Dollar Shortages, CIP Deviations, and the Safe Haven Role of the Dollar*

Philippe Bacchetta†, J. Scott Davis‡ and Eric van Wincoop§

November 30, 2023

Abstract

Since 2007, an increase in risk or risk aversion has resulted in a US dollar appreciation and greater deviations from covered interest parity (CIP). In contrast, prior to 2007, risk had no impact on the dollar, and CIP held. To explain these phenomena, we develop a two-country model featuring (i) market segmentation, (ii) limited CIP arbitrage (since 2007), and (iii) global dollar dominance. During periods of heightened global financial stress, dollar shortages in the offshore market emerge, leading to increased CIP deviations and a dollar appreciation. The appreciation occurs even in the absence of global dollar demand shocks. Central bank swap lines mitigate these effects.

Keywords: Dollar shortages, CIP deviations, Central bank swap lines

JEL Classifications: E44, F31, G15

* We gratefully acknowledge financial support from the Bankard Fund for Political Economy and the Swiss National Science Foundation. This paper represents the views of the authors, which are not necessarily the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

†Philippe Bacchetta, University of Lausanne, Swiss Finance Institute, and CEPR, philippe.bacchetta@unil.ch.
‡J. Scott Davis, Federal Reserve Bank of Dallas, scott.davis@dal.frb.org.
§Eric van Wincoop, University of Virginia and NBER, ev4n@virginia.edu.
1 Introduction

Prior to the Global Financial Crisis (GFC), the US dollar funding market was globally integrated, driven by effective international arbitrage. However, in the aftermath of the GFC, a significant transformation occurred, leading to a segmentation between onshore and offshore dollar markets. This segmentation is reflected in persistent cross-currency basis spreads or deviations from covered interest rate parity (CIP). In periods of heightened financial stress, offshore dollar funding markets experience dollar shortages that are associated with an increase in CIP deviations. At the same time, the US dollar has assumed the role of a safe haven currency, appreciating in periods of financial stress. Table 1 documents how CIP deviations and dollar appreciations are linked to financial stress. It reports regressions of the monthly log change in the US dollar nominal effective exchange rate or the monthly change in the Libor CIP deviation on the monthly change in one of eight measures of risk or risk aversion. Prior to 2007, changes in risk had no effect on the dollar exchange rate and CIP deviation. But over the 2007-2021 period, a rise in risk leads to a statistically significant dollar appreciation and rise in the CIP deviation in every column. The addition of the risk measure also leads to a sizable increase in the adjusted R-squared for the exchange rate.

While there is a large literature documenting and analyzing these features separately, there is little analysis attempting to explain the relationship between dollar shortages, CIP deviations and the safe haven properties of the US dollar. To shed light on these features, we develop a two-country general equilibrium model where both the spot exchange rate and the CIP deviation are determined endogenously. We show that conditions in the offshore dollar funding market affect the swap market and can lead to changes in both the CIP deviation and exchange rate. In a fully integrated global dollar market, an appreciation of the dollar can be explained by an increase in the global demand for dollar assets. With segmented markets, shocks to offshore dollar funding affect the dollar exchange rate, even with no change in the global demand of dollar assets.

In our two-country model, we refer to the Home country as the US and
Table 1: Regression of dollar exchange rate or CIP deviations on measures of risk

<table>
<thead>
<tr>
<th></th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent Variable: $\Delta s_{US,t}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_{US,t-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta Risk_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.786</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1.028)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Risk_{t-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.406</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.505)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.009</td>
<td>-0.007</td>
<td>0.001</td>
<td>0.020</td>
<td>0.001</td>
<td>-0.013</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent Variable: $\Delta CIP_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$CIP_{t-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.438***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.117)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta Risk_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.002</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.016)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Risk_{t-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.007</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.004)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.206</td>
<td>0.199</td>
<td>0.192</td>
<td>0.194</td>
<td>0.193</td>
<td>0.201</td>
<td>0.208</td>
<td>0.243</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1999-2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007-2021</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Notes: $s_{US,t}$ and $q_{US,t}$ are the logs of the nominal and real US effective exchange rate against advanced country currencies (written as USD/FCU); both exchange rate indices are calculated by the Fed Board of Governors. $CIP_t$ is an index of CIP deviations relative to the dollar, computed as the log of the swap rate plus the difference between the foreign and US 3-month Libor and aggregated using the same trade weights as in the effective exchange rate. $Risk_t$ is the level of one of eight risk measures: (1) the log of the VIX, (2) the log of the MOVE index, (3) the log of the risk aversion index from Bekaert et al. (2021), (4) the log of the uncertainty index from Bekaert et al. (2021), (5) the normalized intermediary capital risk factor from He et al. (2017), (6) the normalized intermediary value weighted investment return from He et al. (2017), (7) the log of the bond spread on senior unsecured debt of nonfinancial firms from Gilchrist and Zakrajšek (2012), (8) the normalized excess bond premium from Gilchrist and Zakrajšek (2012). The operator $\Delta$ is the month-over-month change. For scaling, all risk variables in the regression are divided by 100. All regressions include a constant and robust standard errors are written in parentheses, ***/**/* denotes significance at the 1/5/10% level.
the Foreign country as Europe where the currency is the euro, although it is meant to signify a broader rest of the world. In each country, there are lenders and borrowers in domestic and foreign currency. Apart from an initial period that captures the past, it is a two-period model. Prices are preset in the first period. Central banks provide liquidity in the domestic market at a fixed interest rate.

Markets are segmented in the sense that, for both currencies, there is an onshore and an offshore market and there may be imperfect arbitrage between the two. Global financial intermediaries, referred to as CIP arbitrageurs, are the only agents that arbitrage between onshore and offshore markets. In a frictionless world, arbitrage is perfect, so that onshore and offshore rates are equal. We assume that CIP arbitrageurs could fully arbitrage CIP deviations before 2007. As widely documented in the literature, balance sheet constraints have limited arbitrage since then, in part due to new leverage regulations. This means that the offshore markets may not clear at the same interest rate as the onshore markets.

The offshore dollar market is composed of both direct and synthetic dollar lending and borrowing. Synthetic dollar borrowing implies borrowing euros and swapping them into dollars by buying dollar swaps. The buyer of a dollar swap exchanges euros for dollars at the current spot exchange rate and sells these dollars in exchange for euros next period at the forward rate. Similarly, synthetic euro borrowing in the offshore euro market in the US leads to sales of dollar swaps. Global CIP arbitrageurs borrow dollars in the onshore US market and lend synthetic dollars to Europe. This also implies a supply of dollar swaps.

There is a dollar shortage when there is an excess demand for dollar swaps with a zero CIP deviation. This happens when the net demand for synthetic offshore dollar funding (borrowing minus lending) is higher than the net demand for offshore euro funding. This raises the offshore dollar interest rate, leading to a positive CIP deviation. Therefore, CIP deviations are an indicator of dollar shortages.

Asymmetries between the US and the rest of the world can cause both persistent and fluctuating dollar shortages, leading to both persistent and
fluctuating CIP deviations (as seen in the data). One such asymmetry is dollar dominance. We assume that the dollar is the dominant currency for invoicing in international trade. To see how this can lead to a persistent dollar shortage and therefore CIP deviation, consider the implications for foreign currency exposure. Dollar dominance in trade implies higher dollar money balances in Europe than euro balances in the US, leading to higher foreign currency exposure in Europe than the US. Hedging foreign currency exposure leads to larger foreign currency borrowing and lower foreign currency lending. This leads to a greater excess demand for offshore dollar funding than offshore euro funding, giving rise to a dollar shortage.

Shocks to the model can lead to time-varying dollar shortages, which lead to time-varying CIP deviations. We consider various shocks related to increased financial stress that lead to both a higher CIP deviation and dollar appreciation, consistent with Table 1. These shocks all involve a reallocation between offshore and onshore dollar assets, without changing the overall demand for dollar assets. With perfect CIP arbitrage, such a reallocation has no macroeconomic effects. Only the aggregate demand for dollar assets affects the exchange rate.\footnote{Under perfect CIP arbitrage the distinction between onshore and offshore dollar markets is irrelevant. The swap market plays no macroeconomic role either. The swap rate is redundant and is just equal to the interest rate differential.}

One example of such a shock is an increased global demand for money due to a preference for liquidity in a more uncertain environment (a “dash for cash”). With dollar dominance, this implies a larger increase in the demand for dollar money (a “dash for dollars”). To obtain higher onshore dollar money balances, lenders in Europe reduce offshore dollar lending, while borrowers increase offshore dollar borrowing. This does not change the total demand for dollar assets, but it leads to an excess demand for offshore dollar funding (borrowing rises, lending drops). This leads to a dollar shortage, which raises the synthetic dollar interest rate.

A higher offshore dollar rate in turn leads to a dollar appreciation. Borrowers and lenders both need dollars, for example to hold desired dollar money balances, pay for imports invoiced in dollars, repay dollar debt from the pre-
vious period or make new dollar loans. Borrowers can acquire these dollars either by borrowing or buying them on the spot market. A higher cost of dollar borrowing leads borrowers to buy more dollars on the spot market, causing an appreciation. Lenders wish to lend more dollars when the offshore dollar interest rate rises. To be able to do so, they need to buy more dollars on the spot market, also causing a dollar appreciation. With full CIP arbitrage, this liquidity shock would have no impact on the dollar since the global demand for dollar assets is unchanged.

Other examples of shocks that we consider are reduced CIP arbitrage during periods of increased financial stress and a portfolio reallocation by US agents from lending to Europe to lending to the US.\(^2\) Both of these shocks again involve a shift from offshore dollar assets to onshore dollar assets. These shocks all lead to stresses in the offshore dollar market that lead to a purchase of dollars in the spot market. This is a very different explanation for the safe haven role of the dollar than theories that are based on an overall increase in demand for dollar assets due to the appeal of certain US assets at times of heightened risk (e.g., Treasuries). The safe haven theory proposed here instead emphasizes the role of imperfect CIP arbitrage.\(^3\)

When CIP arbitrage is limited, it can be substituted by central bank swap lines. Beginning with the GFC, the Fed set up swap lines with the major foreign central banks exactly to provide offshore dollar lending. Thus, an excess demand for dollars in Europe can be met by the Fed, rather than through CIP arbitrageurs and the swap market. Our framework allows us to analyze the impact of these central bank swap lines. Consistent with the literature, we show that by alleviating dollar shortages, these swap lines limit CIP deviations and put a downward pressure on the dollar.

With the assumptions of dollar dominance, market segmentation, and heterogeneous agents, our model differs from most of the literature. While these assumptions give a relatively more complex model, they make it possible to talk about dollar shortages and understand their implications. Moreover, with

\(^2\)An example of the latter is the reduced lending by US money market funds to European banks during the 2008 global financial crisis and the European sovereign debt crisis.

\(^3\)In the data this safe haven role indeed started with the onset of CIP deviations, as shown in Table 1.
a set of convenient assumptions, the equilibrium of the model can be summarized by equilibrium in two markets: the spot and swap markets. This equilibrium, and the impact of shocks, can then be represented graphically and most of our results are derived analytically.

After a review of related literature, we describe the model in Section 2. It leads to equilibrium in the spot and swap markets, which jointly determine the equilibrium exchange rate and the synthetic dollar rate. Section 3 analyzes the implications of the model for pre-shock CIP deviations and explains the role of dollar dominance. Section 4 analyzes the response of the exchange rate and CIP deviations to shocks related to increased financial stress. Section 5 examines central bank swaps and Section 6 concludes.

**Related Literature**

The structure of the model is based on the literature describing recent dollar shortages and CIP deviations. The globally dominant role of the US dollar is well known. In an environment where the dollar is so widely used, dollar shortages can easily develop when global credit declines during times of global financial stress.

McCaulley and McGuire (2009) discuss the impact of such dollar shortages on CIP deviations and dollar appreciation during the GFC. As it became expensive to roll over maturing dollar debts through synthetic dollar borrowing (higher CIP deviation), firms bought dollars in the spot market (dollar appreciation). Ivashina et al. (2015) document how stress in dollar funding markets during the European sovereign debt crisis led to dollar shortages that increased CIP deviations.

Cesa-Bianchi et al. (2023) describe dollar shortages at the start of the Covid crisis. They write that “heightened economic and financial market uncertainty sparked a global dash for cash,” which led to a dash for dollars because of the dominance of the dollar as a funding currency. This led to a larger selloff of dollar-denominated than foreign currency-denominated assets. They show that, as a result, dollar corporate spreads rose faster than foreign currency corporate spreads.

There is an extensive literature documenting the limits to CIP arbitrage since the GFC, including Du et al. (2018), Diamond and Van Tassel (2023),
Rime et al. (2022), Boyarchenko et al. (2020) and Cenedese et al. (2021). Du and Schreger (2022) provide a survey of the literature on CIP deviations. Several papers, including Du et al. (2018) and Cenedese et al. (2021), provide evidence that tighter bank leverage regulations since the GFC have led to a higher cost of financial intermediation that is responsible for the CIP deviations since that time.

Borio et al. (2016) and Borio et al. (2018) describe in detail the link between persistent CIP deviations since the GFC and persistent imbalances in the FX swap market. On the one hand, there has been an increase in the demand for dollar swaps, in particular by foreign international institutional investors holding dollar assets. On the other hand, the supply of dollar swaps by global banks (CIP arbitrageurs) has been more limited due to the new leverage regulations since the GFC. Avdjiev et al. (2020) show that during the Covid-19 crisis this mechanism was accompanied by a significant increase in offshore dollar borrowing by corporate borrowers. Other papers documenting the increase in dollar hedging and imbalances in FX markets include Du and Huber (2023) and Hau and Bräuer (2022).

The results in our Table 1 are consistent with the literature. Several papers find, as we do, that higher risk has led to a dollar appreciation after 2007, but not before (e.g., Habib and Stracca, 2012; Georgiadis et al., 2021; Lilley et al., 2022). These papers do not simultaneously consider the relationship between risk and CIP deviations. Cerutti et al. (2021) do find that an increase in various risk measures has raised the CIP deviation since 2007.

Several papers have analyzed the impact of central bank swap lines that were established by central banks during and after the GFC (see Choi et al. (2022) for a description). They find in particular that central bank swap lines reduce CIP deviations (Bahaj and Reis, 2022; Cerutti et al., 2021; Rime et al., 2022; Ferrara et al., 2022; Goldberg and Ravazzolo, 2022). Moreover, Kekre and Lenel (2023b) show that swap line announcements over the 2007-2010 and 2020-2021 periods led to a dollar depreciation. Bahaj and Reis (2022) propose an interesting partial equilibrium model to understand the impact of central bank swaps on CIP deviations. Eguren-Martin (2020) proposes a two-country DSGE model to examine the impact of central bank swap lines, but does not
consider CIP deviations.

Some have argued that US government bonds have liquidity or collateral properties that are particularly attractive during times of crisis. The convenience yield perspective has been analyzed by Kekre and Lenel (2023a), Engel and Wu (2023), Jiang et al. (2023), Jiang et al. (2021), Bianchi et al. (2021) and Devereux et al. (2023). The literature has provided convincing evidence of a relationship between changes in relative convenience yields and exchange rates. However, this approach cannot explain why the effect of risk on the dollar was not present before 2007. Moreover, while there is evidence that convenience yields rise during periods of increased financial stress, Diamond and Van Tassel (2023) show that this is not the case for the relative US convenience yield.\footnote{Consistent with this, we show in the Online Appendix that most of the risk variables in Table 1 are unrelated to the relative US convenience yield.}

There are few papers examining the link between CIP deviations and spot exchange rates. Avdjiev et al. (2019) describe a triangular empirical relationship between the broad dollar exchange rate, CIP deviations and cross-border bank lending. While they make no claims about causality and do not develop a theory, they propose an explanation related to the risk-taking channel of exchange rates. They argue that a dollar appreciation weakens financial positions of global financial intermediaries by weakening the balance sheet of unhedged foreign dollar borrowers. This leads to both reduced CIP arbitrage by global financial intermediaries, and therefore a higher CIP deviation, and reduced cross-border banking flows. However, since this is based on an exogenous shock to the exchange rate, it is difficult to compare the pre- and post-2007 periods.

Liao and Zhang (2020) and Tsiang (1959) consider increased hedging of dollar exposures by foreign investors or borrowers due to increased foreign exchange risk. They find that this leads either to a higher CIP deviation and a dollar depreciation or a lower CIP deviation and dollar appreciation. While changes in hedging ratios are empirically important, this cannot explain the simultaneous increase in the CIP deviation and dollar appreciation associated with a rise in financial stress, as documented in Table 1. As we will see, a rise
in currency risk as in Liao and Zhang (2020) and Tsiang (1959) impacts both swap and spot market equilibria. In contrast, shocks leading to a reallocation between onshore and offshore dollar positions only affect the swap market equilibrium. We show that such shocks are capable of generating both a higher CIP deviation and a dollar appreciation.

Finally, Fang and Liu (2021) consider a framework with US financial intermediaries who arbitrage CIP and UIP deviations. An increase in uncertainty tightens the borrowing constraint of the intermediaries, which reduces arbitrage and increases both CIP and UIP deviations. The increase in the UIP deviation operates like an increase in the risk premium on the foreign currency, which leads to a dollar appreciation. The US intermediaries are the only agents operating in the FX swap market. In our model it is the interplay between limited arbitrage by such CIP arbitrageurs and time-varying demand for dollar swaps by other agents that is key. This allows us to consider different types of shocks, such as those that lead to dollar shortages in offshore dollar funding markets.

2 Model Description

There are two countries (Home and Foreign). We think of the Home country as the US and the Foreign country as the rest of the world. For convenience we will often refer to the latter as Europe and the currency as the euro. Although there are three periods (0, 1 and 2), it is more like a two period model (periods 1 and 2) as period 0 is the past. We take asset prices and financial holdings in period 0 as given. Our main focus will be on financial decisions and prices in period 1. Figure 1 presents a flow diagram that includes the agents and financial markets in the model. It shows how funds flow from lenders at the top of the diagram through financial markets in the middle to borrowers at the bottom.

In both countries there are borrowers and lenders of dollars and euros. In the US there is an onshore dollar market and an offshore euro market.

\footnote{In a related framework, Bacchetta et al. (2023) provide a model that links CIP deviations to the appreciation of other safe haven economies, e.g., Japan and Switzerland.}
In Europe there is an onshore euro market and an offshore dollar market. Offshore borrowing and lending can happen both through offshore bonds and through synthetic funding. For example, a European borrower can issue a dollar bond or borrow dollars synthetically by issuing a euro bond and then using the swap market to swap it into dollars. Figure 1 therefore shows that synthetic borrowing and lending is connected to the swap market. Consistent with empirical findings in Liao (2020), we assume arbitrage by borrowers, so that the interest rate is the same for the two types of offshore funding (offshore bonds and synthetic funding).

What is key is that the markets are segmented. Figure 1 shows that only CIP arbitrageurs arbitrage between US and European markets. With a positive CIP deviation, they wish to borrow dollars in the onshore US market and lend synthetic dollars to Europe. Borrowers only borrow in their domestic market, reflecting well-known frictions of accessing foreign credit markets. We allow two types of lenders in each country. The first are domestic lenders that provide funding in their domestic market in both currencies. The second are...
foreign lenders, who provide funding in both currencies in the foreign market. One could alternatively think of them as independently operated domestic and foreign branches of the same lender. Separating them allows us to avoid a situation where these institutions conduct CIP arbitrage by having a US branch of a European institution borrowing dollars in the US and then the European branch of the same institution lending these dollars in Europe.\footnote{Since we already have CIP arbitrageurs, it is of limited interest to introduce other agents that arbitrage between the onshore and offshore markets in the same currency. During the GFC, European banks extensively shifted dollar funding from US branches to European headquarters (see Cetorelli and Goldberg, 2012). Within the context of our model, one can interpret this as a form of CIP arbitrage done by CIP arbitrageurs. But since the GFC such arbitrage through internal capital markets has been discouraged through regulatory guidance. Our benchmark model does not allow for UIP arbitrage either (arbitrage between the two onshore markets in the respective currencies). However, we show in the Online Appendix that UIP arbitrage does not qualitatively affect the results as long as UIP arbitrageurs are risk averse.}

The other agents in Figure 1 are central banks. They only provide funding in the domestic onshore market. We assume that they provide enough liquidity to clear the onshore markets at the desired policy rate, which for simplicity we set equal to zero. Key to the model is that central banks do not provide liquidity to offshore markets. We will somewhat relax this in Section 5, when introducing central bank swap lines.

The empirical literature on FX hedge demand and its effect on the CIP deviation, such as Borio et al. (2016) and Borio et al. (2018), describes various institutions that can be thought of as the real world counterparts to the borrowers and lenders in the model. For example, non-US institutional investors often invest in US onshore assets, while swapping part of it back into their own currencies. This corresponds to foreign lenders in the model. Foreign corporations that issue dollar bonds outside the US are an example of domestic borrowers. We will not explicitly model banks. One can think of them as intermediating between lenders and borrowers, but they do not play a special role.

The remainder of this section is organized as follows. We start by introducing notation. After that we provide an overview of the financial markets, including the key role played by the swap market equilibrium. We next discuss the goods market in periods 1 and 2 and associated period 1 money demand.
This is followed by a discussion of portfolio decisions by borrowers and lenders in both countries. We finish with a discussion of equilibrium in the spot market.

2.1 Notation

Other than the central banks, there are three types of agents in each country, which we denote \( j = 1, 2, 3 \). Here \( j = 1 \) refers to borrowers, \( j = 2 \) to domestic lenders and \( j = 3 \) to foreign lenders. There is a continuum of agents on the interval \([0,1]\) in each country. The share of agents of type \( j \) is denoted \( \alpha_j \). We assume \( \alpha_1 = 1 - n \), \( \alpha_2 = n\lambda \) and \( \alpha_3 = n(1 - \lambda) \). This means that a fraction \( 1 - n \) are borrowers and a fraction \( n \) are lenders. Of the lenders, a fraction \( \lambda \) lend to domestic borrowers and a fraction \( 1 - \lambda \) lend to borrowers in the other country.

We denote \( W_{H,j,t} \) and \( W_{F,j,t} \) as financial wealth, excluding money balances, of a type \( j \) agent at the start of period \( t \) in respectively the Home country in dollars and Foreign country in euros. Aggregate consumption of these agents is denoted \( C_{H,j,t} \) and \( C_{F,j,t} \). Dollar and euro money holdings by type \( j \) agents in country \( h = H, F \) are \( M^\$_{h,j,t} \) and \( M^e_{h,j,t} \).

Onshore bond holdings are \( B^{\$}_{h,j,t} \) in the US in dollars and \( B^{e}_{h,j,t} \) in Europe in euros. These are negative for borrowers and positive for lenders. Global CIP arbitrageurs borrow \( B^{\$}_{CIP,H,t} \) dollars in the onshore dollar market.

We denote \( B^{\$}_{h,j,t} \) as the position in the European dollar funding market, including both offshore dollar bonds and synthetic dollar funding. Similarly, \( B^{e}_{h,j,t} \) is the position in the US euro funding market, again including both offshore euro bonds and synthetic euro funding.

The dollar spot and forward exchange rates are denoted \( S_t \) and \( F_t \). These are dollars per euro. The log exchange rate is denoted \( s_t = \log(S_t) \). The buyer of dollar swaps at time \( t \) exchanges euros for dollars at time \( t \) at the exchange rate \( S_t \) and sells these dollars back at time \( t + 1 \) at the exchange rate \( F_t \).

For \( t = 0, 1 \), dollar and euro interest rates in the onshore markets are \( i^\$_{t} \) and \( i^e_{t} \). The interest rates in the offshore dollar funding market in Europe and euro funding market in the US are \( i^{\$}_{t} \) and \( i^{e}_{t} \). These are the same for
offshore bonds and synthetic offshore funding, so that

$$1 + i^S_t = \frac{F_t}{S_t} \left(1 + i^F_t\right) = \frac{F_t}{S_t} = SR_t$$

(1)

$$1 + i^F_t = \frac{S_t}{F_t} \left(1 + i^H_t\right) = \frac{S_t}{F_t} = \frac{1}{SR_t}$$

(2)

The second equality uses that the onshore rates are set equal to zero by the central banks, i.e., $i^S_t = 0$ and $i^F_t = 0$. The synthetic dollar rate (plus 1) is then equal to $F_t/S_t$, which is the swap rate $SR_t$ in the swap market. Analogously, the synthetic euro rate (plus 1) is the inverse of the swap rate.

### 2.2 Overview of Financial Markets

#### 2.2.1 Two Equilibrium Conditions

As we will see, the model will be solved from two equilibrium conditions, for the spot and swap FX markets. The spot market equilibrium refers to the pure spot market, separate from the spot component of swap transactions. It is discussed in Section 2.6. The swap market equilibrium is discussed below in Section 2.2.2. These two equilibrium conditions will determine the spot rate $s_1$ and the swap rate, or analogously the synthetic dollar interest rate $i^S_1$.

However, there are 6 other financial markets, the 4 onshore and offshore markets in Figure 1 as well as two money markets that equate money demand to money supply. We do not need to be concerned with the onshore markets as we have assumed that the central banks provide sufficient funding in these markets to keep their interest rates equal to zero. We have also seen that the interest rates in both offshore markets depend on the swap rate, which is determined by equilibrium in the swap market.

The relative money market equilibrium condition equates relative money demand of dollars to euros to relative money supply of dollars to euros. As we show in the Online Appendix, we can rewrite this as the foreign exchange market equilibrium condition that equates the current account to net capital outflows. But the Online Appendix also shows that the same foreign exchange market clearing condition follows from the spot and swap market equilibrium.
conditions. Finally, the remaining money market clearing condition can be ignored due to Walras’ Law.

2.2.2 Connection between Offshore Markets and the Swap Market

It is important to understand how the two offshore markets are connected via the swap market. Consider the period 1 offshore dollar funding market in Europe. The demand for dollar funding by European borrowers is $-\alpha_1 B_{F,1,1}^S$. The supply of dollar funding in Europe by European and US lenders is $\alpha_2 B_{F,2,1}^S + \alpha_3 B_{H,3,1}^S$. The excess demand for dollar funding in Europe is then

$$D_{F,1}^{S,syn} = -\alpha_1 B_{F,1,1}^S - \alpha_2 B_{F,2,1}^S - \alpha_3 B_{H,3,1}^S$$  \hfill (3)

There is a superscript $\text{syn}$ as any excess demand for offshore dollar funding corresponds to an excess demand for synthetic dollar funding since the non-synthetic offshore dollar bond market must clear. When $D_{F,1}^{S,syn}$ is positive, there is net borrowing of offshore dollars. This leads to a demand for dollar swaps as euro borrowing is swapped into dollars.

Analogously, the excess demand for synthetic euro funding in the US is

$$D_{H,1}^{E,syn} = -\alpha_1 B_{H,1,1}^E - \alpha_2 B_{H,2,1}^E - \alpha_3 B_{F,3,1}^E$$  \hfill (4)

When positive, there is net borrowing of offshore euros. This creates a supply of dollar swaps as dollar borrowing is swapped into euros.

Finally, CIP arbitrageurs borrow $D_{CIP,1}^S$ dollars in the US and then lend the same amount of dollars in Europe through the synthetic dollar market. The latter involves lending euros in the onshore euro market in Europe and swapping into dollars by selling dollar swaps.

Appendix A discusses the exact swap market transactions that are associated with synthetic offshore borrowing and lending. Aggregating all these swap market transactions, we obtain the following period 1 swap market equilibrium:

$$\frac{F_1}{S_1} D_{F,1}^{S,syn} - S_1 D_{H,1}^{E,syn} - \frac{F_1}{S_1} D_{CIP,1}^{S,H} = 0$$  \hfill (5)

As discussed, a positive excess demand for offshore dollars (first term) leads
to a demand for dollar swaps, while both a positive excess demand for offshore euros (second term) and synthetic dollar funding by CIP arbitrageurs (last term) lead to the sale of dollar swaps.

We speak of a dollar shortage when there is an excess demand for dollar swaps at a zero CIP deviation. This means that the left hand side of (5) is positive when $i_1^{S,F} = i_1^{S,H}$. This excess demand leads to a rise in the swap rate, which implies a higher offshore dollar interest rate. This can for example be caused by an excess demand for offshore dollar funding. It is theoretically possible that the excess demand for offshore dollar funding is equal to the excess demand for offshore euro funding. In that case there is no excess demand for dollar swaps, so that there is no dollar shortage. However, we will consider asymmetries where the excess demand for offshore dollar funding dominates.

2.3 Goods Market and Money Demand

We first discuss the period 1 goods market, where prices are set in advance, and then the period 2 goods market, where prices are flexible. Consumption demand in period 1 also leads to proportional money demand expressions. There is no money demand in period 2.

2.3.1 Period 1 Goods Market

Home and Foreign agents produce differentiated goods. Prices are preset at 1 in the currency of invoicing. Goods sold domestically are invoiced in the domestic currency. Some of the goods sold abroad are invoiced in dollars and some in euros. Specifically, a fraction $a^e$ of US goods sold to Europe is invoiced in euros and a fraction $a^d$ of European goods sold to the US is invoiced in dollars. We would have $a^d = a^e = 0$ under PCP (Producer Currency Pricing) and $a^d = a^e = 1$ under LCP (Local Currency Pricing). However, here we will assume dollar dominance (DCP) in international goods trade by assuming $a^d > a^e$. The extreme case where all international trade is invoiced in dollars corresponds to $a^d = 1$ and $a^e = 0$. 

15
The period 1 consumption index in the Home and Foreign country is

\[ C_{H,j,1} = \left( (1 - \omega) \theta (C_{HH,j,1})^{\theta - 1} + (\omega (1 - a^S)) \theta (C_{HF,j,1})^{\theta - 1} + (\omega a^S) \theta (C_{H,H,j,1})^{\theta - 1} \right)^{\frac{\theta - 1}{\theta}} \]

\[ C_{F,j,1} = \left( (1 - \omega) \theta (C_{FF,j,1})^{\theta - 1} + (\omega (1 - a^E)) \theta (C_{FH,j,1})^{\theta - 1} + (\omega a^E) \theta (C_{F,H,j,1})^{\theta - 1} \right)^{\frac{\theta - 1}{\theta}} \]

Here the notation \( C_{HF,j,1} \) has a HF subscript, referring to the countries of the buyer and seller. So this means consumption by a type \( j \) Home agent of a Foreign good invoiced in euros. Analogous notation applies to the others.

Given these consumption indices, Home and Foreign consumer price indices in respectively dollars and euros are

\[ P_1 = \left( (1 - \omega) + \omega a^S + \omega (1 - a^S) S_1^{1-\theta} \right)^{\frac{1}{1-\theta}} \]

\[ P_1^* = \left( (1 - \omega) + \omega a^E + \omega (1 - a^E) S_1^{\theta-1} \right)^{\frac{1}{1-\theta}} \]

Using that prices are set at 1 in the currency of invoicing, consumption by Home agents of Home goods and the two types of Foreign goods (distinguished by currency of invoicing) is

\[ C_{HH,j,1} = (1 - \omega) \left( \frac{1}{P_1} \right)^{-\theta} C_{H,j,1} \]  

\[ C_{HF,j,1} = \omega (1 - a^S) \left( \frac{1}{S_1 P_1} \right)^{-\theta} C_{H,j,1}; \quad C_{H,H,j,1} = \omega a^S \left( \frac{1}{P_1} \right)^{-\theta} C_{H,j,1} \]  

Similarly, consumption by Foreign agents is

\[ C_{FF,j,1} = (1 - \omega) \left( \frac{1}{P_1^*} \right)^{-\theta} C_{F,j,1} \]

\[ C_{FH,j,1} = \omega (1 - a^E) \left( \frac{1}{S_1 P_1^*} \right)^{-\theta} C_{F,j,1}; \quad C_{F,H,j,1} = \omega a^E \left( \frac{1}{P_1^*} \right)^{-\theta} C_{F,j,1} \]

This is analogous to Betts and Devereux (2000). One piece that we are not explicit about here is that goods are differentiated by agents producing them, giving them price setting power. But all agents producing the same good will end up setting the same price, which we normalize to 1 in the currency of invoicing.
Production of the goods corresponds to demand. All agents in each country receive the same income from production. The resulting income of Home agents in dollars and Foreign agents in euros is denoted respectively \( Y_{H,1} \) and \( Y_{F,1} \):

\[
Y_{H,1} = \sum_{j=1}^{3} \alpha_j \left( C_{HH,j,1} + C_{FH,j,1}^S + S_1 C_{FH,j,1}^e \right)
\]

\[\text{(12)}\]

\[
Y_{F,1} = \sum_{j=1}^{3} \alpha_j \left( C_{FF,j,1} + \frac{1}{S_1} C_{HF,j,1}^S + C_{HF,j,1}^e \right)
\]

\[\text{(13)}\]

### 2.3.2 Money Demand

We assume that money demand in period \( t = 1 \) is equal to a fraction \( \psi \) of consumption of goods invoiced in the corresponding currency:\(^8\)

\[
M_{H,j,1}^S = \psi (1 - \omega + \omega a^S) P_1^\theta C_{H,j,1}
\]

\[\text{(14)}\]

\[
M_{H,j,1}^e = \psi (1 - \omega) S_1^{-\theta} P_1^\theta C_{H,j,1}
\]

\[\text{(15)}\]

\[
M_{F,j,1}^S = \psi (1 - a^e) (S_1 P_1^* )^{\theta} C_{F,j,1}
\]

\[\text{(16)}\]

\[
M_{F,j,1}^e = \psi (1 - \omega + \omega a^e) (P_1^* )^{\theta} C_{F,j,1}
\]

\[\text{(17)}\]

This for example implies that a larger quantity of dollar invoiced imports leads to larger dollar money balances in Europe. This is a feature also present in Gopinath and Stein (2021).

### 2.3.3 Period 2 Goods Market

In period 2 prices are flexible. There is a Home good and a Foreign good, with aggregate endowments of

\[
Q_{H,2} = e^{\kappa H \epsilon_q}
\]

\[
Q_{F,2} = e^{-\kappa F \epsilon_q}
\]

---

\(^8\)An infinitesimal cost of holding money is sufficient to make sure that bonds dominate money even at the ZLB.
where $\kappa_H + \kappa_F = 1$ and $\epsilon_q$ is a period 2 endowment shock with mean of zero. In both countries lenders receive a fraction $a_l$ of the endowment and borrowers a fraction $a_b$, with $na_l + (1 - n)a_b = 1$. Borrowers receive higher period 2 income ($a_b > a_l$), allowing them to repay their debt. There is a CES period 2 consumption index with equal weight to both goods and an elasticity of substitution of $\bar{\theta}$. Central banks target a price of 1 of the domestic good in the domestic currency.

We leave further details regarding the period 2 goods market equilibrium to Appendix B. In equilibrium $s_2 = \epsilon_q/\bar{\theta}$, where $s_2$ is the log exchange rate in period 2. Therefore $E(s_2) = 0$. Let $a_j = a_b$ when $j = 1$ (borrower) and $a_j = a_l$ when $j = 2, 3$ (lender). The period 2 income of lenders and borrowers in both countries is then

$$Y_{H,j,2} = a_j e^{\kappa_H \bar{\theta}s_2} \quad (18)$$

$$Y_{F,j,2} = a_j e^{-\kappa_F \bar{\theta}s_2} \quad (19)$$

Both countries then have exposure to the foreign currency through non-asset income, with a weaker foreign currency lowering their income.

### 2.4 CIP Arbitrageurs

CIP arbitrageurs borrow $D_{CIP,t}^{S,H}$ dollars in the US, convert them to $(1/S_t)D_{CIP,t}^{S,H}$ euros, which is invested in zero interest euro bonds in Europe. These euros are sold forward at $t+1$ for $(F_t/S_t)D_{CIP,t}^{S,H}$ dollars. The spot and forward transactions are part of an FX swap.

As the interest on dollar borrowing in the US is zero, the period $t+1$ profit is then

$$\Pi_{t+1} = \left( \frac{F_t}{S_t} - 1 \right) D_{CIP,t}^{S,H} = i_t s_t F_{CIP,t} D_{CIP,t}^{S,H} \quad (20)$$

We introduce a quadratic regulatory cost that limits arbitrage, equal to $0.5\phi \left( D_{CIP,t}^{S,H} \right)^2$. It is assumed not to affect aggregate resources of the economy. Arbitrageurs
then maximize $\Pi_{t+1} - 0.5\phi \left( D_{CIP,t}^{S,H} \right)^2$, so that

$$D_{CIP,t}^{S,H} = \frac{i_t^{S,F}}{\phi}$$  \hspace{1cm} (21)

Profits are

$$\Pi_{t+1} = \frac{(i_t^{S,F})^2}{\phi}$$  \hspace{1cm} (22)

We assume that half of these profits are associated with Home CIP arbitrageurs and the other half with Foreign CIP arbitrageurs. These profits are transferred equally to all Home and Foreign agents. We denote Home profits in dollars as $\Pi_{H,t+1} = 0.5\Pi_{t+1}$ and Foreign profits in euros as $\Pi_{F,t+1} = 0.5\Pi_{t+1}/S_{t+1}$.

## 2.5 Portfolios of Borrowers and Lenders

Borrowers and lenders need to make a portfolio choice between dollar and euro assets. Onshore assets are always onshore bonds. Offshore assets can be either offshore bonds or synthetic positions. For example, agents in the US market could invest in onshore dollar and offshore euro bonds. But they could also just invest in onshore dollar bonds and hedge part of it by swapping it into euros. Either way, they achieve the same overall dollar and euro asset positions.

For illustrative purposes we focus here on the portfolio problem of European borrowers ($j = 1$) and European lenders to the domestic market ($j = 2$). The Online Appendix derives the portfolios for all other agents. We assume that period 1 portfolios are determined by maximizing a simple mean-variance objective related to the log of period 2 consumption:

$$Ec_{F,j,2} - 0.5\gamma var(c_{F,j,2})$$  \hspace{1cm} (23)

When consumption is log-normal, this is analogous to maximizing $EC_{F,j,2}^{1-\bar{\gamma}}/(1 - \bar{\gamma})$ with $\gamma = \bar{\gamma} - 1$. To see this, we can write $EC_{F,j,2}^{1-\bar{\gamma}}/(1 - \bar{\gamma}) = -(1/\gamma) Ec_{F,j,2}^{-\gamma} + 0.5\gamma^2 var(c_{F,j,2})$. Maximizing this is equivalent to maximizing (23).
allocation. Since our focus is on financial markets, we simplify period 1 consumption. After assuming that period 1 consumption is perfectly smoothed with expected period 2 consumption in a pre-shock equilibrium, we hold period 1 consumption constant after introducing financial shocks in Section 4.

The period 2 budget constraint for Foreign agents \( j = 1, 2 \) is

\[
P_2^{*}C_{F,j,2} = Y_{F,j,2} + \Pi_{F,2} + \frac{1}{S_2}M_{F,j,1}^S + M_{F,j,1}^E + W_{F,j,1} + \left( \frac{1 + i_1^{S,F}}{S_2} - \frac{1}{S_1} \right) B_{F,j,1}^{S,F}
\]

(24)

where \( W_{F,j,1} = (1/S_1)B_{F,j,1}^{S,F} + B_{F,j,1}^{E,F} \) is period 1 wealth. Period 2 consumption is equal to period 2 income \( Y_{F,j,2} + \Pi_{F,2} \) plus the period 2 value of period 1 money balances, plus period 1 financial wealth, plus the excess return on period 1 dollar asset holdings.

We log-linearize the second period budget constraint (24) around \( s_2 = 0, \ i_1^{S,F} = 0 \) and \( C_{F,j,2} = \bar{C}_{F,j,2} \), which is the pre-shock second period consumption level at \( s_2 = 0 \) discussed below. We then have \(^{10}\)

\[
\bar{C}_{F,j,2} + C_{F,j,2}(c_{F,j,2} - \bar{c}_{F,j,2}) = a_j - \rho_{F,j}s_2 + \Pi_{F,2} + M_{F,j,1}^E + (1 - s_2)M_{F,j,1}^S + W_{F,j,1} + (i_1^{S,F} - s_2 + s_1)B_{F,j,1}^{S,F}
\]

(25)

where \( \rho_{F,j} = \kappa_F a_j \bar{\theta} - 0.5 \bar{C}_{F,j,2} \).

Maximizing the mean-variance second period consumption objective (23) then gives for \( j = 1, 2 \)

\[
B_{F,j,1}^{S,F} = -\rho_{F,j} - M_{F,j,1}^S + \bar{C}_{F,j,2}\frac{i_1^{S,F} + s_1}{\gamma \text{var}(s_2)}
\]

(26)

We find the same portfolio expression for \( B_{F,j,1}^{S,H} \) (dollar lending in the US by European lenders). The analogous equation for euro borrowing and lending

\(^{10}\)This uses that the log Foreign period 2 price level in Appendix B is linearized as \(-0.5s_2\) and second period income \( Y_{F,j,2} \) is linearized as \( a_j - \kappa_F a_j \bar{\theta} s_2 \).
by US agents is\footnote{Here we use that $i_{1}^{e,H} = -i_{1}^{s,F}$. This follows from log-linearizing the expression $1 + i_{1}^{e,H} = 1/(1 + i_{1}^{s,F})$ that follows from (1)-(2).}

$$
B_{H,j,1}^{e,H} = -\rho_{H,j} - M_{H,j,1}^{e} - C_{H,j,2}^{e} \frac{i_{1}^{s,F} + s_{1}}{\gamma \text{var}(s_{2})} \tag{27}
$$

where $\rho_{H,j} = \kappa_{H} a_{j} \bar{\theta} - 0.5 \bar{C}_{H,j,2}$. This holds for $j = 1, 2$ and is the same for $B_{H,3,1}^{e,F}$ (euro lending in Europe by US lenders).

The last term in both (26) and (27) captures expected excess returns. In both countries the expected excess return of dollars over euros is $i_{1}^{s,F} + s_{1}$, using that $E(s_{2}) = 0$. An increase leads Europeans to hold more dollar assets and US agents to hold fewer euro assets.

The first two terms are hedge terms. The terms $\rho_{F,j}$ and $\rho_{H,j}$ capture foreign currency exposure through period 2 non-asset income and the period 2 consumer price index. Higher foreign currency exposure lowers the dollar asset position of European lenders and increases dollar borrowing by European borrowers. Similarly, European lenders reduce their dollar asset position when they hold more dollar money, while European debtors increase their dollar debt. This offset is one-for-one. Holding more dollar money balances leads to an equal drop in dollar lending.

Du and Huber (2023) find that foreign lenders to the US hedge their dollar positions much more than US lenders abroad. This is consistent with the portfolios above as long as there is dollar dominance. Dominance of the dollar in trade invoicing ($a^{e} > a^{s}$) implies $M_{F,j,1}^{s} > M_{H,j,1}^{e}$.

2.6 Spot Market Equilibrium

Define $Q_{F,j,1}^{s,spot}$ as period 1 spot market purchases of dollars by European agents, and $Q_{H,j,1}^{e,spot}$ as period 1 spot market purchases of euros by US agents. These are pure spot market transactions, separate from the spot component of swap
transactions. Equilibrium in the spot market is then\(^\text{12}\)

\[
\sum_{j=1}^{3} \alpha_j Q_{F,j,1}^{\text{spot}} = S_1 \sum_{j=1}^{3} \alpha_j Q_{H,j,1}^{\text{spot}} \tag{28}
\]

In the Online Appendix we discuss the spot market transactions of all agents. For illustrative purposes, we focus here on agents of type \(j = 1\) in Europe, which are European borrowers. For each European agent, period 1 dollar income from dollar invoiced exports is

\[
Y_{F,1}^\$ = \sum_{j=1}^{3} \alpha_j C_{H,F,j,1}^\$ \tag{29}
\]

Purchases of dollars on the spot market by European borrowers \((j = 1)\) in period 1 are

\[
Q_{F,j,1}^{\$\text{,spot}} = dM_{F,j,1}^\$ - \Pi_{F,1} - Y_{F,1}^\$ + C_{F,H,j,1}^\$ - (1 + i_{F,0}^\$)B_{F,j,0}^\$,F + B_{F,j,1}^\$,F \tag{30}
\]

Consider the terms on the right hand side of (30). A rise in desired dollar money balances raises demand for dollars. Dollar profits from CIP arbitrageurs and dollar income from dollar invoiced exports reduce demand for dollars. Dollar invoiced imports of US goods raise demand for dollars. Finally, when payments \(-(1 + i_{F,0}^\$)B_{F,j,0}^\$,F\) on dollar debt from the previous period are higher than new dollar borrowing \(-B_{F,j,1}^\$,F\), there will be a demand for dollars on the spot market to meet these obligations.

After deriving analogous spot market transactions for all agents, and then imposing spot market equilibrium (28), the Online Appendix derives the fol-

\(^{12}\)As described in Appendix A, synthetic borrowing and lending of dollars includes a small spot market transaction as well. This is a technical issue related to the definition of a swap. We will abstract from it here, but it is included when deriving the spot market equilibrium in the Online Appendix.
ollowing spot market equilibrium:

\[
\begin{align*}
\sum_{j=1}^{3} \alpha_j dM^S_{F,j,1} - S_1 \sum_{j=1}^{3} \alpha_j dM^\epsilon_{H,j,1} + T A^S_{H,1} - \Pi_{F,1} \\
- \sum_{j=1}^{3} \alpha_j dB^{S,F}_{F,j,0} + S_1 \epsilon_{q,0} \sum_{j=1}^{2} \alpha_j B^{\epsilon,H}_{H,j,0} \\
+ \sum_{j=1}^{2} \alpha_j dB^{S,F}_{F,j,1} + \alpha_3 dB^{S,H}_{F,3,1} - S_1 \sum_{j=1}^{2} \alpha_j dB^{\epsilon,H}_{H,j,1} - \alpha_3 S_1 dB^{\epsilon,F}_{H,3,1} = 0
\end{align*}
\]  

where \( T A^S_{H,1} \) is the US trade account in period 1.

\section{Pre-Shock Equilibrium}

Before introducing period 1 shocks, we solve the pre-shock equilibrium. Given any set of model parameters, including the values of period 0 variables that we take as given, we can solve for the period 1 equilibrium. However, we limit ourselves to parameters that generate a sort of pre-shock steady state, with the following features: (1) equilibrium period 1 variables are equal to period 0 variables, (2) consumption is smoothed in that period 1 consumption of all agents is equal to period 2 consumption when the period 2 shock \( \epsilon_q \) is zero. We also assume that \( s_0 = 0 \), so that \( s_1 = 0 \) as well. Appendix C discusses how to compute such pre-shock equilibria.

In the next section we will discuss the impact of shocks both when the CIP deviation is zero in the pre-shock equilibrium and when it is positive. The advantage of a zero pre-shock CIP deviation is that we can solve the spot and swap market equilibria analytically after linearizing. But a positive pre-shock CIP deviation fits more closely with reality since 2007. In that case we pick a particular set of parameters shown in Table 2.

We show in Appendix C that the pre-shock CIP deviation is zero when

\[
\bar{\theta} (\kappa_F - \kappa_H) + \psi \omega (a^S - a^\epsilon) = 0
\]  

If instead the left hand side is positive, the pre-shock CIP deviation is positive.
Table 2: Benchmark Parameters

<table>
<thead>
<tr>
<th></th>
<th>$\kappa_F$</th>
<th>$\omega$</th>
<th>$\phi$</th>
<th>$\psi$</th>
<th>$a^s$</th>
<th>$a^e$</th>
<th>$a^b$</th>
<th>$a^l$</th>
<th>$\sigma_s$</th>
<th>$\psi$</th>
<th>$\lambda$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_H$</td>
<td>0.50</td>
<td>0.3</td>
<td>2</td>
<td>0.3</td>
<td>0.5</td>
<td>0</td>
<td>1.8</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>0.7</td>
<td>0.5</td>
</tr>
</tbody>
</table>

To understand this, consider a zero pre-shock CIP deviation. In that case $i_1^{\text{F},F} = 0$ and $F_1/S_1 = 1$. The swap market equilibrium (5) then implies that $D_{F,1}^{\text{syn}} = D_{H,1}^{\text{syn}}$. The excess demand for offshore dollar funding must be equal to excess demand for offshore euro funding at $(i_1^{\text{F},F}, s_1) = (0, 0)$. Using the expressions for the optimal portfolios, Appendix C shows that this is the case when (32) is satisfied.

The pre-shock CIP deviation is related to dollar dominance. Consider the case where $\kappa_H = \kappa_F$ and $a^s > a^e$, which is assumed in Table 2. The left hand side of (32) is positive in this case, so that there is a positive pre-shock CIP deviation.

Intuitively, since the dollar is the dominant invoicing currency, demand for dollar money balances in Europe is larger than demand for euro money balances in the US. To hedge the associated exchange rate risk, this reduces foreign currency lending and raises foreign currency borrowing more in Europe than in the US, implying a higher excess demand for synthetic dollar funding than synthetic euro funding. At a zero CIP deviation, there will then be an excess demand for dollar swaps. This raises the swap rate in equilibrium. The resulting higher synthetic dollar rate implies a positive CIP deviation.\textsuperscript{13}

Other than $a^s > a^e$, it is useful to comment on some of the other the parameters in Table 2. The values of $a_b$ and $a_l$ imply that the initial financial wealth of Home and Foreign borrowers is about -1.1, while the initial financial wealth of each of the four types of lenders is 0.5. We assume that half of the

\textsuperscript{13}It is immediate from (32) that $\kappa_F > \kappa_H$ also leads to a positive CIP deviation. The logic is very similar. In this case there is higher foreign currency exposure in Europe through non-asset income rather than dollar money balances.
agents are borrowers and half lenders, while 70% of lenders provide domestic funding and 30% provide funding in the foreign market. These parameters lead to a pre-shock equilibrium where $i^{S,F} = 0.0107$. This is a bit larger than CIP deviations observed in the data. However, as a result of the stylized two-period nature of the model, we see the model as providing qualitative, not quantitative, insights.

4 Shocks

We now discuss the response of the synthetic dollar rate and exchange rate to period 1 shocks. We start by describing the equilibrium spot and swap market schedules. We first ask how these schedules would need to shift to account for the evidence in Table 1. We conclude that we should consider shocks that affect the swap market schedule and leave the spot market schedule unchanged. An upward shift of the swap market schedule creates a dollar shortage, while an unchanged spot market schedule implies that there is no change in the global demand for dollar assets. We then consider three such shocks: a liquidity preference shock, reduced CIP arbitrage and a shift by US lenders from offshore to onshore dollar lending. All three are plausibly related to a more uncertain environment and deliver results consistent with Table 1.

After that, we consider two other shocks that can be related to an increase in risk or risk aversion. One is a relative US convenience yield shock. The other is a rise in the risk aversion parameter $\gamma$. However, these two shocks also affect the global demand for dollar assets, which shifts the spot market schedule. We show that these shocks are not consistent with the evidence of Table 1.

4.1 Spot and Swap Market Equilibria

We can derive the spot and swap market equilibrium schedules analytically in the special case where the pre-shock CIP deviation is zero. We will derive these schedules numerically when starting from a positive pre-shock CIP deviation, but they look the same.
Without any shocks, and assuming a zero pre-shock CIP deviation, Appendix D shows that the spot and swap market equilibria can be linearized as

\[
\nu_1 s_1 + 2 \frac{s_{F,1} + s_1}{\gamma \text{var}(s_2)} = 0 \quad (33)
\]

\[
(2\alpha_3 \nu_1 - \nu_2) s_1 + 2 \frac{s_{F,1} + s_1}{\gamma \text{var}(s_2)} + \frac{1}{\phi} s_{F,1} = 0 \quad (34)
\]

where

\[
\nu_1 = \omega (1 - \omega) \theta (2 - a^g - a^e) + \omega (a^e + a^g - 1)
\]

\[
\nu_2 = \bar{\psi} \omega \theta [(1 - \omega + \omega a^e)(1 - a^e) + (1 - \omega + \omega a^g)(1 - a^g)]
\]

The coefficient \(\nu_2\) is clearly positive. The coefficient \(\nu_1\) is positive as well, assuming \(\theta > 1\) (necessary for price-setting) and \(\omega < 0.5\) (goods home bias). The linearized US trade account in period 1 is \(TA = \nu_1 s_1\), so that a positive \(\nu_1\) implies a standard rise in the trade account in response to a dollar depreciation due to expenditure switching.

We can express \(i_{1,F}^s\) as linear functions of \(s_1\). From (33) the spot market equilibrium schedule is clearly negatively sloped. From (34) the swap market equilibrium schedule is also negatively sloped when \(2/[\gamma \text{var}(s)] + 2\alpha_3 \nu_1 > \nu_2\).

In this case, a sufficient condition for it to be less negatively sloped than the spot market equilibrium is that lenders are biased towards the domestic market (so that \(\lambda > 0.5\), implying \(\alpha_3 < 0.5\)). Figure 2 shows the spot and swap market equilibrium schedules when the swap market equilibrium schedule is negatively sloped. The case where the swap market equilibrium is positively sloped only occurs under extreme parameter values\(^{15}\) and even in that case it will not affect the main conclusions below.

From Figure 2 we see that there are two types of shifts of these schedules that both raise the CIP deviation \(i_{1,F}^s\) and cause a dollar appreciation (drop in \(s_1\)). These are an upward shift in the swap market schedule and a downward

\(^{14}\)To simplify further, we make the additional assumption in Appendix D that the pre-shock excess demand for offshore funding is zero in both countries.

\(^{15}\)For example, for the parameters in Table 2 we need to raise risk aversion above 3500.
shift in the spot market schedule. However, a downward shift in the spot market schedule implies that the dollar appreciates even when there is perfect CIP arbitrage, in which case $\phi \to 0$ and the swap market schedule becomes horizontal. This is inconsistent with the pre-2007 evidence in Table 1.

We therefore consider shocks with three characteristics. First, the shock shifts the swap market schedule upwards. This creates a dollar shortage. Second, it does not shift the spot market schedule. This means that there is no shift in the global demand for dollar assets. Finally, it must be plausibly related to a more uncertain global environment or higher risk aversion. The first two characteristics, when taken together, imply a reallocation from net dollar lending (lending minus borrowing) in Europe to net dollar lending to the US. This creates a dollar shortage in Europe, while it does not affect the overall demand for dollar assets. We now consider shocks with these features.
4.2 Liquidity Preference Shock

A flight to liquidity is common during periods of increased financial stress.\(^{16}\) Here we model a global liquidity preference shock through a rise in \(\psi\), which we can think of as a global dash for cash.

The portfolio expressions show that there is a drop in lending and a rise in borrowing in both currencies that is equal to the desired increase in money balances. Lending in both currencies declines as lenders switch from bonds to cash. Borrowing increases to raise desired money balances in both currencies.\(^{17}\)

As a result of the dollar dominance assumption \(a^d > a^e\), a dash for cash implies a dash for dollars. All money balances rise proportionately when \(\psi\) rises. But when \(a^d > a^e\), the dollarization of trade leads to larger dollar than euro money balances globally. This implies a larger dash for dollars than for euros. An increase in \(\psi\) in period 1 in deviation from its pre-shock value is denoted \(\hat{\psi}\). Appendix D then shows that the spot and swap market equilibria become

\[
\nu_1 s_1 + \frac{2 t_1^{S,F} + s_1}{\gamma var(s_2)} = 0
\]

\[
(2\alpha_3 \nu_1 - \nu_2) s_1 + \frac{2 t_1^{S,F} + s_1}{\gamma var(s_2)} + \frac{1}{\phi} t_1^{S,F} - \hat{\psi} \omega [a^d - a^e] = 0
\]

It is immediate from (35)-(36) that a rise in \(\psi\) in period 1 (\(\hat{\psi} > 0\)) shifts the swap market equilibrium schedule upwards, while it leaves the spot market schedule unchanged. The unchanged spot market schedule is a result of a reallocation between onshore and offshore dollar assets, without any change

\(^{16}\)In the context of the model here, liquidity refers to money. This is also the case in Cesa-Bianchi et al. (2023), who refer to it as a “dash for cash”. In Bianchi et al. (2021), who refer to it as “scrambling for dollars” during times of increased funding risk, it refers broadly to liquid assets of banks, including both Treasury bills and reserves of banks at the Fed. In other contexts it refers to an increased demand for assets that are easily convertible into money. This is the case in Longstaff (2004) and Vayanos (2004), who both refer to it as a “flight to liquidity” in uncertain times. While these papers have in mind investors and banks, we also see an increased demand for cash by firms during increased uncertainty. See for example Li (2019).

\(^{17}\)While not explicitly modeled, it is natural for lenders to become more cautious during periods of increased uncertainty and for borrowers concerned with funding risk to develop precautionary cash reserves.
in the total demand for dollar assets. For example, for offshore dollar lenders the rise in \( \psi \) implies a rise in dollar money balances in the US and an equal drop in offshore dollar lending in Europe. Similarly, European borrowers also hold more dollar money balances in the US and a more negative offshore dollar position (more dollar borrowing). Figure 3 illustrates the effect of a rise in \( \psi \). It leads to a rise in the synthetic dollar rate \( i_{1}^{s,F} \) (and therefore a higher CIP deviation) and an appreciation of the dollar (drop in \( s_{1} \)).

![Figure 3: Liquidity preference shock, rise in \( \psi \)](image)

The dash for dollars leads to a dollar shortage as it is associated with increased dollar borrowing and reduced dollar lending in the offshore dollar market. The resulting higher net synthetic dollar borrowing raises the synthetic dollar rate.\(^{18}\) The higher synthetic dollar interest rate raises offshore dollar lending and lowers offshore dollar borrowing. For both lenders and borrowers, this reduces their dollar money balances below desired levels. This leads to an increased demand for dollars in the spot market, causing the dollar to appreciate. The dash for liquidity, and specifically dollars, during times of financial stress therefore implies both a rise in the synthetic dollar rate and

\(^{18}\)There is also a rise in net synthetic euro borrowing in the US, but this is smaller as a result of dollar dominance.
a dollar appreciation, consistent with empirical evidence for the 2007-2021 period in Table 1.

The solution to the linear system (35)-(36) is

\[ s_1 = -\frac{\omega(a^S - a^E)}{(1 - 2\alpha_3)\nu_1 + \nu_2 + (1/\phi)(1 + 0.5\gamma \text{var}(s_2)\nu_1)} \hat{\psi} \] (37)

\[ \frac{s_{i_1}}{i_1} = \frac{(1 + 0.5\gamma \text{var}(s_2)\nu_1)\omega(a^S - a^E)}{(1 - 2\alpha_3)\nu_1 + \nu_2 + (1/\phi)(1 + 0.5\gamma \text{var}(s_2)\nu_1)} \hat{\psi} \] (38)

It is immediate that when \( \phi \to 0 \), so there there is perfect CIP arbitrage, the shock affects neither the dollar synthetic rate nor the exchange rate. The swap market schedule remains horizontal at \( i_{1,F}^S = 0 \). This is consistent with empirical evidence prior to 2007 in Table 1 that an increase an risk affected neither the exchange rate nor the CIP deviation at that time.

So far we have discussed a broad increase in \( \psi \) for all agents and both currencies. We can also consider cases where \( \psi \) only rises in period 1 for borrowers or only for lenders. There may also be a rise in \( \psi \) limited to dollar money balances, which leads to an even stronger dash for dollars. If we refer to the additive term \( \hat{\psi}\omega[a^S - a^E] \) in the swap market equilibrium schedule (36) as shock, in the case of a shock to \( \psi \) for borrowers only (\( \text{shock}^b \)), for lenders only (\( \text{shock}^l \)) and for dollar balances only (\( \text{shock}^s \)), we need to replace shock by respectively

\[ \text{shock}^b = \alpha_1 \hat{\psi}\omega[a^S - a^E] \]

\[ \text{shock}^l = (\alpha_2 + \alpha_3) \hat{\psi}\omega[a^S - a^E] \]

\[ \text{shock}^s = \hat{\psi}((\alpha_1 + \alpha_2)\omega(1 - a^E) + \alpha_3(1 - \omega + \omega a^S)) \]

In all cases the swap market schedule shifts upward, leading to the same result. The biggest effect quantitatively occurs when there is a dollar liquidity shock (\( \text{shock}^s > \text{shock} \)). It remains the case that the spot market schedule is unaffected as the total demand for dollar assets is unaffected.

Returning to the case with a uniform rise in \( \psi \), Figure 4A provides a numerical illustration for the parameters in Table 2, so that we start from a positive pre-shock CIP deviation. Both the spot and swap market equilibrium
schedules in Figure 4A are computed numerically. Chart A on the left considers a rise in $\psi$ from 0.3 to 0.4 in period 1 for all agents. As in the analytical solution above, the shock leads to a rise in the synthetic dollar rate and an appreciation of the dollar. Figure 4A also illustrates an interesting interaction between the spot and swap market equilibria that amplifies the impact of the shock. Holding the exchange rate constant, the rise in the synthetic dollar rate needed to clear the swap market in response to the rise in $\psi$ corresponds to the upward shift of the swap market schedule. But this is much smaller than the ultimate increase in the synthetic dollar rate. This amplification is illustrated through the stairs from point A to the new equilibrium at point B in the left chart.

Figure 4: Numerical illustration of shocks to $\psi$ and $\phi$

Notes: Both panels report the impact of shocks when starting from the pre-shock equilibrium associated with the model parameters reported in Table 2. Panel A reports the effect of a rise in $\psi$ from 0.3 to 0.4, while panel B reports the effect of a rise in $\phi$ from 2 to 10. The black line is the spot market equilibrium, while the two red lines show the swap market equilibrium before and after the shock.

To understand this interaction between the spot and swap market equilib-
ria, start again with the small initial increase in the synthetic dollar rate when holding the exchange rate fixed. The higher synthetic dollar rate raises dollar lending and reduces dollar borrowing, which leads to a need to buy dollars on the spot market. The resulting dollar appreciation implies an expected dollar depreciation over the next period as the expected period 2 spot rate remains zero. The resulting lower return on dollar funding in Europe reduces dollar lending and increases dollar borrowing. This further increases the excess demand for synthetic dollars. To clear the swap market, the synthetic dollar rate needs to rise further. This process continues until we eventually reach the new equilibrium at point B.

4.3 Shock to CIP Arbitrage

We next consider a rise in $\phi$, which leads to reduced CIP arbitrage. This may for example be a result of reduced intermediary capital during times of financial stress. It only has an effect when we start from a positive CIP deviation as the position of CIP arbitrageurs is $i^{8,F}/\phi$. We therefore again start from the parameters in Table 2. In Figure 4B we raise $\phi$ from 2 to 10 in period 1. We hold the other parameters fixed at their levels in Table 2, so that the pre-shock equilibrium is the same as in Figure 4A.

Figure 4 shows that this shock has a very similar effect as a rise in $\psi$. Both shocks shift up the swap market schedule, raising the synthetic dollar rate and appreciating the dollar.\footnote{While the swap market equilibrium schedule shifts straight up under a rise in $\psi$, it instead rotates around the point where $i^{8,F} = 0$ when $\phi$ rises. But this rotation point does not interest us and is left a bit out of view in Figure 4B.}

4.4 Reallocation by US lenders

During times of increased global financial stress, US lenders tend to reallocate from offshore dollar lending to onshore dollar lending. An example is the reduced dollar lending by US money market funds to European banks during both the GFC and the European debt crisis.

In the model we have separated US lenders that lend to the onshore dollar
market from US lenders that lend to the offshore dollar market. We assumed that a fraction $\lambda$ of lenders lend to the onshore market and a fraction $1-\lambda$ lend to the offshore market. Now think of these domestic and foreign lenders as being separate branches of a bigger organization, which decides to shift more resources to the domestic market, away from the foreign market, during times of increased stress. We can think of this as a rise in $\lambda$. We refer to it as $\lambda_H$ as we assume no change in the $\lambda$ for European lenders. We show in the Online Appendix that this has no effect on the spot market schedule. As discussed in Section 2.5, domestic and foreign US lenders allocate their portfolio the same way between dollar and euro assets. Increasing the relative size of domestic lenders therefore does not change the overall demand for dollar assets.

The Online Appendix also shows that the new swap market schedule becomes

$$
(2\alpha_1 U_1 - \nu_2) s_1 + 2 \frac{i_1^{F,E} + s_1}{\gamma \text{var}(s_2)} + \frac{1}{\phi} i_1^{S,E} - n W_{L,0} \lambda_H = 0
$$

Therefore a rise in $\lambda_H$ again shifts up the swap market schedule. This is because reduced dollar lending to the offshore dollar market creates a dollar shortage. The upward shift of the swap market schedule has the same effect as for the other two shocks (higher CIP deviation and dollar appreciation).

Although the three shocks we have considered are quite different, they have in common a portfolio reallocation from offshore dollar assets to onshore dollar assets, without a change in the overall demand for dollar assets. The former implies a dollar shortage in Europe (upward shift of swap market schedule), while the latter implies that the spot market schedule is unaffected.

### 4.5 Other Shocks

#### 4.5.1 Convenience Yield Shock

To save space, we analyze a convenience yield shock in the Online Appendix. We do so by introducing UIP arbitrageurs that arbitrage between onshore dollar assets and onshore euro assets. The onshore dollar assets have a convenience benefit equivalent to $\eta$ in terms of returns. As is familiar from the convenience yield literature (e.g., Jiang et al., 2023; Engel and Wu, 2023;
Valchev, 2020; Kekre and Lenel, 2023a), $\eta$ leads to an additive term in the UIP equation. In our case

$$t^{S,H}_1 - t^{E,F}_1 - E(s_2) + s_1 = r_{p1} - \eta$$  \hspace{1cm} (40)

The left hand side is the expected excess return of onshore dollars over onshore euros. With our assumptions, this is simply equal to $s_1$. The right hand side has the risk premium and the convenience yield. The risk premium of UIP arbitrageurs is $r_{p1} = \tilde{\gamma}var(s_2)B^{S,H}_{UIP,1}$, with $\tilde{\gamma}$ and $B^{S,H}_{UIP,1}$ the risk aversion and dollar position of UIP arbitrageurs.

An increase in $\eta$ lowers the expected excess return that UIP arbitrageurs demand on onshore dollar assets. This leads to an increase in demand for onshore dollar assets, which raises demand for dollars in the spot market. The spot market schedule shifts down, while the swap market schedule is unaffected. The shock implies an aggregate portfolio shift from euro to dollar assets, without affecting borrowing and lending in the offshore dollar market.

Figure 5: Increase in U.S. convenience yield

![Diagram showing the effect of an increase in U.S. convenience yield on the spot and swap market schedules.](chart5.png)
This is illustrated in Figure 5, which shows the case of imperfect CIP arbitrage (after 2007) in chart A on the left and perfect CIP arbitrage (before 2007) in chart B on the right. The downward shift in the spot market schedule leads to a dollar appreciation and a rise in the CIP deviation in the case of imperfect CIP arbitrage. Also in line with empirical findings in the convenience yield literature, the dollar appreciation is greater after 2007 than before 2007 (see Table 4 in Engel and Wu (2023) and Table 3 in Jiang et al. (2021)). This larger dollar appreciation happens because of the interaction between the spot and swap market equilibrium schedules discussed earlier.

The literature has convincingly shown that convenience yield shocks are an important driver of exchange rates. But this does not necessarily mean that they can account for the evidence in Table 1 related to changes in global financial stress. First, a relative US convenience yield shock implies a dollar appreciation even prior to 2007. But Table 1 shows that financial stress did not affect the dollar exchange rate before 2007. Second, Diamond and Van Tassel (2023) show that while convenience yields rise during financial crises, the difference between the US and foreign convenience yields generally does not rise. In the Online Appendix we provide further evidence showing that the US convenience yield relative to that of other countries is unrelated to the risk variables in Table 1.

4.5.2 Rise in Risk or Risk Aversion

All the shocks in this section can be thought of as related to an increase in risk or risk aversion. However, here we consider one specific aspect of this, related to the impact on portfolios of a rise in $\gamma$ or $\text{var}(s_2)$. They enter portfolios as a product through the term $(i_1^{S,F} + s_1)/[\gamma \text{var}(s_2)]$. $i_1^{S,F} + s_1$ is the expected excess return on dollars in both Europe and the US. Consider a rise in $\gamma$. Just as with a rise in $\phi$, this only affects portfolios when there is a positive CIP deviation to start with. We therefore again use the parameters from Table 2.

Figure 6 illustrates the impact of an increase in $\gamma$ from 50 to 70. Both the swap and spot market schedules shift upward. The CIP deviation rises,  

---

20The excess return on dollars in the US is $i_1^{S,H} + s_1 - s_2 - i_1^{E,H}$, so that the expected excess return is $s_1 - i_1^{E,H}$. This is equal to $i_1^{S,F} + s_1$ since $i_1^{E,H} = -i_1^{S,F}$. 

35
while the dollar depreciates. The rise in the CIP deviation is the result of the upward shift of the swap market schedule. Intuitively, the attractiveness of dollar assets, associated with their positive expected excess return in the pre-shock equilibrium, is reduced by the rise in risk aversion. This lowers lending and raises borrowing in the offshore dollar market. The resulting excess demand for offshore dollar funding raises $i_{1}^{S,F}$ further, increasing the CIP deviation.

At the same time, the rise in $\gamma$ leads to a portfolio shift in both Europe and the US from dollar to euro assets. This shifts the spot market schedule up and leads to a depreciation of the dollar.

These conclusions are in line with Liao and Zhang (2020), who consider such shocks in the context of their effect on hedge ratios. But the resulting dollar depreciation is clearly at odds with the evidence in Table 1. We conclude that this is not the main channel through which an increase in financial stress exerts its impact on the exchange rate.
5 Central Bank Swaps

The US central bank clears the onshore dollar market at the policy interest rate, but does not participate in the offshore market. When CIP arbitrage is frictionless, the offshore dollar market clears at the same interest rate. But when $\phi > 0$ and CIP arbitrage is limited, a dollar funding shortage can occur that raises the cost of dollar funding in the offshore markets.

During the GFC, central banks developed new instruments to address these shortages. The first dollar liquidity swap lines during the crisis were set up between the Fed and the European Central Bank (ECB) and the Swiss National Bank (SNB) in December 2007. Initially these were temporary and the total dollar value of the swap was capped. The Bank of England (BoE), the Bank of Japan (BoJ) and the Bank of Canada (BoC), as well as nine smaller central banks were added to these temporary swap arrangements in the Fall of 2008.\footnote{These nine smaller central banks were the Banks of Norway, Sweden, Denmark, Australia, New Zealand, Korea, Singapore, Mexico, and Brazil} These closed once the 2008 crisis abated, but the lines with the five major central banks were restarted during the Eurozone debt crisis in May 2010. Over the course of 2010 and 2011 these facilities changed from offering a fixed amount of dollar liquidity to offering unlimited dollar liquidity at a fixed price. This price was initially set at the dollar OIS rate plus 100 basis points. In November 2011 the rate was lowered to dollar OIS plus 50 basis points. The swap lines between the Fed and these five major central banks were made permanent in 2013. At the onset of the Covid crisis, these permanent swap lines were already in place. One of the first policy actions during the Covid crisis was to lower the swap rate to OIS plus 25 basis points.\footnote{The central banks also announced in March 2020 that the swap lines would begin offering liquidity with an 84-day maturity in addition to the usual 7-day maturity. The frequency of operations was changed from weekly to daily and the temporary swap lines with the nine smaller central banks that operated during the GFC were reestablished.}

Under dollar liquidity swaps, the foreign central bank and the Fed exchange the foreign currency for dollars at the market exchange rate. The Fed holds the foreign currency in an account at the foreign central bank. At the same time, the two central banks enter into a binding agreement to reverse the transaction at the end of a short period of time. The foreign central bank can then provide
dollar liquidity to their domestic borrowers. Basically, the Fed loans dollars to European borrowers using the foreign central bank as an intermediary.

In terms of the model, the dollar liquidity swap \( D_{CB\text{swap},1}^s \) is provided by the Fed to the ECB. This will be lent to European borrowers in period 1. It reduces the excess demand for offshore dollar funding by an equal amount, so that

\[
D_{F,t}^{s,\text{syn}} = -\alpha_1 B_{F,1,t}^s - \alpha_2 B_{F,2,t}^s - \alpha_3 B_{H,3,t}^s - D_{CB\text{swap},1}^s
\]  

(41)

The ECB provides \( \frac{1}{S_1} D_{CB\text{swap},1}^s \) euros to the Fed, which sits on the Fed’s balance sheet. We can then follow the same steps as before and arrive at a linearized system of the spot and swap market equilibrium schedules:

\[
\nu_1 s_1 + 2 \frac{i_1^s,F}{\gamma \var(s_1)} + s_1 = 0
\]  

(42)

\[
(2\alpha_3 \nu_1 - \nu_2) s_1 + 2 \frac{i_1^s,F}{\gamma \var(s_1)} + \frac{1}{\phi} i_1^s,F - \hat{\psi} \omega [a^s - a^e] + D_{CB\text{swap},1}^s = 0
\]  

(43)

where \( \nu_1 \) and \( \nu_2 \) are the same as before.

It is easy to see that an exogenous increase in \( D_{CB\text{swap},1}^s \) shifts down the swap market schedule, as shown in Figure 7, Panel A. This leads to both a smaller CIP deviation and a weaker dollar. This is in line with the empirical evidence mentioned in the literature review.

There is an incentive to activate central bank swap lines when CIP deviations are large. Assume the interest rate on dollar borrowing from swap lines is \( i_1^s,H + \tau \). Since \( i_1^s,H = 0 \), European dollar borrowers prefer borrowing through the swap line a soon as \( i_1^s,F > \tau \). If we assume that the European central bank is always ready to activate the swap line, this imposes a ceiling \( \tau \) on the synthetic dollar interest rate, as shown in Bahaj and Reis (2022).\(^{23}\) This implies that \( D_{CB\text{swap},1}^s = 0 \) in equation (43) when \( i_1^s,F < \tau \), but that equation (43) is replaced by \( i_1^s,F = \tau \) when we reach this threshold.

Panel B of Figure 7 shows the impact of an increase in global liquidity preference \( \hat{\psi} \) in this case. Without central bank liquidity swaps, the swap

\(^{23}\)In reality swap lines are not necessarily activated in function of CIP deviations. Allen et al. (2017) provide empirical evidence that money market liquidity is more important than CIP deviations.
market schedule simply shifts up and we move from A to C. The shock leads to a rise in the CIP deviation and an appreciation of the dollar. If instead the Fed offers unlimited dollar liquidity at an interest rate $\tau$, the swap market schedule becomes horizontal once $i_1^{S,F}$ reaches $\tau$. In this case the rise in $\psi$ leads to a new equilibrium at B instead of C. The central bank swap line therefore reduces the rise in the CIP deviation and also reduces the appreciation of the dollar.

6 Conclusion

New leverage regulations in the aftermath of the GFC have lead to more limited CIP arbitrage. While CIP arbitrage was close to perfect until 2007, we have seen persistent CIP deviations since then. At the same time the behavior of the dollar exchange rate has changed. Prior to the GFC, the dollar exchange rate was not significantly affected by changes in risk and risk aversion. However, since 2007 a more uncertain environment has led to both a dollar appreciation
and an increased CIP deviation.

We have developed a model to account for these stylized facts. Key to the model is limited CIP arbitrage since the GFC. This creates market segmentation that separates the dollar funding market in the United States from that in the rest of the world. We have shown that conditions in the offshore dollar funding market can exert an effect on the dollar exchange rate even when there is no overall change in demand for dollar assets. The model is summarized by two equilibrium equations, one for the spot FX market equilibrium and another for the swap FX market equilibrium. These jointly determine the spot exchange rate and the swap rate or CIP deviation.

Dollar dominance can account for persistent CIP deviations as seen in the data. A variety of shocks that are plausibly related to an increase in risk or financial stress can account for the appreciation of the dollar and rise in CIP deviation seen in the data during such episodes. These shocks have in common shortages in offshore dollar funding due to a reallocation from offshore to onshore dollar assets. Dollar dominance again plays a key role in some of these shocks. Central banks swap lines can mitigate the effects of these shocks by reallocating dollars from onshore to offshore markets.

Future work should consider extending the model in several directions. First, all assets in the model are safe and short term. Introducing risky borrowing would allow us to analyze the implications for dollar and foreign currency corporate bond yields that have been discussed in the literature associated with dollar shortages. Introducing different maturities, we could also study the implications for the term structure. Finally, banks play no role in the model other than a veil that connects lenders and borrowers. Introducing banks that conduct maturity transformation can lead to bank liquidity problems during periods of dollar shortages when short-term dollar liabilities are not rolled over. This potentially leads to even greater importance of central bank swap lines.
Appendix

A Swap Market Equilibrium

Table A1 describes how to construct synthetic asset positions. First consider lending 1 dollar synthetically at time $t$. Creating this synthetic dollar asset has three parts: lending $1/S_t$ euros in the onshore euro bond market, buying a dollar swap of $Q_{t,\text{swap}}^S = -F_t/S_t$ and a small spot market transaction of buying $(F_t/S_t) - 1$ dollars in exchange for $(1/S_t)[(F_t/S_t) - 1]$ euros. The lending of onshore euros involves the payment of $1/S_t$ euros at time $t$ and receipt of $1/S_t$ euros at time $t+1$. A swap transaction of $Q_{t,\text{swap}}^S = -F_t/S_t$ implies that at time $t$ you pay $-Q_{t,\text{swap}}^S = F_t/S_t$ dollars in exchange for $(1/S_t)(F_t/S_t)$ euros and at time $t+1$ you receive $F_t/S_t$ dollars in exchange for $1/S_t$ euros. Finally, the small spot market transaction implies that you receive $(F_t/S_t) - 1$ dollars at time $t$ in exchange for $(1/S_t)[(F_t/S_t) - 1]$ euros. Adding up these three transactions, on net you pay 1 dollar today and receive $F_t/S_t = 1 + \frac{i_t}{S_t}$ dollars at $t+1$. It is therefore analogous to making a one dollar loan with an interest rate of $i_t/S_t$.

Table A1: Synthetic Assets and the Swap Market

<table>
<thead>
<tr>
<th>Synthetic Dollars: $\Delta B_t^S, F = 1$</th>
<th>Synthetic Euros: $\Delta B_t^E, H = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta P_t^{\text{F}, F} = 1/S_t$</td>
<td>$\Delta P_t^{\text{E}, H} = S_t$</td>
</tr>
<tr>
<td>$\Delta Q_{t,\text{swap}}^S = -F_t/S_t$</td>
<td>$\Delta Q_{t,\text{swap}}^E = S_t$</td>
</tr>
<tr>
<td>$\Delta Q_{t,\text{spot}}^S = (F_t/S_t) - 1$</td>
<td>$\Delta Q_{t,\text{spot}}^E = 0$</td>
</tr>
</tbody>
</table>

Notes: The table reports what is needed to create a synthetic dollar (left column) and euro (right column) asset (lending respectively 1 dollar and 1 euro synthetically at time $t$). The Table shows how the synthetic dollar asset is created by combining onshore euro lending, a swap market transaction and a small spot market transaction. Similarly, it shows how the synthetic euro asset is created by combining onshore dollar lending and a swap market transaction.

Next consider lending 1 euro synthetically. As described in Table A1, this has two parts: lending $S_t$ dollars in the onshore dollar bond market and buying
a dollar swap of $Q_{t,\text{swap}}^{\$,} = S_t$. Lending $S_t$ dollars in the onshore bond market implies that you pay $S_t$ dollars at time $t$ and receive $S_t$ dollars at time $t+1$. The swap transaction of $S_t$ implies that at time $t$ you receive $S_t$ dollars in exchange for 1 euro, while at time $t+1$ you pay $S_t$ dollars in exchange for $S_t/F_t$ euros. The net effect is that you pay 1 euro at time $t$ and receive $S_t/F_t = 1 + i_t^{e,H}$ euros at time $t+1$. This is equivalent to making a 1 euro loan with an interest rate of $i_t^{e,H}$.

Define $Q_{H,j,1}^{\$,\text{swap}}$ and $Q_{F,j,1}^{\$,\text{swap}}$ as the period 1 swap market transaction by agents of type $j$ in respectively the Home and Foreign country. Period 1 swap market equilibrium is then

$$\sum_{j=1}^{3} \alpha_j Q_{H,j,1}^{\$,\text{swap}} + \sum_{j=1}^{3} \alpha_j Q_{F,j,1}^{\$,\text{swap}} + Q_{CIP,1}^{\$,\text{swap}} = 0 \quad (A.1)$$

The last term is the swap market transaction by CIP arbitrageurs.

Define $B_{h,j,1}^{\$,s}$ as the synthetic dollar position of agents of type $j$ in country $h = H, F$. Analogously, define $B_{h,j,1}^{e,H,s}$ as the synthetic euro position of agents of type $j$ in country $h = H, F$. CIP arbitrageurs have a synthetic dollar position in Europe of $D_{CIP,1}^{\$,H}$. Using Table A1, these synthetic positions imply the following swap market transactions:

$$Q_{H,j,1}^{\$,\text{swap}} = S_1 B_{H,j,1}^{e,H,s} \quad j = 1, 2 \quad (A.2)$$

$$Q_{H,j,1}^{\$,\text{swap}} = -\frac{F_1}{S_1} B_{H,j,1}^{\$,F,s} \quad j = 3 \quad (A.3)$$

$$Q_{F,j,1}^{\$,\text{swap}} = -\frac{F_1}{S_1} B_{F,j,1}^{\$,F,s} \quad j = 1, 2 \quad (A.4)$$

$$Q_{CIP,1}^{\$,\text{swap}} = -\frac{F_1}{S_1} D_{CIP,1}^{\$,H} \quad (A.5)$$

Substituting (A.2) to (A.6) into (A.1), we have

$$-\frac{F_1}{S_1} \left( \alpha_1 D_{F,1,1}^{\$,F,s} + \alpha_2 D_{F,2,1}^{\$,F,s} + \alpha_3 D_{H,3,1}^{\$,F,s} \right) + S_1 \left( \alpha_1 B_{H,1,1}^{e,H,s} + \alpha_2 B_{H,2,1}^{e,H,s} + \alpha_3 B_{F,3,1}^{e,H,s} \right) - \frac{F_1}{S_1} D_{CIP,1}^{\$,H} = 0 \quad (A.7)$$

42
Since the offshore bond markets clear, we have

\[
\alpha_1 B_{F,1,1}^S + \alpha_2 B_{F,2,1}^S + \alpha_3 B_{H,3,1}^S = \alpha_1 B_{F,1,1}^S + \alpha_2 B_{F,2,1}^S + \alpha_3 B_{H,3,1}^S = -D_{F,1}^S_{syn} \\
\alpha_1 B_{H,1,1}^E + \alpha_2 B_{H,2,1}^E + \alpha_3 B_{F,3,1}^E = \alpha_1 B_{H,1,1}^E + \alpha_2 B_{H,2,1}^E + \alpha_3 B_{F,3,1}^E = -D_{H,1}^E_{syn}
\]

The swap market equilibrium (A.7) then becomes

\[
\frac{F_1}{S_1} D_{F,1}^S_{syn} - S_1 D_{H,1}^E_{syn} - \frac{F_1}{S_1} D_{CIP,1}^S_{H} = 0 \quad (A.8)
\]

which is equation (5) in the text.

## B Period 2 Goods Market Equilibrium

Agents of type \( j \) in country \( h \) receive an endowment of \( a_j Q_{h,2} \) of the good of country \( h \). The period 2 consumption index for agents from both countries is

\[
C_2 = \left( (0.5)^{\frac{1}{\bar{\theta}}} C_{H,2} + (0.5)^{\frac{1}{\bar{\theta}}} C_{F,2} \right)^{\frac{\bar{\theta}}{\phi - 1}} \quad (B.1)
\]

Here \( C_{H,2} \) is consumption of the Home good and \( C_{F,2} \) is consumption of the Foreign good. The parameter \( \bar{\theta} \) is the elasticity of substitution among the two goods. Central banks target a price of \( P_{H,2} = 1 \) for the Home good in dollars and a price of \( P_{F,2} = 1 \) for the Foreign good in euros. The price index of consumption in dollars is then

\[
P_2 = \left( 0.5 + 0.5 S_2 \right)^{-\bar{\theta}} C_2 \quad (B.2)
\]

and the price index in euros is \( P_2^* = P_2/S_2 \). The standard intratemporal first-order conditions imply consumption of Home and Foreign goods of

\[
C_{H,2} = 0.5 \left( \frac{1}{P_2} \right)^{-\bar{\theta}} C_2 \quad (B.3)
\]

\[
C_{F,2} = 0.5 \left( \frac{S_2}{P_2} \right)^{-\bar{\theta}} C_2 \quad (B.4)
\]
for agents from both countries.

Let \( C_{h,j,2} \) be period 2 aggregate consumption by agents of type \( j \) in country \( h \). Using the expressions for the supply \( Q_H \) and \( Q_F \) of Home and Foreign goods, period 2 goods market clearing implies

\[
e^{\kappa_H \epsilon_q} = 0.5 \left( \frac{1}{P_2} \right)^{-\bar{\theta}} \sum_{j=1}^{3} \alpha_j (C_{H,j,2} + C_{F,j,2}) \tag{B.5}
\]

\[
e^{-\kappa_F \epsilon_q} = 0.5 \left( \frac{S_2}{P_2} \right)^{-\bar{\theta}} \sum_{j=1}^{3} \alpha_j (C_{H,j,2} + C_{F,j,2}) \tag{B.6}
\]

Let \( \bar{C}_{h,j,2} \) be second period consumption when \( s_2 = 0 \), so that \( S_2 = 1 \). In that case \( P_2 = 1 \). Linearizing (B.5)-(B.6) around \( \epsilon_q = s_2 = 0 \), we get

\[
1 + \kappa_H \epsilon_q = 0.5 \sum_{j=1}^{3} \alpha_j (C_{H,j,2} + C_{F,j,2}) + 0.25 \sum_{j=1}^{3} \alpha_j (\bar{C}_{H,j,2} + \bar{C}_{F,j,2}) \bar{\theta} s_2 \tag{B.7}
\]

\[
1 - \kappa_F \epsilon_q = 0.5 \sum_{j=1}^{3} \alpha_j (C_{H,j,2} + C_{F,j,2}) - 0.25 \sum_{j=1}^{3} \alpha_j (\bar{C}_{H,j,2} + \bar{C}_{F,j,2}) \bar{\theta} s_2 \tag{B.8}
\]

First set \( \epsilon_q = 0 \). It follows immediately by first subtracting and then adding these equations that \( \epsilon_q = s_2 = 0 \) and

\[
\sum_{j=1}^{3} \alpha_j (\bar{C}_{H,j,2} + \bar{C}_{F,j,2}) = 2 \tag{B.9}
\]

Using this equation, subtracting (B.8) from (B.7) implies (using \( \kappa_H + \kappa_F = 1 \)) that \( \epsilon_q = \bar{\theta} s_2 \) or \( s_2 = \epsilon_q / \bar{\theta} \).

### C Pre-Shock Equilibrium

In the pre-shock equilibrium saving of all agents is zero, so that wealth is the same in period 1 as in period 0. Setting period 1 saving equal to zero for all
agents, we have

\begin{align*}
P_{1}C_{H,j,1} &= Y_{H,1} + \Pi_{H,1} + S_{1}i_{0}^{e,H}B_{H,j,0}^{e,H} \quad j = 1, 2 \quad (C.1) \\
P_{1}C_{F,j,1} &= Y_{F,1} + \Pi_{F,1} + \frac{1}{S_{1}}i_{0}^{s,F}B_{F,j,0}^{s,F} \quad j = 1, 2 \quad (C.2) \\
P_{1}^{*}C_{F,j,1} &= Y_{F,1} + \Pi_{F,1} + i_{0}^{e,H}B_{F,j,0}^{e,H} \quad j = 3 \quad (C.3) \\
P_{1}^{*}C_{F,j,1} &= Y_{F,1} + \Pi_{F,1} + i_{0}^{e,H}B_{F,j,0}^{e,H} \quad j = 3 \quad (C.4)
\end{align*}

This sets period 1 consumption equal to income, which is the sum of income from production, transfers from CIP arbitrageurs and interest income. It can be shown that one of these equations is redundant as we get an identity when adding them up, with weight \( \alpha_{j} \) for agent of type \( j \) and converting euros to dollars. So we remove the last equation.

In the pre-shock equilibrium we also have consumption smoothing: \( C_{h,j,1} = \bar{C}_{h,j,2} \). Substituting this into the period 2 budget constraints (see Online Appendix) with \( S_{2} = 1 \) (so that also \( P_{2} = P_{2}^{*} = 1 \)), linearizing the return on the foreign currency bond, and replacing \( \bar{i}_{e,H} = -i_{1}^{s,F} \), we have

\begin{align*}
C_{H,j,1} &= a_{j} + \Pi_{H,2} + M_{H,j,1}^{s} + M_{H,j,1}^{e} + W_{H,j,1} - B_{H,j,1}^{e,H}(i_{1}^{s,F} + s_{1}) \quad j = 1, 2 \quad (C.5) \\
C_{H,j,1} &= a_{j} + \Pi_{H,2} + M_{H,j,1}^{s} + M_{H,j,1}^{e} + (1 + i_{1}^{s,F})W_{H,j,1} - B_{H,j,1}^{e,F}(i_{1}^{s,F} + s_{1}) \quad j = 3 \quad (C.6) \\
C_{F,j,1} &= a_{j} + \Pi_{F,2} + M_{F,j,1}^{s} + M_{F,j,1}^{e} + W_{F,j,1} + B_{F,j,1}^{s,F}(i_{1}^{s,F} + s_{1}) \quad j = 1, 2 \quad (C.7) \\
C_{F,j,1} &= a_{j} + \Pi_{F,2} + M_{F,j,1}^{s} + M_{F,j,1}^{e} + (1 + i_{1}^{e,H})W_{F,j,1} + B_{F,j,1}^{s,H}(i_{1}^{s,F} + s_{1}) \quad j = 3 \quad (C.8)
\end{align*}

The last two equations needed to derive the pre-shock equilibrium are

\begin{align*}
\sum_{j=1}^{3} \alpha_{j}(C_{H,j,1} + C_{F,j,1}) &= 2 \quad (C.9) \\
(1 + i_{1}^{s,F})D_{F,1}^{s,F} - S_{1}D_{H,1}^{e,sym} - (1 + i_{1}^{s,F})D_{CIP,1}^{s,H} &= 0 \quad (C.10)
\end{align*}
These correspond to the period 2 world goods market equilibrium (B.9), replacing $\bar{C}_{h,j,2} = C_{h,j,1}$, and the period 1 swap market equilibrium (5). We then have a total of 13 equations: (C.1)-(C.3) and (C.5)-(C.10). Note that some equations are used twice, for both $j = 1$ and $j = 2$. This system can be solved by substituting expressions for CIP profits, money balances and portfolio holdings and setting $i_1^{s,F} = i_0^{s,F} = i^{s,F}$, $s_1 = s_0 = 0$ and $W_{h,j,1} = W_{h,j,0}$. We then have 13 equations in 13 variables: the 6 period 1 consumption levels of all 6 types of agents (3 types in each country), the 6 initial wealth levels $W_{h,j,0}$ and $i^{s,F}$.

Now consider the special case where

$$\tilde{\theta}(\kappa_H - \kappa_F) = \psi\omega(a^s - a^e) \tag{C.11}$$

This delivers a simple pre-shock equilibrium where $i^{s,F} = s = 0$, $C_{h,j,1} = 1$ for $h = H, F$ and $j = 1, 2, 3$ and $W_{h,j,0} = 1 - a_j - \psi$. To see this, we need to check the equations above. It is immediate that period 1 price levels are 1. From the period 1 goods market clearing conditions (12)-(13) it then also follows that $Y_{H,1} = Y_{F,1} = 1$. Period 1 and 2 profits of CIP arbitrageurs are zero when the synthetic dollar rate is zero in period 0 and 1. It is then immediate that (C.1)–(C.3) are satisfied.

Money demand is $M_{H,j,1}^s = \psi(1 - \omega + \omega a^s)$, $M_{H,j,1}^e = \psi\omega(1 - a^s)$, $M_{F,j,1}^s = \psi\omega(1 - a^e)$ and $M_{F,j,1}^e = \psi(1 - \omega + \omega a^e)$. Substituting these into (C.5)–(C.8), it is immediate that these are satisfied as well. (C.9) is clearly also satisfied as $C_{h,j,1} = 1$ for all agents. Finally, the swap market equilibrium (C.10) is satisfied when $D_{H,1}^{s,syn} = D_{H,1}^{e,syn}$. This holds when

$$\alpha_1B_{F,1,1}^{s,F} + \alpha_2B_{F,2,1}^{s,F} + \alpha_3B_{H,3,1}^{s,F} = \alpha_1B_{H,1,1}^{e,H} + \alpha_2B_{H,2,1}^{e,H} + \alpha_3B_{F,3,1}^{e,H} \tag{C.12}$$

Using $s_1 = i_1^{s,F} = 0$, the portfolio expressions and money demand expressions,
we have

\begin{align*}
B_{F,j,1}^{\delta,F} &= -\kappa_F a_j \bar{\theta} + 0.5 - \psi \omega (1 - a^e) \quad j = 1, 2 \\
B_{H,j,1}^{\delta,F} &= W_{H,j,0} - B_{H,j,1}^{\epsilon,F} = 1 - a_j - \psi + \kappa_H a_j \bar{\theta} - 0.5 + \psi \omega (1 - a^e) \quad j = 1, 2 \\
B_{H,j,1}^{\delta,H} &= -\kappa_H a_j \bar{\theta} + 0.5 - \psi \omega (1 - a^e) \quad j = 1, 2 \\
B_{F,j,1}^{\epsilon,H} &= W_{F,j,0} - B_{F,j,1}^{\delta,H} = 1 - a_j - \psi + \kappa_F a_j \bar{\theta} - 0.5 + \psi \omega (1 - a^e) \quad j = 3
\end{align*}

Substituting these into (C.12), using that \( \alpha_1 = 1 - n, \alpha_2 = \lambda n, \alpha_3 = (1 - \lambda)n, \) \( a_1 = a_b, a_2 = a_3 = a_l \) and \( na_l + (1 - n)a_b = 1 \), it follows that (C.12) holds as long as the condition (C.11) is satisfied. We therefore have a pre-shock equilibrium.

\section*{D Linearized Model}

Here we linearize the model for the special case where \( \bar{D}_{F,1}^{\delta,\text{syn}} = \bar{D}_{H,1}^{\epsilon,\text{syn}} = 0 \), where a bar refers to the pre-shock equilibrium. We linearize around \( s_1 = 0 \), interest rates in periods 0 and 1 equal to 0 and \( \psi = \bar{\psi} \). A hat will denote a deviation from the pre-shock equilibrium.

Log-linearization of the period 1 consumer price indices (6)-(7) gives

\begin{align*}
\hat{p}_1 &= \omega (1 - a^e) s_1 \quad \text{(D.1)} \\
\hat{p}^*_1 &= -\omega (1 - a^e) s_1 \quad \text{(D.2)}
\end{align*}

Linearization of Home consumption levels (8)-(9) gives (using that period 1 consumption remains equal to the pre-shock equilibrium values \( C_{H,j,1} = C_{F,j,1} = 1 \))

\begin{align*}
\dot{C}_{HH,j,1} &= (1 - \omega) \theta \dot{p}_1 \\
\dot{C}_{HF,j,1}^{\epsilon} &= \omega (1 - a^e) \theta (\dot{p}_1 - s_1); \quad \dot{C}_{HF,j,1}^{\delta} = \omega a^e \theta \dot{p}_1
\end{align*}
Linearization of Foreign consumption levels (10)-(11) gives

\[
\hat{C}_{F,j,1} = (1 - \omega)\theta \hat{p}_1^* \\
\hat{C}_{FH,j,1}^s = \omega(1 - a^e)\theta (s_1 + \hat{p}_1^*) \\
\hat{C}_{FH,j,1}^e = \omega a^e \theta \hat{p}_1^*
\]

Linearization of the income levels (12)-(13) gives

\[
\hat{Y}_{H,1} = \omega(1 - \omega)\theta \left( 2 - a^s - a^e \right) s_1 + \omega a^e s_1 \\
\hat{Y}_{F,1} = -\omega(1 - \omega)\theta \left( 2 - a^e - a^s \right) s_1 - \omega a^s s_1
\]

Money demand expressions (14)-(17) linearize as

\[
\hat{M}_{H,j,1}^s = \hat{\psi}(1 - \omega + \omega a^s) + \bar{\psi}(1 - \omega + \omega a^s)\omega(1 - a^s)\theta s_1 \\
\hat{M}_{H,j,1}^e = \hat{\psi}(1 - a^s) - \bar{\psi}(1 - \omega + \omega a^s)\omega(1 - a^s)\theta s_1 \\
\hat{M}_{F,j,1}^s = \hat{\psi}(1 - \omega + \omega a^s) - \bar{\psi}(1 - \omega + \omega a^s)\omega(1 - a^e)\theta s_1 \\
\hat{M}_{F,j,1}^e = \hat{\psi}(1 - \omega + \omega a^s) - \bar{\psi}(1 - \omega + \omega a^s)\omega(1 - a^e)\theta s_1
\]

Using the portfolio expressions (26)-(27) and denoting deviations from the pre-shock equilibrium with a hat, we have

\[
\hat{B}_{F,j,1}^{s,F} = -\hat{M}_{F,j,1}^s + \frac{t_1^{s,F} + s_1}{\gamma \text{var}(s_2)} \\
\hat{B}_{H,j,1}^{e,H} = -\hat{M}_{H,j,1}^e - \frac{t_1^{e,H} + s_1}{\gamma \text{var}(s_2)} \\
\hat{B}_{H,j,1}^{e,F} = -\hat{M}_{H,j,1}^e - \frac{t_1^{e,F} + s_1}{\gamma \text{var}(s_2)} \\
\hat{B}_{F,j,1}^{s,H} = -\hat{M}_{F,j,1}^s + \frac{t_1^{s,H} + s_1}{\gamma \text{var}(s_2)}
\]

For the swap market equilibrium we also need expressions for \( \hat{B}_{H,3,1}^{s,F} \) and \( \hat{B}_{F,3,1}^{e,H} \). Using that \( W_{H,3,1} = B^{s,F}_{H,3,1} + S_1 B^{e,F}_{H,3,1} \) and \( W_{F,3,1} = \frac{1}{S_1} B^{s,H}_{F,3,1} + B^{e,H}_{F,3,1} \), it
follows that

\[
\begin{align*}
\hat B_{H,3,1}^{\delta,F} &= \hat W_{H,3,1} - s_1 \bar B_{H,3,1}^{e,F} - \hat B_{H,3,1}^{\epsilon,F} \\
\hat B_{F,3,1}^{\delta,H} &= \hat W_{F,3,1} + s_1 \bar B_{F,3,1}^{\delta,H} - \hat B_{F,3,1}^{\epsilon,H}
\end{align*}
\]  
(D.13)  
(D.14)

Using the wealth accumulation equations, setting period 1 consumption equal to 1 and period 0 interest rates equal to 0, we have

\[
\begin{align*}
W_{H,3,1} &= W_{H,3,0} + Y_{H,1} + \Pi_{H,1} - P_1 + (S_1 - S_0)B_{H,3,0}^{\delta,F} + (M_{H,3,0}^s - M_{H,3,1}^s) + S_1 (M_{H,3,0}^e - M_{H,3,1}^e) + \left( \frac{1}{S_1} - \frac{1}{S_0} \right) B_{F,3,0}^{\delta,H} + (M_{F,3,0}^e - M_{F,3,1}^e) + \frac{1}{S_1} (M_{F,3,0}^s - M_{F,3,1}^s)
\end{align*}
\]

Notice that \( Y_{H,1} - P_1 = T A_{H,1}^s \) and \( Y_{F,1} - P_1^* = T A_{F,1}^e \). Trade accounts are zero in the pre-shock equilibrium. We also have \( T A_{H,1}^s = -S_1 T A_{F,1}^e \). The latter linearizes as \( T A_{H,1}^s = -T A_{F,1}^e \), so that \( Y_{F,1} - P_1^* = -T A_{H,1}^s \). It follows that

\[
\begin{align*}
\hat W_{H,3,1} &= T A_{H,1}^s + \bar B_{H,3,0}^{\delta,F} s_1 - \hat M_{H,3,1}^s - \hat M_{H,3,1}^e \\
\hat W_{F,3,1} &= -T A_{H,1}^s - \bar B_{F,3,0}^{\delta,H} s_1 - \hat M_{F,3,1}^e - \hat M_{F,3,1}^s
\end{align*}
\]

Substituting these wealth expressions into (D.13)-(D.14), using that pre-shock asset positions are the same in periods 0 and 1, we have

\[
\begin{align*}
\hat B_{H,3,1}^{\delta,F} &= T A_{H,1}^s - \hat M_{H,3,1}^s - \hat M_{H,3,1}^e - \hat B_{H,3,1}^{\epsilon,F} \\
\hat B_{F,3,1}^{\delta,H} &= -T A_{H,1}^s - \hat M_{F,3,1}^e - \hat M_{F,3,1}^s - \hat B_{F,3,1}^{\epsilon,H}
\end{align*}
\]

(D.17)  
(D.18)

Substituting the expressions for \( \hat B_{H,3,1}^{\epsilon,F} \) and \( \hat B_{F,3,1}^{\delta,H} \), we have

\[
\begin{align*}
\hat B_{H,3,1}^{\delta,F} &= T A_{H,1}^s - \hat M_{H,3,1}^s + \frac{i_{1}^{s,F} + s_1}{\gamma var(s_2)} \\
\hat B_{F,3,1}^{\delta,H} &= -T A_{H,1}^s - \hat M_{F,3,1}^e - \frac{i_{1}^{s,F} + s_1}{\gamma var(s_2)}
\end{align*}
\]

(D.19)  
(D.20)
Using the results above for period 1 output and the price level, we have

\[ TA^s_H = Y_H,1 - P_1 = \hat{Y}_{H,1} - \hat{P}_1 = \nu_1 s_1 \]  

(D.21)

where

\[ \nu_1 = \omega (1 - \omega) \theta (2 - a^s - a^e) + \omega (a^e + a^s - 1) \]

We will assume \( \theta \geq 1 \). Since \( a^s + a^e \leq 2 \), we have (using \( \theta \geq 1 \))

\[ \nu_1 \geq \omega (1 - \omega) \leq \omega (2 - \omega) > 0 \]

This follows since \( \omega < 0.5 \) due to home bias in trade.

First consider the spot market equilibrium (31). Linearizing gives

\[ \hat{M}^s_{F,j,1} - \hat{M}^e_{H,j,1} + TA^s_{H,1} + \sum_{j=1}^2 \alpha_j \hat{B}^s_{F,j,1} + \alpha_3 \hat{B}^s_{H,3,1} - \sum_{j=1}^2 \alpha_j \hat{B}^e_{H,j,1} - \alpha_3 \hat{B}^e_{F,3,1} = 0 \]  

(D.22)

The time zero variables are the same as the pre-shock levels for time 1.

After substituting the portfolio expressions and expression for the trade account, we have

\[ \nu_1 s_1 + \frac{2 \gamma \var(s_1)}{\gamma \var(s_2)} = 0 \]  

(D.23)

where \( \nu_1 > 0 \) as discussed.

The swap market equilibrium is

\[ \left( 1 + \hat{i}^s_{F,1} \right) D^s_{H,1} - S_1 D^e_{H,1} = \left( 1 + \hat{i}^s_{1} \right) D^s_{CIP,1} = 0 \]  

(D.24)

This is linearized as

\[ \hat{D}^s_{F,1} - \hat{D}^e_{H,1} = \hat{D}^s_{CIP,1} = 0 \]  

(D.25)

We have

\[ \hat{D}^s_{F,1} - \hat{D}^e_{H,1} = - \sum_{j=1}^2 \alpha_j \hat{B}^s_{F,j,1} - \alpha_3 \hat{B}^s_{H,3,1} + \sum_{j=1}^2 \alpha_j \hat{B}^e_{H,j,1} + \alpha_3 \hat{B}^e_{F,3,1} \]  

(D.26)
Using the portfolio expressions, this is equal to

\[
\begin{align*}
\hat{D}_{F,1}^{\text{syn}} - \hat{D}_{H,1}^{\text{syn}} &= (\alpha_1 + \alpha_2)(\hat{M}_{F,j,1}^{\text{syn}} - \hat{M}_{H,j,1}^{\text{syn}}) - 2 \frac{i_{1,1}^{F,\text{syn}} + s_1}{\gamma \text{var}(s_2)} \\
&- 2\alpha_3 T A_{H,1}^{\text{syn}} + \alpha_3 (\hat{M}_{H,3,1}^{\text{syn}} - \hat{M}_{F,3,1}^{\text{syn}})
\end{align*}
\]

Substituting money demand expressions, we can write this as

\[
D_{F,1}^{\text{syn}} - D_{H,1}^{\text{syn}} = \hat{\psi}\omega [a^F - a^H] + \nu_2 s_1 - 2 \frac{i_{1,1}^{F,\text{syn}} + s_1}{\gamma \text{var}(s_2)} - 2\alpha_3 T A_{H,1}^{\text{syn}} \tag{D.27}
\]

where

\[
\nu_2 = \bar{\psi}\omega \theta \left[ (1 - \omega + \omega a^F)(1 - a^F) + (1 - \omega + \omega a^H)(1 - a^H) \right]
\]

Swap market equilibrium is then

\[
\hat{\psi}\omega [a^F - a^H] + \nu_2 s_1 - 2 \frac{i_{1,1}^{F,\text{syn}} + s_1}{\gamma \text{var}(s_2)} - 2\alpha_3 \nu_1 s_1 - \frac{1}{\phi} i_{1,1}^{\text{syn}} = 0 \tag{D.28}
\]
References


53


