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The Output Effects of Government Consumption: A Note

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Abstract This paper presents a simplified analysis of the effects of government consumption in the context of the neoclassical growth model. The analysis complements the recent paper of Aiyagari, Christiano and Eichenbaum (1992), and provides a simpler demonstration of one of their main results that there is an analog to the Keynesian multiplier in such a model.

JEL Classification numbers: E13, E62

1. Introduction

A recent paper in this journal by Aiyagari, Christiano and Eichenbaum (1992) demonstrated the possibility of a multiplier effect from government purchases to private sector output in the context of the neoclassical growth model for increases in government purchases that are persistent. The analysis in their paper is complicated by their use of a stochastic growth model. In this note I show that it is possible to derive this result in the nonstochastic version of the model by simply differentiating the conditions that characterize the steady-state equilibrium of the model. The appropriate interpretation of the results obtained from differentiating steady-state conditions is that they show the long run effects of permanent changes in government purchases. By showing the relationship between the output multiplier and the underlying parameters of tastes and technology, I demonstrate the crucial role played by the supply of and demand for capital in steady-state equilibrium in generating multiplier effects in this model.

2. The model

It is well known that competitive equilibrium allocations in the one-sector neoclassical growth model with government consumption spending financed by lump sum taxes are given by the solution to the following planning problem:

$$\max_{C_t, N_t, K_{t+1}} \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

subject to

$$L_t + N_t = 1$$

$$F(K_t, N_t) + (1-\delta)K_t = C_t + K_{t+1} + G_t$$

$$K_0 = \bar{K}_0$$

where C_t and L_t denote consumption and leisure (or non-market activities), both of which are assumed to be normal goods, N_t denotes time spent at private sector production (market activities), K_t denotes capital supplied to private sector production, and G_t denotes government purchases of private sector output. $1 \geq \beta \geq 0$ denotes the discount rate of future utility, and $1 \geq \delta \geq 0$ denotes the rate of depreciation of the capital stock.

It is straightforward to show that the following conditions characterize the solution to this planning problem:

$$D_1U(C_t, 1-N_t) = \lambda_t \tag{1}$$

$$D_2U(C_t, 1-N_t) = \lambda_t D_2F(K_t, N_t) \tag{2}$$

$$\beta\lambda_{t+1}[(1-\delta) + D_1F(K_{t+1}, N_{t+1})] = \lambda_t \tag{3}$$

$$F(K_t, N_t) + (1-\delta)K_t = C_t + K_{t+1} + G_t \tag{4}$$

along with the boundary conditions $K_0 = \bar{K}_0$ and $\lim_{t\to\infty} \beta^t \lambda_t K_{t+1} = 0$. D_i denotes differentiation with respect to the i'th argument of a function. The steady-state equilibrium of this model is then characterized by dropping time subscripts and rearranging terms to obtain the following system:

$$D_1U(C,1-N) = \lambda \tag{5}$$

$$D_2U(C,1-N) = \lambda D_2F(K,N) \tag{6}$$

$$\beta[(1-\delta)+D_1F(K,N)] = 1 \tag{7}$$

$$F(K,N) = C + \delta K + G \tag{8}$$

To analyze the effects of changes in government purchases, we simply linearize the system around its (initial) steady-state equilibrium. Log-differentiating the system we obtain

$$\xi_{CC}\hat{C}-\xi_{CL}\frac{N}{1-N}\hat{N} = \hat{\lambda} \tag{9}$$

$$\xi_{LC}\hat{C}-\xi_{LL}\frac{N}{1-N}\hat{N} = \hat{\lambda} + \gamma_{NK}\hat{K} + \gamma_{NN}\hat{N} \tag{10}$$

$$\gamma_{KK}\hat{K} + \gamma_{KN}\hat{N} = 0 \tag{11}$$

$$\theta_K\hat{K} + \theta_N\hat{N} = s_C\hat{C} + \delta\frac{K}{Y}\hat{K} + s_G\hat{G} \tag{12}$$

where ξ^i_j = the elasticity of the marginal utility of i with respect to j for i,j = C,L. Concavity of preferences implies that ξ_{CC} and $\xi_{LL} < 0$, and $\xi_{CC}\xi_{LL} - \xi_{LC}\xi_{CL} > 0$. The assumption that both consumption and leisure are normal goods implies that $\xi_{CC} - \xi_{LC} < 0$ and that $\xi_{LL} - \xi_{CL} < 0$. γ_{ij} denotes the elasticity of the marginal product of i with respect to j for i,j = K,N. The requirement that the production function be concave implies that γ_{NN} and $\gamma_{KK} < 0$, and $\gamma_{NN}\gamma_{KK} - \gamma_{NK}\gamma_{KN} \geq 0$. Under constant returns to scale, this holds with strict equality. θ_i denotes the elasticity of output with respect to i for i = K,N and is always positive. The assumption of constant returns to scale means that θ_i also denotes the share of factor i in the output of the private sector, that $\theta_N + \theta_K = 1$ and that $\gamma_{KK} + \gamma_{KN} = \gamma_{NN} + \gamma_{NK} = 0$. Constant returns to scale also means that $\gamma_{NN} = - \gamma_{NK} = \theta_K/\sigma_{KN}$ where σ_{KN} is the elasticity of substitution between capital and labor. Finally s_j is the share of final output allocated to j for j = C,G. The hats "^" denote percentage deviations from equilibrium. All of the elasticity and share parameters are evaluated at their initial steady-state equilibrium values.

Note that the steady-state capital-output ratio can be written $K/Y = \beta\theta_K/(1-\beta(1-\delta))$, and the share of private sector consumption in steady-state output $s_C = 1-(\delta K/Y)-s_G = (1-\beta(1-\delta(1-\theta_K)))/(1-\beta(1-\delta))-s_G$.

It is obvious that this framework is quite general and can incorporate a wide variety of assumptions about tastes and technology. Logarithmic utility, zero and 100% percent depreciation of the capital stock (and all values in between), fixed labor supply, indivisible labor, and Cobb-Douglas production technology are all special cases of the above.

If the private production technology exhibits constant returns to scale, the steady-state capital-output and capital-labor ratios are tied down by parameters of tastes and technology (see equation (7) above). This is not true, however, of the steady-state capital stock. From equation (11) above we can see that a given proportionate change in steady-state employment will call forth an equiproportionate change in the steady-state capital stock, that is, $\hat{K} = \hat{N}$. It follows immediately from the assumption of constant returns to scale that $\hat{Y} = \hat{N}$. This is the essential source of the difference between the results below and those of, for example, Hoon (1992) who also studies the effect of permanent changes in government consumption purchases using this model. Hoon (implicitly) assumes a fixed labor supply: this assumption, in conjunction with (3), ties down the steady-state capital stock, thereby eliminating the possibility of any change in output.

To fully characterize the steady-state effects of permanent changes in government purchases of private sector output we simplify the system (9)-(12) and solve for $\hat{C}$ and $\hat{N}$ to obtain

$$\hat{C} = -\frac{1}{\Delta_2} s_G(\xi_{LL} - \xi_{CL}) \frac{N}{1-N}\hat{G} \tag{13}$$

$$\hat{N} = \frac{1}{\Delta_2} s_G(\xi_{CC} - \xi_{LC})\hat{G} \tag{14}$$

where

$$\Delta_2 = (\xi_{CC} - \xi_{LC}) \frac{1-\beta(1-\delta(1-\theta_K))}{1-\beta(1-\delta)} + s_C(\xi_{LL} - \xi_{CL}) \frac{N}{1-N} \leq 0$$

as long as consumption and leisure are both normal goods.

An increase in government purchases of private sector output unambiguously lowers private sector consumption. If we assume that the marginal utility of consumption is constant and that the utility function is separable, $\xi_{CC} = \xi_{CL} = \xi_{LC} = 0$, the offset is one for one: each extra dollar of government purchases of private sector output crowds out one dollar of private sector consumption. Under these assumptions about preferences, the supply of effort to private sector production is unchanged by a change in government purchases, so output is unchanged also. More generally, private sector output increases in response to an increase in government purchases. Note that with separable preferences, the assumption of constant marginal utility of consumption is equivalent to assuming that there is no income effect on leisure. Aiyagari, Christiano and Eichenbaum (1992) point out that the existence of such an income effect is crucial to generating multiplier effects in the neoclassical model.

3. Multipliers

The steady-state output effects of changes in government purchases are determined by the response of private sector employment and capital:

$$\epsilon_G^Y = \theta_N \epsilon_G^N + \theta_K \epsilon_G^K$$

However the assumption that the private sector technology exhibits constant returns to scale implies that the capital-labor ratio is constant, which in turn implies that capital and employment respond equiproportionately to changes in government purchases, and that $\epsilon_G^Y = \epsilon_G^N$.

The steady-state output multiplier associated with a permanent change in government purchases of private sector output is

$$\frac{dY}{dG} = \frac{1}{\Delta_2}(\xi_{CC} - \xi_{LC}) = \frac{1}{s_C \frac{(\xi_{LL}-\xi_{CL})}{(\xi_{CC}-\xi_{LC})} \frac{N}{1-N} + \frac{1-\beta(1-\delta(1-\theta_K))}{1-\beta(1-\delta)}} =$$

$$= \frac{1}{s_C \frac{(\xi_{LL}-\xi_{CL})}{(\xi_{CC}-\xi_{LC})} \frac{N}{1-N} + (1-s_I)} \gtrsim 0$$

where s_I denotes the share of investment in steady state output. This is a somewhat more general version of Aiyagari, Christiano and Eichenbaum's equation (15). If we assume that the marginal utility of consumption is constant and that the utility function is separable, $\xi_{CC} = \xi_{CL} = \xi_{LC} = 0$, a permanent increase in government purchases will leave steady-state private sector output unchanged. All of the increase in government purchases is offset by an equal decline in private consumption. More generally, output will increase, and possibly by more than the increase in purchases, giving rise to a genuine "multiplier" analogous to that found in Keynesian models of output determination. Finally, note that if we assume that the utility function is homothetic, $(\xi_{LL}-\xi_{CL})/(\xi_{CC}-\xi_{LC}) = 1$, and the only parameter of the point-in-time utility function that matters for the size of the multiplier is the one that ties down hours worked in the steady state, N.

Three limiting values of this multiplier are of special interest, and illuminate the importance of endogenous capital accumulation in generating multiplier effects in this model. As $\beta \to 0$ or $\delta \to 0$ or $\theta_K \to 0$, the output multiplier approaches

$$\frac{dY}{dG} = \frac{1}{s_C \frac{(\xi_{LL}-\xi_{CL})}{(\xi_{CC}-\xi_{LC})} \frac{N}{1-N} + 1} < 1$$

The intuition for these results is as follows. If capital never depreciates, a permanent change in military purchases of private sector output will have no effect on steady-state investment. There will be a temporary change in investment purchases to move the capital stock to its new steady-state level, but investment demand in the new steady-state equilibrium (as in the old) will be zero. Steady-state output changes only to the extent that private consumption does not offset the change in military purchases. We might note here that the conclusion of Fisher and Turnovsky (1992) using essentially the same model as the above that "...in the long run, an increase in government expenditure leads to a less-than-equal increase in output" (p.6) depends crucially on their assumption that capital never depreciates.

As $\theta_K \to 0$, output is unresponsive to changes in the capital input (which is therefore optimally set equal to zero), and consequently there is no change in investment demand either temporarily or in the steady state in response to a permanent change in military purchases.

The conditions $\delta \to 0$ and $\theta_K \to 0$ mean that the steady state <u>demand</u> for capital approaches zero. In contrast, the condition $\beta \to 0$ means that the steady state <u>supply</u> of capital approaches zero. If households attach zero weight to future utility (are infinitely impatient), they will never save and so will not alter their savings in response to changes in government purchases. Note that all three limiting cases imply that the share of investment in steady-state output $s_I = 0$.

Figure 1a shows the general relationship between the size of the multiplier and the elasticity of output with respect to capital under the assumption of homothetic preferences for different values of the rate of depreciation of capital. Figure 1b shows the same relationship under different assumptions about the rate of time preference. Note that if constant returns to scale is an accurate characterization of private sector technology, and we accept the commonly used estimate that the share of labor in private sector output is around two-thirds (and therefore θ_K is around one third), Figure 1 suggests that, empirically, the steady-state multiplier effect from a permanent change in military purchases is likely to be quite small.

4. Conclusions

This note provided an alternative simplified derivation of one of the key results in Aiyagari, Christiano and Eichenbaum (1992), namely that there is an analog to the Keynesian output multiplier in the neoclassical growth model. The key to generating multiplier effects was shown to be capital accumulation: absent a meaningful supply of or demand for capital in the steady state, the multiplier is always less than 1. I also showed that the empirical likelihood of a significant multiplier effect is small, based on a plausible parameterization of the model.

References

Aiyagari, S. Rao, Lawrence J. Christiano and Martin Eichenbaum, 1992, "The output, employment and interest rate effects of government consumption," Journal of Monetary Economics, 30, 73-86.

Fisher, Walter H., and Stephen J. Turnovsky, 1992, "Fiscal policy and the term structure of interest rates: an intertemporal optimizing analysis," Journal of Money, Credit and Banking, 24, 1-26.

Hoon, Hian Teck, 1992, "Defense spending cutbacks and real interest rates in the neoclassical one-sector and two-sector models," Economics Letters, 39, 443-448.

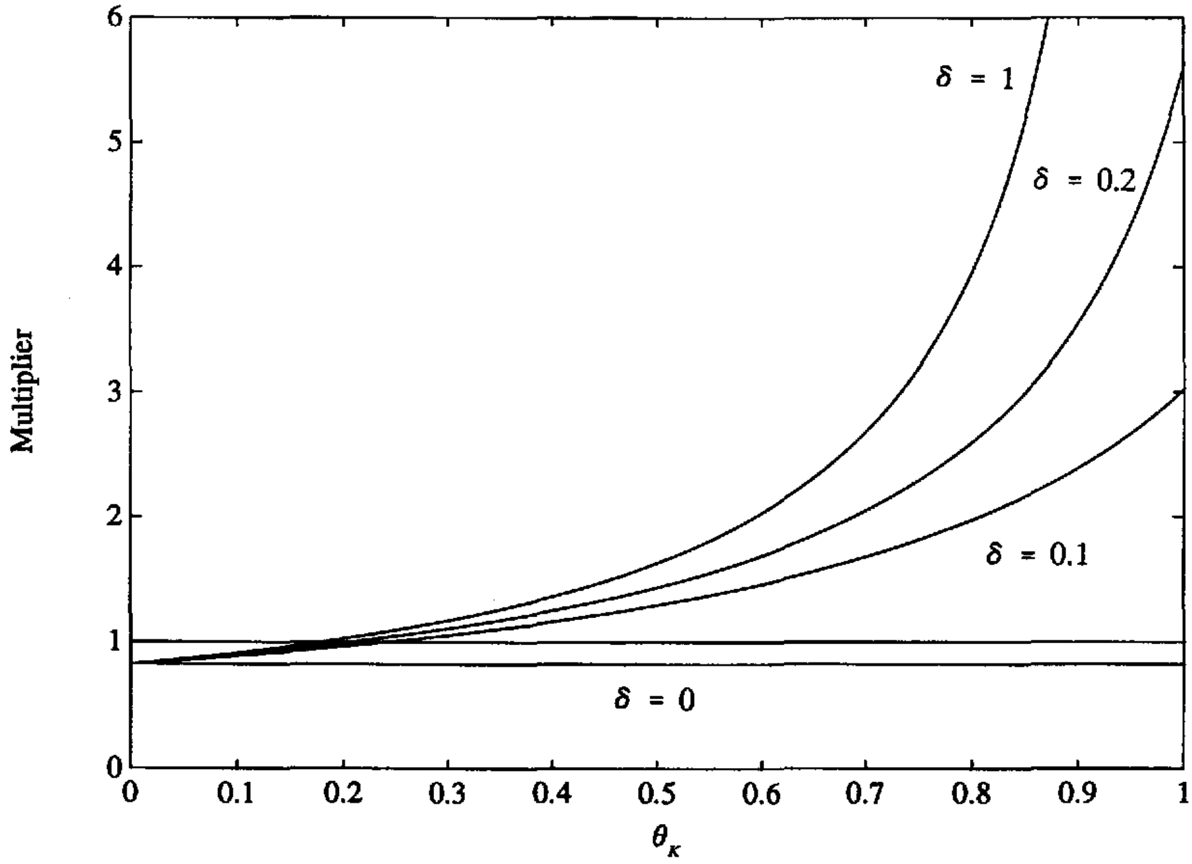

Figure 1a. Parameter values: β = 0.96, N = 0.2, s_G = 0.15.

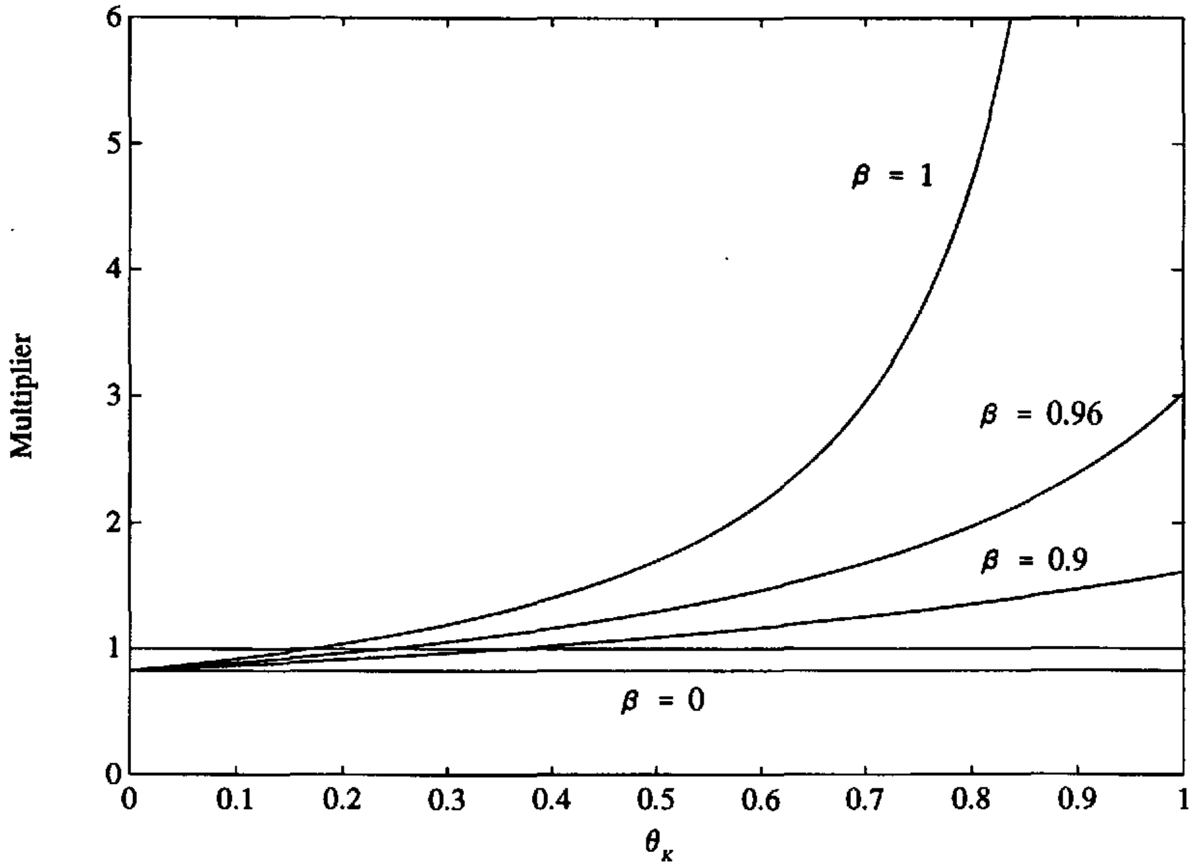

Figure 1b. Parameter values: N = 0.2, δ = 0.1, s_G = 0.15.

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