Money, Output, and Income Velocity

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The views expressed in this article are solely those of the authors and should not be attributed to the Federal Reserve Bank of Dallas or to the Federal Reserve System.
MONEY, OUTPUT, AND INCOME VELOCITY

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Abstract
This paper attempts to assess empirically the contribution of three structural shocks—monetary, institutional (financial and fiscal), and technological—to output and velocity fluctuations in the national bank era and the post-1973 period. To identify these shocks we impose only long-run restrictions, derived from a monetary growth model. We find that higher money growth increases (decreases) velocity in the first (second) period, depending crucially on the resulting changes in the transactions frequency. Credit-enhancing financial or expansionary fiscal shocks have a permanent positive effect on velocity and a hump-shaped effect on output, whereas technological shocks cause velocity to decrease in the short run and output to move to a permanently higher level.

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I. Introduction

The sources of output fluctuations as well as the links between the real and the monetary sector have always been at the heart of macroeconomics. This paper revisits these issues by focusing on the movements of income velocity and output. We estimate a structural vector autoregressive (VAR) model to empirically assess the contribution of monetary, institutional, and technological shocks to fluctuations in the aforementioned macroeconomic aggregates. Following Blanchard and Quah (1989) and Ahmed, Ickes, Wang, and Yoo (forthcoming), we impose only long-run restrictions, based on the theoretical predictions of a monetary growth model, to identify the structural disturbances, and let the data determine the short-run dynamics. The study provides additional evidence on the money-output correlation, as well as on the causes of money velocity movements. The results obtained can generate significant policy implications, since any unpredictable changes in velocity cast serious doubts on the desirability of monetary targets pursued by central banks.

Traditionally, movements of money velocity have been attributed to factors affecting the demand for real balances. Based on the standard inventory model, e.g., Baumol (1952), the velocity of money is often expressed as a function of real income and the nominal interest rate. An increase in the nominal interest rate increases the cost of holding money, reduces the demand for real balances and, given a constant level of real income, increases

\[ \frac{M}{P} = \frac{Y}{R} \]

1 Blanchard and Quah's original work, which examines the importance of aggregate supply relative to aggregate demand shocks, has been extended to the study of labor market disturbances [Shapiro and Watson (1988)] and of international business cycles [Ahmed et al. (forthcoming)]. The interaction between the real and the monetary sector, however, remains an open issue.

2 For example, the recent instability of the M1 velocity was a major factor in the Federal Reserve's deemphasis of M1 as a monetary target.
velocity. On the other hand, changes in real income affect money velocity positively (negatively) only if the income elasticity of money demand is less (greater) than one. Recently, Bordo and Jonung (1987, 1990) have found that institutional variables which affect the level of financial development are significant determinants of money velocity in five advanced countries including the United States. Also, Small and Porter (1989) argue that much of the short-run variability in M2-velocity can be explained by changes in the opportunity cost of holding money balances. Nevertheless, the behavior of money velocity has not been examined within a general equilibrium framework which explicitly takes into account the dynamic interactions between the real and the monetary sector.

To study the movements of velocity within a general equilibrium framework, one needs to examine first the effects of money growth on real macroeconomic aggregates and especially on output, a channel which has not been considered in the existing empirical literature on velocity. Tobin (1965), using a model in which money is treated as an asset to hold, argues that more rapid money growth leads to higher holdings of capital relative to money, and, hence, increases output and consumption, which is known as the "Tobin effect." However, if money is treated as a factor of production or if it is required prior to purchases of the capital good [Stockman (1981)], then the opposite result emerges, usually referred to as the "reverse Tobin effect." Furthermore, the real-business-cycle models claim that monetary shocks do not play any significant role as a source of persistent output movements. The money-output correlation is instead due to a reverse causation via the increase in the demand for transaction services as output increases [King and Plosser (1984)].

This paper emphasizes the transactions role of money by constructing a
dynamic general equilibrium model in which money is introduced through a modified cash-in-advance constraint. We alter the traditional framework in two ways. First, we do not restrict the consumption interval to coincide with the money holding interval and second, we allow only a fraction of the capital good to be subject to the cash-in-advance constraint. This fraction is assumed to depend on institutional changes, either in the financial markets or in the fiscal structure, hereafter simply referred to as "institutional shocks". Such shocks can be treated as combinations of money demand (financial) and IS (fiscal) disturbances. These institutional shocks then together with any money supply shocks constitute what economists conventionally refer to as interest rate innovations.

Our theoretical results suggest that a technological or institutional improvement will lead to higher output. The former, if it is Harrod-neutral, will not affect the steady-state income velocity of money. Furthermore, in the presence of large variations in the transactions frequency, the Tobin effect will emerge, contrary to the standard cash-in-advance models [e.g., Stockman (1981)]. If, on the other hand, the transactions frequency does not vary much, a reverse Tobin effect will be present and velocity, contrary to standard beliefs, may decrease as the money growth rate increases.

Based on the long-run predictions of the theoretical model, we perform a structural VAR analysis using quarterly U.S. data to examine the short-run and long-run interactions between the growth rate of money, the M2-velocity of money, and output. The data used cover part of the national bank era (1880:1-1912:II), as well as the post-1973, flexible-exchange-rate period (1974:1-1990:IV). These sample periods are of special interest since business cycles occurred more often than in any other period [see Morgenstern (1959) and Dornbusch and Fischer (1986), respectively]. Also the correlation of
business cycles across countries has been found to be small [see Mitchell (1929) and Dornbusch and Fischer (1986), respectively], as opposed to the interwar period and the post-World War II period under the Bretton Woods regime. This enables us to restrict our attention to a closed-economy framework without introducing a significant bias. Furthermore, our focus on these two periods is justified by the fact that they differ considerably not only with respect to the exchange-rate regime but also the average inflation rate. The latter is expected to have a significant impact on the frequency of transactions and thus the velocity of money.\(^3\)

In the national bank era, we find that higher money growth reduces both output and velocity, indicating that the effect of anticipated inflation on the transactions frequency is negligible. In the post-1973 period, on the other hand, persistent inflation highlights the role of transactions frequency. As a result, velocity responds positively to the growth rate of money, corroborating standard beliefs. We also find that any institutional change which enhances credit transactions increases velocity permanently and has a short-run positive effect on output. Furthermore, a technological improvement increases output smoothly and gradually, but tends to reduce the M2-velocity in the short run. The latter suggests that M2 is a luxury good. In contrast to the traditional inventory model, however, even if money is a luxury good, velocity may still be positively correlated with output in the case of monetary or institutional changes.

The remainder of the paper is organized as follows. Section II develops

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\(^3\) The average inflation rate over the period 1974-1990 was 5.9\% (0.026), as opposed to -0.25\% (0.024) which occurred during the period 1880-1912 (standard deviations in parentheses). Moreover, the small standard deviation relative to the average in the post-1973 period indicates that the inflation experienced was relatively persistent.
the theoretical model which is used to illustrate the propagation mechanism of the structural shocks. Section III outlines the empirical methodology. Section IV presents the empirical findings obtained from the analyses of impulse responses and variance decompositions. Finally, section V provides a brief summary and some suggestions for future research.

II. An Illustrative Model

The following general equilibrium model is used to illustrate the propagation mechanism of the structural shocks. It also provides some theoretical foundations of the long-run causal ordering of the three macroeconomic aggregates mentioned above—the growth rate of money, the velocity of money, and output. This ordering is used below to identify the structural VAR system.

A. The Model

Consider an economy in which a representative agent with perfect foresight seeks to maximize her lifetime utility, \( U = \sum_{t=0}^{\infty} \beta^t u(c_t) \), where \( c_t \) is per capita consumption and \( \beta \in (0,1) \) is the constant discount rate. In each period \( t \), she produces a certain amount of output, \( Y_t \). The production technology is described by \( Y_t = A_t f(k_t) \), where \( k_t \) denotes per capita capital stock and \( A_t \) is simply a technological parameter. Any technical innovation will increase \( A_t \) and thus will enlarge the production possibilities set. In

4. In the presence of two endogenous state variables, capital and money, it is impossible to solve analytically a stochastic model. We construct instead a model with perfect foresight which can be viewed as the certainty-equivalent version of a stochastic economy.

5. We assume that both the felicity, \( u(\cdot) \), and the production function, \( f(\cdot) \), have the usual neoclassical properties: they are increasing, strictly concave, twice continuously differentiable, and they satisfy the Inada conditions. Moreover, capital is essential in production and thus \( f(0) = 0 \).
addition to factor payments, at the beginning of each period the individual receives a lump sum cash transfer $s_t$ (in real terms) from the government. Let $P_t$ be the price level and $M_t$ be the level of nominal money holdings at the beginning of period $t$. We can then write the agent's budget constraint (in real terms) as:

$$c_t + m_{t+1} + k_{t+1} = A_t f(k_t) + \frac{m_t}{1 + \pi_t} + s_t,$$  \hspace{1cm} (1)

where $m_t = M_t / P_{t-1}$ denotes real money holdings and $\pi_t$ denotes the inflation rate, defined as $\pi_t = (P_t - P_{t-1}) / P_{t-1}$. To simplify the algebra, we have assumed that the capital stock depreciates fully at the end of each period.\(^6\)

The representative agent is also subject to a generalized cash-in-advance or liquidity constraint: all purchases of current consumption and a fraction, $\theta$, of investment must be made using cash.\(^7\) We allow the fraction $\theta$ to depend on institutional changes which enhance credit transactions. The following constraint must therefore be satisfied:

$$c_t + \theta(\phi_t) k_{t+1} = \tau(\pi) \left\{ \frac{m_t}{1 + \pi_t} + s_t \right\},$$  \hspace{1cm} (2)

where $\phi$ captures credit-enhancing financial and/or expansionary fiscal policies, and $\tau(\pi) \in (0, \infty)$ denotes the frequency of transactions. An increase in $\phi$ will result in a decrease in $\theta$, i.e., $\theta(\phi) < 0$. In practice, credit transactions may be enhanced by the use of bills of exchange, letters of credit and credit guarantees.\(^8\) In a similar way, any expansionary fiscal policy, such as an

\(^6\) All the results remain qualitatively unchanged in the case where the depreciation rate, $\delta$, is less than one, as long as the net rate of return on capital is positive, i.e., $A f_k - \delta > 0$.

\(^7\) The cases $\theta = 0$ and $\theta = 1$ are associated with Lucas (1980) and Stockman (1981), respectively. Stockman indeed characterizes both cases, demonstrating that money is superneutral in the first but not in the second case. The intermediate case, on the other hand, is analyzed in Koenig (1987).

\(^8\) Financial developments which encourage the use of cash, such as the establishment of NOW and super-NOW accounts, can be viewed as a reduction in $\phi$. 
increase in the government size, will release part of the cash requirements, since federal government money holdings are not included in money measures, and thus it will also decrease $\theta$ [for a further discussion, see Barro (1978)]. The transactions frequency, $\tau$, on the other hand, is assumed to depend positively on the endogenously determined inflation rate, i.e., $\tau > 0$. This generalizes the cash-in-advance constraint by allowing the consumption interval (frequency of payments) to differ from the money interval (frequency of sales).

In summary, the representative agent seeks to maximize her utility by choosing consumption, capital accumulation, and real money holdings, subject to the private budget and the cash-in-advance constraints, (1) and (2) respectively, taking as given the paths of prices and government transfers. To close the model, we next specify the government budget constraint: $s_t = m_t - \frac{m_t}{1+\pi}$, and the money supply process: $M_{t+1} = (1+\mu)M_t$.

B. Steady-State Analysis and Comparative Statics

The following equations describe the steady-state equilibrium of this economy (see the appendix): 11

$$Af_k(k) = \frac{1}{\tau(\mu)} \left[ \frac{1+\mu-\beta}{\beta^2} \theta(\phi) + \frac{1}{\beta} \right], \quad (3)$$

$$Af(k) + [\theta(\phi) - 1]k = \tau(\mu)m. \quad (4)$$

Equation (3) determines capital accumulation and is analogous to the

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9 This transactions frequency is inversely related to the interval between trips to the bank [see, Baumol (1952)]. For generalizations of the inventory model and discussions on the $\tau > 0$ assumption, see Fried (1973), Clower and Howitt (1978), and Romer (1986).

10 The introduction of $\tau$ in our model closes the gap between expenditure and cash holdings and hence the cash-in-advance constraint must hold with equality.

11 It can be shown that the stationary point of this economy is unique and saddle-point stable.
modified-golden-rule condition.\textsuperscript{12} Equation (4) demonstrates the steady-state relationship between real money balances and the capital stock. Utilizing (4) and $\mu=\pi$, we can also determine the nominal interest rate $i$ endogenously:

$$i = \Lambda f_k(k) + \pi = \frac{1}{\tau(\mu)} \frac{1+\mu-\beta}{\beta^2} \theta(\phi) + \frac{1}{\beta} + \mu,$$

(5)

which depends on money growth and institutional (financial and fiscal) changes.

In terms of the model presented above, the income velocity of money is defined as:

$$V = \frac{Y}{m} = \frac{c+k}{m} = \tau(\pi) \frac{c+k}{c+\theta(\phi)k} = \tau(\pi) \frac{1}{1-(1-\theta)(k/Y)}.$$

(6)

It is then apparent that velocity depends positively on $\tau$ and $k/Y$ but negatively on $\theta$. Differentiating (3) and (4), one can derive the effects of $\mu$, $\phi$, and $\Lambda$ on the steady-state level of output (see the appendix for the details):

$$\frac{dY}{d\mu} = \frac{\theta \Lambda f_k}{\Lambda} \left[ (1+\mu-\beta) \frac{\tau \pi}{\tau} - 1 \right] \geq 0,$$

(7a)

$$\frac{dY}{d\phi} = \frac{\Lambda f_k}{\Lambda} \left[ - \theta (1+\mu-\beta) \right] > 0,$$

(7b)

$$\frac{dY}{d\Lambda} = \frac{f_k^2}{f_{kk}} > 0,$$

(7c)

where $\Lambda = -\tau \beta^2 \Lambda f_k > 0$. Furthermore, using (5) and (6), we obtain:

$$\frac{dl}{d\mu} = 1 - \frac{\theta}{\tau \beta^2} \left[ (1+\mu-\beta) \frac{\tau \pi}{\tau} - 1 \right] \geq 0,$$

(8a)

\textsuperscript{12}In general, the cash-in-advance economy will accumulate less capital than the one corresponding to the modified golden rule. This follows from the fact that part of savings is retained in the form of money. If $\theta=0$, however, or if $\beta=1+\mu$, then equation (3) simplifies to $\Lambda f_k=1/\beta$, which is the modified golden rule condition in an economy with fully depreciated capital and constant population size.
\[ \frac{dj}{d\phi} = \frac{\theta\phi}{\tau \beta^2} (1+\mu-\beta) < 0 \]  
(8b)

\[ \frac{dj}{dA} = 0 \]  
(8c)

\[ \frac{dV}{d\mu} = \frac{V}{\tau} \left[ (1-\theta) \frac{V}{\tau} \frac{d(k/Y)}{d\mu} + \frac{\tau}{\nu} \right] > 0, \]  
(9a)

\[ \frac{dV}{d\phi} = \frac{V^2}{\tau} \left[ \frac{d(k/Y)}{d\phi} \frac{k}{Y} \phi \right] > 0, \]  
(9b)

\[ \frac{dV}{dA} = \frac{V^2}{\tau} \frac{d(k/Y)}{dA} = 0, \]  
(9c)

where (8c) and (9c) hold if technical progress enters the production function in a Harrod-neutral way.\(^\text{13}\)

A higher money growth rate has two opposing effects on capital, real money holdings, and velocity. First, it yields a higher rate of inflation, which raises the cost of holding money and thus decreases the net rate of return on capital, given the cash-in-advance constraint. This will in turn cause a decrease in nominal interest rate, capital, consumption, real money balances, and output. Under diminishing returns, the capital-output ratio falls, and thus from (6) a lower money velocity will emerge. This implies that real money balances decrease proportionately less than output. Second, more rapid money growth will increase the transactions frequency, \(\tau\), which releases part of money holdings. In equilibrium, this increases nominal interest rate, capital and output and, under dynamic efficiency, consumption must also rise. Further, from (6), higher transactions frequency has a direct positive effect

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\(^{13}\) The assumption of Harrod-neutral technical progress is very common in the literature since it is required for the existence of a steady-state equilibrium [for example, see King et al. (1988a)]. Furthermore, if the production function takes the Cobb-Douglas form then the Harrod-neutral technical progress becomes identical to Hicks-neutral and capital-augmenting.
In summary, the overall effects of a higher \( \mu \) on nominal interest rate, capital, consumption, real balances, and money velocity are ambiguous.

Any institutional change captured by an increase in \( \phi \) has a direct positive effect on money holdings and results in a higher marginal cost of capital. This decreases the nominal interest rate and leads to lower levels of output, consumption and capital, which, under the cash-in-advance constraint, requires less money to facilitate transactions. Overall, its net effect on real money balances is ambiguous. Nevertheless, velocity will decrease because even if money balances decrease, output falls proportionately more.

Finally, a technological improvement increases the marginal product of capital and thus increases the steady-state level of capital and output. Under dynamic efficiency, consumption will increase as well. Hence, to finance the higher levels of consumption and investment, real money holdings must also rise. In the case of a Harrod-neutral technical progress, output and real balances are found to increase proportionately and thus, from (6), money velocity will remain unchanged.

It is worth noting that these results differ from those in the existing literature in several aspects. First, contrary to the standard inventory model, output and velocity may move in the same direction, even if money is a luxury good. In our framework, the direction in which these two variables move depends on the parameter (shock) that initiates the movement. Second, in contrast to conventional beliefs, money velocity and inflation may be negatively correlated when the transactions frequency does not change significantly. Third, unlike the standard cash-in-advance model, rapid money

\[14\] This is the mechanism emphasized in the traditional partial equilibrium literature.
growth may promote capital accumulation, i.e., the Tobin effect may be present. This will occur in the case where inflation increases significantly the frequency of transactions and thus relaxes the cash-in-advance constraint. Finally, in contrast to reduced-form money demand models, our general equilibrium framework allows us to decompose the effect of interest rate on money velocity into two parts; money supply growth and institutional (financial or fiscal) changes.

III. The Empirical Methodology

We next use the structural vector autoregression technique developed recently by Blanchard and Quah (1989). This technique is different from Sims' VAR approach (Sims 1980) under which the decompositions of the shocks are not unique and depend on the ad hoc ordering of the variables. It also differs from Bernanke’s estimation method [Bernanke (1986)], which imposes restrictions on the short-run coefficients. In contrast, Blanchard and Quah’s method relies only on long-run restrictions. Since most macroeconomic debates regarding the interactions between the real and the monetary sector are about the short-run effects, the use of the Blanchard and Quah’s method seems more appropriate for this study. Furthermore, we estimate the same system over two sample periods with different monetary and exchange rate regimes. Thus, we need to identify the system without relying on restrictions that would be appropriate for one regime but not for the other. This seems more likely if we use long-run rather than short-run restrictions.

We therefore estimate a structural VAR by imposing only long-run restrictions based on our theoretical model. This procedure enables us to

identify the econometric model and to examine both the short-run and long-run effects of three types of shocks—monetary, institutional, and technological—on velocity and output.

A. Identification

In this section, we outline the identification procedure introduced by Ahmed et al. (forthcoming). Accordingly, consider any structural model which, in a moving-average representation, can be written as:

\[ X_t = G(L)\varepsilon_t \]  

where \( X_t \) is a (N×1) vector of stationary variables, \( G \) is a non-singular matrix polynomial in the lag operator \( L \), and \( \varepsilon_t \) is a vector of \( N \) independent, serially uncorrelated shocks.

Suppose that the theoretical framework implies a triangular matrix of the long-run coefficients, \( G(1) \). This and the orthogonality of the structural disturbances (shocks) are then sufficient for identifying the system. To see it, rewrite (10) in VAR form as:

\[ H(L)X_t = \varepsilon_t \]  

where \( H(L) \) is the inverse of \( G(L) \) and thus \( H(1) \) is also lower triangular. By separating the long-run component, we have:

\[ H^*(L)\Delta X_t = -H(1)X_{t-1} + \varepsilon_t \]  

where \( H^*(L) = (1-L)^{-1}[H(L)-H(1)L] \) and \( \Delta \) denotes the difference operator. Equations (10), (11), and (12) represent different forms of the same structural model. They can all be retrieved as shown below.

Consider the reduced form of (12),

\[ B(L)\Delta X_t = QX_{t-1}^* + \varepsilon_t^* \]  

where \( B(0) \) is the identity matrix. This can be estimated equation-by-equation using OLS and regressing \( \Delta X_t \) on its lagged values and on \( X_{t-1} \). Moreover, manipulation of (13) yields:
\[ JQ^{-1}B(L)AX_t = JX_{t-1} + JQ^{-1}e_t^* \]  

(14)

where \( J \) is the inverse of the Cholesky factor of \((Q^{-1})\Sigma(Q^{-1})'\). Since \( J \) is lower triangular and \( \text{Var}(JQ^{-1}e_t^*) \) is diagonal, (14) provides estimates of the structural parameters in (12) and hence the structural form (10) can also be retrieved. In particular, notice that the estimated \( G(1) \) matrix is equal to \( J^{-1} \). It is well known that the Cholesky factor is unique up to the signs of its diagonal elements. Since these signs must be determined within the theoretical setup, this recovery process is unique, indicating that (12) and hence (10) are also uniquely identified.

B. The Estimated Long-Run Model

Our estimated system includes three macroeconomic variables: the growth rate of money, the income velocity of money, and output. The three structural shocks are a money-supply shock \((c_\mu)\), an institutional shock which affects cash/credit transactions \((c_\phi)\), and a technological shock \((c_A)\). In terms of our theoretical framework, these shocks correspond to changes in \( \mu, \phi, \) and \( A \).

Note, in particular, that a negative \( c_\phi \) shock indicates a credit-enhancing financial or fiscal policy.

We use quarterly U.S. data covering the national bank era, 1880:1-1912:II as well as the post-1973 period, 1974:1-1990:IV.\(^{16}\) These periods differ at least in two aspects: the exchange rate regime and, more importantly for the purpose of this paper, the average inflation rate. As mentioned, the second period exhibits persistently higher inflation rate (see footnote 3).

\(^{16}\)Although the data for the national bank era are available for a little longer period we truncate them for the following reasons. After the Civil War the federal government slowly reduced the money supply, and by the year 1879 the dollar returned to convertibility with gold at the pre-war rate. Thus, our data series start from 1880. On the other hand, the year 1912 is chosen to avoid the establishment of a different regime, i.e., the Federal Reserve System.
The data series for the first period are taken from Gordon (1986) and for the second, from the Citibase data tape. The three macroeconomic variables of the system are measured as follows. The first, the money growth rate, is measured as the first difference of the logarithm of the monetary base. The second variable, the velocity of money, is calculated as the ratio of nominal GNP to M2. Finally, the third variable, output, is measured by real GNP. We use LM, LV, and LY to denote the monetary base, M2-velocity, and output in log levels and attach an initial D to these variable names to indicate first differences (DLM, DLV, and DLY). (All series are plotted in Figures A1 and A2 in the appendix.)

To implement the estimation procedure described in section III.A, all the variables in equation (10) must be stationary. Using the augmented Dickey-Fuller test, we find the monetary base, M2-velocity, and real GNP to be integrated of order one, i.e., they are all stationary in the first differences.

In order to identify the model, we assume that long-run movements in money growth are independent of institutional or output shocks, i.e., the elements $G_{12}$ and $G_{13}$ of the long-run coefficient matrix $G(1)$ in equation (10) are zero. During the post-1879 national bank era, the gold standard regime was adopted and changes in the monetary base were tied closely to the exogenous supply of gold. Thus, at least in the long run, we can treat the monetary base

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17. We use M2 as a measure of nominal money demand because the M1 data series in the first period is not available. Although the M2 measure involves precautionary in addition to transactions use of money, we capture this by allowing for a variable frequency of transactions [see equation (2)].

18. The $t$-statistics for the monetary base, the money velocity, and output are -2.21, -2.24, and -2.76 in the first period, and -3.31, -1.90, and -2.26 in the second. All of them are in absolute value below the critical value -3.45, at 5 percent significance level.
as being independent of the institutional or output shocks.\textsuperscript{19} The exogeneity of
the monetary base in the post-1973 period however is less obvious. Notice
though that the adoption of the flexible-exchange-rate regime, over this
period, makes the assumption about the long-run behavior of money supply more
plausible, as compared to the Bretton Woods system. Under the latter regime,
output shocks are likely to lead to changes in money demand and thus changes
in money supply as well, in order for the Central Bank to maintain a fixed
exchange rate. Furthermore, by using the monetary base as a measure of the
money stock we can avoid any criticism regarding reverse causation, i.e., an
output shock affects money through its effect on the demand for transaction
services.\textsuperscript{20}

Another identifying restriction is that a technological shock has no
long-run effect on the velocity of money, and so the element $G_{23}$ of $G(1)$ is
also zero. This is based upon the prediction derived from the theoretical
model [see equation (9c)].

In summary, the structural model (10), in its long-run form, can be
specified as:

\begin{equation}
\begin{bmatrix}
DLM \\
DLV \\
DLY \\
\end{bmatrix} = \begin{bmatrix}
m_0 \\
v_0 \\
y_0 \\
\end{bmatrix} + \begin{bmatrix}
G_{11} & 0 & 0 \\
G_{21} & G_{22} & 0 \\
G_{31} & G_{32} & G_{33} \\
\end{bmatrix}\begin{bmatrix}
\epsilon_{\mu} \\
\epsilon_{\phi} \\
\epsilon_{A} \\
\end{bmatrix}
\end{equation}

\text{(15)}

where $m_0$, $v_0$, and $y_0$ denote constant terms and all the diagonal elements of
$G(1)$ are positive by construction.

\textsuperscript{19} During the national bank era, most of the major output innovations were
common world shocks, which under the gold standard regime, generated no
country-specific feedback effects on money supply. Nevertheless, we do not
preclude from money accommodations in the short run. Further support of the
argument that the money supply is inelastic with respect to output in the
national bank era can be found in Barro (1989).

\textsuperscript{20} We will further comment on this assumption after performing the variance
decomposition analysis (see section IV below).
IV. Results

Theoretically, the fact that G(1) is lower triangular implies that it is also non-singular. In practice, to guarantee the existence of three stochastic trends, driving the short-run and long-run paths of LM, LV, and LY, we need to test the hypothesis that there is no co-integration among these variables. We test various co-integration relationships using the method outlined in Engle and Yoo (1987). More specifically, we test whether velocity or output are co-integrated with money. The former will be true under a Cagan-like money demand setup in which velocity is exclusively driven by money growth or anticipated inflation. The latter, on the other hand, reflects perfect money-output correlation. We also test whether there exists a co-integration relationship among all three variables. Velocity may be co-integrated with money and output if the institutional shock does not generate any permanent effect, while output may be co-integrated with the other two variables if the technological shock has only a transitory impact. In all cases, we fail to reject the null hypothesis of no co-integration at the 5 percent significance level.

We are now prepared to estimate the reduced form and then retrieve the structural model, as given by equation (10). Based on the Schwarz (Bayesian) information criterion, we have chosen the lag length to be two in both sample periods.

21 The t-statistics in testing co-integration between LV-LM, LY-LN, LV-LN-LY, and LY-LN-LY are -1.79 (-1.95), -2.89 (-2.59), -1.93 (-1.95), and -3.01 (-2.58), respectively, for the first (second) period. All of them are in absolute value below the critical values, 3.39, in the two-variable case, and 3.93, in the three-variable case (see Engle and Yoo (1987), Table 2).

22 Lütkepohl (1985) presents the results of a simulation study in which various criteria are compared with other classical test procedures. He finds that the
We focus below on the estimated long-run responses, impulse responses, and variance decompositions of velocity and output.

A. Long-Run Responses

By normalizing all diagonal terms to one, the elements of the estimated long-run moving average matrix polynomial, $G(I)$, show the long-run responses of each transformed variable to a one standard-deviation change in each of the shocks.

Consider first the national bank era. The long-run responses of the growth rate of velocity and output to a money growth shock are -0.611 and -0.050, respectively, while the response of output growth to an institutional shock is 0.639. These results suggest that a reverse Tobin effect is present, i.e., higher money growth retards output growth, and that the transactions frequency effect is small; thus velocity decreases as money growth increases. Moreover, credit enhancement caused by either a financial or a fiscal shock is found to increase output, which is consistent with our theoretical model. In general, computer simulated standard errors, using 1000 replications, for off-diagonal estimates are large. Nevertheless, all of our results except for the reverse Tobin effect are significant at 95 percent confidence level.\(^\text{23}\)

In the second period, in which sustained inflation was present, we find remarkably different long-run responses. Specifically, the long-run responses of velocity and output to monetary shocks are positive (0.261 and 0.200, respectively), whereas the response of output to institutional shocks is

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\(^{23}\) The best criterion is that of Schwerz, in terms of the frequency of the determination of the true order and the smallest value of the error prediction.

\(^{23}\) The standard errors for the elements $G_{21}$, $G_{31}$, and $G_{32}$ of $G(I)$ are 0.26, 0.31, and 0.16, respectively.
essentially zero (-0.056). Due to large standard errors, however, we will not draw any conclusion before examining the short-run responses.

B. Short-Run Dynamics

The impulse response functions show how the dependent variables respond over a 20-quarter horizon to a one standard-deviation change in each shock. The impulse responses of the first differences of velocity and output for the first period are shown in Figure 1a and for the second, in Figure 2a. Starting from the top, the figures display the responses to the first (monetary), the second (institutional), and the third (output) shock, respectively. The one-standard-error bands are also plotted in these figures. These bands are not too wide, indicating that the impulse responses are reasonably precise. We have also plotted the responses of the levels, obtained by accumulating the first difference responses, which display the results more clearly (Figures 1b and 2b).

Responding to a monetary shock, output and velocity move in the same direction. In the first period, they decrease significantly over the first two or three quarters and then increase slightly for the next four quarters as they approach a permanently lower level. The adverse effect of a monetary shock on output diminishes quickly, whereas the adjustment of velocity is smooth and gradual. The result indicates that, in terms of our theoretical model, money growth rate affects velocity indirectly through its general equilibrium effect on the capital stock and on output, a channel that has not been considered in the existing literature. The dynamic responses over the second period, on the other hand, show that both velocity and output respond

24 The standard errors for the elements $C_{21}$, $C_{31}$, and $C_{32}$ are 5.05, 2.61, and 0.40.
positively to the monetary shock after the immediate impact effect. The adjustment of velocity is again smoother than that of output. Based on our theoretical model, we conclude that, in the case of persistently higher inflation, the Tobin effect is present. Notice, however, that the positive impact of money on output starts diminishing after five quarters. Furthermore, since the transactions frequency plays a more significant role as inflation rises, the conventional positive relationship between velocity and money growth emerges. Using the criterion of one standard error, similar to Blanchard and Quah (1989), we find that these dynamic effects of the monetary shock on velocity and output are significant at least over the first few quarters.

In both periods, institutional shocks, such as credit-enhancing financial innovations or expansionary fiscal policies, increase velocity and lead to higher output in the short run. This is consistent with our theoretical predictions. More specifically, the dynamic responses of output are hump-shaped: the effects are initially big and then go down quickly. In contrast to the first period, the positive effect of the institutional shock on output diminishes more rapidly in the second period, approaching a negative but small long-run value. In general, these results conform with the theory, developed by Greenwood and Jovanovic (1990), that financial development increases the rate of return on capital and thus enhances economic growth. It also conforms with Rush (1985) who finds that during the gold standard era fluctuations in output are associated with fluctuations in the level of financial intermediation.

Finally, consider the responses to an output shock. In the short run, velocity tends to decrease, indicating that M2 is a luxury good. The effect, however, dies out within two years, in both sample periods. The response of
output to its own shock, on the other hand, is found to be similar to that in the empirical study of Shapiro and Watson (1988) and that in the calibration analysis of King, Plosser, and Rebelo (1988). Specifically, it approaches smoothly and gradually a permanently higher long-run value. Notice, however, that this long-run value is much smaller in the second period than it is in the first.

C. Variance Decompositions

The variance decompositions are used to evaluate the importance of each shock in explaining changes in the dependent variables. The estimates in Tables 1a-1c (2a-2c) give the percentage of the forecast error variance for each variable which is accounted for in the first (second) period by a one standard deviation change in each shock. The selected horizon ranges from 1 to 20 quarters.

The first noteworthy result is that even in the short run, the money growth rate in the first period is virtually exogenous with respect to the institutional and technological shocks. Specifically, more than 96 percent of the variance of money growth is explained by its own shocks. In the second period, however, about 14 percent of money growth variations are due to institutional changes. With this feedback effect, our empirical study may underestimate the role of the institutional shock in the last period.

Second, money velocity is explained exclusively by the monetary and institutional shocks. Although their initial adverse effects are less influential, monetary shocks in the first period account for more than 20 percent of velocity movements over a longer horizon. On the contrary, in the second period, their importance decreases as we increase the horizon length. Moreover, in both periods, institutional shocks account for more than 75 percent of velocity movements. This is generally in accordance with the
findings of Bordo and Jonung (1987, 1990) who conclude that institutional changes have been crucial determinants of money velocity. Moreover, since monetary and institutional shocks constitute the interest rate disturbances, our result explains why the nominal interest rate is the main driving force of the velocity of money.

Finally, in contrast to the monetary, the institutional shock is very influential in explaining output growth, e.g., at the 4-quarter horizon, it accounts for about 50 percent in the first and 20 percent in the second period.

D. Further Remarks

In late 1890s, there were some major discoveries of gold in South Africa, Alaska, and Colorado. Such discoveries not only backed money supply, but also stimulated an increase in prices and output. Thus, spurious co-movements between money and output may possibly be observed. To circumvent this problem, we re-estimate the structural VAR for the 1880:1-1896:IV sample period. (The impulse responses are shown in Figures A3-A4 and the variance decompositions in Tables A1-A3 in the appendix.)

The results are qualitatively consistent with those found using the whole sample period, indicating that our estimation is likely to be stable. However, by eliminating the possibly spurious money-output co-movements, the reverse Tobin effect is now larger. As a consequence, money growth rate becomes more

25 Starting from late 1890's to 1914, the gold stock increased by an average rate of 3.5 percent per year. By contrast, in the previous twenty years it increased by an average of 1.5 percent per year [see Friedman and Schwartz (1963)].

26 During the 1880-1896 period, the price declined at a rate of about one percent a year. Over the 1897-1912 period, on the other hand, the price level increased by two percent per year, as a result of the major gold discoveries which took place after 1896.
influential on velocity. To be specific, the long-run responses of velocity and output to monetary shocks are now -0.892 (0.43) and -0.209 (0.42), respectively (standard errors in parentheses), which are much larger in magnitude than the responses obtained previously. From the impulse response diagrams, it can be clearly seen that the reverse Tobin effect is not only bigger but also more persistent. Furthermore, the variance decompositions show that monetary shocks account for about 8 percent of output movements within 4 quarters, as well as for 17 to 51 percent of velocity changes over the 20-quarter horizon. 27

V. Conclusions

This paper uses a structural VAR approach to study the sources of output and velocity fluctuations in the national bank era as well as in the post-1973 period. The estimated system includes three structural disturbances--monetary, institutional, and technological--which are identified using only long-run restrictions.

We find that in the first period the reverse Tobin effect is present and that money growth creates a significant negative effect on velocity. In the second period, however, the persistent inflation highlights the role of transactions frequency and thus higher money growth leads to an increase in velocity. Other institutional changes which enhance credit transactions have a permanently positive effect on velocity, in accordance to Bordo and Jonung (1987, 1990), and a hump-shaped effect on output, lending support to Greenwood and Jovanovic (1990). A technological shock causes a short-run decrease in M2-velocity, while its positive effect on output is smooth and gradual.

27 Although the standard errors are large, the latter effect, after four quarters, is significant even at 95% confidence level.
corroborating with the findings of Shapiro and Watson (1988) as well as with the calibration results in King, Plosser, and Rebelo (1988). Furthermore, our findings, regarding money supply and institutional (financial and/or fiscal) shocks being the main sources of fluctuations in velocity, are consistent with the conventional beliefs that the velocity of money has mainly been driven by the nominal interest rate.

Finally, we find that in response to a monetary or an institutional shock, output and velocity move in the same direction. In response to a technological shock, on the other hand, they move in opposite directions. Thus, the observed negative correlation between the two variables over the first sample period is apparently due to technological disturbances, whereas the positive correlation in the second period is a consequence of monetary or institutional shocks.

In a recent simulation study, Hodrick, Kocherlakota, and Lucas (1991) conclude that a conventional stochastic cash-in-advance model cannot generate enough variations in velocity to capture what economists have observed. However, our modified cash-in-advance framework, which incorporates institutional and transactions frequency changes, enables us to examine possible sources of fluctuations in velocity as well as in output. Along this line, it would be interesting to expand the VAR model in order to examine the interactions between these variables and the variability of relative prices, as well as the paradoxical relationship between prices and interest rates, better known as the "Gibson paradox."
References


Gordon, R. J., 1986, The American Business Cycle: Continuity and Change, NBER,
Rush, M., 1985, Unexpected Monetary Disturbances during the Gold Standard Era,
Table 1. Variance Decompositions (1880:I-1912:II)†

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Structural Disturbances</th>
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<tbody>
<tr>
<td></td>
<td>Monetary ($\mu_e$)</td>
</tr>
<tr>
<td>a. Variance Decomposition of Money Growth (%)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>99.52 (4.26)</td>
</tr>
<tr>
<td>2</td>
<td>97.49 (4.00)</td>
</tr>
<tr>
<td>20</td>
<td>96.66 (5.02)</td>
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<tr>
<td>b. Variance Decomposition of Velocity (%)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7.44 (6.01)</td>
</tr>
<tr>
<td>2</td>
<td>14.65 (8.25)</td>
</tr>
<tr>
<td>4</td>
<td>20.58 (10.16)</td>
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<tr>
<td>8</td>
<td>21.95 (11.89)</td>
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<tr>
<td>20</td>
<td>22.60 (13.35)</td>
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<td>c. Variance Decomposition of Output (%)</td>
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<tr>
<td>1</td>
<td>5.75 (6.06)</td>
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<tr>
<td>2</td>
<td>5.52 (6.33)</td>
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<tr>
<td>4</td>
<td>3.70 (6.15)</td>
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<td>8</td>
<td>1.79 (5.68)</td>
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<td>20</td>
<td>0.79 (6.24)</td>
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Table 2. Variance Decompositions (1974:I-1990:IV)†

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</thead>
<tbody>
<tr>
<td></td>
<td>Monetary ($\mu_e$)</td>
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<tr>
<td>a. Variance Decomposition of Money Growth (%)</td>
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<tr>
<td>1</td>
<td>84.76 (19.96)</td>
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<td>85.05 (13.82)</td>
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<td>b. Variance Decomposition of Velocity (%)</td>
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<tr>
<td>1</td>
<td>40.27 (22.03)</td>
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<tr>
<td>2</td>
<td>21.93 (17.74)</td>
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<tr>
<td>4</td>
<td>9.82 (13.71)</td>
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<td>8</td>
<td>4.55 (12.17)</td>
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<tr>
<td>20</td>
<td>2.66 (14.95)</td>
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<tr>
<td>c. Variance Decomposition of Output (%)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7.28 (9.74)</td>
</tr>
<tr>
<td>2</td>
<td>2.95 (6.74)</td>
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<tr>
<td>4</td>
<td>2.93 (8.70)</td>
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<tr>
<td>8</td>
<td>3.83 (12.97)</td>
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<tr>
<td>20</td>
<td>2.34 (16.12)</td>
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</table>

† Numbers in parentheses are simulated standard errors of the estimates from 1000 replications. Sums may not add to 100 due to rounding.
Figure 1a. Impulse Responses of Log Differences of Velocity and Output (1880:1-1912:II)

(1) To a Standard Deviation Monetary Shock

(2) To a Standard Deviation Institutional Shock

(3) To a Standard Deviation Output Shock
Figure 1b. Impulse Responses of Log Levels of Velocity and Output (1880:1-1912:II)

(1) To a Standard Deviation Monetary Shock

(2) To a Standard Deviation Institutional Shock

(3) To a Standard Deviation Output Shock
Figure 2a. Impulse Responses of Log Differences of Velocity and Output (1974:I-1990:IV)

1. To a Standard Deviation Monetary Shock

2. To a Standard Deviation Institutional Shock

3. To a Standard Deviation Output Shock
Figure 2b. Impulse Responses of Log Levels of Velocity and Output (1974:I-1990:IV)

(1) To a Standard Deviation Monetary Shock

(2) To a Standard Deviation Institutional Shock

(3) To a Standard Deviation Output Shock
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