The Algebra of Price Stability

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The Algebra of Price Stability

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I. Introduction

Legislation recently introduced by Congressman Stephen Neal would mandate that price stability receive the highest priority among the goals of the Federal Reserve. Because of the renewed policy interest in price stability, the potential costs and benefits of achieving it have attracted much attention. One of the more important benefits attributed to price stability is that it would lessen the uncertainty associated with the price level and the detrimental effects this uncertainty can have on long-term contracting and resource allocation (Hall 1981, Black 1990, Parry 1990, Hoskins 1991, Summers 1991). Proponents of price stability argue that because money is an intertemporal store of value, an uncertain price level causes people to devote resources to protecting themselves against potential declines in the value of money. Therefore, the elimination of price level uncertainty would allow a more efficient allocation of resources. A second possible benefit from price stability is a reduction in the discretion afforded policymakers in setting monetary policy. Proponents argue that if price stability is a central bank's primary responsibility, then this goal would reduce policymakers' pursuit of other discretionary goals. The costs of achieving price stability are primarily associated with the short-term adjustment costs of moving from the current inflationary regime to the price stability regime, such as the output lost when actual inflation is less than anticipated.

While the subject of price stability has received much attention, there has been relatively little discussion about specific monetary policies that

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1 House of Representatives Joint Resolution 409.

would enable a central bank to achieve and maintain price stability or the
degree to which price stability would conflict with other policy objectives.
Therefore, rather than add to the debate about the costs and benefits of price
stability, in this paper we investigate the conditions under which monetary
policy would ensure price stability and examine the effectiveness of a broad
class of monetary rules in achieving price stability. We also examine whether
price stability constrains the monetary policy authorities' pursuit of short-
run stabilization objectives. Throughout the paper, the focus is on the
restrictions that price stability imposes on the conduct of monetary policy,
rather than on the desirability or optimality of price stability.

Unfortunately, a precise definition of price stability has not emerged
from recent literature. Fortunately, however, all advocates and opponents of
price stability appear to have either one of two types of definitions in mind:
some form of targeting the price level, which we refer to as strong price
stability (SPS), or some form of targeting inflation at zero (or at least at a
very low rate), which we refer to as weak price stability (WPS). In order to
make our analysis of price stability concrete, we propose formal definitions
for both of these versions of price stability. At a minimum, strong price
stability requires that the price level be a stationary stochastic process
while weak price stability requires that inflation be a stationary stochastic
process. As we show below, these definitions imply quite different long-run
behavior of prices.

For each definition of price stability, we outline the long-run
restrictions on the behavior of the monetary aggregates needed to achieve
price stability and demonstrate conditions under which specific monetary rules
are consistent with price stability. The monetary rules examined include
monetary aggregate targeting, nominal GNP targeting, price level targeting, and interest rate targeting. We show that the monetary aggregate restrictions associated with SPS are much more powerful than those associated with WPS. In fact, implementing WPS would not necessarily require any changes in the stationarity properties of the price level or the monetary aggregates. For SPS, we show that targeting the price level and, to a lesser extent, nominal GNP are more likely to be consistent with strong price stability than is targeting monetary aggregates or interest rates.

In addition, we use a simple linear rational expectations macro model to explore how the conditions for price stability may constrain other monetary policy objectives. We find that the conditions for SPS and WPS impose few constraints on the ability of the monetary authorities to pursue short-run stabilization objectives. Intuitively, this result follows from the fact that the conditions for price stability are of a long-run nature while stabilization objectives are of a short-run nature.

The distinction between short- and long-runs also raises the question of what is the relevant time horizon for evaluating the performance of policies that satisfy the alternative definitions of price stability. We show that the main difference between WPS and SPS is that the latter results in a reduction of long-term uncertainty about the price level. In fact, advocates of SPS cite this attribute of SPS as the main reason for pursuing it. In our definitions of price stability, however, the relevant time horizon is essentially infinity. Shorter horizons may be more relevant for business planning horizons over which the benefits of reduced price level uncertainty are likely to accrue. We find that while SPS helps to reduce price level uncertainty at shorter time horizons, it is not a sufficient condition for
minimizing price level uncertainty. For longer time horizons, SPS is a necessary condition for minimizing price level uncertainty.

The paper is organized as follows. The second section defines and discusses the characteristics of strong and weak price stability. Section three outlines the restrictions on the monetary aggregates that are consistent with our definitions of price stability and section four evaluates specific monetary rules for their ability to achieve price stability. In section five, a macro model is introduced to examine whether achieving price stability precludes the pursuit of short-run stabilization policies. Section five also examines the connections between strong price stability and price level uncertainty at horizons less than infinity. Conclusions and discussion follow.

II. Defining Price Stability

Before we examine the implications of price stability for monetary policy, we must be more precise about the possible interpretations of the term "price stability." A careful reading of the literature indicates that the different interpretations of price stability fall into two groups. One emphasizes that price stability requires the long-run level of prices be targeted (SPS) [e.g., Black (1990), Gavin and Stockman (1988), Hetzel (1990), Hoskins (1991), and Parry (1990)], while the other emphasizes that price stability entails targeting only the inflation rate (WPS) [e.g., Corrigan (1990), McCallum (1990b), Neumann (1991), and Summers (1991)].

Under SPS, the monetary authorities ensure that the price level is described by a stochastic process of the form:

3 Not all the WPS advocates support a "zero" targeted inflation rate.
\( p_t = p^* + \epsilon_{pt} \)

where \( p^* \) is the targeted long-run value of the price level and \( \epsilon_{pt} \) is a mean zero stationary stochastic process. Because \( \epsilon_{pt} \) is a mean zero stationary stochastic process, \( p^* \) is the unconditional mean or long-run average value of the price level under strong price stability. The important characteristic of pursuing this target is that the price level will not display drift. Depending on the specifics of the price stability mandate, gaps between \( p_t \) and \( p^* \) will eventually be closed.

Under WPS, the monetary authorities essentially pursue an inflation target. This yields an inflation process of the form

\[ \pi_t = \mu + \epsilon_{\pi t}. \]

where \( \mu \) is the inflation target over a given time interval and \( \epsilon_{\pi t} \) is a mean zero stationary stochastic process. For the remainder of the paper we consider the zero inflation target (that is, set \( \mu \) equal to zero). Because there are no other restrictions on \( \epsilon_{\pi t} \), weak price stability means that while inflation will be a stationary stochastic process the (log) price level itself may be a nonstationary or integrated process. This is clearly seen for the case where \( \epsilon_{\pi t} \) is white noise. In this case, inflation is white noise but the price level is a random walk. Price level drift occurs because, WPS by targeting the inflation rate fails to reverse shocks to the price level. Under a policy that targets inflation, the monetary authorities take the price level as given at the beginning of each period and then attempt to achieve the target inflation rate. They do not attempt to reverse shocks that occur to the price level because of monetary control errors and macroeconomic shocks that drive a wedge between the actual and targeted inflation rates.

Note that under both definitions of price stability the average
inflation rate or unconditional expectation of inflation, \( E(\pi_t) \), is zero.\(^4\) However, long-term price level uncertainty is going to be substantially greater under weak price stability than under strong price stability. In contrast to the constant variance of the price level rule, the uncertainty associated with long-run forecasts of the price level under the zero inflation rate rule grows linearly with the forecast horizon.\(^5\) This difference arises because unexpected changes in the price level under an inflation rule are permanent. After a shock, the monetary authorities are interested only in achieving zero inflation from that time hence; they do not offset the price shock. Therefore, there is substantially more long-term uncertainty about the price level under the inflation rule than under the price level rule. The short-term uncertainty depends on the structure of the economy, the information available to the monetary authority, and the sources of shocks to the economy. As we show below, in general, there are no clear predictions about which type of rule would generate the most short-term uncertainty.

The perceived additional reduction in long-run uncertainty gained from a price level rule is one of the main issues that divides those who argue for

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\(^4\) However, in any given period, strong price stability does not imply that inflation should be equal to zero.

\(^5\) To illustrate that the price level rule results in less long-run uncertainty, let \( E(p_{t+k}|I_t) \) be the optimal forecast of \( p_{t+k} \), given the information set at time \( t \). For case where \( \epsilon_{pt} \) and \( \epsilon_{pt} \) are white-noise errors, under weak price stability price level the variance of k-horizon price level forecast is

\[
\text{var}[p_{t+k} - E(p_{t+k}|I_t)] = \sigma^2_{\pi k} ,
\]

while for strong price stability the variance of k-horizon price level forecast is

\[
\text{var}[p_{t+k} - E(p_{t+k}|I_t)] = \sigma^2_p .
\]

The assumption of white-noise errors is not crucial. We could assume that \( \epsilon_{pt} = a_p(L)\epsilon_{pt} \) and \( \epsilon_{pt} = a_\pi(L)\epsilon_{\pi t} \), where \( \epsilon_{pt} \) and \( \epsilon_{\pi t} \) are stationary stochastic processes. The comparison of long-run volatility is not substantially changed by this more general assumption about the \( \epsilon \)'s.
the SPS (e.g. Hoskins 1991) and those who argue for WPS (e.g. McCallum 1990b). Advocates of WPS do not view the additional reduction in long-run uncertainty as substantial relative to the gain from reducing average inflation to zero. The advocates of SPS, on the other hand, see this benefit as one of the main reasons for pursuing price stability.

III. Price Stability and Monetary Policy

This section examines the restrictions that the alternative definitions of price stability impose on the conduct of monetary policy. Strong price stability requires that the long-run level of prices fluctuate around a particular level; in other words, the price level is a stationary stochastic process. The requirement that prices be stationary—meaning that prices follow an integrated process of order zero, I(0)—imposes very strong restrictions on the long-run conduct of monetary policy. Consider the simple quantity theory relationship (in logarithms)

\[ p_t = m_t + v_t - q_t, \]

(3)

where \( m_t \) is a particular monetary aggregate, \( v_t \) is velocity, and \( q_t \) is real GNP in logarithms. Strong price stability requires that monetary policy must be conducted in such a way that \([m_t + v_t - q_t]\) is I(0).

For example, in order to achieve strong price stability when \( v_t \) and \( q_t \) are integrated of order 1, I(1), monetary policy must follow a feedback rule in which the money supply, velocity, and real GNP are cointegrated.

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6 The basic insights of this section hold for more general models of the macroeconomy. The appendix shows in a standard rational expectations macro model that the order of integration of the price level essentially depends on the order of integration of \( m_t \), shocks to aggregate supply, and shocks to aggregate demand. To achieve price stability, monetary policy must offset all sources of nonstationarity that otherwise affect the price level.
An example of such a feedback rule is:

\[
(4) \quad m_t = -v_{t-1} + q_{t-1} + \text{[I(0)]},
\]

where \text{[I(0)]} contains other terms that are I(0).  

Weak price stability requires that the inflation rate be I(0) with mean zero. To achieve weak price stability where \( v_t \) and \( q_t \) are I(1), the monetary authority need only set \( m_t \) to offset the deterministic drift in \( v_t - q_t \). Aside from the positive mean, U.S. inflation since 1946 has arguably satisfied the conditions for weak price stability. This suggests that aside from the positive drift in prices, current monetary policy is consistent with our definition of weak price stability.

On the face of it, the restrictions imposed by the two versions of price stability do not appear to be very powerful; yet, they can rule out entire classes of monetary policies. If \( v_t \) or \( q_t \) is not I(0), then under strong price stability the money supply must be chosen so that it will offset the nonstationarity of velocity and output; in other words, the actual money supply must follow some sort of feedback rule. In the long run, monetary policy must offset the effects of permanent shocks to

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velocity and real GNP.

IV. Evaluating Various Monetary Rules

This section examines some well-known monetary rules to determine whether they satisfy the conditions for strong price stability. We first consider the implications of price stability for money supply targets. We then consider the effects of target cones and target bands on the feasibility of price stability. Finally, we examine several feedback rules for monetary policy—including targeting nominal GNP, the price level, and interest rates—and determine under what conditions these rules ensure price stability.

IV.1 Money supply targets

Under k-percent growth rules, the monetary authority attempts to control money growth so that it increases at a k-percent annual rate in each time period, implying \( \mathbb{E}(\Delta m_t | I_{t-1}) = k \). In this case, actual money supply growth is given by

\[
(5) \quad m_t - m_{t-1} = k + \psi_t,
\]

where \( \psi_t \) is a control error (which is presumably a stationary stochastic process, typically assumed to be white noise). Note that if control errors are not offset, the money supply exhibits "base drift," meaning that control errors have a permanent effect on the money supply. Here, the money supply is an I(1) process independent of the behavior of \( v_t \) and \( q_t \).

This type of monetary rule is unlikely to meet the necessary condition
for strong price stability. Because \( m_t \) for the \( k \)-percent growth rule is \( I(1) \) regardless of the behavior of \( v_t \) and \( q_t \), \([m_t - v_t + q_t]\) is unlikely to be \( I(0) \) and, hence, strong price stability will not hold. However, as long as \( k \) is set equal to the deterministic drift in \( q_t - v_t \), the \( k \)-percent growth rule will satisfy the conditions for weak price stability.

In principle, target bands or target cones can force the monetary authority to offset control errors; the money supply is contracted if it exceeds the band and is expanded if it is below the band. However, the current method of specifying target cones (in addition to allowing the target to drift) is unlikely to yield strong price stability. The reason is that target cones are not a particularly binding constraint on the money supply in the long run.

For example, consider the case in which the money supply is given by a random walk with drift—that is, an on-average \( k \)-percent rule. The expected value of the future money supply and its one standard deviation confidence band are given by

\[
E(m_t | I_0) \pm SD = m_0 + kt + \sigma_\phi \sqrt{t},
\]

where \( m_0 \) is the initial money supply level and \( \sigma_\phi \) is the standard deviation of the control error. Compare this with a target cone of the form

\[
m_0 + kt \pm \delta t,
\]

where \( \delta t \) is the width of the cone. Regardless of the size of the cone \( \delta \) and the variance of control error \( \sigma_\phi^2 \), eventually the target cone will envelop the confidence band for \( m_t \). This characteristic implies that a

\[ \text{This critique of base drift has been mentioned by several authors, including Poole (1970).} \]

\[ \text{Only in the very unlikely case in which } v_t \text{ and } q_t \text{ automatically offset any permanent change in the money supply will prices be stationary.} \]
target cone is not a particularly binding constraint in the long run, when the money supply follows a random walk with drift. Indeed, a target cone that is never readjusted is insufficient to yield strong price stability, even if velocity and output are themselves I(0). This result suggests that the current monetary aggregate targeting procedure, which combines a target cone and base drift, is unlikely to produce strong price stability.

On the other hand, a target band of the form

\[ m_0 + kt \pm \theta, \]

where \( \theta \) is the band width, is consistent with strong price stability as long as \( v_t \) and \( q_t \) are stationary. A target band of the form given in equation (8) ensures that the money supply is stationary around a deterministic trend. Provided \( k \) is chosen to offset the deterministic drift in \( q_t \) and \( v_t \), the money supply will satisfy the conditions for strong price stability. Of course, when \( v_t \) and \( q_t \) are nonstationary, a target band will not achieve strong price stability.

IV.2 Nominal GNP targets

Several researchers have suggested using nominal GNP targeting as an intermediate target for monetary policy [e.g., Hall (1983), Tobin (1983), Gordon (1985), Taylor (1985), McCallum (1988, 1989, 1990a, 1990b)]. The motivation for this type of targeting is that it avoids many of the problems associated with monetary aggregate targeting, such as velocity instability.

We should mention several general points about nominal GNP targeting. First, by targeting the nominal GNP growth rate, the monetary authority can only be sure that nominal GNP (in logarithms), \( p_t + q_t \), is I(1); there is
no way of ensuring that prices are stationary. Thus, to achieve strong price stability, the appropriate nominal GNP target must be directed at the level of nominal GNP. On the other hand, to achieve weak price stability, the monetary authority need only target nominal GNP growth.

Second, if the monetary authority targets the level of nominal GNP, the attainment of strong price stability depends on the behavior of output, the nominal GNP target, and the implied real GNP and price level targets. To illustrate this point, let the logarithm of nominal GNP be given by $x_t = p_t + q_t$. Consider a simplified version of the nominal GNP rule considered by McCallum (1988, 1989, 1990a, 1990b), in which the money growth is determined by the feedback rule:

$$\Delta m_t = - \Delta y_{t-1} + \Delta x^*_t - \lambda (x_{t-1} - x^*_{t-1}),$$

where $x^*_t$ is the nominal GNP target and $0 < \lambda < 1$. Here, the money supply is set to offset past deviations of nominal GNP from its target.

Using the quantity theory relationship and the money supply equation to solve for prices yields:

$$p_t = 1/[1-(1-\lambda)L] \left[ \Delta^2 y_t + \Delta q^*_t - \Delta q_t + \Delta p^*_t - \lambda (q_{t-1} - q^*_{t-1}) \right]$$

$$+ \lambda p^*_{t-1} \right],$$

where $q^*_t$ and $p^*_t$ are the implied targets for real GNP and the price level, respectively (note: $x^*_t = q^*_t + p^*_t$). For strong price stability, the

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12 McCallum's nominal GNP rule looks like

$$\Delta m_t = 0.0075 - (\nu_{t-1} - \nu_{t-17})/16 - \lambda (x_{t-1} - x^*_{t-1}),$$

where the money supply is the monetary base, the time index represents quarters, and 0.0075 is the quarterly growth rate in potential or target real GNP, or $E(\Delta q^*_t)$.

13 The solution for $p_t$ given above ignores the complementary solution implied by the price level difference equation. To get the general solution for $p_t$, add the term

$$(1-\lambda) \left( p_0 - p^* - 1/[1-(1-\lambda)L] \left[ \Delta^2 y_0 + \Delta q^*_0 - \Delta q_0 + \Delta p^*_0 - \lambda (q_{-1} - q^*_{-1}) \right] \right).$$
appropriate price target is a constant—that is, $p^*_t = p^*$. Thus, reducing (10) yields

$$p_t = p^* + q^*_t - q_t + \left[ \frac{1}{1 - (1 - \lambda) L} \right] \Delta^2 v_t. \quad (11)$$

If $1 > \lambda > 0$, strong price stability is feasible as long as velocity is I(2) or less and $q^*_t - q_t$ is I(0). As long as the nominal GNP target (given the constant price target, this means the implicit real GNP target, $q^*_t$) adjusts so that $q^*_t - q_t$ is I(0), then the price level will be an I(0) process. This result suggests that nominal GNP targeting is capable of achieving strong price stability, even if real GNP is nonstationary, as long as the implied real GNP target takes into account this nonstationarity.

Hall (1983) and McCallum (1989) suggest selecting a once and for-all-time target path for nominal GNP. Their motivation is to ensure that the nominal GNP rule is credible. Gordon (1985) and Tobin (1983) suggest periodically reevaluating the nominal GNP target to take into account changes in potential GNP. This debate about how to choose the nominal GNP target is not a trivial matter as far as price stability is concerned. As the above algebra suggests, the method of choosing the GNP target can be quite important in determining the feasibility of price stability. Only with a periodic reevaluation of the nominal GNP target can nominal GNP targeting ensure strong price stability when real GNP is I(1).

IV.3 Price level targets

where the $p_0$ is the price level in the initial time period.
Possible problems associated with choosing the nominal GNP target have led some to suggest targeting the price level directly (Barro 1986 and McCallum 1990b). Indeed, price level rules appear to be the most direct and flexible monetary rules for achieving strong price stability. For weak price stability, some sort of inflation target rule must be employed.

McCallum (1990b) suggests a price level rule of the form
\[ \Delta m_t = \Delta q^*_t - \Delta v_{t-1} + \Delta q_{t-1} - \lambda (p_{t-1} - p^*) , \]
where \( \Delta q^*_t \) is the targeted quarterly real GNP growth rate. Solving McCallum's price rule yields
\[ p_t = p^* + \left[ \frac{1}{(1-(L-(r-r)L)} \right] \left[ \Delta q^*_t + \Delta^2 v_t - \Delta^2 q_t \right] . \]
Therefore, for McCallum's rule to yield price stability, \( v_t \) and \( q_t \) must be I(2) or less. Indeed, any price level rule that includes feedback terms for velocity and real GNP growth, as in equation (13), will yield price stability if velocity and real GNP are I(2) or less.

IV.4 Interest rate targets

In general, interest rate targets do not ensure strong price stability. Goodfriend (1987) and VanHoose (1989) have shown that the desire to smooth nominal interest rates leads to price level nonstationarity. Hence, interest rate targeting tends to be inconsistent with strong price stability.\(^{14}\)

Recently, Hetzel (1990) has offered an interesting proposal in which

\(^{14}\) McCallum (1990b) has shown that a pure interest rate peg does not constitute a well-formulated monetary policy. Some additional specification of the money supply process is needed—for example, a money supply rule. However, the stochastic process for prices changes depends on the money supply rule even though the nominal interest rate is pegged.
the government issues indexed bonds as well as nominal bonds and uses the spread between the two types of bonds as a guide for monetary policy. We can formalize his proposal as a simple expected inflation target, or

\[ m_t - m_{t-1} = -\theta [E(p_{t+1} | I_t) - p_t], \]

where \( E(p_{t+1} | I_t) \) is the rational expectation of the price level at \( t + 1 \), given time period \( t \) information. Using the quantity theory equation, we can solve for the inflation rate (assuming \( |\theta| < 1 \)):

\[ \Delta p_t = \sum_{i=0}^{\infty} (-\theta)^i E[(\Delta v_{t+1} - \Delta q_{t+1}) | I_t]. \]

The presence of the expectations term prevents the difference operator on both sides from canceling out. Therefore, the price level can be nonstationary even if velocity and real GNP are stationary. Thus, the Hetzel interest rate target is not likely to generate strong price stability. It is, however, consistent with weak price stability. This is to be expected since Hetzel's rule can be used to pursue a zero inflation target.

V. Price stability in a macroeconomic model

V.1 Price stability and short-run stabilization policy

While strong price stability imposes strong restrictions on the long-term behavior on prices and monetary policy, what do these restrictions imply for short term stabilization policy? In general, the long-run restrictions implied by price stability will impose constraints on the ability of the monetary authority to engage in short-term stabilization. However, as we show below, it is not clear that these constraints are particularly binding.
The restrictions that strong price stability impose on short-term stabilization policy depends on the short-term dynamics of the economy. In order to analyze these constraints we must be more specific about the economic structure than in the previous section. In this regard, we consider a simple linear rational expectations model of the economy given by:

\( m_t - p_t = -a_0 i_t + a_1 q_t - v_t \)  
\( q_t = -b_1 [ i_t - (tE p_{t+1} - p_t) ] + e_t \) 
\( q_t = q^* + c_1 (p_t - tE p_t) + z_t \)

where \( a_0, a_1, b_1, \) and \( c_1 \) are positive constants. \( m_t \) is the (log) money supply, \( p_t \) is the log of the price level, \( q_t \) is the log of output, \( i_t \) is the nominal interest rate, \( v_t \) is a shock to money demand, \( e_t \) is a shock to commodity demand, \( q^* \) is the natural or trend level of output, and \( z_t \) is a supply shock. Equation (16) is a simple money demand equation, (17) is commodity demand equation (IS curve), and (18) is an aggregate supply equation.

Substituting and rewriting yields:

\( -a_1 tE p_{t+1} + (1+a_1+a_2) p_t = \alpha_2 tE p_t = m_t + v' - \alpha_3 (q^*_t + z_t) \),

where \( \alpha_1 = a_0, \alpha_2 = c_1(a_1+a_0/b_1), \alpha_3 = a_1+a_0/b_1, v' = a_0 e_t/b_1 + v_t. \)

Strong price stability requires \( [m_t + v' - \alpha_3 (q^*_t + z_t)] \) be I(0) (see appendix). This condition is qualitatively similar to the condition described in the previous section and, hence, may of the insights of the previous section will carry over to this model. To make further analysis of the effect of price stability on short run stabilization policy interesting, yet keep the

\[ \text{we combine the money and commodity demand shocks into a single random variable for notational and algebraic ease. Separating the two shocks would not alter insight derived from the analysis below.} \]
algebra tractable, we assume that \( q^* \) and \( v'_t \) are given by \( q^*_t = q^*_{t-1} + e^q_t \) and \( v'_{t} = v'_{t-1} + e^v_t \), where \( e^q_t \) and \( e^v_{t} \) as well as \( z_t \) are white noise.

Suppose that the monetary policy is governed by the following reaction function:

\[
(20) \quad m_t = \theta_0 + \theta_1 m_{t-1} + \theta_2 e^q_t + \theta_3 q^*_{t-1} + \theta_4 e^v_t + \theta_5 v'_{t-1} + \theta_6 z_t + \lambda (p^* - p_{t-1}).
\]

Note that this reaction function encompasses many of the rules described above. For example, if \( \theta_1 = 1 \), \( \theta_3 = \theta_5 = 0 \), and \( \lambda > 0 \) then monetary policy is following a price level target similar to that described by McCallum with \( \lambda (p^* - p_{t-1}) \) representing the error correction term. If \( \lambda = 0 \) as well, then monetary policy would be consistent with the expected inflation target suggested by Hetzel.

Given the monetary reaction function described by equation (20), the equilibrium price process has the form

\[
(21) \quad p_t = \delta_0 + \delta_1 m_t + \delta_2 e^q_t + \delta_3 q^*_{t-1} + \delta_4 e^v_t + \delta_5 v'_{t-1} + \delta_6 z_t.
\]

The \( \delta \)'s can be solved for by the method of undetermined coefficients and are presented in the appendix. Using the monetary reaction function and equation (20), we can rewrite the equilibrium price process as

\[
(22) \quad [1-(\theta_1-\lambda \theta_1)L]p_t = \delta_0(1-\theta_1) + \delta_1 \theta_0 + \delta_1 \lambda p^* + [\delta_1 \theta_2 + \delta_2 - \delta_2 \theta_1 L]e^q_t + [\delta_1 \theta_3 + \delta_3 - \delta_3 \theta_1 L]q^*_{t-1} + [\delta_1 \theta_4 + \delta_4 - \delta_4 \theta_1 L]e^v_t + [\delta_1 \theta_5 + \delta_5 - \delta_5 \theta_1 L]v'_{t-1} + [\delta_1 \theta_6 + \delta_6 - \delta_6 \theta_1 L]z_t.
\]

Recall that the term \( L \) represents a lag operator.

In order for strong price stability to hold (that is, prices to be \( I(0) \)), the money supply reaction function must satisfy the following restrictions:

(i) \( \{\delta_1 \theta_3 + \delta_3 - \delta_3 \theta_1 \} = 0 \),

(ii) \( \{\delta_1 \theta_5 + \delta_5 - \delta_5 \theta_1 \} = 0 \), and
(iii) \( |\theta_1 - \lambda \delta_1| < 1 \).

Using the values of \( \delta_1, \delta_3, \) and \( \delta_4 \) (see appendix), the first two conditions can be rewritten as (i') \( \theta_1 = \alpha_3(1-\theta_1) \) and (ii') \( \theta_5 = -(1-\theta_1) \). Note that, in general, the presence of the error correction term \( (\lambda) \) is neither a necessary nor sufficient condition for price stability.

Note also that price stability places no restrictions on the other parameters of the money supply reaction function, in particular \( \theta_2, \theta_4, \) and \( \theta_6 \). Yet, these parameters are typically the ones most useful for short run stabilization policy. For example, consider the monetary authority who wanted to minimize \( \text{Var}(q_t-\bar{q}_t) \). For the above model,

\[
(23) \quad \text{Var}(q_t-\bar{q}_t) = \left[ c_1(\delta_2-\delta_1 \theta_2) \right]^2 \text{Var}(e_t) + \left[ c_1(\delta_4-\delta_1 \theta_4) \right]^2 \text{Var}(e^*_t)
\]

\[ + \left[ c_1(\delta_6-\delta_1 \theta_6)+1 \right]^2 \text{Var}(z_t). \]

One can choose \( \theta_2, \theta_4, \) and \( \theta_6 \) so that \( \text{Var}(q_t-\bar{q}_t) = 0 \) regardless of the values of \( \theta_1, \theta_3, \) and \( \theta_5 \) (see appendix). Thus, in this example strong price stability imposes no restrictions whatsoever on short-term stabilization policy. In fact, even if we did not allow the monetary authority to use current information in setting its money supply rule (i.e. \( \theta_2 = \theta_4 = \theta_6 = 0 \)), the \( \text{Var}(q_t-\bar{q}_t) \) can still be set equal to zero and price stability hold. In this case, \( \theta_1 = \lambda/(1+\alpha_1) \) and \( -(1+\alpha_1)(2\alpha_1+1)/\alpha_1 < \lambda < -(1+\alpha_1)/\alpha_1 \). This money supply rule, even though it cannot directly offset current period shocks, is able to stabilize output by moving future money supply in reaction to the shocks.\(^{16} \) This in turn, causes expectations of future prices to change and, hence, causes commodity demand to move to offset current period shocks. Interestingly, in this case, the error

\(^{16} \) We are of course assuming that the monetary authority can credibly commit to its state contingent rule.
The correction term is negative. Note that a price level target similar to that examined by McCallum ($\theta_1 - 1$ and $0 < \lambda < 1$ with $\theta_2 = \theta_4 = \theta_6 = 0$) while achieving price stability does not minimize the $\text{Var}(q_t - q^*_t)$. Perhaps this is why the price level target in McCallum's analysis is less effective than other types of rules (specifically nominal GNP targeting) at stabilizing real GNP.

V.2 Price stability and price level uncertainty

While the definition of strong price stability proposed above has the advantage of an being an unambiguous mathematical or algebraic definition of price stability, it is not necessarily a definition that captures the economic essence of what some people consider to be price stability—namely price uncertainty does not distort economic decision making. In other words, while our definition of strong price stability imposes long-run restrictions on the properties on the price level, the relevant economic time horizon may not be infinity.

For the above economic model and monetary policy reaction function, price level uncertainty at an arbitrary time horizon, $k$, is given by

\begin{equation}
\text{Var}(p_{t+k} - \mathbb{E} p_{t+k}) = \\
(A_1 + A_2(1-(\theta_1 - \lambda \delta_1)^2(k-2)) + A_3(1-(\theta_1 - \lambda \delta_1)^{k-2}) + A_4(k-2) \text{Var}(e^q_t)) \\
+ (B_1 + B_2(1-(\theta_1 - \lambda \delta_1)^2(k-2)) + B_3(1-(\theta_1 - \lambda \delta_1)^{k-2}) + B_4(k-2) \text{Var}(e^y_t)) \\
+ (C_1 + C_2(1-(\theta_1 - \lambda \delta_1)^2(k-2)) \text{Var}(z_t)).
\end{equation}

In general, $\text{Var}(p_{t+k} - \mathbb{E} p_{t+k})$ is a nonlinear function of the money reaction function parameters (see appendix), and that for an arbitrary time horizon, $k$, strong price stability does not necessarily minimize this variance. However, under strong price stability, $A_3 = A_4 = B_3 = B_4 = 0$. Eliminating
these terms is particularly useful at longer time horizons. Furthermore, $A_4$ and $B_4$ are only equal to zero when strong price stability holds. Therefore, while strong price stability is not sufficient to minimize price uncertainty at shorter time horizons, it is necessary at long time horizons. Note that the short-term behavior of the money reaction function as captured by the $\theta_2$, $\theta_4$, and $\theta_6$ parameters, which are not constrained by strong price stability, can be used in conjunction with strong price stability to minimize price uncertainty at an arbitrary time horizon.

The fact that strong price stability while imposing restrictions on the long-run conduct of monetary policy does not necessarily imply strong restrictions on short-term stabilization policy is not peculiar to the above economic model or monetary reaction function. Walsh (1986), who examines a slightly different economic model than the one above, considers a monetary rule that consists of a monetary target ($\gamma_t$) and stabilization term $[\gamma(q_t - t^{-1}E q_{t+1})]$. In his model, the restrictions implied by strong price stability are entirely reflected in the choice of monetary target ($\gamma_t$); given strong price stability, the optimal short-run stabilization parameter $\gamma$ is independent of the monetary target (Walsh 1986, p. 697).

In conclusion, strong price stability does not necessarily hamper the conduct of short-term stabilization policy whether the goal is output stabilization or minimizing price uncertainty. Moreover, strong price stability is useful for minimizing price uncertainty at any time horizon and is necessary for minimizing uncertainty at long time horizons. It

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17 Several studies examine the transition costs of moving to zero inflation, in terms of lost output (e.g., Howitt 1990, Aiyagari 1990). These studies are not inconsistent with our results: for short horizons the monetary authorities still face a potential tradeoff of minimizing price level uncertainty or minimizing output variability.

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seems that while strong price stability implies restrictions on long-run monetary policy stringent enough to rule out some common monetary policies (as demonstrated in section IV), it does not necessarily imply strong restrictions on short-term monetary policy. This may leave the monetary authority enough flexibility to consider short-term stabilization policy while still achieving strong price stability in the long-run.

Discussion and Conclusions

Achieving strong price stability, defined as targeting of the long-run price level, requires strong long-run restrictions on the conduct of monetary policy. In general, the money supply must follow some sort of feedback rule that offsets the nonstationarity in velocity and/or real GNP. Among the alternative types of monetary policy rules considered, price level targeting and nominal GNP targeting (provided the nominal GNP target adjusts to account for changes in trend real GNP) show the most promise for generating strong price stability. Simple money supply targeting yields strong price stability in fewer circumstances than does targeting either the price level or nominal GNP. Indeed, the current monetary aggregate procedures that include target cones and base drift are almost guaranteed not to result in strong price stability.

In contrast to strong price stability, achieving weak price stability, defined as targeting the inflation rate, imposes relatively weak restrictions on the conduct of monetary policy. In fact, attaining weak price stability would not necessarily require any changes in the stationarity properties of the monetary aggregates. The relatively weak long-run restrictions on the monetary aggregates associated with weak price
stability reflects that stochastic price level shocks need not be reversed under this policy. This implies that the reduction in long-run uncertainty about the price level is much smaller under weak price stability than under strong price stability.

Advocates of strong price stability cite the reduction of long-run price level uncertainty under such a policy as the main reason it should be pursued. While our explicit definition of strong price stability implies minimizing price level uncertainty at an infinite horizon, there are close connections between strong price stability and price level uncertainty at shorter horizons which may be more relevant for business planning horizons. Specifically, in the context of a simple linear rational expectations model, the attainment of strong price stability helps to minimize uncertainty at shorter time horizons but it is not a sufficient condition. For longer time horizons, strong price stability is a necessary condition for minimizing uncertainty.

Somewhat surprisingly, the attainment of strong price stability does not preclude the monetary authorities from pursuing other short-run stabilization objectives. This follows from the fact that the conditions for price stability are long-run conditions which are not inconsistent with most short-run behavior of the monetary aggregates. This result does, however, raise to the forefront issues of credibility and optimality which this paper does not address. Namely, does the pursuit of short-run stabilization policies damage the credibility of the authorities' long-run commitment to price stability?
Appendix

Solving for the equilibrium price level.

Starting with equation (19), taking expectations based on information set dated t-1, and solving the resulting difference equation for expected prices yields:

\[ (A1) \quad t^{-1}E_t p_t = \sum_{\ell=0}^{\infty} \left( \frac{\alpha_1}{1+\alpha_1} \right)^\ell t^{-1}E_t [m_{t+1} + v'_{t+1} - \alpha_3(q^*_{t+1}+z_{t+1})]/(1+\alpha_1). \]

Using (A1) we can obtain \( t^{-1}E_t p_{t+1} \) and substituting for \( t^{-1}E_t p_t \) and \( t^{-1}E_t p_{t+1} \), we find that equilibrium prices are given by

\[ (A2) \quad p_t = [m_t + v'_t - \alpha_3(q^*_t+z_t)] + \sum_{\ell=0}^{\infty} \left( \frac{\alpha_1}{1+\alpha_1} \right)^\ell t^{-1}E_t [m_{t+1} + v'_{t+1} - \alpha_3(q^*_{t+1}+z_{t+1})]/(1+\alpha_1+\alpha_2). \]

Note that, because \( \frac{\alpha_1}{1+\alpha_1} < 1 \), the order of integration for \( p_t \) is determined by the order of integration of \( m_t + v'_t - \alpha_3(q^*_t+z_t) \).

Given the money reaction function given by equation (20), equilibrium prices can be solved for by method of undetermined coefficients:

\[ (21) \quad p_t = \delta_0 + \delta_1 m_t + \delta_2 e^q_t + \delta_3 q^*_{t-1} + \delta_4 e^v_t + \delta_5 v'_{t-1} + \delta_6 z_t. \]

where

\[
\begin{align*}
\delta_0 &= \alpha_1 \delta_1 (\lambda p^*+\theta_0)/(1+\alpha_1 \delta_1) \\
\delta_2 &= -\alpha_3 - \alpha_2 \delta_1 \theta_2 + \alpha_1 \delta_1 \theta_3 + \alpha_3 \delta_3)/(1+\alpha_1 + \alpha_2 + \alpha_1 \delta_1) \\
\delta_3 &= (-\alpha_2 + \alpha_1 \delta_1 \theta_3)/(1+\alpha_1 \delta_1) \\
\delta_4 &= (1-\alpha_2 \delta_1 \theta_3 + \alpha_1 \delta_1 \theta_2 + \alpha_1 \delta_3)/(1+\alpha_1 + \alpha_2 + \alpha_1 \delta_1) \\
\delta_5 &= (1+\alpha_1 \delta_1 \theta_5)/(1+\alpha_1 \delta_1) \\
\delta_6 &= -\alpha_3 - \alpha_2 \delta_1 \theta_6)/(1+\alpha_1 + \alpha_2 + \alpha_1 \delta_1) \\
\alpha_1 \lambda \delta_1^2 + (1+\alpha_1 (1-\theta_1)) \delta_1 - 1 &= 0.
\end{align*}
\]

Using the conditions (i) and (ii) for strong price stability and the equations for \( \delta_1 \) and \( \delta_2 \) yields:

\[
\begin{align*}
\alpha_1 \lambda \delta_1 \delta_2 + (1+\alpha_1 (1-\theta_1)) \delta_2 + \alpha_3 &= 0 \\
\alpha_1 \lambda \delta_1 \delta_5 + (1+\alpha_1 (1-\theta_1)) \delta_5 - 1 &= 0.
\end{align*}
\]
Thus, $\delta_3 = -\alpha_3\delta_1$ and $\delta_2 = \delta_1$. From conditions (i) and (ii), this yields (i') $\theta_3 = \alpha_3(1-\theta_1)$ and (ii') $\theta_3 = -(1-\theta_1)$.

**Effect of strong price stability on VAR($q_t - q^*$)**

Suppose the nonetary authority wished to minimize

$\text{(23) } \text{Var}(q_t - q^*_t) = [c_1(\delta_2 - \delta_1\theta_2)]^2 \text{Var}(e^*_t) + [c_1(\delta_1 - \delta_1\theta_4)]^2 \text{Var}(e^*_t)$

$+ [c_1(\delta_6 - \delta_1\theta_6) + 1]^2 \text{Var}(z_t)$.  

Because $\delta_1$, $\delta_3$, and $\delta_5$ are independent of $\theta_2$, $\theta_4$, and $\theta_6$, it is possible to choose $\theta_2$, $\theta_4$, and $\theta_6$ so that $\text{Var}(q_t - q^*_t) = 0$.

When $\theta_2 = \theta_4 = \theta_6 = 0$,

$\text{Var}(q_t - q^*_t) = [c_1\delta_2]^2 \text{Var}(e^*_t) + [c_1\delta_4]^2 \text{Var}(e^*_t) + [c_1\delta_6 + 1]^2 \text{Var}(z_t)$.  

Under strong price stability,

$\delta_2 = \alpha_3(l+\alpha_1\delta_1\theta_1)/(l+\alpha_1+\alpha_2+\alpha_1\delta_1\lambda)$, $\delta_4 = (l+\alpha_1\delta_1\theta_1)/(l+\alpha_1+\alpha_2+\alpha_3\delta_1\lambda)$, and $c_1\delta_6 + 1 = (l+\alpha_1+\alpha_1\delta_1\lambda)/(l+\alpha_1+\alpha_2+\alpha_1\delta_1\lambda)$. By setting $\theta_1 = -l/\alpha_1\delta_1 = \lambda/(1+\alpha_1)$, then $\delta_2 = \delta_4 = 0$. Because $\delta_1$ in this case is equal to $-(l+\alpha_1)/(\alpha_1\lambda)$, then $c_1\delta_6 + 1 = 0$ as well.

**Derivation of k-horizon forecast variance.**

From equation (22) and after some algebra, we obtain

\[ P_{t+k} - \tau E P_{t+k} = [\delta_1\theta_2 + \delta_2]_{1=0}^{\delta k-1} (\theta_1 - \delta_1\lambda)^i e^*_{t+k-i} \]

$+ (-\delta_2\theta_1 + \delta_1\theta_3 + \delta_3)_{1=0}^{\delta k-2} (\theta_1 - \delta_1\lambda)^i e^*_{t+k-1-i}$

$+ (\delta_1\theta_3 + \delta_3 - \delta_5\theta_1)_{1=0}^{\delta k-3} (\theta_1 - \delta_1\lambda)^i (q^*_{t+k-2-i} - \tau E q^*_{t+k-2-i})$

$+ [\delta_1\theta_4 + \delta_4]_{1=0}^{\delta k-1} (\theta_1 - \delta_1\lambda)^i e^*_{t+k-1} + (\delta_4\theta_1 + \delta_1\theta_3 + \delta_3)_{1=0}^{\delta k-2} (\theta_1 - \delta_1\lambda)^i e^*_{t+k-1-i}$

$+ (\delta_1\theta_5 + \delta_5 - \delta_5\theta_1)_{1=0}^{\delta k-3} (\theta_1 - \delta_1\lambda)^i (v^*_{t+k-2-i} - \tau E v^*_{t+k-2-i})$

$+ [\delta_1\theta_6 + \delta_6]_{1=0}^{\delta k-1} (\theta_1 - \delta_1\lambda)^i z_{t+k-1} + (\delta_6\theta_1 + \delta_1\theta_5 + \delta_5)_{1=0}^{\delta k-2} (\theta_1 - \delta_1\lambda)^i z_{t+k-1-i}$.  

After some algebra, we obtain

$\text{(24) } \text{Var}(P_{t+k} - \tau E P_{t+k}) =$
\[\{A_1 + A_2(1-(\theta_1-\delta_1)^2)^{(k-2)} + A_3(1-(\theta_1-\delta_1)^2)^{k-2}) + A_4(k-2) \} \text{Var}(e^q_t)\]
\[+ (B_1 + B_2(1-(\theta_1-\delta_1)^2)^{(k-2)} + B_3(1-(\theta_1-\delta_1)^2)^{k-2}) + B_4(k-2) \} \text{Var}(e^e_t)\]
\[+ (C_1 + C_2(1-(\theta_1-\delta_1)^2)^{(k-2)} \} \text{Var}(z_t),\]

where

\[A_1 = \{\delta_1 \theta_2+\delta_2\}^2 + \{(\delta_1 \theta_2+\delta_2)(\theta_1-\delta_1\lambda)-\delta_2 \theta_1+\delta_1 \theta_3+\delta_3\}^2\]

\[A_2 = (\theta_1-\delta_1\lambda)^2 \times
\frac{[(\delta_1 \theta_2+\delta_2)(\theta_1-\delta_1\lambda)-\delta_2 \theta_1+\delta_1 \theta_3+\delta_3-(\delta_1 \theta_3+\delta_3 \theta_1)/(1-(\theta_1-\delta_1\lambda))]^2}{(1-(\theta_1-\delta_1\lambda)^2)}\]

\[A_3 = 2(\theta_1-\delta_1\lambda)(\delta_1 \theta_3+\delta_3 \theta_1) \times
\frac{[(\delta_1 \theta_2+\delta_2)(\theta_1-\delta_1\lambda)-\delta_2 \theta_1+\delta_1 \theta_3+\delta_3-(\delta_1 \theta_3+\delta_3 \theta_1)/(1-(\theta_1-\delta_1\lambda))]^2}{(1-(\theta_1-\delta_1\lambda)^2)}\]

\[A_4 = (\delta_1 \theta_3+\delta_3 \theta_1)^2/(1-(\theta_1-\delta_1\lambda)^2)\]

\[B_1 = \{\delta_1 \theta_4+\delta_4\}^2 + \{(\delta_1 \theta_4+\delta_4)(\theta_1-\delta_1\lambda)-\delta_4 \theta_1+\delta_1 \theta_5+\delta_5\}^2\]

\[B_2 = (\theta_1-\delta_1\lambda)^2 \times
\frac{[(\delta_1 \theta_4+\delta_4)(\theta_1-\delta_1\lambda)-\delta_4 \theta_1+\delta_1 \theta_5+\delta_5-(\delta_1 \theta_5+\delta_5 \theta_1)/(1-(\theta_1-\delta_1\lambda))]^2}{(1-(\theta_1-\delta_1\lambda)^2)}\]

\[B_3 = 2(\theta_1-\delta_1\lambda)(\delta_1 \theta_5+\delta_5 \theta_1) \times
\frac{[(\delta_1 \theta_4+\delta_4)(\theta_1-\delta_1\lambda)-\delta_4 \theta_1+\delta_1 \theta_5+\delta_5-(\delta_1 \theta_5+\delta_5 \theta_1)/(1-(\theta_1-\delta_1\lambda))]^2}{(1-(\theta_1-\delta_1\lambda)^2)}\]

\[B_4 = (\delta_1 \theta_5+\delta_5 \theta_1)^2/(1-(\theta_1-\delta_1\lambda)^2)\]

\[C_1 = \{\delta_1 \theta_6+\delta_6\}^2 + \{(\delta_1 \theta_6+\delta_6)(\theta_1-\delta_1\lambda)-\delta_6 \theta_2\}^2\]

\[C_2 = (\theta_1-\delta_1\lambda)^2[(\delta_1 \theta_6+\delta_6)(\theta_1-\delta_1\lambda)-\delta_6 \theta_1]^2/(1-(\theta_1-\delta_1\lambda)^2).\]

Note that under strong price stability:

\[(\delta_1 \theta_3+\delta_3 \theta_1) = (\delta_1 \theta_5+\delta_5 \theta_1) = 0,\]

and, hence, \(A_3 - A_4 - B_3 - B_4 = 0.\)
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