No. 9212

THE ANALYSIS OF FISCAL POLICY IN NEOCLASSICAL MODELS

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August 1992

Research Paper

Federal Reserve Bank of Dallas
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Abstract

This paper presents an analysis of the effects of changes in government purchases in the context of a simple static neoclassical model. I show why there can never be a multiplier in such a model under standard assumptions about tastes and technology when the capital stock is held fixed. The standard analysis is extended to include an examination of the effects of changes in public sector employment. The introduction of public sector employment means that we must be careful in choosing between alternative empirical measures of the theoretical concept of aggregate output. Under current national income accounting conventions, GNP may in fact fall in response to increased government purchases.
1. Introduction

The existence or otherwise of a "multiplier" effect from changes in government purchases to aggregate output has been a central issue in macroeconomics since Keynes (1936). The textbook Keynesian model (see, for example, Dornbusch and Fischer (1978)) predicts that an increase in government purchases of goods and services will call forth an even greater increase in production of these goods and services. However the existence of a multiplier effect from government purchases to aggregate output was called into question by Barro (1981) and Hall (1980). Reasoning from an equilibrium framework, they argued that increases in government purchases always lead to smaller (i.e. less than one-for-one) increases in aggregate output. They emphasized the distinction between temporary and permanent changes in government purchases, and argued that temporary increases (such as those associated with military spending during wars) will have a greater output effect than permanent increases. But even temporary increases, they concluded, will raise output by a less than equal amount by crowding out some component of private demand. Estimates in Hall (1986), for example, suggest that each extra dollar of military spending raises GNP by only sixty two cents.

Recently this conclusion has been called into question by Baxter and King (1990) and Aiyagari, Christiano and Eichenbaum (1990). Both groups of authors show that in the context of a fully specified dynamic general equilibrium model, permanent increases in government purchases will raise output by more than temporary increases, and by more than the increase in purchases. The two groups of authors stress different reasons for the difference between their results and those of Barro and Hall. Baxter and King
emphasize the importance of endogenizing the capital accumulation decision, while Aiyagari, Christiano and Eichenbaum emphasize the existence of an income effect on leisure and complementarity between labor and capital in the production function.

Some of the confusion on this issue stems from the fact that it is generally impossible to find exact solutions to dynamic general equilibrium models of the type commonly used for neoclassical analysis except under certain restrictive assumptions. However static versions of the same models are easier to deal with, and can help to clarify some of the issues associated with the analysis of fiscal policy in the more complicated dynamic models. Below I will illustrate the effects of changes in government purchases in a simple static representative agent ("Robinson Crusoe") economy. I will consider changes in both government purchases of goods and purchases of services. About half of total government purchases of goods and services consists of compensation of employees. Typically, analyses of the effects of fiscal policy do not distinguish between government purchases of private sector output and government purchases of labor services. Below I will show that in the context of a neoclassical model the distinction is quite important.

The analysis is best thought of as illustrating the effects of changes in what Baxter and King refer to as "basic" government purchases, namely purchases that do not augment the utility of consumers (except possibly in an additively separable manner) nor augment the productivity of private factors of production. The standard example of such purchases is of course those undertaken by the military. I extend the standard analysis by introducing public sector employment in a manner symmetric with purchases of output.
Public sector employment detracts from the representative agent's time endowment in a manner similar to time spent at private production. Furthermore, the time spent working for the government does not enhance the productivity of private factors of production.

Once public sector employment is introduced into the model, we need to be careful about choosing empirical counterparts for the theoretical concepts of aggregate output and government purchases of goods and services. Gross National Product (GNP) is the measure of aggregate output most commonly used in macroeconomic analysis.\(^2\) It is intended to measure the total output of all factors of production owned or supplied by a country's residents. It is also intended to include the output of the country's government. The output of the government sector is defined to be equal to the compensation of government employees because of the unobservable nature of the government's product. Government purchases of goods and services consist of the sum of government purchases of final goods and services from the private sector and compensation of government employees. The existence or otherwise of a multiplier effect from government purchases to aggregate output is then usually assessed by looking at the relationship between GNP and total government purchases of goods and services. However, because of the inclusion of compensation of government employees in both series, the size of the multiplier effect is potentially mismeasured.

Below I will show that in the context of a static neoclassical model where increased government purchases of final output increase private production, GNP as conventionally measured may in fact fall. Likewise increased public sector employment which necessarily depresses private sector output will also have an ambiguous effect on GNP. On the basis of simple
estimates of some key parameters, however, it seems unlikely that we would observe the perverse result of GNP falling in response to increased government purchases. These same parameter estimates also suggest that multiplier estimates based on GNP understate the expansionary effect of increased government purchases on private sector output by about a third. The distinction between GNP and private sector output also changes our impression of how the U.S. economy behaved during World War I and World War II. The best available data suggest that private sector output fell in both 1917 and 1918. During World War II, the rate of growth of private sector output tapered off rapidly after 1941, while GNP continued to grow at robust rates until 1944. The post-World War II “depression” in 1946 looks a lot less severe when judged in terms of the performance of private sector output (a 4.8-percent decline) than when it is judged in terms of the performance of GNP (a massive 19.0-percent decline). The paper concludes with some tentative estimates of the differential effects of the two types of government purchases on aggregate output.

2. How government is treated in the national accounts

The invention of national income accounting is one of the great achievements of twentieth century economics. However, the practice of measuring the aggregate output of goods and services produced in a country is fraught with difficulties, and inevitably relies on accounting conventions that seem arbitrary and result in strange anomalies. The treatment of government is one of the more troublesome aspects of national accounting. This stems from the fact that the output of government is not exchanged on any market, thus making measurement of its value and volume quite difficult.
Under current national income accounting conventions (see, for example, U.S. Department of Commerce (1985)) output of the government sector is equated with compensation of employees in that sector. Furthermore, all of the output produced by the government is classified as final as opposed to intermediate output. Thus it is added to the output of the business and household sectors to arrive at aggregate output.

There are two obvious problems with the way government is treated in the national income accounts. First, there can be little doubt that much of government output currently classified as final output is in fact intermediate output (Kuznets 1948, pp 156-157). The example most commonly used to illustrate this point is the provision of policing and security services. Police services provided by the government are treated as final output (measured as the compensation of employees in police forces) while the same services provided by private security companies are classified as intermediate output used in the production of final output by the firms who hire them. Leffler (1978) explored the consequences for international comparisons of output of reallocating some of the components of government output from final to intermediate classification. Horz and Reich (1982) and Reich (1986) explore in greater detail the scope for separating intermediate from final product for the government sector. Eisner and Nebhut (1981) and Eisner (1989), on the other hand, adopt the more radical approach of classifying all government output as intermediate.

The second obvious shortcoming of the current treatment of government output in the national accounts has to do with the fact that no imputation is made for the services of government capital. Eisner and Nebhut (1981) and Eisner (1989) make such imputations for the United States for the post World
War II period and find that this correction, in conjunction with a number of other corrections, yields a figure for net government product that is more than 50 per cent larger than the BEA measure (Eisner and Nebhut (1981), p.41).

In what follows, I want to explore the implications of the current measurement conventions for the analysis of fiscal policy in a neoclassical model. Thus the output of the government sector will be equated with compensation of employees in government, and will all be classified as final output.

3. Model

Consider an economy where the final output of the private sector, $Y$, is produced using capital employed in the private sector, $K$, and labor employed in the private sector, $N$, by means of a standard constant returns to scale production function $F(K,N)$. The stock of capital is fixed, while the level of the labor input is determined by a standard labor-leisure trade off. Final output is either consumed, $C$, or appropriated by the government, $G$. The single representative agent derives utility from consumption and leisure, $L$. His preferences are summarized by the utility function $U(C,L)$, which is assumed to satisfy the usual concavity requirements. The agent is endowed with a single unit of time each period, which must be divided between leisure, private productive effort and working for the government, $N^g$. Formally,

$$1 = L_t + N_t + N^g_t$$

(1)

Thus, working for the government is no more distasteful in a utility sense than working at the private production technology.

What is the role of government in this economy? Since we are assuming
no market imperfections, any and all government activity in this economy reduces welfare. Government purchases of private sector output do not contribute to private welfare by substituting for private consumption purchases. Nor do they enhance the productivity of factors employed in the private sector. Likewise the labor employed by the government does not produce anything of value to the private sector. These admittedly extreme assumptions considerably simplify the algebra in what follows without altering the substance. 3

Assuming that the government finances all of its activities using lump-sum taxes, the competitive equilibrium in this economy can be obtained as the solution to a simple planning problem. The first order conditions that characterize the optimal allocations are

\[ D_{1}U(C_{t}, 1 - N_{t} - N_{t}^{g}) = \lambda_{t} \]  

(2)

\[ D_{2}U(C_{t}, 1 - N_{t} - N_{t}^{g}) = \lambda_{t}D_{2}F(K_{t}, N_{t}) \]  

(3)

\[ F(K_{t}, N_{t}) = C_{t} + G_{t} \]  

(4)

where \( D_{i} \) denotes differentiation with respect to the \( i \)'th argument and \( \lambda_{t} \) is the Lagrange multiplier for the period \( t \) resource constraint. The interpretation of these conditions is straightforward. The only important decision that the representative agent faces is how much to work at the private productive activity, given the demands of the government on his time and final output and the available technology for converting the labor input.
into final output. The optimal supply of labor to private production equates the marginal product of private effort to the marginal rate of substitution between consumption and leisure.

Following current national income accounting conventions, total government purchases of goods and services in this economy are given by

$$\Gamma_t = G_t + w_t N^g_t = G_t + D_2 F(K_t, N_t) N^g_t$$

(5)

where $w_t$ is the real wage, which in equilibrium is simply the marginal product of labor. Clearly, this aggregate is not exogenous: shocks to the production function that alter the marginal productivity of labor will cause changes in $\Gamma_t$ even in the absence of changes in $G_t$ or $N^g_t$. It also should be noted that $\Gamma_t$ measures the true value (in real terms) of the resources appropriated by the government. Government purchases of goods and services in the national accounts may understate this total at times. Thus if the labor employed by the government consists solely of military conscripts paid some fixed wage, $\bar{w}_t$, below the market wage, the measured total understates the true total by the amount $(w_t - \bar{w}_t) N^g_t$. This discrepancy is likely to be particularly important in wartime but is of tangential importance for what follows. 4

In the context of this model, GNP is defined as the sum of Gross National Private Product (GNPP) and Gross National Government Product (GNGP): 5,6
GNPP is simply the output produced by the supply of factors to private production activity (=F(K,N)). GNGP is by convention defined to be compensation of employees in the government sector, which in this model simply consist of wage payments, w_tN^g. This convention reflects the fact that the output of government is unobservable. Thus under current national accounting conventions we can rewrite the expression for GNP as

\[ GNP_t = F(K_t,N_t) + w_tN^g_t = F(K^g_t,N^g_t) + D_2F(K^g_t,N^g_t)N^g_t \]  

where I have simply substituted the marginal product of labor for the equilibrium real wage. Under ideal circumstances, we would assume that the output of the government sector is a function of the capital, K^g, and labor, N^g, employed in the sector, summarized by \( F^g(K^g,N^g) \). In the special case where \( F = F^g \), GNP would be unaffected by the split of production between the private and public sectors. We generally do not believe this to be the case, however, if for no other reason than the absence of competition in the government sector will typically cause factors to be employed relatively inefficiently.

4. Analysis

To analyze the effects of changes in government purchases of final
output or employment, it is convenient to linearize the model around the equilibrium allocations. Thus we obtain

\[ \xi_{cc} \dot{C}_t - \xi_{cl} \frac{N}{1 - N - N^g} \dot{N}_t - \xi_{cl} \frac{N^g}{1 - N - N^g} \dot{N}^g_t = \lambda_t \]  

(8)

\[ \xi_{lc} \dot{C}_t - \xi_{ll} \frac{N}{1 - N - N^g} \dot{N}_t - \xi_{ll} \frac{N^g}{1 - N - N^g} \dot{N}^g_t = \lambda_t + \gamma_{NN} \dot{K}_t + \gamma_{NN} \dot{N}_t \]  

(9)

\[ \theta_k \dot{K}_t + \theta_N \dot{N}_t = \theta_c \dot{C}_t + \theta_d \dot{C}_t \]  

(10)

where \( \xi_{ij} \) is the elasticity of the marginal utility of i with respect to j for i, j = C, L. Concavity of preferences implies that \( \xi_{CC} \) and \( \xi_{LL} \) < 0, and

\( \xi_{cc} \xi_{ll} - \xi_{lc} \xi_{cl} > 0 \). \( \gamma_{ij} \) denotes the elasticity of the marginal product of i with respect to j for i, j = K, N. The requirement that the production function be concave implies that \( \gamma_{NN} \) and \( \gamma_{KK} \) < 0, and \( \gamma_{NN} \gamma_{KK} - \gamma_{NK} \gamma_{KN} > 0 \). \( \theta_i \) denotes the elasticity of output with respect to i for i = K, N and is always positive. The assumption of constant returns to scale means that \( \theta_i \) also denotes the share of factor i in the output of the private sector. Finally \( \theta_j \) is the share of final output allocated to j for j = C, G. The hats "^\wedge" denote percentage deviations from equilibrium. All of the elasticity and share parameters are evaluated at their equilibrium values.
The system becomes more tractable if we make the simplifying assumption that the utility function is separable. In this case $\xi_{cl} = \xi_{lc} = 0$, and we only need to solve for $\hat{C}_t$ and $\hat{N}_t$ simultaneously, since $\hat{\lambda}_t$ is determined recursively. Solving the system under this assumption gives us the following expressions for consumption and private effort:

\[\hat{C}_t = -\frac{1}{\Delta} \Theta_G (\xi_{ll} \frac{N}{1 - N - N^g} + \gamma_{nn}) \hat{C}_t - \frac{1}{\Delta} \Theta_N \xi_{ll} \frac{N^g}{1 - N - N^g} \hat{N}_t^g - \frac{1}{\Delta} (\Theta_N \gamma_{nk} - \Theta_K (\xi_{ll} \frac{N}{1 - N - N^g} + \gamma_{nn}))(\hat{K}_t)
\]

\[\equiv \epsilon_{Ct}^c \hat{C}_t + \epsilon_{Nt}^c \hat{N}_t^g + \epsilon_{Kt}^c \hat{K}_t^g \quad (11)\]

\[\hat{N}_t = \frac{1}{\Delta} \xi_{cc} \Theta_G \hat{C}_t - \frac{1}{\Delta} \Theta_c \xi_{ll} \frac{N^g}{1 - N - N^g} \hat{N}_t^g - \frac{1}{\Delta} (\Theta_C \gamma_{nk} + \xi_{cc} \Theta_K) \hat{K}_t
\]

\[\equiv \epsilon_{Ct}^n \hat{C}_t + \epsilon_{Nt}^n \hat{N}_t^g + \epsilon_{Kt}^n \hat{K}_t^g \quad (12)\]

where

\[\Delta = \Theta_N \xi_{cc} + \Theta_C (\xi_{ll} \frac{N}{1 - N - N^g} + \gamma_{nn}) \leq 0.\]

and $\epsilon_{ij}^k$ denotes the elasticity of $i$ with respect to $j$. $\epsilon_C^C$ and $\epsilon_N^N$ are both positive: an increase in the endowment of capital raises the equilibrium level
of consumption; an increase in government purchases of private output raises
private employment. $\xi^C_G$, $\xi^C_N$, and $\xi^N_N$ are all negative: an increase in
government purchases of final output lowers consumption; an increase in
government employment also lowers consumption; and an increase in government
employment lowers private sector employment. The sign of $\xi^N_N$ is ambiguous,
and depends among other things on the elasticity of substitution between
capital and labor.

Note that there is a certain amount of symmetry in the response of
consumption to increased government purchases and the response of private
sector employment to increased government employment. In the special case
where the marginal utility of consumption is constant, $\xi_{CC} = 0$ and increased
government purchases of private sector output are offset one-for-one by a
reduction in consumption. With all of the extra government spending absorbed
in lower consumption, there is no need to increase production or work effort.
In this case,

$$\xi_G^c = -\frac{1}{\Delta} \left( \xi_{LL} \frac{N}{1 - N - N^G} + \gamma_{NN} \right) = \frac{-\theta_G}{\theta_C}$$

so the multiplier, $dC/dG$, is equal to -1. Thus, higher government purchases
call forth no increase in production. It is worth noting that with separable
utility the condition that $\xi_{CC} = 0$ implies that there is no income effect on
leisure. Aiyagari, Christiano and Eichenbaum (1990) emphasize that the
existence of such an income effect is essential to generating multiplier
effects on output in a neoclassical model.
Likewise, if the marginal utility of leisure is constant, $\xi_{LL} = 0$, and an increase in public sector employment is offset one-for-one by a reduction in leisure. Private sector employment is unchanged, as is private sector output and consumption. Letting $\epsilon^L$ denote the elasticity of leisure with respect to public sector employment, we obtain

$$\epsilon^L = -\frac{N}{1 - N - N^g} \left( \frac{1}{\Delta \xi_{LL}} \frac{N^g}{1 - N - N^g} + \frac{N^g}{N} \right)$$

Noting that $L = 1 - N - N^g$, the relevant multiplier is easily shown to be

$$dL/dN^g = -1.$$ 

Since labor is the only variable factor of production, the response of private sector output (GNPP) to an increase in government purchases of output will simply be the product of the elasticity of output with respect to labor and the elasticity of private sector employment with respect to $G$ (i.e. $\epsilon^G_{GNPP} = \Theta^N \epsilon^N_G$). Likewise the response of private sector output to a change in public sector employment will be the product of the elasticity of output with respect to labor and the elasticity of private sector employment with respect to public sector employment. Thus we obtain
The point to note here is that the signs of these elasticities are unambiguous: increased government purchases of private product raise private sector output, while increased public sector employment lowers private sector output. These results are intuitively plausible. The idea that increased government purchases raise private sector output is a staple of macroeconomic theories of all stripes. That increased government employment should lower private sector output is also plausible, as long as we keep in mind that labor is the only variable input to the production process.10

It is straightforward to show that there is no real "multiplier" effect of increased government purchases on private sector output. While output rises in response to higher government purchases, it must always do so less than one-for-one. As long as preferences exhibit some degree of curvature, some part of any increase in government purchases will be absorbed in the form of lower consumption, with the result that output increases less than government purchases. Noting that $\varepsilon^\text{GNPP}_g = \Theta_0 dGNPP / dG$, substituting for $\Delta$ and rearranging we can rewrite equation (13) as

$$\frac{d\text{GNPP}}{dG} = \frac{\Theta_N \varepsilon^\text{CC}}{\Theta_N \varepsilon^\text{CC} + \Theta_C \varepsilon^\text{LL} \left( \frac{N}{1 - N - N^g} + \gamma_{NN} \right)}$$

From this expression it is clear that the "multiplier" must lie between 0 and
1. With a constant marginal utility of consumption, all of the extra output demanded by the government comes out of a reduction in consumption, and production is unchanged. If both the marginal utility of leisure and the marginal productivity of labor are constant, $\xi_{LL} = \gamma_{NN} = 0$, and all of the extra output demanded by the government is met by a one-for-one increase in production with private consumption unchanged. In the more standard case with diminishing marginal products and marginal utilities, the multiplier is less than 1.

Analogously, increased public sector employment will generally produce a less than one-for-one increase in total employment. If we let $E$ denote total employment (i.e. $N + N^g$) then it is straightforward to show that

$$\frac{dE}{dN^g} = \frac{\Theta_N \xi_{CC} + \Theta_C \gamma_{NN}}{\Theta_N \xi_{CC} + \Theta_C \gamma_{NN} + \Theta_C \xi_{LL}} \frac{N}{1 - N - N^g}$$

With a constant marginal utility of leisure, $\xi_{LL} = 0$ and an increase in government employment raises total employment one-for-one. If both the marginal utility of consumption and the marginal productivity of labor are constant, $\xi_{CC} = \gamma_{NN} = 0$, and all of the extra public sector employment is offset by a decline in private sector employment. Leisure remains unchanged. More generally, the employment effects of an increase in government employment will be to raise total employment by less.

5. Multiplier effects on GNP

Let us now examine the response of GNP to changes in government
purchases and employment. From the definition of GNP in equation (7) above we can show that

\[ e_G^{GNP} = s_p e_G^{GNPP} + (1-s_p) e_w^w \]

\[ e_N^w = s_p e_N^{GNPP} + (1-s_p) (e_N^w + 1) \]

where \( s_p \) denotes the share of private sector output in total GNP and \( e_i^w \) denotes the elasticity of the real wage with respect to \( i = G, N^g \).

Substituting for \( e_i^w \) and rearranging terms we obtain the following expressions

\[ e_G^{GNP} = \frac{G^{GNP}_t}{G_t} \bigg|_{\dot{k}_N = 0} = \frac{\Phi}{\theta_N} e_G^{GNPP} \]

\[ e_N^{GNPP} = \frac{G^{GNP}_t}{N^g_t} \bigg|_{\dot{k}_N = \dot{\theta}_N = 0} = \frac{\Phi}{\theta_N} e_N^{GNPP} + (1-s_p) \]

where \( \Phi = s_p \theta_N + (1-s_p) \gamma_{NN} \). Note that with constant returns to scale, \( \gamma_{NN} = -\gamma_{NK} = -\theta_N / \sigma_{NK} \), where \( \sigma_{NK} \) denotes the elasticity of substitution between capital and labor in private sector production. The signs of these elasticities are in general ambiguous because of the ambiguous sign of \( \Phi \).
The sign of the elasticity of GNP with respect to increases in government purchases of final output depends on the sign of $\Phi$. With a constant marginal product of labor, $\gamma_{NN} = 0$ and an increase in government purchases of private sector output will always raise GNP. Furthermore, the multiplier effects will be the same, i.e. $d\text{GNP}/dG = d\text{GNPP}/dG$. More generally the multipliers are related by $d\text{GNP}/dG = (\Phi/s_P\theta_N)d\text{GNPP}/dG$.

The intuition for the critical role played by the marginal product of labor is as follows. An increase in government purchases must be accompanied by an increase in taxes, which lowers household income or wealth. With leisure assumed to be a normal good, the supply of labor increases. The demand for labor is fixed by the predetermined capital stock, so the effect on the real wage is determined by the slope of the demand curve for labor. If the labor demand curve is flat (as it will be if labor has a constant marginal product), the real wage remains constant, which in turn means that the imputed output of the government sector $(WN^9)$ is constant. In this case GNP rises one-for-one with the increase in private sector output. More generally, the real wage will fall in response to an increase in government purchases of private sector output, leaving the net effect on GNP ambiguous.

Table 1 lists the values of the critical term $\Phi$ for different values of $\sigma_{KN}$, $\theta_N$ and $s_P$. The possibility that increased government purchases of final output will lower GNP is greater, the lower the elasticity of substitution between capital and labor, the smaller the share of private sector output in GNP and the smaller the elasticity of private sector output with respect to
How likely is it that we will observe the perverse result of GNP falling in response to an increase in government purchases? Table 2 presents estimates of the parameters $\theta_N$ and $s_p$ for a small number of countries for which the necessary data are available. Estimates of the elasticity of substitution between capital and labor are harder to come by, so we will assume that it is equal to 1. This assumption causes the expression for $\Phi$ to simplify to $\Phi = \theta_N - (1-s_p)$. For developed countries it seems that the most likely combination of parameter values would have $\theta_N > (1-s_p)$. The average value of $\theta_N$ for the nine countries shown is 0.47, while the average value of $s_p$ is 0.88.

While the possibility of different signed multipliers for GNP and GNPP seems small, it is still the case that the multipliers will generally differ in magnitude. Using the averages of the figures in Table 2, the elasticity of GNP with respect to changes in government purchases of final product will be about three-quarters (0.35/0.47) the elasticity of GNPP.

We have already noted that the elasticities of GNP and GNPP with respect to public sector employment are related by

$$\epsilon_{N^*}^{\text{GNP}} = (1-s_p) + \frac{\Phi}{\theta_N} \epsilon_{N^*}^{\text{GNPP}}$$

Thus, even in the special case where the effects of a change in government purchases of private sector output has the same effects on GNP and GNPP (i.e.
when \( \gamma_{Nk} = 0 \) the effects of a change in the amount of labor employed by the government on GNP and GNPP will not be the same. In this case, the multipliers are related by

\[
\frac{d \text{GNP}}{d N^g} = \frac{\phi}{s_p \theta^N} \frac{d \text{GNPP}}{d N^g} + (1-s_p) \frac{\text{GNP}}{N^g}
\]

Furthermore, it is theoretically possible that GNP will increase in response to an increase in \( N^g \). Assuming that \( \sigma_{K_N} = 1 \), this possibility comes down to the requirement that \( \theta_K \) is a lot bigger than \( \theta_N \), i.e. that the technology for producing private sector output be relatively capital intensive. The parameter values in Table 2 suggest that increases in public sector employment lead to increases in GNP.

5. How it matters: Output during wartime

The distinction between GNP and GNPP is likely to be most important in wartime, as it is during wars that we see the most dramatic variation in government employment (and hence government "output"). The rates of growth of GNP and GNPP from 1938 to 1955 are plotted Figure 1. This period covers World War II and the Korean War. Note the large differences in the estimates of the growth rate of the economy during and immediately after World War II. The rate of growth of private sector output fell steadily after 1941, while the rate of growth of GNP exceeded 15.0 percent in 1942 and 1943, dropping to 8.2 percent in 1944. GNPP declined 2.1 percent in 1945 and 4.8 percent in 1946, and then grew 2.6 percent in 1947. By contrast, GNP fell 1.9 percent in
1945, a massive 19.0 percent in 1946, and 2.8 percent in 1947. Clearly, the impression we get of how the economy behaved during and immediately after World War II is substantially different depending on which measure of aggregate output we focus on. Specifically, the 4.8-percent decline in GNPP in 1946 looks a lot less like a postwar "depression" than the 19.0-percent decline in GNP. Note that estimates of growth for the Korean War period are less affected by the distinction.

Official estimates of GNP on an annual basis are only available from 1929. For the period prior to this date, there are three major alternative unofficial series of figures going back to 1869, these being the estimates of Kendrick (1961), Romer (1989) and Balke and Gordon (1989). The estimates of Balke and Gordon, and Romer are considered superior to the older estimates of Kendrick. Figure 2 illustrates growth rates of two measures of aggregate output around World War I using the Balke and Gordon and Romer series. According to Romer's estimates, GNP grew 5.2 percent in 1918 after falling slightly (0.5 percent) in 1917 (the United States declared war on April 6, 1917). By contrast, GNPP fell 3.4 percent in 1917 and 4.9 percent in 1918, and then grew 8.4 percent in 1919. The Balke-Gordon estimates show GNP flat in 1917, growing 7.7 percent in 1918, and then contracting in each of the three subsequent years. The Balke-Gordon series shows the smallest decline in GNPP over the course of the war (2.8 percent in 1917, 1.8 percent in 1918) and smaller growth in GNPP in 1919 (3.1 percent) than the 8.4 percent estimated by Romer. In short, while the details differ between the two series, the basic point that the "shape" of the war-induced cycle in aggregate activity is different depending on whether we look at total product or private sector product continues to hold.
7. Multiplier estimates

Hall (1986) presents empirical results that suggest that there is no multiplier effect from military spending to GNP. His estimates indicate that each extra dollar of military purchases raise GNP by only 62 cents, i.e. less that one-for-one. The analysis above suggests that military spending should be decomposed into its goods and services components before estimating multiplier type relationships, and that GNPP rather than GNP is a more appropriate concept of aggregate output. Unfortunately, attempts to obtain multiplier estimates based on a decomposition of military spending into goods purchases and factor payments run into serious data problems. Constant dollar estimates of total defense purchases and the various components thereof (durable goods, nondurable goods etc.) are only available from 1972. The period since 1972 encompasses the end of the Vietnam war and the Carter-Reagan defense buildup. Constant (1987) dollar estimates of compensation of employees engaged in national defense and national defense purchases of goods are plotted in Figure 3. Casual inspection of these plots should make us pessimistic about finding strong relationships between the components of military spending and aggregate output over this sample period - there is just not enough variation in the key explanatory variables. This is borne out by the regression results in Table 3. There I present estimates of the impact effects of the two different types of military spending on GNP and GNPP. Consistent with the predictions of the theory, the absolute magnitude of the effects of military spending on GNPP are greater than the effects on GNP. The coefficient signs are as predicted: higher purchases of goods raise output, increased appropriation of effort lowers it. However, the standard errors associated with the coefficient estimates are large, and we are unable to
reject the hypothesis that the coefficients are equal to zero.

7. Summary and Conclusions

In this paper I outlined how government purchases of goods and services affect output in a simple static neoclassical model. The composition of government purchases was shown to be important in terms of how the output of the private sector responds: increased purchases of private sector output generally lead to increased production, while increased public sector employment generally depresses private production. I also showed that under standard assumptions about tastes and technology, there can never be a multiplier effect in a static neoclassical model. As Baxter and King correctly pointed out, output can only increase more than one-for-one with increased government purchases when the capital accumulation decision is endogenous.

The introduction of public sector employment into the analysis necessitates that we distinguish between GNP as currently measured and private sector output. Under current national accounting conventions, GNP understates the expansionary effects on output of increased government purchases of goods. Rotemberg and Woodford (1989) look at the effects of fiscal policy on Private Value Added, which they define to be GNP less all government purchases of goods and services. This is going too far, as government purchases of goods are part of private product. It is not just in the analysis of fiscal policy that care must be taken when confronting the predictions of neoclassical models with aggregate data. Recent work by Benhabib, Rogerson and Wright (1991) and Greenwood and Hercowitz (1991) has explored the implications of extending the basic neoclassical real business cycle model to incorporate
household production. The concept of aggregate market output in those models is not the same as GNP because GNP includes an imputation for the services provided by owner-occupied houses. Benhabib, Rogerson and Wright and Greenwood and Hercowitz recognize this when comparing the predictions of their models with the data.

The distinction between GNP and private sector production is likely to be most important when there are large changes in public sector employment (and thus measured government output). Such changes typically occur in and around wartime. Casual empiricism yields a somewhat different impression of U.S. output performance during World Wars I and II when we focus on private sector production rather than GNP.

Finally I presented some evidence of the differential effects of military purchases on output. While simple least squares parameter estimates are consistent with the predictions of the theory, they are statistically insignificant.
Table 1

Values of $\Phi = s_p \theta_n + (1-s_p)\gamma_{NN}$

<table>
<thead>
<tr>
<th>$\sigma_{Kn}$</th>
<th>$s_p$</th>
<th>0.5</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0.33</td>
<td>0.33</td>
<td>-0.505</td>
<td>-0.171</td>
<td>-0.004</td>
<td>0.163</td>
<td>0.33</td>
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<tr>
<td>0.50</td>
<td>0.50</td>
<td>-0.25</td>
<td>0.05</td>
<td>0.20</td>
<td>0.35</td>
<td>0.50</td>
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<tr>
<td>0.67</td>
<td>0.67</td>
<td>0.005</td>
<td>0.271</td>
<td>0.404</td>
<td>0.537</td>
<td>1.0</td>
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<td>$\sigma_{Kn} = 1$</td>
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<tr>
<td>$\theta_n = 0.33$</td>
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<td>0.13</td>
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<td>0.33</td>
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<td>0.50</td>
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<td>0.67</td>
<td>0.17</td>
<td>0.37</td>
<td>0.47</td>
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<td>0.67</td>
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<tr>
<td></td>
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<td>$s_p$</td>
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<td>United States</td>
<td>0.55</td>
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<td>0.92</td>
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<td>0.87</td>
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<td>0.89</td>
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<tr>
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<tr>
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<td>0.87</td>
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<tr>
<td>Spain</td>
<td>0.40</td>
<td>0.89</td>
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<tr>
<td>United Kingdom</td>
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<td>0.86</td>
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Note to Table 2. Data are from OECD Department of Economics and Statistics National Accounts 1975-87 Volume II: Detailed Tables. $\theta_N$ is calculated as the ratio [compensation of employees paid by resident producers to resident households less compensation of employees in general government]/[GDP less GDP originating in the government sector]. $s_p$ is calculated as the ratio [GDP less GDP originating in the government sector]/[GDP]. All figures are for 1985.
<table>
<thead>
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<th>Dependant variable</th>
<th>Δlog GNP</th>
<th>Δlog GNPP</th>
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<tr>
<td>Constant</td>
<td>0.351E-2</td>
<td>0.366E-2</td>
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<td>(0.134E-2)</td>
<td>(0.149E-2)</td>
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<tr>
<td>Δlog GNPₙ₋₁</td>
<td>0.368</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.113)</td>
<td></td>
</tr>
<tr>
<td>Δlog GNPPₙ₋₁</td>
<td>-</td>
<td>0.366</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.113)</td>
</tr>
<tr>
<td>Δlog MILG</td>
<td>0.038</td>
<td>0.044</td>
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<td></td>
<td>(0.030)</td>
<td>(0.034)</td>
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<tr>
<td>Δlog MILN</td>
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<td>-0.146</td>
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<td></td>
<td>(0.168)</td>
<td>(0.188)</td>
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<tr>
<td>R²</td>
<td>0.12</td>
<td>0.12</td>
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<tr>
<td>DW</td>
<td>1.97</td>
<td>1.96</td>
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<tr>
<td>Durbin's h</td>
<td>0.04</td>
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<tr>
<td>S.E.</td>
<td>0.009</td>
<td>0.011</td>
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<td>T</td>
<td>1972:3-1990:4</td>
<td>1972:3-1990:4</td>
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</tbody>
</table>

Notes to Table 3. All data are from U.S. Department of Commerce (1986), and Survey of Current Business, various issues.
References


1. I would like to thank my colleagues at the Federal Reserve Bank of Dallas for comments on an earlier draft. The views expressed in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

2. The US Commerce Department recently switched from emphasizing GNP to GDP as its principal indicator of aggregate economic activity. In a closed economy model such as the one considered in this paper the two measures are of course identical. The points to be made in what follows apply to GDP as well.

3. It is possible to allow both G and $N^0$ to augment utility in an additive manner, by writing the utility function as $U(C,L)+V(G,N^0)$, without altering any of what follows.

4. See Oneal (1991) for a recent attempt to adjust estimates of military spending in NATO countries to allow for conscription. Oneal shows that the monetary value of conscription averaged 9.2 percent of military spending in 1974, declining to 5.7 percent in 1987. Eisner (1989) Table 5 presents estimates of the uncompensated factor services of draftees for the period 1947-75 for the United States. In 1968, at the height of the Vietnam War, compensation of employees in the military as reported in Table 3.7.A of U.S. Department of Commerce (1986) was $21.7 billion. Eisner estimates the uncompensated services of draftees that same year as amounting to $13.4 billion.

5. Note that in Table 40 of National Income 1954 Edition GNP is split into "Gross government product" (what I am calling GNGP) and "Other gross product" (what I am calling GNPP).

6. GNP as computed by the Commerce Department also includes the imputed output of the stock of residential housing. For the sake of simplicity, we will abstract from the existence of a housing stock in this model.


8. King, Plosser and Rebelo (1988a,b) analyze dynamic versions of models similar to the one in this paper by linearizing around the steady state equilibrium. The advantage of this approach, in addition to its simplicity, is the ease with which we can see the relationship between the reduced form parameters of the (linearized) model and the more fundamental parameters of tastes and technology.

9. With nonseparable utility the signs of the various elasticities of interest are unchanged as long as $\xi_{LC}$ and $\xi_{CL}$ are both positive.

10. In a dynamic setting the capital input would also be variable.

11. This assumption is consistent with the observed constancy of factor shares in total income over long periods of time.
12. These figures do not reflect any adjustment for the effects of the Draft during either war. Thus GNP growth is probably understated, while GNPP growth is probably overstated.

13. GNPP was estimated by subtracting Kendrick’s estimates of Government Product (Kendrick (1961) Table A-III) from the Romer and Balke-Gordon GNP estimates.

14. Constant dollar estimates of compensation of employees in national defense are available from 1952.

15. Note that the constant dollar series for compensation of employees engaged in national defense is the relevant variable for measuring the amount of effort appropriated by the government, as opposed to the number of people employed in the military. The constant dollar compensation of employees series is constructed by extrapolating base-year compensation by an index of employment for military personnel and an index of hours worked for civilians employed in national defense. In both cases, the extrapolators are adjusted for changes in the quality of the workforce. Thus the constant dollar compensation of employees series captures both increases in the numbers of workers employed in national defense and increases in the quality of workers employed in national defense. It should be obvious that such a series is preferable to one measuring only the numbers without any regard to quality. Further we would expect that quality changes in military personnel were particularly significant following the abolition of the draft in 1975. The construction of constant dollar compensation of employees series is explained in U.S. Department of Commerce (1988).
Figure 1
Rate of Growth of Gross National Product (GNP) and Gross National Private Product (GNPP), 1938-1955

Note to Figure 1: Data are from Table 40 of U.S. Department of Commerce (1954).
Figure 2(a)
Rate of Growth of Gross National Product (GNP) and Gross National Private Product (GNPP), 1914-1922
(Using Romer Estimates)

Note to Figure 2(a): Data are from Romer (1989).
Figure 2(b)
Rate of Growth of Gross National Product (GNP) and Gross National Private Product (GNPP), 1914-1922
(Using Balke-Gordon Estimates)

Note to Figure 2(b): Data are from Balke and Gordon (1989).
Figure 3


Billions of 1987 Dollars

Note to Figure 3: Data are from Citibase.
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