

**Problems of Testing Fiscal Solvency in High Inflation Economies:  
Evidence from Argentina, Brazil, and Mexico**

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**Abstract**

Most cointegration tests of dynamic government solvency use a measure of seignorage that is significantly biased for high inflation. Using a more appropriate measure, cointegration tests indicate government solvency in Argentina, Brazil, and Mexico during the 1980s.

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The question of government solvency has received a lot of attention in recent years especially with the large U.S. fiscal deficits and the fiscal distress suffered by Latin American governments since the onset of the "debt crisis" in the 1980s. In fact, such distress led to foreign debt moratoria (Argentina, Brazil and Mexico in 1982 and Brazil in 1987) and internal debt (Argentina 1989 and 1990 and Brazil 1990) due to the untested diagnosis that these public sectors were "insolvent."

Tests of solvency usually concentrate on testing whether the components of the real budget deficit form a stationary linear combination [Hamilton and Flavin (1986), Hakkio and Rush (1991), and Trehan and Walsh (1988 and 1991)]. In other words, recent studies concentrate on testing whether the real non-interest fiscal deficit, the level of real (domestic and foreign) public debt, and real seignorage are cointegrated. But as Trehan and Walsh (1991) point out, this approach assumes that real interest rates on public sector debt are constant. They suggest that testing the stationarity of the first difference of public debt is equivalent to testing for cointegration. The assumed constancy of real interest rates on government debt, however, is not the only problem with cointegration tests of government solvency.

I argue here these tests suffer another significant drawback in high inflation economies: discrete time estimates of seignorage collection will be biased downward. Since seignorage is collected continuously, the bias will be larger the higher the rate of money growth. Most studies of fiscal solvency have limited their analysis to the United States and Europe where the bias is relatively small because these regions have experienced relatively low inflation rates since World War II.<sup>1</sup> This is not true in Latin America where chronic inflation has been the norm in most countries.

#### **Tests of Public Sector Solvency**

The methodology for testing solvency is based upon the stochastic characteristics of the components of the budget constraint. Briefly, if we assume interest rate parity and that, on average, the exchange rate follows the domestic rate of inflation, the dynamic government budget constraint is

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<sup>1</sup>In fact Ahmed and Rogers (1993) analyze fiscal budget balance in the United States ignoring seignorage completely.

$$d_{t+1} = \delta_{t+1} + (1 + \rho_r)d_t - \sigma_{t+1} \quad (1)$$

where the real level of debt is  $d_t = D_t/P_t$ , the real primary government deficit is  $\delta_{t+1} = (G_{t+1} - T_{t+1})/P_{t+1}$ , the real value of seignorage is  $\sigma_{t+1} = \Delta M_{t+1}/P_{t+1}$ ,  $D_t$  is (domestic plus foreign) government debt,  $M_t$  is monetary base,  $G_t$  is the totality of government non-interest spending,  $T_t$  is the totality of government non-interest revenues,  $\pi_t$ <sup>2</sup> is the expected rate of inflation, and  $\rho_r$  is the average real rate of interest on government debt.

Suppose the time series vector  $X_t = [\delta_{t+1}, d_t, \sigma_{t+1}]$  is first difference stationary. By the Wold decomposition theorem,  $X_t$  can be represented

$$(1 - L)X_t = \lambda + C(L)v_t \quad (2)$$

where  $C(L)$  is a  $3 \times 3$  matrix in the lag operator,  $\lambda$  is a drift term, and  $v_t$  is a vector white noise process with  $v_t = [\nu_{1,t}, \nu_{2,t}, \nu_{3,t}]$ . We can form the inclusive of debt interest government deficit by multiplying  $X_t$  by the cointegrating vector  $B' = [1, \rho, -1]$ . This yields the following expression<sup>2</sup>

$$(1 - L)B'X_t = B'\lambda + B'C(L)v_t \quad (3)$$

One can use equation (4) to rationally forecast the value of future government debt. Substituting equation (4) into equation (2) and iterating forward, one finds the solution to the value of  $d_t$ . As Trehan and Walsh (1991) show, equation (4) implies that if intertemporal budgets are satisfied (no bubbles), real government debt will follow the following process

$$(1 - L)d_{t+1} = \delta_{t+1} + \rho_r d_t - \sigma_{t+1} = \frac{B'\lambda}{\rho} + D(L)v_t \quad (4)$$

where  $D(L)v_t$  is stationary. Equation (5) implies that for dynamic budget balance to obtain the first difference

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<sup>2</sup>If one separates internal debt and external debt, as I do below for Argentina and Brazil,  $X_t = [\delta_{t+1}, d_t, d'_t, \sigma_{t+1}]$ ,  $C(L)$  is a  $4 \times 4$  matrix in the lag operator, and  $v_t$  is a vector white noise process with  $v_t = [\nu_{1,t}, \nu_{2,t}, \nu_{3,t}, \nu_{4,t}]$ . Real internal public sector debt now equals  $d_t$  while real foreign debt equals  $d'_t$ . Assuming that these governments could not borrow internationally, the inclusive of debt interest government deficit will equal  $B'X_t$  where the cointegrating vector  $B' = [1, \rho, \rho^*, -1]$ , where  $\rho$  is the real interest rate on internal debt and  $\rho^*$  is the real interest rate on external debt.

of real debt must be stationary or, equivalently, the primary deficit, the stock of internal debt, the stock of foreign debt, and seignorage are cointegrated with cointegrating vector  $\beta' = [1, \rho_p, -1]$ . Most studies test for government insolvency by testing whether such cointegration exists. Unfortunately, this approach suffers from two severe limitations. The first pointed out by Trehan And Walsh (1991) is that one must assume that the real interest rate on government debt is constant. Second, the test necessitates the measurement of real seignorage, which is problematic at best.

#### The Bias in Conventional Measures of Seignorage

Simple discrete measurement of seignorage generates a biased measure of the real resource flow from money creation as money growth is more or less a continuous process [Welch, Primo Braga, and André (1987) and Cukierman (1988)]. To see this, consider the continuous time amount of seignorage collected at time  $t$

$$\hat{\sigma}_t = \frac{\dot{M}_t}{P_t} \quad (5)$$

where variables are defined as above and the dot represents an instantaneous time derivative. Note that

$$\hat{\sigma}_t = \mu \frac{M_t}{P_t} = \mu \frac{M_0}{P_0} e^{(\mu - \pi)t} \quad (6)$$

where  $\mu$  now represents the instantaneous rate of nominal money growth<sup>3</sup> and  $\pi$  is the instantaneous rate of inflation.

Integrating equation (8) from  $t$  to  $t+1$  yields

$$\sigma_{t+1}^* = \int_t^{t+1} \hat{\sigma}_t dt = \frac{M_t}{P_t} \left[ \frac{\mu}{\mu - \pi} (e^{\mu - \pi} - 1) \right] \quad (7)$$

On the other hand, discrete time measurement of seignorage gives

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<sup>3</sup>The instantaneous rate of growth of money,  $\mu$ , can be approximated by  $\ln(1+\mu')$  where  $\mu'$  is the discrete time rate of growth. Note that for the period of time selected, money growth is assumed constant. The instantaneous inflation rate can be approximated in a similar fashion.

$$\sigma_{t+1} = \frac{M_t}{P_t} \left[ \frac{M_{t+1}}{M_t} - 1 \right] = \frac{M_t}{P_t} [e^{\mu} - 1] \quad (8)$$

Subtracting equation (8) from (9) yields an expression for the bias inherent usual measures of seigniorage<sup>4</sup>

$$\sigma_{t+1} - \sigma_{t+1}^* = \frac{M_t}{P_t} \left[ \frac{\mu}{\mu - \pi} (e^{\mu - \pi} - 1) - e^{\mu} + 1 \right] \quad (9)$$

This non-linear bias becomes larger the larger the money growth rate and the larger the divergence of inflation from the money growth rate. A better measure is to approximate equation (8) by

$$\sigma_t^* \approx \ln(1 + \mu_t^*) \frac{M_t}{P_t} \quad (10)$$

where  $\mu^*$  is the discrete time measure of money growth  $(M_{t+1}/M_t) - 1$ .

#### Empirical Evidence

First, I will compare the test results for Mexico of using the two methods of calculating seigniorage in cointegration tests of government budget balance. The Mexican data covers the period 1980:2 to 1988:12. Table 1 shows augmented Dickey and Fuller (1979) tests of stationarity of levels and first differences of the real primary government deficit,<sup>5</sup> real government debt outstanding, and the two measures of real seigniorage. All variables have unit roots but are significantly stationary in first differences. The fact that real debt is first difference stationary implies that the dynamic government budget constraint in Mexico was fulfilled from 1980 to 1988 [Trehan and Walsh (1991)].

Cointegration tests yield different results. Table 2 shows the augmented Dickey-Fuller tests of stationarity of the linear combination of the real primary deficit, real debt, and real seigniorage with cointegrating vector [1, 11.943, -1] The value of the real interest rate, 11.943, corresponds to the weighted average interest

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<sup>4</sup>The analysis extends to the inflation tax in a straight forward way.

<sup>5</sup>The primary deficit includes all non-financial revenues and expenditures of the Mexican public sector.

rate on foreign and domestic government debt as calculated by Feliz and Torres (1991). If one uses the discrete measure of seignorage, these variables are not significantly cointegrated. The continuous time approximation, however, yields significant cointegration which is consistent with the fact that the first difference of real debt is also stationary.

How do the higher inflation countries of Argentina and Brazil compare to Mexico? Data on primary deficits and debt in Argentina and Brazil is scarce so I will concentrate on testing the stationarity of the first difference of real government domestic debt. The tests that appear in Table 3 show significant stationarity of the changes in real internal government debt. Even in these countries, dynamic budget balance holds; seignorage adjusts to satisfy the government's budget constraint in spite of a shrinking real monetary base.

#### **Final Comments**

Continuous time approximations of seignorage revenue indicate that seignorage adjusts to fulfill the government's dynamic budget constraint in high inflation countries. Such a conclusion is important not only in terms of determining the source of inflationary pressure in these countries but also in determining if these countries reached a point where seignorage revenue could not "finance" the real resources the governments of these countries. In other words, some have argued that these countries would move into a debt lead hyperinflation the real deficit was larger than the peak of the so-called "inflation Laffer-curve." Theoretically in such a scenario, the government would not be able generate enough inflation tax to meet its necessities without a continuous acceleration of inflation and internal debt because the real monetary base would shrink faster than the government could create base money. Such a notion motivated the partial internal debt moratoria implemented in Argentina (December 1989 and January 1990) and Brazil (March 1990). The evidence presented here, however, shows that neither Argentina and Brazil, let alone Mexico, never reached such a point prior to their moratoria.

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## Data Appendix

Argentina: INDEC

Brazil: All data comes from the Fundação Getúlio Vargas and the Banco Central do Brasil.

Mexico: Data on Mexican primary surplus, money growth, external and internal debt, and prices come from the Banco de Mexico data base Sie-Sat.



**Table 1**  
**Mexico: Tests of a Unit Root and Time Trend 1986:3-1990:2**  
**Real Internal Government Debt**

a. Null Hypothesis: Variable has a Unit Root		
Variable	Augmented Dickey-Fuller Test <sup>(a)</sup>	
	T-ratio	
	with time trend	without time trend
Primary Surplus (G-T)	-1.87	-1.54
(Discrete) Seignorage	-2.37	-0.504
real government debt <sup>(b)</sup>	-2.11	-1.33
$\Delta$ Primary Surplus (G-T)	-6.01***	-8.12***
$\Delta$ (Discrete) Seignorage	-8.12***	-8.16***
$\Delta$ Real Government Debt <sup>(b)</sup>	-11.24***	-11.26***

- Notes: (a) Six lags were used in these tests of stationarity for all variables except debt which used two lags. The lag structure was chosen by adding lags until the Q(30) statistic did not reject the null hypothesis of autocorrelated residuals. The test results were not sensitive to the choice of lag length.
- (b) Variable significantly violates normality assumption either because of skewness or kurtosis using the tests developed in Jarque and Bera (1980).

**Table 2**  
Mexico: Tests for Unit Root on  $\delta_t + 11.943d_t - \sigma_t$

a. Null Hypothesis: Variable has a Unit Root		
	Augmented Dickey-Fuller Test <sup>(a)</sup>	
	T-ratio	
	with time trend	without time trend
With discrete measure of seignorage	-2.12	-1.863
With continuous measure of seignorage	-3.96**	-68.73***

Notes: (a) Six lags were used without a trend in these tests of stationarity. The lag structure was chosen by adding lags until the Q(22) statistic did not reject the null hypothesis of autocorrelated residuals.

\* signifies significance at the  $\alpha=0.10$  level, \*\* signifies significance at the  $\alpha=0.05$  level, and \*\*\* signifies significance at the  $\alpha=0.01$  level.

**Table 3a**

Argentina: Tests for a Unit Root 1986:3-1990:2  
Real Internal Government Debt

a. Null Hypothesis: Variable has a Unit Root (with time trend)		
Variable	Phillips-Perron Test T-ratio	Augmented Dickey-Fuller Test <sup>(a)</sup> T-ratio
$\Delta$ real government debt	-4.17***	-4.17***
b: Null Hypothesis: Variable has Unit Root (no time trend)		
Variable	Phillips-Perron Test T-ratio	Augmented Dickey-Fuller Test <sup>(a)</sup> T-ratio
$\Delta$ real government debt	-4.07***	-4.07***

**Table 3b**

Brazil: Tests of a Unit Root and Time Trend 1986:3-1990:2  
Real Internal Government Debt

a. Null Hypothesis: Variable has a Unit Root (with time trend)		
Variable	Phillips-Perron Test T-ratio	Augmented Dickey-Fuller Test <sup>(a)</sup> T-ratio
$\Delta$ real government debt(b)	-5.02***	-4.56***
b: Null Hypothesis: Variable has Unit Root (with no time trend)		
Variable	Phillips-Perron Test T-ratio	Augmented Dickey-Fuller Test <sup>(a)</sup> T-ratio
$\Delta$ real government debt(b)	-4.94***	-4.50***

Notes: (a) Zero lags were used in the Argentine tests and one lag was used in the Brazilian tests of stationarity. The lag structure was chosen by adding lags until the Q(22) statistic did not reject the null hypothesis of autocorrelated residuals.

(b) Variable significantly violates normality assumption either because of skewness or kurtosis using the tests developed in Jarque and Bera (1980).

\* signifies significance at the  $\alpha=0.10$  level, \*\* signifies significance at the  $\alpha=0.05$  level, and \*\*\* signifies significance at the  $\alpha=0.01$  level.

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