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The Impact of Differential Human Capital Stocks on Club Allocations

by Lori L. Taylor

Units of human capital come equipped with utility functions, no extra charge. This characteristic inseparability of human and human capital creates significant complications for club theorists. For a variety of production functions, maximizing output requires clubs that mix individuals with different human capital stocks. On the other hand, when tastes differ, maximizing utility requires homogeneous clubs. Therefore, when people must live and work in the same jurisdiction, differences in human capital endowments force a choice between production and consumption inefficiencies. When the production inefficiencies of mixing types outweigh the consumption inefficiencies, differences in human capital endowments lead to mixed clubs.

Eitan Berglas approaches the question of mixed clubs in his 1976 article "Distribution of Tastes and Skills and the Provision of Local Public Goods." In Berglas' model there are two types of individuals, each endowed with different tastes and types of human capital. The two types of human capital are the only factors, and the production function is homogeneous of degree one in each of the human capital types. The number of clubs is undefined. Berglas sets up the model so that the two human capital types are essential complements in production, thereby forcing the model to a mixed clubs solution.

In a more recent article, "Tastes, Skills and Local Public Goods," Jan Brueckner (1989) expands on Berglas' model to analyze clubs in which skill types are complementary, but nonessential. Brueckner's model allows the homogeneous-clubs allocation to be optimal. He finds that mixed clubs Pareto-dominate homogeneous clubs whenever the production efficiencies from imperfect labor complementarities exceed the consumption inefficiencies of mixing. Without some labor complementarities, homogeneous clubs are Pareto-optimal in Brueckner's model.

This paper expands Berglas' and Brueckner's analysis to examine club allocations when human capital endowments differ and skills are perfect substitutes. The model presented here also differs from previous work by restricting the number of possible jurisdictions and by introducing land into the analysis. I find that even when skills are perfect substitutes, mixed clubs can Pareto-dominate homogeneous clubs.

I also argue that horizontal equity is necessary but not sufficient for stability when individuals have Nash conjectures and moving is costless. I construct additional stability conditions and find that while identical, mixed clubs are always stable, homogeneous clubs become both less optimal and less stable as wages diverge between jurisdictions. Therefore, when individuals accumulate human capital at different rates over time -- causing the human capital stocks to diverge -- a homogeneous-clubs allocation generally becomes unstable.

Setting up the Model

Consider a perfect-information world in which there are two jurisdictions and two types of individuals. The jurisdictions have identical production technologies and endowments of land. The social planner collects all rents and redistributes them to residents in lump sums. Rents need not be equally distributed between the two types of persons, but all individuals of type i in jurisdiction j must be treated equally. The two types of individuals are distinguished by different utility functions (denoted U and V for type one and two, respectively), and endowments of human capital (denoted h_U and h_V, respectively). By assumption, utility functions are concave, continuous, and twice differentiable. There is only one type of human capital and the numbers of type 1 and type 2 agents are equal.

Labor supply for both types is fixed at one time unit per person. However, firms pay wages that depend on the quantity of human capital offered. Wage income therefore varies according to type within each jurisdiction.

Individuals have Nash conjectures about the wages paid to human capital, the goods prices and the rent redistribution function. The social planner can make transfers between jurisdictions if necessary to support an optimal allocation.

Each individual must rent one unit of land (that produces no utility) and all individuals must be located in one of the two jurisdictions.

Following Stiglitz (1977) the total output of each jurisdiction can be used either for private consumption (X) or for public consumption (S). Output is produced from two factors -- land (L) and human capital (H). By assumption, the production function is continuous, twice differentiable, and homogeneous of degree one.

The Social Planning Problem

Subject to the usual constraints, the utilitarian social planner maximizes total utility (TU)

$$TU = \alpha N_1 U_1(x_{1U}, S_1) + \beta M_1 V_1(x_{1V}, S_1) + \alpha N_2 U_2(x_{2U}, S_2) + \beta M_2 V_2(x_{2V}, S_2)$$

where the subscripts denote the jurisdiction; N_i indicates the number of type one individual in jurisdiction i; M_i indicates the number of type two individuals in jurisdiction i; x_{ij} indicates the consumption of private good X by an individual of type j in jurisdiction i and the α and β are utility weights for type one and two individuals, respectively.[1] Let $\alpha+\beta=1$. The constraints are goods-clearing constraints.

$$(1.1)\quad X_1+X_2 = N_1x_{1U} + M_1x_{1V} + N_2x_{2U} + M_2x_{2V}$$

$$(1.2)\quad F(H_i, L_i) = S_i + X_i \qquad i=1,2$$

where $H_i = N_ih_U + M_ih_V$ and $L_i = L^* - N_i - M_i$ for i=1,2. Because there are an equal number of agents of each type, $N_1 + N_2 = M_1 + M_2 = \bar{N}$.

Optimality Conditions

The conditions for optimality under these specifications can be divided into two categories -- conditions for the optimal provision of goods within a jurisdiction and conditions for the optimal allocation of individuals to jurisdictions. Appendix 1 presents the Kuhn-Tucker conditions that lead to these optimality conditions.

The marginal condition for optimally providing goods within a jurisdiction is the expected Samuelson (1954) condition. The sum of the marginal rates of substitution must equal the marginal rate of transformation.

$$(2.1)\quad N_i \frac{\partial U_i/\partial S_i}{\partial U_i/\partial x_{i1}} + M_i \frac{\partial V_i/\partial S_i}{\partial V_i/\partial x_{i2}} = \frac{\partial S_i/\partial F_i}{\partial X_i/\partial F_i} = 1 \qquad i=1,2.$$

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[1] Given concave utility, any Pareto optimum can be described in this utilitarian framework by varying the utility weights. This approach is therefore equivalent to Berglas' Pareto optimization approach when there are only two jurisdictions.

The possibility of corner solutions complicates the optimal assignment of individuals to jurisdictions. Three types of allocations might arise -- an allocation with two homogeneous clubs, an allocation with one mixed and one homogeneous club, and an allocation with two mixed clubs. Each case will be analyzed in turn, and then decision rules that indicate the optimal case under various circumstances will be discussed.

In all cases, the optimal allocation of individuals to jurisdictions requires that

$$(3.1a)\quad TU = \alpha\lambda_U[X_1 + X_2 - w_1H_1 + r_1T_1 - w_2H_2 + r_2T_2]$$

where w_i and r_i are marginal productivity wages and rents, respectively, in jurisdiction i, $T_i = N_i + M_i$, and $\alpha\lambda_U = \beta\lambda_V$ is the marginal utility of additional unit of either consumption good, adjusted by the weight of each type in the social welfare function.

Because the production function (F) is linearly homogeneous, optimal allocation condition 3.1a reduces to the requirement that total rents exceed total provision of public goods (assuming non-satiation and positive utility).[2]

$$(3.1b)\quad TU = \alpha\lambda_U[\ (r_1 + r_2)L - S_1 - S_2\ ].$$

Although the social planner will collect sufficient rents to completely

[2] If private landlords were in the model, rather than a social planner who collects all of the rents, condition 3.1a implys that a tax on rents would be sufficient to finance the local public good at the optimum.

cover the cost of the public good, some tax or transfer mechanism still may be necessary to support the optimal allocation. Depending on the utility function, the landlord/social planner may need to redistribute rent income to one or both types of individuals. The social planner also may need to tax the wage income of one type of individual to finance a subsidy for the other type. Optimization does not require equal tax/transfer treatments for the two types.

Not all allocations that satisfy the requirements for optimality will be stable equilibria. With Nash conjectures about the level of public good provided and about the wage, rent and tax/transfer functions, stability requires more than horizontal equity. When moving costs are zero, stability requires that individuals expect higher utility in their assigned jurisdictions than in any other feasible jurisdiction, *including jurisdictions in which no member of their type is present*. In each case, the conditions under which optima are also stable equilibria will be discussed.

Case 1 -Two Homogeneous Clubs

Only one allocation occurs with two homogeneous clubs. For notational simplicity, consider the assignment of type 1 individuals to jurisdiction 1 and of type 2 individuals to jurisdiction 2. This allocation, however, may not be stable.

Stability

When moving costs are zero, the types must be unable to increase their utility by changing jurisdictions. Otherwise, the allocation will not be stable. Therefore, each type must receive greater utility from the assigned allocation than they would expect in the other jurisdiction. In other words, stability requires that each type 1 (2) individual expect higher utility in a homogeneous jurisdiction of type 1 (2) than in a homogeneous jurisdiction of type 2 (1) individuals. Thus, assuming marginal productivity factor payments and Nash conjectures about the level of the public good provided and about the rent, wage, and transfer functions, stability requires that

(4.1) $U_1(x_{1U}, S_1) \geq U(x_{2V} + w_2(h_u - h_v), S_2)$

(4.2) $V_2(x_{2V}, S_2) \geq V(x_{1U} - w_1(h_u - h_v), S_1)$.

If the human capital endowments are sufficiently disparate, or the relative utility elasticities sufficiently similar, then the homogeneous clubs allocation is not stable.[3]

<u>Case 2 - One Mixed and One Homogeneous Club</u>

Optimality

The marginal conditions for optimally assigning individuals, given that one of the jurisdictions must be a homogeneous club (the partially-mixed clubs solution) are

(3.1) $TU = \alpha\lambda_U[X_1 + X_2 - w_1H_1 + r_1T_1 - w_2H_2 + r_2T_2]$

(5.1) $[V_1(x_{1V}, S_1) - V_2(x_{2V}, S_2)] = \lambda_V[x_{1V} - \partial F_1/\partial M_1] - \lambda_V[x_{2V} - \partial F_2/\partial M_2]$

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[3] Moving costs would add to the stability of the homogeneous clubs allocation. If moving costs were positive, then stability would require that

(4.1) $U_1(x_{1U}, S_1) \geq U(x_{2V} + w_2(h_u - h_v), S_2) - c$
(4.2) $V_2(x_{2V}, S_2) \geq V(x_{1U} - w_1(h_u - h_v), S_1) - c$

where c is the utility-denominated moving cost.

where λ_j is the marginal utility of x_{ij} and $\alpha\lambda_U=\beta\lambda_V$. This specification assumes that the homogeneous club will be of type 2, but the analysis for a homogeneous club of type 1 is perfectly symmetric.

Stability

When moving costs are zero, the types must be unable to increase their utility by changing jurisdictions or else the allocation will not be stable. Clearly, this requires that type two individuals receive the same level of utility in each jurisdiction ($V_1=V_2$). The utility requirement of the type 1 individual is not well specified, however. Brueckner approaches this problem by comparing type 1's utility in the internal allocation to his utility in a homogeneous allocation of type 1 individuals. Such a comparison makes sense when land does not limit the size or number of allocations, but it does not make sense here. A homogeneous allocation of type 1 individuals is not a basis for comparison in this case, and therefore comparing the utility of type 1 individuals in this allocation to their utility in a homogeneous allocation would be inappropriate. Given that the allocation will have one mixed and one homogeneous club, the planner must compare the utility of the type 1 individual in the mixed allocation to that individual's expected utility in a homogeneous jurisdiction of type 2 individuals. The type 1 individual must receive greater utility from the assigned allocation than he would expect in the other jurisdiction.

Assuming marginal productivity factor payments and Nash conjectures about the level of the public good provided and about the rent, wage, and transfer functions, stability requires that

$$(5.3)\quad x_{1V} - w_1 h_v + r_1 = x_{2V} - w_2 h_v + r_2$$

$$(5.4)\quad U(x_{2U}, S_2) \geq U(x_{1V} + w_1(h_u - h_v), S_1)$$

$$(5.5)\quad V(x_{1V}, S_1) = V(x_{2V}, S_2)$$

where w_1 is the prevailing wage in the homogeneous jurisdiction and w_2 is the prevailing wage in the mixed jurisdiction.[4]

If the utility function is CES or log-linear, then $V_1 = V_2$ implies that $x_{1V} = x_{2V}$ and that $S_1 = S_2$. Equation 5.2a reduces to a requirement that the type 2 individuals be distributed so as to maximize transformed output, given that one of the jurisdiction will be homogeneous. Further, the requirement that $S_1 = S_2$ reduces the stability constraint on the utility of type 1 individuals

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[4] Again, moving costs would increase the stability of this allocation. If moving costs were positive, then stability would require that

$$(5.3a)\quad |x_{1V} - w_1 h_v + r_1 - x_{2V} + w_2 h_v - r_2| \leq c$$
$$(5.4a)\quad U_2(x_{2U}, S_2) \leq U(x_{1V} + w_1(h_u - h_v), S_1) - c$$

where c is again the utility-denominated costs of moving between jurisdictions.

(equation 5.4) to a requirement that

(5.3b) $x_{2U} > x_{2V} + w_1(h_u - h_v)$.

Total output in this partially-mixed clubs allocation will be less than or equal to total output in a totally-mixed clubs allocation, because the partially-mixed allocation represents a constrained version of the totally-mixed allocation. Under partial mixing, only the type of individual allocated to a homogeneous jurisdiction will be allocated for productive efficiency.

<u>Case 3 - Two Mixed Clubs</u>

Optimality

Assuming marginal productivity wages and rents, the marginal conditions for optimally assigning individuals to jurisdictions (interior solution) reduce to

(3.1) $TU = \alpha\lambda_U[X_1 + X_2 - (w_1H_1 - r_1T_1) - (w_2H_2 - r_2T_2)]$

(6.1) $[V_1(x_{1V}, S_1) - V_2(x_{2V}, S_2)] = \lambda_V[x_{1V} - \partial F_1/\partial M_1] \quad - \lambda_V[x_{2V} - \partial F_2/\partial M_2]$

(6.2) $[U_1(x_{1U}, S_1) - U_2(x_{2U}, S_2)] = \lambda_U[x_{1U} - \partial F_1/\partial N_1] \quad - \lambda_U[x_{2U} - \partial F_2/\partial N_2]$

where λ_j is the marginal utility of x_{ij} and $\alpha\lambda_U=\beta\lambda_V$.

Stability

When moving is costless, stability requires that $V_1=V_2$ and $U_1=U_2$. [5] Clearly, not all allocations that satisfy the above optimality conditions are

[5] Again, sufficient moving costs could sustain any optimal allocation.

stable equilibria. As an interior solution, however, an identical mixed-clubs allocation is both optimal and stable, regardless of the utility specification.

Other allocations can be both stable and optimal under certain conditions. If the utility function is CES or log-linear, then the equal utility constraint requires that the level of private good consumption for each type be independent of the assigned jurisdiction. In other words, $x_{1j} = x_{2j}$ for each type. Conditions 6.1 and 6.2 therefore reduce to simple conditions requiring that the allocation maximize total consumption $(X_1+X_2+S_1+S_2)$.

(6.1a) $\partial F_1/\partial M_1 = \partial F_2/\partial M_2$

(6.2a) $\partial F_1/\partial N_1 = \partial F_2/\partial N_2$

Assuming marginal productivity wages and rents, equations 6.1a and 6.2a further reduce to simple conditions that wages and rents equalize across the two jurisdictions.

Choosing Between Cases

To identify the dominant case, it is necessary to specify the utility function. The following example using log-linear utility functions illustrates the conditions under which each of the above cases solves the social planning problem.

Let the utility function for each type of individual be log-linear such that $U_i = \phi \ln(x_{iU}) + \theta \ln(S_i)$ and $V_i = \hat{\phi} \ln(x_{iV}) + \hat{\theta} \ln(S_i)$.

Case 1

The homogeneous clubs allocation leads to the following consumption allocations

$x_{11}=\alpha\phi F_T/N$; $x_{22}=\beta\hat{\phi}F_T/N$; $S_1=\alpha\theta F_T$; $S_2=\beta\hat{\theta}F_T$.

where $F_T = F_1 + F_2$ is the total transformed output of the two homogeneous jurisdictions. Total consumption in jurisdiction one with its population of type one individuals would be αF_T, while total consumption in jurisdiction two would be βF_T. Unless by chance the weights (α and β) are proportionate to the relative human capital endowments, this allocation will require a transfer of output between jurisdictions.

The homogeneous clubs allocation yields the following level of total utility

(8.1) $TU= N[\alpha\phi\ln(\alpha\phi/N) + \alpha\theta\ln(\alpha\theta) + \beta\hat{\phi}\ln(\beta\hat{\phi}/N) + \beta\hat{\theta}\ln(\beta\hat{\theta}) + \ln(F_T)]$.

This level of utility may be impossible to achieve, however. If the allocation is unstable, then agents will move out of their assigned jurisdictions, and the allocation will devolve to a stable equilibrium which might yield a lower level of utility. As discussed above, stability requires that each type expect lower utility in the other jurisdiction than he receives in the assigned jurisdiction. Therefore, equations 4.1 and 4.2 lead to the following stability conditions for homogeneous clubs

(9.1) $\Sigma_1 = (\phi/\theta)[\ln(\alpha\phi F_T) - \ln(\beta\hat{\phi}F_T + Nw_2(h_u - h_v))] + \ln(\alpha\theta/\beta\hat{\theta}) \geq 0.$

(9.2) $\Sigma_2 = (\hat{\phi}/\hat{\theta})[\ln(\beta\hat{\phi}F_T) - \ln(\alpha\phi F_T - Nw_1(h_u - h_v))] - \ln(\alpha\theta/\beta\hat{\theta}) \geq 0.$

Whenever either stability condition (equation 9.1 or 9.2) is violated the allocation degenerates into a heterogeneous clubs allocation, even if the homogeneous clubs allocation produces more total utility.

Case 2

The allocation with one homogeneous club and one mixed club leads to the same allocation rule as does the identical mixed-clubs allocation. However, the identical mixed-clubs allocation produces more output because it leads to a productively efficient allocation of both types of human capital. The identical mixed-clubs allocation therefore dominates the allocation with one mixed and one homogeneous club.

Case 3

The identical mixed-clubs allocation leads to the following consumption allocation[6]

$x_{11}=x_{21}=\alpha\phi 2\hat{F}/N;\ x_{12}=x_{22}=\beta\hat{\phi}2\hat{F}/N;\ S_1=S_2=(\alpha\theta+\beta\hat{\theta})\hat{F}$

where $\hat{F}=F([h_u+h_v]N/2,L-N)$. This allocation does not require any transfer of output between jurisdictions, and yields the following level of total utility

$$(8.2)\quad TU=N[\alpha\phi\ln(\alpha\phi/N)+(\alpha\theta+\beta\hat{\theta})\ln(\alpha\theta+\beta\hat{\theta})-(\alpha\theta+\beta\hat{\theta})\ln(2)+\beta\hat{\phi}\ln(\beta\hat{\phi}/N)+\ln(2\hat{F})]$$

Conditions Under Which Mixed Clubs Dominate Homogeneous Clubs

The identical mixed-clubs allocation produces greater total utility whenever the output gains from efficiently allocating the human capital types

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[6] Because the identical jurisdictions allocation is both optimal and stable it will be the only allocation considered here.

exceed the consumption inefficiencies of mixing.[7] Equivalently, the identical mixed-clubs allocation leads to greater total utility whenever the indicator function I (equation 10) is positive.

(10) $$I = \{(\alpha\theta + \beta\hat{\theta})\ln(\alpha\theta+\beta\hat{\theta}) - \alpha\theta\ln(2\alpha\theta) - \beta\hat{\theta}\ln(2\beta\hat{\theta})\} + \{\ln(2\hat{F}) - \ln(F_T)\}.$$

The term in the first set of brackets represents the utility loss from mixing while the term in the second set of brackets represents the productivity gain from mixing.

Comparative Statics of the Indicator Function

Comparative statics of the indicator function illustrate intuitively appealing characteristics.

(11.1) $$\partial I/\partial\alpha = (\theta - \hat{\theta})\ln(\alpha\theta+\beta\hat{\theta}) - \theta\ln(2\alpha\theta) + \hat{\theta}\ln(2\beta\hat{\theta})$$

(11.2) $$\partial I/\partial\theta = \alpha\ln[\alpha\theta+\beta\hat{\theta}] - \alpha\ln(2\alpha\theta)$$

(11.3) $$\partial I/\partial\hat{\theta} = \beta\ln[\alpha\theta+\beta\hat{\theta}] - \beta\ln(2\beta\hat{\theta}).$$

and, assuming a constant elasticity form for the production function (F)

(11.4) $$\partial I/\partial h_i = \xi_F[(h_u+h_v)^{-1} - F(Nh_i, L-N)/h_i F_T] \quad i=u,v.$$

where $\xi_F = (\partial F_i/\partial H_i)(H_i/F_i)$.

If $(\beta\hat{\theta}>\alpha\theta)$ - implying either that the planner strongly favors type two

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[7] The identical mixed-clubs allocation also dominates the homogeneous clubs allocation as an equilibrium (if not as an optimum) whenever the homogeneous jurisdictions allocation is unstable.

individuals ($\beta>\alpha$) or that type two individuals have a relatively strong taste for the public good ($\hat{\theta}>\theta$) (or both) - then increasing the weight placed on type two individuals (β) decreases the likelihood of an interior solution. Intuitively, if the social planner favors the type with a relative taste for the public good then increasing the weight given to the favored type makes an interior solution less likely, ceteris paribus. Because the consumption inefficiency from mixing arises from the consumption of the public good, favoring the type with a taste for the public good amplifies the inefficiency in the view of the planner.

Similarly, changing the utility parameters to bring them closer together -either by decreasing $\hat{\theta}$ or increasing θ when ($\beta\hat{\theta}>\alpha\theta$) or by increasing $\hat{\theta}$ or decreasing θ when ($\beta\hat{\theta}<\alpha\theta$) -increases the likelihood of an interior solution. Intuitively, the more similar the tastes for the public good ($\hat{\theta}$ and θ), the less consumption inefficiency from mixing.

Assuming that the marginal productivity of land is positive, bringing the human capital endowments closer together reduces the likelihood of an interior allocation (reduces I). Similarly, separating the human capital endowments increases the likelihood of an interior allocation (increases I). Intuitively, the more dissimilar the human capital endowments, the greater the productivity gain from efficiently allocating human capital.

Implications of the Analysis Over Time

The research presented here analyses the allocation question in a static setting, but the model also lends itself to multi-period analysis. In the model, S represents a generic public good. Define it now as schooling, and let human capital endowments in period t be a function of S and h_i. Assuming continuous time, $\dot{h}_i(t)=h_i(t)[\delta S_i(t)]$.[8]

If the identical mixed-clubs allocation is optimal in the first period, then all individuals receive the same level of schooling, S. The human capital ratio remains constant over time, implying that if the utility functions are linearly homogeneous, or log-linear, then the sign of the indicator function does not change with time. Therefore, for these utility specifications, if the identical mixed-clubs allocation is optimal in any period it is optimal in all subsequent periods.

On the other hand, if the homogeneous or partially-mixed clubs allocation is optimal in the first period, then individuals receive different levels of schooling and the human capital stocks diverge over time. As the human capital stocks diverge, the productivity gains from efficiently allocating human capital increase. If tastes are sufficiently similar so that the indicator function is positive at the maximum level of productivity gains, then the identical mixed-clubs allocation is optimal in the long run.

Even if the productivity gains from mixing never dominate the utility costs, the identical mixed-clubs allocation may still dominate on stability grounds. As the human capital stocks diverge over time, the incentive to move from the assigned jurisdiction to the other jurisdiction increases. Appendix 2 illustrates the time-instability of homogeneous-clubs allocations when the utility functions are log linear.

Conclusions

The research presented here contributes to our understanding of the club

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[8] This functional form is adapted from Lucas (1988). Here, schooling consumption substitutes for Lucas' measure of labor time set aside for human capital accumulation. By assumption, $\delta S_i(t) > 1$.

allocation process in a number of ways. The analysis demonstrates that human capital differences can lead to optimal mixed clubs even when the human capital stocks of each type are perfect substitutes. The analysis also demonstrates that when the number of jurisdictions is fixed, an allocation with identical, mixed clubs is always stable, even though it may not be optimal. Finally, the analysis explores an additional source of instability for homogeneous clubs, and demontrates that homogeneous clubs tend to become both less stable and less optimal as human capital stocks diverge.

References

Berglas, E. (1976), "Distribution of Tastes and Skills and the Provision of Local Public Goods," Journal of Public Economics, 6 (4): 409-423.

Brueckner, J. (1989), "Tastes, Skills, and Local Public Goods," BEBR Working Paper no. 89-1610. (October).

Samuelson, P. (1954), "The Pure Theory of Public Expenditure," Review of Economics and Statistics, 36 (4): 387-389.

Stiglitz, J. (1977), "The Theory of Local Public Goods," in The Economics of Public Services, M.S. Feldstein and R.P. Inman (eds), Macmillan, London.

Appendix 1

The Kuhn-Tucker Conditions

Maximizing the social welfare function implies maximizing the following Lagrangian

$$L = \alpha N_1 U_1 + \beta M_1 V_1 + \alpha N_2 U_2 + \beta M_2 V_2$$
$$+ \lambda(X_1 + X_2 - N_i x_{iu} - M_1 x_{iv} - N_2 x_{2u} - M_2 x_{2v})$$
$$+ \delta_1 (F(H_1, L_1) - S_i - X_i) \quad + \delta_2 (F(H_2, L_2) - S_2 - X_2)$$
$$+ \Delta(N_1 + N_2 - M_1 - M_2).$$

The resulting first-order conditions are

$$(A.1) \quad \frac{\partial L}{\partial x_{iu}} : \alpha \frac{\partial U_i}{\partial x_{iu}} = \lambda \qquad i=1,2$$

$$(A.2) \quad \frac{\partial L}{\partial x_{iv}} : \beta \frac{\partial V_i}{\partial x_{iu}} = \lambda \qquad i=1,2$$

$$(A.3) \quad \frac{\partial L}{\partial S_i} : \alpha N_i \frac{\partial U_i}{\partial S_i} = \beta M_i \frac{\partial V_i}{\partial S_i} = \delta_i \qquad i=1,2$$

$$(A.4) \quad \frac{\partial L}{\partial X_i} : \delta_i = \lambda \qquad i=1,2$$

$$(A.5) \quad \frac{\partial L}{\partial N_i} : \alpha U_i - \lambda x_{iu} + \delta_i \frac{\partial F_i}{\partial N_i} + \Delta \geq 0$$

$$\alpha N_i U_i - \lambda N_i x_{iu} + \delta_i \frac{\partial F_i}{\partial N_i} N_i + N_i \Delta = 0 \qquad i=1,2$$

$$(A.6) \quad \frac{\partial L}{\partial M_i} : \beta V_i - \lambda x_{iv} + \delta_i \frac{\partial F_i}{\partial M_i} - \Delta \geq 0 \qquad i=1,2$$

$$\beta M_i V_i - \lambda M_i x_{iv} + \delta_i \frac{\partial F_i}{\partial M_i} M_i - \Delta M_i = 0$$

Substituting equations A.1 and A.2 into equation A.3 yields the expected Samuelson conditions

$$(2.1) \quad N_i \frac{\partial U_i/\partial S_i}{\partial U_i/\partial \chi_{iu}} + M_i \frac{\partial V_i/\partial S_i}{\partial V_i/\partial \chi_{iv}} = 1 \qquad i=1,2.$$

Adding equations A.5 and A.6 for both jurisdictions (evaluated as equalities) yields equation A.7

$$(A.7) \quad [\alpha N_i U_i + \beta M_i V_i + \alpha N_2 U_2 + \beta M_2 V_2]$$

$$- \lambda [N_i x_{iu} + M_i x_{iv} + N_2 x_{2u} + M_2 x2_v]$$

$$+\lambda \quad \frac{\partial F_1}{\partial N_1} N_1 + \frac{\partial F_1}{\partial M_1} M_1 + \frac{\partial F_2}{\partial N_2} N_2 + \frac{\partial F_2}{\partial M_2} M_2 \qquad = 0$$

The first term in brackets respresents total utility while the second term in brackets represents total consumption of the private good between the two jurisdictions. Therefore, substituting marginal productivity wages and rents into equation A.7 yields equation 3.1a

$$(3.1a) \quad TU = \alpha \lambda_U [X_1 + X_2 - w_1 H_1 + r_1 T_1 - w_2 H_2 + r_2 T_2].$$

Appendix 2

The Stability of Homogeneous Allocations Across Time: An Example Using Log-Linear Utility

To illustrate the time-instability of homogeneous clubs, assume that the utility functions are log-linear such that $U_i = \phi \ln(x_{iU}) + \theta \ln(S_i)$ and $V_i = \hat{\phi} \ln(x_{iV}) + \hat{\theta} \ln(S_i)$. Under this utility specification, $S_1 = \alpha\theta F_T$ and $S_2 = \beta\hat{\theta} F_T$ where $F_T = F_1 + F_2$ is the total transformed output of the two homogeneous jurisdictions. Therefore, $\dot{h}_u(t) = h_u \delta \alpha \theta F_T$ and $\dot{h}_v(t) = h_v \delta \beta \hat{\theta} F_T$.

Stability requires that no type expect greater utility in the other jurisdiction than in the assigned jurisdiction. In other words, stability conditions 9.1 and 9.2 must both hold.

$$(9.1) \qquad \Sigma_u = \phi[\ln(\alpha\phi F_T) - \ln(\beta\hat{\phi} F_T + Nw_2(h_u - h_v))] + \theta \ln(\alpha\theta/\beta\hat{\theta}) \geq 0.$$

$$(9.2) \qquad \Sigma_v = \hat{\phi}[\ln(\beta\hat{\phi} F_T) - \ln(\alpha\phi F_T - Nw_1(h_u - h_v))] - \hat{\theta} \ln(\alpha\theta/\beta\hat{\theta}) \geq 0.$$

Differentiating stability conditions 9.1 and 9.2 with respect to time yields:

$$\frac{\partial \Sigma_u}{\partial h_u} = \frac{\phi\, N^2 w_2\, [w_1(h_u - h_v)(\xi_F - 1) - h_v(w_1 + w_2)]}{[\beta\hat{\phi} F_T + Nw_2(h_u - h_v)] F_T \xi_F}$$

$$\frac{\partial \Sigma_v}{\partial h_u} = \frac{\hat{\phi}\, N^2 w_1 h_v [w_2(h_u - h_v)(\xi_F - 1) + h_u(w_1 + w_2)]}{[\alpha\phi F_T + Nw_2(h_u - h_v)] h_u F_T \xi_F}$$

$$\frac{\partial \Sigma_v}{\partial h_v} = \frac{-\hat{\phi}\, N^2 w_1 [w_2(h_u - h_v)(\xi_F - 1) + h_u(w_1 + w_2)]}{[\alpha\phi F_T + Nw_2(h_u - h_v)] F_T \xi_F}$$

$$\frac{\partial \Sigma_u}{\partial h_v} = \frac{\phi\, N^2 w_2 h_u [h_v(w_1 + w_2) - w_1(h_u - h_v)(\xi_F - 1)]}{[\beta\hat{\phi} F_T + Nw_2(h_u - h_v)] h_v F_T \xi_F}$$

$$\frac{\partial \Sigma_u}{\partial t} = C_u \quad [w_1(h_u - h_v)(\xi_F - 1) - h_v(w_1 + w_2)] h_u \delta [\alpha\theta - \beta\hat{\theta}]$$

$$\frac{\partial \Sigma_v}{\partial t} = C_v \quad [w_2(h_u - h_v)(\xi_F - 1) + h_u(w_1 + w_2)] h_v \delta [\alpha\theta - \beta\hat{\theta}]$$

If $S_1 > S_2$ $(S_2 > S_1)$ then eventually, $h_u > h_v$ $(h_v > h_u)$ and the human capital stocks diverge over time. If $S_1 > S_2$ $(\alpha\theta - \beta\hat{\theta})$ and $h_u > h_v$ then stability condition 9.1 decreases over time and (assuming that ϕ and $\hat{\phi}$ are not zero) the allocation becomes unstable. If $S_2 > S_1$ and $h_v > h_u$ then stability condition 9.2 decreases over time and again the allocation becomes unstable.