Problems of Testing Fiscal Solvency in High Inflation Economies: Evidence from Argentina, Brazil, and Mexico

John H. Welch, Research Department
Federal Reserve Bank of Dallas

The views expressed in this article are solely those of the authors and should not be attributed to the Federal Reserve Bank of Dallas or to the Federal Reserve System.
Problems of Testing Fiscal Solvency in High Inflation Economies: Evidence from Argentina, Brazil, and Mexico

John H. Welch
Federal Reserve Bank of Dallas

February 1993
Revised August 1993

Abstract

Most cointegration tests of dynamic government solvency use a measure of seignorage that is significantly biased for high inflation. Using a more appropriate measure, cointegration tests indicate government solvency in Argentina, Brazil, and Mexico during the 1980s.

*Senior Economist, Federal Reserve Bank of Dallas, Station K, Dallas, TX 75007, Tel. (214)922-5165, Fax. (214)922-5194. I would like to thank Shengyi Guo for careful research assistance and Raúl A. Feliz for his comments and for providing me with his data. Of course, any remaining errors and omissions are the responsibility of the author. The views expressed in this article are those of the author and should not be attributed to the Federal Reserve Bank of Dallas or the Federal Reserve System.
The question of government solvency has received a lot of attention in recent years especially with the large U.S. fiscal deficits and the fiscal distress suffered by Latin American governments since the onset of the "debt crisis" in the 1980s. In fact, such distress led to foreign debt moratoria (Argentina, Brazil and Mexico in 1982 and Brazil in 1987) and internal debt (Argentina 1989 and 1990 and Brazil 1990) due to the untested diagnosis that these public sectors were "insolvent."

Tests of solvency usually concentrate on testing whether the components of the real budget deficit form a stationary linear combination [Hamilton and Flavin (1986), Hakkio and Rush (1991), and Trehan and Walsh (1988 and 1991)]. In other words, recent studies concentrate on testing whether the real non-interest fiscal deficit, the level of real (domestic and foreign) public debt, and real seignorage are cointegrated. But as Trehan and Walsh (1991) point out, this approach assumes that real interest rates on public sector debt are constant. They suggest that testing the stationarity of the first difference of public debt is equivalent to testing for cointegration. The assumed constancy of real interest rates on government debt, however, is not the only problem with cointegration tests of government solvency.

I argue here these tests suffer another significant drawback in high inflation economies: discrete time estimates of seignorage collection will be biased downward. Since seignorage is collected continuously, the bias will be larger the higher the rate of money growth. Most studies of fiscal solvency have limited their analysis to the United States and Europe where the bias is relatively small because these regions have experienced relatively low inflation rates since World War II. This is not true in Latin America where chronic inflation has been the norm in most countries.

Tests of Public Sector Solvency

The methodology for testing solvency is based upon the stochastic characteristics of the components of the budget constraint. Briefly, if we assume interest rate parity and that, on average, the exchange rate follows the domestic rate of inflation, the dynamic government budget constraint is

---

1In fact Ahmed and Rogers (1993) analyze fiscal budget balance in the United States ignoring seignorage completely.
\[ d_{t+1} = \delta_{t+1} + (1 + \rho)\rho d_t - \sigma_{t+1} \]  

(1)

where the real level of debt is \( d_t = D_t/P_t \), the real primary government deficit is \( \delta_{t+1} = (G_{t+1} - T_{t+1})/P_{t+1} \), the real value of seignorage is \( \sigma_{t+1} = \Delta M_{t+1}/P_{t+1} \), \( D_t \) is (domestic plus foreign) government debt, \( M_t \) is monetary base, \( G_t \) is the totality of government non-interest spending, \( T_t \) is the totality of government non-interest revenues, \( \pi_t^* \) is the expected rate of inflation, and \( \rho \) is the average real rate of interest on government debt.

Suppose the time series vector \( X_t = [\delta_{t+1}, d_t, \sigma_{t+1}] \) is first difference stationary. By the Wold decomposition theorem, \( X_t \) can be represented

\[ (1 - L)X_t = \lambda + C(L)\nu_t \]  

(2)

where \( C(L) \) is a 3 x 3 matrix in the lag operator, \( \lambda \) is a drift term, and \( \nu_t \) is a vector white noise process with \( \nu_t = [\nu_1, \nu_2, \nu_3] \). We can form the inclusive of debt interest government deficit by multiplying \( X_t \) by the cointegrating vector \( B' = [1, \rho, r, -1] \). This yields the following expression

\[ (1 - L)\beta'X_t = \beta'\lambda + \beta' C(L)\nu_t \]  

(3)

One can use equation (4) to rationally forecast the value of future government debt. Substituting equation (4) into equation (2) and iterating forward, one finds the solution to the value of \( d_t \). As Trehan and Walsh (1991) show, equation (4) implies that if intertemporal budgets are satisfied (no bubbles), real government debt will follow the following process

\[ (1 - L)d_{t+1} = \delta_{t+1} + (1 + \rho)\rho d_t - \sigma_{t+1} = \frac{\beta'\lambda}{\rho} + D(L)\nu_t \]  

(4)

where \( D(L)\nu_t \) is stationary. Equation (5) implies that for dynamic budget balance to obtain the first difference

\[ 2^2 \text{If one separates internal debt and external debt, as I do below for Argentina and Brazil, } X_t = [\delta_{t+1}, d_t, d'_{t+1}, \sigma_{t+1}], C(L) \text{ is a 4 x 4 matrix in the lag operator, and } \nu_t \text{ is a vector white noise process with } \nu_t = [\nu_1, \nu_2, \nu_3, \nu_4]. \text{ Real internal public sector debt now equals } d_t \text{ while real foreign debt equals } d'_{t+1}. \text{ Assuming that these governments could not borrow internationally, the inclusive of debt interest government deficit will equal } B'X_t, \text{ where the cointegrating vector } B' = [1, \rho, \rho', -1]. \text{ where } \rho \text{ is the real interest rate on internal debt and } \rho' \text{ is the real interest rate on external debt.} \]
of real debt must be stationary or, equivalently, the primary deficit, the stock of internal debt, the stock of foreign
debt, and seignorage are cointegrated with cointegrating vector $\beta' = [1, \rho_\mu - 1]$. Most studies test for government
insolvency by testing whether such cointegration exists. Unfortunately, this approach suffers from two severe
limitations. The first pointed out by Trehan And Walsh (1991) is that one must assume that the real interest
rate on government debt is constant. Second, the test necessitates the measurement of real seignorage, which
is problematic at best.

The Bias in Conventional Measures of Seignorage

Simple discrete measurement of seignorage generates a biased measure of the real resource flow from money
creation as money growth is more or less a continuous process [Welch, Primo Braga, and André (1987) and
Cukierman (1988)]. To see this, consider the continuous time amount of seignorage collected at time $t$

$$\hat{\sigma}_t = \frac{M_t}{P_t}$$

where variables are defined as above and the dot represents an instantaneous time derivative. Note that

$$\hat{\sigma}_t = M_t = \frac{M_0}{P_0}e^{\mu \pi t}$$

where $\mu$ now represents the instantaneous rate of nominal money growth and $\pi$ is the instantaneous rate of
inflation.

Integrating equation (8) from $t$ to $t+1$ yields

$$\sigma_{t+1}^* = \int_t^{t+1} \hat{\sigma}_t dt = \frac{M_t}{P_t} \left[ \frac{\mu}{\mu - \pi} (e^{\mu \pi} - 1) \right]$$

On the other hand, discrete time measurement of seignorage gives

---

3The instantaneous rate of growth of money, $\mu$, can be approximated by $\ln(1+\mu^\tau)$ where $\mu^\tau$ is the discrete
time rate of growth. Note that for the period of time selected, money growth is assumed constant. The
instantaneous inflation rate can be approximated in a similar fashion.
\[
\sigma_{t+1} = \frac{M_t}{P_t} \left( \frac{M_{t+1}}{M_t} - 1 \right) = \frac{M_t}{P_t} (e^\mu - 1) \tag{8}
\]

Subtracting equation (8) from (9) yields an expression for the bias inherent usual measures of seigniorage:

\[
\sigma_{t+1} - \sigma_{t+1}^* = \frac{M_t}{P_t} \frac{\mu - \pi}{\mu + \pi} (e^\mu - 1 - e^\mu + 1) \tag{9}
\]

This non-linear bias becomes larger the larger the money growth rate and the larger the divergence of inflation from the money growth rate. A better measure is to approximate equation (8) by

\[
\sigma_t^* = \ln(1 + \mu_t) \frac{M_t}{P_t} \tag{10}
\]

where \(\mu_t\) is the discrete time measure of money growth \((M_{t+1}/M_t) - 1\).

**Empirical Evidence**

First, I will compare the test results for Mexico of using the two methods of calculating seigniorage in cointegration tests of government budget balance. The Mexican data covers the period 1980:2 to 1988:12. Table 1 shows augmented Dickey and Fuller (1979) tests of stationarity of levels and first differences of the real primary government deficit, real government debt outstanding, and the two measures of real seigniorage. All variables have unit roots but are significantly stationary in first differences. The fact that real debt is first difference stationary implies that the dynamic government budget constraint in Mexico was fulfilled from 1980 to 1988 [Trehan and Walsh (1991)].

Cointegration tests yield different results. Table 2 shows the augmented Dickey-Fuller tests of stationarity of the linear combination of the real primary deficit, real debt, and real seigniorage with cointegrating vector \([1, 11.943, -1]\). The value of the real interest rate, 11.943, corresponds to the weighted average interest

---

*The analysis extends to the inflation tax in a straightforward way.

*The primary deficit includes all non-financial revenues and expenditures of the Mexican public sector.
rate on foreign and domestic government debt as calculated by Feliz and Torres (1991). If one uses the discrete measure of seignorage, these variables are not significantly cointegrated. The continuous time approximation, however, yields significant cointegration which is consistent with the fact that the first difference of real debt is also stationary.

How do the higher inflation countries of Argentina and Brazil compare to Mexico? Data on primary deficits and debt in Argentina and Brazil is scarce so I will concentrate on testing the stationarity of the first difference of real government domestic debt. The tests that appear in Table 3 show significant stationarity of the changes in real internal government debt. Even in these countries, dynamic budget balance holds; seignorage adjusts to satisfy the government's budget constraint in spite of a shrinking real monetary base.

Final Comments

Continuous time approximations of seignorage revenue indicate that seignorage adjusts to fulfill the government's dynamic budget constraint in high inflation countries. Such a conclusion is important not only in terms of determining the source of inflationary pressure in these countries but also in determining if these countries reached a point where seignorage revenue could not "finance" the real resources the governments of these countries. In other words, some have argued that these countries would move into a debt lead hyperinflation if the real deficit was larger than the peak of the so-called "inflation Laffer-curve." Theoretically in such a scenario, the government would not be able generate enough inflation tax to meet its necessities without a continuous acceleration of inflation and internal debt because the real monetary base would shrink faster than the government could create base money. Such a notion motivated the partial internal debt moratoria implemented in Argentina (December 1989 and January 1990) and Brazil (March 1990). The evidence presented here, however, shows that neither Argentina and Brazil, let alone Mexico, never reached such a point prior to their moratoria.
References


Data Appendix

Argentina: INDEC

Brazil: All data comes from the Fundação Getúlio Vargas and the Banco Central do Brasil.

Mexico: Data on Mexican primary surplus, money growth, external and internal debt, and prices come from the Banco de Mexico data base Sie-Sat.
Table 1
Mexico: Tests of a Unit Root and Time Trend 1986:3-1990:2
Real Internal Government Debt

<table>
<thead>
<tr>
<th>Variable</th>
<th>Augmented Dickey-Fuller Test(a)</th>
<th>T-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with time trend</td>
<td>without time trend</td>
</tr>
<tr>
<td>Primary Surplus (G-T)</td>
<td>-1.87</td>
<td>-1.54</td>
</tr>
<tr>
<td>(Discrete) Seignorage</td>
<td>-2.37</td>
<td>-0.504</td>
</tr>
<tr>
<td>real government debt(b)</td>
<td>-2.11</td>
<td>-1.33</td>
</tr>
<tr>
<td>△Primary Surplus (G-T)</td>
<td>-6.01***</td>
<td>-8.12***</td>
</tr>
<tr>
<td>△(Discrete) Seignorage</td>
<td>-8.12***</td>
<td>-8.16***</td>
</tr>
<tr>
<td>△Real Government Debt(b)</td>
<td>-11.24***</td>
<td>-11.26***</td>
</tr>
</tbody>
</table>

Notes: (a) Six lags were used in these tests of stationarity for all variables except debt which used two lags. The lag structure was chosen by adding lags until the Q(30) statistic did not reject the null hypothesis of autocorrelated residuals. The test results were not sensitive to the choice of lag length.

(b) Variable significantly violates normality assumption either because of skewness or kurtosis using the tests developed in Jarque and Bera (1980).
| Table 2  
Mexico: Tests for Unit Root on $\delta_t + 11.943d_t - \sigma_t$ |
|---------------------------------------------------------------|

**a. Null Hypothesis: Variable has a Unit Root**

### Augmented Dickey-Fuller Test\(^{(a)}\)

<table>
<thead>
<tr>
<th></th>
<th>T-ratio with time trend</th>
<th>T-ratio without time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>With discrete measure of</td>
<td>-2.12</td>
<td>-1.863</td>
</tr>
<tr>
<td>seignorage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With continuous measure of</td>
<td>-3.96(^{**})</td>
<td>-68.73(^{***})</td>
</tr>
<tr>
<td>seignorage</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** (a) Six lags were used without a trend in these tests of stationarity. The lag structure was chosen by adding lags until the Q(22) statistic did not reject the null hypothesis of autocorrelated residuals.

* signifies significance at the $\alpha=0.10$ level, ** signifies significance at the $\alpha=0.05$ level, and *** signifies significance at the $\alpha=0.01$ level.
### Table 3a

Argentina: Tests for a Unit Root 1986:3-1990:2
Real Internal Government Debt

<table>
<thead>
<tr>
<th>Variable</th>
<th>Phillips-Perron Test</th>
<th>Augmented Dickey-Fuller Test&lt;sup&gt;(a)&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T-ratio</td>
<td>T-ratio</td>
</tr>
<tr>
<td>Δreal government debt</td>
<td>-4.17&lt;sup&gt;***&lt;/sup&gt;</td>
<td>-4.17&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

**Notes:** (a) Zero lags were used in the Argentine tests and one lag was used in the Brazilian tests of stationarity. The lag structure was chosen by adding lags until the Q(22) statistic did not reject the null hypothesis of autocorrelated residuals.
(b) Variable significantly violates normality assumption either because of skewness or kurtosis using the tests developed in Jarque and Bera (1980).

* signifies significance at the $\alpha=0.10$ level, ** signifies significance at the $\alpha=0.05$ level, and *** signifies significance at the $\alpha=0.01$ level.

### Table 3b

Brazil: Tests of a Unit Root and Time Trend 1986:3-1990:2
Real Internal Government Debt

<table>
<thead>
<tr>
<th>Variable</th>
<th>Phillips-Perron Test</th>
<th>Augmented Dickey-Fuller Test&lt;sup&gt;(a)&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T-ratio</td>
<td>T-ratio</td>
</tr>
<tr>
<td>Δreal government debt&lt;sup&gt;(b)&lt;/sup&gt;</td>
<td>-5.02&lt;sup&gt;***&lt;/sup&gt;</td>
<td>-4.56&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Phillips-Perron Test</th>
<th>Augmented Dickey-Fuller Test&lt;sup&gt;(a)&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T-ratio</td>
<td>T-ratio</td>
</tr>
<tr>
<td>Δreal government debt&lt;sup&gt;(b)&lt;/sup&gt;</td>
<td>-4.94&lt;sup&gt;***&lt;/sup&gt;</td>
<td>-4.50&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

**Notes:** (a) Zero lags were used in the Argentine tests and one lag was used in the Brazilian tests of stationarity. The lag structure was chosen by adding lags until the Q(22) statistic did not reject the null hypothesis of autocorrelated residuals.
(b) Variable significantly violates normality assumption either because of skewness or kurtosis using the tests developed in Jarque and Bera (1980).

* signifies significance at the $\alpha=0.10$ level, ** signifies significance at the $\alpha=0.05$ level, and *** signifies significance at the $\alpha=0.01$ level.
<table>
<thead>
<tr>
<th>No.</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>9201</td>
<td>Are Deep Recessions Followed by Strong Recoveries?</td>
<td>Mark A. Wynne and Nathan S. Balke</td>
</tr>
<tr>
<td>9202</td>
<td>The Case of the &quot;Missing M2&quot;</td>
<td>John V. Duca</td>
</tr>
<tr>
<td>9203</td>
<td>Immigrant Links to the Home Country: Implications for Trade, Welfare,</td>
<td>David M. Gould</td>
</tr>
<tr>
<td></td>
<td>and Factor Rewards</td>
<td></td>
</tr>
<tr>
<td>9204</td>
<td>Does Aggregate Output Have a Unit Root?</td>
<td>Mark A. Wynne</td>
</tr>
<tr>
<td>9205</td>
<td>Inflation and Its Variability: A Note</td>
<td>Kenneth M. Emery</td>
</tr>
<tr>
<td>9206</td>
<td>Budget Constrained Frontier Measures of Fiscal Equality and Efficiency</td>
<td>Shawna Grosskopf, Kathy Hayes, Lori Taylor, William</td>
</tr>
<tr>
<td></td>
<td>in Schooling</td>
<td>Weber</td>
</tr>
<tr>
<td>9207</td>
<td>The Effects of Credit Availability, Nonbank Competition, and Tax</td>
<td>John V. Duca and Bonnie Garrett</td>
</tr>
<tr>
<td></td>
<td>Reform on Bank Consumer Lending</td>
<td></td>
</tr>
<tr>
<td>9208</td>
<td>On the Future Erosion of the North American Free Trade Agreement</td>
<td>William C. Gruben</td>
</tr>
<tr>
<td>9209</td>
<td>Threshold Cointegration</td>
<td>Nathan S. Balke and Thomas B. Fomby</td>
</tr>
<tr>
<td>9210</td>
<td>Cointegration and Tests of a Classical Model of Inflation in Argentina,</td>
<td>Raúl Aníbal Feliz and John H. Welch</td>
</tr>
<tr>
<td></td>
<td>Bolivia, Brazil, Mexico, and Peru</td>
<td></td>
</tr>
<tr>
<td>9211</td>
<td>Nominal Feedback Rules for Monetary Policy: Some Comments</td>
<td>Evan F. Koenig</td>
</tr>
<tr>
<td>9212</td>
<td>The Analysis of Fiscal Policy in Neoclassical Models</td>
<td>Mark Wynne</td>
</tr>
<tr>
<td>9213</td>
<td>Measuring the Value of School Quality</td>
<td>Lori Taylor</td>
</tr>
<tr>
<td>9214</td>
<td>Forecasting Turning Points: Is a Two-State Characterization of the</td>
<td>Kenneth M. Emery &amp; Evan F. Koenig</td>
</tr>
<tr>
<td></td>
<td>Business Cycle Appropriate?</td>
<td></td>
</tr>
</tbody>
</table>
9216 An Analysis of the Impact of Two Fiscal Policies on the Behavior of a Dynamic Asset Market (Gregory W. Huffman)

9301 Human Capital Externalities, Trade, and Economic Growth (David Gould and Roy J. Ruffin)

9302 The New Face of Latin America: Financial Flows, Markets, and Institutions in the 1990s (John Welch)

9303 A General Two Sector Model of Endogenous Growth with Human and Physical Capital (Eric Bond, Ping Wang, and Chong K. Yip)

9304 The Political Economy of School Reform (S. Grosskopf, K. Hayes, L. Taylor, and W. Weber)

9305 Money, Output, and Income Velocity (Theodore Palivos and Ping Wang)

9306 Constructing an Alternative Measure of Changes in Reserve Requirement Ratios (Joseph H. Haslag and Scott E. Hein)

9307 Money Demand and Relative Prices During Episodes of Hyperinflation (Ellis W. Tallman and Ping Wang)

9308 On Quantity Theory Restrictions and the Signalling Value of the Money Multiplier (Joseph Haslag)

9309 The Algebra of Price Stability (Nathan S. Balke and Kenneth M. Emery)

9310 Does It Matter How Monetary Policy is Implemented? (Joseph H. Haslag and Scott E. Hein)

9311 Real Effects of Money and Welfare Costs of Inflation in an Endogenously Growing Economy with Transactions Costs (Ping Wang and Chong K. Yip)

9312 Borrowing Constraints, Household Debt, and Racial Discrimination in Loan Markets (John V. Duca and Stuart Rosenthal)

9313 Default Risk, Dollarization, and Currency Substitution in Mexico (William Gruben and John Welch)

9314 Technological Unemployment (W. Michael Cox)

9315 Output, Inflation, and Stabilization in a Small Open Economy: Evidence From Mexico (John H. Rogers and Ping Wang)
Price Stabilization, Output Stabilization and Coordinated Monetary Policy Actions (Joseph H. Haslag)

An Alternative Neo-Classical Growth Model with Closed-Form Decision Rules (Gregory W. Huffman)

Why the Composite Index of Leading Indicators Doesn’t Lead (Evan F. Koenig and Kenneth M. Emery)

Allocative Inefficiency and Local Government: Evidence Rejecting the Tiebout Hypothesis (Lori L. Taylor)

The Output Effects of Government Consumption: A Note (Mark A. Wynne)

Should Bond Funds be Included in M2? (John V. Duca)

Recessions and Recoveries in Real Business Cycle Models: Do Real Business Cycle Models Generate Cyclical Behavior? (Mark A. Wynne)

Retaliation, Liberalization, and Trade Wars: The Political Economy of Nonstrategic Trade Policy (David M. Gould and Graeme L. Woodbridge)


Growth and Equity with Endogenous Human Capital: Taiwan’s Economic Miracle Revisited (Maw-Lin Lee, Ben-Chieh Liu, and Ping Wang)

Clearinghouse Banks and Banknote Over-issue (Scott Freeman)

Coal, Natural Gas and Oil Markets after World War II: What’s Old, What’s New? (Mine K. Yücel and Shengyi Guo)

On the Optimality of Interest-Bearing Reserves in Economies of Overlapping Generations (Scott Freeman and Joseph Haslag)

Retaliation, Liberalization, and Trade Wars: The Political Economy of Nonstrategic Trade Policy (David M. Gould and Graeme L. Woodbridge) Reprint of 9323

On the Existence of Nonoptimal Equilibria in Dynamic Stochastic Economies (Jeremy Greenwood and Gregory W. Huffman)

The Credibility and Performance of Unilateral Target Zones: A Comparison of the Mexican and Chilean Cases (Raul A. Feliz and John H. Welch)
9332  Endogenous Growth and International Trade (Roy J. Ruffin)
9333  Wealth Effects, Heterogeneity and Dynamic Fiscal Policy (Zsolt Becsi)
9334  The Inefficiency of Seigniorage from Required Reserves (Scott Freeman)
9335  Problems of Testing Fiscal Solvency in High Inflation Economies: Evidence from Argentina, Brazil, and Mexico (John H. Welch)