Federal Reserve Bank of Dallas

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Globalization Institute Working Paper 423 **September 2023**

Research Department

https://doi.org/10.24149/gwp423

Mean Group Distributed Lag Estimation of Impulse Response Functions in Large Panels*

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September 17, 2023

Abstract

This paper develops Mean Group Distributed Lag (MGDL) estimation of impulse responses in large panels with one or two cross-section dimensions. Sufficient conditions for asymptotic consistency and asymptotic normality are derived, and satisfactory small sample performance is documented using Monte Carlo experiments. MGDL estimators are used to estimate the effects of crude oil price increases on US city- and product-level retail prices.

Keywords: Large panels, impulse response functions, estimation and inference, mean group distributed lag approach (MGDL)

JEL Classification: C23

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*The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

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1 Introduction

Estimation of impulse-response functions (IRFs) has garnered growing interest in the recent literature. In the specific case when the shock of interest is assumed to be observed, it is now widely recognized that there exist numerous auxiliary regression specifications within a time series framework that can be employed for estimating the impulse response function, either directly or through iterative methods. These include (i) the local projection regressions popularized by Jordà (2005), (ii) the distributed lag approach (e.g., Kimball et al. (2006), Kilian (2008a, 2009), Romer and Romer (2010), and Baumeister and Kilian (2014)), and (iii) iterative approaches utilizing ARDL regression specifications (e.g., Anzuini et al. (2013), Bachmeier and Cha (2011), Coibion (2012), Kimball et al. (2006), Romer and Romer (2004, 2010) or Kilian (2008a, 2008b), among others) or the multivariate VAR or VARX specifications. Relative merits of these approaches in finite samples of interest were investigated by Choi and Chudik (2019). Extending these specifications to panel data setting is not straightforward, in part due to the time series bias (popularized by Nickell (1981) in a dynamic panel data context) as well as cross-sectional dependence.

This paper considers large panels with possibly two cross-section dimensions (motivated by our application), labeled as M and N, and a time dimension, T. It is assumed that the shock of interest is observed, common, and the impulse-responses follow a random coefficient specification. We derive sufficient conditions for consistency and asymptotic normality of the Mean Group (MG) estimator based on distributed lag specifications (or MGDL for short). Consistency result is obtained as $(M, N, T) \to \infty$ jointly without any restrictions on the relative rates N/T and M/T. In contrast, our asymptotic normality result requires $N/T \to 0$ and $M/T \to 0$ as $(M, N, T) \to \infty$ jointly. No restrictions on cross-section dependence are imposed. Similar results are obtained for conventional panels with single cross-section dimension.

Monte Carlo experiments show satisfactory finite sample performance for the selected sample sizes of interest, $M = N \in \{30, 40, 50, 100\}$, and $T \geq 50$. Estimation of the cumulative sums of the impulse response function (also referred to as cumulative multipliers) is also investigated, including the possibility of direct estimation based on regressions with cumulated variables, $\xi_{ijt} = x_{ijt} + \xi_{i,j,t-1}$. Our results suggest cumulative multipliers are more reliably estimated by cumulating estimated impulse response estimates, as opposed to using regressions featuring cumulated variables.

MGDL approach is then applied to estimate the effects of crude oil price shocks, utilizing a quarterly retail price dataset at the city and product levels, provided by the Council for Community and Economic Research (C2ER, https://www.c2er.org/redt/). Generic crude oil price increases are associated with significant increase in retail gasoline prices. The estimated pass-through is fast and complete, in line with the existing crude oil pass-through studies. Oil price shocks are associated with significant effects on other product categories as well. This interesting finding may result from the observation that fluctuations in crude oil prices are influenced by demand shocks that have a broader impact on both economic activity and prices.

The remainder of the paper is organized as follows. Section 2 introduces the model, proposes the MGDL estimators, provides asymptotic results, and discusses potential extensions. Section 3 reports on finite sample evidence. Section 4 presents empirical application to US city- and product-level prices. Section 5 concludes the paper. Mathematical derivations and proofs, and additional estimation results are provided in an appendix.

A few words on the notations. Throughout the paper, K and $K_0, K_1, ...$ indicate finite generic positive constants that do not depend on the sample size (M, N, T) nor on the subscripts (i, j, t). These constants could take different values at different instances in the paper. The symbols '$\rightarrow_p$' and '$\rightarrow_d$' respectively denote the convergence in probability and distribution. '$\rightarrow_j$' denotes joint convergence. All vectors are column vectors, represented by bold lower case letters. Matrices are bold upper case letters. $\|\mathbf{A}\| = \sqrt{\varrho\left(\mathbf{A}'\mathbf{A}\right)}$ is the spectral norm of matrix $\mathbf{A}$,[1] $\varrho\left(\mathbf{A}\right) \equiv \max_{1\leq i\leq n} \left\{\left|\lambda_i\left(\mathbf{A}\right)\right|\right\}$ is the spectral radius of $\mathbf{A}$, and $|\lambda_1(\mathbf{A})| \geq |\lambda_2(\mathbf{A})| \geq ... \geq |\lambda_n(\mathbf{A})|$ are the eigenvalues of $\mathbf{A}$.

2 MGDL estimator

We consider a panel data with two cross-section dimensions (M, N) and a time dimension (T). Let x_{ijt} be a variable for the cross-section unit (i, j) in period t, observed for $i = 1, 2, ..., M$, $j = 1, 2, ..., N$, and $t = 1, 2, ..., T$. In the application developed in Section 4, the index i refers to individual price categories and the index j refers to geographic locations. For future reference, we define the $M \times 1$ vectors $\mathbf{x}_{iot} = (x_{i1t}, x_{i2t}, ..., x_{iMt})'$ and the $NM \times 1$ vector collecting all units,

[1] Note that if $\mathbf{x}$ is a vector, then $\|\mathbf{x}\| = \sqrt{\varrho\left(\mathbf{x}'\mathbf{x}\right)} = \sqrt{\mathbf{x}'\mathbf{x}}$ corresponds to the Euclidean length of vector $\mathbf{x}$.

$\mathbf{x}_t = (\mathbf{x}'_{1\circ t}, \mathbf{x}'_{2\circ t}, ..., \mathbf{x}'_{N\circ t})'$.

Let v_t be a common shock observed for $t = 1, 2...., T$. We assume x_{ijt} can be decomposed as

$$x_{ijt} = a_{ij} + \sum_{\ell=0}^{\infty} b_{ij\ell} v_{t-\ell} + z_{ijt}, \tag{1}$$

and we make the assumptions below on the fixed effects (a_{ij}), the impulse-response coefficients $(b_{ij\ell})$, the shock (v_t), and the process (z_{ijt}). Let h be a selected maximum horizon of interest that does not depend on the sample size (N, M, T). We collect $b_{ij\ell}$ for $\ell = 0, 1, ..., h$ into an $(h+1) \times 1$ vector $\mathbf{b}_{ij} = (b_{ij0}, b_{ij1}, ..., b_{ijh})'$. The dependence of the dimension of $\mathbf{b}_{ij}$ on h is suppressed to simplify the notations.

ASSUMPTION 1 *(Random coefficient assumption)* $\mathbf{b}_{ij} = \mathbf{b}_i + \mathbf{c}_j + \boldsymbol{\omega}_{ij}$, *where* $\mathbf{b}_i$ *and* $\mathbf{c}_j$ *are non-random and* $\boldsymbol{\omega}_{ij} \sim IID\left(\mathbf{0}, \boldsymbol{\Omega}_{ij}\right)$. *Furthermore,* $\sum_{j=1}^{M} \mathbf{c}_j = \mathbf{0}$, *and there exist constants* K, K_0, K_1 *and* $0 \leq \rho < 1$ *such that* $\|\mathbf{b}_i\| < K$, $\|\mathbf{c}_j\| < K$, $K_0 < \|\boldsymbol{\Omega}_{ij}\| < K_1$, *and* $E\left(b_{ij\ell}^2\right) < \rho^{\ell} K$ *for all* $\ell > h$.

ASSUMPTION 2 *(Component z_{ijt}) There exist constants K and $0 \leq \rho < 1$, such that $E\left|z_{ijt} z_{ijt'}\right| < K\rho^{|t-t'|}$ for all i, j, t, t'.*

ASSUMPTION 3 *(Shock v_t) v_t is independent of $v_{t'}$ and $z_{ijt'}$ for any i, j and $t \neq t'$. In addition, there exist constants K_0, K_1 such that $K_0 < E\left(v_t^4\right) < K_1$.*

The object of interest is the estimation of parameter vectors $\mathbf{b}_i$ and $\mathbf{c}_j$. Assumptions 1-3 are rather general. First, the commonly used stationary VAR representations for the $NM \times 1$ vector $\mathbf{x}_t$ would satisfy our assumptions. Hence $\mathbf{x}_t$ could be given by a high-dimensional VAR model. $\mathbf{x}_t$ can alternatively be represented by a high-dimensional MA(∞) process that falls outside the VAR representations. In addition, unobserved common shocks or factors are allowed. In fact the cross-section dependence is left unrestricted in Assumption 2, and z_{ijt} can be expected to be strongly cross-sectionally correlated. Our assumptions are also compatible with heteroskedasticity in all dimensions (including the time), and the requirement $E\left|z_{ijt} z_{ijt'}\right| < K\rho^{|t-t'|}$ can also accommodate the possibility of certain breaks.

One limitation of this setup is that it assumes v_t to be observed, thus significantly constraining the applicability of this approach. In addition, Assumptions 1-3 rule out I(1) processes, but our approach can be extended to integrated processes of order 1. This extension is discussed below in Section 2.2 and in the Monte Carlo section.

This paper considers a mean group approach built on the unit-specific DL regressions given by

$$x_{ijt} = a_{ij} + \sum_{\ell=0}^{h} b_{ij\ell} v_{t-\ell} + e_{hijt}, \tag{2}$$

where $e_{hijt} = \sum_{\ell=h+1}^{\infty} b_{ij\ell} v_{t-\ell} + z_{ijt}$. Denote the corresponding LS estimates of unknown parameters $b_{ij\ell}$ as $\hat{b}_{ij\ell}$, which we collect in an $(h+1) \times 1$ vector $\hat{\mathbf{b}}_{ij} = \left(\hat{b}_{ij0}, \hat{b}_{ij1}, ..., \hat{b}_{ijh}\right)'$.

Mean Group Distributed Lag (MGDL) estimators of $\mathbf{b}_i$ and $\mathbf{c}_j$, respectively, are given by

$$\hat{\mathbf{b}}_i = N^{-1} \sum_{j=1}^{N} \hat{\mathbf{b}}_{ij}, \text{ for } i = 1, 2, ..., M, \tag{3}$$

and

$$\hat{\mathbf{c}}_j = M^{-1} \sum_{i=1}^{M} \left(\hat{\mathbf{b}}_{ij} - \hat{\mathbf{b}}_i\right), \text{ for } j = 1, 2, ..., N. \tag{4}$$

Inference can be conducted using the non-parametric variance estimators,

$$Var\left(\hat{\mathbf{b}}_i\right) = \frac{1}{N(N-1)} \sum_{j=1}^{N} \hat{\boldsymbol{\omega}}_{ij} \hat{\boldsymbol{\omega}}'_{ij}, \tag{5}$$

and

$$Var\left(\hat{\mathbf{c}}_j\right) = \frac{1}{M(M-1)} \sum_{i=1}^{M} \hat{\boldsymbol{\omega}}_{ij} \hat{\boldsymbol{\omega}}'_{ij}, \tag{6}$$

where $\hat{\boldsymbol{\omega}}_{ij} = \hat{\mathbf{b}}_{ij} - \hat{\mathbf{b}}_i - \hat{\mathbf{c}}_j$. The following theorem establishes sufficient conditions for consistency and asymptotic normality of the MGDL estimators.

Theorem 1 *(Consistency) Let x_{ijt} be generated by (1) and Assumptions 1-3 hold. Consider $\hat{\mathbf{b}}_i$ and $\hat{\mathbf{c}}_j$ given by (3) and (4), respectively. Let $h \geq 0$ be a fixed integer that does not depend on the sample size (M, N, T), and suppose $M, N, T \to_j \infty$ without any restrictions on M/T and N/T.*

Then,

$$\hat{\mathbf{b}}_i \to_p \mathbf{b}_i \text{ and } \hat{\mathbf{c}}_j \to_p \mathbf{c}_j. \tag{7}$$

If $M, N, T \to_j \infty$ *such that* $M/T \to 0$ *and* $N/T \to 0$*, then we have*

$$\sqrt{N}\left(\hat{\mathbf{b}}_i - \mathbf{b}_i\right) \to_d N\left(0, \bar{\mathbf{\Omega}}_{i\circ}\right), \text{ and } \sqrt{M}\left(\hat{\mathbf{c}}_j - \mathbf{c}_j\right) \to_d N\left(0, \bar{\mathbf{\Omega}}_{\circ j}\right), \tag{8}$$

where $\bar{\mathbf{\Omega}}_{i\circ} = N^{-1}\sum_{j=1}^{N}\mathbf{\Omega}_{ij}$*, and* $\bar{\mathbf{\Omega}}_{\circ j} = M^{-1}\sum_{i=1}^{M}\mathbf{\Omega}_{ij}$*.*

Proofs are presented in Appendix.

The MGDL estimator is consistent regardless of the relative rate of expansion of (M, N, T). In contrast, for asymptotic distribution to depend only on $\mathbf{\Omega}_{ij}$, we require $N/T \to 0$ (for $\hat{\mathbf{b}}_i$) and $M/T \to 0$ (for $\hat{\mathbf{c}}_j$). According to the Monte Carlo evidence in Section 3, however, the finite sample performance is satisfactory in samples of interest when (N, M) is non-negligible relative to T.

2.1 MGDL estimator in large panels with a single cross section dimension

Suppose $M = 1$. Hence, we have data (dropping subscript $i = 1$) on x_{jt}, generated according to

$$x_{jt} = a_j + \sum_{\ell=0}^{\infty} b_{j\ell} v_{t-\ell} + z_{jt}. \tag{9}$$

In this setting, we replace the random coefficient Assumption 1 with the following assumption on the elements of $\mathbf{b}_j = (b_{j0}, b_{j1}, ..., b_{jh})'$.

ASSUMPTION 4 *(Random coefficient assumption for a single cross-section dimension)* $\mathbf{b}_j = \mathbf{b} + \boldsymbol{\omega}_j$*, where* $\boldsymbol{\omega}_j \sim IID\left(\mathbf{0}, \mathbf{\Omega}_j\right)$*. Furthermore, there exist constants* K_0, K_1 *and* $0 \leq \rho < 1$ *such that* $\|\mathbf{b}\| < K$*,* $K_0 < \|\mathbf{\Omega}_j\| < K_1$*, and* $E\left(b_{j\ell}^2\right) < \rho^\ell K$ *for all* $\ell > h$*.*

The following proposition establishes asymptotic normality of the MGDL estimator. The asymptotic analysis is similar in the case of single-cross-section panels.

Proposition 1 *Let* $M = 1$*, and (dropping the subscript* $i = 1$*) suppose* x_{jt} *is generated by (9) and* *let Assumptions 2-4 hold. Consider the MGDL estimator*

$$\hat{\mathbf{b}} = N^{-1} \sum_{j=1}^{N} \hat{\mathbf{b}}_j, \tag{10}$$

where $\hat{\mathbf{b}}_j$ *is the unit-specific LS estimator of* $\mathbf{b}_j$ *using the regression,*

$$x_{jt} = a_j + \sum_{\ell=0}^{h} b_{j\ell} v_{t-\ell} + e_{hjt}. \tag{11}$$

Let $h \geq 0$ *be a fixed integer that does not depend on the sample size* (N, T)*, and suppose* $N, T \rightarrow_j \infty$ *such that* $N/T \rightarrow 0$*. Then,*

$$\sqrt{N} \left(\hat{\mathbf{b}} - \mathbf{b} \right) \rightarrow_d N \left(0, \bar{\mathbf{\Omega}} \right), \tag{12}$$

where $\bar{\mathbf{\Omega}} = N^{-1} \sum_{j=1}^{N} \mathbf{\Omega}_j$*.*

2.2 Extensions

We have considered mean group estimation based on the DL regressions (2). Other unit-specific estimation approaches could be utilized in place of the DL regression specification. One possibility is to augment specification (2) with appropriately lagged dependent variable,

$$x_{ijt} = a_{ij} + \sum_{\ell=0}^{h} b_{ij\ell} v_{t-\ell} + \varphi x_{i,j,t-h-1} + e_{hit}. \tag{13}$$

In the case of I(1) variables, such augmentation will be necessary. We refer to regressions (13) as augmented DL (or ADL for short), and the resulting mean group estimator is referred to as MGADL. Additional lags of the dependent variable, or lags of other covariates could also be included. There are also several other approaches that could be considered as alternatives to DL or ADL. These include the local projection approach popularized by Jordà (2005), or the iterative ARDL, VARDL, or VARX* approaches. See Choi and Chudik (2019) for a discussion of the strengths and weaknesses of these approaches.

Another important consideration is the object of interest. In some empirical applications, the primary focus can be on the mean cumulative response,

$$\delta_{i,h} = \sum_{\ell=0}^{h} b_{i\ell},$$

where $\delta_{i,h}$ is also commonly referred to as the cumulative multiplier. These multipliers can be estimated by cumulating estimates of $b_{i\ell}$, namely

$$\hat{\delta}_{i,h} = \sum_{\ell=0}^{h} \hat{b}_{i\ell}.$$

Alternatively, these multipliers can be estimated directly using the ADL regressions applied to the $I(1)$ variables generated by cumulating x_{ijt},

$$\xi_{ijt} = x_{ijt} + \xi_{i,j,t-1},$$

with the initial value ξ_{ij0} set to zero, without any loss of generality. Direct MGADL estimator of $\hat{\delta}_{i,h}$ can be obtained using the following specification.

$$\xi_{ijt} = a_{ij} + \sum_{\ell=0}^{h} \delta_{ij\ell} v_{t-\ell} + \varphi \xi_{i,j,t-h-1} + \theta \Delta \xi_{i,j,t-h-1} + e_{hit}, \quad (14)$$

which is similar to (13) but utilizes the I(1) variable ξ_{ijt} in place of the stationary first differences. We have effectively included two lags of the dependent variable in (14) to facilitate the exposition in the MC section below, where we compare the direct and indirect MGADL estimators of $\delta_{i,h}$, assuming the random coefficient specification $\delta_{ij\ell} = \delta_{i,h} + c_{j,h}^{\delta} + \omega_{ij\ell}^{\delta}$ implied by Assumption 1.

3 Monte Carlo Evidence

This section investigates the small sample performance of the MGDL estimators of impulse-response coefficients $\mathbf{b}_i = (b_{i0}, b_{i1}, ..., b_{ih})'$ and the cumulative multipliers $\delta_{i,h} = \sum_{\ell=0}^{h} b_{i\ell}$. We set the horizon $h = 8$ matching the horizon selected in the empirical application below. We consider the MGDL estimator $\hat{\mathbf{b}}_i$ given by (3), which is based on the DL regressions (2). Additionally, we consider the benefits of augmenting regressions in (2) by $x_{i,j,t-h-1}$, and the three direct options for the estimation of $\delta_{i,h}$ as discussed in Section 2.2. Subsections 3.1-3.3 below respectively provide the Monte Carlo simulation design, the description of adopted estimators, and statistics of interest. The last subsection presents a summary of the Monte Carlo findings.

3.1 Data Generating Process

We generate x_{ijt} based on (1), namely

$$x_{ijt} = a_{ij} + \sum_{\ell=0}^{\infty} b_{ij\ell} v_{t-\ell} + z_{ijt}, \quad \text{for} i = 1, 2, ..., M, \ j = 1, 2, ..., N, \text{ and } t = 1, 2, ..., T, \tag{15}$$

where the shock v_t is generated as $v_t \sim N(0, 1)$ and the fixed effects are generated as $a_{ij} \sim N(1, 1)$. z_{ijt} is generated to be persistent as

$$z_{ijt} = \rho_{ij} z_{ij,t-1} + \sqrt{1 - \rho_{ij}^2} e_{ijt},$$

for $t = -B+1, -B+2, ..., 0, 1, 2, ..., T$, in which $B = 100$, $\rho_{ij} \sim U[0.3, \rho_{\max}]$ with two choices for $\rho_{\max} = 0.5$ and $\rho_{\max} = 0.9$. We generate e_{ijt} according to

$$e_{ijt} = \gamma_{ij} f_t + \varepsilon_{ijt},$$

where $f_t \sim N\left(0, 0.1^2\right)$, $\gamma_{ij} = U[0, 1]$, and $\varepsilon_{ijt} \sim N(0, 1)$.

The IRF coefficients, $b_{ij\ell}$, are generated based on the following random coefficient specification,

$$b_{ij\ell} = b_{i\ell} + c_{j\ell} + \omega_{ij\ell},$$

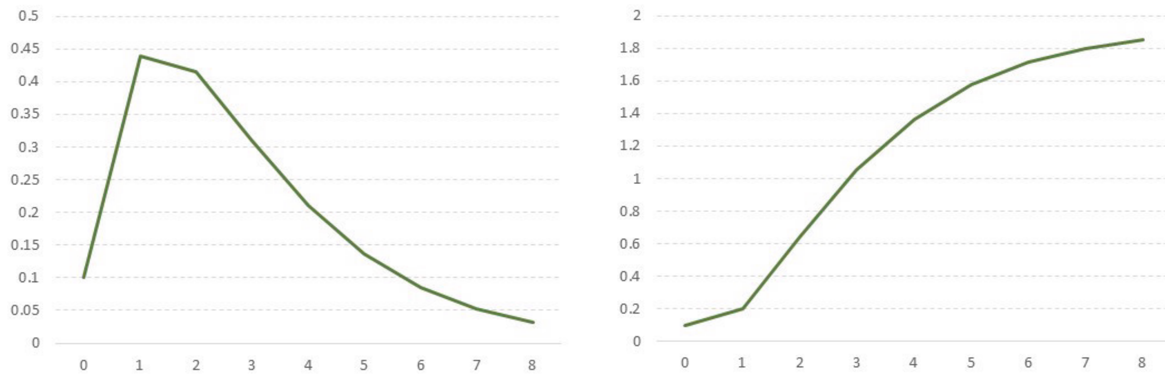
for $\ell = 1, 2, ..., B$, where $B = 100$,

$$b_{i\ell} = H_1 \lambda_1^{\ell} + H_2 \lambda_2^{\ell},$$

$\lambda_1 = 0.6$, $\lambda_2 = 0.4$, $H_1 = 2$ and $H_2 = -1.9$. This parameterization resembles a hump-shaped IRF that is common in many applications (see Figure 1 for plots of $b_{i\ell}$ and $\delta_{i,h} = \sum_{\ell=0}^{h} b_{i\ell}$).

Figure 1: Plots of $b_{i\ell}$ and the cumulative multipliers ($\delta_{i,h} = \sum_{\ell=0}^{h} b_{i\ell}$)

(a) Plot of $b_{i\ell}$, $\ell = 0, 1, ..., 8$ (b) Plot of $\delta_{i,h} = \sum_{\ell=0}^{h} b_{i\ell}$, $h = 0, 1, ..., 8$



We generate $c_{j\ell} = 0.1\kappa_\ell \alpha_j$, $\kappa_\ell = 0.8^\ell$, and $\alpha_j = 1 - 2\,(j-1)\,/\,(N-1)$. This ensures $\sum_{j=1}^{N} c_{j\ell} = 0$, as required for identification. $\omega_{ij\ell}$ is generated as $\omega_{ij\ell} = \kappa_\ell \times \Delta_{ij}$, where $\Delta_{ij} \sim U\,[-0.2, 0.2]$. We set $b_{ij\ell} = 0$ for $\ell > B$ (and $B = 100$). Note that due to an exponential decay, $b_{ij\ell}$ are all negligible for $\ell > B$.

We consider $M = N \in \{30, 40, 50, 100\}$, and $T = \{50, 100, 150, 500\}$ and compute $R_{MC} = 5000$ Monte Carlo replications. Values of $M = N = 40$ and $T = 100$ resemble the dimensions of quarterly data in our application in Section 4, whereas the largest sample size ($T = 500$) is more useful when validating the theoretical results.

3.2 Estimators

For the estimation of $b_{i\ell}$, we consider the MGDL estimator with and without augmentation by lagged dependent variable. The former is based on (2) and the latter is based on the augmented regression (13).

For the estimation of the cumulative multiplier ($\delta_{i,h}$), we consider cumulating the MGDL estimates, namely $\hat{\delta}_{i,h} = \sum_{\ell=0}^{h} \hat{b}_{i\ell}$, which gives us two estimators depending on whether or not the lagged dependent variable is included in the regression - see (2) and (13). Furthermore, we consider direct estimation based on the ADL regression (16), which is based on the cumulated variable, $\xi_{ijt} = x_{ijt} + \xi_{i,j,t-1}$.

We consider three different regression specifications for this exercise obtained from (14). The first specification imposes $\theta = 0$ and is given by

$$\xi_{ijt} = a_{ij} + \sum_{\ell=0}^{h} \delta_{ij\ell} v_{t-\ell} + \varphi \xi_{i,j,t-h-1} + e_{hit}, \tag{16}$$

which is similar to (13) but utilizes the I(1) variable ξ_{ijt} in place of x_{ijt}. The second specification is based on (16) with restrictions $\varphi = 1$ and $\theta = 0$, namely

$$\xi_{ijt} = a_{ij} + \sum_{\ell=0}^{h} \delta_{ij\ell} v_{t-\ell} + \xi_{i,j,t-h-1} + e_{hit}. \tag{17}$$

The third specification imposes $\varphi = 1$ only,

$$\xi_{ijt} = a_{ij} + \sum_{\ell=0}^{h} \delta_{ij\ell} v_{t-\ell} + \xi_{i,j,t-h-1} + \theta \Delta \xi_{i,j,t-h-1} + e_{hit}. \tag{18}$$

Confidence intervals are based on the nonparametric estimator in (5) and we use Bonferroni correction to control for the family-wise error rate, as proposed by Dunn (1961).

3.3 Objectives

Our focus here is twofold: (i) the estimation of $b_{i\ell}$; and (ii) the estimation of the cumulative multiplier $\delta_{i,h} = \sum_{\ell=0}^{h} b_{i\ell}$. Regarding the first objective, we report the overall estimation bias computed as

$$bias_b = \frac{1}{(h+1)\, NR} \sum_{r=1}^{R} \sum_{i=1}^{N} \sum_{\ell=0}^{h} \left(\hat{b}_{i\ell}^{(r)} - b_{i\ell} \right), \tag{19}$$

the overall RMSE computed as

$$rmse_b = \left[\frac{1}{(h+1)\, NR} \sum_{r=1}^{R} \sum_{i=1}^{N} \sum_{\ell=0}^{h} \left(\hat{b}_{i\ell}^{(r)} - b_{i\ell} \right)^2 \right]^{1/2}, \tag{20}$$

and the 95% family-wise (across both $\ell = 0, 1, ..., h$ and $i = 1, 2, ..., N$) confidence intervals coverage rate

$$FWCR_{b,0.95} = \frac{1}{R} \sum_{r=1}^{R} I \left\{ \sum_{i=1}^{N} \sum_{\ell=0}^{h} I \left[\hat{b}_{i\ell}^{(r)} \in CI \left(\hat{b}_{i\ell}^{(r)} \right) \right] = N(h+1) \right\}, \tag{21}$$

where $CI\left(\hat{b}_{i\ell}^{(r)}\right)$ is the 95% family-wise confidence interval for $\hat{b}_{i\ell}^{(r)}$.

In the case of estimation of the cumulative multiplier $\delta_{i,h}$, we focus on $h = 8$, and report the bias,

$$bias_{\delta,h} = \frac{1}{NR}\sum_{r=1}^{R}\sum_{i=1}^{N}\left(\hat{\delta}_{i,h}^{(r)} - \delta_{i,h}\right), \tag{22}$$

the RMSE

$$rmse_{\delta,h} = \left[\frac{1}{NR}\sum_{r=1}^{R}\sum_{i=1}^{N}\left(\hat{\delta}_{i,h}^{(r)} - \delta_{i,h}\right)^2\right]^{1/2}, \tag{23}$$

as well as the 95% family-wise (across $i = 1, 2, ..., N$) confidence intervals coverage rate.

$$FWCR_{\delta,h,0.95} = \frac{1}{R}\sum_{r=1}^{R} I\left[\sum_{i=1}^{N} I\left[\hat{\delta}_{i,h}^{(r)} \in CI\left(\hat{\delta}_{i,h}^{(r)}\right)\right] = N\right]. \tag{24}$$

3.4 Monte Carlo Findings

Table 1 summarizes the simulation results for the bias ($bias_b \times 100$), RMSE ($rmse_b \times 100$), and the family-wise coverage rate of the 95% confidence intervals ($FWCR_{b,0.95} \times 100$) for the estimation of non-cumulative IRF parameters $\mathbf{b}_i$. Overall, the bias of the MGDL estimator is small and diminishes with T. The coverage rate approaches 95 percent with the increase in sample size. For $N = M = 40$ and $T = 100$, the reported coverage rate is 82.7 and 83.82 percent, depending on the persistence parameter $\rho_{\max}$. The choice of $\rho_{\max}$ turns out to be inconsequential to the performance of the MGDL estimator, although it affects the performance of the MGADL estimator, with a slightly higher bias for a larger value of $\rho_{\max}$. The RMSE results in Table 1 suggest that the MGDL estimator is more accurate, but the RMSE reduction from augmenting lagged dependent variable is marginal, except for small values of T.

The results for the estimation of the cumulative multiplier $\delta_{i,h}$ (for $h = 8$) are presented in Table 2 for the simulations with $\rho_{\max} = 0.5$ and in Table 3 for $\rho_{\max} = 0.8$. Regardless of the choice of $\rho_{\max}$, the direct estimators of $\delta_{i,8}$ based on regressions (16)-(18) have consistently poorer performances in terms of the bias, RMSE, and the coverage rates, compared to the indirect estimator based on (2) or its augmented counterpart based on (13). The RMSE difference between the direct and indirect estimators is strikingly large, in the range of 41 and 381 percent for $T = 50$. Although the performance gap between the two estimators appear to diminish with an increase in T, it still remains large even for $T = 150$. This outcome leads us to focus on the indirect estimators of the cumulative multiplier in the empirical application in the next section. Overall, our Monte Carlo experiments show satisfactory performance of the MGDL and MGADL estimators for the IRF means $\mathbf{b}_i$ and their cumulative sums.

Table 1: MC results for the estimation of $\mathbf{b}_i$ in experiments with low and high persistence ($\rho_{\max} = 0.5$ and $\rho_{\max} = 0.9$)

	Bias (×100)				**RMSE (×100)**				**Coverage Rate (%)**			
T:	**50**	**100**	**150**	**500**	**50**	**100**	**150**	**500**	**50**	**100**	**150**	**500**
	(a) Experiments with low persistence ($\rho_{\max} = 0.5$)											
$N = M$	MGDL											
30	-0.16	-0.08	-0.04	-0.02	4.37	3.52	3.29	3.02	77.00	78.92	81.30	86.18
40	-0.17	-0.08	-0.04	-0.02	3.78	3.05	2.86	2.60	82.62	82.70	84.20	89.32
50	-0.17	-0.09	-0.05	-0.02	3.40	2.74	2.56	2.34	83.06	85.80	86.22	90.70
100	-0.17	-0.07	-0.05	-0.01	2.41	1.94	1.82	1.65	87.40	88.56	89.94	93.28
	MGADL											
30	0.07	0.01	0.02	0.00	4.52	3.55	3.31	3.02	77.06	79.58	81.74	86.34
40	0.06	0.01	0.01	0.00	3.91	3.07	2.87	2.61	82.10	82.92	84.22	89.46
50	0.07	0.00	0.01	0.00	3.52	2.75	2.57	2.34	82.72	86.24	86.68	90.78
100	0.06	0.02	0.01	0.00	2.51	1.95	1.82	1.65	85.20	89.52	90.60	93.54
	(b) Experiments with high persistence ($\rho_{\max} = 0.9$)											
	MGDL											
30	-0.16	-0.08	-0.04	-0.02	4.34	3.52	3.29	3.02	78.92	81.02	83.10	87.38
40	-0.18	-0.08	-0.04	-0.02	3.76	3.05	2.85	2.60	83.42	83.82	85.48	90.40
50	-0.17	-0.09	-0.05	-0.02	3.38	2.73	2.56	2.34	84.52	87.26	87.74	91.20
100	-0.17	-0.07	-0.05	-0.01	2.39	1.94	1.82	1.65	88.08	89.34	91.00	93.92
	MGADL											
30	0.22	0.09	0.07	0.01	4.55	3.56	3.31	3.02	78.32	82.16	83.00	87.52
40	0.21	0.09	0.06	0.01	3.95	3.08	2.87	2.61	81.64	84.04	85.52	90.28
50	0.21	0.08	0.06	0.01	3.56	2.76	2.57	2.34	82.60	87.12	87.94	90.80
100	0.21	0.09	0.06	0.02	2.55	1.96	1.82	1.66	83.54	89.80	91.70	93.26

Notes: This table reports the overall bias and RMSE (both ×100), as defined in (19) and (20), respectively, and the family-wise coverage rate of Bonferroni-corrected 95 percent confidence intervals, as defined by (21). MGDL estimator is based on DL regressions (2). MGADL estimator is based on the augmented DL regressions (13). Reported results are based on 5000 replications.

Table 2: MC results for the estimation of the cumulative multiplier $\delta_{i,h}$ $(h = 8)$ in experiments with low persistence $(\rho_{\max} = 0.5)$

	Bias (×100)				**RMSE (×100)**				**Coverage Rate (%)**			
T:	**50**	**100**	**150**	**500**	**50**	**100**	**150**	**500**	**50**	**100**	**150**	**500**
$N = M$	(a) MGDL, $\hat{\delta}_{i,h} = \sum_{\ell=0}^{h} \hat{b}_{i\ell}$											
30	-1.45	-0.68	-0.36	-0.15	27.15	24.22	23.65	23.06	86.38	85.88	86.22	85.76
40	-1.56	-0.72	-0.40	-0.14	23.48	21.01	20.50	19.91	88.32	87.04	88.02	88.12
50	-1.49	-0.81	-0.45	-0.17	21.11	18.84	18.35	17.87	89.02	89.34	89.40	88.84
100	-1.56	-0.67	-0.41	-0.12	14.95	13.35	13.02	12.63	90.44	90.36	91.10	91.14
	(b) MGADL, $\hat{\delta}_{i,h} = \sum_{\ell=0}^{h} \hat{b}_{i\ell}$											
30	0.65	0.12	0.14	-0.02	28.17	24.41	23.75	23.08	85.94	85.66	86.84	85.92
40	0.58	0.10	0.08	-0.01	24.38	21.17	20.58	19.92	87.82	87.28	88.02	88.10
50	0.60	0.02	0.06	-0.03	21.92	18.98	18.43	17.88	88.34	89.34	89.36	88.96
100	0.54	0.15	0.06	0.00	15.62	13.43	13.05	12.63	89.06	90.46	91.22	91.28
	(c) Direct MGDL estimation based on (16) (φ is not restricted)											
30	-53.52	-21.83	-13.04	-3.38	74.81	45.78	37.33	26.91	17.66	33.98	44.30	68.72
40	-53.32	-21.49	-13.25	-3.44	74.77	43.92	35.45	24.36	16.24	31.02	40.72	66.70
50	-53.32	-22.07	-13.60	-3.39	74.02	43.81	34.56	22.80	14.32	27.94	36.28	63.68
100	-52.98	-21.40	-12.27	-3.23	71.82	41.79	31.77	18.96	10.04	20.78	27.74	51.08
	(d) Direct MGDL estimation based on (17) (φ is restricted to one)											
30	-15.03	-6.33	-3.76	-1.17	50.36	35.98	31.60	25.57	35.86	50.08	59.02	74.78
40	-15.20	-6.54	-4.18	-1.30	49.16	33.78	29.28	22.79	32.02	46.48	54.84	74.58
50	-14.78	-6.73	-4.19	-1.18	47.98	32.92	27.90	21.03	30.94	42.96	50.44	72.20
100	-14.62	-6.16	-3.40	-1.10	45.02	29.95	24.63	16.78	22.54	32.38	39.34	61.24
	(e) Direct MGDL estimation based on augmented regression (18)											
30	-11.29	-4.11	-2.10	-0.71	41.85	30.11	27.24	24.10	49.16	63.94	71.54	81.52
40	-11.45	-4.33	-2.50	-0.79	39.95	27.57	24.61	21.09	45.14	61.14	69.44	82.58
50	-11.01	-4.36	-2.45	-0.65	38.72	26.17	22.80	19.14	41.78	57.88	66.54	81.40
100	-10.87	-4.10	-2.14	-0.72	34.76	22.27	18.75	14.40	32.06	47.14	56.18	76.38

Notes: This table reports the bias and rmse (both ×100), defined in (22) and (23), respectively, and the family-wise coverage rate of 95 percent confidence intervals defined in (24). Direct MGDL estimators are based on regression specifications (16), (17) and (18), which are special cases of (14) with restrictions on φ and θ. Reported results are based on 5000 replications.

Table 3: MC results for the estimation of the cumulative multiplier $\delta_{i,h}$ ($h = 8$) in experiments with high persistence ($\rho_{\max} = 0.9$)

	Bias (×100)				**RMSE (×100)**				**Coverage Rate (%)**			
T:	**50**	**100**	**150**	**500**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
$N = M$	(a) MGDL, $\hat{\delta}_{i,h} = \sum_{\ell=0}^{h} \hat{b}_{i\ell}$											
30	-1.46	-0.69	-0.36	-0.15	28.87	25.16	24.28	23.25	86.74	85.72	86.62	85.64
40	-1.58	-0.73	-0.39	-0.14	25.04	21.83	21.04	20.07	89.20	87.26	87.92	88.52
50	-1.49	-0.82	-0.45	-0.17	22.49	19.57	18.84	18.01	89.24	88.98	89.36	88.82
100	-1.55	-0.67	-0.41	-0.12	15.89	13.86	13.36	12.73	90.96	90.56	91.04	91.06
	(b) MGADL, $\hat{\delta}_{i,h} = \sum_{\ell=0}^{h} \hat{b}_{i\ell}$											
30	1.97	0.80	0.61	0.11	30.54	25.48	24.42	23.27	85.60	86.16	86.70	85.68
40	1.91	0.78	0.56	0.13	26.56	22.11	21.16	20.08	87.02	87.40	87.96	88.40
50	1.91	0.72	0.53	0.11	23.88	19.81	18.96	18.03	87.62	89.02	89.20	88.68
60	1.86	0.84	0.50	0.14	17.11	14.04	13.43	12.75	87.88	90.80	91.58	91.04
	(c) Direct MGDL estimation based on (16) (φ is not restricted)											
30	-54.30	-22.25	-13.30	-3.44	76.16	46.59	37.90	27.10	18.40	35.14	45.24	68.62
40	-54.14	-21.95	-13.51	-3.50	76.00	44.60	35.93	24.51	16.74	32.10	41.76	67.24
50	-54.12	-22.52	-13.89	-3.45	75.16	44.45	34.98	22.93	15.02	29.06	37.06	64.10
60	-53.78	-21.84	-12.57	-3.28	72.81	42.26	32.07	19.04	11.08	21.50	28.90	51.44
	(d) Direct MGDL estimation based on (17) (φ is restricted to one)											
30	-15.03	-6.33	-3.76	-1.17	51.35	36.63	32.08	25.73	38.40	51.64	59.84	75.10
40	-15.21	-6.54	-4.17	-1.30	49.92	34.31	29.66	22.93	33.94	48.08	55.66	75.06
50	-14.78	-6.73	-4.20	-1.18	48.59	33.35	28.22	21.15	33.08	44.10	51.34	72.44
60	-14.62	-6.16	-3.40	-1.10	45.35	30.18	24.82	16.86	24.00	33.00	40.36	61.80
	(e) Direct MGDL estimation based on augmented regression (18)											
30	-8.54	-2.67	-1.05	-0.42	40.63	29.81	27.17	24.13	54.18	67.86	74.38	82.02
40	-8.69	-2.93	-1.46	-0.50	38.13	27.09	24.42	21.08	50.02	64.90	73.14	83.86
50	-8.26	-2.84	-1.37	-0.32	36.76	25.49	22.46	19.10	46.14	62.18	69.68	82.88
60	-8.21	-2.71	-1.29	-0.46	32.35	21.24	18.23	14.32	35.68	51.56	60.22	78.28

Notes: See the notes in Table 2.

4 The Effects of Oil Price Shocks

This section utilizes a quarterly retail price data provided by the Council for Community and Economic Research (C2ER, https://www.c2er.org/redt/), formerly known as American Chamber of Commerce Researchers Association. Originally created for comparing the cost of living for mid-level managers in various metropolitan areas in the U.S., the C2ER dataset contains retail prices of a large number of individual goods and services in dollars and cents (see Choi, Choi, and Chudik (2020)). Our dataset covers retail prices for 43 products ($M = 43$) in 41 metropolitan areas ($N = 41$), spanning from the first quarter of 1990 to the fourth quarter of 2015 ($T = 104$).

Here we investigate the impacts of crude oil price shocks, computed as the first difference of log of crude oil prices sequentially sampled at the end of each quarter, on the retail prices. To this end, we utilize the daily West Texas Intermediate (WTI) crude oil prices obtained from FRED database (https://fred.stlouisfed.org, series DCOILWTICO). The object of our ultimate interest is the cumulative multipliers of the oil shock, measured at the horizon of $h = 8$ quarters.

Guided by the MC results in Section 3, we focus on the MGDL and MGADL approaches based on the specifications (2) and (13) with $h = 8$. Table 4 reports the estimates of cumulative multipliers, $\hat{\delta}_{i,h} = \sum_{\ell=0}^{h} \hat{b}_{i\ell}$, together with Bonferroni-corrected 95% confidence intervals. Significant entries are highlighted by asterisks. Not surprisingly, in retail gasoline price (product #26) we note a large pass-through effect of oil price shocks (59.9% in MGDL). To interpret, a 1% rise in WTI crude oil prices on average results in an approximate 0.6% increase in retail gasoline prices across US cities. Given that the crude oil accounts for about a half of the cost of producing gasoline (Baumeister and Kilian (2014)), our results indicate a full pass-through.

Table 4: Estimates of oil price shocks cumulative multipliers ($h = 8$)

	Product category	MGDL		MGADL	
		$\hat{\delta}_{i,8}$	Conf. Interval$^{\lozenge}$	$\hat{\delta}_{i,8}$	Conf. Interval$^{\lozenge}$
1	TBONESTEAK	0.126*	[0.068,0.184]	0.121*	[0.067,0.175]
2	GROUNDBEEF	0.193*	[0.123,0.263]	0.194*	[0.123,0.264]
3	FRYINGCHICKEN	0.008	[-0.075,0.090]	0.005	[-0.084,0.093]
4	CANNED TUNA	0.042	[-0.035,0.119]	0.043	[-0.034,0.120]
5	WHOLEMILK	0.158*	[0.101,0.214]	0.152*	[0.095,0.209]
6	EGGS	0.251*	[0.188,0.315]	0.250*	[0.185,0.316]
7	MARGARINE	0.071	[-0.016,0.158]	0.073	[-0.014,0.161]
8	CHEESE	0.065*	[0.010,0.120]	0.064*	[0.011,0.118]
9	POTATOES	0.449*	[0.346,0.552]	0.447*	[0.345,0.550]
10	BANANAS	0.194*	[0.131,0.258]	0.194*	[0.126,0.262]
11	LETTUCE	-0.027	[-0.105,0.050]	-0.028	[-0.109,0.054]
12	BREAD	0.172*	[0.104,0.241]	0.172*	[0.104,0.239]
13	COFFEE	0.201*	[0.147,0.255]	0.201*	[0.145,0.257]
14	SUGAR	0.146*	[0.094,0.198]	0.144*	[0.094,0.194]
15	CORNFLAKES	0.009	[-0.037,0.054]	0.008	[-0.037,0.052]
16	CANNED PEAS	0.109*	[0.014,0.204]	0.105*	[0.012,0.199]
17	CANNED PEACHES	0.052*	[0.005,0.099]	0.052*	[0.003,0.100]
18	TISSUES	0.151*	[0.105,0.196]	0.150*	[0.103,0.197]
19	DETERGENT	0.123*	[0.073,0.174]	0.125*	[0.075,0.176]
20	SHORTENING	0.246*	[0.197,0.295]	0.241*	[0.191,0.291]
21	FROZEN CORN	0.028	[-0.047,0.102]	0.028	[-0.048,0.103]
22	SOFT DRINK	0.068	[-0.010,0.145]	0.069	[-0.009,0.148]
23	HOME PRICE	0.003	[-0.030,0.037]	0.004	[-0.031,0.038]
24	PHONE	0.009	[-0.034,0.051]	0.005	[-0.041,0.050]
25	AUTO MAINTENANCE	-0.519*	[-0.568,-0.469]	-0.523*	[-0.572,-0.475]
26	GASOLINE	0.599*	[0.558,0.640]	0.569*	[0.528,0.610]
27	DOCTOR VISIT	-0.006	[-0.050,0.037]	-0.006	[-0.051,0.038]
28	DENTIST VISIT	-0.095*	[-0.144,-0.046]	-0.094*	[-0.144,-0.044]
29	MCDONALD'S HAMBURGER	0.048*	[0.005,0.091]	0.048*	[0.003,0.092]
30	PIZZA	-0.027	[-0.071,0.017]	-0.029	[-0.074,0.016]
31	FRIED CHICKEN	0.018	[-0.054,0.089]	0.023	[-0.052,0.098]
32	HAIRCUT	0.030	[-0.017,0.077]	0.030	[-0.017,0.077]
33	BEAUTY SALON	0.028	[-0.031,0.087]	0.024	[-0.036,0.085]
34	TOOTHPASTE	-0.022	[-0.094,0.050]	-0.015	[-0.097,0.067]
35	DRY CLEANING	0.018	[-0.021,0.057]	0.016	[-0.023,0.055]
36	MAN'S SHIRT	-0.092*	[-0.181,-0.003]	-0.093*	[-0.181,-0.004]
37	APPLIANCE REPAIR	0.081*	[0.030,0.131]	0.083*	[0.033,0.134]
38	NEWSPAPER	-0.052	[-0.140,0.037]	-0.054	[-0.141,0.033]
39	MOVIE	0.020	[-0.009,0.049]	0.020	[-0.009,0.048]
40	BOWLING	-0.055	[-0.140,0.031]	-0.049	[-0.133,0.035]
41	TENNIS BALLS	-0.014	[-0.080,0.051]	-0.010	[-0.076,0.057]
42	BEER	0.072*	[0.039,0.104]	0.071*	[0.039,0.104]
43	WINE	0.044	[-0.010,0.099]	0.047	[-0.010,0.104]

Notes: ($^{\lozenge}$) 95 percent family-wise confidence intervals are reported.
(*) Statistically significant estimates are highlighted by asterisk.

This table reports MGDL and augmented MGDL cumulative multiplier estimates $\hat{\delta}_{i,h}$ at horizon $h = 8$ quarters for the crude oil price shocks. MGDL estimates are based on DL regressions and augmented MGDL regressions $x_{ijt} = a_{ij} + \sum_{\ell=0}^{h} b_{ij\ell} v_{t-\ell} + e_{hit}$, and augmented MGDL estimates are based on regressions $x_{ijt} = a_{ij} + \sum_{\ell=0}^{h} b_{ij\ell} v_{t-\ell} + \varphi x_{i,j,t-h-1} + e_{hit}$, where x_{ijt} is log-difference of price for product category i in city j in period t from C2ER dataset, which spans $M = 43$ reported categories over $N = 41$ cities, covering $T = 104$ quarterly periods from 1990Q1 to 2015Q4.

Further inspection of the impulse response coefficients $\hat{b}_{i\ell}$ suggests that the full pass-through takes place within one quarter, in line with our economic intuition. Interestingly, the cumulative multiplier estimates are also statistically significant in many other products (21 out of the remaining 42), mostly in traded goods. The cumulative multiplier estimates are, however, a bit smaller in the non-gasoline product prices, with the mean cumulative multiplier of 10.1% and the median of 12.6%. Nevertheless, these significant secondary effects of crude oil prices are somewhat surprising, which is better aligned with the notion that changes in crude oil prices are primarily influenced by demand shocks that have a broader impact on economic activity and overall price levels. The estimates of location effects (c_j) are reported in Table A1 in Appendix, which are economically very small and insignificant in the vast majority of locations under study.

5 Conclusion

This paper develops the estimators of impulse-responses in a panel setting with one or two cross-sections under assumptions that the shock is common, observed, and the impulse responses follow a random coefficient specification. We derive sufficient conditions for consistency and asymptotic normality of mean group estimation based on the distributed lag and augmented distributed lag specifications. These estimators are found to have satisfactory small sample performance. We then illustrate the empirical relevance of our analysis by examining how shocks in crude oil prices affect the retail prices of diverse products in metropolitan areas across the United States.

Several potential extensions remain for future research. Specifically, within the context of panel data, alternative methodologies beyond the distributed lag specifications investigated in this paper warrant further exploration. Some of these alternative approaches may yield panel estimators with improved performance in small samples.

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A Appendix

This appendix consists of two sections. Section A.1 presents lemmas and proofs. Section A.2 presents additional empirical results.

A.1 Lemmas and Proofs

Lemma A.1 *Let Assumptions 1-3 hold, $h \geq 0$ be a fixed integer that does not depend on the sample size, and define $T-h \times 1$ vectors $\mathbf{v}_\ell = (v_{h+1-\ell}, v_{h+2-\ell}, ..., v_{T-\ell})'$ for $\ell = 0,1,2,...,h$, and $\mathbf{e}_{hij} = (e_{h,i,j,h+1}, e_{h,i,j,h+2}, ..., e_{h,i,j,T})'$, where $e_{hijt} = \sum_{\ell=h+1}^{\infty} b_{ij\ell} v_{t-\ell} + z_{ijt}$. Then there exists a constant K that does not depend on (i,j,T), such that*

$$E \left\| \frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h} \right\|^2 < \frac{K}{T}, \tag{A.1}$$

where $\mathbf{V} = (\boldsymbol{\tau}, \mathbf{v}_0, \mathbf{v}_1, ..., \mathbf{v}_h)'$ and $\boldsymbol{\tau}$ is $T-h \times 1$ vector of ones.

Proof. Consider individual elements of $(h+2) \times 1$ vector $\mathbf{V}'\mathbf{e}_{hij}/(T-h)$. Let $\mathbf{s}_\ell$ be $(h+2) \times 1$ selection vector for the $(\ell+2)$-th element, for $\ell = -1, 0, 1, ..., h$. For $\ell = -1$, we have

$$\frac{\mathbf{s}_{-1}'\mathbf{V}'\mathbf{e}_{hij}}{T-h} = \frac{1}{T-h} \sum_{t=h+1}^{T} e_{hijt}, \tag{A.2}$$

and for $\ell = 0,1,2,...,h$, we have

$$\frac{\mathbf{s}_{\ell}'\mathbf{V}'\mathbf{e}_{hij}}{T-h} = \frac{1}{T-h} \sum_{t=h+1}^{T} v_{t-\ell} e_{hijt}. \tag{A.3}$$

We derive stochastic upper bounds for (A.2) and (A.3) next. Recall $e_{hijt} = \sum_{\ell=h+1}^{\infty} b_{ij\ell} v_{t-\ell} + z_{ijt}$. Using Assumption 2 and noting h is fixed, we have

$$E \left(\frac{1}{T-h} \sum_{t=h+1}^{T} z_{ijt} \right)^2 = \frac{1}{(T-h)^2} \sum_{t=h+1}^{T} \sum_{t'=h+1}^{T} E \left(z_{ijt} z_{ijt'} \right)^2 < \frac{K}{T}. \tag{A.4}$$

where K does not depend on (i, j, T). Furthermore, under Assumptions 1 and 3, we have

$$E\left(\frac{1}{T-h}\sum_{t=h+1}^{T}\sum_{\ell'=h+1}^{\infty} b_{ij\ell'}v_{t-\ell'}\right)^2 < \frac{K}{T}, \tag{A.5}$$

where K does not depend on (i, j, T). Using (A.4) and (A.5), we obtain

$$E\left\|\frac{\mathbf{s}'_{-1}\mathbf{V}'\mathbf{e}_{hij}}{T-h}\right\|^2 < \frac{K}{T}.$$

Consider (A.3) next. We have

$$\frac{1}{T-h}\sum_{t=h+1}^{T} v_{t-\ell}e_{hijt} = \frac{1}{T-h}\sum_{t=h+1}^{T} v_{t-\ell}z_{ijt} + \frac{1}{T-h}\sum_{t=h+1}^{T} v_{t-\ell}\sum_{\ell'=h+1}^{\infty} b_{ij\ell'}v_{t-\ell'}.$$

For the first term, using Assumptions 2 and 3, we obtain (noting that $E\left(v_{t-\ell}v_{t'-\ell}z_{ijt}z_{ijt'}\right) = E\left(v_{t-\ell}v_{t'-\ell}\right)E\left(z_{ijt}z_{ijt'}\right)$ and $E\left(v_{t-\ell}v_{t'-\ell}\right) = 0$ for $t \neq t'$)

$$\begin{aligned} E\left(\frac{1}{T-h}\sum_{t=h+1}^{T} v_{t-\ell}z_{ijt}\right)^2 &= \frac{1}{(T-h)^2}\sum_{t=h+1}^{T}\sum_{t'=h+1}^{T} E\left(v_{t-\ell}v_{t'-\ell}z_{ijt}z_{ijt'}\right) \\ &= \frac{1}{(T-h)^2}\sum_{t=h+1}^{T} E\left(v_{t-\ell}^2\right)E\left(z_{ijt}^2\right) < \frac{K}{T}. \end{aligned}$$

Similarly, for the second term we obtain under Assumptions 1 and 3,

$$E\left(\frac{1}{T-h}\sum_{t=h+1}^{T} v_{t-\ell}\sum_{\ell'=h+1}^{\infty} b_{ij\ell'}v_{t-\ell'}\right)^2 < \frac{K}{T}.$$

These results imply

$$E\left\|\frac{\mathbf{s}'_{-\ell}\mathbf{V}'\mathbf{e}_{hij}}{T-h}\right\|^2 < \frac{K}{T}, \text{ for } \ell = 0, 1, 2, ..., h.$$

This completes the proof. ■

Lemma A.2 *Let x_{ijt} be generated by (1) and Assumptions 1-3 hold. Let $h \geq 0$ be a fixed integer that does not depend on the sample size, and let $\boldsymbol{\epsilon}_{ij} = \hat{\mathbf{b}}_{ij} - \mathbf{b}_{ij}$, where $\hat{\mathbf{b}}_{ij}$ is the LS estimator of*

$(h+1)\times 1$ vector $\mathbf{b}_{ij}=(b_{ij0},b_{ij1},...,b_{ijh})'$ in the DL regression (2). Then

$$\boldsymbol{\epsilon}_{ij}=O_p\left(T^{1/2}\right), \tag{A.6}$$

uniformly in i,j. In addition, let $N_T=N(T)$ and $M_T=M(T)$ be any nondecreasing positive-integer-valued functions of T. Then,

$$N_T^{-1}\sum_{j=1}^{N_T}\boldsymbol{\epsilon}_{ij}=O_p\left(T^{1/2}\right), \tag{A.7}$$

$$M_T^{-1}\sum_{i=1}^{M_T}\boldsymbol{\epsilon}_{ij}=O_p\left(T^{1/2}\right), \tag{A.8}$$

and

$$M_T^{-1}N_T^{-1}\sum_{j=1}^{N_T}\sum_{i=1}^{M_T}\boldsymbol{\epsilon}_{ij}=O_p\left(T^{1/2}\right). \tag{A.9}$$

Proof. Let $\mathbf{x}_{ij}=(x_{ij,h+1},x_{ij,h+2},...,x_{ijT})'$, $\mathbf{v}_\ell=(v_{h+1-\ell},v_{h+2-\ell},...,v_{T-\ell})'$ for $\ell=0,1,2,...,h$, $\mathbf{e}_{hij}=(e_{h,i,j,h+1},e_{h,i,j,h+2},...,e_{h,i,j,T})'$, and let $\boldsymbol{\tau}$ be the $T-h\times 1$ vector of ones. Define also the $T-h\times(h+2)$ matrix $\mathbf{V}=(\boldsymbol{\tau},\mathbf{v}_0,\mathbf{v}_1,...,\mathbf{v}_h)'$. Vectors $\mathbf{x}_{ij}$, $\mathbf{v}_\ell$, $\mathbf{e}_{hij}$, and $\boldsymbol{\tau}$, and the matrix $\mathbf{V}$ all depend on T, but subscript T is omitted to simplify the notations. We also omitted the subscript h from $\mathbf{x}_{ij}$, $\mathbf{v}_\ell$, $\boldsymbol{\tau}$, and $\mathbf{V}$, although these vectors and matrix depend also on h. Using this notation, the DL regression (2) can be written in a matrix form as

$$\mathbf{x}_{ij}=\mathbf{V}\mathbf{b}^*_{ij}+\mathbf{e}_{hij}, \tag{A.10}$$

where $\mathbf{b}^*_{ij}=\left(a_{ij},\mathbf{b}'_{ij}\right)'$. Consider $\boldsymbol{\epsilon}^*_{ij}=\hat{\mathbf{b}}^*_{ij}-\mathbf{b}^*_{ij}$, where $\hat{\mathbf{b}}^*_{ij}$ is the LS estimator of $\mathbf{b}^*_{ij}$ given by

$$\mathbf{b}^*_{ij}=\left(\mathbf{V}'\mathbf{V}\right)^{-1}\mathbf{V}'\mathbf{x}_{ij}. \tag{A.11}$$

Since $\boldsymbol{\epsilon}_{ij}$ is part of $\boldsymbol{\epsilon}_{ij}^{*} = \left(\hat{a}_{ij} - a_{ij}, \boldsymbol{\epsilon}_{ij}'\right)'$, a sufficient condition for (A.6) to hold is $\boldsymbol{\epsilon}_{ij}^{*} = O_p\left(T^{1/2}\right)$. Substituting (A.10) in (A.11), we obtain

$$\begin{aligned}\boldsymbol{\epsilon}_{ij}^{*} &= \mathbf{b}_{ij}^{*} - \mathbf{b}_{ij}^{*} = \left(\frac{\mathbf{V}'\mathbf{V}}{T-h}\right)^{-1} \frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h} \\ &= \left(\hat{\boldsymbol{\Sigma}}_v^{-1} - \boldsymbol{\Sigma}_v^{-1}\right) \frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h} + \boldsymbol{\Sigma}_v^{-1} \frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h},\end{aligned}$$

where $\hat{\boldsymbol{\Sigma}}_v = \frac{\mathbf{V}'\mathbf{V}}{T}$, and under Assumption 3 we have $\text{plim}_{T\to\infty} \hat{\boldsymbol{\Sigma}}_v = \boldsymbol{\Sigma}_v$, where $\boldsymbol{\Sigma}_v$ is full rank. It follows $\left(\hat{\Sigma}_v^{-1} - \Sigma_v^{-1}\right) = o_p(1)$ and $\left\|\boldsymbol{\Sigma}_v^{-1}\right\| < K$. In addition, result (A.1) of Lemma A.1 implies

$$\frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h} = O_p\left(T^{1/2}\right),$$

uniformly in i, j. It now follows that $\boldsymbol{\epsilon}_{ij}^{*} = O_p\left(T^{1/2}\right)$, uniformly in i, j, which implies (A.6). To establish (A.7), consider first,

$$\begin{aligned}N_T^{-1}\sum_{j=1}^{N_T} \boldsymbol{\epsilon}_{ij}^{*} &= \left(\hat{\Sigma}_v^{-1} - \Sigma_v^{-1}\right)\left(N_T^{-1}\sum_{j=1}^{N_T} \frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h}\right) \\ &\quad + \Sigma_v^{-1}\left(N_T^{-1}\sum_{j=1}^{N_T} \frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h}\right).\end{aligned}$$

Lemma A.1 established $E\left\|\mathbf{V}'\mathbf{e}_{hij}/(T-h)\right\|^2 < K/T$, where the constant K does not depend on i, j. Hence, for any positive integer-valued function $N_T = N(T)$,

$$E\left\|N_T^{-1}\sum_{j=1}^{N_T} \frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h}\right\|^2 \leq N_T^{-1}\sum_{j=1}^{N_T} E\left\|\frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h}\right\|^2 < \frac{K}{T},$$

and therefore

$$N_T^{-1}\sum_{j=1}^{N_T} \frac{\mathbf{V}'\mathbf{e}_{hij}}{T-h} = O_p\left(T^{1/2}\right).$$

Since also $\left(\hat{\boldsymbol{\Sigma}}_v^{-1} - \boldsymbol{\Sigma}_v^{-1}\right) = o_p(1)$, and $\|\boldsymbol{\Sigma}_v\| < K$, it follows that $N_T^{-1}\sum_{j=1}^{N_T} \boldsymbol{\epsilon}_{ij} = O_p\left(T^{1/2}\right)$. This completes the proof of (A.7). Results (A.8)-(A.9) can be established using the same steps. ■

Proof of Theorem 1.. Let $\boldsymbol{\epsilon}_{ij} = \hat{\mathbf{b}}_{ij} - \mathbf{b}_{ij}$. We have

$$\hat{\mathbf{b}}_i = N^{-1}\sum_{j=1}^{N}\hat{\mathbf{b}}_{ij} = N^{-1}\sum_{j=1}^{N}\mathbf{b}_{ij} + N^{-1}\sum_{j=1}^{N}\boldsymbol{\epsilon}_{ij}.$$

Substituting $\mathbf{b}_{ij} = \mathbf{b}_i + \mathbf{c}_j + \boldsymbol{\omega}_{ij}$ (Assumption 1), we obtain

$$\hat{\mathbf{b}}_i = \mathbf{b}_i + N^{-1}\sum_{j=1}^{N}(\mathbf{c}_j + \boldsymbol{\omega}_{ij}) + N^{-1}\sum_{j=1}^{N}\boldsymbol{\epsilon}_{ij}.$$

But $N^{-1}\sum_{j=1}^{N}\mathbf{c}_j = 0$. In addition, $Var\left(N^{-1}\sum_{j=1}^{N}\boldsymbol{\omega}_{ij}\right) = N^{-2}\sum_{j=1}^{N}\boldsymbol{\Omega}_{ij}$, and $\|\boldsymbol{\Omega}_{ij}\| < K$ under Assumption 1. Hence, $N^{-1}\sum_{j=1}^{N}\boldsymbol{\omega}_{ij} = O_p\left(N^{-1/2}\right)$. Furthermore, $N^{-1}\sum_{j=1}^{N}\boldsymbol{\epsilon}_{ij} = O_p\left(T^{-1/2}\right)$ by result (A.7) of Lemma A.2. Hence,

$$\hat{\mathbf{b}}_i - \mathbf{b}_i = O_p\left(N^{1/2}\right) + O_p\left(T^{-1/2}\right).$$

The consistency result $\hat{\mathbf{b}}_i \to_p \mathbf{b}_i$ follows. Consider next the probability limit of $\hat{\mathbf{c}}_j$. We have

$$\begin{aligned}\hat{\mathbf{c}}_j &= M^{-1}\sum_{i=1}^{M}\left(\hat{\mathbf{b}}_{ij} - \hat{\mathbf{b}}_i\right) = M^{-1}\sum_{i=1}^{M}\left(\hat{\mathbf{b}}_{ij} - N^{-1}\sum_{j=1}^{N}\hat{\mathbf{b}}_{ij}\right) \\ &= M^{-1}\sum_{i=1}^{M}\hat{\mathbf{b}}_{ij} - M^{-1}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}\hat{\mathbf{b}}_{ij}\end{aligned}$$

Using $\hat{\mathbf{b}}_{ij} = \mathbf{b}_{ij} + \boldsymbol{\epsilon}_{ij}$ and $\mathbf{b}_{ij} = \mathbf{b}_i + \mathbf{c}_j + \boldsymbol{\omega}_{ij}$, we obtain

$$\begin{aligned}\hat{\mathbf{c}}_j &= M^{-1}\sum_{i=1}^{M}(\mathbf{b}_i + \mathbf{c}_j + \boldsymbol{\omega}_{ij} + \boldsymbol{\epsilon}_{ij}) - M^{-1}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}(\mathbf{b}_i + \mathbf{c}_j + \boldsymbol{\omega}_{ij} + \boldsymbol{\epsilon}_{ij}) \\ &= \mathbf{c}_j + M^{-1}\sum_{i=1}^{M}(\mathbf{b}_i + \boldsymbol{\omega}_{ij} + \boldsymbol{\epsilon}_{ij}) - M^{-1}\sum_{i=1}^{M}\mathbf{b}_i - M^{-1}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}(\boldsymbol{\omega}_{ij} + \boldsymbol{\epsilon}_{ij}) \\ &= \mathbf{c}_j + M^{-1}\sum_{i=1}^{M}(\boldsymbol{\omega}_{ij} + \boldsymbol{\epsilon}_{ij}) - M^{-1}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}(\boldsymbol{\omega}_{ij} + \boldsymbol{\epsilon}_{ij}).\end{aligned}$$

Under Assumption 1, we have

$$M^{-1}\sum_{i=1}^{M}\boldsymbol{\omega}_{ij}=O_p\left(M^{-1/2}\right), \text{ and } M^{-1}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}\boldsymbol{\omega}_{ij}=O_p\left(M^{-1/2}N^{-1/2}\right).$$

These results, together with results (A.8) and (A.9) of Lemma A.2 establish

$$\hat{\mathbf{c}}_j=\mathbf{c}_j+O_p\left(M^{-1/2}\right)+O_p\left(M^{-1/2}N^{-1/2}\right)+O_p\left(T^{-1/2}\right),$$

hence $\hat{\mathbf{c}}_j\rightarrow_p\mathbf{c}_j$ as required.

We establish asymptotic distribution next. We have

$$\sqrt{N}\left(\hat{\mathbf{b}}_i-\mathbf{b}_i\right)=N^{-1/2}\sum_{j=1}^{N}\boldsymbol{\omega}_{ij}+N^{-1/2}\sum_{j=1}^{N}\boldsymbol{\epsilon}_{ij}$$

Since $\boldsymbol{\omega}_{ij}\sim IID\left(\mathbf{0},\boldsymbol{\Omega}_{ij}\right)$, $K_0<\left\|\boldsymbol{\Omega}_{ij}\right\|<K_1$, under Assumption 1, we have

$$N^{-1/2}\sum_{j=1}^{N}\boldsymbol{\omega}_{ij}\rightarrow_d N\left(0,\bar{\boldsymbol{\Omega}}_{i\circ}\right),$$

where $\bar{\boldsymbol{\Omega}}_{i\circ}=N^{-1}\sum_{j=1}^{N}\boldsymbol{\Omega}_{ij}$. In addition, result (A.7) of Lemma A.2 implies $N^{-1/2}\sum_{j=1}^{N}\boldsymbol{\epsilon}_{ij}=O_p\left(N^{1/2}T^{-1/2}\right)$. Hence for any i, as $N,T\rightarrow_j\infty$ such that $N/T\rightarrow 0$, we obtain

$$\sqrt{N}\left(\hat{\mathbf{b}}_i-\mathbf{b}_i\right)\rightarrow_d N\left(0,\bar{\boldsymbol{\Omega}}_{i\circ}\right).$$

Consider the asymptotic distribution of $\hat{\mathbf{c}}_j$ next. We have

$$\begin{aligned}\sqrt{M}\left(\hat{\mathbf{c}}_j-\mathbf{c}_j\right) &= M^{-1/2}\sum_{i=1}^{M}\boldsymbol{\omega}_{ij}+M^{-1/2}\sum_{i=1}^{M}\boldsymbol{\epsilon}_{ij}-M^{-1/2}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}\boldsymbol{\omega}_{ij}\\ &\quad -M^{-1/2}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}\boldsymbol{\epsilon}_{ij}.\end{aligned}$$

Using the same arguments as before, we have

$$M^{-1/2}\sum_{i=1}^{M}\boldsymbol{\omega}_{ij}\rightarrow_d N\left(\mathbf{0},\bar{\boldsymbol{\Omega}}_{\circ j}\right),$$

and

$$M^{-1/2}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}\boldsymbol{\omega}_{ij}=O_p\left(N^{-1/2}\right),$$

where $\bar{\boldsymbol{\Omega}}_{\circ j}=M^{-1}\sum_{i=1}^{M}\boldsymbol{\Omega}_{ij}$. Using results (A.8) and (A.9) of Lemma A.2, we obtain

$$M^{-1/2}\sum_{i=1}^{M}\boldsymbol{\epsilon}_{ij}=O_p\left(M^{1/2}T^{-1/2}\right),$$

and

$$M^{-1/2}N^{-1}\sum_{i=1}^{M}\sum_{j=1}^{N}\boldsymbol{\epsilon}_{ij}=O_p\left(M^{1/2}T^{-1/2}\right).$$

Hence, for any j, as $M,T\rightarrow_j\infty$ such that $M/T\rightarrow 0$,

$$\sqrt{M}\left(\hat{\mathbf{c}}_j-\mathbf{c}_j\right)\rightarrow_d N\left(\mathbf{0},\bar{\boldsymbol{\Omega}}_{\circ j}\right).$$

This completes the proof. ■

Proof of Proposition 1.. Proof of Proposition 1 follows the same lines of arguments as the proof of Theorem 1. Result (A.6) of Lemma A.2 implies $\hat{\mathbf{b}}_j=\mathbf{b}_j+\boldsymbol{\epsilon}_j=\mathbf{b}_j+O_p\left(T^{-1/2}\right)$, and we have

$$\hat{\mathbf{b}}=N^{-1}\sum_{j=1}^{N}\hat{\mathbf{b}}_j=N^{-1}\sum_{j=1}^{N}\mathbf{b}_j+O_p\left(T^{-1}\right)=N^{-1}\sum_{j=1}^{N}\boldsymbol{\omega}_j+O_p\left(T^{-1/2}\right),$$

where we substituted $\mathbf{b}_j=\mathbf{b}+\boldsymbol{\omega}_j$ (Assumption 4). Noting that $N^{-1}\sum_{j=1}^{N}\boldsymbol{\omega}_j=O\left(N^{-1/2}\right)$, we obtain $\hat{\mathbf{b}}\rightarrow_p\mathbf{b}$, as $N,T\rightarrow_j\infty$, without any restrictions on N/T. Consider next

$$\sqrt{N}\left(\hat{\mathbf{b}}-\mathbf{b}\right)=N^{-1/2}\sum_{j=1}^{N}\boldsymbol{\omega}_{ij}+O_p\left(N^{1/2}T^{-1/2}\right).$$

But since $\boldsymbol{\omega}_j \sim IID\left(\mathbf{0}, \boldsymbol{\Omega}_j\right)$, and $K_0 < \|\boldsymbol{\Omega}_j\| < K_1$, under Assumption 4, we have $N^{-1/2}\sum_{j=1}^{N}\boldsymbol{\omega}_j \rightarrow_d N\left(\mathbf{0}, \bar{\boldsymbol{\Omega}}\right)$, where $\bar{\boldsymbol{\Omega}} = N^{-1}\sum_{j=1}^{N}\boldsymbol{\Omega}_j$. Hence as $N, T \rightarrow_j \infty$ such that $N/T \rightarrow 0$, we obtain

$$\sqrt{N}\left(\hat{\mathbf{b}} - \mathbf{b}\right) \rightarrow_d N\left(\mathbf{0}, \bar{\boldsymbol{\Omega}}\right).$$

This completes the proof. ■

A.2 Additional Empirical Results

Table A1: Estimates of IRF location effects ($c^{\delta}_{j,h}$) for oil price shocks cumulative multipliers at horizon $h = 8$

	City or Metro Area, State	MGDL $\hat{c}^{\delta}_{j,8}$	MGDL Conf. Interval$^{\Diamond}$	Augmented MGDL $\hat{c}^{\delta}_{j,8}$	Augmented MGDL Conf. Interval$^{\Diamond}$
1	Amarillo, TX	0.012	[-0.013,0.036]	0.007	[-0.016,0.030]
2	Atlanta, GA	0.018	[-0.006,0.042]	0.017	[-0.007,0.042]
3	Cedar Rapids, IA	-0.031*	[-0.052,-0.010]	-0.029*	[-0.051,-0.007]
4	Charlotte-Gastonia-Rock Hill, NC-SC	-0.004	[-0.024,0.016]	-0.006	[-0.027,0.016]
5	Chattanooga, TN-GA	-0.041*	[-0.064,-0.017]	-0.043*	[-0.067,-0.019]
6	Cleveland-Akron, OH	-0.017	[-0.041,0.006]	-0.017	[-0.040,0.007]
7	Colorado Springs, CO	-0.001	[-0.029,0.027]	0.000	[-0.029,0.029]
8	Columbia, MO	0.004	[-0.021,0.030]	0.005	[-0.021,0.031]
9	Columbia, SC	-0.030*	[-0.049,-0.011]	-0.033*	[-0.051,-0.014]
10	Dallas-Fort Worth, TX	0.000	[-0.023,0.023]	-0.002	[-0.024,0.020]
11	Denver-Boulder-Greeley, CO	-0.032*	[-0.061,-0.003]	-0.028	[-0.058,0.003]
12	Dover, DE	0.030*	[0.009,0.051]	0.030*	[0.008,0.052]
13	Houston-Galveston-Brazoria, TX	-0.022	[-0.048,0.005]	-0.026	[-0.052,0.001]
14	Huntsville, AL	0.024	[-0.003,0.051]	0.023	[-0.005,0.051]
15	Jonesboro, AR	-0.004	[-0.025,0.017]	-0.006	[-0.029,0.016]
16	Joplin, MO	-0.029*	[-0.050,-0.009]	-0.029*	[-0.050,-0.007]
17	Knoxville, TN	0.030*	[0.011,0.049]	0.031*	[0.010,0.052]
18	Lexington, KY	0.049*	[0.030,0.068]	0.050*	[0.029,0.071]
19	Los Angeles-Riverside-Orange County, CA	-0.013	[-0.032,0.005]	-0.010	[-0.029,0.008]
20	Louisville, KY-IN	0.004	[-0.013,0.021]	0.002	[-0.018,0.022]
21	Lubbock, TX	0.000	[-0.021,0.022]	0.000	[-0.021,0.022]
22	Memphis, TN-AR-MS	0.012	[-0.013,0.036]	0.010	[-0.016,0.035]
23	Montgomery, AL	0.009	[-0.019,0.036]	0.006	[-0.022,0.034]
24	Odessa-Midland, TX	0.021	[-0.001,0.044]	0.016	[-0.007,0.039]
25	Oklahoma City, OK	0.015	[-0.012,0.042]	0.015	[-0.011,0.042]
26	Omaha, NE-IA	-0.031*	[-0.057,-0.004]	-0.033*	[-0.058,-0.008]
27	Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	0.008	[-0.016,0.033]	0.007	[-0.019,0.034]
28	Phoenix-Mesa, AZ	-0.047*	[-0.074,-0.019]	-0.041*	[-0.070,-0.011]
29	Portland-Salem, OR-WA	-0.011	[-0.048,0.026]	-0.008	[-0.047,0.031]
30	Raleigh-Durham-Chapel Hill, NC	0.008	[-0.014,0.031]	0.010	[-0.015,0.034]
31	Reno, NV	0.039*	[0.015,0.064]	0.040*	[0.015,0.065]
32	Salt Lake City-Ogden, UT	0.022	[-0.005,0.048]	0.022	[-0.006,0.049]
33	San Antonio, TX	0.029*	[0.000,0.059]	0.036*	[0.009,0.064]
34	South Bend, IN	0.033	[-0.002,0.068]	0.033	[-0.002,0.069]
35	Springfield, IL	-0.027	[-0.056,0.003]	-0.029	[-0.059,0.001]
36	St. Cloud, MN	0.045*	[0.020,0.070]	0.045*	[0.019,0.071]
37	St. Louis, MO-IL	-0.055*	[-0.080,-0.029]	-0.051*	[-0.077,-0.025]
38	Tacoma, WA	-0.005	[-0.035,0.024]	-0.005	[-0.035,0.025]
39	Tucson, AZ	-0.013	[-0.042,0.016]	-0.012	[-0.042,0.019]
40	Waco, TX	-0.013	[-0.037,0.011]	-0.014	[-0.039,0.011]
41	York, PA	0.011	[-0.012,0.035]	0.015	[-0.008,0.038]

Notes: ($\Diamond$) 95 percent family-wise confidence intervals are reported.
(*) Statistically significant estimates are highlighted by asterisk.
Cumulative location effects are defined as $c^{\delta}_{j,h} = \sum_{\ell=0}^{h} c_{jh}$, where c_{jh} are the location effects defined in Assumption 1. This table reports MGDL and augmented MGDL cumulative location effects estimates $\hat{c}^{\delta}_{i,h} = \sum_{\ell=0}^{h} \hat{c}_{ih}$ at horizon $h = 8$ quarters for the crude oil price shocks. MGDL estimates are based on DL regressions and augmented MGDL regressions $x_{ijt} = a_{ij} + \sum_{\ell=0}^{h} b_{ij\ell} v_{t-\ell} + e_{hit}$, and augmented MGDL estimates are based on regressions $x_{ijt} = a_{ij} + \sum_{\ell=0}^{h} b_{ij\ell} v_{t-\ell} + \varphi x_{i,j,t-h-1} + e_{hit}$, where x_{ijt} is log-difference of price for product category i in city j in period t from C2ER dataset, which spans $M = 43$ reported categories over $N = 41$ cities, covering $T = 104$ quarterly periods from 1990Q1 to 2015Q4.