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ABSTRACT: In this paper we analyze a class of monetary policy rules that provide complete insulation for large economies from foreign real and monetary disturbances. The rule requires the central bank to float the exchange rate and manage the interest rate. In particular, the interest rate is targeted in such a way that the domestic excess demand for money is independent of domestic interest rate variations that may result from foreign disturbances transmitted through integrated capital markets.

The paper focuses on a combination interest rate and exchange rate policy because we recognize that there are two channels through which foreign disturbances can impinge on the domestic economy. These are through commodity markets and capital markets. To successfully eliminate such disturbances the central bank needs two instruments. These are purchases and sales of domestic assets and foreign assets.

Under almost all circumstances the insulating policy is found to be suboptimal in terms of minimizing the variability of domestic output. This is true because by insulating the domestic economy from foreign disturbances the central bank forgoes the opportunity to export domestically sourced disturbances to foreign countries. Optimal policy almost always requires the central bank to manage both the exchange rate and the interest rate.

* The views expressed in this paper are solely those of the authors and should not be attributed either to the Federal Reserve Bank of Dallas or the Federal Reserve System.
1. INTRODUCTION

During the era of fixed exchange rates it was generally felt that a movement to flexible exchange rates would reduce the international transmission of disturbances and allow countries to pursue more independent monetary policies. Experience with flexible exchange rates after 1973, however, indicated little perceptible change in the degree of cross-country correlation in real economic activity [see Shafer and Loopesko (1983), Artus and Young (1979) and Frenkel (1980)]. This led to a more eclectic approach to exchange rate management. It has been claimed that the apparent failure of flexible exchange rates to provide adequate insulation strengthened the case for activist exchange market intervention policies [Goldstein (1980), pp. 48-54, for example]. This paper investigates policies that do provide complete insulation for large economies in a world with perfect capital mobility. We identify a combination of interest rate and exchange rate rules that insulates a country from foreign real and monetary disturbances transmitted through both the current and capital account. We discuss the optimality of following such a policy.

There is already a rather extensive literature on the insulating properties of exchange rates for the small open economy.¹ This paper is different in that we examine a two (large) country version of the typical small country framework. More importantly, we acknowledge the major role played by integrated capital markets as a source of foreign disturbances in the domestic economy. Thus we allow the central banks in each country to employ interest rate rules as well as exchange rate rules as they seek to attain independence from foreign disturbances. Our major conclusions are as follows. Exchange rate management alone is capable of eliminating the transmission of either foreign real or foreign monetary disturbances but is
incapable of simultaneously eliminating both. Hence, when considering exchange rate policies alone, one concludes with the usual recommendation to float or fix the exchange rate depending on the nature of the foreign disturbance. However, a policy wherein the exchange rate floats and the interest rate is managed is capable of eliminating the transmission of both types of disturbance. The foreign central bank may at the same time pursue such a policy and eliminate the transmission of domestic disturbances to its own economy. In other words, flexible exchange rates combined with interest rate management in each country is a feasible joint policy combination which provides monetary autonomy and eliminates completely the international transmission of disturbances.

The intuition behind these results is that, in a completely integrated trading world, there are just two channels of transmission of disturbances: the goods market and the capital market. Ideally the Fed would require two instruments to target each source of foreign disturbance. While the Fed controls only the money supply, since it has two components -- domestic assets and foreign assets -- there are in fact two proximate instruments for independent use. Directing the exchange rate, or purchases of foreign assets, towards the commodity market and directing the interest rate, or purchases of domestic assets, towards the capital market is sufficient for complete insulation.

Though feasible, it generally will not be optimal for the central bank to pursue a policy of complete insulation. This is because, given the fixed behavior of the foreign central bank, it is always possible to export some domestic disturbances abroad and reduce the overall variance of domestic output. In support of this claim we show that the only time an insulating policy is optimal is when there are no domestic disturbances and only
foreign disturbances. In general, central banks will always want manage both
the exchange rate and the interest rate to differing degrees.

The remainder of this paper is as follows. Section 2 outlines the
assumptions and basic setup of the model. Section 3 postulates central bank
behavior and introduces the money supply processes. The solution for output
and price in each country is obtained in Section 4. In Section 5 a variety
of policies are considered and their implications for output variability are
examined. Section 6 briefly discusses the optimality of various policies and
concluding comments are presented in Section 7.

2. THE MODEL

We consider a stylized two country model with perfect commodity and
capital mobility. The motivation for this approach is that it examines an
integrated trading world wherein foreign and domestic disturbances are
transmitted internationally with the utmost ease. If insulating policies
exist in such an environment then they surely exist in a world with less
than perfect commodity and/or asset arbitrage.

To this end consider two countries that produce and consume the same
perfectly substitutable good. Production occurs in spatially distinct areas
but, in the absence of transportation costs or other impediments to trade,
we can consider a single world market where prices adjust to ensure market
clearing period by period. Consequently, purchasing power parity always
holds.

Worldwide commodity demand is assumed to depend on the real interest
rate in each country. For simplicity we ignore the influence of aggregate
income on the demand for commodities and instead focus on the intertemporal
substitution possibilities facing consumers.
Commodity supply in each country follows the standard wage contracting specification where only unanticipated price disturbances produce output movements.

There is one type of bond perfectly mobile worldwide so that uncovered interest rate parity is an equilibrium condition of the model.

The demand for money in each country is assumed to be proportionately related to the price level, positively related to the level of real income and inversely related to the nominal rate of interest.

These assumptions are summarized in the following model:

\[
Y_t^S = Y_t + \theta(p_t - E_t^{-1}p_t) + \varepsilon_t
\]
\[
M_t^d = p_t + Y_t - \beta_i t
\]
\[
i_t = r_t + E_t p_{t+1} - p_t
\]
\[
Y_t^* = Y_t^* + \theta^*(p_t^* - E_t^{-1}p_t^*) + \varepsilon_t^*
\]
\[
M_t^s = p_t^* + Y_t^* - \beta_i^* t
\]
\[
i_t^* = r_t^* + E_t p_{t+1}^* - p_t^*
\]
\[
Y_t + Y_t^* = A_n - c r_t - c^* r_t^*
\]
\[
p_t = p_t^* + \varepsilon_t
\]
\[
i_t = i_t^* + E_t \varepsilon_t + \varepsilon_t
\]

With the exception of \(i_t\), \(r_t\), \(Y_t\) and \(Y_t^*\), all variables are expressed as in logarithms. An asterisk denotes a foreign variable, \(t\) denotes the time period and superscripts \(d\) and \(s\) denote demand and supply respectively. The definitions of the variables are:

- \(M\) = the nominal stock of money
- \(p\) = the price level
- \(Y\) = real income
- \(i\) = the nominal interest rate
\( r = \) the real interest rate

\( e = \) the exchange rate, the domestic price of foreign currency.

\( E_t(\cdot) \) represents the conditional expectation of variable \( \cdot \) given period \( t \) information. Equations (1) and (4) are the commodity supply relations. Equations (2) and (5) are the money demand functions while (3) and (6) specify the Fisher equations. Worldwide commodity market clearing occurs according to equation (7). The two international arbitrage conditions are the PPP relationship (8) and the UIP condition (9). The variables \( \varepsilon_t \) and \( \varepsilon^*_t \) are iid zero mean finite variance \( (\sigma^2_\varepsilon \text{ and } \sigma^2_{\varepsilon^*} \text{ respectively}) \) random variables.

3. THE MONEY SUPPLY PROCESSES

It remains to specify the money supply processes of the home and foreign central banks. For simplicity we abstract from the banking sector and assume that the money stock in each country is equal to its monetary base made up of central bank holdings of international reserves and domestic credit. Because there are the two separate components to the money supply the central banks are able to manage the exchange rate by buying and selling international reserves in response to deviations in the actual exchange rate from its target value. Concurrently the central bank may pursue a nominal interest rate target by open market sales and purchases of domestic assets. Together these policies imply the following money supply processes:

\[
M_t = \rho_t + m_e(t_e - e_t) + m_i(i_t - i^*_t) + u_t \\
M^*_t = \rho^*_t - m_e(t_e^* - e_t) + m_i(i_t^* - i^*_t) + u_t.
\]

\( t_e \) is the domestic target exchange rate and \( i_t \) is the target interest rate. \( \rho \) is the systematic growth rate of the domestic money supply and \( u_t \) is an independently distributed zero mean random disturbance with variance \( \sigma^2_u \). The
foreign variables are similarly defined.

The parameter \( m_e \) determines the degree to which the domestic central bank is committed to or supports the target exchange rate. For \( m_e > 0 \) exchange rate policy in the home country can be described as leaning against the wind. As \( m_e \) becomes larger the exchange rate becomes progressively more fixed. For \( m_e = 0 \) the exchange rate is freely floating. Similarly, \( m_i \) measures the degree to which the central bank targets the domestic nominal interest rate. For \( m_e = 0 \) and \( m_i = 0 \) the central bank is following a mixed policy of managing both the domestic interest rate and the exchange rate. Similar central bank behavior is presumed for the foreign country.

The target exchange rate in the home economy is given by

\[ \dot{e}_t = e_0 + \eta_t, \tag{12} \]

where \( \eta \) is the selected rate of growth (devaluation) of the target exchange rate. \( \eta > 0 \) would correspond to a crawling peg or sliding parity regime. Similarly, for the foreign country,

\[ \dot{e}^*_t = e^*_0 + \eta^*_t. \tag{13} \]

In choosing a target for the nominal interest rate we require that it in fact be achievable by the appropriate selection of the parameter \( m_i \). Define \( \pi \) to be the anticipated or systematic rate of inflation. That is,

\[ \pi = E_{t-1}(p_{t+1} - p_t). \]

Then

\[ E_{t-1}i_t = \pi + i_n, \]

where \( i_n \) is the natural rate of interest, or that interest rate in the home country which ex ante clears the goods market. For \( m_i = \infty \) it must be true that \( i_t = \pi + i_n \). In other words, the nominal interest rate target cannot be chosen independently of the expected equilibrium rate of inflation. In general, there is freedom of choice regarding the value of \( \pi \) and therefore
i_t (see below). Similarly, \( i_t^* \) must be chosen compatible with \( z^* \) given \( r_t^* \).

4. SOLUTION OF THE MODEL

The solution involves making use of the market equilibrium conditions to generate a set of expectational equations. These are solved using the method of undetermined coefficients (see Appendix A). Of interest are the resulting equilibrium distributions of the endogenous variables. For prices these are:

\[
P_t = K_0 + K_1 t + K_2 T + K_3 X_t
\]

where

\[
P_t = [P_t \ P_t^*]^T, \ T = [e_0 \ i \ e_0^* \ i^*]^T \text{ and } X_t = [\epsilon_t \ \nu_t \ \epsilon_t^* \ \nu_t^*]^T.
\]

The coefficient matrices are defined:

\[
K_0 = \begin{bmatrix}
\pi_1^* [(\beta^*+m_1^*)(r_n^*+\pi^*) - Y_n^*] \\
(1-\pi_1^* ) [(\beta^*+m_1^*)(r_n^*+\pi^*) - Y_n^*] \\
\end{bmatrix}
\]

\[
K_1 = \begin{bmatrix}
\pi_1^* (p^*+m_e^*) + (1-\pi_1^*) (p^*-m_e^* \eta^*) \\
(1-\pi_1^* ) (p^*+m_e^*) + \pi_1^* (p^*-m_e^* \eta^*) \\
\end{bmatrix}
\]

\[
K_2 = \begin{bmatrix}
\pi_2^* m_e - \pi_1^* m_i - (1-\pi_1^*) m_e^* - (1-\pi_1^*) m_i^* \\
(1-\pi_1^* ) m_e - (1-\pi_1^*) m_i - \pi_1^* m_e^* - \pi_1^* m_i^* \\
\end{bmatrix}
\]

\[
K_3 = \begin{bmatrix}
-\pi_2^* (1+\beta+m_i^*) - \pi_3^* (\beta^*+m_1^*) & \pi_3^* & -\pi_2^* (\beta^*+m_i^*) - \pi_3^* (1+\beta^*+m_1^*) & \pi_3^* \\
-\pi_2^* (1+\beta+m_i^*) - \pi_3^* (\beta^*+m_1^*) & \pi_3^* & -\pi_2^* (\beta^*+m_i^*) - \pi_3^* (1+\beta^*+m_1^*) & \pi_3^* \\
\end{bmatrix}
\]

where

\[
\pi_1 = \frac{1+m_e}{1+m_e+m_e^*} \quad \pi_1^* = \frac{1+m_e^*}{1+m_e+m_e^*}
\]

\[
\pi_2 = (1+\theta)(1+\beta+m_i^*) + m_e \quad \pi_2^* = (1+\theta^*)(1+\beta^*+m_1^*) + m_e^*
\]

\[
\pi_3 = m_e - \theta^* (\beta^*+m_i^*) \quad \pi_3^* = m_e^* - \theta(\beta^*+m_1^*)
\]

\[
\pi_4 = \pi_2^* \pi_2 - \pi_3 \pi_3^*.
\]

It is evident that the systematic and unanticipated components of the
price level in each country depends on the systematic and unanticipated components of policy in each country. Observe also that the pair \((\nu_2, \nu_3)\) is simple linear transformation of the policy parameter pair \((m_e, m_1)\). We exploit this transformation in Section 6 below.

From equations (14) we have the anticipated rates of inflation:
\[
\pi = \nu_1(\rho + m_\epsilon \eta) + (1 - \nu_1)(\rho^* - m_\epsilon^* \eta^*)
\]  
and
\[
\pi^* = \nu_1(\rho^* - m_\epsilon^* \eta^*) + (1 - \nu_1)(\rho + m_\epsilon \eta),
\]
for all \(t\).

We noted above that an achievable interest rate target must be compatible with the expected or systematic component of inflation. The central bank, therefore, must be sure that it can, in fact, control domestic inflation. Note that
\[
\lim_{m_\epsilon \to -\nu_1} \pi = (1 + m_\epsilon^*) \eta + (\rho^* - m_\epsilon^* \eta^*) \quad \lim_{m_\epsilon \to +\nu_1} \pi = (r + m_\epsilon \eta) - m_\epsilon \eta^*
\]
and
\[
\lim_{m_\epsilon \to -\nu_1} \pi^* = m_\epsilon^* \eta + (\rho^* - m_\epsilon^* \eta^*) \quad \lim_{m_\epsilon \to +\nu_1} \pi^* = (\rho + m_\epsilon \eta) - (1 + m_\epsilon) \eta^*.
\]
In other words, as long as at least one of \(m_\epsilon\) and \(m_\epsilon^*\) is finite (only one of the two countries attempts to fix the exchange rate), then the central bank indeed has control over the systematic rate of inflation with the appropriate choice of \(\rho\) and/or \(\eta\).

Solving for the domestic nominal interest rate yields:
\[
it = r_n + \pi + \sigma_1 e_t + \sigma_2 v_t + \sigma_3 e^*_t + \sigma_4 v^*_t
\]
where
\[
\sigma_1 = -[(1 + \theta) K_{11} + \theta^* K_{21}^* + 1] \quad \sigma_2 = -[(1 + \theta) K_{12} + \theta^* K_{22}^*]
\]
\[
\sigma_3 = -[(1 + \theta) K_{13} + \theta^* K_{23}^* + 1] \quad \sigma_4 = -[(1 + \theta) K_{14} + \theta^* K_{24}^*].
\]
Each \(K_{ij}^*\) is the appropriate element of the matrix \(K_3\). Taking limits gives
\[ \lim_{\mu \to \infty} i_t = r_n + \pi, \]
so that a central bank policy of fixing the nominal interest rate with a target \( i_t \) is achievable. Similar results hold for the exchange rate.

5. **EXCHANGE RATE AND INTEREST RATE POLICIES**

We focus on exchange rate and interest rate policies and the international transmission of disturbances. Of interest are the implications of independent exchange rate and interest rate policies. In the spirit of pre-1973 policy discussions (i.e. before the complete breakdown of the Bretton-Woods system) the goal of the central bank in each country is presumed to be to reduce the international transmission of disturbances -- both real and monetary -- so that pursuit of an independent monetary policy is possible. For the home central bank pursuit of this goal involves the selection of the interest rate management parameter \( (m_i) \), the exchange rate management parameter \( (m_e) \), the target rate of growth of the exchange rate \( (\eta) \) and the rate of growth of the money supply \( (\rho) \). The foreign central bank chooses similarly.

5.1 **Exchange Rate Policies**

Consider a policy where the domestic central bank floats the interest rate but fixes the exchange rate \( (m_i = 0, m_e = \infty) \). In this instance the unanticipated price shocks in each country are equal and given by

\[ p_t - E_{t-1}p_t = p_t^* - E_{t-1}p_t^* = -(\pi_2^* - \pi_3^*)^{-1}[(\rho^* + m_i^*)\epsilon_t + (1+\rho^* + m_i^*)\epsilon_t^* - \nu_t^*]. \]

The anticipated rates of inflation are equal and given by

\[ \pi = \pi^* = \rho^* - m_e^* \eta^*. \]

Thus under a fixed exchange rate the domestic economy is open to foreign real and monetary disturbances and the domestic anticipated inflation rate is fully determined by foreign systematic policies. The domestic central
bank loses control over the domestic money supply which becomes demand determined.

If the exchange rate is allowed to float freely \( (m_i = 0, e = 0) \) then \( \pi = \rho \) so expected inflation at home depends only on the rate of growth of the domestic money supply. However, it is evident from (14) that all of the disturbances impinge on domestic output so that a floating exchange rate does not insulate the economy from any foreign disturbances.

However, consider the following policy: \( (m_i = 0, e = \theta^s\beta) \). This would be a dirty float where the central bank leans against the wind with the exchange rate. Such a policy eliminates the transmission of foreign monetary shocks to the domestic economy through manipulation of the exchange rate alone. Alternatively, \( (m_i = 0, e = -\beta[1-m_e^s(1+\beta^s+m_i^s)^{-1}] ) \) insulates the domestic economy from foreign real disturbances. This requires a leaning with the wind exchange rate policy.

Each of the above exchange rate policies eliminates one source of foreign disturbance but is incapable of eliminating both. Moreover, there is no coordination of exchange rate policies (i.e. any combination of \( m_e \) and \( m_e^s \) for \( m_i = m_i^s = 0 \)) that will do the job.

5.3 Interest Rate Policies

We now consider interest rate policies. A pure interest rate policy is a policy that ignores the exchange rate \( (e = 0) \) and keys solely on the interest rate. From (14) it is clear that a policy \( (m_i = -\beta, e = 0) \) implies that \( \pi_3 = 0 \) and thus the elements \( K_{13} \) and \( K_{14} \) of \( K_3 \) disappear. In this instance we have

\[
P_t - E_{t-1}P_t = (1+\theta)^{-1}(\nu_t - \epsilon_t).\]

Thus the influence of unanticipated foreign monetary and real disturbances on domestic output is eliminated. Further, \( e = 0 \) implies that \( \pi_i^s = 1 \) so
that \( \pi = \rho \). Or, the influence of systematic foreign disturbances is also eliminated. Thus, irrespective of the behavior of the foreign central bank, a policy of floating the exchange rate and managing the interest rate completely insulates the domestic economy from foreign disturbances.

The money supply process implied by such a rule is

\[
M_t = \rho t - \beta (i_t - i_t) + \nu_t.
\]

When the interest falls below the target value the domestic central bank increases the money supply through open market purchases of assets by an amount exactly equal to the increase in the home demand for money. This leaning with the wind policy fully accommodates any interest rate induced change in the demand for money.

It also follows that the foreign central bank can independently pursue such a policy. That is \( (m_1^* = -\beta^*, m_2^* = 0) \) is a feasible fully insulating policy for the foreign central bank. Thus, flexible exchange rates, combined with an interest rate rule provide for national monetary autonomy, without the need for coordination.

5.3 Transmission Channels

It is evident that a policy of fixing the exchange rate and floating the interest rate, the policy usually considered, cannot insulate the domestic economy from foreign disturbances. This follows because of the nature of the economic linkages in an integrated trading world. These linkages are through the commodity market -- via commodity arbitrage or PPP -- and the capital market -- via assets arbitrage or UIP. To close the domestic economy to foreign disturbances requires breaking these two links. The exchange rate clearly ties domestic and foreign prices together through the purchasing power parity relation. Allowing the exchange rate to float breaks this link. The domestic interest rate is the variable through which
foreign capital market disturbances impact on the domestic economy. If the money supply is managed in such a way that the excess demand for money is independent of the interest rate then foreign capital market disturbances become irrelevant, as occurs under our proposed rule. Thus while only the interest rate is actively managed under a complete insulation regime, in fact, both elements in the central bank's arsenal of instruments are directed towards different aspects of economic integration. It is probably no coincidence, then, that central banks are often accused of being overly concerned with domestic interest rates. These form an important component of policies designed to provide central banks with monetary autonomy.

6. OPTIMAL POLICIES

To this point we have considered policies optimal in the sense of providing insulation from foreign disturbances and complete monetary autonomy. However, there are other criteria of optimality that might be considered. In particular, do insulating policies entail costs in terms of increased output variability? This might be an issue because if the domestic country insulates itself from foreign disturbances it may be foregoing the opportunity to export some of its own domestically sourced disturbances to the foreign country, thereby sacrificing an opportunity to reduce the variance of domestic output.

Consider now the goal of minimizing the variance of domestic output. Within the present framework, this is the same as minimizing the variance of the forecast error on output and very nearly the same as minimizing the variability of unanticipated inflation. From (1) and (14) we have

\[ Y_t = Y_n + \theta (K_1^{11} + 1/\theta) \epsilon_t + K_3^{12} \nu_t + K_3^{13} \epsilon_t^* + K_3^{14} \nu_t^*. \]

Thus,
\[
\text{var}(Y_t) = \theta^2 [(K_3^{11} + 1/\theta)^2 \sigma_\zeta^2 + (K_3^{12})^2 \sigma_\delta^2 + (K_3^{13})^2 \sigma_{\zeta*}^2 + (K_3^{14})^2 \sigma_{\delta*}^2].
\]

However, noting that \( \pi_2 \) and \( \pi_3 \) are linear transformations of \( \pi_e \) and \( \pi_i \) yields a transformation of \( \text{var}(Y_t) \) that is somewhat easier to work with. Specifically, denote

\[
\Sigma = (1/\theta)^2 \text{var}(Y_t)
\]

Then

\[
\Sigma = a - \frac{b \pi_2^2 - c \pi_3}{\pi_2^2 \pi_3} + \frac{(e \pi_2^2 - d \pi_2 \pi_3 + f \pi_3^2)}{\pi_2^2 \pi_3^2}.
\]

(18)

The coefficients are defined in the Appendix. Determining the optimal \( \pi_2 \) and \( \pi_3 \) yields the optimal \( \pi_e \) and \( \pi_i \) through the reverse transformation.

It is a straightforward matter to show that maximizing \( \Sigma \) over \( \pi_2 \) and \( \pi_3 \) yields

\[
\pi_3 = E \pi_2^*
\]

(19)

and

\[
\pi_2 = E \pi_3^* + F.
\]

(20)

From (14) and the supporting definitions, this in turn implies that

\[
\pi_e = -((1+\theta+\theta^*)^{-1}[-(1+\theta)E \pi_2^* - \theta^* E \pi_3^* - \theta^* F + \theta^*(1+\theta)]
\]

and

\[
\pi_i = -((1+\theta+\theta^*)^{-1}[E \pi_2^* - E \pi_3^* - F + (1+\theta+\theta^*) \beta + (1+\theta)],
\]

where

\[
E = -\frac{[\theta^*(1+\theta^*) + \theta(1+\theta)]^2 \sigma_{\zeta*}^2 + [(1+\theta^*)^2 \sigma_{\zeta*}^2 + \theta^* \sigma_{\zeta*}^2 + \theta(1+\theta+\theta^*)^2 \sigma_{\zeta*}^2 \sigma_{\delta*}^2}{(1+\theta+\theta^*)^2 \theta^* (1+\theta^*) \sigma_{\zeta*}^2 \sigma_{\delta*}^2 + \theta(1+\theta) \sigma_{\zeta*}^2 \sigma_{\delta*}^2 + \theta^* \sigma_{\zeta*}^2 \sigma_{\delta*}^2 + \theta(1+\theta+\theta^*) \sigma_{\zeta*}^2 \sigma_{\delta*}^2}
\]

and

\[
F = \theta^* \frac{[\theta^*(1+\theta^*)^2 \sigma_{\zeta*}^2 + \theta^2 \sigma_{\zeta*}^2 + \theta^* \sigma_{\zeta*}^2 + \theta^2 \sigma_{\zeta*}^2 + (1+\theta^*)^2 \sigma_{\zeta*}^2 \sigma_{\delta*}^2 + \theta(1+\theta+\theta^*) \sigma_{\zeta*}^2 \sigma_{\delta*}^2]}{[\theta^*(1+\theta^*) \sigma_{\zeta*}^2 \sigma_{\delta*}^2 + \theta(1+\theta) \sigma_{\zeta*}^2 \sigma_{\delta*}^2 + \theta^* \sigma_{\zeta*}^2 \sigma_{\delta*}^2 + \theta(1+\theta+\theta^*) \sigma_{\zeta*}^2 \sigma_{\delta*}^2]}
\]

In general, therefore, the optimal \( \pi_2 \) and \( \pi_3 \), and hence \( \pi_e \) and \( \pi_i \), will be functions of all the structural parameters and disturbances in the model. The general behavior of the central bank will be to manage both the exchange rate and the interest rate under almost all circumstances. Furthermore, substituting (19) and (20) back into (18) yields

\[
\text{var}(Y_t) = \theta^2 [(K_3^{11} + 1/\theta)^2 \sigma_\zeta^2 + (K_3^{12})^2 \sigma_\delta^2 + (K_3^{13})^2 \sigma_{\zeta*}^2 + (K_3^{14})^2 \sigma_{\delta*}^2].
\]
\[
\Sigma = -a + \frac{fb^2 - bcd + c^2e}{4fe - d^2}
\] (21)

so that, when following an optimal policy the domestic central bank can always find some rule that yields it a constant level of utility. In other words, no matter what the actions of the foreign central bank, the domestic central bank can act in such a way as to attain some minimal level of utility.

To gain more insight in the nature of the optimal policies, consider the following special cases:

**Case 1**: \(\sigma_k = \sigma_\epsilon = 0\) -- the only disturbances in the economy are from the foreign country. One might expect the insulating policy to be optimal in this instance. This is verified by substituting for \(\sigma_k^2\) and \(\sigma_\epsilon^2\) into \(E\) and \(F\) to obtain \(E = 0\) and \(F = (1 + \theta)\). Substituting into \(m_\epsilon\) and \(m_i\) yields \(m_\epsilon = 0\) and \(m_i = -\beta\), which is the insulating policy. This is the only time that an insulating policy is optimal.

**Case 2**: \(\sigma_k^* = \sigma_\epsilon^* = 0\) -- the only disturbances in the economy are domestic. In this case \(F = \infty\) and \(E = -\infty\). The optimal policy requires the central bank to fix one or both the interest rate and the exchange rate. It is interesting to note that fixing the interest rate alone \((m_i = \infty)\), for any exchange rate regime, will be optimal. By doing so some of the domestic disturbances are passed on to the foreign economy through the capital markets.

**Case 3**: \(\sigma_\epsilon = \sigma_\epsilon^* = 0\) -- there are only real disturbances and no monetary shocks. The optimal policy here is the same as for Case 2.

**Case 4**: \(\sigma_k^* = \sigma_\epsilon^* = 0\) -- that is a world with just monetary disturbances. In this case one can show that

\[
E = - \frac{[(1 + \theta^*)^2 \sigma_\epsilon^2 + \theta^2 \sigma_\epsilon^2]}{\theta^* (1 + \theta^*) \sigma_\epsilon^* + \theta (1 + \theta) \sigma_\epsilon^*}
\]
Thus, both the interest rate and the exchange rate are managed to a degree that depends on the underlying variances of the monetary disturbances. It is interesting to note, however, that in this case \( \Sigma = 0 \). That is when there are just monetary disturbances, the central bank can find a policy that will completely eliminate output variability, should it so desire.

Optimal policies considered previously in the literature have dealt with just exchange rate management. This is a special case of the present model with \( m_i = 0 \) and policy choices restricted to \( m_e \). Our analysis indicates that such policies are always suboptimal when the central bank has sufficient domestic assets on hand to enable it to pursue a nominal interest rate target.

7. CONCLUSIONS

This paper considers interest rate and exchange rate management policies that reduce the international transmission of disturbances and allow countries to pursue more independent monetary policies. We find that exchange rate management alone is incapable of simultaneously eliminating the transmission of both foreign real and monetary disturbances to the domestic economy. The elimination of foreign monetary disturbances is possible with an exchange rate policy of leaning against the wind whereas the elimination of foreign real disturbances requires leaning with the wind. Each policy is a dirty float.

However, an interest rate policy combined with a floating exchange rate is capable of simultaneously eliminating the transmission of both foreign real and monetary disturbances. By accommodating interest rate induced
changes in the demand for money capital market disturbances are removed. By floating the exchange rate commodity market disturbances are eliminated. It is through these two markets that foreign disturbances are transmitted.

The cost of achieving autonomy will nearly always be increased output and price variability. Only when there are no domestic disturbances will an insulating policy be optimal. In general the central bank will want to manage both the exchange rate and the interest rate. Thus it should not be surprising that central banks quickly became disillusioned with fully flexible exchange rate regimes following the general move to floating rates during 1973. Nor should it be surprising that central banks appear to be preoccupied with interest rates and the management thereof. Such behaviour has been shown to be optimal under general circumstances in this paper.
References


**Appendix**

I. In this Appendix we outline the method used to derive the solutions (14). First we demonstrate that \( r_t = r_t^* \), which follows from the fact that there is a single worldwide goods market. From (3) and (6)

\[
r_t - r_t^* = (i_t - i_t^*) - (E_t p_{t+1} - E_t p_{t+1}^*) + (p_t - p_t^*). \tag{A1}
\]

Interest rate parity (9) and purchasing power parity (8) imply

\[
i_t - i_t^* = (E_t p_{t+1} - E_t p_{t+1}^*) - (p_t - p_t^*),
\]

so that the RHS of (A1) is identically zero.

Using \( r_t = r_t^* \) and letting \( c + c^* = 1 \), substituting (1) and (4) into (7) yields

\[
r_t = r_n - \theta(p_t - E_{t-1}p_t) - \theta^*(p_t^* - E_{t-1}p_t^*) - (\epsilon_t + \epsilon_t^*), \tag{A2}
\]

where \( r_n = A_n - Y_n - Y_t^* \) is the real interest rate that ex ante clears the goods market. Next it is simply a matter of substituting (A2), (1) or (4) and (8) into each of the money market clearing conditions

\[
M_t^D = M_t^E
\]

and

\[
M_t^{D*} = M_t^{E*},
\]

to derive the quasi-reduced form solutions:

\[
\begin{bmatrix}
\text{m}_e + (1+\theta)(1+\beta+m_i) & \theta^*(\beta+m_i) - m_e & [P_t] \\
\theta(\beta^*+m_{i}^*) - m_e^* & m_e^* + (1+\theta^*)(1+\beta^*+m_{i}^*) & [P_t^*]
\end{bmatrix} =
\begin{bmatrix}
(\beta+m_i)(r_n + \pi) - Y_n \\
(\beta^*+m_{i}^*)(r_n^* + \pi^*) - Y_n^*
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
m_e - m_i & 0 & [e_0] \\
0 & -m_e^* - m_{i}^* & [e_0^*]
\end{bmatrix} + \begin{bmatrix}
\theta - (1+\theta^*) & \theta^*(\beta+m_i) & [E_{t-1}p_t] \\
\theta(\beta^*+m_{i}^*) & \theta^*(1+\beta^*+m_{i}^*) & [E_{t-1}p_t^*]
\end{bmatrix} + \begin{bmatrix}
-(1+\beta+m_i) & 1 & [\epsilon_t] \\
-(\beta^*+m_{i}^*) & 0 & [\epsilon_t^*]
\end{bmatrix}
\]

or,

\[
AP_t = R_0 + R_1t + R_2T + R_3E_{t-1}p_t + R_4X_t. \tag{A3}
\]

\( P_t, T \) and \( X_t \) are all defined in the text. Guessing a solution
\[ P_t = K_0 + K_1 t + K_2 T + K_3 X_t, \]  

it is apparent that
\[ E_{t-1} P_t = K_0 + K_1 t. \]

Substituting (A4) and (A5) into (A3) and equating coefficients yields:
\[ K_0 = (A - R_3)^{-1} R_0 \]
\[ K_1 = (A - R_3)^{-1} R_1 \]
\[ K_2 = (A - R_3)^{-1} R_2 \]
and
\[ K_3 = A^{-1} R_4. \]
Solving these matrix equations gives the solution presented in (14) of the text.

II. Here we provide the definitions of the coefficients in (18).
\[
\begin{align*}
    a &= \left[ (1+\theta^*)^2 \sigma_{\xi}^2 + \theta^2 \sigma_{\tilde{\xi}^*}^2 \right] / \theta^2 (1+\theta+\theta^*)^2 \\
    b &= 2[\theta^*(1+\theta^*) \sigma_{\xi}^2 + \theta(1+\theta) \sigma_{\tilde{\xi}^*}^2] / \theta(1+\theta+\theta^*)^2 \\
    c &= 2[(1+\theta^*)^2 \sigma_{\xi}^2 + \theta^2 \sigma_{\tilde{\xi}^*}^2] / \theta(1+\theta+\theta^*)^2 \\
    d &= 2[\theta^*(1+\theta^*) \sigma_{\xi}^2 + \theta(1+\theta) \sigma_{\tilde{\xi}^*}^2] / (1+\theta+\theta^*)^2 \\
    e &= \left[ \theta^*2 \sigma_{\xi}^2 + (1+\theta)^2 \sigma_{\tilde{\xi}^*}^2 + (1+\theta+\theta^*)^2 \sigma_{\hat{\xi}}^2 \right] / (1+\theta+\theta^*)^2 \\
    f &= \left[ (1+\theta^*)^2 \sigma_{\xi}^2 + \theta^2 \sigma_{\tilde{\xi}^*}^2 + (1+\theta+\theta^*)^2 \sigma_{\hat{\xi}}^2 \right] / (1+\theta+\theta^*)^2.
\end{align*}
\]
1 Turnovský (1984) provides a useful, representative discussion.

2 We could have expressed $Y_t$ and $Y_t^*$ in logs and then modified the LHS of (7) to contain the share weighted logs of real output. Doing so however does not add to the intuition behind this model and merely complicates the algebra.