No. 9116

PUBLIC DEBTS AND DEFICITS IN MEXICO: A COMMENT

by

John H. Welch*

September 1991

Research Paper

Federal Reserve Bank of Dallas

No. 9116

PUBLIC DEBTS AND DEFICITS IN MEXICO:
A COMMENT

by

John H. Welch*

September 1991

*Economist, Research Department, Federal Reserve Bank of Dallas. The views expressed in this paper are those of the author and do not reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

The question of government solvency has received a lot of attention in recent years especially with the large U.S. fiscal deficits and the fiscal distress suffered by Latin American governments since the onset of the "debt crisis" in the beginning of the 1980s. Feliz and Torres (1991) test whether the dynamic government budget constraint of Mexico is binding for the period 1981 to 1988. They find that intertemporal budget balance was violated over this period. The methodology is based upon the stochastic characteristics of the components of the budget constraint. Briefly, if we assume interest rate parity and that, on average, the exchange rate follows the domestic rate of inflation, the dynamic government budget constraint is[1]

$$\frac{D_{t+1}}{P_{t+1}} = \frac{(G_t - T_t)}{P_{t+1}} + \frac{(1 + \rho_t)(1 + \pi_t^e)}{(1 + \pi_t^e)} \frac{D_t}{P_t} - \frac{\Delta M_{t+1}}{P_{t+1}} \quad (1)$$

where D_t is (domestic plus foreign) government debt, M_t is monetary base, G_t is the totality of government spending, T_t is the totality of government non-borrowed revenues, π_t^e is the expected rate of inflation, and ρ_t is the average real rate of interest on government debt.

Let the real level of debt be d_t, the real primary government deficit be δ_t, and the real value of seignorage be $\sigma_{t+1} = \Delta M_{t+1}/P_{t+1}$. Equation (1) can be rewritten as

[1]The model follows the discussion of Welch, Primo Braga, and André (1987) and Trehan and Walsh (1988).

$$d_{t+1} = \delta_t + (1 + \rho_t)d_t - \sigma_{t+1} \tag{2}$$

Suppose the time series vector X_t = [δ_t, d_t, σ_{t+1}] is first difference stationary. By the Wold decomposition theorem, X_t can be represented

$$(1 - L)X_t = \lambda + C(L)\nu_t \tag{3}$$

where C(L) is a 3 x 3 matrix in the lag operator, λ is a drift term, and ν_t is a vector white noise process with ν_t = [$\nu_{1,t}$, $\nu_{2,t}$, $\nu_{3,t}$]. We can form the inclusive of debt interest government deficit by multiplying X_t by the cointegrating vector ß' = [1, ρ, -1]. This yields the following expression

$$(1 - L)\beta' X_t = \beta'\lambda + \beta' C(L)\nu_t \tag{4}$$

One can use equation (4) to rationally forecast the value of future government debt. Substituting equation (4) into equation (2) and iterating forward, one finds the solution to the value of d_t. As Trehan and Walsh (1991) show, equation (4) implies that if intertemporal budgets are satisfied (no bubbles), real government debt will follow the following process[2]

---

[2]This particular solution assumes a constant real interest rate. The test below are based upon an extension by Trehan and Walsh (1991) to variable real interest rates.

$$(1 - L)d_{t+1} = \delta_t + \rho_t d_t - \sigma_{t+1} = \frac{\beta'\lambda}{\rho} + D(L)\nu_t \qquad (5)$$

where $D(L)\nu_t$ is stationary. Equation (5) implies that for dynamic budget balance to obtain, the primary deficit, the stock of internal debt, the stock of foreign debt, and seignorage are cointegrated with cointegrating vector $\beta' = [1, \rho_t, -1]$. Feliz and Torres (1991) choose to test for government insolvency by testing whether such cointegration exists. Unfortunately, this approach suffers from two severe limitations. The first is that one must assume that the real interest rate on government debt is constant. Secondly, the test necessitates the measurement of real seignorage which is problematic at best. I will discuss this issue further below.

A better approach is to perform an equivalent test which looks at the stationarity of the first difference of the real government debt which, from equation (5) is equal to the cointegration relationship between the primary deficit, the level of real debt, and real seignorage or, in other words, the deficit inclusive of interest payments.[3] If the first difference of real debt is stationary, budget balance holds. Table 1 shows the results of augmented Dickey-Fuller and Phillips-Perron tests of stationarity using Feliz and Torres' data on real government debt in Mexico. All tests significantly reject the null hypothesis of non-stationary first differences of real debt. Hence, the evidence strongly indicates that

---

[3]Trehan and Walsh (1991) extend their results of (1988) to the case where the real interest rate on government debt is not constant, as in this case. Their results show that if ρ_t is a stochastic process bound strictly below by $\lambda > 0$ in expected value and $(1-L)d_t$ is a stationary process, then intertemporal budget balance is satisfied. Real interest rates on internal government debt were positive in both Argentina and Brazil over the period.

Table 1
Mexico: Tests of a Unit Root and Time Trend 1986:3-1990:2
Real Internal Government Debt

a. Null Hypothesis: Variable has a Unit Root (with time trend)

Variable	Phillips-Perron Test T-ratio	Augmented Dickey-Fuller Test[a] T-ratio
Δreal government debt[b]	-14.03***	-13.73***

b: Null Hypothesis: Variable has Unit Root (with no time trend)

Variable	Phillips-Perron Test T-ratio	Augmented Dickey-Fuller Test[a] T-ratio
Δreal government debt[b]	-13.73***	-13.73***

Notes:

(a) One lag was used in these tests of stationarity. The lag structure was chosen by adding lags until the Q(22) statistic did not reject the null hypothesis of autocorrelated residuals.

(b) Variable significantly violates normality assumption either because of skewness or kurtosis using the tests developed in Jarque and Bera (1980).

* signifies significance at the $\alpha=0.10$ level, ** signifies significance at the $\alpha=0.05$ level, and *** signifies significance at the $\alpha=0.01$ level.

intertemporal budget balance was maintained over this period.

What accounts for the discrepancy of between these findings and those of Feliz and Torres (1991)? As discussed in Welch, Primo Braga, and André (1987) and Cukierman (1988), simple discrete measurement of seignorage generates a biased measure of the real resource flow from money creation as money growth is more or less a continuous process. To see this, consider the continuous time amount of seignorage collected at time t

$$\hat{\sigma}_t = \frac{\dot{M}_t}{P_t} \tag{6}$$

where variables are defined as above and the dot represents an instantaneous time derivative. Note that

$$M_t = M_0 e^{\mu t} \tag{7}$$

where μ now represents the instantaneous rate of nominal money growth.[4] Hence

$$\dot{M}_t = \mu M_0 e^{\mu t} \tag{8}$$

Equation 5 now becomes

[4]The instantaneous rate of growth of money, μ, can be approximated by $\ln(1+\mu^*)$ where μ^* is the discrete time rate of growth. Note that for the period of time selected, money growth is assumed constant.

$$\hat{\sigma}_t = \mu \frac{M_0}{P_0} e^{(\mu-\pi)t} \tag{9}$$

where π is the instantaneous rate of inflation.[5]

Integrating equation (5) from t to t+1 yields

$$\begin{aligned} \sigma^*_{t+1} &= \int_t^{t+1} \hat{\sigma}_t dt = \frac{\mu}{(\mu - \pi)} \frac{M_0}{P_0} e^{(\mu-\pi)t} \left[e^{(\mu-\pi)} - 1 \right] \\ &= \frac{M_t}{P_t} \left[\frac{\mu}{\mu - \pi} \left(e^{\mu-\pi} - 1 \right) \right] \end{aligned} \tag{10}$$

On the other hand, discrete time measurement of seignorage gives

$$\begin{aligned} \sigma_{t+1} &= \frac{M_t}{P_t} \left[\frac{M_{t+1}}{M_t} - 1 \right] = \frac{M_t}{P_t} \left[\frac{M_o e^{\mu(t+1)}}{M_o e^{\mu t}} - 1 \right] \\ &= \frac{M_t}{P_t} \left[e^{\mu} - 1 \right] \end{aligned} \tag{11}$$

Subtracting equations (9) from (10) yields an expression for the bias inherent in Feliz and Torres' measure of seigniorage

$$\sigma_{t+1} - \sigma^*_{t+1} = \frac{M_t}{P_t} \left[\frac{\mu}{\mu - \pi} \left(e^{\mu-\pi} - 1 \right) - e^{\mu} + 1 \right] \tag{12}$$

---

[5]The instantaneous inflation rate can be approximated in a similar fashion as described in footnote 3.

This non-linear bias becomes larger the larger the money growth rate and the larger the divergence of inflation from the money growth rate. A Taylor series expansion also shows that the bias can reverse sign depending upon the relative difference between inflation and money growth.

In summary, the conclusion of this comment is that somehow seignorage (and the other parts of the interest inclusive deficit) behaves to keep dynamic budget balance. This conclusion also holds for Argentina and Brazil, countries which have an historical inflation rate much higher than Mexico's and consequently have a smaller monetary base [Welch 1991]. The major problem in Latin American countries in a long term framework, however, remains to shift from monetary financed deficits to tax financed deficits. Fortunately, Mexico seems to have crossed this hurdle.

I would like to thank Shengyi Guo for careful research assistance and Raúl A. Feliz for his comments and for providing me with his data. Of course, any remaining errors and omissions are the responsibility of the author. The views expressed in this article are those of the authors and should not be attributed to the Federal Reserve Bank of Dallas or the Federal Reserve System.

References

Cukierman, A., 1988, Rapid inflation - deliberate policy or miscalculation?, Carnegie-Rochester Conference Series on Public Policy 29, 11-75.

Dickey, D. A. and W.A. Fuller, 1979, Distribution of the estimators of autoregressive time series with a unit root, Journal of the American Statistical Association 74:66, 427-431.

Engle, R. F. and C.W.J. Granger, 1987, Co-integration and error correction: representation, estimation, and testing," Econometrica 55, 251-276.

Feliz, R.A. and F.A. Torres, 1991, Deuda y déficit público en Mexico, Estudios Economicos, 6:1.

Fuller, W.A. (1976): Introduction to Statistical Time Series. (John Wiley and Sons, New York).

Granger, C.W.J. and P. Newbold, 1986, Forecasting Economic Time Series (Academic Press, New York).

Hakkio, C., and M. Rush, 1991, Is the budget deficit 'too large'? Economic Inquiry 29, 429-445.

Jarque, C.M. and A.K. Bera, 1980, Efficient tests for normality, homoscedasticity, and serial dependence of regression residuals, Economics Letters 6, 255-259.

Phillips, P.C.B. and P. Perron, 1988, Testing for a unit root in time series regression, Biometrica 75:2, 335-346.

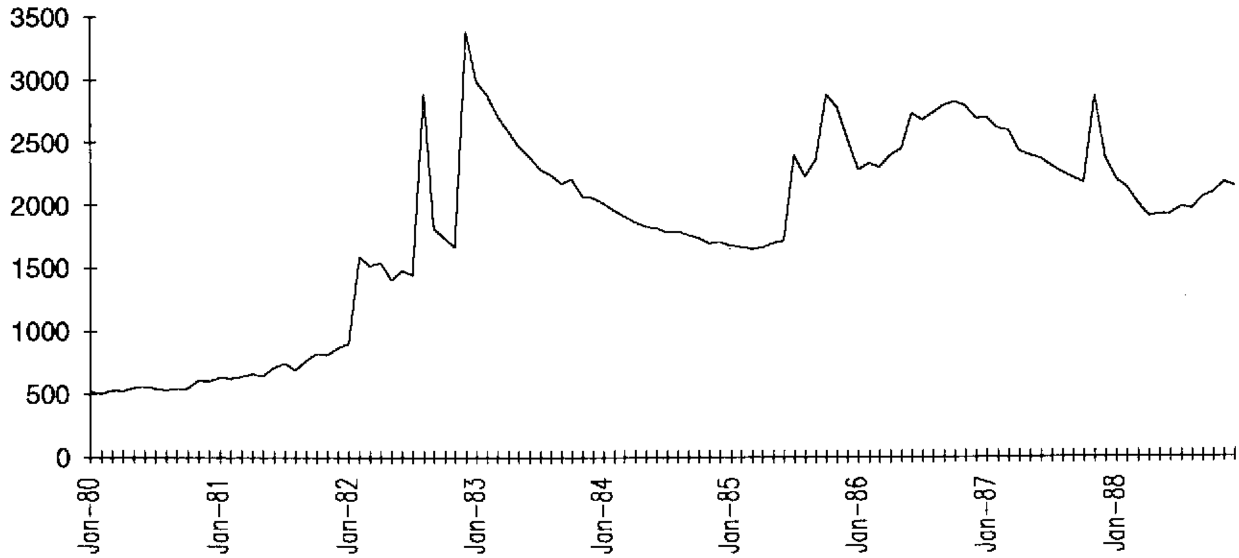
Trehan, B. and C.E. Walsh, 1988, Common trends, the government's budget constraint, and revenue smoothing, Journal of Economic Dynamics and Control 12, 425-444.

-----, 1991, Testing intertemporal budget constraints: theory and applications to U.S. budget and current account deficits, Journal of Money, Credit, and Banking 23, 206-223.

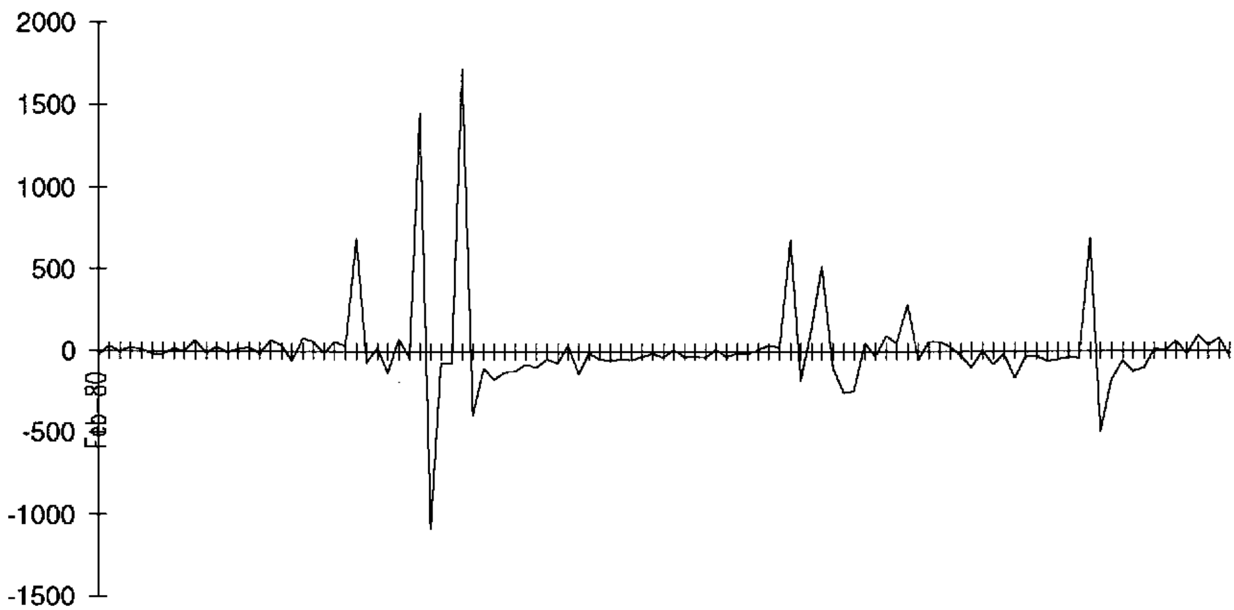
Welch, J.H., 1991, Rational inflation and real internal debt bubbles in Argentina and Brazil?, working paper no. 9113, Federal Reserve Bank of Dallas, August.

Welch, J.H., C.A. Primo Braga, and P.T.A. André, 1987, Brazilian public sector disequilibrium, World Development 15:8, 1045-1052.

Figure 1
Mexico: Real Public Debt 1980 to 1988
(in Billions of 1980 Pesos)



Mexico: First Differences of Real Public Debt 1980 to 1988



RESEARCH PAPERS OF THE RESEARCH DEPARTMENT
FEDERAL RESERVE BANK OF DALLAS

Available, at no charge, from the Research Department
Federal Reserve Bank of Dallas, Station K
Dallas, Texas 75222

9001 Another Look at the Credit-Output Link (Cara S. Lown and Donald W. Hayes)

9002 Demographics and the Foreign Indebtedness of the United States (John K. Hill)

9003 Inflation, Real Interest Rates, and the Fisher Equation Since 1983 (Kenneth M. Emery)

9004 Banking Reform (Gerald P. O'Driscoll, Jr.)

9005 U.S. Oil Demand and Conservation (S.P.A. Brown and Keith R. Phillips)

9006 Are Net Discount Ratios Stationary?: The Implications for Present Value Calculations (Joseph H. Haslag, Michael Nieswiadomy and D.J. Slottje)

9007 The Aggregate Effects of Temporary Government Purchases (Mark A. Wynne)

9008 Lender of Last Resort: A Contemporary Perspective (George G. Kaufman)

9009 Does It Matter How Monetary Policy is Implemented? (Joseph H. Haslag and Scott E. Hein)

9010 The Impact of Differential Human Capital Stocks on Club Allocations (Lori L. Taylor)

9011 Is Increased Price Flexibility Stabilizing? The Role of the Permanent Income Hypothesis (Evan F. Koenig)

9012 Fisher Effects and Central Bank Independence (Kenneth M. Emery)

9013 Methanol as an Alternative Fuel (Mine K. Yucel)

9101 Large Shocks, Small Shocks, and Economic Fluctuations: Outliers in Macroeconomic Time Series (Nathan S. Balke and Thomas B. Fomby)

9102 Immigrant Links to the Home Country: Empirical Implications for U.S. and Canadian Bilateral Trade Flows (David M. Gould)

9103 Government Purchases and Real Wages (Mark Wynne)

9104 Evaluating Monetary Base Targeting Rules (R.W. Hafer, Joseph H. Haslag and Scott E. Hein)

9105 Variations in Texas School Quality (Lori L. Taylor and Beverly J. Fox)

9106 What Motivates Oil Producers?: Testing Alternative Hypotheses (Carol Dahl and Mine Yucel)

9107 Hyperinflation, and Internal Debt Repudiation in Argentina and Brazil: From Expectations Management to the "Bonex" and "Collor" Plans (John H. Welch)

9108 Learning From One Another: The U.S. and European Banking Experience (Robert T. Clair and Gerald P. O'Driscoll)

9109 Detecting Level Shifts in Time Series: Misspecification and a Proposed Solution (Nathan S. Balke)

9110 Underdevelopment and the Enforcement of Laws and Contracts (Scott Freeman)

9111 An Econometric Analysis of Borrowing Constraints and Household Debt (John V. Duca and Stuart S. Rosenthal)

9112 Credit Cards and Money Demand: A Cross-Sectional Study (John V. Duca and William C. Whitesell)

9113 Rational Inflation and Real Internal Debt Bubbles in Argentina and Brazil? (John H. Welch)

9114 The Optimality of Nominal Contracts (Scott Freeman and Guido Tabellini)

9115 North American Free Trade and the Peso: The Case for a North American Currency Area (Darryl McLeod and John H. Welch)

9116 Public Debts and Deficits in Mexico: A Comment (John H. Welch)

Public Debts and Deficits in Mexico: A Comment

Revised November 1991

John H. Welch
Federal Reserve Bank of Dallas