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THE ALGEBRA OF PRICE STABILITY

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# Research Paper

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Federal Reserve Bank of Dallas

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## I. Introduction

Legislation recently introduced by Congressman Stephen Neal mandates that price stability receive the highest priority of the Federal Reserve.<sup>1</sup> Because of the renewed policy interest in price stability, the potential costs and benefits of achieving it have attracted much attention.<sup>2</sup> Among the envisioned benefits of price stability are the reduction of inflation-induced distortions that result because of the nonindexation of the tax system and the reduction of transaction costs (or shoe-leather costs) of changing interest-bearing assets to cash and vice versa. One of the more important benefits attributed to price stability is that it lessens the uncertainty associated with the long-run price level and the detrimental effects this uncertainty can have on long-term contracting and resource allocation (Hall 1981; Black 1990; Parry 1990; Hoskins 1991; Summers 1991). Proponents of price stability argue that because money is an intertemporal store of value, an uncertain price level causes people to devote resources to protecting themselves against potential declines in the value of money. Therefore, eliminating price level uncertainty allows a more efficient allocation of resources. The costs of achieving price stability are primarily associated with the short-term adjustment costs of moving from the current inflationary regime to the price stability regime--such as the lost output resulting from inflation being less than anticipated.

While the subject of price stability has received much attention, there has been relatively little discussion about the specific monetary policies that would enable the Federal Reserve to achieve and maintain price stability.

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<sup>1</sup> House of Representatives Joint Resolution 409

<sup>2</sup> See Aiyagari (1990, 1991) and Hoskins (1991).

Rather than add to the debate about the costs and benefits of price stability, this paper examines the conditions under which monetary policy would ensure price stability and examines a broad class of monetary rules to determine whether these policies would enable the Federal Reserve to achieve price stability.

Taking the operational definition of price stability to be stability of prices around a particular price level, we show that achieving price stability implies strong, long-run restrictions on the behavior of monetary aggregates. We outline these restrictions and demonstrate conditions under which specific monetary rules are consistent with price stability. The monetary rules examined include monetary aggregate targeting, nominal GNP targeting, price level targeting, and interest rate targeting. We show that price level targeting, and to a lesser extent, nominal GNP targeting are more likely to be consistent with price stability than is targeting monetary aggregates or targeting interest rates. Finally, while we make no judgments about the relative merits of the various alternative monetary policy rules, we do suggest how one might incorporate the goal of price stability into a formal analysis of optimal monetary policy.

## II. Defining Price Stability

Before we examine the implications of price stability for monetary policy, it is necessary to clear up some ambiguity about the precise definition of "price stability." There seem to be two alternative notions of price stability floating around in the literature. The first notion is that price stability means zero inflation--that is, the monetary authority should

strive for zero average inflation.<sup>3</sup> The second notion is that price stability requires the long-run level of prices to be stable around a particular level.

We argue that the second notion of price stability better captures the long-term benefits envisioned by the proponents of price stability--namely, reducing long-term uncertainty about the price level. The reason for this is that stabilizing prices around a particular level versus just maintaining a zero average inflation rate can imply substantially different levels of long-run uncertainty. While zero average inflation is a necessary condition for achieving the reduction of long-term uncertainty about the price level, it is not sufficient.

To illustrate this point, suppose the monetary authority can control the money supply so that the following price level behaviors are feasible: the simple price level rule

$$(1) \quad p_t = p^* + \epsilon_{pt}$$

and the zero, "on average," inflation rule

$$(2) \quad \pi_t = \epsilon_{\pi t}.$$

For simplicity, let  $\epsilon_{pt}$  and  $\epsilon_{\pi t}$  be white-noise errors with mean zero and variance  $\sigma_p^2$  and  $\sigma_\pi^2$  respectively. Note that under both rules, the average inflation rate or unconditional expectation of inflation,  $E(\pi_t)$ , is zero. However, the conditional expectation of inflation for the price level rule,  $E(\pi_{t+1}|I_t)$ , is  $-\epsilon_{pt-1}$ .

The short-term uncertainty inherent for these two price rules, as measured by  $\text{var}[p_{t+1} - E(p_{t+1}|I_t)]$  is given by  $\sigma_p^2$  and  $\sigma_\pi^2$ . This short-term

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<sup>3</sup> The debate between Aiyagari and Hoskins seems to be set in the context of a zero inflation target.

uncertainty depends on the structure of the economy, the information available to the monetary authority, and the sources of shocks to the economy. In general, there are no clear predictions about which type of rule would generate the most short-term uncertainty.<sup>4</sup> However, the inflation rule is almost certain to generate more long-term uncertainty. Let  $E(p_{t+k}|I_t)$  be the optimal forecast of  $p_{t+k}$ , given the information set at time  $t$ . For the inflation rule,

$$(3) \quad \text{var}[p_{t+k} - E(p_{t+k}|I_t)] = \sigma_\pi^2 k,$$

while for the price level rule

$$(4) \quad \text{var}[p_{t+k} - E(p_{t+k}|I_t)] = \sigma_p^2.$$

In contrast to the constant variance of the price level rule, the uncertainty associated with long-run forecasts of the price level under the zero inflation rate rule grows linearly with the forecast horizon.<sup>5</sup> Thus, the price level rule and the zero inflation rate rule imply very different levels of long-run uncertainty.

The difference in the long-term uncertainty inherent in the two notions of price stability arises because unexpected changes in the price level under a zero average inflation rule policy are permanent. After a shock has

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<sup>4</sup> For example, suppose the economy is characterized by the following equations:

$$y_t = m_t - p_t + v_t$$

and

$$y_t = [p_t - E(p_t|I_{t-1})] + z_t.$$

If the monetary authority's (and the public's) information set includes knowledge of economic variables dated  $t-1$ , then the price level target and the zero inflation target would result in the same short-term variability,  $\epsilon_{pt} = \epsilon_{\pi t}$ .

<sup>5</sup> The assumption of white-noise errors is not crucial. We could assume that  $\epsilon_{pt} = a_p(L)e_{pt}$  and  $\epsilon_{\pi t} = a_\pi(L)e_{\pi t}$ , where  $\epsilon_{pt}$  and  $\epsilon_{\pi t}$  are stationary stochastic processes. The comparison of long-run volatility is not substantially changed by this more general assumption about the  $\epsilon$ 's.

occurred, the monetary authorities are only interested in achieving zero inflation from that point in time; they do not offset the price shock. Under a constant price level policy, however, unexpected price level shocks must be reversed. Therefore, there is substantially less long-term uncertainty about the price level under the price level rule than under the inflation rate rule.

### III. Price Stability and Monetary Policy

In this section, we examine the restrictions that the goal of price stability places on the conduct of monetary policy. Throughout, we focus on whether particular policies are consistent with price stability and not on the desirability, or "optimality", of these particular policies.

Because "price stability" requires that the long-run level of prices fluctuate around a particular level, it follows that the price level will be a stationary stochastic process. The requirement that prices be stationary--or, in other words, that prices follow an integrated process of order zero,  $I(0)$ , imposes restrictions on the long-run conduct of monetary policy. Consider the simple quantity theory relationship (in logarithms)

$$(5) \quad p_t = m_t + v_t - q_t,$$

where  $m_t$  is a particular monetary aggregate,  $v_t$  is the associated velocity aggregate, and  $q_t$  is real GNP in logarithms. Price stability requires  $m_t + v_t - q_t$  be  $I(0)$ . Thus, monetary policy must be conducted in such a way that  $m_t + v_t - q_t$  is  $I(0)$ .<sup>6</sup>

For example, when  $v_t$  and  $q_t$  are integrated of order 1,  $I(1)$ , then to achieve price stability, monetary policy must follow a feedback rule in

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<sup>6</sup> The basic insights of this section hold for more general models of the macroeconomy. To achieve price stability, monetary policy must offset all sources of nonstationarity that otherwise affect the price level.

which the money supply, velocity, and real GNP are cointegrated. An example of a feedback that satisfies this restriction is

$$(6) \quad m_t = -v_{t-1} + q_{t-1} + [I(0)]$$

where  $[I(0)]$  contains other terms that are  $I(0)$ .

On the face of it, this restriction does not appear to be very powerful; yet it can rule out entire classes of monetary policies. If  $v_t$  or  $q_t$  is not  $I(0)$ , then the money supply must be chosen so that it will offset the nonstationarity of velocity and output; in other words, the actual money supply must follow some sort of feedback rule. That is, in the long run, monetary policy must offset the effects of permanent shocks to velocity and real GNP.

Is it possible that  $m_t + v_t - q_t$  is  $I(0)$  regardless of the behavior of the money supply? In addition to not being the case empirically, this possibility is highly unlikely at a theoretical level because it requires velocity, output, or both to offset the behavior of the money supply. Long-run money neutrality of output rules out this type of behavior.

#### IV. Evaluating Various Monetary Rules: Money Supply Targets

Given that price stability requires  $m_t + v_t - q_t$  to be  $I(0)$ , we examine some well-known monetary rules to determine whether they meet this necessary and sufficient condition for price stability. We first consider the implications of price stability for money supply targets. Of the money supply targets, we consider two types of "k percent" growth rules--one in which control errors are offset and one in which control errors are not offset. We also consider the effects of target cones on the feasibility of price stability. Finally, we consider the debate about the appropriate



choice of monetary aggregate to target.

#### A. On-average k percent growth rule

Under k percent growth rules, the monetary authority attempts to control money growth so that it increases at a k percent annual rate in each time period, implying  $E(\Delta m_t | I_{t-1}) = k$ . Note that if control errors are not offset, actual money supply growth is given by

$$(7) \quad m_t - m_{t-1} = k + \psi_t,$$

where  $\psi_t$  is a control error (which is presumably a stationary stochastic process--typically assumed to be white noise). The money supply, then, is an I(1) process independent of the behavior of  $v_t$  and  $q_t$ .

This type of monetary rule is very unlikely to meet the necessary condition for price stability because  $m_t - v_t + q_t$  is unlikely to be I(0) regardless of the behavior of  $v_t$  and  $q_t$ .<sup>7</sup> Thus, a k percent rule with base drift that does not offset past control errors is incapable of achieving price stability. This critique of base drift has been mentioned by several authors including Poole (1970).

#### B. k percent growth rule that offsets control errors

Unlike the base drift case above, it is possible to construct monetary growth rules in which over long time horizons, money grows at k percent but control errors are offset. An example of a k percent rule that offsets control errors is given by

$$(8) \quad m_t - m_{t-1} = k + \lambda [m_0 + k(t-1) - m_{t-1}] + \psi_t,$$

where  $m_0$  is the level of money supply at the time the rule was implemented. The term  $\lambda$  describes how quickly control errors are offset. The money

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<sup>7</sup> Only in the very unlikely case in which  $v_t$  and  $q_t$  automatically offset any permanent change in the money supply will prices be stationary.

growth rule described by equation (8) results in a money supply that is stationary around a deterministic trend. Still, this rule only achieves price stability if velocity and output are stationary. In general, this rule does not ensure that  $m_t + v_t - q_t$  will be  $I(0)$ .

### C. Target cones

In principle, target bands for money supply can force the monetary authority to offset control errors; the money supply is contracted if it exceeds the band and is expanded if it is below the band. However, the current method of specifying target cones (in addition to allowing the target to drift) is unlikely to yield price stability. The reason is that target cones are not a particularly binding constraint on the money supply in the long run.

For example, consider the case in which the money supply is given by a random walk with drift--that is, an on-average  $k$  percent rule. The expected value of the future money supply and its one standard deviation confidence band are given by

$$(9a) \ E(m_t | I_0) \pm SD = m_0 + kt \pm \sigma_\psi \sqrt{t},$$

where  $m_0$  is the initial money supply level and  $\sigma_\psi$  is the standard deviation of the control error.

Compare this with a target cone of the form

$$(9b) \ m_0 + kt \pm \delta t,$$

where  $\delta t$  is the width of the cone. Regardless of the size of the cone ( $\delta$ ) and the variance of control error ( $\sigma_\psi^2$ ), eventually the target cone will envelop the confidence band for  $m_t$ . This characteristic implies that a target cone is not a particularly binding constraint in the long run when the money supply follows a random walk with drift. Indeed, a target cone

that is never readjusted is insufficient to yield price stability even if velocity and output are themselves  $I(0)$ . This result suggests that the current monetary aggregate targeting procedure, which combines a target cone and base drift, is unlikely to produce price stability.

#### D. Choosing the appropriate monetary aggregate

The above analysis implies that a monetary aggregate target can only achieve price stability if it corrects for past control errors, if real GNP is stationary, and if the aggregate's velocity is stationary. This implication raises the issue of which monetary aggregate to target if such a rule is to achieve price stability. Currently, many economists advocate targeting either M2 or the monetary base (MB). Empirically, however, the M2 aggregate's velocity has been stationary over the post-World War II period, while base velocity has not. This finding suggests using an M2 rule if price stability is the ultimate goal.<sup>8</sup>

It should be noted, however, that a stationary  $k$  percent rule for M2 implies a feedback rule for MB that also satisfies conditions for price stability. Assume that M2 velocity is  $I(0)$ , while MB velocity is  $I(1)$ . Because M2 is related to the MB through the money multiplier, M2 (in logarithms) equals

$$(10) \quad m2_t = mb_t + mm_t,$$

where  $mb_t$  is the logarithm of the monetary base and  $mm_t$  is the logarithm of the money multiplier. Using the quantity relationship for M2 and equation (10), we can write a quantity relationship for the monetary base in which base velocity equals

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<sup>8</sup> Still, it should be noted that M2 velocity may not always remain a stationary time series. Indeed, the stationarity of M2 velocity appears to be a post-World War II phenomenon.

$$(11) \quad vb_t = vm2_t + mm_t.$$

Because  $vb_t$  is  $I(1)$  and  $vm2_t$  is  $I(0)$ ,  $mm_t$  must be  $I(1)$ : the money multiplier is nonstationary. If M2 follows some sort of  $k$  percent rule, then the monetary base must be given by a feedback rule that adjusts for changes in velocity. Indeed, the monetary base and its velocity would be cointegrated, which satisfies the conditions for price stability if  $q_t$  is also stationary.

#### V. Evaluating Various Monetary Rules: Alternative Feedback Rules

Given that pure money supply targeting meets the requirements of price stability only if velocity and real GNP are stationary and if the money supply itself is stationary, we would like to evaluate whether other classes of monetary rules are more likely to ensure price stability. In this section, we examine several feedback rules for monetary policy--including nominal GNP targeting, price level targeting, and interest rate targeting--and determine under what conditions these rules ensure price stability.

##### A. Simple feedback rules

For the case in which velocity and real GNP are nonstationary and are integrated of order 1,  $I(1)$ , we showed in Section III that a feedback rule like equation (6) would be necessary to generate price stability. In fact, several well-known feedback rules have the same basic form as equation (6). Walsh (1986) examines a model with an aggregate supply curve and a quantity theory aggregate demand curve, as well as a nominal money supply equation. Walsh's optimal money supply equation can be rewritten so that it has the

same form as equation (6).

Meltzer (1984) also proposes a feedback rule that is similar to that in equation (6). Meltzer's rule is given by

$$(12) \Delta m_t = - (v_{t-1} - v_{t-1-k})/k + (q_{t-1} - q_{t-1-k})/k,$$

where  $k$  is a prespecified and constant lag length. The term  $\Delta$  is the difference operator, or  $\Delta = 1 - L$  where  $L$  is a lag operator with  $Lz_t = z_{t-1}$ .

Meltzer's rule can be rewritten as

$$(12a) \quad m_t = m_0 - \sum_{i=1}^k v_{t-i}/k + \sum_{i=1}^k q_{t-i}/k,$$

where  $m_0$  is the initial money supply and is treated as a constant.

Equation (12a) essentially has the same form as equation (6), except Meltzer's rule uses a moving average of past velocity and real GNP values. Note that Meltzer's rule and the simple feedback rule given by equation (6) ensure price stability only if velocity and real GNP are integrated of order 1,  $I(1)$ , or less. If velocity or real GNP are integrated of orders greater than 1, the Meltzer rule will not guarantee price stability.

## B. Nominal GNP targets

Several researchers--Hall (1983); Tobin (1983); Gordon (1985); Taylor (1985); McCallum (1988, 1989, 1990a, 1990b)--have suggested using nominal GNP targeting as an intermediate target for monetary policy. The motivation for this type of targeting is that it avoids many of the problems associated with monetary aggregate targeting, such as velocity instability. We consider several well-known nominal GNP rules, as well as nominal GNP targeting in general.

### 1. General nominal GNP rules

A couple of general points about nominal GNP targeting are worth

making. First, if the monetary authority targets the nominal GNP growth rate, then it can only be sure that nominal GNP,  $p_t + q_t$ , is  $I(1)$ ; there is no way of ensuring that prices are stationary. Thus, to achieve price stability, the appropriate nominal GNP target must be directed at the level of nominal GNP.

Second, if the monetary authorities target the level of nominal GNP, then price stability is possible, depending on the behavior of output, the nominal GNP target, and the implied real GNP and price level targets. To illustrate this point, let nominal GNP be given by  $x_t = p_t + q_t$ . Consider the case in which the monetary authority knows the current velocity (or can calculate velocity); then a simple example of a nominal GNP rule would be one in which the money supply is set so that

$$(13) \Delta x_t = \Delta x_t^* - \lambda(x_{t-1} - x_{t-1}^*)$$

or

$$(14) \Delta m_t = -\Delta v_t + \Delta x_t^* - \lambda(x_{t-1} - x_{t-1}^*)$$

where  $x_t^*$  is the nominal GNP target and  $0 < \lambda < 1$ . Thus, the money supply is set to offset past deviations of nominal GNP from its target.

If we use the quantity theory relationship and the money supply equation to solve for prices, we find that

$$(15) p_t = 1/[1-(1-\lambda)L] [\Delta q_t^* - \Delta q_t + \Delta p_t^* - \lambda(q_{t-1} - q_{t-1}^*) + \lambda p_{t-1}^* ],$$

where  $q_t^*$  and  $p_t^*$  are the implied targets for real GNP and the price level, respectively (note:  $x_t^* = q_t^* + p_t^*$ ).<sup>9</sup> For price stability, the

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<sup>9</sup> The solution for  $p_t$  given above ignores the complementary solution implied by the price level difference equation. To get the general solution for  $p_t$ , add the term

$$(1-\lambda)^t (p_0 - 1/[1-(1-\lambda)L] [\Delta q_0^* - \Delta q_0 + \Delta p_0^* - \lambda(q_{-1} - q_{-1}^*) + \lambda p_{-1}^* ])$$

appropriate price target is a constant--that is,  $p_t^* = p^*$ . Thus,

$$(16) p_t = p^* + [1/(1-(1-\lambda)L)] [\Delta q_t^* - \Delta q_t - \lambda(q_{t-1} - q_{t-1}^*)] ,$$

or

$$(16') p_t = p^* + q_t^* - q_t.$$

As long as the nominal GNP target (given the constant price target, this means the implicit real GNP target,  $q_t^*$ ) adjusts so that  $q_t^* - q_t$  is  $I(0)$ , then the price level will be an  $I(0)$  process. This result suggests that nominal GNP targeting is capable of achieving price stability even if real GNP is nonstationary as long as the implied real GNP target takes into account this nonstationarity.

Hall (1983) and McCallum (1989) suggest picking a target path for nominal GNP once and for all and keeping it fixed. Their motivation is to ensure that the nominal GNP rule is credible. Gordon (1985) and Tobin (1983) suggest periodically reevaluating the nominal GNP target to take into account changes in potential GNP. This debate about how to choose the nominal GNP target is not a trivial matter as far as price stability is concerned. As the above algebra suggests, how the GNP target is chosen can be quite important for determining whether price stability is feasible. Only with a periodic evaluation of the nominal GNP target can nominal GNP targeting ensure price stability when real GNP is  $I(1)$ .

For the case in which velocity is unknown at the time monetary policy is conducted, a feedback rule of the form

$$(17) \Delta m_t = - \Delta v_{t-1} + \Delta x_t^* - \lambda(x_{t-1} - x_{t-1}^*)$$

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where the  $p_0$  is the price level in the initial time period.

may satisfy the necessary conditions for price stability.

Solving for  $p_t$  yields

$$(18) p_t = p^* + q_t^* - q_t + [1/(1-(1-\lambda)L)] \Delta^2 v_t.$$

If  $1 > \lambda > 0$ , price stability is feasible as long as velocity is  $I(2)$  or less and  $q_t^* - q_t$  is  $I(0)$ .

## 2. Specific nominal GNP rules

McCallum (1988, 1990a, 1990b) in various articles suggests using the following GNP rule:

$$(19) \Delta m_t = 0.0075 - (v_{t-1} - v_{t-17})/16 - \lambda(x_{t-1} - x_{t-1}^*),$$

where the money supply is the monetary base, the time index represents quarters, and 0.0075 is the quarterly growth rate in potential or target real GNP, or  $E(\Delta q_t^*)$ .

It is clear that McCallum's rule is very similar to the feedback rule described in equation (17). With the quantity theory equation, solving McCallum's nominal GNP rule yields

$$(20) p_t = p^* + [1/(1-(1-\lambda)L)] [0.0075 + \sum_{i=1}^{16} (\Delta v_t - \Delta v_{t-i})/16 - \Delta q_t - \lambda(q_{t-1} - q_{t-1}^*)],$$

or

$$(20a) p_t = p^* + [1/(1-(1-\lambda)L)] [0.0075 + \sum_{i=1}^{16} \sum_{j=0}^{i-1} (\Delta^2 v_{t-j})/16 - \Delta q_t - \lambda(q_{t-1} - q_{t-1}^*)]$$

Because McCallum's nominal GNP target is a deterministic trend, in order for the McCallum rule to yield price stability,  $v_t$  must be  $I(2)$  or less and  $q_t$  must be  $I(0)$ . Price stability occurs only if real GNP is trend-stationary.

Taylor (1985) suggests setting nominal GNP so that

$$(21) q_t - q_t^* = -\beta(p_t - p_{t-1}),$$



and  $q_t^*$  represents natural (target) level of real GNP where  $\beta \geq 0$ .

This rule can be rewritten as

$$(22) \Delta x_t = \Delta q_t^* - (q_{t-1} - q_{t-1}^*) + (1 - \beta)(p_t - p_{t-1}).$$

Solving for the price level yields

$$(23) p_t = p_0 + 1/\beta (Q_t),$$

where  $Q_t = \sum_{i=0}^t (q_i^* - q_i)$ . Here, the price level is a function of past deviations of real GNP from its target level. Taylor's rule is unlikely to yield price stability because even if  $q_i^* - q_i$  is  $I(0)$ ,  $Q_t$  is still  $I(1)$ .

### C. Price level targets

Possible problems associated with choosing the nominal GNP target have led some (Barro 1986 and McCallum 1990b) to suggest targeting the price level directly. Indeed, price level rules appear to be the most direct and flexible monetary rules for achieving price stability.

McCallum (1990b) suggests a price level rule of the form

$$(24) \Delta m_t = 0.0075 - (v_{t-1} - v_{t-17})/16 + (q_{t-1} - q_{t-17})/16 \\ - \lambda(p_{t-1} - p_{t-1}^*),$$

where 0.0075 is the targeted quarterly real GNP growth rate. Setting the price target ( $p_t^*$ ) equal to a constant ( $p^*$ ) and solving McCallum's price rule yields

$$(25) p_t = p^* + [1/(1-(1-\lambda)L) [.0075 \\ + \sum_{i=1}^{16} \sum_{j=0}^{i-1} \Delta^2 v_{t-j}/16 - \sum_{i=1}^{16} \sum_{j=0}^{i-1} \Delta^2 q_{t-j}/16] .$$

Therefore, for McCallum's rule to yield price stability,  $v_t$  and  $q_t$  must be  $I(2)$  or less. Indeed, any price level rule that includes feedback terms for velocity and real GNP growth, as in equation (24), will yield price

stability if velocity and real GNP are I(2) or less.

McCulloch (1991) has suggested a price level target of the form

$$(26) m_t = g + \bar{m}_{t-1} + b(p^* - p_{t-1}),$$

where  $0 < b \leq 1$ ,  $g$  is related to the growth rate of natural real GNP, and  $\bar{m}_{t-1}$  is a weighted-average of past money supply, or  $\bar{m}_{t-1} = (1 - a)/(1 - aL) m_{t-1}$ , with  $0 < a < 1$ . It is possible to rewrite McCulloch's rule as

$$(26a) \Delta m_t = g(1 - a) + b(1 - aL)(p^* - p_{t-1}).$$

Note that McCulloch's rule is similar to McCallum's price level target. They differ in that McCallum's rule includes a velocity adjustment and a real GNP adjustment, while McCulloch's rule responds not only to deviations of last period's prices from the target ( $p^* - p_{t-1}$ ) but also to deviations two periods ago ( $p^* - p_{t-2}$ ). Using the quantity theory equation, the McCulloch rule implies a price level given by

$$(27) p_t = p^* + g/b + [1 - (1 - b)L - abL^2]^{-1} (\Delta v_t - \Delta q_t).$$

From equation (27), it is clear that the McCulloch rule results in price stability only if velocity and real GNP are integrated of order 1 or less.

#### D. Interest rate targets

In general, interest rate targets do not ensure price stability. Goodfriend (1987) and Van Hoose (1989) have shown that the desire to smooth nominal interest rates leads to price level nonstationarity. Hence, interest rate targeting tends to be inconsistent with price stability.<sup>10</sup>

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<sup>10</sup> McCallum has shown that a pure interest rate peg does not constitute a well-formulated monetary policy. Some additional specification of the money supply process is needed--for example, a money supply rule. However, the stochastic process for prices changes depends on the money supply rule even though the nominal interest rate is pegged.

Recently, Hetzel (1990) has offered an interesting proposal in which the government issues indexed bonds as well as nominal bonds and uses the spread between the two types of bonds as a guide for monetary policy. We can formalize his proposal as a simple feedback rule of the form

$$(28) m_t - m_{t-1} = -\theta(i^{NI}_t - i^I_t),$$

where  $i^{NI}_t$  is the yield on nominal bonds while  $i^I_t$  is the yield on indexed bonds. This is a nominal interest rate target, where the target is the real rate of interest as reflected by the indexed bond yield.

Making use of the Fisher equation, we can rewrite equation (28) as an expected inflation target, or

$$(28a) m_t - m_{t-1} = -\theta[E(p_{t+1}|I_t) - p_t],$$

where  $E(p_{t+1}|I_t)$  is the rational expectation of the price level at  $t + 1$ , given time period  $t$  information. Using the quantity theory equation, we can solve for the inflation rate (assuming  $|\theta| < 1$ ):

$$(29) \Delta p_t = \sum_{i=0}^{\infty} (-\theta)^i E[(\Delta v_{t+i} - \Delta q_{t+i})|I_t].$$

The presence of the expectations term prevents the difference operator on both sides from canceling out. Therefore, the price level can be nonstationary even if velocity and real GNP are stationary. Thus, the Hetzel interest rate target is not likely to generate price stability.

## VI. "Optimal" Versus "Feasible" Monetary Policy

The analysis in the previous sections focused on whether price stability was feasible under various monetary rules. We made no attempt to examine the character of the optimal monetary rule. Determining the optimal monetary rule will require a careful consideration of the "costs and benefits" of the various policy proposals. In general, the optimal

monetary rule will depend on the monetary authority's objective function (the monetary authority may wish to stabilize real output as well as prices); the information available to the monetary authority when it sets its policy instruments; and the structure of the economy--not only in terms of long-run dynamics, such as the orders of integration of important stochastic processes, but in terms of the short-term dynamics as well. Rather than explicitly examining the optimal monetary rule, we suggest how price stability considerations could be formally introduced into the analysis of optimal monetary policy.

Many of the analyses of monetary policy--such as Goodfriend (1987), Barro (1989) and Van Hoose (1989)--typically have an objective (or loss) function for monetary authority of the form

$$(30) \alpha \text{var}[p_t - E(p_t | I_{t-1})] + \beta \text{var}[E(p_{t+1} | I_t) - p_t] + \dots,$$

where  $\alpha$  and  $\beta$  are weights involving output variability and expected inflation variability.<sup>11</sup> Price stability imposes strong, long-run restrictions on the conduct of monetary policy but not nearly as strong restrictions on the short-term conduct of monetary policy. Unfortunately, objective functions like equation (30) place little emphasis on long-term uncertainty of prices. Thus, the degree to which price stability holds typically receives little weight in the analysis of optimal monetary policy.

To introduce price stability considerations into the analysis, long-term uncertainty about the price level must be factored into the monetary authority's objective function. This could be done by introducing the

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<sup>11</sup> The first term in equation (30) typically reflects the loss due to output variability, where output is determined by an expectations-augmented Phillips curve or by a Lucas supply function.

variance of long-term forecasts--that is,  $\text{var}[p_{t+k} - E(p_{t+k}|I_t)]$ -- into the monetary authority's objective function. For example, the monetary authority's objective function might be given by

$$(31) \sum_{i=0}^{\infty} [\alpha_i \text{var}\{p_{t+i} - E(p_{t+i}|I_{t-1})\} + \beta_i \text{var}\{E(p_{t+i+1}|I_t) - p_t\}] + \dots,$$

where the  $\alpha_i$ 's and the  $\beta_i$ 's reflect the relative weights the monetary authority places on short-term uncertainty and long-term uncertainty. An objective function such as (31) would require monetary policy to take into account long-term dynamics as well as short-term dynamics. Of course, determining the weights in an objective function such as (31) requires a clear understanding of the costs and benefits associated with reducing long-run uncertainty about the price level.

#### VII. Concluding Remarks

As we demonstrated above, the goal of price stability implies strong, long-run restrictions for the conduct of monetary policy. In general, the money supply must follow some sort of feedback rule that offsets the nonstationarity in velocity, real GNP, or both. Among the alternative types of monetary targets, price level targeting and nominal GNP targeting (provided the nominal GNP target adjusts to account for changes in trend real GNP) show the most promise for generating price stability. Pure money supply targeting yields price stability in fewer circumstances than do either price level targets or nominal GNP targets. Indeed, the current monetary aggregate procedures that include target cones and base drift are almost guaranteed not to result in price stability. Our analysis focused on the feasibility of price stability under these alternative policy rules.

To determine the "optimality" of these rules, a better understanding of the effects of long-term price uncertainty is needed.

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