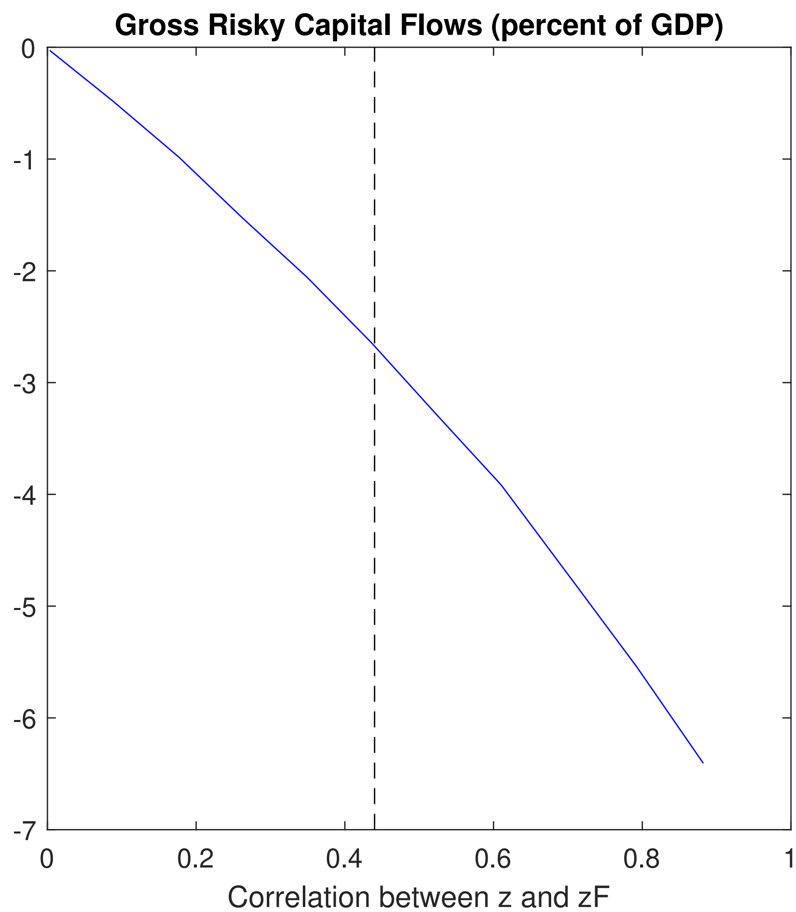
**Miranda-Agrippino, Silvia, and Hélene Rey.** 2022. "The Global Financial Cycle." In *Handbook of International Economics*. Vol. 6, , ed. Gita Gopinath, Ethan Helpman and Kenneth Rogoff, 1–43. Elsevier Science B.V.

**Quinn, Dennis P, and Hans-Joachim Voth.** 2008. “A century of global equity market correlations.” *American Economic Review Papers and Proceedings*, 98(2): 535–40.

**Tille, Cedric, and Eric van Wincoop.** 2010. “International capital flows.” *Journal of International Economics*, 80(2): 157–175.

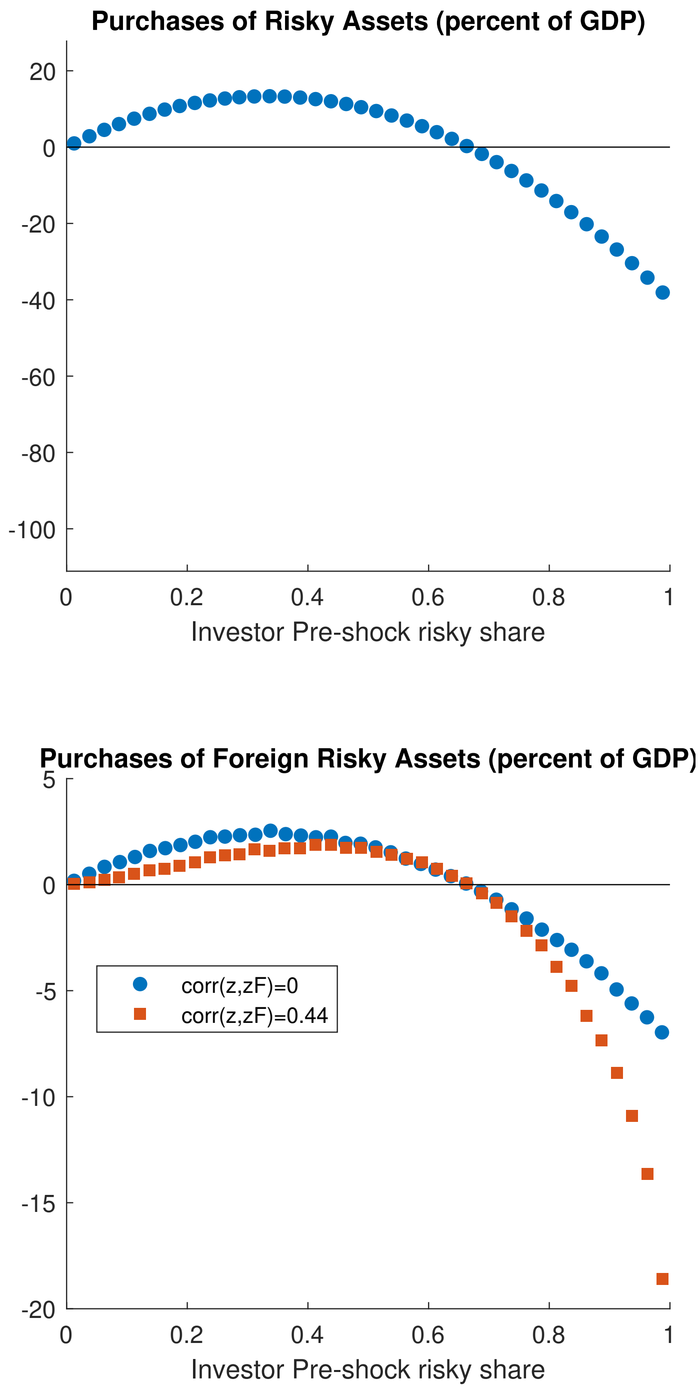
**Tille, Cedric, and Eric van Wincoop.** 2014. “International capital flows under dispersed private information.” *Journal of International Economics*, 93(1): 31–49.

Figure 1: Gross Flows (Outflows plus Inflows) Risky Assets after 10% Fall Risky Asset Prices




Notes: The horizontal axis shows the correlation between the risky and foreign shares, which is varied in the model by changing the correlation between the risky share and the Sharpe ratio loss that is used in the calibration. The vertical dashed line marks the correlation between the risky share and the foreign share of 0.44 in the calibrated version of the model, which corresponds to a correlation between the risky share and the Sharpe ratio loss of -0.49.

Figure 2: Purchases of Risky and Foreign Risky Assets after 10% Fall Risky Asset Prices




Notes: The 100,000 investors in a country are sorted by pre-shock risky shares and grouped into 40 bins of 2500 investors. Each dot represents the sum of purchases of risky assets or purchases of foreign risky assets across the 2500 investors in that bin. The bottom chart reports results both for the calibrated model, where the correlation between the risky and foreign share is 0.43, and the case where this correlation is zero.