Recessions and Recoveries in Real Business Cycle Models: Do Real Business Cycle Models Generate Cyclical Behavior?

by

Nathan S. Balke, Department of Economics
Southern Methodist University
and
Research Department, Federal Reserve Bank of Dallas

and

Mark A. Wynne, Research Department
Federal Reserve Bank of Dallas

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Recessions and recoveries in real business cycle models:
Do real business cycle models generate reasonable cyclical behavior?\textsuperscript{1}

Nathan S. Balke* and Mark A. Wynne**

\textsuperscript{*}Department of Economics
Southern Methodist University
and
Research Department
Federal Reserve Bank of Dallas

\textsuperscript{**}Research Department
Federal Reserve Bank of Dallas

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Comments welcome.

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Recessions and recoveries in real business cycle models: Do Real Business Cycle Models Generate Reasonable Cyclical Behavior?

Nathan S. Balke and Mark A. Wynne

Abstract

This paper evaluates the performance of some simple general equilibrium business cycle models in terms of their ability to generate "real world" business cycles. By "real world" business cycles we mean business cycles in the classical NBER sense of the term, where recessions are periods of absolute declines in economic activity. We date turning points in economic activity using the algorithm devised by Bry and Boschan (1971) to mimic the informal procedures used by the NBER to date peaks and troughs in economic activity. We further divide the cycle into the various "phases" used by Burns and Mitchell (1946) in order to determine the "shape" of the business cycle as well as look for asymmetries between expansions and contractions. We conclude that while simple Real Business Cycle models can generate business cycles of plausible duration and depth, they do not match the actual "shape" of the business cycle. In particular, they do not produce the concave expansions--declining output growth over the course of the expansion--apparent in actual business cycles. On the other hand, the Real Business Cycle models do a better job of mimicking growth cycles--cycles in detrended data.
1. Introduction

Do general equilibrium business cycle models generate business cycles that look like the business cycles observed in a modern industrial economy? The early real business cycle literature (for example Kydland and Prescott (1982), Hansen (1985), Prescott (1986) and the papers in the March/May 1988 issue of the Journal of Monetary Economics) evaluated the ability of dynamic general equilibrium models to mimic the cyclical behavior of real world data by comparing selected second moments of the data generated by the models with those generated using real world data. The models were judged to be successful if the second moments generated by the model were "close" (in some informal sense) to the second moments found in the data.²

In this paper we propose an alternative method for evaluating the ability of these models to generate cyclical behavior. Specifically we propose a comparison of the cycles generated in artificial model economies with the cycles in U.S. economic activity identified in the NBER business cycle chronology. We argue that it is business cycles in this latter sense that policy makers, the press and the general public are typically concerned with. Periods of recession and expansion as identified by the NBER business cycle chronology denote periods of absolute declines and increases in economic activity. While periods of sub-par growth do receive attention and are a source of concern to policy makers, it is nonetheless the case that the qualitative nature of policy deliberations changes when the economy experiences absolute declines in economic activity. For this reason alone we should be interested in the NBER business cycle concept.

²More recently Christiano and Eichenbaum (1992) and Watson (1991) have attempted more formal evaluations of how well these models fit the data.
In contrast, standard practice in the contemporary literature on business cycles considers the deviations of output from some secular trend as the cyclical movement that needs to be explained by a business cycle model. For example, in his essay "Understanding Business Cycles" Lucas (1977) poses the question "Why is it that, in capitalist economies, aggregate variables undergo repeated fluctuations about trend, all of essentially the same character?" (emphasis added) as the question that business cycle theory is supposed to answer. The appropriate definition of the business cycle has also been addressed by Beveridge and Nelson (1981), who argue that "...it is not clear in the context of a growing and perhaps inflationary economy that to declare a "cyclical" downturn one necessarily should require that indicators show actual declines." (Beveridge and Nelson (1981) p152). They go on to propose a definition of the cyclical component of economic activity as consisting of fluctuations about an appropriately defined trend. This conception of the business cycle is more akin to the growth-cycle concept developed by the NBER in the postwar period. Among others, Zarnowitz (1992) has argued that it is important to retain a distinction between growth cycles and business cycles, arguing that "The policy implications of ... a deceleration in economic growth are entirely different from those of a recession, which always depresses real incomes and spending, outputs, and employment." (Zarnowitz, 1992, p.30), in part because the impact of decelerations in growth tend to be more localized, concentrated in areas of particular cyclical sensitivity, such as housing. Falk (1986) provides an interesting example of how trend removal can obscure important features of the cycle. He points out that if an intrinsic feature of the mechanism propagating business cycles is that successive peaks are higher, conventional
trend removal methods will tend to bias tests of business cycle symmetry.

There is another reason why we might want to focus on the traditional notion of the business cycle rather than the growth cycle, and this has to do with the problem of operationalizing the concept of trend growth. A realized GDP growth rate of say 2.5 percent may denote an expansion or contraction in the growth cycle sense depending on whether the trend rate of growth is 2.0 percent or 3.0 percent. Yet operationalizing the concept of trend output or activity is difficult. Is the trend level of output at a point in time simply the extrapolation of past values of output? Or is it the maximum "sustainable" level of GNP, something more akin to potential output? In contrast there is no ambiguity about the concept of absolute declines in economic activity: after all, zero is zero. It is interesting to note that pre-World War II business cycle theorists were quite sensitive to the problem of trend measurement and removal. As Joseph Schumpeter noted in his review of Mitchell's 1927 book on business cycles:

"...if trend-analysis is to have any meaning, it can derive it only from previous theoretical considerations, which must not only guide us in interpreting results, but also in choosing the method. Failing this, a trend is no more than a descriptive device summing up past history with which nothing can be done. It lacks economic connotation. It is, in fact, merely formal. We can apply the familiar methods just as well to e.g. a few successive years of a prosperity-phase, as to the whole of the material we may happen to have (as, again, to a period of political commotion). The result has the same claim in every case to be called a trend in the

\[\text{See also Frickey (1934).}\]
statistical sense, and may in each case be decomposed into component elements in an indefinite number of ways which have no rational connection to each other—unless it be supplied by the theory of the subject under research.” (Schumpeter 1930, pp. 166-167)

Schumpeter’s comments are as relevant today as they were sixty years ago: the almost universally applied Hodrick-Prescott filter is in no sense a better theory of trend growth than is a simple deterministic trend.

Furthermore, the more traditional conception of the business cycle underlies many recent statistical models of economic fluctuations. For example, the two-state Markov switching model of Hamilton (1989) when applied to real GNP characterizes output as switching between positive growth states and negative growth states—expansions and contractions. In fact, Hamilton obtains regime switch dates that are similar to the NBER Business Cycle chronology. Recent extensions of Hamilton’s regime switching model by Boldin (1990) and Sichel (1992) have tended to confirm the presence of traditional business cycles.

In this paper, we examine whether simple real business cycle models are able to generate cyclical behavior consistent with the traditional-NBER conception of business cycles. That is, do these models produce business cycles of appropriate duration and shape? We adopt the concept of business-cycle time introduced by Burns and Mitchell (1946), dividing recessions and expansions in calendar time into separate business cycle “phases”. Using this framework, we show that for real GNP expansions tend to be concave and recessions linear. That is, growth tends to be faster earlier in the expansion and slower later in the expansion, while the rate of decline is not
significantly different over the course of the recession. Furthermore, output tends to have "round" peaks and "pointed" troughs. In addition to aggregate output, we consider the "shape" of consumption of nondurables and services, fixed investment, hours, and real wages. While not every variable displays concave expansions, "round" peaks and "pointed" troughs are present in most of these variables.

Two simple real business models considered in King, Plosser, and Rebelo (1988)—a variant of the Long-Plosser (1983) model and of the Hansen (1985) and Rogerson (1988) models—are then used to generate artificial economic histories. The algorithm developed by Bry and Boschan (1971) to mimic the business cycle dating procedure of the NBER is used to date business cycle peaks and troughs in the artificial data. We then compare the nature of business cycles implied by these simple real business cycle models to that of actual business cycles. We find that while these models are adequate at capturing the duration and the amplitude of actual business cycles, they do not capture the entire shape of the business cycle. In particular, they fail to produce concave expansions for aggregate output and investment, and peaks tend to be "pointed" rather than "round". On the other hand, the real business cycle models do a much better job of mimicking the actual growth cycles.

The work reported below is closely related to work by King and Plosser (1989) on the test of the Adelmans. The objective of the King and Plosser paper is to evaluate a simple real business cycle models using the methods developed by Burns and Mitchell (1946) to characterize the business cycle. To

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4Emery and Koenig (1992) note a similar phenomenon in the Commerce Department's Leading and Coincident Indexes.
this end they date peaks and troughs in economic activity in the artificial
economy they study using the Bry-Boschan (1971) algorithm, and further
dividing the cycles thus obtained into various phases. King and Plosser then
compare the qualitative features of the cycles found in the real business
cycle model with corresponding features of real world series and conclude that
they are unable to distinguish between the two (compare their figures 3 and
10). This is the sense in which they conclude that the real business cycle
model passes the test of the Adelmans, although they note reservations about
the power of this type of test in their conclusion. While King and Plosser
stay as close as possible to the letter of Burns and Mitchell methods in
evaluating a real business cycle model, we simply use their methods as a point
of departure for our analysis, combining their phase classification with
formal statistical tests to evaluate our artificial economies. Thus while
King and Plosser are unable to reject the possibility that the real world data
are generated by a simple real business cycle model, we are able to reject
this possibility. We do this by showing that various statistics of interest
calculated for the real world lie in the tails of the implied Monte Carlo
distributions generated by the models we consider.

2. The shape of the business cycle

In their monumental study of business cycles, Burns and Mitchell (1946)
divided the business cycle into nine phases. The first phase in this
classification is defined as the three months centered on the initial trough,
while the fifth phase is defined as the three months centered on the
subsequent peak. The ninth phase is defined as the three months centered on
the trough marking the end of the recession, and is also the first phase of
the next business cycle. The second, third and fourth phases break the expansion into three time intervals of equal length, while the sixth, seventh and eighth phases break the subsequent recession into three time intervals of equal length. The Burns and Mitchell phases allow for the possibility that business cycles evolve according to economic or business cycle time (Stock (1987)) rather than calendar time.

It is possible to think of alternative ways to characterize the business cycle, such as a multi-state Markov switching model, that are not as ad-hoc or as ex post in nature as the Burns and Mitchell phase characterization. However, the Burns and Mitchell characterization does have a long history of use in business cycle analysis and, while ex post, does have the advantage of being a relatively simple way of describing certain features of the business cycle. Furthermore, the computational ease of calculating the Burns and Mitchell phases is attractive when we try to evaluate the ability of artificial economies to replicate features of real world cycles.

We use the Burns and Mitchell phase classification to characterize two features of the business cycle. First, we examine the overall "shape" of the business cycle as reflected in certain key aggregate variables. By the shape of the cycle we mean the pattern of variation in growth rates of the key aggregates over the course of expansions and recessions. Second we use the Burns and Mitchell phase characterization to examine the question of business

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5See for example Hamilton (1989) and Boldin (1992).

6The shape of the business cycle has been studied by Neftci (1993) and Sichel (1993) among others.
cycle symmetry. Specifically, we will consider the extent to which recessions are simple negative expansions.

2.1 The shape of post-World War II business cycles

In our analysis of post-World War II business cycles, we follow Burns and Mitchell (1946) and divide the business cycle into eight phases. Since we use quarterly data, the first phase is defined as the quarter of the initial trough, while the fifth phase is defined as the quarter of the subsequent peak. The second, third and fourth phases break the expansion into three time intervals of equal length, while the sixth, seventh and eighth phases break the subsequent recession into three time intervals of equal length.

To determine the shape of the business cycle, we simply regress the growth rate of each series against dummy variables that break business cycle into the different phases described above. The coefficient estimates then represent the average growth rate of the series (per quarter) during the different phases of the business cycle. To keep things manageable we restrict our attention to five key real macroeconomic aggregates: GNP, consumption of nondurable goods and services, fixed investment, and hours worked (all in per capita terms), and real wages.

Table 1 presents the results of regressing the growth rates of the various series against the Burns and Mitchell phase dummies over the 1948-1992 period.

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7The issue of asymmetry in business cycles has previously been addressed by Blatt (1980), DeLong and Summers (1986), Neftci (1984), Fak (1986), and Sichel (1989). Blatt (1980) explicitly points out the implications of asymmetric (growth) cycle for "Frisch-type" models of the sort we will consider below.

8 See Appendix A for details of how the phase dummies were set.
period. Table 2 presents p-values for various hypotheses about the nature of business cycle phases. The first set of hypotheses test whether various phases have the same growth rates, and is designed to show whether the business cycle has a distinctive shape. The second set of hypotheses considers whether business cycles are symmetric. The symmetry hypothesis implies that the average rate of growth in a recession phase is just the negative of the growth rate in the corresponding expansion phase (correcting for trend growth). Thus, for example, symmetry implies:

$$\Delta y_{\text{Expansion}} + \Delta y_{\text{Contraction}} = 2(\text{Trend growth rate})$$

where $\Delta y$ denotes the average rate of growth of $Y$ during the indicated business cycle stage. We can make the symmetry tests more elaborate using the Burns and Mitchell phases:

$$\Delta y_{\text{Phase 1}} + \Delta y_{\text{Phase 5}} = 2(\text{Trend growth rate})$$

$$\Delta y_{\text{Phase 2}} + \Delta y_{\text{Phase 6}} = 2(\text{Trend growth rate})$$

$$\Delta y_{\text{Phase 3}} + \Delta y_{\text{Phase 7}} = 2(\text{Trend growth rate})$$

$$\Delta y_{\text{Phase 4}} + \Delta y_{\text{Phase 8}} = 2(\text{Trend growth rate})$$

If the trend rate of growth is zero, then symmetry implies that growth in recessions is just the negative of growth in expansions.10

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9 We also ran versions employing the White (1980) consistent covariance matrix estimator with the Newey-West (1987) correction for serial correlation. The results were qualitatively similar.

10 The trend rate of growth for output, consumption, and investment was estimated by fitting a common linear time trend to these series as in King, Plosser, and Rebelo (1988). The trend growth rate for wages was calculated separately using a linear time trend.
From Tables 1 and 2, we find that all the series with the exception real wages reject the simple two-phase characterization of the business cycle (that is, we can reject the null hypothesis: $\text{Ph} 2=3=4=5$ and $\text{Ph} 1=6=7=8$). Output, investment, and, to a lesser extent, consumption display concave-shaped expansions (that is, Phase 2 is greater than Phase 3 which is in turn greater than Phase 4); growth in these series is highest in the early phases of the expansion and falls as the expansion progresses. The behavior of output and investment is consistent with a "recovery" effect.\textsuperscript{11} If the economy does indeed "recover" from a recession, we would expect the growth rate to be greater in the second phase of the business cycle (the first third of the recovery) than in the third and fourth phases. No recovery effect is apparent in either hours or real wages. In contrast to the concavity of expansions, recessions appear to be more or less linear. Aside from consumption, there is little evidence that the recession phases are statistically different. For consumption, the trough phase is quite different from the rest of the recession phases. Thus, the Burns and Mitchell phase regressions seem to imply that for aggregate output and investment the "shape" of the business cycle is characterized by concave expansions and linear recessions. The combination of a concave expansion and a linear recession supports a characterization of traditional business cycles as having "rounded" peaks and "pointed" troughs. There is also strong evidence against symmetric business cycles. With the exception of real wages, all of the series reject symmetry hypotheses either individually or jointly.

These results are similar to those of Sichel (1992) who examined the

\textsuperscript{11}Elsewhere we have examined the relationship between the rate of growth during the recovery period and various measures of the severity of the prior recession. See Wynne and Balke (1992, 1993).
behavior of quarterly real GNP. He broke the business cycle into a recession phase, a recovery phase, and a rest-of-expansion phase and considered different lengths for recovery phase. Like us, he found a significant recovery effect and argued for a three-phase characterization of the business cycle that includes a high growth recovery phase. However, the three-phase characterization of the business cycle only captures part of the concavity that we find.

2.2 Bry-Boschan dating of business cycles

The peak and trough dates that make up the official NBER business cycle chronology are determined by the business cycle dating committee of the NBER. To evaluate the ability of an artificial economy to mimic certain features of real world business cycles, we need to be able to replicate the NBER business cycle dating procedure using time series generated in the artificial economy. To this end we employ the business cycle dating algorithm devised by Bry and Boschan (1971) to automate the rules used by the business cycle dating committee in picking peak and trough dates. The structure of this algorithm is described in Appendix B. Essentially, the Bry-Boschan algorithm involves finding local maximums and minimums of a smoothed version of a time series subject to restrictions on the length of the entire cycle and on the length on expansion and recession phases. An obvious and important question is how well does the Bry-Boschan algorithm mimic the procedures used by the NBER committee. Table 3 reports the peak and trough dates selected by the algorithm using per capita GNP, along with the actual NBER business cycle.

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12 For a description of the business cycle dating process see Moore and Zarnowitz (1986) and Hall (1992).
dates. The Bry-Boschan dates are surprisingly close to the NBER dates. The algorithm tends to perform best in terms of picking trough dates, matching 7 of the 9 in the NBER Chronology. One trough is dated one quarter after the NBER date, and one is dated three quarters after the NBER date. The algorithm is a little less successful in picking peaks, matching only 3 of the NBER dates perfectly. Two peaks are dated one quarter before the NBER dates, and four are dated two quarters before the NBER dates. Recessions tend to be slightly longer in the Bry-Boschan chronology (4.9 quarters versus 3.6 quarters for the NBER dates) and expansions slightly shorter (16.1 quarters versus 17.1 quarters for NBER dates).13

Further insights into how well the Bry-Boschan algorithm performs can be obtained by comparing the properties of the cycles generated by the two chronologies. Tables 4 and 5 present the Burns and Mitchell phase regressions for the case where turning point dates were determined by the Bry-Boschan dating procedure. Again, for the most part peaks tend to be rounded while trough are pointed. Output and investment have business cycle shapes characterized by concave expansions and linear recessions, although the evidence is not quite as strong as was the case for the NBER dates. Interestingly, with the Bry-Boschan dates, hours seem to be more concave in recessions than in expansions as the growth rate of hours gets more negative as the recession deepens. As was the case for the NBER dates, real wages do not show significant phase behavior. Also, the Bry-Boschan cycles appear to be asymmetric; only real wages fail to strongly reject the hypothesis of

13Note that the turning points reported in Table 3 differ slightly from those reported in Table 2 of King and Plosser (1989). The difference arises from the fact that while we both use the same algorithm to date peaks and troughs, King and Plosser look at the behavior of per capita GNP while we look at GNP per head of population aged 16 and over.
symmetry.

On the basis of these comparisons we are reasonably confident that the Bry-Boschan algorithm does a good job at imitating the actual procedures used by the NBER committee. In the concluding section we will outline ways in which we think the algorithm could be improved upon.

2.3 The shape of growth cycles.

For the sake of comparison we also decided to examine the shape and symmetry of the growth cycles experienced by the US economy in the postwar period, and consider the ability of prototypical real business cycle models to replicate these features of the actual data.

Table 3 reports the Bry-Boschan turning points for growth cycles, along with the NBER growth cycle chronology. Not surprisingly, there are more growth cycles than traditional business cycles — ten versus eight for the period for which the two official chronologies overlap. Furthermore, unlike traditional business cycles, the Bry-Boschan growth cycle expansions and contractions have nearly the same average duration; the expansions average 7.5 quarters while the recessions average 7.4 quarters. Note that while the Bry-Boschan algorithm does a reasonable job of detecting NBER growth cycles (it misses only one), the Bry-Boschan algorithm does less well at matching NBER growth cycle dates than it does at matching business cycle dates.¹⁴

Tables 6 and 7 present results from the Burns and Mitchell phase regressions for the growth cycle characterization of business cycles. Unlike the traditional business cycle conception, the growth cycle seems to be adequately characterized by two phases: recessions and expansions. There is little evidence of concavity (or convexity) for either expansions or recessions. Furthermore, the hypothesis of symmetry is not rejected for any of the variables. Thus, in contrast to traditional business cycles, growth cycles appear to characterized by "linear" expansions and contractions, and by symmetry. For nearly every variable, there is less negative growth in the first recession phase (phase 6) than in the other recession phases, but it is not statistically significant.

3. Business and growth cycles in two simple artificial economies

The question we are concerned with in this paper is to what extent can simple artificial economies of the type developed in the current real business cycle literature generate traditional business cycles. What is required for such an exercise is a fully articulated dynamic general equilibrium model that can be calibrated to real world data and simulated to generate time path for the various economic aggregates of interest. Prototypical versions of models of this type that are driven by real shocks are studied in some detail in King, Plosser and Rebelo (1988a,b). We will consider two of the basic models discussed in that paper that are driven by transitory shocks to productivity, these being a variant of the Long and Plosser (1983) model with realistic depreciation, and the Rogerson (1988) and Hansen (1985) models with data match the NBER growth cycle dates more often than the dates obtained using deterministic detrended data, the Hodrick-Prescott detrended data yields three extra growth cycles that are not present in the NBER chronology.
indivisible labor.

3.1 Two simple model economies

The structure of the basic neoclassical model we examine is as follows. Household preferences are assumed to be defined over consumption, $C_t$, and leisure, $L_t$, and to have the standard time-separable form

$$\sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

Preferences are restricted to the following class (consistent with balanced growth)

$$U(C_t, L_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} \nu(1-N_t)$$

for $0 < \sigma < 1$ and $\sigma > 1$, and

$$U(C_t, L_t) = \log(C_t) + \nu(1-N_t)$$

for $\sigma = 1$. Here we will restrict ourselves to the log specification.

The only difference between the two economies we study has to do with the divisibility of labor. We consider the basic neoclassical model with divisible labor, and the variant on that model with indivisible labor associated with Rogerson (1988) and Hansen (1985). In the Rogerson-Hansen economy, households can work either some fixed number of hours, $\bar{N}_0$, or not at all. Optimal allocations in this economy involve trading in lotteries over consumption and leisure, which in turn yield a preference specification for the representative household that is linear in hours worked:
\[ U(C_t, L_t) = \log(C_t) + \psi N_t \]

where \( \psi = -\theta(\log(1-N_t))/\bar{N}_0 \) if \( \nu(1-N_t) = \theta \log(1-N_t) \) (see Hansen (1985) pp.315-318).

Output, \( Y_t \), is produced with capital, \( K_t \), and labor, \( N_t \), by means of standard constant returns to scale technology

\[ Y_t = A_t F(K_t, X_t, N_t) \]

where \( A_t \) denotes a transitory productivity shock and \( X_t \) represents labor augmenting technological progress. We further restrict ourselves to a Cobb-Douglas specification of the production function: \( F(K_t, X_t N_t) = K_t^{\alpha} (X_t N_t)^{\gamma} \). We assume that technological progress occurs at some exogenously determined rate, \( \gamma_t = X_{t+1}/X_t \), as the simplest way to induce nonstationarity in our model.

Capital accumulation follows the standard process

\[ K_{t+1} = (1-\delta_k)K_t + I_t \]

where \( I_t \) denotes investment and \( \delta_k \) denotes the rate of depreciation of the capital stock. Resource constraints on time and output are specified as follows

\[ L_t + N_t = 1 \]

and

\[ C_t + I_t = Y_t \]

It is straightforward to show that both of these models can be...
accommodated in the following linear system\textsuperscript{15}

\[
\begin{bmatrix}
    k_{t+1} \\
    \hat{A}_{t+1}
\end{bmatrix} =
\begin{bmatrix}
    \mu_1 & \pi_{kA} \\
    0 & \rho
\end{bmatrix}
\begin{bmatrix}
    k_t \\
    \hat{A}_t
\end{bmatrix} +
\begin{bmatrix}
    0 \\
    \epsilon_{A,t+1}
\end{bmatrix} = M s_t + \epsilon_{t+1} 
\]  

(3)

where lower case letters are simply the stationary (trend adjusted) versions of their upper case equivalents. \( \epsilon_A \) denotes a technology shock. Hats \(^\wedge\) are used to denote (percentage) deviations from steady state equilibrium levels. The elements of the matrices \( \Pi \) and \( \Pi_A \) are functions of the underlying parameters of tastes and technology. Table 8 below gives the values for the elements of these matrices for the two representative real business cycle models that we study, along with the more primitive parameters of tastes and technology.

3.2 The shape of the business cycle for the model economies

We simulated the system described by (3) and (4) above to generate a series for (detrended) output lasting 180 periods (which corresponds to the number of quarters in the post World War II sample). To examine traditional business cycles, we then restored the trend to the output series to generate the path of the level of output. The Bry-Boschan algorithm was then used to

\textsuperscript{15}See King Plosser and Rebelo (1988a,b) and Hansen and Sargent (1991).
pick peak and trough dates in the output series to obtain an NBER-style business cycle chronology. Using these dates we broke the expansions and contractions up into the Burns and Mitchell phases, and ran the phase regressions and symmetry tests for the generated data. We also considered the ability of these models to generate growth cycles. For these tests we applied the Bry-Boschan algorithm to the detrended output series generated by the artificial economies, and proceeded as before. These exercises were repeated 1000 times to arrive at Monte Carlo distributions of the various statistics of interest.\footnote{Our approach here is similar to that suggested by Gregory and Smith (1991).}

Table 9 provides some summary statistics for the business cycles generated in the two real business cycle models. The mean and standard deviation of the Monte Carlo distribution is presented as well as the percentile of the Monte Carlo distribution in which the actual business cycle statistic is placed. Both the basic Long-Plosser model and the Hansen-Rogerson model tend to generate more cycles (whether traditional business cycles or growth cycles) than were actually present in the post-World War II sample. However, because the actual business cycle statistics are not in the extreme tails of the Monte Carlo distributions implied by the two models (with the possible exception of the expansion duration of the HR model), these models do a reasonable job of producing recessions and expansions of with durations and amplitudes consistent with actual business cycles. King and Plosser (1989) show essentially the same result – see their figures 7 and 8.
3.3 Monte carlo phase results

To determine whether standard RBC models can replicate the "shape" of the business cycle, we ran the Burns and Mitchell phase regressions for each replication and tabulated the Monte Carlo distribution. Table 10 presents the mean and the standard deviation of the Monte Carlo distribution of the phase growth rates implied by the Long-Plosser model. The percentile of the Monte Carlo distribution in which the actual phase growth rate is present is presented as well. Table 11 tabulates the percentage of replications in which the various hypotheses about the shape and symmetry of the business cycle are rejected at 0.05 significance level for the Long-Plosser model. Tables 12 and 13 present analogous results for the traditional business cycle phases and the Hansen-Rogerson model.

From Tables 10-13, we see that the Long-Plosser and Hansen-Rogerson RBC models do not match the phase behavior of actual business cycles. Actual phase growth rates are often in the tails of the Monte Carlo distribution. For most of the variables, the actual growth rates in Phases 1, 2, 5, and 6 are in the extreme tails of the Monte Carlo distribution. Typically, the rates of decline in the trough (Phase 1) are too high in the model economies relative to actual growth rates, but the growth rates early in the expansion (Phase 2) implied by the model economies are too low. Similarly, the peak (Phase 5) the growth rates implied by the model economies are too high while for output, investment, and hours, the model economies imply a much sharper decline at the onset of a recession (Phase 6) than is the case for actual recessions. Furthermore, actual consumption is in the tails for almost every phase for both the LP and the HR models; the growth rate of actual consumption is higher in expansions and lower in recessions than those implied by the
model economies. This is consistent with the well-known result that
consumption is not volatile enough in these classes of models. In contrast,
the business cycle shape of investment suggests greater volatility in these
models than in actual investment; another well known characteristic of these
models.

In addition, the RBC models do not mimic the overall shape of business
cycles particularly well. For output, investment, and hours, the model
economies imply "spiked" peaks and troughs while in actual business cycles the
peaks are more "round". Focusing on aggregate output, concave expansions are
not a feature of these RBC models. From Table 11, the proportion of
replications in which hypothesis linear expansions (Phase 2 = 3 = 4) is
rejected is close to the nominal size of the test. Similarly, investment in
these models does not show concave business cycle shape. Consumption in the
model economies, on the other hand, is actually convex over the expansion as
the growth rate of consumption increases as the expansion progresses. For all
the variables, aside from perhaps the trough phase, linear recessions appear
to be a characteristic of these models. In fact, evidence against the basic
two phase cycle with just expansions and contractions is due primarily to the
"spiked" nature of peaks and troughs; the greatest rates of growth and decline
occur at peaks and recessions.

Tables 14, 15, 16, and 17 present phase results for growth cycle phases
for the Long-Plosser model and Hansen-Rogerson model. The RBC models seem to
mimic the shape of the growth cycle better than the shape of the business
cycle. Like the actual growth cycle, the RBC growth cycle is symmetric.17

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17 This is not surprising given that the variables were generated by a
linear approximation around the steady state.
However, the for RBC models there appears to be more evidence against the two phase characterization of the growth cycle than in the actual data. This is primarily due to the peak and trough phases of the RBC model. As was the case above, phases 1, 2, 5, and 6 from the actual growth cycle are typically in the tails of the Monte Carlo distributions. Again, the model economies generate peaks and troughs that are "spiked". Both models also generate consumption series that are not volatile enough while investment is too volatile.

Figures 1 through 5 display graphically the shape for both the business and growth cycle. For ease of comparison, the figures have been normalized so that same cyclical duration and trend hold for the actual and both models.

4. Conclusion and extensions

What have we learned from this exercise? Our objective in this paper was to evaluate the ability of simple real business cycles to generate classical business cycles (absolute increases and decreases in output) by breaking the cycle up into the expansion and contraction phases used by Burns and Mitchell in their monumental empirical study. We have shown that this way of looking at the cycle yields interesting insights about fluctuations in economic activity, and illuminates some shortcomings of basic real business cycle models. In some respects, the basic neo-classical model does quite well in capturing actual business cycles. It generates cycles with expansion and recession durations and growth rates that are not too different (in a statistical sense) from actual business cycles. However, with respect to the subtle shape of the business cycle, this model fails to generate the concave expansions and "rounded" peaks typical of actual business cycles. Second, the
difference between business cycles and growth cycles matters in the evaluation of the adequacy of the RBC model: the RBC model does a better job matching the shape of growth cycles than of business cycles. It is apparent that the business cycle is not just a growth cycle with a (deterministic) trend tacked on.

4.1. Extensions to the real business cycle model

The fact that the simple RBC models examined here were better able to replicate the shape nature of growth cycles than of business cycles suggests that there may be an interaction between the trend and cycle that neither of the artificial economies considered above adequately captures. Two extensions to the above analysis readily suggest themselves. The first is to allow for a common stochastic trend rather than a common deterministic trend as in King, Plosser and Rebelo (1988b). While the additional persistence implied by this model is likely to lengthen the duration of the cycle, it not clear that this extension would generate concave expansions or "round off" peaks. The second is to allow for external increasing returns to scale as in Baxter and King (1991). They have found that productive externalities improved the ability of the neo-classical model to match actual economic data.\(^\text{18}\) However, increasing returns to scale may not be capable of generating concave expansions and rounded peaks. The effect of the externality will be largest when aggregate output is high, such as during the late stages of the expansion, which may actually work against generating concave expansions.

A further extension would be to add sectors to the basic real business

\(^{18}\) They also include a government sector and preference shocks in the model.
cycle model to reflect government spending, net exports, inventory investment, and consumer durables. To determine whether the exclusion of these sectors is potentially important, we considered the phase behavior of just consumption of nondurables and services plus fixed investment. Tables 18 and 19 present the phase regressions and hypothesis tests for consumption of nondurables and services plus fixed investment (C+I) for both NBER dates and Bry-Boschan dates chosen on the basis of C+I (instead of GNP). The evidence of concave expansions in output, while still present, becomes substantially weaker depending on whether NBER dates or Bry-Boschan dates are used. While actual government spending and net exports have business cycle shapes that would, if anything, make it more difficult to generate concave expansions (Balke and Wynne (1992)), they would introduce an alternative source of shocks into the real business cycle model that has in other contexts improved "the fit" of the basic neo-classical model (Christiano and Eichenbaum (1992)). Inventory investment and expenditures on consumer durables, on the other hand, have very strong Burns and Mitchell phase behavior—they are both have very strong recovery effects and rounded peaks (see Balke and Wynne (1992) and Sichel (1992)). Nonetheless given the tendency of the basic RBC model to generate "spiked" peaks and troughs, it is not clear that adding these sectors will generate desired business cycle shape.

4.2. Statistical extensions

One possible extension is the consideration of alternative methods of dating business cycles. In this paper, we essentially used a univariate approach to dating business cycles: business cycle dates were determined by peaks and troughs in aggregate output (per capita). However, one the
fundamental characteristics of business cycles is the comovement of economic variables. The Bry-Boschan dating procedure, while relatively simple to implement, is univariate and, hence, may not reflect the multi-variate nature of the business cycle.

There are, at least, two possible alternatives to the univariate approach taken in this paper. One is to statistically identify a common factor among economic variables (for example Stock and Watson (1989)) and then use the Bry-Boschan procedure to pick peaks and troughs in the common factor.\(^{19}\) This would be applied to both the actual data and to the artificial data generated by the model economies. A second approach is the construction of a multivariate Markov regime switching model in which the common factor is the Markov regime variable. Peak and trough dates would then be chosen on the basis of whether the probability of being in a particular regime is above some arbitrary value (see Hamilton (1989)). Again, the approach would need to be applied to both the actual and artificial data. While these alternative methods are more satisfactory from a theoretical point-of-view, both methods are substantially more computer intensive than the univariate Bry-Boschan algorithm—an important drawback for repeated simulations of the artificial economies. Furthermore, it is unclear whether the business cycle dates chosen by these methods would be dramatically different from those chosen by the Bry-Boschan procedure given that it chooses dates that are roughly consistent with the NBER business cycle dates.

\(^{19}\) Note that, Stock and Watson (1989) estimate a common factor model in terms of first differences of the data. Picking peaks and troughs on the bases of the identified common factor (without adding the trends back in) would be more akin to identifying growth cycles rather than business cycles.
business cycles and those implied by the artificial economies. In this paper, we examined the Burns and Mitchell phase behavior to compare actual and artificial business cycles. While the Burns and Mitchell phases provide a relatively rich and flexible way to model the shape of the business cycle, they do not provide a single criterion by which we can evaluate whether a particular shape is present in the either the actual data or in the artificial data. One possible solution to this problem is to employ the approach recently developed by Neftci (1993) to test for particular shapes of the business cycle. His actual test applies to detrended data—hence, growth cycles—and would need to modified to allow for trend. Also, it is not clear how to translate the Neftci test to a multi-variate context. Nonetheless, using his approach may be a fruitful way to summarize whether artificial economies can generate business cycles apparent in the actual data.

4.3 Departures from linear models.

The typically concave shape of expansions and the linear shape of recessions suggest an asymmetry over the business cycle that is inconsistent with linear models of the business cycle (see Blatt (1980) and Sichel (1989)). Nonlinear models of the cycle that utilize the concept of a ceiling on output, such as Friedman's "plucking model" of business cycles (Friedman (1969, 1988)) and Hicks' model of the trade cycle (Hicks (1950)) (which also places a floor on output), may be consistent with the concavity of output over expansions. So too may neoclassical models with irreversible investment as analyzed by Sargent (1980) and Dow and Olson (1992).
Appendix A

The phase dummies were created as follows:

(i) \( \text{phase 1} = 1 \) if \( t = \text{trough} \), 0 otherwise;

(ii) the length of phase 2, phase 3, phase 4 = \( \text{integer}\[(\text{peak-trough})/3]\). If the remainder is 1, the extra month is assigned to phase 3. If the remainder is 2, an extra month is assigned to phase 2 and phase 4.

(iii) \( \text{phase 5} = 1 \) if \( t = \text{peak} \), 0 otherwise;

(iv) the length of phase 6, phase 7, phase 8 = \( \text{integer}\[(\text{trough - peak})/3]\). If the remainder is 1, the extra month is assigned to phase 7. If the remainder is 2, an extra month is assigned to phase 6 and phase 8.

Note that the rules for allocating remainder months between the different phases follows Burns and Mitchell (1946), pp. 145-146.
Appendix B

The Bry-Boschan Algorithm for Dating Business Cycles

(1.) Eliminate extreme values of the raw series and replace them with values obtained from a Spencer curve fitted to the series. Extreme values are defined as values of the ratio of the actual series to the Spencer curve that are more than 3.5 standard deviations from the sample mean of the ratio of the actual series to the Spencer curve. A Spencer curve is a symmetric filter with declining weights - essentially a centered 15-month moving average with weights -0.0094, -0.0188, -0.0156, 0.0094, 0.0656, 0.1438, 0.2094, 0.2313, 0.2094, 0.1438, 0.0656, 0.0094, -0.0156, -0.0188, -0.0094.

(2.) Calculate a 12-month (centered) moving average of the adjusted series. Find the local maxima and minima. Take the dates of these local maxima and minima as tentative peak and trough dates, making sure that peaks and troughs alternate. If peaks and troughs do not alternate, eliminate those peaks (troughs) with the lowest (highest) value.

(3.) Fit a Spencer curve to the adjusted series from (1.). Find the highest values of the Spencer curve that are within five months of the tentative peaks identified in step 2. Likewise find the lowest values of the Spencer curve that are within five months of the tentative troughs identified in step 2. Make sure that the new peak and trough dates alternate and that cycle duration is at least fifteen months.

(4.) Calculate a four-month moving average of the adjusted series from (1.). Find the highest values of this series that are within five months of the peaks identified in step 3. Likewise find the lowest values of this series that are within five months of the troughs identified in step 3. Make sure that the new peak and trough dates alternate and that cycle duration is at least fifteen months.
least fifteen months.

(5.) Using the raw series adjusted for extremes, find the highest values within four months of the peaks identified in step 4. Likewise find the lowest values of this series that are within four months of the troughs identified in step 4. Be sure that no peak or trough date is within six months of the beginning or end of the sample, that peak and trough dates alternate, that cycle duration is at least fifteen months, and that expansion and contraction phases are at least five months long. The resulting peak and trough dates are taken as the business cycle turning points.
References


Rogerson, Richard, 1988, "Indivisible labor, lotteries and equilibrium," Journal of Monetary Economics, 21, 3-16.


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<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
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<td>(3.82)</td>
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<td>(1.77)</td>
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<td>(Peak)</td>
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<td>(1.32)</td>
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Notes to Table 1. Standard errors in parentheses. Output is defined as GNP per capita, where population is defined as the total civilian noninstitutional population over 16 years of age. Consumption is consumption of nondurables and services per capita. Investment is gross private fixed investment, again in per capita terms. Hours is defined as the product of total employment, as measured by the Household Survey, times the average weekly hours of all workers from the Household Survey, divided by the total civilian noninstitutional population over 16. Wages are measured as the gross average hourly earnings production workers. All data is from CITIBASE.
Table 2

Hypothesis Tests - Shape and Symmetry of the Business Cycle
Burns and Mitchell Phases, NBER Dates

<table>
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<tr>
<th>Hypothesis</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
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<td>0.713</td>
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Tests of Symmetry; NBER Dates

<table>
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<tr>
<th>Hypothesis</th>
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<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
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Notes to Table 2.
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Notes to Table 3. NBER business cycle dates are from Moore and Zarnowitz (1986) Table A.3 and NBER (1992). NBER growth cycle dates are from Moore and Zarnowitz (1986) Table A.8. The NBER growth cycle chronology ends with the growth recession of 1978-82. Bry-Boschan business cycle dates were obtained by applying the Bry-Boschan business cycle dating rules to per capita GNP. The Bry-Boschan growth cycle dates were obtained by applying the Bry-Boschan algorithm to detrended per capita GNP.
<table>
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Notes to Table 4. See notes to Table 1.
Table 5
Hypothesis Tests - Shape and Symmetry of the Business Cycle
Burns and Mitchell Phases
Bry-Boschan Dates

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<tr>
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<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
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Tests of Symmetry
Burns and Mitchell Phases; Bry-Boschan Dates

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<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession/ Expansion</td>
<td>0.000</td>
<td>0.006</td>
<td>0.000</td>
<td>0.001</td>
<td>0.087</td>
</tr>
<tr>
<td>Phase 5/ Phase 1</td>
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<td>0.135</td>
<td>0.006</td>
<td>0.520</td>
</tr>
<tr>
<td>Phase 6/ Phase 2</td>
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<td>0.404</td>
<td>0.627</td>
<td>0.359</td>
<td>0.051</td>
</tr>
<tr>
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<td>0.012</td>
<td>0.100</td>
<td>0.309</td>
</tr>
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<td>Phase 8/ Phase 4</td>
<td>0.001</td>
<td>0.006</td>
<td>0.006</td>
<td>0.000</td>
<td>0.393</td>
</tr>
<tr>
<td>Joint</td>
<td>0.000</td>
<td>0.010</td>
<td>0.003</td>
<td>0.000</td>
<td>0.201</td>
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Notes to Table 5.
<table>
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<th>Phase</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
</tr>
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<td>-4.51</td>
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<td>-5.00</td>
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</tr>
<tr>
<td>(Trough)</td>
<td>(1.03)</td>
<td>(0.74)</td>
<td>(3.60)</td>
<td>(0.92)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>Phase 2</td>
<td>3.48</td>
<td>0.91</td>
<td>5.47</td>
<td>0.45</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>(0.70)</td>
<td>(0.50)</td>
<td>(2.44)</td>
<td>(0.66)</td>
<td>(0.68)</td>
</tr>
<tr>
<td>Phase 3</td>
<td>3.00</td>
<td>1.39</td>
<td>5.80</td>
<td>1.93</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>(0.71)</td>
<td>(0.51)</td>
<td>(2.49)</td>
<td>(0.67)</td>
<td>(0.68)</td>
</tr>
<tr>
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<td>1.22</td>
<td>5.25</td>
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<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(0.70)</td>
<td>(0.50)</td>
<td>(2.44)</td>
<td>(0.66)</td>
<td>(0.68)</td>
</tr>
<tr>
<td>Phase 5</td>
<td>3.14</td>
<td>0.04</td>
<td>5.99</td>
<td>2.65</td>
<td>0.87</td>
</tr>
<tr>
<td>(Peak)</td>
<td>(1.03)</td>
<td>(0.74)</td>
<td>(3.60)</td>
<td>(0.92)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>Phase 6</td>
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<td>-0.22</td>
<td>-1.27</td>
<td>-0.39</td>
<td>-0.34</td>
</tr>
<tr>
<td></td>
<td>(0.77)</td>
<td>(0.55)</td>
<td>(2.67)</td>
<td>(0.72)</td>
<td>(0.74)</td>
</tr>
<tr>
<td>Phase 7</td>
<td>-2.16</td>
<td>-1.01</td>
<td>-7.20</td>
<td>-1.07</td>
<td>-0.52</td>
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<tr>
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<td>(0.70)</td>
<td>(0.50)</td>
<td>(2.44)</td>
<td>(0.66)</td>
<td>(0.68)</td>
</tr>
<tr>
<td>Phase 8</td>
<td>-3.66</td>
<td>-1.57</td>
<td>-7.06</td>
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<td>-0.84</td>
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<td>(0.55)</td>
<td>(2.67)</td>
<td>(0.72)</td>
<td>(0.74)</td>
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<tr>
<td>$\bar{R}^2$</td>
<td>0.41</td>
<td>0.12</td>
<td>0.15</td>
<td>0.18</td>
<td>-0.01</td>
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<tr>
<td>DW</td>
<td>1.74</td>
<td>2.40</td>
<td>1.63</td>
<td>1.86</td>
<td>1.69</td>
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</table>
Table 7
Hypothesis Tests: Burns and Mitchell Phases
Growth Cycle (Deterministic detrending and Bry-Boschan Dates)

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ph 2=3=4=5</td>
<td>0.196</td>
<td>0.311</td>
<td>0.760</td>
<td>0.182</td>
<td>0.896</td>
</tr>
<tr>
<td>and Ph 6-7-8=1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ph 2=3=4=5</td>
<td>0.429</td>
<td>0.480</td>
<td>0.998</td>
<td>0.214</td>
<td>0.908</td>
</tr>
<tr>
<td>Ph 2-3-4</td>
<td>0.264</td>
<td>0.794</td>
<td>0.988</td>
<td>0.216</td>
<td>0.790</td>
</tr>
<tr>
<td>Ph 2=3</td>
<td>0.634</td>
<td>0.503</td>
<td>0.925</td>
<td>0.119</td>
<td>0.605</td>
</tr>
<tr>
<td>Ph 2=4</td>
<td>0.112</td>
<td>0.667</td>
<td>0.950</td>
<td>0.144</td>
<td>0.891</td>
</tr>
<tr>
<td>Ph 3=4</td>
<td>0.271</td>
<td>0.807</td>
<td>0.876</td>
<td>0.908</td>
<td>0.514</td>
</tr>
<tr>
<td>Ph 6-7-8=1</td>
<td>0.118</td>
<td>0.200</td>
<td>0.347</td>
<td>0.221</td>
<td>0.642</td>
</tr>
<tr>
<td>Ph 6-7-8=1</td>
<td>0.231</td>
<td>0.252</td>
<td>0.193</td>
<td>0.154</td>
<td>0.891</td>
</tr>
<tr>
<td>Ph 6-7</td>
<td>0.866</td>
<td>0.331</td>
<td>0.103</td>
<td>0.489</td>
<td>0.854</td>
</tr>
<tr>
<td>Ph 6-8</td>
<td>0.124</td>
<td>0.098</td>
<td>0.127</td>
<td>0.058</td>
<td>0.635</td>
</tr>
<tr>
<td>Ph 7=8</td>
<td>0.150</td>
<td>0.447</td>
<td>0.968</td>
<td>0.194</td>
<td>0.755</td>
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</table>

Tests of Symmetry

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession/</td>
<td>0.934</td>
<td>0.549</td>
<td>0.988</td>
<td>0.766</td>
<td>0.764</td>
</tr>
<tr>
<td>Expansion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase 5/</td>
<td>0.347</td>
<td>0.855</td>
<td>0.846</td>
<td>0.703</td>
<td>0.257</td>
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<tr>
<td>Phase 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase 6/</td>
<td>0.152</td>
<td>0.397</td>
<td>0.247</td>
<td>0.957</td>
<td>0.767</td>
</tr>
<tr>
<td>Phase 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase 7/</td>
<td>0.402</td>
<td>0.590</td>
<td>0.687</td>
<td>0.367</td>
<td>0.730</td>
</tr>
<tr>
<td>Phase 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase 8/</td>
<td>0.091</td>
<td>0.634</td>
<td>0.619</td>
<td>0.587</td>
<td>0.987</td>
</tr>
<tr>
<td>Phase 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>0.167</td>
<td>0.865</td>
<td>0.772</td>
<td>0.867</td>
<td>0.819</td>
</tr>
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</table>

Notes to Table 7.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Long-Plosser with realistic depreciation</th>
<th>Hansen-Rogerson (infinite elasticity of labor supply)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.953</td>
<td>0.947</td>
</tr>
<tr>
<td>$\sigma_{kA}$</td>
<td>0.137</td>
<td>0.164</td>
</tr>
<tr>
<td>$\sigma_{ck}$</td>
<td>0.617</td>
<td>0.598</td>
</tr>
<tr>
<td>$\sigma_{cA}$</td>
<td>0.298</td>
<td>0.337</td>
</tr>
<tr>
<td>$\sigma_{nk}$</td>
<td>-0.294</td>
<td>-0.424</td>
</tr>
<tr>
<td>$\sigma_{NA}$</td>
<td>1.048</td>
<td>1.579</td>
</tr>
<tr>
<td>$\sigma_{yA}$</td>
<td>0.249</td>
<td>0.174</td>
</tr>
<tr>
<td>$\sigma_{IA}$</td>
<td>1.608</td>
<td>1.916</td>
</tr>
<tr>
<td>$\sigma_{ik}$</td>
<td>-0.629</td>
<td>-0.838</td>
</tr>
<tr>
<td>$\sigma_{ik}$</td>
<td>4.733</td>
<td>5.683</td>
</tr>
<tr>
<td>$\sigma_{wA}$</td>
<td>0.544</td>
<td>0.598</td>
</tr>
<tr>
<td>$\sigma_{wA}$</td>
<td>0.560</td>
<td>0.337</td>
</tr>
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</table>

Notes to Table 8. Data are from King, Plosser and Rebelo (1988a), Table 3. Underlying parameters of tastes and technology are as follows: $\beta = 0.988$, $\theta = 0.2$, $\gamma_x = 1.004$, $\alpha = 0.58$. 
### Table 9
Summary Statistics for Monte Carlo Business Cycles
Average (standard deviation) of Monte Carlo Distribution, and [percentile] in the Monte Carlo distribution that actual business cycle statistic occupies.

<table>
<thead>
<tr>
<th></th>
<th>Actual (8-B dates)</th>
<th>Long-Plosser</th>
<th>Hansen-Rogerson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Peaks</td>
<td>8</td>
<td>11</td>
<td>9.6 (1.7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[0.25]</td>
</tr>
<tr>
<td>Number of troughs</td>
<td>9</td>
<td>11</td>
<td>9.5 (1.6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[0.51]</td>
</tr>
<tr>
<td>Average Duration of</td>
<td>16.1 (3.9)</td>
<td>7.5 (1.3)</td>
<td>13.8 (3.0)</td>
</tr>
<tr>
<td>Expansion</td>
<td></td>
<td></td>
<td>[0.80]</td>
</tr>
<tr>
<td></td>
<td>4.9 (0.6)</td>
<td>7.4 (1.4)</td>
<td>4.3 (0.8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[0.78]</td>
</tr>
<tr>
<td>Average Rate of</td>
<td>3.17 (0.31)</td>
<td>2.84 (0.39)</td>
<td>2.96 (0.34)</td>
</tr>
<tr>
<td>Growth in Expansion</td>
<td></td>
<td></td>
<td>[0.76]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.72 (0.56)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[0.25]</td>
</tr>
<tr>
<td>Average Rate of</td>
<td>-3.17 (0.55)</td>
<td>-2.79 (0.41)</td>
<td>-2.82 (0.56)</td>
</tr>
<tr>
<td>Growth in Recession</td>
<td></td>
<td></td>
<td>[0.25]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.72 (0.56)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[0.25]</td>
</tr>
</tbody>
</table>

Notes to Table 9. Standard errors in parentheses. Percentile of Monte Carlo distribution in which the actual datum lies is reported in square brackets.
Table 10
Burns and Mitchell Business Cycle Phase Regressions
Long-Plosser (with Trend)

<table>
<thead>
<tr>
<th>Phase 1 (Trough)</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-4.22</td>
<td>0.24</td>
<td>-14.87</td>
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<td>-0.64</td>
</tr>
<tr>
<td></td>
<td>(1.01)</td>
<td>(0.19)</td>
<td>(3.08)</td>
<td>(0.70)</td>
<td>(0.33)</td>
</tr>
<tr>
<td></td>
<td>[0.40]</td>
<td>[1.00]</td>
<td>[1.00]</td>
<td>[0.59]</td>
<td>[1.00]</td>
</tr>
<tr>
<td>Phase 2</td>
<td>3.15</td>
<td>1.58</td>
<td>6.89</td>
<td>1.25</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>(0.62)</td>
<td>(0.17)</td>
<td>(1.80)</td>
<td>(0.40)</td>
<td>(0.24)</td>
</tr>
<tr>
<td></td>
<td>[0.98]</td>
<td>[1.00]</td>
<td>[0.82]</td>
<td>[0.86]</td>
<td>[0.98]</td>
</tr>
<tr>
<td>Phase 3</td>
<td>2.58</td>
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<td>4.42</td>
<td>0.62</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>(0.62)</td>
<td>(0.18)</td>
<td>(1.80)</td>
<td>(0.40)</td>
<td>(0.25)</td>
</tr>
<tr>
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<td>[1.00]</td>
<td>[0.31]</td>
<td>[0.99]</td>
<td>[0.96]</td>
</tr>
<tr>
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<td>2.17</td>
</tr>
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<td>(0.19)</td>
<td>(1.68)</td>
<td>(0.37)</td>
<td>(0.25)</td>
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<tr>
<td></td>
<td>[0.13]</td>
<td>[0.51]</td>
<td>[0.17]</td>
<td>[0.51]</td>
<td>[0.29]</td>
</tr>
<tr>
<td>Phase 5 (Peak)</td>
<td>4.72</td>
<td>2.40</td>
<td>10.25</td>
<td>1.86</td>
<td>2.87</td>
</tr>
<tr>
<td></td>
<td>(1.07)</td>
<td>(0.24)</td>
<td>(3.17)</td>
<td>(0.71)</td>
<td>(0.39)</td>
</tr>
<tr>
<td></td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.01]</td>
</tr>
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<td>0.99</td>
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<td>0.11</td>
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<tr>
<td></td>
<td>(1.01)</td>
<td>(0.21)</td>
<td>(3.09)</td>
<td>(0.70)</td>
<td>(0.34)</td>
</tr>
<tr>
<td></td>
<td>[0.93]</td>
<td>[0.00]</td>
<td>[0.98]</td>
<td>[0.99]</td>
<td>[0.01]</td>
</tr>
<tr>
<td>Phase 7</td>
<td>-2.09</td>
<td>0.93</td>
<td>-9.31</td>
<td>-2.42</td>
<td>0.33</td>
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<tr>
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<td>(1.27)</td>
<td>(0.24)</td>
<td>(3.85)</td>
<td>(0.87)</td>
<td>(0.42)</td>
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<tr>
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<td>[0.08]</td>
<td>[0.00]</td>
<td>[0.42]</td>
<td>[0.14]</td>
<td>[0.44]</td>
</tr>
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<td>-1.68</td>
<td>0.78</td>
<td>-7.54</td>
<td>-1.97</td>
<td>0.29</td>
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<tr>
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<td>(1.20)</td>
<td>(0.25)</td>
<td>(3.59)</td>
<td>(0.81)</td>
<td>(0.42)</td>
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<tr>
<td></td>
<td>[0.14]</td>
<td>[0.00]</td>
<td>[0.15]</td>
<td>[0.01]</td>
<td>[0.89]</td>
</tr>
</tbody>
</table>

Notes to Table 10.
<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ph 2<del>3</del>4=5 and Ph 6<del>7</del>8=1</td>
<td>0.16</td>
<td>0.82</td>
<td>0.18</td>
<td>0.20</td>
<td>0.32</td>
</tr>
<tr>
<td>Ph 2<del>3</del>4=5</td>
<td>0.12</td>
<td>0.74</td>
<td>0.12</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>Ph 2<del>3</del>4=5</td>
<td>0.04</td>
<td>0.54</td>
<td>0.07</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Ph 2~4=5</td>
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<td>0.25</td>
<td>0.09</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
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<td>0.07</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Ph 6<del>7</del>8~1</td>
<td>0.12</td>
<td>0.34</td>
<td>0.12</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>Ph 6<del>7</del>8~1</td>
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Tests of Symmetry

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Notes to Table 12.
### Table 13
Hypothesis Tests: Hansen-Rogerson Model (with trend)
Percentage of replications in which reject at the 0.05 significance level

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### Tests of Symmetry

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Tests of Symmetry

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<td>0.03</td>
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<td>0.03</td>
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<tr>
<td>Phase 8/Phase 4</td>
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Notes to Table 15.
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<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
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<td>[0.00]</td>
<td>[0.99]</td>
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<td>(3.10)</td>
<td>(0.88)</td>
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<td>(0.71)</td>
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<td>[0.34]</td>
<td>[1.00]</td>
<td>[1.00]</td>
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<tr>
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<td>-7.09</td>
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</table>

Notes to Table 16.
Table 17
Hypothesis Tests: Symmetry of Growth Cycles
Hansen-Rogerson Model (without trend)
Percentage of replications in which reject at the 0.05 significance level

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ph 2-3-4-5 and Ph 6-7-8-1</td>
<td>0.42</td>
<td>0.99</td>
<td>0.44</td>
<td>0.48</td>
<td>0.99</td>
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<tr>
<td>Ph 2-3-4</td>
<td>0.09</td>
<td>0.45</td>
<td>0.16</td>
<td>0.21</td>
<td>0.45</td>
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<td>Ph 2-3</td>
<td>0.13</td>
<td>0.14</td>
<td>0.19</td>
<td>0.23</td>
<td>0.14</td>
</tr>
<tr>
<td>Ph 2-4</td>
<td>0.08</td>
<td>0.58</td>
<td>0.18</td>
<td>0.26</td>
<td>0.58</td>
</tr>
<tr>
<td>Ph 3-4</td>
<td>0.04</td>
<td>0.24</td>
<td>0.04</td>
<td>0.04</td>
<td>0.24</td>
</tr>
<tr>
<td>Ph 6-7-8-1</td>
<td>0.30</td>
<td>0.90</td>
<td>0.32</td>
<td>0.34</td>
<td>0.90</td>
</tr>
<tr>
<td>Ph 6-7-8</td>
<td>0.13</td>
<td>0.42</td>
<td>0.23</td>
<td>0.29</td>
<td>0.42</td>
</tr>
<tr>
<td>Ph 6-8</td>
<td>0.15</td>
<td>0.13</td>
<td>0.22</td>
<td>0.26</td>
<td>0.13</td>
</tr>
<tr>
<td>Ph 7-8</td>
<td>0.13</td>
<td>0.54</td>
<td>0.25</td>
<td>0.34</td>
<td>0.54</td>
</tr>
<tr>
<td>Ph 6-7</td>
<td>0.04</td>
<td>0.18</td>
<td>0.04</td>
<td>0.05</td>
<td>0.18</td>
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</table>

Tests of Symmetry

<table>
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<tr>
<th>Hypothesis</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession/Expansion</td>
<td>0.01</td>
<td>0.08</td>
<td>0.01</td>
<td>0.01</td>
<td>0.08</td>
</tr>
<tr>
<td>Phase 5/Phase 1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Phase 6/Phase 2</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
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<tr>
<td>Phase 7/Phase 3</td>
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<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>Phase 8/Phase 4</td>
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<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
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<tr>
<td>Joint</td>
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<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
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Notes to Table 17.
<table>
<thead>
<tr>
<th>Phase</th>
<th>C+I NBER dates</th>
<th>C+I B-B dates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Trough)</td>
<td>(Peak)</td>
</tr>
<tr>
<td>Phase 1</td>
<td>0.30 (1.08)</td>
<td>4.34 (1.02)</td>
</tr>
<tr>
<td></td>
<td>2.47 (0.48)</td>
<td>2.88 (0.48)</td>
</tr>
<tr>
<td>Phase 4</td>
<td>1.65 (0.49)</td>
<td>2.67 (0.48)</td>
</tr>
<tr>
<td>Phase 5</td>
<td>-0.22 (1.08)</td>
<td>1.73 (1.08)</td>
</tr>
<tr>
<td>Phase 6</td>
<td>-3.30 (1.23)</td>
<td>-2.27 (0.88)</td>
</tr>
<tr>
<td>Phase 7</td>
<td>-3.22 (1.08)</td>
<td>-1.89 (1.25)</td>
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<tr>
<td>Phase 8</td>
<td>-3.92 (1.23)</td>
<td>-2.41 (0.88)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.32</td>
<td>0.38</td>
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<tr>
<td>DW</td>
<td>1.89</td>
<td>1.74</td>
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Notes to Table 18.
### Table 19
Hypothesis Tests - Shape and Symmetry of the Business Cycle
Burns and Mitchell Phases

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<th>Hypothesis</th>
<th>C+I NBER dates</th>
<th>C+I B-B dates</th>
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</thead>
<tbody>
<tr>
<td>Ph 2=3=4=5 and Ph 6=7=8=1</td>
<td>0.000</td>
<td>0.423</td>
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<tr>
<td>Ph 2=3=4</td>
<td>0.002</td>
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<td>Ph 2=3</td>
<td>0.019</td>
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<tr>
<td>Ph 2=4</td>
<td>0.001</td>
<td>0.237</td>
</tr>
<tr>
<td>Ph 3=4</td>
<td>0.235</td>
<td>0.756</td>
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<tr>
<td>Ph 6=7=8=1</td>
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<td>0.344</td>
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<td>Ph 6=7=8</td>
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<td>0.944</td>
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<td>Ph 6=7</td>
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<td>0.805</td>
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<td>Ph 6=8</td>
<td>0.720</td>
<td>0.911</td>
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<td>Ph 7=8</td>
<td>0.670</td>
<td>0.735</td>
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#### Tests of Symmetry

<table>
<thead>
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<th>Hypothesis</th>
<th>C+I NBER dates</th>
<th>C+I B-B dates</th>
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</thead>
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<tr>
<td>Recession/ Expansion</td>
<td>0.000</td>
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<td>Phase 5/ Phase 1</td>
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<tr>
<td>Joint</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Notes to Table 19**


Figure 1

Business Cycle Shape for Output

- Actual (BB dates)
- LP
- HR

Growth Cycle Shape for Output

- Actual (BB dates)
- LP
- HR
Figure 2

*Business Cycle Shape for Consumption*

- **Actual (BB dates)**
- **LP**
- **HR**

---

**Growth Cycle Shape for Consumption**

- **Actual (BB dates)**
- **LP**
- **HR**
Figure 3
Business Cycle Shape for Investment

- Actual (BB dates)
- LP
- HR

Figure 4
Growth Cycle Shape for Investment

- Actual (BB dates)
- LP
- HR
Figure 4
Business Cycle Shape for Hours

Growth Cycle Shape for Hours
Figure 5

Business Cycle Shape for Real Wages

- Actual (88 dates)
- LP
- HR

Growth Cycle Shape for Real Wages

- Actual (88 dates)
- LP
- HR

The graphs illustrate the business cycle and growth cycle for real wages, showing actual and model-based estimates.
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