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The Contribution of Foreign Holdings of U.S. Treasury Securities to the U.S. Long-Term Interest Rate: An Empirical Investigation of the Impact of the Zero Lower Bound[*]

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Abstract

We find empirical evidence of a possible structural break in the relationship between the foreign holdings of U.S. Treasury securities and the U.S. long-term interest rate occurring at the time when U.S. monetary policy became constrained at the zero-lower bound (ZLB). The estimated marginal effect of the foreign holdings ratio on the U.S. long-term interest rate, particularly its long-run effect, appears to have become stronger during the ZLB regime than it was before. We argue that the leading explanation of this apparent break is the nonlinearity introduced by the ZLB. Motivated by theory, we propose a flexible nonlinear specification to deal with the ZLB—a threshold single-equation error-correction model splitting the sample in two regimes, pre-ZLB and ZLB, which replaces the observed Fed Funds rate with a shadow Fed Funds rate derived from a Tobit-IV model to incorporate a broader measure of the stance of monetary policy. With this setup, we find no significant structural break in the relationship between foreign holdings and long-term rates at the ZLB. Therefore, we argue that the ZLB is a leading cause of the apparent shift in the empirical relationship. We also show that the estimated effects are not just statistically significant, but also economically significant. Through counterfactual analysis, we show that changes in China's holdings of U.S. Treasury securities played an important role in explaining the 2004-2006 interest rate conundrum period and kept the long-term interest rate from going even lower in the recent ZLB period.

JEL Codes: C24, E43, E58, F21

Keywords: long-term interest rates, foreign holdings of notes and bonds, expectations hypothesis, structural break, zero lower bound, monetary policy, China

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Resumen

Encontramos evidencia empírica robusta de una ruptura estructural en la relación entre las tenencias extranjeras de obligaciones y bonos del Tesoro y la tasa de interés a largo plazo de EE. UU. probablemente como consecuencia de que la política monetaria se vio restringida un su límite inferior de cero (LIC) a fines de 2008. Argumentamos que esto se puede modelar mejor en forma no lineal usando un modelo de corrección de errores de una sola ecuación y con la tasa de fondos federales como la variable umbral que divide endógenamente la muestra en dos regímenes pre-LIC y LIC. Encontramos que el efecto marginal estimado de la relación de tenencias extranjeras en la tasa de interés a largo plazo de EE. UU. es mayor en valor absoluto durante el régimen LIC que en el régimen anterior, especialmente su efecto a largo plazo. Por el contrario, usando la tasa sombra de fondos federales derivada de un modelo Tobit-IV, no encontramos una ruptura estructural significativa entre ambos regímenes. Por lo tanto, sostenemos que el LIC es una de las causas principales del cambio estructural estimado. Además, investigamos el impacto concurrente de las compras de valores del Tesoro por parte de la Reserva Federal a través de un caso hipotético que no asume intervenciones de flexibilización cuantitativa (QE por sus siglas en inglés) después de 2008. Nuestros resultados sugieren que las tres rondas de QE pueden haber reducido la tasa de interés a largo plazo entre 38 y 55 puntos básicos en promedio. También encontramos que los cambios en las tenencias de China de obligaciones y bonos del Tesoro de los EE. UU. jugaron un papel importante a la hora de explicar la paradoja de las tasas de interés del período 2004-2006 y, de hecho, su comportamiento evitó que la tasa de interés a largo plazo fuera aún más baja en el reciente período LIC.

Códigos de clasificación JEL: C24, E43, E58, F21

Palabras clave: tasas de interés a largo plazo, tenencias extranjeras de obligaciones y bonos, hipótesis de expectativas, ruptura estructural, límite inferior de cero, flexibilización cuantitativa, China.

1. Introduction

The U.S. long-term interest rate (to be precise, the 10-year Treasury yield) has trended lower over the past three decades (Council of Economic Advisers (2015)). The literature studying the determinants of the long-term rate has recognized that foreign holdings of U.S. long-term Treasury securities are an important empirical factor.[1] The large amount of purchases of Treasuries by foreign investors and institutions with excess savings (mainly East Asian countries and Middle-East oil-exporting countries) has given credence to the hypothesis that global forces do play a role explaining the path followed by long yields (e.g., the so-called global savings glut hypothesis of Bernanke (2005, 2015)).

The empirical evidence has documented a significant negative relationship between the long-term interest rate and foreign holdings (or, in some cases, net purchases) of U.S. Treasury securities (Warnock and Warnock (2009), Bandholz *et al.* (2009), Beltran *et al.* (2013), among others). Our paper contributes to this literature with novel evidence on the stability of the relationship even after conventional interest rate policy ran up against the zero-lower bound (henceforth, ZLB) in 2008m12 and the Federal Reserve turned to balance sheet policies (BSPs) and forward guidance (FG).

We adopt as our baseline model the trademark reduced-form linear specification in error-correction form commonly used in the existing literature (e.g., Warnock and Warnock (2009)). We motive this model based on a weak form representation of the expectations hypothesis of the term structure of interest rates and the Fisher equation expressing the nominal yield in terms of the real rate and inflation expectations, we derive a long-run cointegrating relationship between the long-term interest rate, the short-term rate, the long- and short-run inflation expectations, and the term premia which is partly endogenous and dependent on a measure of the size of foreign holdings.

Our key measure of foreign holdings is the ratio of foreign official holdings of U.S. long-term Treasury securities as a share of marketable U.S. Treasury securities (where marketable Treasury securities are defined as the total outstanding Treasury securities excluding the Federal Reserve's holdings). This foreign holdings' ratio increased from around 11% in early 1994 to 56% at the end of 2008. Since then, this ratio has declined—and it would have declined even more without the Federal Reserve's multiple BSP actions in the aftermath of the 2008-09 financial recession absorbing part of the increase in total outstanding Treasuries.

We document that the series for the long-term interest rate, the short-term rate, the long- and short-run inflation expectations, and the foreign holdings ratio are cointegrated of order one and provide evidence of one cointegrating relationship between them consistent with the baseline linear error-

[1] Treasury securities include both Treasury notes issued in two-, three-, five- and 10-year terms and Treasury bonds issued with longer terms of more than 10 years. Treasury securities do not include Treasury bills (T-bills) which are short-term obligations with a term of one year or less.

correction model. Using this linear baseline specification and a battery of stability tests (Hao and Inder (1996), Hansen (1992) and Gregory and Hansen (1996) tests), we find robust evidence of a breakpoint in the long-run cointegrating relationship at the end of 2008.

We use a threshold single-equation error-correction model to estimate both the short-run and the long-run effects of the foreign holdings ratio on the long-term interest rate. The endogenously determined threshold value is derived using the policy rate as the threshold variable and doing so the model naturally splits the sample into two regimes, which roughly correspond with the period before the ZLB and the period at the ZLB. Consistent with the evidence of parameter instability in the relationship, we find that a one percentage point increase (decrease) in the foreign official holdings ratio has an overall long-run impact of reducing (raising) the U.S. long-term interest rate by around 6 basis points at the ZLB period compared to 4 basis points in the pre-ZLB period.[2]

We conjecture that, even though we take into account explicitly the concurrent impact of changes in the Federal Reserve's holdings of outstanding U.S. Treasuries through its BSP actions, the effect of the ZLB on the short-term rate has contributed to this apparent shift in the empirical relationship between the foreign holdings ratio and the long-term interest rate after the 2008-09 financial crisis. To investigate this, we incorporate the ZLB on the short-term rate based on the Black (1995) model into the weak form representation of the expectations hypothesis of the term structure of interest rates that motivates our baseline empirical specification. We show that not taking account of the nonlinearities that arise from the ZLB implies that the baseline linear error-correction model is misspecified. Moreover, this misspecification can be significant and lead conventional empirical estimates to suggest potential structural breaks in the relationship that in fact are not structural in nature.

To flexibly capture the sort of nonlinearities in the specification that theory suggests result from the ZLB, we use the shadow Fed Funds rate (i.e., the policy rate that the Federal Reserve would had implemented without the ZLB) derived from a Tobit-IV model as a replacement of the actual Fed Funds rate in our preferred threshold model. We find no significant break in the impact of the foreign holdings ratio on the long-term rate which implies that the ZLB is the main contributor of the structural change. These results are robust in a smooth transition regression specification based on an autoregressive distributed lag model to characterize the long-term interest rate. The results are also robust to a number of alternative specifications including different ways to define the key measures of foreign holdings and shadow rate, additional macro regressors that can impact the relationship, or even a threshold VECM extension to address concerns about possible endogeneity in the single-equation error-correction specification.

[2] A Wald test indicates that estimated coefficients on the foreign holdings ratio between the two regimes are statistically significant different from each other at the 5% level.

Finally, we illustrate through counterfactual analysis the economic significance of the empirical relationship between foreign holdings and the U.S. long-term interest rate uncovered in the paper. Everything else equal, we argue that China's holdings of U.S. Treasury securities have had a notable impact on the U.S. long-term interest rate. We find that the accelerated pace of increase in China's holdings since 2001 may have lowered the U.S. long-term interest rate by 24 basis points on average during the so-called interest rate conundrum period (2004-06).[3] In contrast, if China's pace of purchases of U.S. long-term Treasury securities had not stalled since 2011m07, the U.S. long-term interest rate could have been driven on average 25 basis points lower between 2011m07 and 2014m12.

The rest of the paper is organized as follows. Section 2 provides an overview of the pertaining literature on the determinants of the long-term interest rate and points at previous evidence of parameter instability in the relationship between the long-term yield and foreign holdings of U.S. Treasuries. In Section 3, we derive a theoretical long-run cointegrating relationship from the weak form of the expectations hypothesis of the term structure of interest rates modified to incorporate the ZLB on the short-term rate as in Black (1995). These derivations motivate our subsequent empirical analysis. In Section 4, we document and extensively discuss the data used in our empirical model. Section 5 presents our empirical findings, and the estimation results under alternative specifications. Section 6 provides our main robustness checks. Section 7 illustrates the economic importance of the shifting role of foreign holdings at the ZLB through a series of counterfactual analysis on the potential effects of China's policy of building up reserves through large purchases of U.S. Treasuries. And Section 8 concludes.

2. Literature Review

2.1 Foreign Holdings of U.S. Treasuries and the U.S. Long-Term Interest Rate

Standard macroeconomic fundamentals—such as inflation expectations, the short-term interest rate, fiscal conditions, etc.—are not sufficient to account for the observed behavior of the U.S. long-term interest rate.[4] Among the other macro factors which can affect the U.S. long-term interest rate, the foreign holdings (or net purchases) of U.S. Treasury securities are thought to be an important one—especially the large holdings (net purchases) of U.S. Treasury securities by foreign central banks. The logic is that higher foreign demand of U.S. Treasury securities pushes their price up and, therefore, lowers Treasury yields. Based on that premise, a number of studies have tried to quantify the effects of foreign holdings (or net

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[3] In seventeen Federal Open Market Committee (FOMC) meetings from June 2004 to June 2006, the Fed increased the federal funds rate from 1% to 5.25% but the long-term interest rate (i.e., the 10-year Treasury yield) remained flat or trended slightly lower. This puzzling behavior was pointed out in testimony by former Fed chairman Alan Greenspan before the U.S. Senate's Committee on Banking, Housing, and Urban Affairs on February 16, 2005 (available here: http://www.federalreserve.gov/boarddocs/hh/2005/february/testimony.htm).

[4] See, e.g., Correia-Nunes and Stemitsiotis (1995), Breedon *et al.* (1999), Caporale and Williams (2002), Dewachter and Lyrio (2006), and Diebold *et al.* (2006) on macroeconomic fundamentals as determinants of long-term yields.

purchases) of U.S. Treasuries on the U.S. long-term yield (Warnock and Warnock (2009), Craine and Martin (2009), Bandholz *et al.* (2009), Bertaut *et al.* (2012), Beltran *et al.* (2013), etc.).

Table 1. Estimated Impact of US$ 100 Billion Foreign Purchases of U.S. Treasury/Agency Securities on the U.S. Long-Term Treasury Yield: An Overview of Previous Studies

Study	Impact (in basis points)	Foreign Variables Measurement	Sample Period
Rudebusch *et al.* (2006)	No significant impact	12-month foreign official flows into Treasury securities (scaled by total outstanding)	1990m05-2005m12
Warnock and Warnock (2009)	-34	12-month foreign official flows into Treasury and Agency securities (scaled by GDP)	1984m01-2005m05
	-16	12-month foreign total flows into Treasury and Agency securities (scaled by GDP)	
Bandholz *et al.* (2009)	-12	Foreign total holdings of Treasury securities (scaled by total outstanding)	1986m01-2006m06
Craine and Martin (2009)	-61	Foreign official holdings of Treasury securities (scaled by personal income)	1/1/1990-12/31/2003
Bertaut *et al.* (2012)	-13	Foreign official holdings of Treasury and Agency securities (scaled by total outstanding)	1980q1-2007q2
Beltran *et al.* (2013)	-46	1-month foreign official flows into Treasury notes and bonds (scaled by total outstanding)	1994m01-2007m06
	-50	1-month foreign official flows into Treasury notes and bonds (scaled by GDP)	
	-16 or -21	Foreign official holdings of Treasury notes and bonds (scaled by total outstanding)	
Kaminska and Zinna (2014)	-4	Foreign official holdings of Treasury notes and bonds (scaled by total outstanding)	2001m01-2012m11

Note: Beltran *et al.* (2013), Craine and Martin (2009) and Kaminska and Zinna (2014) are for the 5-year term premium, 10-year forward rate and 10-year real Treasury yield respectively. All other papers study the 10-year nominal Treasury yield. We choose the end of sample in each reported study as the baseline date to scale the corresponding measure of the foreign variables when computing the reported impacts. The interested reader is also referred to the comparisons made in the previous literature: particularly, Table 4 in Bertaut *et al.* (2012) and Table 6 in Beltran *et al.* (2013).

Table 1 summarizes the evidence from the relevant empirical papers in the literature. The (estimated) marginal effect of foreign purchases equivalent to 100 billion dollars of U.S. Treasury securities on the U.S. long-term yield ranges from no effect to around 60 basis points and is generally statistically significant. The range of the estimated impacts reported across studies varies due to differences in the econometric methods, datasets, and sample periods used.[5]

A number of related studies have investigated the same relationship for other countries too. For example, Carvalho and Fidora (2015) investigate the effect of foreign purchases of government bonds issued by the euro-area countries on their long-term interest rates. Andritzky (2012) and Arslanalp and Poghosyan (2014) use panel data techniques to investigate the effect of foreign demand on sovereign bonds issued by a group of advanced economy countries on their respective long-term sovereign bond yields. Peiris (2010), Pradhan *et al.* (2011), and Ebeke and Lu (2014) study the same type of relationship for a group of emerging economies. In general, all these studies find some effect of the foreign demand of sovereign bonds on the long-term yield in the international data as well, with a one percentage point increase in the share of foreign investors in the government bond market reducing government bond yields by 3 to 10 basis points—albeit (like most of the papers that focus on the U.S.) they largely rely on linear specifications that keep the marginal effects constant over time.

2.2 Parameter Instability and Structural Breaks

The possibility of structural breaks in the relationship between foreign holdings (or net purchases) and the long-term yield has been acknowledged in the literature. In some studies, researchers focus on certain sample periods to avoid the possibility of structural breaks in the estimation. Sierra (2010) restricts its sample from 1994m05 till 2007m12 avoiding the potential break in early 1994 due to China's emergence as a major player in the U.S. Treasury markets and the potential break at the time of the 2008-09 financial recession.[6] Beltran *et al.* (2013) emphasizing the period from 1994m01 till 2007m06 and Goda *et al.* (2013) from 1994m02 to 2007m06 also limit their sample for similar reasons. Using the Quant-Andrew single break point test, Goda *et al.* (2013) find a break date at 1998m11 when foreign official investors moved to a strong accumulation of reserves after the 1997 Asian financial crisis. However, their structural break analysis is based on the exclusion of two other possible breaks around 1994 and around the 2008-09 financial crisis.

[5] The estimated effects of foreign holdings (or foreign purchases) of U.S. Treasury securities on the U.S. long-term interest rate can vary partly due to measurement and specification differences. For instance, some studies consider both Treasury and Agency securities, while others consider foreign total holdings or foreign official holdings. Therefore, a comparison across models is not straightforward.

[6] Sierra (2010) investigates realized excess return (a measure of premia) for U.S. bond yields with maturities from 2 to 10 years using foreign official and private data in his model specification.

In other studies, researchers find that a statistically-significant impact can be detected only during certain sample periods and argue that those effects may be shifting over time as the U.S. Treasury market evolves. Wu (2005) finds that the relationship between the 10-year Treasury yield and foreign official net purchases of U.S. Treasuries (as a percentage of U.S. GDP) is unstable—appearing to be only relevant since the early 2000s when the pace of foreign purchases of U.S. Treasuries notably accelerated. Briere *et al.* (2008) also point out the instability of the impact of their foreign demand variable and, similarly, find that its effect appears to be statistically significant only after 2002.

Some researchers have argued that the impact is time-varying by comparing estimated marginal effects across different subsamples. Mann and Klachkin (2012) compare the regression results between subsamples before the implementation of Quantitative Easing (QE) measures in the aftermath of the 2008-09 financial recession. They find that the negative relationship between the foreign demand and long-term yields disappeared after QE and they even suggest that the Fed's purchases altered this relationship. Beltran *et al.* (2013) find that the estimated effect of foreign purchases is a bit smaller using an extended sample (January 1994 to June 2011) rather than the shorter sample period they prefer (January 1994 to June 2007), which may suggest that BSPs lowered the impact of foreign demand on the long-term rate.

In addition, researchers have also documented varying overall impacts calculated as a constant marginal effect multiplied by the change in the foreign demand over certain periods. Kaminska and Zinna (2014) calculate the overall impacts across different subsamples which show a significantly larger overall impact before QE than during QE. Their estimated (constant) marginal effect is that one percent increase of foreign demand has an impact of 4.9 basis points on lowering the 10-year real Treasury yield. Their calculated overall impacts of the change in foreign demand on the 10-year real Treasury yield are 80.8 basis points for the period from 2001 to 2008 and 2.4 basis points for the periods from March 2009 to November 2009 and from November 2010 to June 2011.[7]

We take all this evidence as suggesting that we need to account for the possibility of structural change when modelling the relationship between the long-term interest rate and foreign holdings (or net purchases) of Treasury securities. This is, in fact, one of the salient points that our paper makes. We model the impact of foreign holdings specifically allowing it to be time-varying to capture potential breaks in the relationship. We argue that, even controlling for the direct effect of QE policies on Treasury securities, the differential impacts of the foreign demand of Treasuries on U.S. long-term interest rates in the environment of near-zero short-term interest rates that constrained conventional monetary policy following the 2008-09 financial recession.

[7] Using overall impacts (also called cumulative impacts) rather than marginal impacts has its limitations. Overall impacts may vary across time simply due to the differences in the magnitudes of the foreign demand changes over the corresponding periods.

3. Modelling the Term Structure of Interest Rates

Let $i_{n,t}$ be the nominal yield of an *n*-period pure discount bond that is bought at time t and matures after *n* periods. The weakest form of the expectations hypothesis of the term structure of interest rates (Campbell and Shiller (1987, 1991), Hall *et al.* (1992), and Campbell (1995)) can be expressed as follows,

$$i_{n,t} = \frac{1}{n}\left[\sum_{j=1}^{n} E_t(i_{1,t+j-1})\right] + \theta_{n,t}, \tag{1}$$

where $\theta_{n,t} \equiv \frac{1}{n}\sum_{j=1}^{n} \varphi_{j,t}$ denotes a term capturing the effects of the premia along the yield curve (accounting for risk, preferences about liquidity, etc.).[8]

We consider the case where the premia $\theta_{n,t}$ can be driven by macro factors and potentially by exogenous shocks too. Based on the literature reviewed in the previous section, we model $\theta_{n,t}$ accordingly as,

$$\theta_{n,t} = \theta_n^0 + \theta_n^1 fh_t + \theta_n^2 \varepsilon_{n,t}, \tag{2}$$

where fh_t is our preferred measure of the foreign demand—the foreign holdings of U.S. Treasury notes and bonds as a percentage of outstanding marketable Treasury notes and bonds (net of Federal Reserve holdings)—and $\varepsilon_{n,t}$ is the exogenous (stationary) component of the premia.[9]

The Fisher equation links nominal yields to real rates and inflation expectations, i.e., it decomposes the nominal one-period yield as

$$i_{1,t} = r_{1,t} + E_t(\pi_{t+1}), \tag{3}$$

where $r_{1,t}$ is the real per-period yield earned on a one-period zero coupon bond at time t and π_{t+1} defines the inflation rate between time t and t+1. Hence, the average of the expected future short-term nominal yields on the right-hand side of (1) can be expressed in terms of averages over expected future real rates and over inflation.

We can rewrite equation (1) by subtracting $i_{1,t}$ on both sides as follows,

[8] The premia is zero for any given maturity—i.e., $\theta_{n,t} = 0$—under the pure expectations hypothesis. A milder version of the expectations hypothesis allows the premia to be constant over time—i.e. $\theta_{n,t} = \theta_n$. Equation (1) describes the weakest form of the expectations hypothesis where the premia varies over time.

[9] The 'global savings glut theory' championed, among others, by former Federal Reserve chairman Bernanke (2005, 2015) suggests that large purchases of U.S. Treasury securities by foreign investors (notably by foreign central banks) can be an important force behind the movements in the premia—particularly during 2004-06. However, we also note that there are other potential structural explanations for this relationship. For instance, the 'theory of safe asset shortages' (Caballero (2006, 2010)) argues that U.S. Treasuries constitute one of the preeminent safe assets but its supply has failed to keep up with global demand. Excess demand for safe U.S. Treasuries from foreign holders can, therefore, influence the liquidity risk and lower the U.S. yields.

$$i_{n,t} - i_{1,t} = \frac{1}{n}\left[\sum_{j=1}^{n} E_t(i_{1,t+j-1})\right] - i_{1,t} + \theta_{n,t}. \tag{4}$$

Rearranging terms, we get

$$i_{n,t} - i_{1,t} = \frac{1}{n}\left[\sum_{m=1}^{n-1}\sum_{j=1}^{m} E_t(\Delta i_{1,t+j})\right] + \theta_{n,t}$$

$$= \sum_{j=1}^{n-1}\left(1-\frac{j}{n}\right) E_t\left(\Delta i_{1,t+j}\right) + \theta_{n,t}, \tag{5}$$

where Δ is the first-difference operator—i.e., $\Delta i_{1,t+j} = i_{1,t+j} - i_{1,t+j-1}$.

The weak form of the expectations hypothesis in (1) together with the specification of premia in (2) and the Fisher equation in (3) have important statistical implications that provide a useful basis for our empirical study of the relationship between foreign demand and the long-term yield. Hence, an important implication of the expectations hypothesis is that the spread between the long and short ends of the yield curve in (5) must be related to a sequence of expectations of future movements on the short-term interest rate as well as to the premia.

Combining the premia in (2) with (5), we derive the following expression

$$i_{n,t} - i_{1,t} - \theta_n^1 fh_t = \sum_{j=1}^{n-1}\left(1-\frac{j}{n}\right) E_t\left(\Delta i_{1,t+j}\right) + \theta_n^0 + \theta_n^2 \varepsilon_{n,t}. \tag{6}$$

Using the Fisher equation in (3) to replace the short-term nominal rates on the right-hand side of (5) together with the premia equation in (2), we get

$$\left(i_{n,t} - \pi_{t,n}^e\right) - \left(i_{1,t} - \pi_{t,1}^e\right) - \theta_n^1 fh_t = \frac{1}{n}\left[\sum_{j=1}^{n} E_t\left(r_{1,t+j-1}\right)\right] - r_{1,t} + \theta_n^0 + \theta_n^2 \varepsilon_{n,t}$$

$$= \sum_{j=1}^{n-1}\left(1-\frac{j}{n}\right) E_t\left(\Delta r_{1,t+j}\right) + \theta_n^0 + \theta_n^2 \varepsilon_{n,t}, \tag{7}$$

where short-term inflation expectations are defined as $\pi_{t,1}^e \equiv E_t(\pi_{t+1})$ and long-term inflation expectations over the lifespan of the bond are given by $\pi_{t,n}^e \equiv \frac{1}{n}\left[\sum_{j=1}^{n} E_t\left(\pi_{t+j}\right)\right]$.[10]

Equation (6) shows the relationship between: the expected future changes in short-term rates over the lifespan of a long-term bond till maturity $E_t\left(\Delta i_{1,t+j}\right), j = 1, \dots, n-1$, and the exogenous component of the premia $\varepsilon_{n,t}$, on the right-hand side; and the yield spread adjusted to incorporate the endogenous component of the premia $(i_{n,t} - i_{1,t} - \theta_n^1 fh_t)$, on the left-hand side. Similar logic applies to the interpretation of the inflation-adjusted (real) yields expression derived in (7).

[10] Mehra (1998) is one of a number of papers suggesting that the nominal long-term bond yield is cointegrated with inflation—in particular, with the one-period current inflation rate. The Fisher equation provides a natural theoretical reference to make explicit such a connection between inflation expectations and nominal yields.

Assuming that $i_{n,t} \sim I(1)$, $i_{1,t} \sim I(1)$, $\pi^e_{t,n} \sim I(1)$, $\pi^e_{t,1} \sim I(1)$, and $fh_t \sim I(1)$, the fact that all these variables are cointegated can be seen from equation (7). The right-hand side of (7) is stationary around its constant mean provided that the short-term real interest rate is $r_{1,t} \sim I(1)$—and hence $\Delta r_{1,t} \sim I(0)$—and also that the exogenous component of the premia is $\varepsilon_{n,t} \sim I(0)$. As a result, the term on the left-hand side of (7) must be stationary.

Given these conditions, the column-vector $\gamma \equiv (1, -\gamma_1, -\gamma_2, -\gamma_3, -\gamma_4)'$ where $\gamma_2 = 1$, $\gamma_1 = -\gamma_3 = \gamma$ $(\gamma = 1)$ and $\gamma_4 = \theta^1_n$ is a cointegrating vector for the vector of observables given by $X_t = (i_{n,t}, i_{1,t}, \pi^e_{t,n}, \pi^e_{t,1}, fh_t)'$. Hence, the weak form of the expectations hypothesis predicts that the long yield ($i_{n,t}$) is cointegrated (up to a constant) with the short yield ($i_{1,t}$), the long-term inflation expectations ($\pi^e_{t,n}$), the short-term inflation expectations ($\pi^e_{t,1}$), and the foreign holdings (fh_t).

3.1 The Linear Error-Correction Specification

Engle and Granger (1987) show that cointegration implies and is implied by an error-correction representation. Given that the vector of observables $X_t = (i_{n,t}, i_{1,t}, \pi^e_{t,n}, \pi^e_{t,1}, fh_t)'$ contains only $I(1)$ variables, the theoretical relationship implied by equation (7) means that a cointegrating vector of constants $\gamma \equiv (1, -\gamma_1, -\gamma_2, -\gamma_3, -\gamma_4)'$ exists such that the linear combination $\gamma' X_t$ is $I(0)$ (up to a constant intercept). Hence, according to the Granger representation theorem in Engle and Granger (1987), there exists a statistical representation for X_t in the form of a vector error-correction model (VECM) that can be expressed as,

$$\Delta X_t = \mu + \Pi X_{t-1} + \sum_{l=1}^{k} \Gamma_l \Delta X_{t-l} + \epsilon_t, \quad (8)$$

where the matrices $\left(\mu, \Pi, \{\Gamma_l\}_{l=1}^{k}\right)$ describe the VECM specification in its most general form with up to $k \geq 1$ lags of the variables in X_t in first differences and with non-zero intercepts.

A single-equation specification in error-correction form for the long-term interest rate ($i_{n,t}$) akin to the specification used in much of the existing empirical literature (e.g., Warnock and Warnock (2009)) also emerges from the cointegrating relationship implied by (7). The single-equation error-correction model (SEECM) can be expressed as,

$$\Delta i_{n,t} = \beta_0 + \alpha(\gamma' X_{t-1}) + \sum_{l=1}^{k} \beta_l{}' \Delta X_{t-l} + e_t, \quad (9)$$

where β_0 is the intercept, $\beta_l = (\beta_{1,l}, \beta_{2,l}, \beta_{3,l}, \beta_{4,l}, \beta_{5,l})'$ are the coefficients on the first-differenced variables lagged *l*-periods, and α determines the speed of adjustment towards the long-run cointegrating relationship.

When the long-run relationship among the variables in X_t is out of equilibrium, either the long-term yield ($i_{n,t}$), the short-term rate ($i_{1,t}$), the long-term inflation expectations ($\pi^e_{t,n}$), the short-term inflation expectations ($\pi^e_{t,1}$), and/or the foreign holdings (fh_t) must adjust. Therefore, as implied by equation (7), the error-correction term $i_{n,t} - \gamma_1 i_{1,t} - \gamma_2 \pi^e_{t,n} - \gamma_3 \pi^e_{t,1} - \gamma_4 fh_t$ should have predictive power for future changes in the long-term interest rate.

3.2 The Nonlinearity at the ZLB and Heckman Selection

To explore the empirical link between the long-term nominal yield ($i_{n,t}$), the short-term nominal yield ($i_{1,t}$), and the foreign holdings ratio (fh_t) over the sample period that includes the recent experience with unconventional monetary policy at the ZLB in the U.S., we require additional modelling assumptions to conceptualize the nonlinearity introduced by the ZLB constraint. For this purpose, we extend the well-known Black (1995)'s interest rate model which postulates a latent or shadow short-term rate that can be negative even though the actual nominal short-term rate is bounded from below. The ZLB constraint arises in this context because the existence of cash (currency), an asset whose interest rate is zero, implies that nominal interest rates on other asset classes (short-term bonds, among others) must be non-negative to rule out arbitrage.

Expanding on Black (1995)'s idea, we specify a bivariate sample selection model (also referred as Heckman sample selection, Probit selection or Type II Tobit model) to specify the one-period nominal yield $i_{1,t}$ which comprises of a participation equation

$$x_{1,t} = \begin{cases} 1 \;\; if\; x^*_{1,t} > 0 \; (Non - ZLB), \\ 0 \;\; if\; x^*_{1,t} \leq 0 \; (ZLB), \end{cases} \tag{10}$$

an outcome equation

$$i_{1,t} = \begin{cases} i^*_{1,t} \; if\; x_{1,t} = 1, \\ 0 \quad if\; x_{1,t} = 0, \end{cases} \tag{11}$$

and a linear model with additive errors for the corresponding (unobserved) latent variables

$$x^*_{1,t} = z'_{1,t}\gamma + u^1_t,$$

$$i^*_{1,t} = z'_{2,t}\alpha + u^2_t. \tag{12}$$

We assume that there exists an economic relationship for the shadow rate $i_{1,t}^*$ given by (12), where $z_{2,t}'$ is a vector of multiple factors.[11] A different latent variable $x_{1,t}^*$, explained by the vector of factors $z_{1,t}'$, determines whether the nominal yield $i_{1,t}$ equates the shadow rate $i_{1,t}^*$ or is constrained at the ZLB.

We observe whether the nominal one-period yield is constrained or not ($x_{1,t}$) and the yield itself (i). Hence, the latent shadow rate $i_{1,t}^*$ is implicitly observed through the nominal yield $i_{1,t}$ if unconstrained (that is, if $x_{1,t} = 1 \leftrightarrow x_{1,t}^* > 0$). The Tobit model is a special case of the bivariate selection model in (10)-(12) when the latent variable in the participation equation $x_{1,t}^*$ exactly equates the shadow rate $i_{1,t}^*$. Our more general specification implies that the opportunity costs of setting short-term rates at the ZLB (given by $x_{1,t}^*$) do not have to be solely determined by the shadow rate ($i_{1,t}^*$); hence, the factors driving both latent variables may differ ($z_{1,t}' = z_{2,t}'$) and even the coefficients on the factors that are common may vary.

We complete the model specification with the additional assumption that the correlated errors are jointly normally distributed and homoscedastic

$$\begin{bmatrix} u_t^1 & \\ u_t^2 & \end{bmatrix} \left| z_{1,t}', z_{2,t}' \right] \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}\right],$$

The normalization $\sigma_1^2 = 1$ is needed here since only the 0-1 variable $x_{1,t}$ is observed and not the magnitude of the opportunity cost $x_{1,t}^*$. The model in (10)-(12) specifies a Tobit regression for the short-term yield and suggests some alternative strategies to analyze the long-term yield under the ZLB constraints.

We assume that there exists a latent relationship for the shadow rate $i_{1,t}^*$ given as follows,

$$i_{1,t}^* = z_t'\alpha + u_t^2, \tag{12}$$

where $z_{2,t}'$ is the vector of multiple factors driving the shadow rate. One way to model the shadow rate consistent with much of the existing monetary literature would be to specify a standard Taylor (1993)-type monetary policy rule. The dependent variable, however, is the nominal short-term interest rate $i_{1,t}$ which is not always observed as it can be stuck at zero. The short-term rate for any given time *t* is observed if

$$i_{1,t} = \begin{cases} 0 \quad if\ z_t^{'}\gamma + u_t^1 \leq 0\ (ZLB), \\ i_{1,t}^*\ if\ z_t^{'}\gamma + u_t^1 > 0\ (Non - ZLB), \end{cases} \tag{13}$$

where we assume the Gaussianity of the error terms, i.e. $u_t^1 | z_t' \sim N(0, \sigma_1^2)$ with the normalization $\sigma_1 = 1$ and $u_t^2 | z_t' \sim N(0, \sigma_2^2)$. The model also allows a bivariate representation where the covariance between the errors terms is given by $\sigma_{12} = \rho\sigma_1\sigma_2$ and possibly different than zero.[12] Under this flexible specification, the

[11] One way to model the shadow rate consistent with much of the existing monetary literature would be to specify a standard Taylor (1993)-type monetary policy rule.

[12] It follows that the shadow rate is distributed as $i_{1,t}^* | z_t' \sim N(z_t'\alpha, \sigma_2^2)$.

vector of factors z_t' can have a different effect on: (Probit part) the likelihood of staying away from the ZLB (i.e., $Prob\left(i_{1,t}^* \leq 0\right) = 1 - \Phi\left(\frac{z_t'\gamma}{\sigma_1}\right)$ where $\Phi(.)$ denotes the cdf of the standard normal distribution); (Truncated regression part) the interest rate response conditional on interest rates being unconstrained. The standard Tobit model arises as a special case if $\frac{\gamma}{\sigma_1} = \frac{\alpha}{\sigma_2}$ and $u_t^1 = u_t^2$.

The expectation conditional on the interest rate being observed away from the ZLB is given by $E\left(i_{1,t} \middle| z_t, z_t'\gamma + u_t^1 > 0\right) = z_t'\alpha + \sigma\lambda\left(\frac{z_t'\gamma}{\sigma_1}\right)$. An observation drawn from the population, which may or may not be censored, implies the following unconditional expectation

$$E\left(i_{1,t} | z_t\right) = \Phi\left(\frac{z_t'\gamma}{\sigma_1}\right)\left(z_t'\alpha + \sigma\lambda\left(\frac{z_t'\gamma}{\sigma_1}\right)\right), \tag{14}$$

where $\lambda\left(\frac{z_t'\gamma}{\sigma_1}\right) \equiv \frac{\phi\left(\frac{z_t'\gamma}{\sigma_1}\right)}{\Phi\left(\frac{z_t'\gamma}{\sigma_1}\right)}$, while $\Phi(.)$ and $\phi(.)$ denote the cdf and pdf of the standard normal distribution respectively. The Heckman selection model implies that the marginal effect on the unobserved shadow rate is $\frac{\partial E\left(i_{1,t}^* | z_t\right)}{\partial z_t} = \alpha$ while the marginal effect on the short-term rate in (14) is given by $\frac{\partial E\left(i_{1,t} | z_t\right)}{\partial z_t} = \alpha\Phi\left(\frac{z_t'\alpha}{\sigma}\right)$. We argue that the censoring at the ZLB has the consequence that it attenuates the marginal effects of the factors driving the shadow rate. Whenever the proportion of non-ZLB observations is large in the sample ($\Phi\left(\frac{z_t'\alpha}{\sigma}\right) \approx 1$), the marginal effects would be largely similar to the case without censored data. In turn, the marginal effect on the observed short-term rate can be decomposed as $\frac{\partial E\left(i_{1,t} | z_t\right)}{\partial z_t} = Prob\left(i_{1,t} > 0\right)\frac{\partial E\left(i_{1,t} | z_t, i_{1,t} > 0\right)}{\partial z_t} + E\left(i_{1,t} \middle| z_t, i_{1,t} > 0\right)\frac{\partial Prob\left(i_{1,t} > 0\right)}{\partial z_t}$. Hence, we see that a change in the factors driving the shadow rate has two consequences: it affects the conditional mean of the shadow rate in the uncensored part of the distribution (away from the ZLB), and it affects the probability that the observation will fall in that part of the distribution.

From the point of view of our work investigating the relationship between the long-term interest rate and the foreign holdings of Treasury securities it is what happens at the ZLB when the observed rates and the shadow rates diverge that matters. The weak form of the expectations hypothesis in equation (1) can be rewritten using (12) instead of the expectation of the shadow rate $E\left(i_{1,t}^* | z_t\right) = z_t'\alpha$ which is unobserved. One empirical strategy we could pursue then is to discard the observations at the ZLB which is the empirical literature has favored in the past focusing on the subsample away from the ZLB. For the subpopulation from which the non-ZLB observations are drawn, we could write the regression implied by (11) as $i_{1,t} \middle| i_{1,t} > 0 = E\left(i_{1,t} \middle| i_{1,t} > 0\right) + e_t = z_t'\alpha + \sigma\lambda_t + e_t$ where e_t is $i_{1,t}$ minus its conditional expectation. Unless we take account of the nonlinear term λ_t in the estimation, all the biases that arise from an omitted variable can be expected.

Key Implications. We define the one-period shadow rate as $i_{1,t}^*$ and set the lower bound on interest rates at zero such that the actual short-term rate $i_{1,t}$ can be expressed as,

$$i_{1,t} = \max\{i_{1,t}^*, 0\} = i_{1,t}^* + \max\{-i_{1,t}^*, 0\}. \tag{10}$$

Equation (10) shows that the observed nominal short-term interest rate $i_{1,t}$ can be interpreted as a call option on the shadow interest rate $i_{1,t}^*$ whose strike price is zero percent. Alternatively, the observed nominal interest rate $i_{1,t}$ can also be expressed as the sum of the shadow interest rate $i_{1,t}^*$ and an option-like value—which we call the floor value—which supports the ZLB constraint by switching funds from short-term bonds to cash when the shadow interest rate $i_{1,t}^*$ becomes negative.

Taking account of the ZLB constraint in (10), the weak form of the expectations hypothesis in equation (1) can be generalized as follows

$$i_{n,t} = \frac{1}{n}\left[\sum_{j=1}^{n} E_t(\max\{i_{1,t+j-1}^*, 0\})\right] = \frac{1}{n}\left[\sum_{j=1}^{n} E_t(i_{1,t+j-1}^*)\right] + l_{n,t} + \theta_{n,t}, \tag{11}$$

rewritten in terms of the shadow rate $i_{1,t}^*$, where $l_{n,t} \equiv \frac{1}{n}\left[\sum_{j=1}^{n} E_t(\max\{-i_{1,t+j-1}^*, 0\})\right]$ defines an additional premium term which equals the option-like floor value on the short-term shadow rate $i_{1,t}^*$ averaged over the maturity of the corresponding *n*-period bond. We draw attention to this specification because the time-varying floor value in (11) has been absent from the existing empirical work motivated by equation (1), even though it may matter also in subsamples where monetary policy is unconstrained by the ZLB and the observed and shadow short-term rates coincide. This may, in turn, bias the existing estimates.

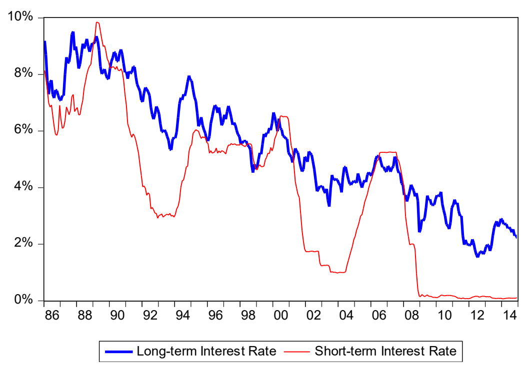
In the remainder of this paper we investigate the empirical evidence of instability in the long-run cointegrating relationship—but also in the short-run dynamics modelled in equation (9)—focusing on the shifting impact of foreign holdings (fh_t). While we conjecture that the ZLB may lead to a structural break in the relationship, we adopt a more agnostic approach on its direction (weakening or strengthening the effect) and magnitude. We view this as a largely empirical question, which to our knowledge we are the first to address, and leave the development of richer theories to account for the empirical evidence presented in the remainder of the paper for future research.

4. Data

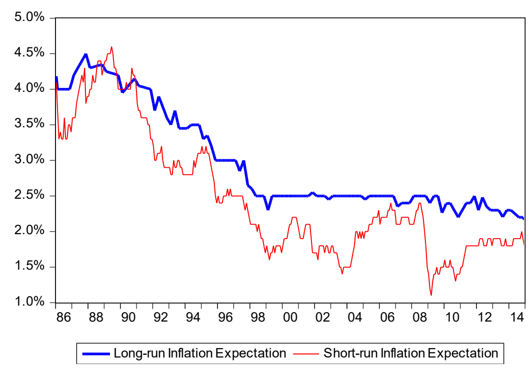
Our dataset covers the sample period after the Volcker disinflation of the early 1980s from 1986m01 till 2014m12. Therefore, our sample covers most of the Great Moderation era and the ZLB period that began in 2008m12 when the Federal Reserve lowered the target level of the Fed Funds rate to the 0 to 25 basis points range (where it stayed until 2015m12). For this period of time, we have complete monthly time series of 348 observations on all variables pertinent to our empirical study.[13] The main explanatory variables in our dataset include the short-term and long-term nominal rates, $i_{1,t}$ and $i_{n,t}$, the short-term and long-term inflation expectations, $\pi^e_{t,1}$ and $\pi^e_{t,n}$, and the foreign holdings fh_t.

Figure 1. Data Plots of Key Variables

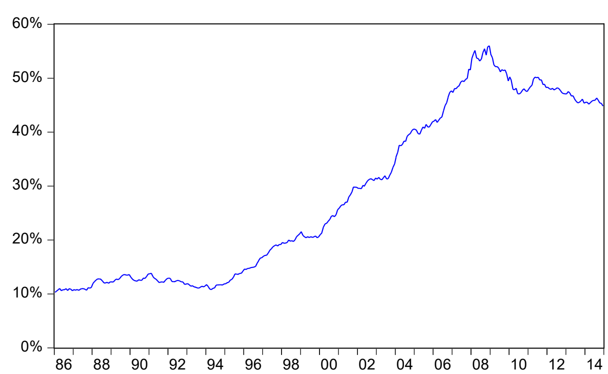
A. Long-term and Short-term Interest Rates



B. Long-run and Short-run Inflation Expectation



C. Foreign Official Holdings as a Share of Outstanding Marketable Treasury Notes and Bonds



Sources: FRED Database, Survey of Professional Forecasters, Blue Chip Economic Indicators, Bertaut and Tryon (2007), Bertaut and Judson (2014), FRB H.4.1., and Department of the Treasury.

[13] Our reference sample period begins in the early years of the Great Moderation after the Volcker disinflation with the development and consolidation of the Greenspan-Bernanke regime of price-based monetary policy which came to be characterized by the Taylor (1993) rule targeting the Fed Funds rate. Our reference sample period also includes the major policy shift towards unconventional monetary policies (and the return of quantity-based policies in the form of Quantitative Easing, QE) that followed the 2008-09 financial recession coinciding with the latter part of Ben Bernanke's tenure and the beginning of Janet Yellen's time at the helm of the Federal Reserve. As summarized in Table 1, other well-known studies in the literature similarly study a sample period beginning in the mid-1980s or early-1990s—the most significant differences in time coverage arising from the fact that our dataset includes data up to 2014m12 helping us explore the stability of the empirical relationship at the ZLB.

For the short-term nominal rate $i_{1,t}$, we use the Fed Funds rate (monthly average) retrieved from the Federal Reserve Bank of St. Louis' FRED database. For the long-term nominal rate $i_{n,t}$, we use the 10-year Treasury yield (monthly average) also retrieved from the FRED database.[14] Both nominal interest rate series are illustrated in Panel A of Figure 1. The short-term inflation expectations data, $\pi^e_{t,1}$, is the monthly one-year-ahead forecast of the (year-over-year) percent change of the quarterly GNP/GDP price deflator from the Blue Chip Economic Indicators survey.[15] The long-term inflation expectations data, $\pi^e_{t,n}$, is from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF), extended with Blue Chip Economic Indicators survey data prior to 1991Q4, and linearly interpolated from quarterly to monthly frequency.[16] The short- and long-term inflation expectations are plotted in Panel B of Figure 1.

The foreign official holdings ratio, fh_t, is constructed from data obtained from Bertaut and Tryon (2007) and expanded by Bertaut and Judson (2014) on benchmark-consistent monthly estimates of foreign official holdings, complemented with total outstanding Treasury notes and bonds data obtained from the Monthly Statement of the Public Debt (MSPD) of the Department of the Treasury and with the Federal Reserve's holdings of U.S. Treasury notes and bonds data available in the Federal Reserve's statistical release H.4.1. The ratio fh_t of foreign official holdings of Treasury notes and bonds as a percentage of the outstanding marketable Treasury notes and bonds excluding the Federal Reserve's holdings of Treasury notes and bonds is plotted in Panel C of Figure 1.

A Closer Look at the Data Used for the Foreign Holdings Ratio (fh_t).

The main source for the foreign holdings (or net purchases) of U.S. Treasury notes and bonds—needed to calculate the ratio fh_t—is the Treasury International Capital (TIC) system of the U.S. The TIC S-Form provides monthly data on foreign official and private investors' net purchases (gross purchases minus

[14] Data sources from the FRED database for the 10-year Treasury yield and the policy rate (the Fed Funds rate): https://research.stlouisfed.org/fred2/series/DGS10, https://research.stlouisfed.org/fred2/series/FEDFUNDS.

[15] The Blue Chip Economic Indicators survey reports forecasts of the percent change (from the prior quarter, annualized) of the quarterly GNP/GDP deflator at monthly frequency. We denote the reported monthly inflation forecast at period t for inflation q-quarters-ahead as $g^{qoq}_{t,q}$, where $q = 0$ indicates the current quarter, and collect those forecasts over the next four quarters—i.e., $\left(g^{qoq}_{t,1}, g^{qoq}_{t,2}, g^{qoq}_{t,3}, g^{qoq}_{t,4}\right)$. Our measure of the short-term inflation expectations at time t is the corresponding one-year-ahead forecast of the percent change in the GNP/GDP price deflator expressed in year-over-year rates, which we denote as g^{yoy}_{t+4}. We compute the one-year-ahead inflation forecast at time t with the following formula: $g^{yoy}_t = 100\left(\left(\prod_{q=0}^{3}\left(1+\frac{g^{qoq}_{t,1+q}}{100}\right)\right)^{\frac{1}{4}} - 1\right)$.

[16] The main data source for long-term inflation expectations is the Survey of Professional Forecasters (SPF) whose data can be accessed here: https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/inflation.xls. The SPF forecasts that we use are the expectations for the annual average rate of CPI inflation over the next 10 years ("INFCPI10YR") which are only available from 1991Q4 onwards and at quarterly frequency. The SPF also makes available additional 10-year-ahead inflation forecasts from other sources going further back in time that can be downloaded here: https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/additional-cpie10.xls. We follow the SPF's own recommendation and extend the INFCPI10YR series with the additional forecasts obtained by SPF from the Blue Chip Economic Indicators survey (Additional-CPIE10.xls). The variable forecasted by Blue Chip Economic Indicators since 1983 is the CPI (with the exception of 1983Q4 where it still is the GNP deflator) and those forecasts are taken twice a year (March and October). The biannual Blue Chip Economic Indicators series and the quarterly SPF series are then linearly interpolated to monthly frequency and combined together back to the beginning of our sample period (1986m01).

gross sales) of U.S. Treasury notes and bonds starting from 1979m01.[17] The TIC Form SLT reports the monthly data of foreign official and private investors' holdings of U.S. Treasury notes and bonds at market value, but has a limited coverage since it only starts on 2011m09. The TIC annual surveys provide the most accurate data on foreign official and private investors' holdings of U.S. long-term Treasury securities (the amounts reported are those for the end of June of each year).[18]

It is well-known that TIC data has limitations, though.[19] First, the monthly data of net purchases from TIC S-Form suffers from transaction bias. It only records the direct buyer or seller of the Treasury securities, not the ultimate buyer or seller. Second, the estimated holdings computed from accumulated monthly TIC net flows are not fully consistent with the holdings data in the annual survey (which is regarded as the benchmark data) because of valuation changes that occur over the course of the year and, potentially, because of transaction bias too. Third, there are also no official reports for the monthly foreign holdings data before September 2011 when TIC Form SLT got started. Only data on net purchases from the TIC S-Form is available prior to September 2011. Fourth, all data for foreign net purchases and holdings available is subject to custodial bias. For example, a foreign official investor can use a custodian in another country to purchase or sell U.S. Treasury securities. Hence, the geographical allocation of foreign holdings (net purchases) may not be accurate and the foreign official holdings (net purchases) may be overestimated/underestimated.

While we cannot overcome the limitations of the TIC data entirely, we rely on a novel dataset developed by Bertaut and Tryon (2007) and expanded by Bertaut and Judson (2014) that introduces a number of adjustments to the available TIC data (the TIC S-Form data, the annual survey data, and the more recent release of TIC Form SLT data).[20] This dataset provides benchmark-consistent monthly estimates of foreign official and private holdings. Hence, we use these benchmark-consistent estimates of foreign holdings to construct the foreign holdings ratio fh_t that we use in our empirical analysis.

In the literature, the foreign net purchases or holdings variables are generally scaled by either nominal GDP or total outstanding/marketable Treasury notes and bonds. Our variable fh_t is the ratio of foreign official holdings of Treasury notes and bonds as a percentage of the outstanding marketable Treasury notes and bonds—i.e., outstanding Treasury notes and bonds excluding the Federal Reserve's holdings of Treasury notes and bonds.[21]

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[17] The foreign official institutions mainly include foreign central banks. A partial list of foreign official institutions used by TIC can be found here: https://www.treasury.gov/resource-center/data-chart-center/tic/Pages/foihome.aspx.

[18] The TIC annual surveys have been conducted each year since 2002. Before 2002, the TIC surveys were also conducted in 1974, 1978, 1984, 1989, 1994, and 2000.

[19] See Bertaut and Tryon (2007), Warnock and Warnock (2009), and Bertaut and Judson (2014) for further details.

[20] The data can be downloaded from http://www.federalreserve.gov/pubs/ifdp/2014/1113/ifdp1113_data.zip.

[21] Bertaut *et al.* (2012) and Beltran *et al.* (2013) also use the same marketable Treasury notes and bonds as the scaling factor for foreign holdings or foreign net purchases.

The data on total outstanding Treasury notes and bonds is available in the Monthly Statement of the Public Debt (MSPD) from the Department of the Treasury which reports the face value of those securities going back to 1952m07.[22] The data on the Federal Reserve's holdings of U.S. Treasury notes and bonds is available from the Federal Reserve statistical release H.4.1 measured at face value.[23] Using the foreign holdings ratio over outstanding marketable Treasury securities, we take account of the foreign demand of Treasury securities relative to the supply of outstanding Treasury securities net of the impact that the Federal Reserve's holdings of Treasury securities has on the supply. A disadvantage of using the ratio of foreign holdings over outstanding marketable Treasuries, fh_t, is that this ratio has a market value in the numerator but a face value in the denominator. As is noted in the annual survey of foreign portfolio holdings of U.S. Treasury securities, it is not possible to obtain the market value of total outstanding Treasuries on the same basis as the data on foreign holdings or net purchases.[24]

A plausible alternative to construct fh_t that avoids the mismatch in valuation terms between the numerator and the denominator is to use the foreign holdings data at face value (instead of at market value). Table 1A of the FRB H.4.1 release ("Factors Affecting Reserve Balances of Depository Institutions and Condition Statement of Federal Reserve Banks") provides the face value of U.S. Treasury securities held in custody at the Federal Reserve Bank of New York by foreign official institutions. This can be used as an alternative data source for the foreign official holdings of U.S. Treasury securities (at face value) to construct a consistent measure of fh_t. However, the FRB H.4.1. release data only partially accounts for all foreign official holdings of U.S. Treasury securities. Therefore, we argue that the foreign holdings ratio we use still offers the most sensible way to scale foreign holdings, given the limitations of the existing data.

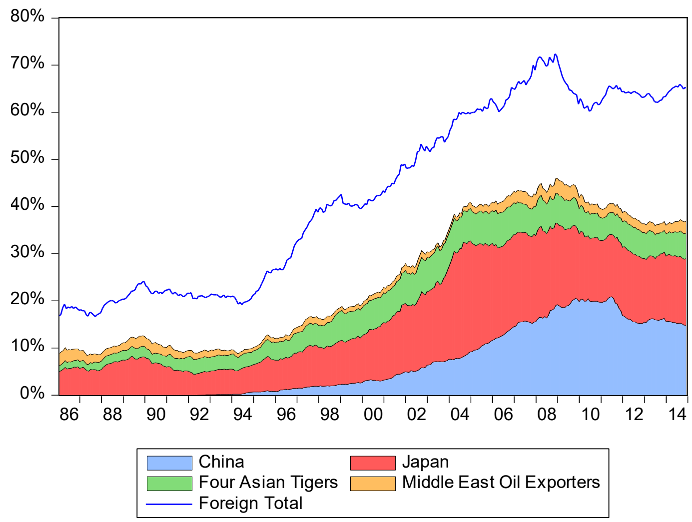
The TIC data reports foreign total (official and private jointly) net purchases or holdings by country.[25] As seen in Panel A of Figure 2, large foreign inflows into U.S. Treasury securities took off in the mid-1990s. Policy changes coupled with the rapid pace of integration of China into the global economy—with large savings exceeding domestic investment opportunities, sizeable current account surpluses, and large foreign exchange reserve accumulation—are noted among the reasons for the dramatic change in foreign holdings of U.S. Treasury securities started around 1994 (e.g., Sierra (2010) and Goda *et al.* (2013)).[26] Japan, other smaller, but fast-growing economies of East Asia (the so-called Four Asian Tigers:

[22] Data source: https://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm. Historical monthly data from the Monthly Statement of the Public Debt (MSPD) is also available for download for the years 1869-1952 from this website, but such data goes beyond the scope of our paper.
[23] Data source: http://www.federalreserve.gov/releases/h41/.
[24] See page 4 on the Foreign Portfolio Holdings of U.S. Securities as of June 30, 2014 released by the U.S. Department of the Treasury.
[25] The data to decompose foreign official or private flows and holdings into different individual foreign countries are not publicly available (is confidential). Hence, we only have individual country data on total (including both official and private) flows and holdings of Treasury securities but not on official and private holdings separately.
[26] Among the cited policy changes, the exchange rate of the Chinese RMB against U.S. dollars move from 5.8 CNY/USD to 8.7 CNY/USD on 1994m02 is generally regarded as signaling a major policy shift and the beginning of a rapid outward transformation for China. China's accession to the World Trade Organization (WTO) in 2001 seems to have further accelerated those trends.

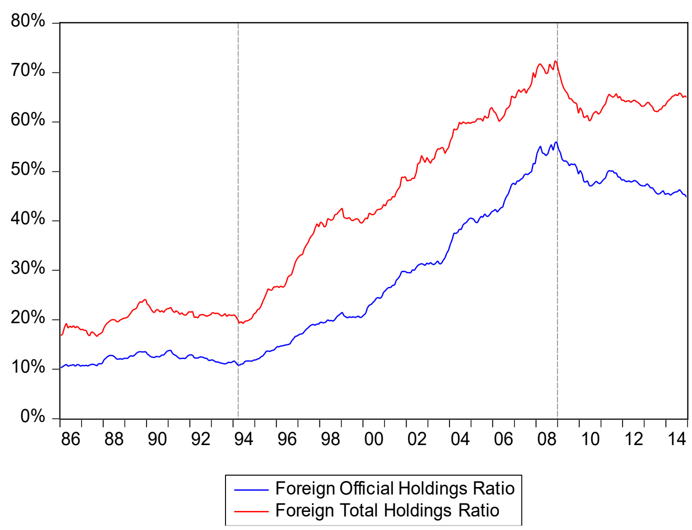
South Korea, Singapore, Hong Kong, and Taiwan), and the oil-exporting countries contributed to a lesser extent to the rise after having been the major foreign players in the U.S. Treasury securities market during the 1980s and the better part of the 1990s.[27]

Figure 2. Foreign Total Holdings of U.S. Long-term Treasury Securities as a Share of Outstanding Marketable Treasury Notes and Bonds

A. By Different Foreign Holders



B. By Ownership Type: Total Holdings vs. Official-Only Holdings



Source: Bertaut and Tryon (2007), Bertaut and Judson (2014), FRB H.4.1., and U.S. Department of the Treasury.

[27] In Panel A of Figure 2, Middle-East Oil Exporters refers to Bahrain, Iran, Iraq, Kuwait, Oman, Qatar, Saudi Arabia, and the U.A.E.

It is possible to analyze foreign holdings (or net flows) by type of investor (official or private) separately. As shown in Panel B of Figure 2, the ratio of foreign total (the sum of official and private) holdings of U.S. Treasury notes and bonds as a percentage of U.S. total outstanding marketable Treasury notes and bonds is largely accounted for by foreign official holdings.[28] From 1994 to 2008, the foreign official holdings ratio dramatically increased from 10% to 55%. China's large accumulation of foreign exchange reserves alone explains a sizeable part of this shift in foreign ownership. After 2008, the increase in the foreign private holdings ratio has mostly made up for the decline in the ratio of foreign official holdings.[29]

The aftermath of the 2008-09 financial recession brought about changes in the U.S. monetary policy framework whereby the Fed Funds rate became constrained at the ZLB and the Federal Reserve switched gears towards other policy tools (BSPs and FG) instead. The large-scale asset purchases by the Federal Reserve under three consecutive rounds of QE had an impact on the share of foreign official holdings in total marketable Treasuries. However, the policy shift required by the ZLB constraint away from conventional monetary policy based on the Fed Funds rate may also have contributed to a structural break in the empirical relationship between foreign holdings and the long-term interest rate. Our paper is the first—to the best of our knowledge—to investigate formally the possibility of such a structural break in order to better quantify and estimate the effects of foreign holdings on long-term yields.

5. Empirical Findings

5.1 Stability of the Long-run Cointegrating Relationship

We argue in Section 3 that the variables $i_{n,t}$, $i_{1,t}$, $\pi^e_{t,n}$, $\pi^e_{t,1}$ and fh_t ought to be cointegrated under the weak form of the expectations hypothesis of the yield curve augmented with a specification of the premia tied to foreign holdings and with the Fisher equation (equations (1)-(3)). We verify the non-stationary properties of these variables in our data to ensure the dataset is consistent with the theory of cointegration. As required by theory, we find that all our variables are I(1) over the full sample period between 1986m01 and 2014m12.[30] In addition, we define the real short-term interest rate based on the Fisher equation, $r_{1,t} = i_{1,t} - E_t(\pi_{t+1}) = i_{1,t} - \pi^e_{t,1}$, by subtracting the short-term inflation expectation

[28] Unlike foreign private holdings, the foreign official holdings are generally treated as exogenous in the existing literature because foreign official institutions generally do not optimize their investment strategy on Treasury securities in response to the prices of Treasuries themselves or U.S. monetary policy (see, e.g. Warnock and Warnock (2009) and Bertaut *et al.* (2012)). We follow the literature in this regard and use foreign official holdings as well. However, we also explore foreign total holdings (official plus private) as a robustness check.

[29] Using Bai-Perron multiple break points test for the regression of the foreign official holdings ratio on a constant and a time trend, two of the breakpoints that we can formally identify occur in 1994m03 and 2009m03. We can informally identify both periods through visual inspection of the time series as well. The Bai-Perron test also indicates break points on 1999m03 and 2004m02.

[30] The concern about mixing I(0) and I(1) variables in our empirical model is that doing so may lead to spurious regression results. The unit root test results for our data are available from the authors upon request. That evidence gives us confidence about our empirical estimation results since we find that we are using a balanced dataset of I(1) variables.

($\pi^e_{t,1}$) from the nominal short-term interest rate ($i_{1,t}$). Consistently with the assumptions in Section 3, the evidence suggests that the real short-term rate $r_{1,t}$ is also an I(1) variable and accordingly $\Delta r_{1,t}$ is an I(0) variable.

Parameter Instability in the Long-Run Cointegrating Relationship.

With all these results, we estimate and make inferences on the long-run cointegrating relationship implied by the benchmark theoretical model, i.e.,

$$i_{n,t} = \gamma_0 + \gamma_1 i_{1,t} + \gamma_2 \pi^e_{t,n} + \gamma_3 \pi^e_{t,1} + \gamma_4 fh_t + \varepsilon_t, \tag{13}$$

with the fully modified OLS (FMOLS) technique developed by Phillips and Hansen (1990). To investigate the parameter stability in the long-run cointegrating relationship posited by equation (13), we implement a battery of tests—the Hao and Inder (1996) FMOLS-based CUSUM test, Hansen (1992)'s instability tests, and Gregory and Hansen (1996)'s no-cointegration tests with a single structural break.

First, we implement the Hao and Inder (1996) FMOLS-based CUSUM test under the null hypothesis of no structural breaks. This test is an extension of the CUSUM test based on OLS residuals proposed by Ploberger and Kramer (1992). Hao and Inder (1996)'s CUSUM test statistic can be computed as $sup_{0<\tau<1} |B(\tau)|$ and $B(\tau) = \frac{1}{\sqrt{\widehat{\omega}^2}\sqrt{T}} \sum_{t=1}^{[T\tau]} \hat{u}_t$, where $\hat{u}_t$ is the FMOLS residual, T is the sample size, $\widehat{\omega}^2$ is the estimated long-run variance, and $[\cdot]$ denotes the integer part. Table 2 reports the test results using different kernels for both the full sample (1986m01-2014m12) and the subsample (1986m01-2008m11) for equation (13). For the full sample, we can reject the null hypothesis of parameter stability at the 1% level of significance; while the FMOLS-based CUSUM test statistics are insignificant at all conventional levels of significance for the subsample excluding the ZLB period.

Second, we also apply Hansen (1992)'s instability tests. The three associated statistics—L_c, MeanF, and SupF—can be used to test the null hypothesis of parameter stability against the alternative of parameter instability in the FMOLS cointegrating equation given by (13). Table 3 presents the Hansen (1992) test results using different kernels, bandwidth selection methods and pre-whitening options for the full sample (1986m01-2014m12). Although we find some insignificant test statistics for the full sample, overall the evidence from Hansen (1992) tests tends to reject the null hypothesis of parameter stability in the long-run cointegrating relationship.

Finally, we also use a battery of cointegration tests proposed by Gregory and Hansen (1996), which introduce three test statistics—i.e., ADF*, Z^*_α, and Z^*_t—to test the null hypothesis of no cointegration against the alternative hypothesis of cointegration allowing for the cointegrating vector to change at a single unknown break date during the sample period. The tests can be applied to three types of structural break models: level shift, level shift with trend, and both level shift and slope shift (called regime shift). These test statistics are helpful in detecting a break in the cointegrating relationship especially when the conventional cointegrating tests (e.g., Engle-Granger or Phillips-Ouliaris) cannot reject the null hypothesis of no cointegration. A byproduct of these tests is the estimated breakpoint date, although these tests only allow for one breakpoint.[31]

The Engle and Granger (1987) and Phillips and Ouliaris (1990) cointegration tests (tau-statistic and z-statistic) both indicate that the variables, $i_{n,t}$, $i_{1,t}$, $\pi^e_{t,n}$, $\pi^e_{t,1}$ and fh_t, appear cointegrated for the full sample period. To further examine possible changes in the long-run cointegrating relation given by equation (13), we also conduct these tests using a smaller subsample between 1986m01 and 2008m11. The cointegration test results for the subsample also confirm the evidence of cointegration.

Since the Engle-Granger and Phillips-Ouliaris cointegration tests reject the null of no cointegration for the full sample period (1986m01-2014m12), not surprisingly the three test statistics of Gregory and Hansen (1996) reject the null hypothesis of no cointegration—for all three types of (structural break) models at the 5% level, except with the Z^*_α test statistic in the regime shift model (this test statistic is only a little bit less negative than its corresponding 5% critical value). Although both conventional cointegrating tests and Gregory and Hansen (1996) tests reject the null of no cointegration, the latter provide some meaningful break dates and corroborating evidence of a break at the end of 2008 (Table 4). We focus on the results for the regime shift type model, where the estimated break point for the Z^*_t test statistic comes as early as 2009m02.

Early 1994 signals a major increase of the foreign holdings ratio (fh_t) in no small part due to China's policy shift towards accumulating U.S. Treasury notes and bonds. Therefore, we conduct similar stability tests—Hao and Inder (1996), Hansen (1992) and Gregory and Hansen (1996) tests—over the subsample period 1986m01-2008m11 to examine its statistical significance. Table 3 shows that none of the three test statistics in Hansen (1992) can reject the null hypothesis of parameter stability at the 5% level for this subsample except marginally the MeanF statistic. Neither does the Hao and Inder (1996) test. The Z^*_t and Z^*_α of Gregory and Hansen (1996) test statistics for the regime shift model (Table 4) detect 1994m02 as a breakpoint. However, in all cases, this breakpoint is not significant at conventional statistical levels.

[31] The tests were conducted using the Matlab code downloaded from http://www.ssc.wisc.edu/~bhansen/progs/joe_96.html.

Table 2. Hao and Inder (1996) FMOLS-based CUSUM Test for the Long-Run Cointegrating Relationship in Equation (13): Full Sample vs. 1986m01-2008m11

Test Statistic		Critical Values		
Full Sample (1986m01-2014m12)	Subsample (1986m01-2008m11)	10%	5%	1%
Panel A: Non-prewhitened Bartlett kernel and Newey-West fixed bandwidth				
1.225	0.610	0.7687	0.8475	1.0383
Panel B: Prewhitened Quadratic Spectral kernel and Andrew automatic selection bandwidth				
1.219	0.741	0.7687	0.8475	1.0383
Panel C: Prewhitened Bartlett kernel and Andrew automatic selection bandwidth				
1.204	0.740	0.7687	0.8475	1.0383

Note: The critical values for the FOMLS-based CUSUM test under the null that the parameters of the long-run cointegrating relationship are stable are from Hao and Inder (1996).

Table 3. Hansen (1992) Parameter Stability Tests for the Long-Run Cointegrating Relationship in Equation (13): Full Sample vs. 1986m01-2008m11

Test Statistics	Full Sample (1986m01 – 2014m12)		Subsample (1986m01 – 2008m11)	
	Test Statistic Value	P-Value	Test Statistic Value	P-Value
Panel A: Non-prewhitened Bartlett kernel and Newey-West fixed bandwidth				
L_c	0.844	0.075	0.355	≥ 0.20
MeanF	7.156	0.147	5.292	≥ 0.20
SupF	17.175	0.097	12.717	≥ 0.20
Panel B: Prewhitened Quadratic Spectral kernel and Andrew automatic selection bandwidth				
L_c	0.324	≥ 0.20	0.508	≥ 0.20
MeanF	32.902	0.01	9.119	0.049
SupF	330.744	0.01	15.620	0.161
Panel C: Prewhitened Bartlett kernel and Andrew automatic selection bandwidth				
L_c	0.310	≥ 0.20	0.466	≥ 0.20
MeanF	28.473	0.01	8.405	0.074
SupF	279.197	0.01	15.227	0.182

Note: The results for L_c, MeanF and SupF were obtained using the Matlab code downloaded from http://www.ssc.wisc.edu/~bhansen/progs/jbes_92.html. The null hypothesis is parameters are stable in the long-run relationship. The SupF and MeanF statistics were calculated using the trimming range [0.15, 0.85].

Table 4. Gregory and Hansen (1996) Cointegration Tests for the Long-Run Cointegrating Relationship in Equation (13): Full Sample vs. 1986m01-2008m11

Model	5% Critical Value	Full Sample (1986m01 – 2014m12)		Subsample (1986m01 – 2008m11)	
		Test Statistic	Breakpoint	Test Statistic	Breakpoint
Level Shift	-5.56	ADF* = -7.26	2010m07	ADF* = -7.58	1999m12
	-5.56	Z_t^* = -5.92	2010m08	Z_t^* = -5.72	1994m05
	-59.40	Z_α^* = -65.69	2010m08	Z_α^* = -60.21	1994m05
Level Shift with Trend	-5.83	ADF* = -7.14	2010m07	ADF* = -7.26	1991m11
	-5.83	Z_t^* = -6.16	2010m08	Z_t^*= -6.12	2002m10
	-65.44	Z_α^* = -70.44	2010m08	Z_α^* = -66.59	1996m07
Regime Shift	-6.41	ADF* = -7.71	2010m07	ADF* = -7.59	1989m11
	-6.41	Z_t^* = -6.44	2009m02	Z_t^* = -6.12	1994m02
	-78.52	Z_α^* = -77.19	2009m08	Z_α^* = -67.45	1994m02

Note: This table reports the results of testing the null of no cointegration against the alternative of cointegration with allowance of a possible change in the cointegrating vector at a single unknown break point using the three test statistics for each of the three types of models in Gregory and Hansen (1996) using the full sample period of 1986m01-2014m12 and the subsample period of 1986m01-2008m11. The maximum lag length for the ADF* test is 12 and the lag length was selected using the downward t-statistic method. The 5% critical values are from Gregory and Hansen (1996).

FMOLS Estimates of the Long-Run Cointegrating Relationship.

Based on the body of evidence presented, we argue that the long-run cointegrating relationship in equation (13) appears to have broken down during the full sample period (1986m01-2014m12) and find a likely break date at the end of 2008. The FMOLS estimation results for equation (13) over the full sample period (1986m01-2014m12) and over a sub-sample excluding the 2008-09 financial recession and its aftermath of monetary policy stuck at zero (1986m01-2008m11) are presented in Table 5. All the estimated coefficients have the expected signs and, in most cases, are statistically significant at least at the 5% level.

Table 5. FMOLS Estimation Results for Equation (13)

	(1) 1986m01-2014m12	(2) 1986m01-2008m11
$i_{1,t}$	0.404***	0.262***
$\pi^e_{t,n}$	2.218***	1.259***
$\pi^e_{t,1}$	-0.986**	-0.051
fh_t	-0.021***	-0.027***

Note: ***, ** and * represent the level of significance at 1%, 5% and 10%, respectively. The FMOLS regressions were conducted using Bartlett kernel and Newey-West fixed bandwidth. A constant is included but not reported here.

5.2 Linear and Threshold SEECMs

Based on the standard implication of the weak form of the expectations hypothesis posited in the paper, we select the SEECM specification in order to incorporate the evidence of non-stationarity in the data and the long-run cointegrating relationship between the variables into a model for the long-term yield. This specification corresponds to the benchmark linear model posited in equation (9) in Section 3. The single-equation specification permits us to examine both the long-run and short-run effects of the foreign holdings ratio (fh_t) on the long-term yield ($i_{n,t}$) and easily test for and estimate structural breaks. In this subsection, first, we consider a linear SEECM specification without structural breaks; second, we model the structural break using a threshold SEECM specification.

Linear SEECM

Our linear SEECM model with one lag of the differenced variables takes the following form[32]:

$$\Delta i_{n,t} = \beta_0 + \alpha\left(i_{n,t-1} - \gamma_1 i_{1,t-1} - \gamma_2 \pi^e_{t-1,n} - \gamma_3 \pi^e_{t-1,1} - \gamma_4 fh_{t-1}\right) + \cdots$$
$$\beta_1 \Delta i_{n,t-1} + \beta_2 \Delta i_{1,t-1} + \beta_3 \Delta \pi^e_{t-1,n} + \beta_4 \Delta \pi^e_{t-1,1} + \beta_5 \Delta fh_{t-1} + \varepsilon_t. \quad (14)$$

We impose the following two restrictions on the long-run relationship in equation (14). First, we assume the long-run inflation expectation has a one-to-one relationship with the long-term rate.[33] Second, we assume the coefficients on the short-term interest rate and the short-term inflation expectations are equal in absolute value but of opposite sign. In summary, we require that:

[32] Our benchmark SEECM specification is based on the VECM in equation (8) as we discussed in Section 3. The Schwarz information criterion indicates a lag length of one is optimal for the VECM specification. Therefore, here we choose lag length of one for our benchmark SEECM specification too.

[33] Warnock and Warnock (2009) also impose a restriction on the long-run inflation expectation. They argue that the impact of long-run inflation expectation would become inconceivably large without any restriction.

$$\gamma_2 = 1 \text{ and } \gamma_1 + \gamma_3 = 0. \tag{15}$$

These parameter constraints are consistent with the theory laid out in Section 3, but permit us to retain some flexibility in (14) for the estimation (as customarily done in the empirical literature).

We use a nonlinear least-squares method to simultaneously estimate both the short-run dynamics and the long-run cointegrating relationship parameters in equation (14) under the restrictions given in (15). Table 6 displays the estimation results for both the full sample (1986m01-2014m12) and the subsample without the ZLB period (1986m01-2008m11). We also consider different measures of foreign holdings ratio variable, fh_t. In columns (1) and (4) of Table 6, fh_t is the foreign official holdings ratio. In columns (2) and (5), fh_t is the foreign total holdings ratio which combines official and private holdings. Finally, in columns (3) and (6), fh_t is split into a foreign official holdings ratio and a foreign private holdings ratio.

The estimated coefficients on the error-correction term, $\hat{\alpha}$, are all significant at the 5% level across all specifications as shown in Table 6, which provides additional validation for the use of the error-correction model[34]. For the subsample prior to the ZLB episode, the estimated short-run effects of foreign holdings ratio are not significant.[35] Those effects become significant using the full sample. The estimated long-run effects of the foreign holdings ratio are all highly significant (except the long-run effect of foreign private holdings) over the full sample. In addition, the long-run effects are all larger (in absolute value) for the full sample than for the subsample that excludes the period at the ZLB. The differences between the full sample and the subsample estimates reported in Table 6 further motivates the necessity of explicitly considering structural breaks in the specification.

Threshold SEECM

We aim to explore the nonlinearities—if any—on the impact of foreign holdings of U.S. Treasuries (net of Federal Reserve holdings) on the long-term U.S. yield. The onset of the period of monetary policy at the ZLB around the end of 2008 is, in our view, a primary candidate to explain the structural change suggested by the results on the battery of stability tests reported before in Subsection 5.1. Hence, we use the threshold SEECM with the Fed Funds rate (specifically, the simple moving average of the first and second lags, i.e. $(i_{1,t-1} + i_{1,t-2})/2$) as the threshold variable with which to investigate the empirical plausibility of the hypothesis that the nonlinearity arising at the end of 2008 may be due to the ZLB.

[34] The estimated $\hat{\alpha}$ ranges from -0.11 to -0.12 for the full sample, and -0.14 to -0.17 for the subsample, indicating similar adjustment speeds for each sample.

[35] Bandholz *et al.* (2009) use a SEECM specification covering a pre-crisis sample and they also find insignificant short-run effect of the foreign total holdings ratio.

Table 6. Linear and Threshold SEECMs Estimation Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
α	-0.136***	-0.162***	-0.167***	-0.113***	-0.119***	-0.118***	-0.120***
$i_{1,t-1}$	0.248***	0.249***	0.253***	0.339***	0.353***	0.339***	0.259***
$\pi^e_{t-1,n}$	1	1	1	1	1	1	1
$\pi^e_{t-1,1}$	-0.248***	-0.249***	-0.253***	-0.339***	-0.353***	-0.339***	-0.259***
fh_{t-1} (official)	-0.046***		-0.032***	-0.050***		-0.047***	
fh_{t-1} (private)			-0.049**			-0.019	
fh_{t-1} (total)		-0.037***			-0.041***		
$fh_{t-1}d_t$ (official)							-0.061***
$fh_{t-1}(1-d_t)$ (official)							-0.044***
$\Delta i_{n,t-1}$	0.319***	0.324***	0.323***	0.284***	0.278***	0.272***	0.283***
$\Delta i_{1,t-1}$	-0.037	-0.021	-0.015	-0.035	-0.031	-0.035	-0.027
$\Delta\pi^e_{t-1,n}$	0.637	0.553	0.541	0.381	0.322	0.341	0.382
$\Delta\pi^e_{t-1,1}$	0.133	0.130	0.134	0.195	0.178	0.183	0.192
Δfh_{t-1} (official)	-0.040		-0.037	-0.064**		-0.061**	
Δfh_{t-1} (private)			-0.024			-0.051*	
Δfh_{t-1} (total)		-0.030			-0.052**		
$\Delta fh_{t-1}d_t$ (official)							-0.138***
$\Delta fh_{t-1}(1-d_t)$ (official)							-0.048
constant	0.482***	0.625***	0.653***	0.380***	0.441***	0.420***	0.416***

Note: ***, ** and * represent the level of significance at 1%, 5% and 10%, respectively. Columns (1) to (3) use the linear SEECM specification for the subsample 1986m01 – 2008m11. Columns (4) to (6) use the linear SEECM specification for the full sample 1986m01 – 2014m12. Column (7) uses the threshold SEECM specification for the full sample 1986m01 – 2014m12.

We consider one threshold and two policy rate regimes only, and define a dummy variable d_t such that $d_t = 0$ if $(i_{1,t-1} + i_{1,t-2})/2 \geq \tau$, $d_t = 1$ if $(i_{1,t-1} + i_{1,t-2})/2 < \tau$. The coefficients on the foreign holdings ratio in both the short-run and in the long-run are allowed to vary when the simple moving average of the Fed Funds rate ($(i_{1,t-1} + i_{1,t-2})/2$) is above or below the threshold value (τ) as follows:

$$\Delta i_{n,t} = \beta_0 + \alpha\left(i_{n,t-1} - \gamma_1 i_{1,t-1} - \gamma_2 \pi^e_{t-1,n} - \gamma_3 \pi^e_{t-1,1} - \gamma_4 fh_{t-1} d_t - \bar{\gamma}_4 fh_{t-1}(1 - d_t)\right) + \cdots$$

$$\beta_1 \Delta i_{n,t-1} + \beta_2 \Delta i_{1,t-1} + \beta_3 \Delta \pi^e_{t-1,n} + \beta_4 \Delta \pi^e_{t-1,1} + \beta_5 \Delta fh_{t-1} d_t + \bar{\beta}_5 \Delta fh_{t-1}(1 - d_t) + \varepsilon_t. \quad (16)$$

Then, we estimate a nonlinear specification of the SEECM model in threshold form (equation (16) above) with the parameter restrictions in (15).

We consider an endogenously determined threshold value which minimizes the sum of squared residuals of the corresponding specification. We search the optimal threshold value for the simple moving average of the Fed Funds rate over the [0.07, 5]-range in equation (16). The lower bound of the range 0.07 is the minimum value the threshold variable takes in our sample while the upper bound is set at 5 to ensure that our specification is flexible enough to capture the relevant sample split.

The endogenously determined optimal threshold value lies within the range $\tau^* \in (0.68, 0.99)$, which separates the sample into two policy regimes exactly coinciding with the timing of the ZLB.[36] The first regime occurs when $(i_{1,t-1} + i_{1,t-2})/2 \geq \tau^*$ (corresponding to the pre-ZLB period, 1986m01-2008m11). The second regime is when $(i_{1,t-1} + i_{1,t-2})/2 < \tau^*$(corresponding to the ZLB period, 2008m12-2014m12). From column (7) in Table 6 below, for the long-run effect, in the first regime (the pre-ZLB regime), a one percentage point increase (decrease) in the foreign official holdings ratio is associated with around a 4 basis point decrease (increase) in the long-term rate. In contrast, this marginal effect increases from 4 to around 6 basis points in the second regime (the ZLB regime). A Wald test indicates that the long-run estimated coefficients on the foreign official holdings ratio are significantly different from each other at the 5% level. For the short-run effect, it is not significant in the pre-ZLB regime. The short-run effect, however, becomes larger (in absolute value) and significant during the ZLB regime.

5.3 The Smooth Transition Regression ADL Model

As shown in Section 5.2, we find a significant change in the impact of foreign holdings ratio on the U.S. long-term rate around the end of 2008 based on the threshold SEECM. However, our threshold SEECM can only model abrupt changes of the marginal effects. To further consider the possibility of a smooth

[36] Our estimation results using the endogenously determined threshold value are the same as the estimation using an exogenously determined threshold value to impose a break date of 2008m11 to split the sample.

transition of the impact of foreign holdings ratio from the pre-ZLB period to the ZLB period, we use the smooth transition regression (STR) technique proposed by Teräsvirta (1994) in this subsection.

In order to conduct the STR regression, we use the following ADL modelling approach described in Pesaran and Shin (1999):

$$\Delta i_{n,t} = \beta_0 + \rho_0 i_{n,t-1} - \rho_1 i_{1,t-1} - \rho_2 \pi^e_{t-1,n} - \rho_3 \pi^e_{t-1,1} - \rho_4 fh_{t-1} + \beta_1 \Delta i_{n,t-1} + \beta_2 \Delta i_{1,t-1} + \beta_3 \Delta\pi^e_{t-1,n} + \beta_4 \Delta\pi^e_{t-1,1} + \beta_5 \Delta fh_{t-1} + \varepsilon_t. \tag{17}$$

Similar to the restrictions in equation (15), here we impose the following restrictions on equation (17):

$$\rho_0 = \rho_2 \text{ and } \rho_1 + \rho_3 = 0. \tag{18}$$

Compared to the SEECM in equation (14), equation (17) is an ADL model in the unrestricted error-correction form. Equation (17) has the same coefficient estimates as those in equation (14) where the long-run coefficients on $i_{1,t}$, $\pi^e_{t,n}$, $\pi^e_{t,1}$, and fh_t can be computed as ρ_1/ρ_0, ρ_2/ρ_0, ρ_3/ρ_0 and ρ_4/ρ_0 respectively in equation (17) (See columns (1) and (2) in Table 7 for the estimation results).

Pesaran *et al.* (2001) develop a bounds test specifically for the ADL model with which we test the existence of a level relationship (i.e., cointegrating relationship) in equation (17) among $i_{n,t}$, $i_{1,t}$, $\pi^e_{t,n}$, $\pi^e_{t,1}$, and fh_t. The bounds test uses the F-statistic for testing the null hypothesis of $\rho_0 = \rho_1 = \rho_2 = \rho_3 = \rho_4 = 0$. Our F-statistics are 7.71 using the full sample and 7.35 using the subsample 1986m01-2008m11. Both are outside of the 5% critical value bounds (2.86, 4.01) tabulated in Pesaran *et al.* (2001) for the case of unrestricted intercept and no trend. Therefore, we reject the null of the absence of a level relationship in the ADL model, which further supports the view that a long-run cointegrating relationship exists.

Similar to equation (16), using the following threshold ADL model in equation (19), i.e.,

$$\Delta i_{n,t} = \beta_0 + \rho_0 i_{n,t-1} - \rho_1 i_{1,t-1} - \rho_2 \pi^e_{t-1,n} - \rho_3 \pi^e_{t-1,1} - \rho_4 fh_{t-1} d_t - \bar{\rho}_4 fh_{t-1}(1-d_t) + \beta_1 \Delta i_{n,t-1} + \beta_2 \Delta i_{1,t-1} + \beta_3 \Delta\pi^e_{t-1,n} + \beta_4 \Delta\pi^e_{t-1,1} + \beta_5 \Delta fh_{t-1} d_t + \bar{\beta}_5 \Delta fh_{t-1}(1-d_t) + \varepsilon_t. \tag{19}$$

under the restrictions in equation (18), we obtain the same endogenously determined threshold value of $\tau^* \in (0.68, 0.99)$ over the same [0.07, 5] range as before which exactly leads to the pre-ZLB and the ZLB regimes (See column (3) in Table 7 for the estimation results).

Table 7. ADL and STR Models Estimation Results

	(1)	(2)	(3)	(4)
$i_{n,t-1}$	-0.136***	-0.113***	-0.120***	-0.120***
$i_{1,t-1}$	-0.034***	-0.038***	-0.031***	-0.031***
$\pi^e_{t-1,n}$	-0.136***	-0.113***	-0.120***	-0.120***
$\pi^e_{t-1,1}$	0.034***	0.038***	0.031***	0.031***
fh_{t-1} (official)	0.006***	0.006***		
$fh_{t-1}d_t$ (official)			0.007***	
$fh_{t-1}(1-d_t)$ (official)			0.005***	
$\Delta i_{n,t-1}$	0.319***	0.284***	0.283***	0.283***
$\Delta i_{1,t-1}$	-0.037	-0.035	-0.027	-0.027
$\Delta\pi^e_{t-1,n}$	0.637	0.381	0.382	0.382
$\Delta\pi^e_{t-1,1}$	0.133	0.195	0.192	0.192
Δfh_{t-1} (official)	-0.040	-0.064**		
$\Delta fh_{t-1}d_t$ (official)			-0.138***	
$\Delta fh_{t-1}(1-d_t)$ (official)			-0.048	
constant	0.482***	0.380***	0.416***	0.416***
θ_0				0.007***
θ_1				-0.002***
ϕ_0				-0.138***
ϕ_1				0.090
location parameter c_1				0.851
scale parameter γ_1				50***

Note: ***, ** and * represent the level of significance at 1%, 5% and 10%, respectively. Columns (1) and (2) use the linear ADL specification (equation (17)) for the subsample 1986m01 – 2008m11 and the full sample 1986m01 – 2014m12 respectively. Column (3) uses the threshold ADL specification (equation (19)) for the full sample 1986m01 – 2014m12. Column (4) uses the STR specification (equation (20)) for the full sample 1986m01 – 2014m12.

We also explore a possible smooth transition in the impact of foreign holdings ratio on the long-term rate both in the long-run cointegrating relation and the short-run dynamics, therefore, we model the time-varying effect on both the lagged level variable and the lagged first difference variable of the foreign holdings ratio in the following STR specification, i.e., in

$$\Delta i_{n,t} = \beta_0 + \rho_0 i_{n,t-1} - \rho_1 i_{1,t-1} - \rho_2 \pi^e_{t-1,n} - \rho_3 \pi^e_{t-1,1} - \theta_0 fh_{t-1} - \sum_{m=1}^{M} \theta_m f[\gamma_m(q_t - c_m)]\, fh_{t-1} + \beta_1 \Delta i_{n,t-1} + \beta_2 \Delta i_{1,t-1} + \beta_3 \Delta \pi^e_{t-1,n} + \beta_4 \Delta \pi^e_{t-1,1} + \phi_0 \Delta fh_{t-1} + \sum_{m=1}^{M} \phi_m f[\gamma_m(q_t - c_m)] \Delta fh_{t-1} + \varepsilon_t. \quad (20)$$

where $f[\gamma_m(q_t - c_m)] = \frac{1}{1+e^{-\gamma_m(q_t - c_m)}}$ is the transition function which uses the logistic cumulative distribution function; γ_m is the scale parameter (the smaller the γ_m, the smoother of the transition); c_m is the location parameter which indicates the break dates; q_t is the transition variable. To make it more comparable with the threshold model, we impose $M = 1$ so that there are only two limiting regimes and choose the same threshold variable as the transition variable q_t of our STR specification in equation (20).[37] We also restrict the range of location parameter c_m being [0.07, 5], which is the same as the searching range for the threshold variable before. In addition, we impose the restrictions of equation (18) on equation (20).

Column (4) in Table 7 reports the estimation results of equation (20). The estimated location parameter is 0.851. It implies that the break date is when the transition variable q_t equals 0.851, corresponding to the date between 2008m11 and 2008m12. The estimated scale parameter is 50, indicating a very fast transition. From equation (20), we can compute the time-varying long-run marginal effect of the foreign holdings ratio on the long-term rate as $(\theta_0 + \theta_1 f[\gamma_1(q_t - c_1)])/\rho_0$. The estimated long-run marginal effects indicate a very quick transition around the end of 2008. Therefore, the STR result supports our use of the threshold SEECM/ADL specifications in modelling an abrupt change in the impact of the foreign holdings ratio on the long-term rate.

Our threshold ADL model is a dynamic model in the sense that the change in foreign holdings ratio at time t has impacts on future long-term nominal interest rates. Following Shin *et al.* (2014), it is straightforward to derive the dynamic multipliers associated with unit changes in foreign holdings ratio by rewriting equation (19) into ADL-in-levels representation as below.

[37] We also used the scaled time trend (i.e., t/T, where T is the sample size) as the transition variable. The estimated location parameter is 0.8614, corresponding to a break date of 2010m12, and the estimated long-run marginal effect increases gradually from around 4 bps on 2008m12 to around 6 bps until 2012m12. The detailed results are not reported but are available upon request to the authors.

$$i_{n,t} = \beta_0 + (1 + \rho_0 + \beta_1)i_{n,t-1} - \beta_1 i_{n,t-2} + (\beta_2 - \rho_1)i_{1,t-1} - \beta_2 i_{1,t-2} + (\beta_3 - \rho_2)\pi^e_{t-1,n} - \beta_3\pi^e_{t-2,n} + (\beta_4 - \rho_3)\pi^e_{t-1,1} - \beta_4\pi^e_{t-2,1} + (\beta_5 - \rho_4)fh_{t-1}d_t - \beta_5 fh_{t-2}d_t + \left(\bar{\beta}_5 - \bar{\rho}_4\right)fh_{t-1}(1 - d_t) - \bar{\beta}_5 fh_{t-2}(1 - d_t) \tag{21}$$

We compute the dynamic multiplier function and the cumulative dynamic multiplier function defined in equations (22) and (23), respectively, to illustrate the estimated effects over time:

$$DM_s = \frac{di_{n,t+s}}{dfh_t}, \textit{for all } s = 0, 1, 2, \ldots \tag{22}$$

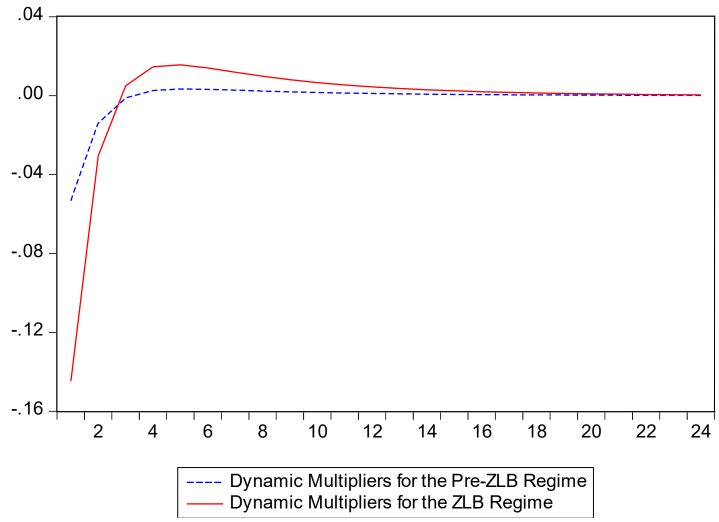
$$Cumulative\_DM_s = \sum_{j=0}^{s} DM_j, \textit{for all } s = 0, 1, 2, \ldots \tag{23}$$

where $DM_0 = 0$, $DM_1 = \beta_5 - \rho_4$, $DM_2 = (1 + \rho_0 + \beta_1)(\beta_5 - \rho_4) - \beta_5$, and $DM_s = (1 + \rho_0 + \beta_1)DM_{s-1} - \beta_1 DM_{s-2}$ for $s = 3, 4, \ldots$ in the ZLB regime (i.e. $d_t = 1$), and similarly for the pre-ZLB regime. Using the estimated parameters of threshold ADL model in column (3) of Table 7, we plot the dynamic multiplier function and the cumulative dynamic multiplier function for each regime in Figure 3.

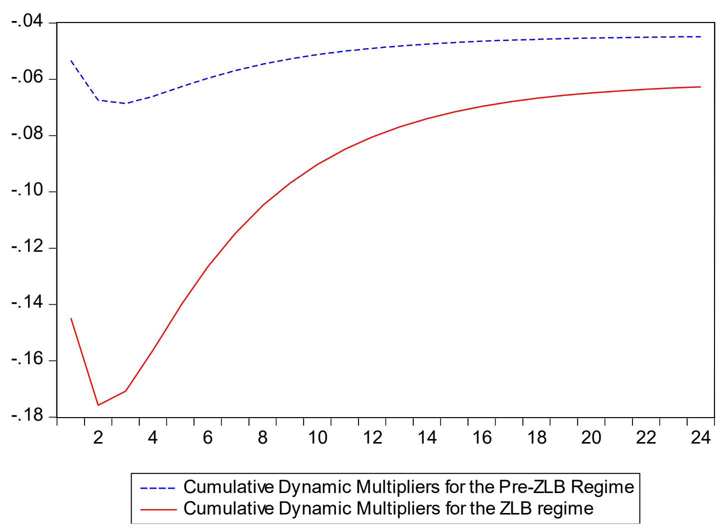
As seen in Panel A of Figure 3, the dynamic multiplier functions in both regimes converge to zero very quickly. For the initial period, a one percentage point increase in the foreign holdings ratio will reduce the long-term rate by around 5.3 basis points in the pre-ZLB regime. During the ZLB regime, a one percentage point increase in the foreign holdings ratio has a relatively larger marginal impact lowering the long-term yield by about 14.5 basis points in the initial period. Panel B of Figure 3 describes the cumulative impacts on the long-term rate over certain period when there is a one percentage point change in the foreign holdings ratio. The value of the cumulative dynamic multiplier function at infinity is called the long-run multiplier (i.e., $\sum_{j=0}^{\infty} DM_j$) and equals the long-run effect in the cointegrating vector shown in Table 7. That means, a one percentage point increase in the foreign holdings ratio has an overall impact reducing the long-term rate by 6.1 basis points in the ZLB regime compared to 4.4 basis points in the pre-ZLB regime.

Figure 3. Dynamic and Cumulative Dynamic Multiplier Functions

A. Dynamic Multiplier Functions



B. Cumulative Dynamic Multiplier Functions



Note: We compute the dynamic and cumulative dynamic multiplier functions defined in equations (22) and (23) based on the ADL-in-level representation as shown in equation (21).

5.4 Tobit-IV SEECM

From the above estimation results, we find that the foreign holdings ratio had a larger (in absolute value) marginal impact on the long-term rate in the ZLB period. In order to further investigate the role of ZLB in leading to the break, based on the discussion in section 3.2, we model the Fed Funds rate using a Tobit model and compute estimates of the latent variable which is the shadow Fed Funds rate that the Fed would had implemented if there was no ZLB. Then, we compare the threshold SEECM results by using the shadow and actual Fed Funds rates.

Based on the monetary policy rules, in the Tobit model of the Fed Funds rate, the explanatory variables are the average of the first and second lags of the Fed Funds rate, the one-year ahead inflation expectation, and the output gap. For the one-year ahead inflation expectation, we use 12-month ahead ex post y-o-y percent change in CPI for all items. For the output gap, we use the HP filter on the log of industrial production index to obtain its cycle series, which is the measurement for the output gap. Data on the CPI for all items and the industrial production index are retrieved from the FRED database. Following Kiesel and Wolters (2014), we consider the inflation expectation and the output gap as endogenous variables and we use their first to six lags as instruments. In addition, as the 0 to 0.25% range of the Fed Funds rate is generally regarded as the ZLB environment, we impose the values of Fed Funds rate within 0 to 0.25% range to be zero in order to accommodate the Tobit model censoring at zero[38]. (i.e., this means that the Fed Funds rate is censored at zero for the observations from 2008m12 to the end of our sample, 2014m12 in our Tobit model.)

From the estimated Tobit-IV model, we can derive the shadow Fed Funds rate. Specifically, we recursively estimate the shadow Fed Funds rate from 2009m02. We also use two well-known shadow Fed Funds rates in the existing literature, Wu and Xia (2016) and Krippner (2013 and 2015), for comparison.

Then, we use these shadow Fed Funds rates as the measurement for the short-term rate in the threshold SEECM (equation (16)) with an exogenous break date of 2008m11. The estimated marginal effects of the foreign holdings ratio in both the short-run and long-run are reported in Table 8. From Table 8, the results of using our Tobit-IV shadow Fed Funds rate are in lines with those using the Wu-Xia and Krippner shadow Fed Funds rates. By using the shadow Fed Funds rate, the differences between the estimated long-run marginal effects of foreign holdings ratio in the pre-ZLB and the ZLB periods become smaller than that using the actual Fed Funds rate.

In addition, through Wald tests, we cannot reject the equivalence of the estimated long-run marginal effects in the pre-ZLB and the ZLB periods at all conventional levels of significance when using the shadow Fed Funds rates. Therefore, we argue that the break in the impact of foreign holdings ratio in the long-run cointegrating relationship is mostly attributable to the effect of the ZLB.

[38] Kim and Mizen (2010) has a similar argument. In their Tobit model, they consider the Japanese short-term policy rate is effectively zero lower bounded if it falls below certain threshold values.

Table 8. Threshold SEECM: Actual vs. Shadow Federal Funds Rates

Federal funds rate (FFR)	Estimated coefficients on the foreign official holdings ratio				Wald Test of the equivalence of coefficients in the long-run (P-Values)
	Short-run		Long-run		
	Pre-ZLB period	ZLB period	Pre-ZLB period	ZLB period	
Actual FFR	-0.048	-0.138***	-0.044***	-0.061***	0.05
Wu-Xia Shadow FFR	-0.048	-0.129***	**-0.045*****	**-0.055*****	0.28
Krippner Shadow FFR	-0.044	-0.131***	**-0.047*****	**-0.052*****	0.53
Tobit-IV Shadow FFR	-0.043	-0.130***	**-0.049*****	**-0.052*****	0.78

Note: ***, ** and * represent the level of significance at 1%, 5% and 10%, respectively.

6. Robustness Checks

Concerns About Endogeneity: Linear VECM and Weak Exogeneity of the Foreign Official Holdings Ratio.

To alleviate the concern of endogeneity in the vector of explanatory variables $X_t = (i_{n,t}, i_{1,t}, \pi^e_{t,n}, \pi^e_{t,1}\ , fh_t)'$, we consider the linear VECM specification straight from our theoretical benchmark in Section 3 (equation (8)). Using an unrestricted VAR model in levels for the full sample (1986m01-2014m12), the Schwarz information criterion indicates a lag length of 2.[39] Therefore, we choose a lag length of $k = 1$ for the VECM specification. In addition, both the Johansen Trace test and Maximum Eigenvalue test indicate one cointegrating relation in the VECM.

By imposing the restriction in (15) on the cointegrating vector of the linear VECM model for estimation, we obtain consistent results corresponding to the equation on the long-term yield ($i_{n,t}$) reported in Table 6.[40] We consider the estimation for the full sample covered by our dataset (1986m01-2014m12) as well as for the subsample that excludes the observations from the ZLB regime identified with the threshold SEECM model (1986m01-2008m11). Similar to the findings for the linear SEECM found in

[39] As an additional robustness check, we also considered an expanded sample starting in 1984m12 but keeping the end period on 2014m12. We find that the result holds true in this case as well. The findings are not reported here due to space constraints, but are available upon request from the authors.
[40] Detailed results are not reported here due to space constraints, but are available upon request from the authors.

Table 6, the estimated coefficient on the foreign official holdings ratio in the short-run is not statistically significant for the subsample excluding the ZLB observations. And the long-run effects are larger (in absolute value) in the full sample than in the subsample.

Following Norrbin *et al.* (1997), we test the weak exogeneity of the foreign official holdings ratio as well as the short-term rate, long-run and short-run inflation expectations in the VECM framework. Table 9 reports the estimated coefficients on the error-correction term α for each equation in the linear VECM specification. As suggested in Norrbin *et al.* (1997), we use the Mackinnon (1991) critical values for the t-statistics. For the full sample, the estimated coefficients on the error-correction term are significant only in the equation of the long-term interest rate which is the basis for our SEECM specification. The evidence suggests that the disequilibrium from the long-run cointegrating relationship can only be adjusted through the long-term interest rates. Therefore, the variables $i_{1,t}$, $\pi^e_{t,n}$, $\pi^e_{t,1}$ and fh_t are weakly exogenous, which provides additional support for the use of the SEECM specification we made in our analysis.

Table 9. Estimated Coefficient α on the Error-Correction Term in the Linear VECM

$\Delta i_{n,t}$	$\Delta i_{1,t}$	$\Delta \pi^e_{t,n}$	$\Delta \pi^e_{t,1}$	Δfh_t
For the subsample 1986m01-2008m11				
-0.107	0.027	-0.001	0.004	-0.029
[-4.014]	[1.230]	[-0.232]	[0.401]	[-0.599]
For the full sample 1986m01-2014m12				
-0.113**	0.020	0.002	0.003	0.068
[-4.800]	[1.121]	[0.678]	[0.327]	[1.484]

Note: t-statistics are in brackets. ***, ** and * represent the level of significance at 1%, 5% and 10%, respectively. From MacKinnon (1991), 5% critical value is -4.456 and 10% critical value is -4.163.

Concerns about Omitted Variable Bias: Threshold ADL Model with Additional Macro Factors

We control for the possibility of omitted variable bias by adding additional macro factors proposed in the previous literature (see, e.g., Warnock and Warnock (2009), Beltran *et al.* (2013), and Goda *et al.* (2013)) into our threshold ADL model. We consider the following control variables: the log of S&P 500 index (sp_t), U.S. budget deficit scaled by GDP (def_t),[41] expected real GDP growth over the next year ($growth_t$),[42] and the log of VXO index (vxo_t).[43]

Table 10. Estimation Results of the Threshold ADL Model with Additional Control Variables

	(1)	(2)	(3)	(4)
$i_{n,t-1}$	-0.131***	-0.135***	-0.124***	-0.117***
$i_{1,t-1}$	-0.034***	-0.041***	-0.028***	-0.031***
$\pi^e_{t-1,n}$	-0.131***	-0.135***	-0.124***	-0.117***
$\pi^e_{t-1,1}$	0.034***	0.041***	0.028***	0.031***
$fh_{t-1}d_t$ (official)	0.006***	0.008***	0.008***	0.007***
$fh_{t-1}(1-d_t)$ (official)	0.004**	0.006***	0.005***	0.005***
$\Delta i_{n,t-1}$	0.292***	0.300***	0.276***	0.276***
$\Delta i_{1,t-1}$	-0.014	-0.018	-0.004	-0.038
$\Delta\pi^e_{t-1,n}$	0.421	0.352	0.403	0.427
$\Delta\pi^e_{t-1,1}$	0.197	0.196	0.200	0.189
$\Delta fh_{t-1}d_t$ (official)	-0.127***	-0.128***	-0.122**	-0.140***
$\Delta fh_{t-1}(1-d_t)$ (official)	-0.048	-0.041	-0.045	-0.050
constant	0.769***	0.432***	0.491***	0.459***
sp_{t-1}	-0.057			
Δsp_{t-1}	0.587**			
def_{t-1}		-0.008		
Δdef_{t-1}		-0.051		
$growth_{t-1}$			-0.022	
$\Delta growth_{t-1}$			0.208*	
vxo_{t-1}				-0.019
Δvxo_{t-1}				-0.072
Wald test for the equivalence of long-run coefficients on foreign holdings ratio (P-values)	0.014	0.028	0.023	0.057

Note: ***, ** and * represent the level of significance at 1%, 5% and 10%, respectively. The sample period is from 1986m01 to 2014m12.

[41] Data for budget deficit are from U.S. Treasury monthly Treasury statement.
Source: https://www.fiscal.treasury.gov/fsreports/rpt/mthTreasStmt/current.htm

[42] The expected real GDP growth rates are computed from the SPF's median forecast data for the level of the real GDP and linearly interpolated from quarterly to monthly. Source: https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/median-forecasts.

[43] We use Chicago Board Options Exchange (CBOE) VXO index instead of CBOE VIX index because the former index has a longer price history back to 1986m01.

We add both the lagged level and lagged first difference terms of these control variables one at a time into our threshold ADL model in equation (19) with restrictions in equation (18).[44] Table 10 shows the estimation results. Only coefficients on the log of S&P 500 index and the expected real GDP growth rate in the lagged first difference terms are significant at the 5% and 10% levels respectively. The endogenously determined threshold values can split the sample into the pre-ZLB regime (1986m01-2008m11) and the ZLB regime (2008m12-2014m12), exactly the same as before. In addition, the Wald tests for the null hypothesis of equivalence of estimated coefficients on $fh_{t-1}d_t$ and $fh_{t-1}(1-d_t)$ indicate that the long-run effects of foreign official holdings ratio on the long-term rate are significantly different between the pre-ZLB and the ZLB regimes (except for the specification in column (4) which is marginally significant at the 5% level). Therefore, our main results from the threshold models still hold when we considering additional macro factors as control variables.

Concerns on the Measurement of Foreign Demand of U.S. Treasuries

As an additional robustness check on the measurement of the foreign variable fh_t, we use the foreign total (sum of official and private) holdings ratio instead of foreign official holdings ratio in our Threshold SEECM specification (equation (16)). The results in Table 11 show that the endogenously determined threshold value splits the sample into the pre-ZLB and ZLB regimes exactly the same as in the case with exogenously determined threshold value. We find that the effects of the foreign total holdings ratio (both short-run and long-run) become larger (more negative) in the ZLB regime than in the pre-ZLB regime. Overall, we obtain similar results as those using the foreign official holdings ratio in Table 6.

Table 11. Threshold SEECM Estimation Results Using Foreign Total Holdings Ratio

Threshold Value	Variable of Interest	when $(i_{1,t-1}+i_{1,t-2})/2 \geq \tau^*$	when $(i_{1,t-1}+i_{1,t-2})/2 < \tau^*$
$\tau^* \in (0.68, 0.99)$ endogenously determined over range [0.07,5]	foreign total holdings (short-run)	-0.036	-0.121***
	foreign total holdings (long-run)	-0.037***	-0.051***

Note: *** represents significance level of 1%. The estimated coefficients on the error-correction terms are significant at the 1% level (not reported here).

[44] We also tried adding all the four control variables into our threshold ADL model simultaneously. None of the coefficients on these control variables are significant.

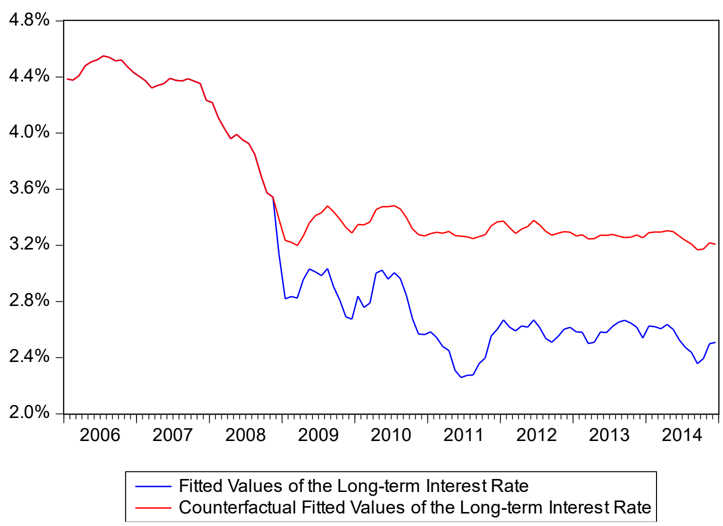
7. Counterfactual Analysis

Counterfactual Analysis 1: Exploring the Effects of the ZLB.

In order to compare the different impacts on the U.S. long-term interest rate due to the different marginal effects of the foreign holdings ratio in the pre-ZLB and the ZLB regimes. We assume that, in the ZLB regime, the estimated coefficients on the foreign official holdings ratio (both short-run and long-run) are the same as those estimated for the pre-ZLB regime based on the estimates from the threshold SEECM in Table 6.

We recursively compute the original and counterfactual fitted values of the long-term interest rate, as shown in Figure 4, to illustrate the magnitude of the change in the effect of the foreign official holdings ratio on the long-term interest rate in the ZLB regime resulting from structural break. From Figure 4, we observe that the average difference between the fitted and counterfactual fitted values of the long-term interest rate is around 66 basis points (if monetary policy accommodation had been kept at the near-zero in line with the actual Fed Funds rate).

Figure 4. Counterfactual Analysis 1 - Fitted Values of the Long-term Interest Rate



Source: Authors' calculation based on our benchmark empirical model.

Note: In the counterfactual case, we assume the estimated coefficients on the foreign official holdings ratio in the ZLB regime of the threshold SEECM keep the same as those in the pre-ZLB regime. The original and counterfactual fitted values of the long-term interest rate are computed based on the estimation results in Table 8 and the counterfactual case.

Counterfactual Analysis 2: Exploring the Effect of China's Policy of Accumulating U.S. Treasuries.

From Panel A of Figure 2, we observe that China's holdings of U.S. Treasury notes and bonds (as a share of outstanding marketable U.S. Treasury notes and bonds) increased dramatically since 1994 and, moreover, China's pace of accumulation further accelerated from 2001 to 2011. As of June 30, 2014, China became the largest foreign holder of U.S. Treasury notes and bonds. Hence, we focus on China's total holdings—most of which are presumed to be official holdings—to investigate its role on the interest rate conundrum period (2004–2006) and the Treasury unwinding period (2011-2014) that followed.

The 'interest rate conundrum' period (2004-06): We consider what would happen to the long-term yield during the conundrum period if the increase in China's holdings since 2001 had not accelerated. Specifically, we assume that the growth of China's total holdings (sum of official and private holdings) of U.S. Treasury notes and bonds from 2001 to 2006 kept the same pace as during the 1994-2001 period.[45] Using the estimation results in the threshold SEECM in Table 6, we recursively compute the fitted values of the long-term interest rate based on the actual and counterfactual foreign total holdings ratio[46] (Panel A of Figure 4). The average differences between the fitted and counterfactual fitted values of the long-term yield is 24 basis points during the conundrum period (2004m06-2006m06) (See Panel B of Table 12). That means, if China's holdings after 2001 would had kept the same pace as in the 1994-2000 period, the long-term yield could have been on average 24 basis points higher. This partially explains the interest rate conundrum during 2004 to 2006.

China's unwinding-of-Treasuries period (2011-2014): We continue to assess the impact of China's holdings on U.S. long-term yields, but focusing on the recent ZLB period instead. China began to reduce its holdings of U.S. Treasury notes and bonds in 2011m07. Hence, the empirical question is what would have happened to the U.S. long-term yield if China had kept increasing their holdings of U.S. Treasuries during the 2011-2014 period. To be more specific, we assume that China's holdings could have kept growing from 2011m07 to 2014m12 at the same rate as during the 2001m01-2011m06 period.[47]

We construct the counterfactual foreign total holdings ratio based on China's counterfactual holdings and derive the fitted and counterfactual fitted values of the long-term interest rate (Panel B of Figure 4) using the Threshold SEECM estimation results in Table 12. The average difference between the

[45] For the construction of the counterfactual China's holdings, we linearly extrapolate the China's holdings from 2001m01 to 2006m12 using the same slope for the period from 1994m01 to 2000m12. The slope was obtained by running an OLS regression of the China's holdings on a constant and a time trend for the sample period 1994m01 to 2000m12. The estimated coefficient on the time trend is the estimated slope. These estimates are not reported due to space constraints, but are available from the authors upon request.
[46] The counterfactual foreign total holdings are constructed using the counterfactual China's holdings and the actual holdings of other foreign countries.
[47] For the construction of the counterfactual China's holdings, we linearly extrapolate the China's holdings from 2011m07 to 2014m12 using the same slope for the period from 2001m01 to 2011m06. The slope was obtained by running an OLS regression of the China's holdings on a constant and a time trend for the sample period 2001m01 to 2011m06. The estimated coefficient on the time trend is the estimated slope. These estimates are not reported due to space constraints, but are available from the authors upon request.

original and counterfactual fitted values of the long-term yield during 2011m07 to 2014m12 is 25 basis points (Panel B of Figure 4 and Panel B of Table 12). This counterfactual analysis suggests that, if China had not reduced its holdings since 2011m07 and had instead continued its path of large purchases of U.S. long-term Treasury securities as before, the U.S. long-term yield could have been on average 25 basis points lower during the 2011m07-2014m12 period.

Table 12. Difference between the Original and Counterfactual Fitted Values of Long-term Rate

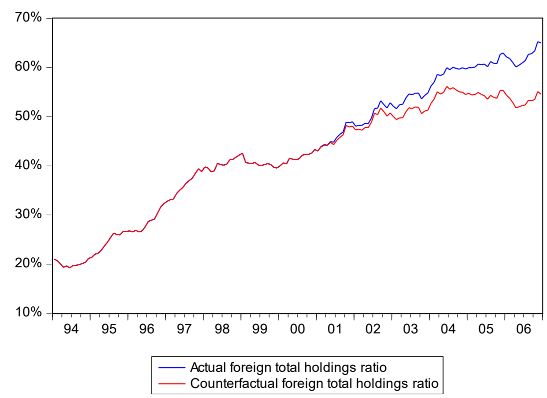
Period		Average	Range
Panel A: Counterfactual 2			
2009m04-2009m11	QE1	38 bps	14 – 49 bps
2010m08-2011m10	QE2	53 bps	12 – 90 bps
2013m01-2014m10	QE3	55 bps	33 – 64 bps
Panel B: Counterfactual 3			
2004m06-2006m06		24 bps	15 – 34 bps
2011m07-2014m12		25 bps	3 – 40 bps

Note: The three QE periods are determined according to the three rounds of expansion in the holdings of Treasury notes and bonds by the Federal Reserve.

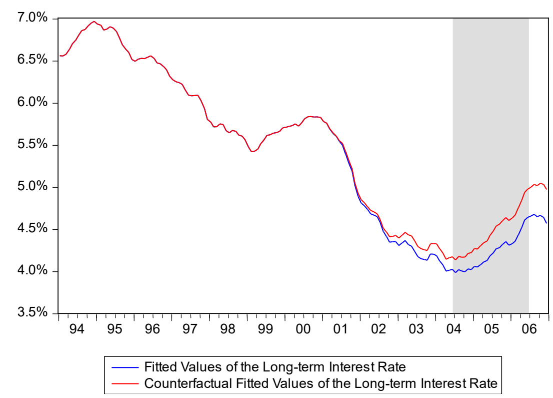
Figure 5. Counterfactual Analysis 2

Panel A: Interest Rate Conundrum Period (2004-06)

Actual and Counterfactual Foreign Total Holdings Ratio

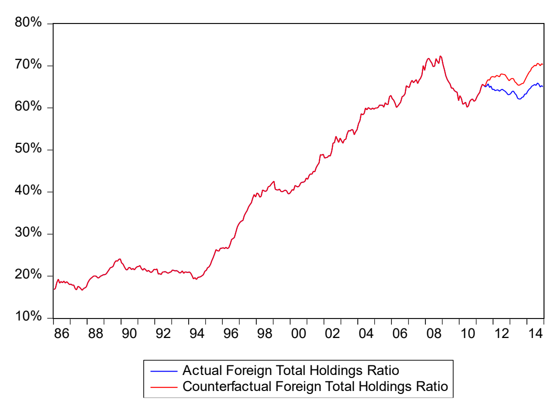


Fitted Values of the Long-Term Interest Rate

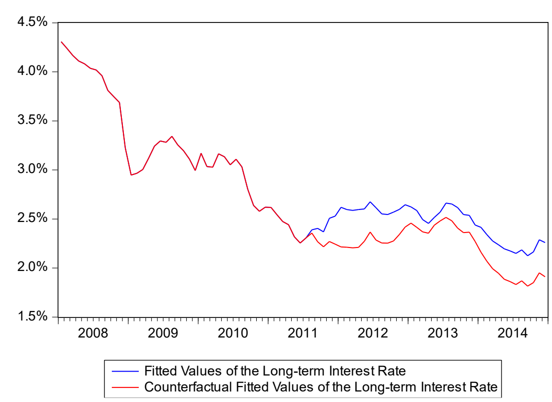


Panel B: China's unwinding-of-Treasuries period (2011-14)

Actual and Counterfactual Foreign Total Holdings Ratio



Fitted Values of the Long-Term Interest Rate



Sources: Bertaut and Tryon (2007), Bertaut and Judson (2014), FRB H.4.1., Department of the Treasury, and authors' calculations based on our benchmark empirical model.

Note: the counterfactual foreign total holdings are the sum of the counterfactual China's holdings and the actual holdings of other foreign countries. In Panel A, the counterfactual of China's holdings is calculated by assuming the growth of China's holdings from 2001m01 to 2006m12 had kept at the same pace as in the period from 1994m01 to 2000m12. In Panel B, the counterfactual of China's holdings is calculated by assuming the growth of China's holdings from 2011m07 to 2014m12 had kept at the same pace as in the period from 2001m01 to 2011m06.

8. Conclusion

In this paper, we expand the literature on the factors that determine the long-term interest rate by considering an extended sample covering the recent ZLB period and putting the spotlight on the apparent shifting role of the foreign demand of U.S. Treasuries. We investigate the possibility of structural breaks in the empirical relationship between U.S Treasury securities and the U.S. long-term yield. Through a battery of stability tests, we find robust empirical evidence supporting the view that a statistically significant breakpoint date appears to have occurred at the end of 2008.

Based on a threshold single-equation error-correction model (SEECM) with Fed Funds rate as the threshold variable, we naturally split our 1986m01-2014m12 sample into two policy regimes—the pre-ZLB and ZLB regimes. The impact of the foreign holdings ratio on the long-term yield shifted at the time the Fed Funds rate became stuck near zero. In other words, we find that the estimated negative marginal long-run effect of the foreign official holdings ratio on the long-term yield became larger (in absolute value) in the ZLB regime than in the pre-ZLB regime.

Furthermore, our evidence suggests that changes in foreign demand of Treasury securities are an economically important factor driving the U.S. long-term yield although its role appears to have shifted at the ZLB when our only monetary policy indicator is the Fed Funds rate itself. In order to support our argument that the ZLB is an important reason for this apparent structural break, we propose a flexible empirical model motivated by theory which suggests that a broader indicator of monetary policy in periods when the ZLB is binding or at least a likely event. Consistent with theory, we derive a shadow Fed Funds rate from an IV-Tobit to replace the nominal Fed Funds rate as a broad indicator of monetary policy. Given this, we find no evidence of structural break in the impact of foreign holdings ratio on the long-term yield.

Our results provide evidence that the effects of foreign holdings are statistically significant, but also economically meaningful. Using a counterfactual analysis assuming monetary policy had not provided additional monetary policy accommodation at the ZLB, we find that the long-term yield could have been about 66 basis points higher on average while the Fed Funds remained stuck at near zero.

We also evaluate the effects of China's holdings on the U.S. long-term yield during the 2004-06 interest rate conundrum period and the recent ZLB period, respectively. Based on a counterfactual analysis assuming slower growth of China's holdings between 2001 and 2006, we find that changes in China's holdings ratio can partially explain the interest rate conundrum having kept the long-term yields lower by about 24 basis points on average. Using another counterfactual assuming continued increases in China's holdings during the 2011m07-2014m12 period, we find that the recent unwinding in China's holdings of Treasuries started in 2011m07 kept the U.S. long-term yield 25 basis points on average higher than otherwise.

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