A COMPARISON OF ALTERNATIVE MONETARY ENVIRONMENTS

Joseph H. Haslag

October 1995

RESEARCH DEPARTMENT

WORKING PAPER

95-11

Federal Reserve Bank of Dallas
A Comparison of

Alternative Monetary Environments

Joseph H. Haslag

Research Department
Federal Reserve Bank of Dallas
and
Department of Economics
Southern Methodist University

Abstract: In many macroeconomic models, agents hold fiat money balances, despite being rate-of-return dominated, to satisfy either a cash-in-advance constraint or reserve requirements. In this paper, I compare the allocations from the two different economies. Despite the inherent differences in these two modelling approaches, the alternative monetary environments are equivalent in the sense that one can obtain identical equilibrium allocations. This equivalence result hold for a particular combination of monetary policy variables; that is, namely, there is a combination policy characterized by the inflation rate and reserve requirement ratio such that the reserve-requirement model is equivalent to other monetary environments.

I have benefitted greatly from conversations with Jerry Dwyer, Scott Freeman, Rik Hafer, Greg Huffman, Evan Koenig, Finn Kydland, and Carlos Zarazaga. Such assistance, however, should not be interpreted as implicating these people as responsible for any remaining errors. The views expressed herein do not necessarily reflect the views of the Federal Reserve Bank of Dallas nor the Board of Governors of the Federal Reserve System.
1. Introduction

In many economic environments, infinitely-lived agents hold fiat money, despite being a comparatively poor store of value, either because they face a cash-in-advance constraint, it saves on transactions costs, or there is a reserve requirement. Woodford (1990), and others, have demonstrated that models with money-in-the-utility function can be more explicitly represented by models in which there is a cash-in-advance constraint. More generally, functional equivalence will hold between the money-in-the-utility function and any model in which money is valued.

The purpose of this paper is to ask, In what sense are different monetary environments equivalent? In practice, I focus on comparing two environments: a model in which money is held to satisfy a cash-in-advance constraint and a model in which a reserve requirement is present. I investigate the similarities between the alternative environments in terms of the allocations obtained in a monetary equilibrium. Thus, one contribution of this paper is to detail similarities between the model with a cash-in-advance constraint and a model with a reserve requirement. In doing so, one also sees the key differences between these two monetary economies.

The main result in this paper is to show that in steady-state, identical allocations can be obtained from the two model economies. What is crucial for this equivalence is the combination of the inflation rate and reserve requirement ratio. More specifically, for a given inflation rate, one can analytically solve for the steady-state level of capital and consumption in the cash-in-

---

1 Well-known examples of models in which a cash-in-advance constraint is present include Lucas and Stokey (1983), Christiano and Eichenbaum (1992), and Fuerst (1992). See Sargent and Wallace (1985), Freeman (1987), and Smith (1991) for models in which reserve requirements are present. The 'transactions costs' category is a very broad class of models intended to include alternative modelling strategies such as Townsend's (1980) model with spatially separated agents and shopping-time models, such as the one specified in Saving (1971).

2 See Feenstra (1985) for development of the functional equivalence between models with money-in-the-utility function and the class of models with transactions costs.
advance model. The identical capital-consumption allocation will be the solution in the reserve-requirement model for the inflation rate and a particular value of the reserve requirement ratio.

Though the equivalence result is obtained in the direct comparison of the cash-in-advance and reserve-requirement models, the rationale for holding money is not critical. The class of monetary economies that are equivalent to the reserve-requirement model (in the sense described above) can be extended to include models in which the steady-state level of capital is between zero and the level obtained in a non-monetary economy. The trade-off between the inflation rate and the reserve requirement ratio plays a key role in the equivalence result. In the reserve-requirement model, monetary policy directly affects the real return on deposits. There are literally an infinite number of inflation rate-reserve requirement ratio combinations that are consistent with the same real return. The flexibility inherent to the reserve-requirement model means that the set of possible solutions for steady-state capital is between zero and that obtained in a non-monetary economy. Consequently, the reserve-requirement model is equivalent to a much broader class of monetary economies than the example considered here.

I also consider equivalence in terms of seignorage revenue in addition to allocations. In the economies studied here, only a few policy combinations satisfy both allocation and seignorage-revenue equivalence. To keep seignorage revenue equal across different economies it is necessary to maintain both the tax rate and tax base. In this example, the tax base differs which imposes restrictions on the set of reserve requirements ratios that will yield seignorage-revenue equality.

In this paper, I examine a general version of the model developed in Stockman (1983) in which fiat money is required to purchase the consumption good and capital. The reserve-requirement model is a hybrid model in which fiat money is required to purchase the consumption good and to satisfy a reserve requirement. Thus, both models share the feature
that the consumption good is a cash good. In addition, fiat money is required for capital purchases in both economies.

In comparing the two economies, I consider two definitions of equivalence. In "model" equivalence, I compare the first-order conditions from the two alternative models, seeking the set of restrictions that will result in the two sets of first-order conditions being identical. I also consider weaker notion, "allocation" equivalence which is satisfied if the two economies have the same equilibrium quantities of capital and consumption. I show that it would purely coincidental for the conditions for model equivalence to be satisfied and, indeed, would generally fail to hold in a steady-state allocation in which the capital stock is strictly positive. As noted above, however, the two model economies are allocation equivalent for a set of policy combinations.

The paper is organized as follows. In Section 2, the economic environment for the model with a cash-in-advance constraint is developed and the equilibrium is characterized. An economy is specified in Section 3 in which there is a reserve requirement and a cash-in-advance constraint on the consumption good. In addition, I propose a strong definition of equivalence, showing that the two alternative environments generally will not satisfy this definition. In Section 4, I focus on the steady-state allocations of capital and consumption for the two models. It is possible to demonstrate that the two models will yield identical steady-state allocations for a particular combination of the reserve requirement ratio and inflation rate. The conditions are derived in Section 5 for equivalent seignorage revenue across the two models that also meet the equivalent allocation definition. Section 6 then briefly summarizes the findings.

2. The cash-in-advance model

First, I will develop the model with the cash-in-advance constraint. The model has is
essentially the one developed in Stockman, with the cash-in-advance constraint generalized to let agents finance a fraction their gross investment spending with credit.

Suppose the economy is populated by a large number of infinitely-lived agents with identical preferences. Time is indexed $t = 0, 1, 2, \ldots$. The agent seeks to maximize the discounted value of utility, represented as

$$\sum_{t=0}^{\infty} \beta^t U(c_t)$$

where $0 < \beta < 1$ is the pure rate of time preference. Utility is maximized subject to two constraints

$$f(k_t) + \frac{m_{t-1} + \tau_t}{P_t} = c_t + k_{t+1} - (1-\delta)k_t + \frac{m^d_t}{P_t}$$

and

$$\frac{m_{t-1} + \tau_t}{P_t} \geq c_t + \eta(k_{t+1} - (1-\delta)k_t)$$

where $k$ denotes the stock of capital, $m$ is the nominal quantity of fiat money balances, $\tau$ denotes the quantity of fiat money balances transferred at the beginning of date $t$, $c$ is the consumption good sold for $P$ pieces of fiat money, $\delta$ is the rate of capital depreciation, and $m^d$ is
quantity of nominal money balances that the agents wishes to carry over from date $t$ to date $t+1$. In this setup, $\eta$ represents the fraction of gross investment purchases that is required to have fiat money balances in advance. With $\eta = 1$, equation (3) is the model is identical to the one studied in Stockman (1981), whereas with $\eta = 0$, the consumption good is the only cash good such as in the specifications used in Greenwood and Huffman (1987) and Cooley and Hansen (1989).

Units of the consumption good are produced using the production technology captured by the function $f(.)$. I assume the production function has the following properties: $f'(.) \geq 0, f'(.) < 0, f'(0) = \infty$ and $f'(\infty) = 0$. Firms maximize profits in a perfectly competitive setting, using capital to produce units of the consumption good. Firms are price-takers in the market for the capital input so that profit maximization yields the condition that $r_c = f'(.) - (1-\delta)$, where $\delta$ is the rental price of capital.

The nominal stock of fiat money at date $t$ is simply the sum of money balances existing last period and date-$t$ transfers; that is, $m_t = m_{t-1} + \tau_t$. Finally, let money grow according to the policy rule $m_t = \theta m_{t-1}$. Government spending is simply the quantity of monetary transfers.

Equation (2) is the agent's date-$t$ budget constraint. Output produced in period $t$ plus the real value of money balances finance purchases of the consumption good, (net) investment spending, and money balances carried over into next period. Equation (3) is the cash-in-advance constraint. Money balances are necessary to purchase the consumption good and for net additions to the physical capital stock.

The maximization problem can be conveniently written as the value function:

$$ V(m_t, k_t, P_t) = \max_{\{c_t, k_{t+1}, m_{t+1}\}} U(c_t) + \beta V(m_{t+1}, k_{t+1}, P_{t+1}) $$
subject to

\[ f(k_t) + \frac{m_t}{P_t} = c_t + k_{t+1} - (1-\delta)k_t + \frac{m_t}{P_t} \] (5)

and

\[ \frac{m_t}{P_t} \geq c_t + \eta (k_{t+1} - (1-\delta)k_t) \] (6)

Next, form the Lagrangean with multipliers \( \lambda_1 \) and \( \lambda_2 \). The first-order conditions for the Kuhn-Tucker problem are then

\[ U'(c_t) = \lambda_{1t} + \lambda_{2t} \] (7)

\[ \beta V_1(c_t) = \frac{\lambda_{1t}}{P_t} \] (8)

\[ \beta V_2(c_t) = \lambda_{1t} + \eta \lambda_{2t} \] (9)

\[ \left[ \frac{m_t}{P_t} - c_t - \eta (k_{t+1} - (1-\delta)k_t) \right] \lambda_{2t} = 0 \] with \([.] \geq 0, \lambda_{2t} \geq 0 \) . (10)

---

3 Throughout this paper, I will adopt the Stockman's conventions with regard to representing the value function. Here, \( m_t \) is the exogenous state variable, while \( k_t \) is an endogenous state variable and \( P_t \) is a decision variable such that the system is fully described by these three arguments.
There are two market-clearing conditions; namely, all nominal money balances clear and the goods market clears. Formally, $m_t = m_t$ and $c_t + k_{t+1} - (1-\delta)k_t = f(k_t) \forall t \geq 0$.

An equilibrium in this economy is a sequence of prices $\{P_t, r_t\}$, real allocations $\{c_t, k_{t+1}, m_t/P_t\}$, and money growth rates $\{\theta_t\}$ such that

(i) Given prices and the rate of money growth, the real allocation solves the household's maximization problem represented by (1) - (3);

(ii) Given prices and the rate of money growth, the allocations solve the firm's date-t maximization problem;

(iii) the market clearing conditions are satisfied.

The necessary conditions for this equilibrium are then given by equations (7) - (10).

3. The reserve requirement model

In this section, I describe an alternative model in which a simple bank structure is introduced. The banks must hold a fraction of their deposits in the form of flat money balances. The question is how this environment differs from the "pure" cash-in-advance setup described above.

The household's objective is to maximize the sum of discounted utilities in which consumption is the sole argument. Also, firms have access to the same production technology,

\footnote{See Smith (1991) for an alternative interpretation of the reserve requirement restriction.}
captured by the function \( f(.) \). There is an additional reason for holding money in this economy. Agent's must hold fiat money balances in order to purchase the consumption. As will seen later, this hybrid model facilitates comparison with the cash-in-advance model. The focus of the difference then will be that fiat money is not necessary to make net additions to the physical capital stock. The key feature of the model with a reserve requirement is to uncover what the restriction means in terms of modelling the relationship between fiat money and capital good purchases.

To motivate the banking structure, I assume that there is a minimum size for capital good purchases. More importantly for my purpose, the minimum purchase size is larger than the maximum saving by individual agents. This form of capital illiquidity is circumvented by introducing banks that pool together the funds from small savers to purchase the capital good. Agents deposit goods in the bank. The bank then purchases either units of the capital good holds fiat money. Banks can perform this function at zero marginal cost and the banking industry is perfectly competitive. To maximize profits, the bank will offer a return on deposits, denoted \( q_t \), that is equal to the return on its portfolio.

At the end of date \( t-1 \), the banks' balance sheet identity is captured by the expression \( d = \frac{r_{t-1}}{P_{t-1}} + k_r \), where \( d \) is the quantity of deposits per agent and \( r \) is the nominal quantity of fiat money (reserves) held by the bank. Thus, the real value of deposits is equal to the sum of the capital good and the real value of bank reserves.\(^5\) I focus on equilibrium in which capital rate-of-return dominates fiat money. To ensure that banks will hold fiat money, a reserve requirement is imposed; that is, \( \frac{r_{t-1}}{P_{t-1}} \geq \gamma_{t-1} d_r \), where \( \gamma \) is the reserve requirement ratio. Note

\(^5\) The date-\( t \) deposits were carried over from date \( t-1 \). Following the convention in Stockman, the end-of-period nominal quantity of fiat money is denoted with the subscript \( t-1 \), whereas capital carried over to date \( t \) is denoted with subscript \( t \). Similarly, I am assuming that capital goods purchased last period generate income via the production technology in the current period. Moreover, the salvage value of capital is its undepreciated value.
that the real value of deposits can be affected by movements in the price level between date \( t-1 \) and date \( t \). Moreover, for cases in which the return on physical capital rate of return dominates fiat money, the reserve requirement constraint is binding and the bank’s balance sheet expression then implies that

\[
 k_t \leq (1-\gamma_{t-1})d_t. \tag{11}
\]

From the reserve requirement constraint, deposits can be written as

\[
d_t \leq r_{t-1}/(\gamma_{t-1}P_{t-1}). \tag{12}
\]

To express the relationship between the capital stock and bank reserves, substitute (12) into (11), and use the banks’ balance sheet identity to obtain

\[
k_t = \frac{1-\gamma_{t-1}}{\gamma_{t-1}} \frac{r_{t-1}}{P_{t-1}}. \tag{13}
\]

What (13) tells us is that the reserve requirement forces banks to (at least partially) back capital goods purchases with fiat money.

In this setup, the total stock of fiat money is equal to the sum of currency, denoted \( s \), and bank reserves; that is, \( m_t = s_t + r_t \). The agents budget constraint is written as

\[
f(k_t) + \frac{r_{t-1}^e + s_{t-1}^e}{P_t} = c_t + k_{t+1} - (1-\delta)k_t + \frac{r_t^d + s_t^d}{P_t}. \tag{14}
\]

Using equation (13), one can rewrite the budget constraint as
The maximization problem is written as the value function

$$V(s, k, P) = \max_{k, s_{t+1}} U(c_t) + \beta V(s_{t+1}, k_{t+1}, P_{t+1})$$

subject to (14') and (15). Next, form the Lagrangean with multipliers $\mu_1$ and $\mu_2$ corresponding to constraints (14') and (15), respectively. The first-order conditions for the Kuhn-Tucker problem are

$$U'(. \cdot) = \mu_1 + \mu_2$$  \hspace{2cm} (17)

$$\beta V_1(. \cdot) = \frac{\mu_1}{P_t}$$  \hspace{2cm} (18)
\[ \beta V_{2}(r, \gamma_{t}) = \mu_{r}(1 + \frac{\gamma_{t}}{1-\gamma_{t}}) \] 

(19)

and

\[ \left[ \frac{s_{t}}{P_{t}} - c_{t} \right] \mu_{z_{t}} = 0 \] 

(20)

with \{\cdot\}, \mu_{z_{t}} \geq 0. In addition, there are two market clearing conditions: \( m^{d}_{t} = m_{t} \) and \( c_{t} + k_{t+1} - (1-\delta)k_{t} = f(k_{t}) \).

An equilibrium in this economy is a sequence of prices \{P_{t}, r_{t}, q_{t}\}, real allocations \{c_{t}, d_{t+1}\}, and monetary policy variables \{\theta_{t}, \gamma_{t}\} such that

(i) Given prices and the monetary policy variables, the real allocation solves the household's maximization problem represented by (1), and (14') and (15);

(ii) Given prices and the monetary policy variables, the allocations solve the firms's date-\( t \) profit maximization problem;

(iii) Given prices and the monetary policy variables, the allocations solve the bank's date-\( t \) profit maximization problem;

(iv) the market clearing conditions are satisfied.
The necessary conditions for this equilibrium are then given by equations (17)-(20).

Now that equilibrium has been characterized for the two monetary environments, I ask if the two economies are equivalent. Note that the condition for market clearing implies that if the capital stock paths are identical, so will the consumption paths. I first consider a version of equivalence in which I compare directly the first-order conditions. The models are equivalent if the first-order conditions can be written as a set of identical expressions. For this type of "model" equivalence to hold, the conditions are stated in the following proposition.

Proposition 1: The cash-in-advance and reserve-requirement models have identical first-order conditions if: \( \mu_{2t} \neq \mu_{1t}, \gamma_1/(1-\gamma_0) \text{ and } \eta(k_{t+1} - (1-\delta)k_t) \neq 0. \)

Proof: Clearly, if the first-order conditions are identical, the two models will generate identical paths for both allocations and prices.

Note that (7)-(8) and equations (17)-(18) are exactly alike. A comparison of equations (9) with (19) shows that with \( \gamma_1 = \eta/(1+\eta) \) then these two first-order conditions are identical. Equations (10) and (20) are identical if either there is no cash-in-advance constraint against gross investment spending or if gross investment spending is zero. Thus, with these two conditions, the first-order conditions from the cash-in-advance model are identical to those expressions obtained from the reserve-requirement model.

With identical first-order conditions, the equilibrium allocations and prices will naturally be identical for any given value of the inflation rate. The expressions in equations (7)-(10) are quite similar to equations (17)-(20). Indeed, if the conditions in Proposition 1 hold, the cash-in-advance and reserve-requirement models are identical for a particular value of the reserve
requirement ratio. Of course, the value of the ratio depends on $\eta$.

The question then is whether there a feature in either or both of the economies that implies the two Proposition-1 conditions will be satisfied. The answer is no. With or without $\eta = 0$, it would be sheer coincidence for the two independent conditions identified in Proposition 1 to be satisfied. Indeed, in a steady state with $k_{t+1} = k_t > 0$, the zero-gross-investment condition will not hold. Consequently, for any $\eta > 0$, the conditions will certainly not hold. At most, therefore, one would say that satisfying these two independent conditions would be sheer coincidence along a transition path.

Proposition 1 offers an insight into the key difference between the two monetary economies. In particular, equations (9) and (19) differ primarily because there is a difference in the timing of the relationship between fiat money and capital good purchases. In (9), the marginal indirect utility of capital is equal to the sum of two shadow prices: one corresponds to an additional unit of income and the other to an additional unit of fiat money balances. As (19) shows, however, only the shadow price of an additional unit of income is present in the reserve-requirement model. In the cash-in-advance model, the agent forgoes consumption at date $t$ to acquire money balances. The money balances can be used to purchase either the consumption good or the capital good at date $t$. Capital goods become productive at date $t+1$. In contrast, the reserve requirement model dictates that money balances are acquired simultaneously with the capital good. Thus, in the reserve requirement model, the money balances required to purchase the capital good are tied up for only one period before the gains from such purchases are realized. In contrast, one must wait two periods before fiat money acquisition and the gains from the capital purchase are realized in the model in which the cash-in-advance constraint is present.

Note that model equivalence is a rather strong notion for comparing two model
economies. If model equivalence did hold, then one could obtain identical steady-state allocations from the two monetary environments by simply picking the appropriate value for the reserve requirement ratio. In the next section, I consider a somewhat weaker notion of equivalence. As you will see, the question amounts to whether there exists a combination of the reserve requirement ratio and the inflation rate that yields identical allocations.\(^6\)

4. Equivalence with policy combinations

In both monetary economies, fiat money is required to purchase capital. Is there a combination policy defined as value of the reserve requirement and inflation rate such that the two models yield identical steady-state allocations? Here, I derive the analytic solutions for steady-state capital for both model economies. I refer to this notion of equivalence as allocation equivalence.

The expression for steady-state capital in the cash-in-advance model is given by the following:

\[
q(l + e)[1 - B(1 - \delta)]
\]

Equation (21) is a reduced-form representation of the first-order condition in which the marginal value of an additional unit of capital is equal to the marginal costs. The marginal costs

\[
f'(k^*) = \frac{1 - \eta}{\beta} + \frac{\eta(1 + \theta)[1 - \beta(1 - \delta)]}{\beta^2}
\]

\(^6\)Koenig (1987) examines the dynamic behavior of the Stockman model. Koenig finds that a short-run Tobin effect will arise as agents respond to high nominal interest rates, provided that net investment purchases are not subject to a "full" cash-in-advance constraint. In short, if as in the Sidrauski (1967) model, investment goods can be purchased with current period earnings, there will be a positive correlation between saving, investment, and the nominal interest rate, where movements in the interest rate reflects changes in the rate of money growth.
arise because the agent forgoes today's consumption. Note that the agent's time rate of preference is squared in (21), reflecting the two-period gap between when one acquires a unit of fiat money spent on capital and the actual output gains are realized.

The steady-state value of capital in the reserve requirement model solves the following expression

\[ f'(k^*) = \left(1 + \frac{\gamma}{1-\gamma} \right) \frac{1}{\beta} - (1-\delta) - \frac{\gamma}{1-\gamma} \frac{1}{\theta} \]  (22)

(see Appendix §A.1 for the derivation of (22)).

From equations (21) and (22), the steady-state capital stocks are equal for the two alternative models if

\[ \frac{1-\eta}{\beta} + \frac{\eta(1+\theta)(1-\beta)(1-\delta)}{\beta^2} = \left(1 + \frac{\gamma}{1-\gamma} \right) \frac{1}{\beta} - (1-\delta) - \frac{\gamma}{1-\gamma} \frac{1}{\theta}. \]  (23)

The following proposition characterizes steady-state allocations in terms of the inflation rate and the reserve requirement ratio.

**Proposition 2:** For a given inflation rate, there exists a value of the reserve requirement ratio such that the steady-state levels of capital (and implicitly consumption) are identical for the

---

7 It is straightforward to show that the steady-state value of capital is also inversely related to changes in the reserve requirement. Let \( \gamma/(1-\gamma) = \phi \). From (24), \( dk/d\phi = (1/\beta - 1/\theta)/f''(\cdot) \). With \( 0 < \beta < 1, dk/d\phi < 0 \). It follows from the definition of \( \phi \) that it is positively related to \( \gamma \). Hence, \( dk/d\gamma \) is negative.
cash-in-advance and reserve requirement models.

Proof: The proof is done indirectly. I look at the steady-state capital levels with the reserve requirement ratios at its two extreme values. With these boundaries established, the proof is completed by characterizing the effects that changes in reserve requirements would have on steady-state capital.

With $0 < \beta, \delta < 1$, equation (21) yields an interior solution for steady-state capital in the cash-in-advance constraint model. Consider the case with $\gamma = 0$. From equation (22), the right-hand-side becomes $1/\beta - 1 + \delta$ in this economy. It is straightforward to show that the right-hand-side of equation (21) is greater than the $1/\beta - 1 + \delta$. With diminishing marginal product of capital, $k^c < k^r$ with a zero reserve requirement ratio.

Next, consider the case in which $\gamma = 1$. Here, capital is completely crowded so that $k^c > k^r$ when the reserve requirement ratio equals 1.

Lastly, I need to show that the steady-state level of capital monotonically decreases in response to an increase in the reserve requirement ratio. Let $\phi = \gamma/(1-\gamma)$. From equation (22), $dk/d\phi = (1/\beta - 1/0)F'(\cdot)$. With $\theta > \beta$, $dk/d\phi < 0$. Clearly, $d\phi/d\gamma$ is $> 0$, so that $dk/d\gamma < 0$. From (22), this value of the reserve requirement ratio will change as the inflation rate changes. Thus, there is a combination of monetary policy parameters such that one can obtain the identical steady-state level of capital in the model with a cash-in-advance constraint as in the reserve-requirement model.

Proposition 2 demonstrates that the two alternative monetary environments can generate identical steady-state allocations and prices by selecting the two policy variables. Though there
is not a single value of the reserve requirement ratio that yields equivalent allocations, there is a unique combination of the inflation rate and reserve requirement ratio that will result in identical allocations.

What is interesting is that the method of proof in Proposition 2 has some implications for a more general class of models than just those with a cash-in-advance constraint. Note that with \( \gamma = 0 \), the steady-state version of capital would be identical to that obtained in a non-monetary environment if the cash-in-advance constraint on the consumption good were omitted. Also, with \( \gamma = 1 \), capital is completely crowded out in the reserve-requirement model [one can see this from equation (13)]. Let \( k^a \) denote the steady-state level of capital from a monetary model with inflation rate, \( \pi_0 \). The proof of Proposition 2 implies that there exists \( \gamma_0 \in [0,1] \) such that the policy combination \( (\pi_0, \gamma_0) \) such that the reserve-requirement economy will obtain \( k^a \).

A simple numerical example shows the combinations of the inflation rate and reserve requirement that yield the same steady-state levels of capital. To implement this example, I use a Cobb-Douglas production technology; that is, \( f(k) = k^\alpha \). The parameter settings used in this numerical exercise are as follows: \( \alpha = 0.35; \beta = 0.99; \) and \( \delta = 0.02 \). In addition, I need to specify the fraction of gross investment spending that requires cash-in-advance. Following Stockman, I consider one case in which \( \eta = 1.0 \). Calibrating the simulation with a 100% cash-in-advance constraint applied to gross investment may seems a bit unsupported by the data, especially if one interprets \( m \) in the model as high-powered money. Consequently, I also use \( \eta \)

---

8 I could include a scale parameter (total factor productivity variable) to the production technology. As the reader will see, I am interested in comparisons of levels, not the levels themselves, rendering the total factor productivity term unimportant for the computational experiments.

9 If one wanted to calibrate the model to account for, say, business cycle facts, then compensating balances and retained earnings stored in liquid financial assets would be the corresponding "money" that applies to the cash-in-advance constraint.
\( = 0.05 \) as a parameter setting. Arguably, even this value is too large. The key value of this numerical exercise is to illustrate the set of policy combinations that yield identical steady-state level of capital.

Figure 1 plots the value of the reserve requirement ratio and inflation rate that results in the same level of steady-state capital in the reserve-requirement model as in the model with a cash-in-advance constraint. Note that the slope of the line is negative. As the inflation rate rises, the reserve requirement ratio falls. The intuition behind this result is straightforward. First, recall the Stockman effect; that is, the steady-state capital stock is inversely related to movements in the inflation rate. This effect is present in both models. If the size of the Stockman effect is greater (smaller) in the reserve-requirement model then the reserve requirement ratio must decrease (increase). In the reserve-requirement model, the inflation rate and reserve requirement ratio affect the agent’s saving decision through the return on deposits, which is expressed as

\[
q = \frac{\gamma}{\delta} + (1 - \gamma) f'() + (1 - \delta).
\]

Equation (24) indicates that the impact that an increase in inflation rate has on the deposit rate depends on the size of the reserve requirement ratio. Thus, one can infer from the negative slope of the inflation rate-reserve requirement locus that the inflation-rate effect on steady state capital is larger in the reserve requirement model than in the model with the cash-in-advance constraint. Differences in the parameter settings would obviously affect the slope of the locus.

5. Seignorage revenue equivalence

The results reported above focus on finding identical steady-state allocations and price
paths for the two alternative monetary economies. The analysis omits government revenue considerations. In this section, equivalence is extended to consider seignorage revenues in the steady state. Specifically, is there a combination policy that holds revenue constant across the two models and also obtains identical allocations?

A general expression for seignorage revenue is

\[ \frac{1}{P_t} [m_t - m_{t-1}] \]  

(25)

Given the monetary policy rule, one can rewrite the seignorage revenue expression as

\[ \frac{m_t}{P_t} [1 - \frac{1}{\theta}] \]  

(25')

where equation (25') is useful for distinguishing between changes in the tax base \( \frac{m_t}{P_t} \) from the tax rate \( 1 - \frac{1}{\theta} \).

In comparing the seignorage revenue outcomes from the cash-in-advance and reserve-requirement models, note that the models differ only in terms of the tax base. From the cash-in-advance model, the tax base is given by the following expression

\[ \frac{m_t}{P_t} = f(k^c) \]  

(26)

while the tax base from the reserve-requirement model is given by
We know that there exists an infinite number of combination policies that will result in the two monetary economies yielding the same value of steady-state capital. As we shall see, imposing seignorage revenue constancy across the cash-in-advance and reserve-requirement economies will dramatically reduces the set of policy combinations.

Proposition 3: The amount of seignorage revenue raised in the model with the cash-in-advance constraint will be equal to that raised in the model with reserve requirements if: $\theta = 1$ or $\gamma = \delta/(1 + \delta)$.

Proof: With $\theta = 1$, the inflation tax rate is zero, hence seignorage revenue is zero in both models if money stock is held constant across time. There is also a reserve requirement ratio that equates the tax base across the two economies. This requires that the second-term in equation (27) equals zero. It is straightforward to show that this condition is satisfied when $\gamma = \delta/(1 + \delta)$.

Proposition 3 identifies the necessary conditions for the two monetary environments to generate the value of steady-state seignorage revenues. With a constant money supply, there is a single value of the reserve requirement ratio that ensures equivalence in the steady-state allocations. If the reserve requirement is fixed so that the tax bases are identical across the two monetary economies, there are two values of the inflation rate that yield equivalent steady-state allocations. (Equation (23) is a quadratic expression in the inflation rate.)
Table 1 reports the policy combinations that satisfy this broadened definition of equivalence. I use the same parameters settings here as in the computational experiments above. The results reported in the top half of Table 1 look at the value of the reserve requirement for various values of \( \eta \). With such a low inflation rate, the computational experiment indicates that the reserve requirement that yields identical steady-state allocations is around 75% when there is a 100% cash-in-advance applied against capital purchases. The reserve requirement goes to around 98% when the ratio of fiat money to capital is 50% of gross investment spending or below.

The bottom half of Table 1 reports the two values of the inflation rate that generates identical steady-state allocations when the reserve requirement ratio is chosen to ensure the same tax bases constant across the two monetary environments. If calibrated to business cycle frequency, the reserve requirement ratio is slightly below 2%. For example, Table 1 reports that with \( \eta = 1.0 \), the inflation rate that satisfies allocation and seignorage-revenue equivalence would be -92%.

6. Discussion

In this paper, I show that one can obtain the same steady-state allocation of capital and consumption in two different monetary economies: a cash-in-advance model and a reserve-requirement model. A model with a cash-in-advance constraint differs from a reserve-requirement model in one meaningful way: timing. More specifically, the cash-in-advance model maintains that fiat money required to purchase capital goods is effectively idle for two periods before it is transformed into more of the consumption good via the production technology. In contrast, the fiat money required to acquire capital in the reserve-requirement model waits only one period before it is transformed into the consumption good. The key result is that for a
given inflation rate, denoted $\pi_o$, there is a steady-state allocation obtained in a model with a cash-in-advance constraint. One can obtain the same steady-state level of capital in the reserve-rental model provided that the reserve requirement ratio is chosen appropriately. Thus, there is a policy combination $(\pi_o, \gamma_o)$ which generates equivalent steady-state solutions to the cash-in-advance and reserve-rental models.

Though I only investigate one pair of monetary environments, the reserve-rental model is equivalent to a broader class of economies. In fact, the models may appear somewhat rigged since I study economies in which there is a cash-in-advance constraint on the consumption good and fiat money is required to obtain capital. The flexibility of the reserve requirement model, however, means that equivalent allocations and seignorage revenue is actually quite easy to obtain. What is important is the combination of the inflation rate and reserve requirement ratio. For a given value of the inflation rate, the reserve-rental model can obtain any value of steady-state capital between zero and that level which would be the solution in a non-monetary economy. The fact that steady-state capital will lie between the boundaries of zero and non-monetary upper bound means that one can find a combination of the reserve requirement ratio and inflation rate resulting in the identical steady-state allocation in the reserve-rental model. The additional policy parameter—the reserve requirement ratio—adds an extra degree of freedom such that the policy combination amounts to finding two values to jointly solve two independent equations. Consequently, other monetary environments—such as those with spatially separated agents and shopping-time models—will also be equivalent to the reserve-rental model in the sense that identical steady-state allocations can be obtained.

The set of policy combinations that yield equivalent steady-state allocations can also be extended to consider equivalent levels of seignorage revenue. There are three policy
combinations that yield identical values for steady-state capital and seignorage revenue across the cash-in-advance and reserve-requirement models. One policy combination is associated with a constant money stock such that the inflation rate tax rate is zero. One can also set the reserve requirement ratio such that the seignorage tax base is identical across the two monetary economies. Because the steady-state level of capital is a quadratic in the inflation rate, there will be two possible policy combinations that generate the same level of the inflation tax base and capital.

In the introduction of this paper, I ask in what sense are different monetary environments equivalent. The answer offered in this paper is that a model with reserve requirements is equivalent to many monetary economies in the sense of matching the steady-state capital and seignorage revenue. The reason behind the answer is the existence of policy combination—an inflation rate-reserve requirement ratio pair—that lets the reserve-requirement model’s solution vary between zero and an upper bound associated with that found in a non-monetary setting.

This paper also identifies differences between the alternative monetary economies studied here. These differences are likely to produce difference in out-of-steady-state behavior. While there are questions for which a reserve-requirement model is uniquely best-suited, obviously future research will judge the merits of alternative models in terms of how well each accounts for observations at business cycle frequencies. What the current paper achieves is deriving the conditions that are necessary for the reserve-requirement model to have the same steady-state allocations as many different monetary economies. Thus, despite possessing particular traits, the combination of the inflation rate and reserve requirement ratio effectively renders the reserve-requirement model equivalent to many alternative monetary models.
Appendix

1. Derivation of the expression for steady-state capital

In the reserve requirement model, note that the indirect utilities of money and capital can be represented as

\[ V_1(\cdot) = \frac{\mu_1 + \mu_2}{P_t} \]  
(A.1)

and

\[ V_2(\cdot) = \mu_1 \left[ f'(\cdot) + (1 - \delta) + \frac{\gamma_{t-1}}{1 - \gamma_{t-1}} \frac{P_{t-1}}{P_t} \right] \]  
(A.2)

Update the expressions in (A.1) and (A.2) and substitute into the first-order conditions to obtain the Euler equations. Notably,

\[ \beta \mu_{t-1} \left[ f'(\cdot) + (1 - \delta) + \frac{\gamma_{t-1}}{1 - \gamma_{t-1}} \frac{P_{t-1}}{P_t} \right] = \mu_1 \left( 1 + \frac{\gamma_t}{1 - \gamma_t} \right) \]  
(A.3)

\[ \frac{\beta \mu_{t-1}}{P_{t-1}} + \frac{\beta \mu_{t-1}}{P_{t+1}} = \frac{\mu_1}{P_t} \]  
(A.4)

together with (19) and (20).

In the steady state, assume that \( \theta \) and \( \gamma \) are constant. Further, consumption and the
capital stock are constant in steady-state. Then, \( f(\cdot) = c + \delta k \). It is possible to deduce that \( \mu_\gamma \neq 0 \) in the steady state. In (A.2), \( \mu_\gamma = 0 \) if and only if, the Friedman rule applies. I assume throughout this analysis that \( \theta > 0 \) holds. (I will show that in steady state the money growth rate and inflation rate are proportionate next section of this appendix.) This is the standard conditions to ensure that the cash-in-advance constraint is binding.

With \( \mu_\gamma \neq 0 \) and with \( \mu_{tt} = \mu_{t+1} \), then (A.4) can be written

\[
f'(k') = \frac{1 + \phi}{\beta} - (1 - \delta) - \frac{\phi}{\theta}
\]

where \( \phi = \gamma/(1-\gamma) \) and \( \theta = \pi = P_{t+1}/P_t \) in steady state.

Thus, (26) is derived.

2. Steady-state relationship between money growth and inflation

With \( \mu_\gamma \neq 0 \),

\[
c_t = \frac{s_t}{P_t}
\]

Recall, that \( m_t = s_t + \text{res}_t \). In steady state, \( c = f(k) - \delta k \). Using the expression for steady-state consumption and the date-t expression relating reserves to capital, one can write
\[ m_t = P_i[f(k) + \left( \frac{\gamma}{1-\gamma} - \delta \right) k] \]  \hspace{1cm} (A.7)

Update (A.7) one period and after cancelling terms, the expression is

\[ \frac{m_{t+1}}{m_t} = \frac{P_{t+1}}{P_t} \]  \hspace{1cm} (A.8)

In steady state, therefore, (A.8) implies that \( \theta = \pi \).
References


Table 1

Inflation rate and Reserve requirement combinations resulting in identical seignorage revenue and capital

Policy Variables

A. Inflation rate tax is zero

\( \theta = 1.0; \eta = 1.0; \gamma = 0.753 \)

\( \theta = 1.0; \gamma = 0.50 \; \gamma = 0.980 \)

\( \theta = 1.0; \gamma = 0.05 \; \gamma = 0.983 \)

B. Tax base is identical

\( \gamma = \delta/(1+\delta) = 0.0196; \eta = 1.0; \theta_1 = 0.086 \) and \( \theta_2 = 0.076 \);

\( \gamma = \delta/(1+\delta) = 0.0196; \eta = 0.5; \theta_1 = 33.483 \) and \( \theta_2 = 1.242 \);

\( \gamma = \delta/(1+\delta) = 0.0196; \eta = 0.05; \theta_1 = 625.045 \) and \( \theta_2 = 12.637 \);
Inflation rate - res req combinations

\[ \eta = 0.05 \]

\[ \eta = 1.0 \]
Please check the titles of the Research Papers you would like to receive:

9201 Are Deep Recessions Followed by Strong Recoveries? (Mark A. Wynne and Nathan S. Balke)
9202 The Case of the "Missing M2" (John V. Duca)
9203 Immigrant Links to the Home Country: Implications for Trade, Welfare and Factor Rewards (David M. Gould)
9204 Does Aggregate Output Have a Unit Root? (Mark A. Wynne)
9205 Inflation and Its Variability: A Note (Kenneth M. Emery)
9207 The Effects of Credit Availability, Nonbank Competition, and Tax Reform on Bank Consumer Lending (John V. Duca and Bonnie Garrett)
9208 On the Future Erosion of the North American Free Trade Agreement (William C. Gruben)
9209 Threshold Cointegration (Nathan S. Balke and Thomas B. Fomby)
9210 Cointegration and Tests of a Classical Model of Inflation in Argentina, Bolivia, Brazil, Mexico, and Peru (Raul Anibal Feliz and John H. Welch)
9211 The Analysis of Fiscal Policy in Neoclassical Models (Mark Wynne)
9212 Measuring the Value of School Quality (Lori Taylor)
9213 Forecasting Turning Points: Is a Two-State Characterization of the Business Cycle Appropriate? (Kenneth M. Emery & Evan F. Koenig)
9215 An Analysis of the Impact of Two Fiscal Policies on the Behavior of a Dynamic Asset Market (Gregory W. Huffman)
9301 Human Capital Externalities, Trade, and Economic Growth (David Gould and Roy J. Ruffin)
9302 The New Face of Latin America: Financial Flows, Markets, and Institutions in the 1990s (John Welch)
9303 A General Two Sector Model of Endogenous Growth with Human and Physical Capital (Eric Bond, Ping Wang, and Chong K. Yip)
9304 The Political Economy of School Reform (S. Grosskopf, K. Hayes, L. Taylor, and W. Weber)
9305 Money, Output, and Income Velocity (Theodore Palivos and Ping Wang)
9306 Constructing an Alternative Measure of Changes in Reserve Requirement Ratios (Joseph H. Haslag and Scott E. Hein)
9307 Money Demand and Relative Prices During Episodes of Hyperinflation (Ellis W. Tallman and Ping Wang)
9308 On Quantity Theory Restrictions and the Signalling Value of the Money Multiplier (Joseph Haslag)
9309 The Algebra of Price Stability (Nathan S. Balke and Kenneth M. Emery)
9310 Does It Matter How Monetary Policy is Implemented? (Joseph H. Haslag and Scott Hein)
9311 Real Effects of Money and Welfare Costs of Inflation in an Endogenously Growing Economy with Transactions Costs (Ping Wang and Chong K. Yip)
9312 Borrowing Constraints, Household Debt, and Racial Discrimination in Loan Markets (John V. Duca and Stuart Rosenthal)
9313 Default Risk, Dollarization, and Currency Substitution in Mexico (William Gruben and John Welch)
9314 Technological Unemployment (W. Michael Cox)
9315 Output, Inflation, and Stabilization in a Small Open Economy: Evidence from Mexico (John H. Rogers and Ping Wang)
9316 Price Stabilization, Output Stabilization and Coordinated Monetary Policy Actions (Joseph H. Haslag)
9317 An Alternative Neo-Classical Growth Model with Closed-Form Decision Rules (Gregory W. Huffman)
9318 Why the Composite Index of Leading Indicators Doesn't Lead (Evan F. Koenig and Kenneth M. Emery)
9319 Allocative Inefficiency and Local Government: Evidence Rejecting the Tiebout Hypothesis (Lori L. Taylor)
9504 Building a Regional Forecasting Model Utilizing Long-Term Relationships and Short-Term Indicators (Keith R. Phillips and Chih-Ping Chang)

9505 Building Trade Barriers and Knocking Them Down: The Political Economy of Unilateral Trade Liberalizations (David M. Gould and Graeme L. Woodbridge)

9506 On Competition and School Efficiency (Shawna Grosskopf, Kathy Hayes, Lori L. Taylor and William L. Weber)

9507 Alternative Methods of Corporate Control in Commercial Banks (Stephen Prowse)

9508 The Role of Intraglobal Adjustment Costs in a Multi-Sector Economy (Gregory W. Huffman and Mark A. Wynne)

9509 Are Deep Recessions Followed By Strong Recoveries? Results for the G-7 Countries (Nathan S. Balke and Mark A. Wynne)

9510 Oil Prices and Inflation (Stephen P.A. Brown, David B. Oppedahl and Mine K. Yücel)

9511 A Comparison of Alternative Monetary Environments (Joseph H. Haslag)

Name: 
Organization: 

Address: 
City, State and Zip Code: 

Please add me to your mailing list to receive future Research Papers: _____ Yes _____ No
Please check the titles of the Research Papers you would like to receive:

1. A Sticky-Price Manifesto (Laurence Ball and N. Gregory Mankiw)
2. Sequential Markets and the Suboptimality of the Friedman Rule (Stephen D. Williamson)
3. Sources of Real Exchange Rate Fluctuations: How Important Are Nominal Shocks? (Richard Clarida and Jordi Gali)
4. On Leading Indicators: Getting It Straight (Mark A. Thoma and Jo Anna Gray)
5. The Effects of Monetary Policy Shocks: Evidence From the Flow of Funds (Lawrence J. Christiano, Martin Eichenbaum and Charles Evans)

Name: 
Organization: 
Address: 
City, State and Zip Code: 
Please add me to your mailing list to receive future Research Papers: 
Yes 
No