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Abstract

This paper analyzes the equilibrium growth paths of two economies that are identical in all respects, except for the organization of their financial systems: in particular, one has a competitive banking system and the other has a monopolistic banking system. In addition, the sources of inefficiencies, as a result of monopoly banking, and their relationship to the existence of credit rationing are explored. Monopoly in banking tends to depress the equilibrium law of motion for the capital stock for either of two reasons. When credit rationing exists, monopoly banks ration credit more heavily than competitive banks. When credit is not rationed, the existence of monopoly banking leads to excessive monitoring of credit financed investment. Both of these have adverse consequences for capital accumulation. In addition, monopoly banking is more likely to lead to credit rationing than is competitive banking. Finally, the scope for development trap phenomena to arise is considered under both a competitive and a monopolistic banking system.

1 Introduction

Recent empirical work in the field of financial intermediation and economic growth has established a strong, positive association between the development of an economy's formal financial sector and its level (or rate of growth) of real activity.[1] There is also a well-developed theoretical literature exploring the nature of this correlation.[2] However, most of the literature on the interrelationships between intermediation and growth considers an economy possessing a competitive banking system. As a practical matter though, different economies display variation in the industrial structure of their banking systems.

Consequently, more recent theoretical works have begun to examine and compare the macroeconomic differences between economies with competitive banking systems and those with monopolistic banking systems.[3] The results of these works are mixed. Partial equilibrium models, such as Riordan (1993), Petersen and Rajan (1995), Caminal and Matutes (1997) and Schnitzer (1998a,b), find that under monopoly, the severity of the particular bank-borrower problem examined is reduced. In contrast, the general equilibrium models find that less competitive systems may be detrimental to the economy. Smith (1998) analyzes the effect of bank structure on income and the business cycle and finds that less competitive systems are bad for the economy. Cetorelli (1995, 1997) explores the effect of bank structure on (a) the financing of credit-constrained firms and the adoption of new and better technologies, and on (b) the screening process for new loans, respectively. In both papers he finds the impact of monopoly in banking to be ambiguous. A monopoly bank facilitates technology adoption and reduces screening costs; however, these benefits are offset by the redistribution of productive resources to the bank in the form of profits.

The purpose of this paper is to further extend this literature by examining how the market structure of the banking system impacts capital accumulation and economic growth — in the context of a simple, general equilibrium model which allows for credit rationing. More specifically, the performance of an economy with a monopolistic banking system is compared to that of an economy whose banking system is competitive. The primary focus of this comparison is the level of long-run real activity. Related to this, this paper also compares the potential for credit to be rationed, the rates of interest paid on loans and deposits, and the quantity of resources required to operate the banking system. The possibility for development trap phenomena to arise under each type of financial system is also explored.

In general, this paper shows that the existence of a monopolistic banking system will be detrimental to both capital accumulation and economic growth. However, of particular interest for this paper is understanding *how* this is related to the existence and nature of any credit rationing that may arise. Specifically, the following five results are obtained. First, the existence of monopoly power in banking tends to depress the equilibrium law of

[1] See, for instance, Cameron (1967), Goldsmith (1969), Atje and Jovanovic (1993), King and Levine (1993a,b), Demirguc-Kunt and Levine (1996), or Levine and Zervos (1996).

[2] See, for example, Greenwood and Jovanovic (1990), Bencivenga and Smith (1991, 1993), and Boyd and Smith (1997, 1998). For an overview of some of this literature see Pagano (1993), Greenwood and Smith (1995), and Levine (1996).

[3] Competitiveness in the banking sector is also explored in Krasa and Villamil (1992), Winton (1995, 1997), and Yanelle (1997). These papers examine the optimal size, number, and capitalization of banks. However, they do not focus on the relationship between bank structure and capital accumulation and growth.

motion for the capital stock and thus reduces the level of long-run real activity, for either of two reasons. When credit rationing arises, a monopoly bank rations credit more heavily than does a competitive banking system and when credit is not rationed, the existence of monopoly banking leads to excessive monitoring of credit financed investment; both of these have adverse consequences for capital accumulation. Second, monopoly banking is more likely to lead to credit rationing. Third, if there does exist credit rationing, the interest rate paid on deposits will be lower under a monopolistic banking system. However, when there is a monopoly bank and credit is not rationed, the interest rate charged on loans will be higher. Fourth, the amount of resources consumed by the banking sector (as a result of monitoring investment projects) will be greatest in an economy with monopoly banking. Finally, while development trap phenomena may arise under either banking system, they are less likely to occur and will be less severe in an economy with a competitive banking system.

This paper differs in focus and methodology from the more recent works comparing competitive and monopolistic banking systems. The primary difference lies in the role that credit rationing (both its existence and nature) plays in the economy. These other recent models either do not include scope for credit rationing to occur, or allow for credit rationing that differs significantly from that presented in this paper.[4] While credit rationing is not the focus of this paper, its presence (or absence) does play a crucial role in understanding the sources and nature of those inefficiencies arising from a monopoly bank. Monopoly in banking (as in most industries) tends to be detrimental to the economy because it drives a wedge between interest rates charged on loans and those paid on deposits; thus the resulting monopoly profits divert resources away from capital accumulation and growth. This paper shows that the nature of this wedge[5] depends crucially on whether there exists credit rationing in the economy and, equally as important, the type of credit rationing present.[6] It also illustrates situations where, in spite of this wedge, the economy is not adversely affected. Thus this paper sheds new light on exactly *why* monopoly in banking is bad when the economy is characterized as one which allows for credit rationing.

In order to analyze these issues, a variant of Diamond's (1965) neoclassical growth model, in which capital investment must be credit financed, is utilized. The activity of producing capital is also subject to a standard costly state verification (CSV) problem of the type introduced by Townsend (1979); the formulation employed here most closely resembles Diamond (1984), Williamson (1986) and Boyd and Smith (1997, 1998). As in those papers, credit is

[4]For example, Petersen and Rajan (1995) allow for credit rationing in a Stiglitz and Weiss (1981) framework. However, the basic framework of their model and underlying reasons for the existence of credit rationing differ significantly from this paper. Consequently, their results, that a monopoly is less likely to lead to credit rationing, are the reverse of the results obtained in this paper. In addition, they do not examine the impact of market structure and credit rationing on capital accumulation and growth. See Williamson (1986) for a discussion of how the underlying framework and assumptions of a Stiglitz and Weiss model differ significantly and substantially from that of the costly state verification type model used in this paper.

[5]The "nature of the wedge" refers to whether the difference in interest rates is a result of the monopoly bank charging a higher rate on loans, or paying a lower rate on deposits, or both as compared to a competitive bank.

[6]In this paper credit rationing can occur for one of two reasons. There could be insufficient income – meaning that if all the income in the economy was added together, there still would not be enough funds to meet borrower's needs – or insufficient savings – meaning that there is enough income, but that not enough of it is being channeled through the formal financial sector to borrowers.

efficiently allocated through financial intermediaries.

In this context, banks accept deposits from a group of individuals (lenders) who earn labor income when young, and save for retirement in old age. Banks make loans to a group of individuals (borrowers) who have access to stochastic investment technologies which convert current output into future capital. Borrowers require external funds to operate these projects and, as noted, the lending involved is subject to a CSV problem. In particular, the return on any borrower's investment can be observed by the lender only if a fixed cost is incurred.

Two possibilities with respect to the industrial organization of the banking system are then considered: banking may be either competitive or monopolistic. It is assumed that a monopoly bank has monopoly power in the market for both loans and deposits. Interestingly, a monopoly bank will always exercise its powers in one of these markets, but not necessarily in both markets simultaneously. The differences between a competitive and monopolistic banking system then derive from the extent to which a monopoly bank chooses to restrict the supply of loans made (the rationing of credit), the amount of deposits raised (the size of the formal financial sector), and the extent to which a monopoly bank chooses to monitor loans (resources required to operate the bank).

The remainder of the paper is arranged as follows. The basic model is outlined in section 2, while section 3 discusses credit and factor markets. A monopoly banking system is described and analyzed in sections 4 and 5, while sections 6 and 7 do the same for a competitive banking system. Section 8 compares the two banking systems while section 9 concludes. Proofs of selected propositions and lemmas are relegated to the appendix.

2 The Model

This paper considers a two period lived, overlapping generations model with production. Time is discrete and indexed by $t = 0, 1, 2...$ In each period a single final good, which is either consumed or invested, is produced using capital and labor inputs according to the constant returns to scale production function $F(K, L)$. Let $f(k) = F(k, 1)$ denote the intensive production function, where $k = K/L$ is the capital-labor ratio. It is assumed that $f(k)$ is increasing, strictly concave, satisfies $f(0) = 0$, and that the standard Inada conditions hold. Finally, it is assumed that capital is used in the production process each period, and that it then depreciates completely.

Each generation is composed of a continuum of agents having unit mass. Within each generation individuals are divided into two groups: borrowers and lenders. All individuals, regardless of the group into which they fall, are identical with respect to their preferences; they value only old age consumption and are risk neutral. It will be assumed that only lenders are endowed with one unit of labor when young. No individuals in either group is endowed with labor when old. In addition, both borrowers and lenders are endowed with an investment opportunity when young; the exact nature of these opportunities is described in greater detail below. The initial old generation has an aggregate capital endowment of $K_0 > 0$, while subsequent generations have no endowments of either capital or final goods.

2.1 Borrowers

A fraction $1-\alpha$ of individuals within any given generation are potential borrowers (entrepreneurs). They have access to a stochastic linear investment technology which, for every unit of final good invested in the technology at date t, yields z units of capital at date $t+1$.[7] The return z is an iid random variable drawn from the distribution $G(\cdot)$, which is assumed to have a differentiable density function $g(\cdot)$ with support $[0,\bar{z}]$. The expected value of z is denoted by

$$\hat{z}=\int_0^{\bar{z}} zg(z)\,dz.$$

Any borrower's investment return is private information. However, any lender can ascertain the return z by incurring a fixed cost $\gamma>0$ units of capital.[8] Finally, it is assumed that each borrower is endowed with only one, indivisible investment project. This technology requires exactly $q>0$ units of funds to operate.[9] Since borrowers have no funds of their own, their capital investments must be financed externally.

2.2 Lenders

Lenders, who constitute a fraction α of each generation, are differentiated from borrowers in two respects. First, unlike borrowers, lenders supply one unit of labor inelastically when young, earning the real wage rate w_t at date t. Since only old age consumption is valued, lenders save their entire young period income and consume the gross return on that savings when old. Savings takes place by either depositing wage earnings in a bank or by investing in an autarkic (individual specific) capital production technology.

Second, lenders' access to these individual specific investment technologies differs from those available to borrowers. In particular, for each unit of the final good invested by lender i at date t, v_i units of capital are received at date $t+1$. The return v_i is known in advance by lender i, but is private information and cannot be verified by other individuals at any cost.

Let $H(v)$ denote the fraction of the lender population with $v_i \leq v$, and let $h(v) \equiv H'(v)$. It is assumed that $h(v)>0$ holds for $v \in [0,\bar{v}]$, and that $h(v)=0$ otherwise. For much of the paper it will be assumed that the values v_i are uniformly distributed in the population, so that $H(v)=v/\bar{v}$. Finally, lenders can operate their projects at any scale.[10]

[7] The capital obtained from investment projects (including that of lenders) is then used in the production process to produce final goods. These final goods are either consumed (by old borrowers and lenders) or used (by young workers) in future investment projects.

[8] The assumption that monitoring consumes capital follows Bernanke and Gertler (1989).

[9] The assumption of a fixed project size follows Williamson (1986, 1987), Bernanke and Gertler (1989), and Boyd and Smith (1997, 1998). For papers that allow for variable loan size (and variable project size), see Gale and Hellwig (1985) and Gray and Wu (1995).

[10] The purpose of giving lenders different investment returns is to generate an upward sloping supply curve of deposits for intermediaries. The device employed here closely follows that utilized by Williamson (1986). In addition, we want to think of lenders' investment opportunities as being inferior to those of borrowers: the "entrepreneurs" in the economy. Parenthetically, the use of autarkic investment technologies by lenders can be thought of as an "informal financial sector."

3 Trade

Trade occurs in one of two markets: competitive factor markets, where capital and labor are traded, and a formal credit market, where funds are transferred from lenders to borrowers. Trade in each market is now described

3.1 Factor Markets

At each date capital and labor are traded in competitive factor markets. Thus, at time t the real wage, w_t, and the rental rate of capital, ρ_t, both equal their marginal products:

$$\rho_t = f'(k_t) \tag{1}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t). \tag{2}$$

Clearly the real wage is an increasing function of the capital-labor ratio, i.e. $w'(k_t) > 0$. It will be assumed that $w(k_t)$ satisfies

$$w'(0) > C, \tag{A.1}$$

and

$$w''(k_t) < 0 \tag{A.2}$$

for all $k_t \geq 0$.[11] Assumption (A.2) is satisfied by any CES production function with an elasticity of substitution greater than or equal to one.[12]

3.2 Credit Market

It is assumed that all transfers of funds between borrowers and lenders are intermediated.[13] This intermediation may be undertaken either by a monopolistic or a competitive banking system; each possibility is considered below. First, however, the contractual relationship between borrowers and financial intermediaries as well as the relationship between lenders and banks are described.

3.2.1 Borrowers

Since borrowers have no young period income, they must obtain external financing to operate their projects. To obtain funding, borrowers can be thought of as entering into contractual relationships with financial intermediaries (banks). Following Williamson (1986, 1987), it is assumed that borrowers announce contract terms, which can then be either accepted or

[11] The value of C is determined by differentiating the right hand side of equation (21) and setting the result greater than one; i.e. $C = 1/\alpha\left[\hat{z} - (\gamma/q)\,G(\eta)\right]$

[12] Assumptions (A.1) and (A.2) are similar to the conditions used to guarantee the existence and uniqueness of a nontrivial steady state in the Diamond (1965) overlapping generations model.

[13] With competitive financial intermediaries, such an outcome will occur in any event to economize on monitoring costs; on this point see Diamond (1984) and Williamson (1986). The possibility of unintermediated lending in the presence of a monopolistic intermediary is not explored here.

rejected by any intermediary. When a borrower's contract announcement is accepted, the borrower obtains q units of the final good and undertakes his project.

Loan contracts are exactly analogous to those described by Williamson (1986, 1987), and specify the following sets of objects. First, there is a set A_t of realized returns for which state verification will occur; by the same token $B_t = [0, \bar{z}] - A_t$ is the set of return realizations for which verification is not undertaken.[14] When state verification does occur ($z \in A_t$), the repayment specified by the contract can meaningfully be made contingent on the project return; this repayment is denoted by $R_t(z)$, per unit borrowed. When state verification does not occur ($z \in B_t$) loan repayments cannot meaningfully be conditioned on project returns; hence the contract calls for an uncontingent payment x_t in this event. Moreover, in order for a borrower to truthfully reveal when a monitoring state has occurred, it is necessary that

$$R_t(z) \leq x_t \tag{3}$$

hold, for all $z \in A_t$. Finally, feasibility of contract announcements requires that loan repayments not exceed the proceeds of a borrower's investment; therefore

$$\begin{aligned} R_t(z) &\leq z\rho_{t+1} \text{ for } z \in A_t\,, \\ x_t &\leq \inf_{z \in B_t} \left(z\rho_{t+1}\right), \end{aligned} \tag{4}$$

must obtain.[15]

3.2.2 Lenders

At date t, young lenders earn the wage income $w(k_t) \geq 0$, all of which is saved. This savings takes place either through a financial intermediary, or through lenders investing autarkically in their individual specific investment projects.

A lender's decision concerning whether to invest in his own project or to deposit his earnings with a financial intermediary depends on the expected returns from both options. Let r_{t+1} be the gross return on deposits between t and $t+1$, which lenders take as given. Each lender compares this return with the return to financing his own investment project; for lender i this return is $\rho_{t+1} v_i$, measured in units of consumption. Therefore, lender i invests in his own project if and only if

$$r_{t+1} < v_i \rho_{t+1}. \tag{5}$$

Otherwise, the lender will deposit his wage income with a financial intermediary. Since $H\left(r_{t+1}/\rho_{t+1}\right)$ gives the fraction of the lender population for which equation (5) is satisfied, the supply of deposits at each date t is given by $\alpha H\left(r_{t+1}/\rho_{t+1}\right) w(k_t)$.

[14] Note that I abstract from stochastic state verification, as in Diamond (1984), Williamson (1986, 1987), and Boyd and Smith (1997, 1998). See Townsend (1979), Border and Sobel (1987), Bernanke and Gertler (1989), Mookherjee and Png (1989), and Boyd and Smith (1994) for papers which discuss stochastic state verification and its implications. Finally, Krasa and Villamil (2000) detail conditions under which simple debt contracts and stochastic contracts are optimal.

[15] In addition, at date t a borrower generates a positive expected return from operating his project if and only if the expected project return (in terms of final goods), $q\hat{z}\rho_{t+1}$, is at least as great as the expected costs of borrowing funds, $q\int_{A_t} R_t(z)\, g(z)\, dz + q\int_{B_t} x_t g(z)\, dz$. Hence, borrowers will undertake their projects if and only if $q\hat{z}\rho_{t+1} - q\int_{A_t} R_t(z)\, g(z)\, dz - q\int_{B_t} x_t g(z)\, dz \geq 0$ holds.

3.2.3 Financial Intermediation

The following functions are performed by the financial intermediary, regardless of the market structure of the banking system. Financial intermediaries take deposits from lenders, make loans to borrowers, and conduct monitoring of project returns as required by loan contracts.

In order to compare the macroeconomic impact of the different bank structures, it will be necessary to ascertain two things. First, under what conditions does one obtain either of the two types of credit rationing possible in the model? Second, what are the interest rates charged on loans and the interest rates paid on deposits? The following four sections are devoted to ascertaining these things for both monopolistic and competitive banking systems.

4 A Monopoly Banking System

For the purposes of this paper, the bank can be thought of as deriving its monopoly power from an exclusive charter granted by the government.[16] It is also assumed that unintermediated borrowing and lending is impossible — either for legal reasons or because the implied duplication of monitoring effort makes unintermediated lending uneconomical. Moreover, since there is only a single bank, it is not only a monopolist in the market for loans, but it is a monopsonist in the market for deposits as well.

Since each funded borrower obtains a loan of size q, the bank's expected real repayment per unit lent (in terms of final goods) is given by

$$\int_{A_t} \left[R_t(z) - \frac{\gamma \rho_{t+1}}{q} \right] g(z)\, dz + \int_{B_t} x_t g(z)\, dz,$$

where $\int_{A_t} R_t(z)\, g(z)\, dz + \int_{B_t} x_t g(z)\, dz$ represents the expected repayment implied by the contract offered by borrowers, and $\int_{A_t} \left(\gamma \rho_{t+1}/q\right) g(z)\, dz$ represents the expected monitoring costs associated with this contract.

Let $\mu_t \in [0,1]$ denote the fraction of borrowers funded by the bank at date t. Then the bank's revenues are given by $(1-\alpha)\, q \mu_t \left\{ \int_{A_t} \left[R_t(z) - \frac{\gamma \rho_{t+1}}{q} \right] g(z)\, dz + \int_{B_t} x_t g(z)\, dz \right\}$ while its costs equal $(1-\alpha)\, q \mu_t r_{t+1}$. Hence the bank chooses values μ_t and r_{t+1} along with contract terms in order to solve the problem

$$\max_{\mu_t, r_{t+1}, x_t, R_t(z)} (1-\alpha)\, \mu_t q \left\{ \int_{A_t} \left(R_t(z) - \frac{\gamma \rho_{t+1}}{q} \right) g(z)\, dz + \int_{B_t} x_t g(z)\, dz - r_{t+1} \right\} \tag{MBP}$$

subject to equations (3), (4),

$$(1-\alpha)\, \mu_t q \leq \alpha w(k_t)\, H\left(\frac{r_{t+1}}{\rho_{t+1}} \right), \tag{6}$$

[16] Similar to Smith (1998), who obtains the bank charter and how they obtain it are not explicitly modeled in this paper. It is assumed, however, that if the charter is sold to an individual or group, the proceeds from the sale of the charter are not rebated to any individuals in the economy. One can think of the charter as being acquired by a young lender at date t, who consumes the profits generated by the bank at date $t+1$. See Krasa and Villamil (1992) for a more detailed discussion regarding the optimal size and market structure of banks in a costly state verification model.

and $\mu_t \in [0,1]$. Equation (6) guarantees that the monopoly bank cannot lend out more funds than it acquires.

The solution to this problem is for borrowers to offer "a standard debt contract." In particular, if it is feasible to repay the fixed amount x_t — principle plus interest on the loan — then the borrower does so. Otherwise he defaults on his debt, the bank verifies the state, and it retains all of the proceeds from his project. More formally we have[17]

Proposition 1 *The optimal loan contract must satisfy*

$$\begin{aligned} R_t(z) &= z\rho_{t+1};\ z \in A_t \\ A_t &= \left[0, x_t / \rho_{t+1}\right). \end{aligned} \tag{7}$$

It remains to describe the (gross) interest rate charged on loans, x_t, by a monopoly lender. Using equations (7), the bank's expected repayment per unit lent can be rewritten as

$$\begin{aligned} & \int\limits_{A_t} \left[R_t(z) - \frac{\gamma\rho_{t+1}}{q}\right] g(z)\,dz + \int\limits_{B_t} x_t g(z)\,dz \\ = \ & \rho_{t+1}\left[\frac{x_t}{\rho_{t+1}} - \frac{\gamma}{q} G\left(\frac{x_t}{\rho_{t+1}}\right) - \int\limits_0^{x_t/\rho_{t+1}} G(z)\,dz\right] \\ \equiv \ & \rho_{t+1}\pi\left(\frac{x_t}{\rho_{t+1}}\right). \end{aligned}$$

The function $\pi(\cdot)$ gives the bank's per unit expected return, inclusive of monitoring costs, (measured in terms of capital) as a function of the loan rate, x_t, and the future price of capital, ρ_{t+1}. Clearly it will be useful to make some assumptions regarding the properties of the function $\pi(\cdot)$. It will be assumed that

$$\pi'(0) > 0 \tag{A.3}$$

and that

$$\pi''\left(\frac{x_t}{\rho_{t+1}}\right) < 0 \tag{A.4}$$

for all x_t / ρ_{t+1}.[18] Assumptions (A.3) and (A.4) imply that $\pi(\cdot)$ has the configuration depicted in Figure 1.[19] Consequently, there exists a unique value of x_t, depending on ρ_{t+1},

[17] The proof of this proposition is similar to the ones found in Gale and Hellwig (1985) and Williamson (1986, 1987); hence it is omitted here.

[18] Assumption (A.4) is equivalent to the more primitive assumption that $g(z) + (\gamma/q)\,g'(z) > 0$ holds for all z.

[19] These assumptions are identical to those found in Boyd and Smith (1998) and result in a graph of the expected return to the bank which is identical (although for different reasons) to that found in Stiglitz and Weiss (1981).

such that the bank's expected repayment per unit lent is maximized. This value, denoted by $\hat{x}_t\left(\rho_{t+1}\right)$, is implicitly defined by

$$\pi'\left(\frac{\hat{x}_t\left(\rho_{t+1}\right)}{\rho_{t+1}}\right) \equiv 1-\frac{\gamma}{q}g\left(\frac{\hat{x}_t\left(\rho_{t+1}\right)}{\rho_{t+1}}\right)-G\left(\frac{\hat{x}_t\left(\rho_{t+1}\right)}{\rho_{t+1}}\right) \equiv 0. \tag{8}$$

Equation (8), along with assumption (A.4), implies that

$$\hat{x}_t\left(\rho_{t+1}\right) \equiv \rho_{t+1}\eta, \tag{9}$$

where $\eta > 0$ is a constant satisfying $1-(\gamma/q)\,g(\eta)-G(\eta)=0$.

From the foregoing discussion, it is clear that a monopoly bank will choose x_t, r_{t+1}, and $\mu_t \in [0,1]$ to maximize

$$(1-\alpha)\,\mu_t q\left\{\rho_{t+1}\pi\left(\frac{x_t}{\rho_{t+1}}\right)-r_{t+1}\right\}$$

subject to equation (6). Obviously, the bank will extract as much surplus as possible from (funded) borrowers: so that $x_t=\hat{x}_t\left(\rho_{t+1}\right)$ will hold at every date t.[20] Then, letting

$$\psi=\pi\left[\frac{\hat{x}\left(\rho_{t+1}\right)}{\rho_{t+1}}\right] \equiv \pi(\eta),$$

it is transparent that r_{t+1} and μ_t must be chosen to maximize

$$(1-\alpha)\,\mu_t q\left[\rho_{t+1}\psi-r_{t+1}\right]$$

subject to equation (6) and the requirement that $1 \geq \mu_t \geq 0$. Rearranging terms in equation (6) and substituting the result into the maximand, it is apparent that r_{t+1} must be chosen to solve the problem

$$\max_{r_{t+1}} \ \alpha w\left(k_t\right)H\left(\frac{r_{t+1}}{\rho_{t+1}}\right)\left[\rho_{t+1}\psi-r_{t+1}\right] \tag{MBP$'$}$$

subject to

$$1 \geq \frac{\alpha w\left(k_t\right)}{(1-\alpha)\,q}H\left(\frac{r_{t+1}}{\rho_{t+1}}\right). \tag{10}$$

Notice that equation (10) will hold with equality if and only if $\mu_t=1$. Also note that there is no reason for the bank to set r_{t+1} above $\bar{v}\rho_{t+1}$, the maximum return obtainable from any lender's investment project; thus $0 \leq r_{t+1} \leq \bar{v}\rho_{t+1}$ will be satisfied.

In order to guarantee that the objective function in (MBP$'$) is concave, I will henceforth make the following assumption on the distribution function H: H satisfies

$$H''\left(\frac{r_{t+1}}{\rho_{t+1}}\right)\left(\psi-\frac{r_{t+1}}{\rho_{t+1}}\right)-2H'\left(\frac{r_{t+1}}{\rho_{t+1}}\right) \leq 0. \tag{A.5}$$

[20] It is straightforward to show that at this interest rate the bank never extracts enough surplus to deter borrowers from operating their projects.

Assumption (A.5) is satisfied if, for example, investment returns are uniformly distributed in the lender population.[21]

If the solution to (MBP$'$) has $\mu_t < (=)\, 1$, then all borrowers are not (are) funded. Following standard usage, this outcome is referred to as exhibiting credit rationing (no credit rationing). Having ascertained the interest rate charged on loans, $\left(\widehat{x}\left(\rho_{t+1}\right) = \eta \rho_{t+1}\right)$, it remains to determine when credit rationing arises and what will be the interest rate paid on deposits, (r_{t+1}).

4.1 Equilibrium Deposit Rate without Credit Rationing

In the absence of credit rationing, $\mu_t^* = 1$ holds. Therefore, a monopoly bank will offer exactly the interest rate on deposits that just enables it to fund all borrowers; hence the equilibrium rate of return on deposits, r_{t+1}^*, must satisfy

$$1 = \frac{\alpha w\left(k_t\right)}{\left(1-\alpha\right) q} H\left(\frac{r_{t+1}^*}{\rho_{t+1}}\right). \tag{11}$$

When investment returns are uniformly distributed in the lender population, equation (11) reduces to

$$r_{t+1}^* = \frac{\left(1-\alpha\right) q}{\alpha w\left(k_t\right)} \bar{v} \rho_{t+1}. \tag{12}$$

4.2 Equilibrium Deposit Rate with Credit Rationing

When credit is rationed, $\mu_t^* < 1$ holds. As a consequence, the constraint (10) does not bind in the bank's profit maximization problem, (MBP$'$). Therefore r_{t+1} is chosen to maximize $\alpha w\left(k_t\right) H\left(r_{t+1} / \rho_{t+1}\right) \left[\rho_{t+1}\psi - r_{t+1}\right]$ subject to $r_{t+1} \leq \bar{v}\rho_{t+1}$. The value of r_{t+1} solving this problem satisfies

$$H'\left(\frac{r_{t+1}^*}{\rho_{t+1}}\right)\left(\psi - \frac{r_{t+1}^*}{\rho_{t+1}}\right) \geq H\left(\frac{r_{t+1}^*}{\rho_{t+1}}\right), \tag{13}$$

and equality obtains if $r_{t+1}^* < \bar{v}\rho_{t+1}$.[22] When the distribution of investment returns in the lender population is uniform, equation (13) reduces to

$$r_{t+1}^* = \begin{cases} \rho_{t+1}\psi / 2\,; & \text{if } \psi \leq 2\bar{v} \\ \rho_{t+1}\bar{v}; & \text{if } \psi \geq 2\bar{v} \end{cases}. \tag{14}$$

Finally, equations (6) and (14) imply that the fraction of borrowers funded at date t, μ_t^*, satisfies

$$\mu_t^* = \begin{cases} \left[aw\left(k_t\right)\psi\right] / \left[\left(1-\alpha\right) q 2\bar{v}\right]\,; & \psi \leq 2\bar{v} \\ \\ \left[aw\left(k_t\right)\right] / \left[\left(1-\alpha\right) q\right]\,; & \psi \geq 2\bar{v} \end{cases}.$$

[21] Assumptions on lenders' returns (A.5) and on lenders' returns relative to borrowers' returns (A.6) are required because, unlike the previous literature, lenders are given non-trivial, differentiated options regarding what to do with their wage income. A uniform distribution is used throughout the remained of the paper only for ease of exposition. Any distribution which satisfies assumption A.5 would work and would not alter the general results.

[22] Clearly there is no reason for the bank to pay an interest rate on deposits in excess of $\bar{v}\rho_{t+1}$; it will pay strictly less than that amount when it is not optimal to fully tap the potential supply of deposits.

4.3 Conditions Necessary for Credit Rationing

There are two possible reasons why credit rationing might arise with a monopoly banking system. One is that the maximum potential supply of funds is inadequate to meet the demand for loans; this will be referred to as *credit rationing due to insufficient income.* This must be the case if

$$(1-\alpha)\,q > \alpha w\,(k_t) \tag{15}$$

holds at date t. However, even if equation (15) fails, credit may still be rationed. This will occur when a monopoly bank views it as too costly to generate enough deposits to fund the complete set of potential borrowers; this will be referred to as *credit rationing due to insufficient saving.* The following lemma states conditions under which equation (15) will fail, and yet a monopoly bank will ration credit.[23]

Lemma 2 *Credit rationing obtains if and only if either equation (15) holds, or*

$$\frac{\psi}{2} < \frac{(1-\alpha)\,q\bar{v}}{\alpha w\,(k_t)}. \tag{16}$$

This lemma follows directly from the monopoly bank's maximization problem. Notice that equation (16) is equivalent to

$$\frac{\psi}{2}\rho_{t+1} < r^*_{t+1}, \tag{17}$$

where r^*_{t+1} is the equilibrium rate of return on deposits in the absence of credit rationing. As equation (17) makes apparent, the monopoly bank will ration credit whenever the rate of return on deposits necessary to avoid credit rationing is "too high" relative to the return on capital.

In addition to understanding when credit rationing will occur, in terms of interest rates, it will also be useful to state conditions on the current capital stock that imply whether or not credit will be rationed at date t. To do so, define k_{SI} (sufficient income capital stock) to be the value of the capital stock such that the wage income of all lenders is exactly equal to the amount of funds demanded by borrowers, thus

$$(1-\alpha)\,q = \alpha w\,(k_{SI}).$$

Clearly, for $k_t \leq k_{SI}$, credit rationing (due to insufficient income) must be observed. In addition, define k_{MSS} (sufficient savings capital stock) by

$$\frac{\psi}{2} = \frac{(1-\alpha)\,q\bar{v}}{\alpha w\,(k_{MSS})}.$$

Then, if $k_{SI} \leq k_t \leq k_{MSS}$, a monopoly bank views the cost of raising enough funds to avoid credit rationing as excessive. Again credit rationing will be observed (due to insufficient saving), even though it is feasible to fund all potential borrowers. It is important to know when $k_{SI} \lesseqgtr k_{MSS}$ holds. This result is stated in the following lemma.

[23] The lemma assumes, as does the remainder of the analysis in this section, that the distribution of returns in the lender population is uniform.

Lemma 3 *$k_{MSS} \geq k_{SI}$ holds if and only if $\psi/2 \leq \bar{v}$.*

Lemma 3 asserts that $k_{MSS} \geq k_{SI}$ is satisfied if and only if a monopoly bank views it as uneconomical ever to tap the complete supply of deposits. For $k_t \leq \max[k_{SI}, k_{MSS}]$, a monopoly bank will ration credit. Low capital stocks are conducive to the rationing of credit because they imply both a low level of savings, and a high opportunity cost of deposits {see equation (12)}. Conversely, for $k_t \geq \max[k_{SI}, k_{MSS}]$, a monopoly bank will fund all potential borrowers.

5 General Equilibrium: Monopolistic Banking System

Having ascertained the various interest rates and when credit is (and is not) rationed, it is now possible to derive the equilibrium laws of motion for the capital stock and to investigate the properties of dynamical equilibria under a monopolistic banking system.[24]

Recall that there is a large number of borrowers at each date t, and that investment returns are iid; hence there is no aggregate uncertainty in the economy. Moreover, the per capita capital stock at date $t+1$ is simply the expected return on the capital investments of funded borrowers, $\hat{z}q(1-\alpha)\mu_t$, plus the capital generated by the direct investment of unintermediated saving, $\alpha\left[1-H\left(r_{t+1}/\rho_{t+1}\right)\right]\int_{r_{t+1}/\rho_{t+1}}^{\bar{v}} yw(k_t)h(y)\,dy$, less the amount of capital consumed in the process of state verification, $\gamma(1-\alpha)\mu_t G\left(x_t/\rho_{t+1}\right)$. Thus the law of motion for the capital stock has the following form,

$$k_{t+1} = \hat{z}q(1-\alpha)\mu_t - \gamma(1-\alpha)\mu_t G\left(\frac{x_t}{\rho_{t+1}}\right) + \alpha\left[1-H\left(\frac{r_{t+1}}{\rho_{t+1}}\right)\right]\int_{\frac{r_{t+1}}{\rho_{t+1}}}^{\bar{v}} yw(k_t)h(y)\,dy. \quad (18)$$

Substituting the equilibrium relations $x_t = \hat{x}\left(\rho_{t+1}\right) = \rho_{t+1}\eta$ and equation (6) into equation (18) yields the equivalent equilibrium condition

$$k_{t+1} = \alpha w(k_t)\left\{H\left(\frac{r_{t+1}}{\rho_{t+1}}\right)\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right] + \left[1-H\left(\frac{r_{t+1}}{\rho_{t+1}}\right)\right]\int_{\frac{r_{t+1}}{\rho_{t+1}}}^{\bar{v}} yh(y)\,dy\right\}. \quad (19)$$

Equation (19) states that next period's per capita capital stock is a weighted average of the capital obtained from borrowers' investments (inclusive of monitoring costs) and the capital obtained from the direct (unintermediated) investments of lenders in the current period. Under the assumption that the distribution of returns in the lender population is uniform, equation (19) reduces to

$$k_{t+1} = \alpha w(k_t)\left\{\frac{r_{t+1}}{\rho_{t+1}\bar{v}}\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right] + \left[1-\frac{r_{t+1}}{\rho_{t+1}\bar{v}}\right]\left[\frac{\bar{v}^2 - \left(\frac{r_{t+1}}{\rho_{t+1}}\right)^2}{2\bar{v}}\right]\right\}. \quad (20)$$

[24]The results from this section, as well as that of section 7, form the basis for comparison of the two banking systems. This comparison is undertaken in section 8.

The exact configuration of this law of motion depends entirely upon whether or not credit is rationed and the resulting interest rates paid on deposits.

5.1 The Equilibrium Law of Motion: Credit Rationing

When there exists credit rationing, the exact nature of equation (20) depends on whether the credit rationing is due to insufficient income or saving (i.e. on whether $k_{SI} \lesseqgtr k_{MSS}$ holds). Each possibility is briefly considered.

CASE MII (Monopoly Insufficient Income): $k_{SI} > k_{MSS}$ $(\psi/2 > \bar{v})$. In this case a monopoly bank offers the return $r^*_{t+1} = \rho_{t+1}\bar{v}$ on deposits. Although the bank views it as profitable to attract all potential depositors, credit is rationed due to the low level of income. At this interest rate equation (20) reduces to

$$k_{t+1} = \alpha w(k_t)\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right]. \tag{21}$$

Assumptions (A.1) and (A.2) imply that equation (21) defines an increasing, concave locus, as depicted in Figure 2.

CASE MIS (Monopoly Insufficient Saving): $k_{MSS} \geq k_{SI}$ $(\psi/2 \leq \bar{v})$. Here the rate of return offered on deposits is $r^*_{t+1} = \rho_{t+1}(\psi/2)$. The bank does not choose to draw all potential deposits into the formal financial system. Substituting $r^*_{t+1} = \rho_{t+1}(\psi/2)$ into equation (20) yields the equilibrium law of motion

$$k_{t+1} = \alpha w(k_t)\left\{\frac{\psi}{2\bar{v}}\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right] + \left[1 - \frac{\psi}{2\bar{v}}\right]\left[\frac{\bar{v}^2 - (\psi/2)^2}{2\bar{v}}\right]\right\}. \tag{22}$$

Clearly $1 \geq \psi/2\bar{v}$ is satisfied, so that equation (22) also defines an increasing, concave locus as depicted in Figure 2. In addition, if the following condition,

$$\hat{z} - \frac{\gamma}{q}G(\eta) > \frac{2\bar{v}}{3}, \tag{A.6}$$

holds, the locus defined by equation (21) lies above the locus defined by equation(22) at each value of k_t. Assumption (A.6) states that the expected amount of capital (inclusive of monitoring costs) generated by any borrower's investment project is more than two-thirds of the highest return yielded by any lender's direct investment opportunity.[25] To fix ideas, it is henceforth assumed that (A.6) obtains, and thus that equation (21) lies everywhere above equation (22).

5.2 The Equilibrium Law of Motion: No Credit Rationing

Credit is not rationed when $k_t \geq \max[k_{SI}, k_{MSS}]$. In the absence of credit rationing, the equilibrium rate of return on deposits is given by

$$r^*_{t+1} = \frac{(1-\alpha)q}{\alpha w(k_t)}\bar{v}\rho_{t+1}, \tag{23}$$

[25] The reason for this assumption is to capture the idea expressed in footnote 10 – namely, borrowers (entrepreneurs) are endowed, on average, with more productive investment opportunities than lenders. Hence channeling funds to borrowers, as opposed to lenders, will result in greater capital accumulation.

for a uniform distribution of returns. Substituting equation (23) into equation (20) generates

$$k_{t+1} = (1-\alpha)\,q\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right] + \left[\alpha w(k_t) - (1-\alpha)\,q\right]\frac{\bar{v}}{2}\left[1 - \left(\frac{(1-\alpha)\,q}{\alpha w(k_t)}\right)^2\right] \equiv \Psi(k_t)\,. \quad (24)$$

The following lemma characterizes some properties of the function $\Psi(k_t)$.

Lemma 4 *The function $\Psi(k_t)$ satisfies*

a) $\Psi(k_{SI}) = \alpha w(k_{SI})\left[\hat{z} - \gamma/q\, G(\eta)\right]$
b) $\Psi(k_{MSS}) = \alpha w(k_{MSS})\left\{\frac{\psi}{2\bar{v}}\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right] + \left[1 - \frac{\psi}{2\bar{v}}\right]\left[\frac{\bar{v}^2 - (\psi/2)^2}{2\bar{v}}\right]\right\}$
c) $\Psi'(k_{SI}) = 0$,
d) $\Psi'(k_t) \geq 0$,
e) $\lim_{k_t \to \infty} \Psi(k_t)/k_t = 0$.

The proof of lemma 4 can be found in the appendix. Parts (a) and (b) of lemma 4 assert that the equilibrium law of motion for k_t is continuous as the transition from credit rationing to a situation of no credit rationing occurs; this is true regardless of whether case MII or case MIS obtains. The remainder of the lemma asserts that the locus defined by equation (24) is a nondecreasing function that must asymptotically lie below the 45° line. Thus, Figure 2 depicts one possible configuration of the equilibrium law of motion for the capital stock in the presence of a monopoly bank.

5.3 Steady State Equilibria

The existence of at least one non-trivial steady state capital stock equilibrium is implied by assumptions (A.1), (A.2), and lemma 4. Whether or not there is more than one depends heavily on parameter values, however. Several possibilities are now illustrated.

(a) Under the assumptions made to date, Figure 3a and 3b depict the possibility of a unique nontrivial steady state equilibrium, either with or without the rationing of credit, respectively. If there is a unique steady state, then clearly it is asymptotically stable.

(b) Figure 4 depicts the potential for the existence of more than one nontrivial steady state. When multiple steady state equilibria exist, at most one of these can display credit rationing (k_L^*). Moreover, if there is more than one steady state, any steady state with credit rationing will necessarily have the lowest of the steady state capital stocks. And, clearly, at least the lowest and highest capital stock steady states (k_L^* and k_H^*) must generically be asymptotically stable. The possibility that more than one asymptotically stable steady state exists raises the specter of development trap phenomena. In Figure 4, economies with capital stocks below (above) k_M^* must converge to k_L^* (k_H^*). Moreover, under the appropriate configuration of parameters, economies trapped at the low real activity steady state will also experience credit rationing. This is consistent with the description given by McKinnon (1973) and Shaw (1973) of a variety of less developed economies.

6 Competitive Banking System

In the following two sections, the methodology of the previous two sections is repeated so as to determine (a) the interest rates charged on loans and paid on deposits, (b) when credit rationing exists, and (c) the properties of equilibria. The basic setup of the economy is identical to the previous sections, except that the banking system is perfectly competitive. I therefore proceed to characterize an equilibrium under the assumption that there is free entry into banking, and more specifically, that any lender can establish a bank.

As before, it is assumed that borrowers announce loan contracts, subject to the same feasibility and incentive constraints (equations (3) and (4)) as previously. It is also assumed that each potential lender (bank) acts as if it can raise any quantity of deposits desired at the market determined gross rate of return on deposits, r_{t+1}. Clearly, then, banks will accept loan contract offers only if they offer an expected return no lower than the bank's opportunity cost of funds, inclusive of monitoring costs. Thus the loan contracts announced by borrowers must satisfy the expected return constraint

$$\int_{A_t} \left[R_t(z) - \frac{\gamma \rho_{t+1}}{q} \right] g(z)\, dz + \int_{B_t} x_t g(z)\, dz \geq r_{t+1}. \tag{25}$$

Equation (25) states that the expected real repayment implied by any loan contract announcement, $\int_{A_t} R_t(z)\, g(z)\, dz + \int_{B_t} x_t g(z)\, dz$, inclusive of the real expected cost of monitoring, $\int_{A_t} \left(\gamma \rho_{t+1} / q\right) g(z)\, dz$, must at least equal the financial intermediaries' costs of obtaining funds, r_{t+1}, per unit borrowed.

The equilibrium contract between borrowers and lenders continues to be a standard debt contract. The borrower repays the real amount x_t if $z \in B_t$. If he obtains a "poor" return on his investment, then the financial intermediary monitors, incurs the cost $\gamma \rho_{t+1}$, and appropriates all of the borrower's returns. These properties of an equilibrium contract are summarized in the following proposition.

Proposition 5 *The optimal loan contract must satisfy*

$$\begin{aligned} R_t(z) &= z\rho_{t+1};\ z \in A_t \\ A_t &= \left[0, \frac{x_t}{\rho_{t+1}}\right) \end{aligned}$$

and

$$\int_{A_t} \left[qR_t(z) - \gamma \rho_{t+1} \right] g(z)\, dz - \int_{B_t} q x_t g(z)\, dz = q r_{t+1}. \tag{26}$$

Finally, in order for a full general equilibrium to obtain, it is necessary for both factor and credit markets to "clear." As before, an equilibrium in factor markets requires that equations (1) and (2) hold. An equilibrium in credit markets requires that two conditions be satisfied. First, the per unit expected rate of return on loans must equal the per unit rate of return on deposits. In other words, there are zero expected profits for an intermediary. Second, the quantity of funds borrowed must be equal to the quantity of funds deposited.

Intermediaries clearly earn zero (expected) profits if and only if equation (26) is satisfied. Equation (26) can, of course, be written more succinctly as

$$\rho_{t+1}\pi\left(\frac{x_t}{\rho_{t+1}}\right) = r_{t+1}. \tag{27}$$

In addition, an equality between the quantity of funds borrowed, $(1-\alpha)\, q\mu_t$, where μ_t is the fraction of borrowers who obtain funding, and the quantity of funds deposited, $\alpha w(k_t) H\left(r_{t+1}/\rho_{t+1}\right)$, requires that

$$(1-\alpha)\, q\mu_t = \alpha w(k_t) H\left(\frac{r_{t+1}}{\rho_{t+1}}\right), \tag{28}$$

where $H\left(r_{t+1}/\rho_{t+1}\right)$ is the fraction of lenders who deposit their wage with the bank.[26] As noted by Williamson (1986), there may or may not be credit rationing in this economy, even in the presence of a competitive banking system. Equilibrium (i.e. the interest rates charged on loans and paid on deposits) in both the presence and absence of credit rationing is now described.

6.1 Equilibrium Interest Rates without Credit Rationing

An equilibrium without credit rationing will satisfy equations (1), (2), (27) and (28) as well as $\mu_t^\star = 1$. Imposing $\mu_t^\star = 1$ in equation (28) yields

$$(1-\alpha)\, q = \alpha w(k_t) H\left(\frac{r_{t+1}^\star}{\rho_{t+1}}\right). \tag{29}$$

When lenders' returns are uniformly distributed, equation (29) is equivalent to

$$r_{t+1}^\star = \frac{(1-\alpha)\, q}{\alpha w(k_t)}\bar{v}\rho_{t+1}. \tag{30}$$

Notice that, in the absence of credit rationing, the equilibrium rate of return on deposits is the *same* under either a competitive or a monopolistic banking system. This is the case since banks must pay a high enough return to fund all potential borrowers; the return required does not depend on how competitive the banking system is. Thus it is not the case that banks always make profits off both margins – lenders and borrowers.

However, the same is not true of the rate of interest paid by borrowers for funds. Under a competitive banking system, the interest rate on loans, $x_t^\star$, is implicitly defined by equations (27) and (30). Thus, $x_t^\star$ is the solution to the equation

$$\pi\left(\frac{x_t^\star}{\rho_{t+1}}\right) = \frac{(1-\alpha)\, q}{\alpha w(k_t)}\bar{v}. \tag{31}$$

[26] As before, it will be assumed that the distribution of returns on own-investments in the lender population is uniform.

6.2 Equilibrium Interest Rates with Credit Rationing

Credit is rationed at date t when there are some unfunded borrowers — that is, when $\mu_t^\star < 1$ holds — and when it is not possible for any unfunded borrower to "bid funds away" from funded borrowers. The latter requirement translates into x_t being set so as to maximize the expected return to a lender; that is

$$x_t^\star = \hat{x}\left(\rho_{t+1}\right) = \rho_{t+1}\eta. \tag{32}$$

This coincides with the loan rate that is charged by a monopoly bank.

Since $\pi\left(\hat{x}\left(\rho_{t+1}\right)/\rho_{t+1}\right) = \psi$, it is immediate from equation (27) that, under credit rationing, the deposit interest rate must satisfy

$$r_{t+1}^\star = \psi\rho_{t+1}. \tag{33}$$

Finally, the fraction of borrowers that can be funded depends upon the supply of deposits elicited by the rate of return in equation (33); in equilibrium

$$\mu_t^\star = \frac{\alpha w\left(k_t\right)}{\left(1-\alpha\right)q} H\left(\frac{r_{t+1}^\star}{\rho_{t+1}}\right) = \frac{\alpha w\left(k_t\right)}{\left(1-\alpha\right)q} H\left(\psi\right). \tag{34}$$

The conditions necessary for credit rationing to obtain are now described.

6.3 Conditions Necessary for Credit Rationing

Credit rationing will obviously be observed if the maximum potential supply of savings is strictly less than the demand for funds (insufficient income) — that is, if $\left(1-\alpha\right)q > \alpha w\left(k_t\right)$ (or in other words $\left(k_t < k_{SI}\right)$). However, even if there is a sufficient supply of income $\left(\left(1-\alpha\right)q \leq \alpha w\left(k_t\right)\right)$, credit rationing will be observed if the value of $\mu_t^\star$ given by equation (34) is less than one. When this transpires, the supply of savings at the equilibrium rate of return is inadequate to meet the demand (insufficient saving). From equation (34) it is transparent that $\mu_t^\star < 1$ will hold if and only if

$$\psi < \frac{\left(1-\alpha\right)q\bar{v}}{\alpha w\left(k_t\right)}. \tag{35}$$

When $\psi \geq \bar{v}$ is satisfied, clearly equation (35) can not hold for any $k_t \geq k_{SI}$; hence credit is rationed if and only if $k_t < k_{SI}$. However, when $\psi < \bar{v}$ holds equation (35) can obtain even if $k_t > k_{SI}$; in this event there is enough savings to fund all potential borrowers, but the equilibrium rate of return is too low to draw these savings into the formal financial system. Defining the value k_{CSS} (sufficient savings capital stock) by

$$\psi = \frac{\left(1-\alpha\right)q\bar{v}}{\alpha w\left(k_{CSS}\right)},$$

then credit rationing obtains under a competitive banking system whenever $k_t < \max\left[k_{SI}, k_{CSS}\right]$ holds. It is also easy to verify that the condition, $k_t < \max\left[k_{SI}, k_{CSS}\right]$ obtains if and only if the value of r_{t+1} given by equation (30) — the interest rate in the absence of credit rationing — is greater than the interest rate given by equation (33) — the interest rate that obtains under credit rationing.

7 General Equilibrium: A Competitive Banking System

As in the case of the monopoly banking system, the equilibrium law of motion for the per capita capital stock is given by

$$k_{t+1} = \hat{z}q(1-\alpha)\mu_t - \gamma(1-\alpha)\mu_t G\left(\frac{x_t}{\rho_{t+1}}\right) + \alpha\left[1 - H\left(\frac{r_{t+1}}{\rho_{t+1}}\right)\right]\int_{\frac{r_{t+1}}{\rho_{t+1}}}^{\bar{v}} yw(k_t)\,h(y)\,dy. \quad (36)$$

Using equation (28) in equation (36) and under the assumption that lenders' returns are uniformly distributed, one obtains

$$k_{t+1} = \alpha w(k_t)\left\{H\left(\frac{r_{t+1}}{\rho_{t+1}}\right)\left[\hat{z} - \frac{\gamma}{q}G\left(\frac{x_t}{\rho_{t+1}}\right)\right] + \left[1 - H\left(\frac{r_{t+1}}{\rho_{t+1}}\right)\right]\left[\frac{\bar{v}^2 - \left(\frac{r_{t+1}}{\rho_{t+1}}\right)^2}{2\bar{v}}\right]\right\}. \quad (37)$$

The properties of this law of motion depend very strongly on whether or not credit is rationed (and also the reason for the credit rationing). Each situation is now considered in turn.

7.1 Equilibrium Law of Motion: Credit Rationing

Even when credit rationing is observed, there are two possibilities regarding the equilibrium law of motion for the capital stock

CASE CII (Competition Insufficient Income): $\psi > \bar{v}$. Here $k_{CSS} < k_{SI}$ holds, and as in case MII with a monopoly bank, all savings by lenders is intermediated. In addition, the equilibrium interest rates on loans and deposits, respectively, satisfy $x_t^\star = \hat{x}(\rho_{t+1}) = \rho_{t+1}\eta$ and $r_{t+1}^\star = \rho_{t+1}\psi$. Substituting these values into equation (37) yields

$$k_{t+1} = aw(k_t)\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right]. \quad (38)$$

It should be noted that this equilibrium law of motion coincides with that observed under a monopoly banking system when $\psi/2 > \bar{v}$. Consequently, equation (38) has the shape depicted in Figure 5.

CASE CIS (Competition Insufficient Saving): $\psi \leq \bar{v}$. Here $k_{CSS} \geq k_{SI}$ holds. In addition, $x_t^\star = \rho_{t+1}\eta$ and $r_{t+1}^\star = \rho_{t+1}\psi$ continue to obtain, but now $\rho_{t+1}\psi \leq \rho_{t+1}\bar{v}$ is satisfied. Therefore not all lenders necessarily deposit their wage income with the bank. Consequently, the equilibrium law of motion is now

$$k_{t+1} = \alpha w(k_t)\left\{\frac{\psi}{\bar{v}}\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right] + \left[1 - \frac{\psi}{\bar{v}}\right]\left[\frac{\bar{v}^2 - (\psi)^2}{2\bar{v}}\right]\right\}. \quad (39)$$

Equation (39) describes an increasing, concave locus, as depicted in Figure 5. By assumption (A.6) this locus lies entirely below the locus defined by equation (38).

7.2 Equilibrium Law of Motion: No Credit Rationing

When $k_t \geq \max[k_{SI}, k_{CSS}]$, credit will not be rationed; $x_t^\star$ is implicitly defined by equation (31) and $r_{t+1}^\star$ is given by equation (30). Consequently, the equilibrium law of motion for the capital stock takes the form

$$k_{t+1} = (1-\alpha) q \left[\hat{z} - \frac{\gamma}{q} G\left(\frac{x_t^\star}{\rho_{t+1}}\right)\right] + [aw(k_t) - (1-\alpha) q] \frac{\bar{v}}{2} \left[1 - \left(\frac{(1-\alpha) q}{\alpha w(k_t)}\right)^2\right] \equiv \Phi(k_t). \tag{40}$$

Some properties of the function Φ are stated in the following lemma.

Lemma 6 *The function Φ satisfies*

a) $\Phi(k_{SI}) = \alpha w(k_{SI}) \left[\hat{z} - \frac{\gamma}{q} G\left(\frac{x_t^\star}{\rho_{t+1}}\right)\right]$,
b) $\Phi(k_{CSS}) = \alpha w(k_{CSS}) \left\{\frac{\psi}{\bar{v}} \left[\hat{z} - \frac{\gamma}{q} G(\eta)\right] + \left[1 - \frac{\psi}{\bar{v}}\right] \left[\frac{\bar{v}^2 - (\psi)^2}{2\bar{v}}\right]\right\}$,
c) $\Phi'(k_{SI}) > 0$,
d) $\Phi'(k_t) \geq 0$,
e) $\Psi(k_t) < \Phi(k_t) \leq \Psi(k_t) + (1-\alpha)\gamma G(\eta)$.

The proof of lemma 6 is found in the appendix. Part (a) of the lemma asserts that the equilibrium law of motion for the capital stock is discontinuous at $k_t = k_{SI}$, when credit rationing is of the case CII type. If case CIS is the relevant law of motion under credit rationing, then part (b) of lemma 6 states that the law of motion for the capital stock is continuous as the economy transits from a situation of credit rationing to one where credit rationing is not observed. Parts (c) and (d) of the lemma assert that equation (40) defines an increasing locus, while part (e) maintains that asymptotically the law of motion for the capital stock must lie below the 45^0 line. Consequently, the equilibrium law of motion for the capital stock in the absence of credit rationing has the general shape depicted in Figure 5

7.3 Steady State Equilibria

As is apparent from Figure 5, assumptions (A.1) and (A.2) and lemma 6 imply the existence of at least one non-trivial steady state equilibrium in the presence of a competitive banking system. Whether or not there is more than one depends heavily on parameter values, however. Several possibilities are now illustrated.

(a) Figure 6a (6b) depicts a situation where there is a unique steady state equilibrium. Credit rationing does (does not) obtain in the steady state. Clearly, if there is a unique steady state equilibrium, that equilibrium must be asymptotically stable.

(b) Figure 7 depicts a situation in which there are multiple steady state equilibria without credit rationing and, in addition, there may be a steady state exhibiting credit rationing as well. If there are any steady state equilibria with credit rationing, there can be at most one; it will necessarily be asymptotically stable. In addition, there may be multiple asymptotically stable steady state equilibria where credit rationing does not arise. Thus, "development trap" phenomena are quite possible; two intrinsically similar economies can easily display quite different levels of long-run real activity. In some cases, economies "trapped" in low-level equilibria will appear to suffer from relatively severe credit market frictions that lead to rationing while better-developed economies will not exhibit credit rationing, and will appear to have better functioning financial systems.

8 A Comparison of Monopolistic and Competitive Banking Systems

The remainder of this paper examines how economies with competitive banking systems and economies with more concentrated banking systems compare in terms of (a) their long-run levels of real activity, (b) their rates of interest on loans and deposits (which will depend on the existence and type of credit rationing), (c) the prevalence of credit rationing, and (d) the quantity of resources consumed by the banking system. As before, it is necessary to consider several cases, distinguished by the existence and type of credit rationing experienced by each banking system.

8.1 Credit Rationing: Insufficient Income

In both economies, credit rationing is solely the result of insufficient income (i.e. parameter values are such that $\psi/2 > \bar{v}$ holds and credit is rationed only when $k_t \leq k_{SI}$). When $k_t \leq k_{SI}$ obtains, the equilibrium law of motion for the capital stock under monopoly (competitive) banking is given by equation (21) [(38)]: obviously these coincide, see Figure 8.

For $k_t > k_{SI}$, the equilibrium evolution of the capital stock is governed by equation (24) [(40)] when the banking system is monopolistic (competitive). The following proposition characterizes the relationship between the two laws of motion when there is no credit rationing.

Proposition 7 *For $k_t > k_{SI}$, the law of motion for a competitive system lies strictly above that for a monopoly system.*

Proposition 7 follows immediately from part (e) of lemma 6.

If credit is rationed under both a competitive and a monopolistic banking system, the result is that the steady state capital stocks coincide. Similarly, for each asymptotically stable steady state capital stock without credit rationing that exists under a monopoly banking system, there exists an asymptotically stable steady state under competitive banking with a higher capital stock. The reason for this lies in the fact that, in the absence of credit rationing, a monopoly bank charges an excessively high rate of interest on loans. The implications of this are examined in the following section. Finally, for an initial capital stock $k_0 > k_{SI}$, the initial growth rate in capital will be greater for a competitive banking system.

8.1.1 A Comparison of Interest Rates

When $\psi/2 > \bar{v}$ holds, a monopoly banking system always results either in a higher rate of interest on loans, or a lower rate of interest on deposits, than would be observed under a competitive banking system. However, only one, rather than both, of these situations obtain at any date, as the following proposition demonstrates.

Proposition 8 *Suppose that $\psi/2 > \bar{v}$ holds.*
a) *If $k_t \leq k_{SI}$ (credit is rationed), then $x_t = \eta\rho_{t+1}$ under either a monopolistic or a competitive banking system. Under monopoly (competition) $r_{t+1} = \bar{v}\rho_{t+1}$ $\left(= \psi\rho_{t+1}\right)$. Thus the deposit rate of interest is lower under monopoly.*
b) *If $k_t > k_{SI}$ (credit is not rationed), then under monopoly (competition) $x_t = \eta\rho_{t+1}$ (x_t is implicitly given by equation (31)). Thus the rate of interest charged on loans is increased by the presence of a monopoly banking system. The rate of interest paid on deposits, $r_{t+1} = (1-\alpha)\, q\bar{v}\rho_{t+1}/\alpha w(k_t)$, is the same under the two systems.*

Proposition 8 follows immediately from an examination of equations (9), (12), (14), (30), (31), (32), and (33).

8.1.2 Remarks

In the case of credit rationing (part (a)), a monopoly bank makes its profits only off one margin – lenders. While there is a wedge between the interest rates paid on deposits and those charged on loans, the reduction in the rate of interest paid on deposits by a monopoly bank (relative to a competitive bank) has no consequences for capital accumulation; it represents a pure redistribution of income from depositors to the owners of the bank.

However, the higher rate of interest charged on loans (part(b)) does have an implication for the capital stock. An increase in the loan rate of interest obligates a monopoly bank to monitor borrowers with a higher probability than would be observed under competition. This is socially wasteful, and also accounts for the reduction in the capital stock. Thus the analysis predicts that, when insufficient income is the only reason for credit rationing, concentration in the banking system is detrimental to capital formation if and only if a monopoly banking system engages in "excessive" state verification; this will occur whenever credit is not rationed. Finally, this also makes it more likely that a development trap will arise when there exists multiple steady state equilibria.

8.2 Credit Rationing: Insufficient Income and Insufficient Saving

In this case, credit rationing occurs for different reasons in the two economies (i.e. parameter values satisfy $\psi > \bar{v} > \psi/2$). For this configuration of parameters, credit rationing occurs under a monopoly banking system as a result of insufficient saving (i.e. if and only if $k_t \leq k_{MSS}$). Credit rationing will arise with competitive banks as a result of insufficient income (i.e. only when $k_t \leq k_{SI} < k_{MSS}$). Thus, for $k_t \in (k_{SI}, k_{MSS})$, the existence of monopoly banking will lead to credit rationing that would not be observed under a competitive financial system. The difference arises because, when $\psi > \bar{v} > \psi/2$ holds, a competitive banking system draws all potential depositors into the formal financial sector. However, a monopolistic bank views it as too costly to do so. Thus more savings must be generated under monopoly — that is, there must be a larger current capital stock and a larger current income level — to fund all potential borrowers than is the case under competition.

For $k_t \leq (\geq)\ k_{MSS}$, the law of motion for the capital stock is given by equation (22) [(24)] under monopoly. For $k_t \leq (\geq)\ k_{SI}$, the law of motion for the capital stock is given by equation (38) [(40)] under competition. These observations, along with assumption (A.6), imply that the equilibrium law of motion for the capital stock of an economy with a competitive banking system lies everywhere above the corresponding law of motion for the capital stock of an otherwise identical economy, but with a monopolistic banking system: see Figure 9.

The immediate implication of this fact is that, for every asymptotically stable steady state equilibrium under monopoly banking, there exists some asymptotically stable steady state equilibrium under competition with a larger capital stock. This result arises for a combination of three reasons. (a) As before, when neither a competitive nor a monopolistic banking system gives rise to credit rationing, excessive state verification occurs in a monopolistic banking system. (b) Moreover, credit rationing is more likely to occur under monopoly banking (in particular, when $k_{SI} < k_t < k_{MSS}$), preventing some relatively productive investments from being undertaken. (c) Finally, when credit is rationed under either banking system, a monopolistic banking system draws fewer depositors into the formal financial system, so that again fewer high productivity investments are undertaken.

8.2.1 A Comparison of Interest Rates

In section 8.1.1 (when $\psi/2 > \bar{v}$), the existence of monopoly power in banking necessarily resulted in either a higher rate of interest charged on loans, or a lower rate of interest paid on deposits. However, *both situations did not occur* simultaneously. The situation is substantially different here, as stated in the following proposition.

Proposition 9 *Suppose that $\psi > \bar{v} > \psi/2$ holds.*
a) *If $k_t \leq k_{SI}$ (credit is rationed under competition as well as under monopoly), then the interest rate charged on loans is identical under either a competitive or a monopoly banking system. A monopolistic banking system pays a lower rate of interest on deposits.*
b) *If $k_t \geq k_{MSS}$ (credit is not rationed, either under competition or monopoly), then a monopolistic banking system charges a higher rate of interest on loans than a competitive banking system, but pays the same rate of interest on deposits.*
c) *If $k_t \in (k_{SI}, k_{MSS})$ (a monopoly banking system rations credit whereas a competitive one does not), then a monopoly banking system charges a higher rate of interest on loans, and pays a lower rate of interest on deposits, than a competitive banking system.*

Proposition 9 follows from equations (9), (12), (14), (30), (31), (32), and (33).

8.2.2 Remarks

Relative to proposition 8, proposition 9 differs only in that there is now an interval of current capital stock (k_{SI}, k_{MSS}), where credit rationing is observed with a monopolistic, but not with a competitive banking system. Overall however, in this case a monopoly tends to be more detrimental to the economy. In particular, as before in part (a) there is a difference (between a monopoly and competitive bank) in interest rates paid on deposits (and not between interest rates charged on loans). However, unlike the previous case, this now results in less capital accumulation and slower growth. This results from a sufficiently low interest rate being paid on deposits by a monopoly bank that some lenders decide to forgo the formal financial markets and undertake their own investment project; this leads to less capital accumulation relative to a competitive system where only (more productive) borrowers undertake investment.

For values of k_t in the interval (k_{SI}, k_{MSS}) (i.e. part (c)), a monopolistic banking system operates to the simultaneous detriment of both borrowers and lenders, ceteris paribus. In this case profits are made off both margins resulting in excess monitoring and less investment being undertaken by (more productive) borrowers. Part (b) is essentially identical to that of the previous section. Finally, as before, given the relative positions of the equilibrium laws of motion for the two banking systems, it is obvious that a competitive system will, for a given initial capital stock, experience greater initial growth and be less likely to converge to the steady state with the lowest level capital stock – thus avoiding a development trap.

8.3 Credit Rationing: Insufficient Saving

In this final case, credit rationing in both economies again occurs for the identical reasons: namely insufficient saving. When $\bar{v} > \psi$ holds, a monopolistic (competitive) banking system rations credit if and only if $k_t \leq k_{MSS}$ (k_{CSS}) is satisfied and thus all credit rationing is the result of insufficient saving. Since $k_{CSS} < k_{MSS}$ holds, again the range of potential current capital stocks can be partitioned into three intervals, see Figure 10. (a) For $k_t \leq k_{CSS}$, credit rationing is observed under both a monopolistic and a competitive banking system. (b) For $k_t \in (k_{CSS}, k_{MSS})$, credit rationing is observed if the banking system is monopolistic, but not if it is competitive. (c) For $k_t \geq k_{MSS}$, credit rationing does not arise under either monopoly or competition. This is qualitatively similar to the situation discussed in section 8.2. And, indeed, all of the results and remarks described in that section apply to this case as well.

9 Conclusion

Recent literature has established a strong positive correlation between financial market development and real economic performance. However, what is less well-understood is how the level of competition in financial markets affects either the development of the financial system, or the level of real activity (or its rate of growth). This paper attempts to shed some light on this issue.

As a practical matter, many economies have banking systems that are less competitive and more monopolistic in nature. The analysis of this paper indicates that in some — but not all — cases, monopoly in banking can be expected to result in both a lower long-run capital stock, and a slower rate of growth (conditional on the level of the current capital stock) than would be the case in the presence of a competitive banking system. Moreover, it has also been shown exactly when a monopolistic financial system will and will not be detrimental to the long-run level of real activity and how this detrimental effect depends on the existence and the particular type of credit rationing which may arise.

Banks with monopoly power will, naturally, manipulate rates of return on loans, and/or deposits, to enhance their own profits. However, as shown, it is not necessarily the case that they always simultaneous manipulate both interest rates. If savings are not very interest elastic, as most evidence suggests, then a reduction in the rate of interest paid on deposits by a monopoly bank does not have a detrimental effect on capital formation, although it does redistribute income from bank depositors to the owner of the bank. However, when monopoly power in banking raises the rate of interest charged on loans, this leads to excessive monitoring by the financial system. This is detrimental to capital formation as well as being socially wasteful.

The existence of monopoly power in banking also has implications for the amount of credit rationing that might be observed, in equilibrium. Credit rationing is a natural phenomenon arising either because demand exceeds the total supply of funds available in the economy, or because banks do not generate enough deposits by their choice of rates of return on deposits. This paper has shown that, in general, there will be situations where a competitive banking system will not ration credit (because they pay a sufficiently high interest rate) while at the same time a monopoly banking system will ration credit (because it is profitable to depress the rate of return on deposits). In this case, a monopoly bank offers both a lower return on deposits and charges a higher interest rate on loans than would be observed with a competitive banking system.

If the economy exhibits credit rationing under both types of banking systems, then it will be the case that borrowers are charged the same interest rate in either system. However, in this case a monopoly bank may offer either the same or a lower interest rate on deposits than a competitive banking system (depending on the relative returns from borrowers' and lenders' investment opportunities). Conversely, when there is no credit rationing in either system, then it is lenders who experience the same rate of return independent of the nature of the banking system, while a monopoly bank charges a higher interest rate on funds borrowed. Consequently, it is not the case that a monopolistic banking system always charges higher prices for funds lent or that it always offers lower returns on funds deposited, when compared with a competitive banking system.

A final result of this paper deals with developing economies and the existence of development trap phenomena. It is possible in an economy with either type of banking system to be stuck at a low capital stock steady state equilibrium characterized by credit rationing. However, because of the potential discontinuities associated with the equilibrium law of motion for the capital stock of an economy with a competitive banking system, it is less likely that economies with competitive banking systems will experience development traps.

There are several dimensions along which the analysis undertaken here can be extended. One obvious extension would be to consider an economy with more than one bank, but in which each bank has some power to affect prices. This intermediate situation between pure monopoly and perfect competition obviously would constitute a better representation of the financial systems of a number of economies. Second, in the present model there is only a single asset, there is no government sector, and banks are not regulated. Exploring the desirability of regulatory intervention, and allowing some scope for fiscal and monetary policy to affect the operation of the financial system would be important topics for further investigation.

10 Appendix

A Proof of Lemma 4

The first two parts of lemma 4 are obtained by simply evaluating $\Psi(k_t)$ at k_{SI} and k_{MSS}, and using the definitions

$$(1-\alpha)q \equiv \alpha w(k_{SI})$$

and

$$\frac{\psi}{2} \equiv \frac{(1-\alpha)q\bar{v}}{\alpha w(k_{MSS})}.$$

For parts (c) and (d), differentiating $\Psi(k_t)$ yields

$$\Psi'(k_t) = w'(k_t)\frac{\alpha\bar{v}}{2}\left\{\left[1-\left(\frac{(1-\alpha)q}{\alpha w(k_t)}\right)^2\right]+2\left[1-\frac{(1-\alpha)q}{\alpha w(k_t)}\right]\left[\frac{(1-\alpha)q}{\alpha w(k_t)}\right]^2\right\}. \quad \text{(AP.1)}$$

From the definition of k_{SI} (as stated above), $\Psi'(k_{SI}) = 0$ follows immediately. Moreover, since $(1-\alpha)q < \alpha w(k_t)$ must obtain in the absence of credit rationing, $\Psi'(k_t) > 0$, for all $k_t \geq k_{SI}$, clearly holds. Part (e) follows immediately from the fact that $\lim_{k_t\to\infty} w(k_t)/k_t = 0$ holds.

B Proof of Lemma 6

Parts (a) and (b) are easily obtained by evaluating $\Phi(k_t)$ at k_{SI} and k_{CSS} respectively, and using the definitions

$$(1-\alpha)q \equiv \alpha w(k_{SI})$$

and

$$\psi \equiv \frac{(1-\alpha)q\bar{v}}{\alpha w(k_{CSS})}.$$

The continuity of the equilibrium law of motion depends on whether or not case CII or case CIS obtains. In a case CII economy, the equilibrium law of motion for the capital stock when $k_t < k_{SI}$ holds (that is, when there is credit rationing) is given by equation (38). Clearly

$$\lim_{k_t\uparrow k_{SI}} k_{t+1} = \alpha w(k_{SI})\left[\hat{z} - \frac{\gamma}{q}G(\eta)\right].$$

However, for $k_t > k_{SI}$, k_{t+1} evolves according to equation (40). It is easy to verify that

$$\lim_{k_t\downarrow k_{SI}} k_{t+1} = \alpha w(k_{SI})\left[\hat{z} - \frac{\gamma}{q}G\left(\frac{x_t^\star}{\rho_{t+1}}\right)\right]$$

where $x_t^\star$ satisfies $\pi\left(x_t^\star/\rho_{t+1}\right) = \bar{v}$. Since η satisfies $\pi(\eta) \equiv \psi$, clearly $x_t^\star/\rho_{t+1} < \eta$ must hold. Therefore the equilibrium law of motion for the capital stock is discontinuous at $k_t = k_{SI}$.

In a case CIS economy, credit rationing does (does not) obtain for $k_t < (>) \; k_{CSS}$. When $k_t < k_{CSS}$ holds, k_{t+1} is given by equation (39), and

$$\lim_{k_t \uparrow k_{CSS}} k_{t+1} = \alpha w\left(k_{CSS}\right)\left\{\frac{\psi}{\bar{v}}\left[\hat{z} - \frac{\gamma}{q}G\left(\eta\right)\right] + \left[1 - \frac{\psi}{\bar{v}}\right]\left[\frac{\bar{v}^2 - \left(\psi\right)^2}{2\bar{v}}\right]\right\}.$$

Similarly, when $k_t > k_{CSS}$, k_{t+1} is given by equation (40), and

$$\lim_{k_t \downarrow k_{CSS}} k_{t+1} = \alpha w\left(k_{CSS}\right)\left\{\frac{\psi}{\bar{v}}\left[\hat{z} - \frac{\gamma}{q}G\left(\eta\right)\right] + \left[1 - \frac{\psi}{\bar{v}}\right]\left[\frac{\bar{v}^2 - \left(\psi\right)^2}{2\bar{v}}\right]\right\}.$$

Thus, in this case the equilibrium law of motion for the capital stock is continuous.

For part (c) and (d), differentiating equation (40) yields

$$\Phi'\left(k_t\right) = -\left(1-\alpha\right)\gamma g\left(\frac{x_t^\star}{\rho_{t+1}}\right)\frac{d\left(\frac{x_t}{\rho_{t+1}}\right)}{dk_t} + \Psi'\left(k_t\right),$$

while differentiating equation (31), one obtains

$$\frac{d\left(\frac{x_t}{\rho_{t+1}}\right)}{dk_t}\left[1 - \frac{\gamma}{q}g\left(\frac{x_t}{\rho_{t+1}}\right) - G\left(\frac{x_t}{\rho_{t+1}}\right)\right] = -\frac{\left(1-\alpha\right)q\bar{v}}{\alpha\left[w\left(k_t\right)\right]^2}w'\left(k_t\right).$$

The right hand side of this equation is negative, and by assumption (A.4) we have

$$1 - \frac{\gamma}{q}g\left(\frac{x_t}{\rho_{t+1}}\right) - G\left(\frac{x_t}{\rho_{t+1}}\right) \geq 0$$

for all $x_t \left/ \rho_{t+1}\right. \leq \eta$. Therefore, $d\left(x_t^\star \left/ \rho_{t+1}\right.\right)/dk_t < 0$. Moreover $\Psi'\left(k_t\right) \geq 0$ is also satisfied, yielding $\Phi'\left(k_t\right) \geq 0$. Part (c) follows immediately from $\Psi'\left(k_{SI}\right) = 0$.

Finally, for part (e) a comparison of equations (24) and (40) indicates that $\Phi\left(k_t\right) > \Psi\left(k_t\right)$ holds if and only if $G\left(\eta\right) > G\left(x_t^\star \left/ \rho_{t+1}\right.\right)$. But this is immediate from the fact that $x_t^\star < \eta\rho_{t+1}$ whenever credit is not rationed. Moreover,

$$\Phi\left(k_t\right) - \Psi\left(k_t\right) = \left(1-\alpha\right)\gamma\left[G\left(\eta\right) - G\left(\frac{x_t^\star}{\rho_{t+1}}\right)\right] \leq \left(1-\alpha\right)\gamma G\left(\eta\right),$$

establishing the result.

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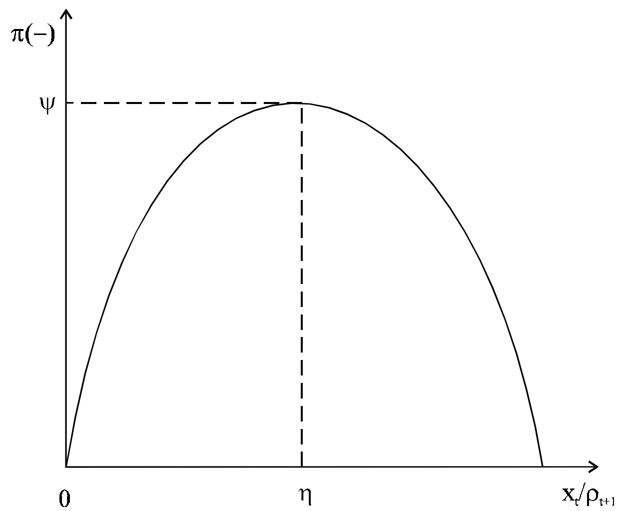

Figure 1: Expected Return to a Lender

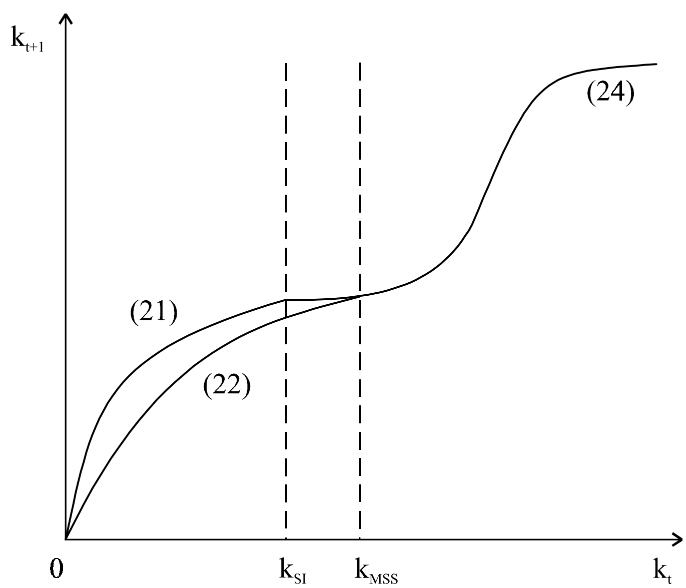

Figure 2: Monopoly Banking System — Equilibrium Laws of Motion

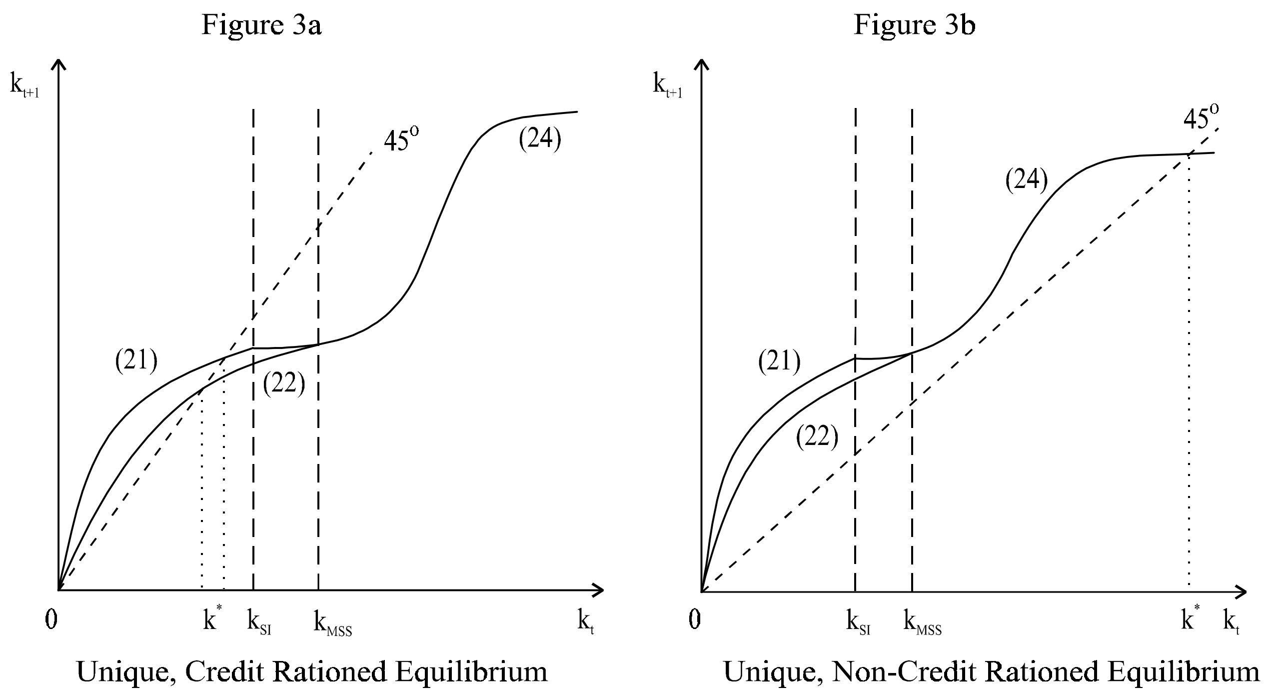

Figure 3: Monopoly Bank: Unique Steady State Equilibria

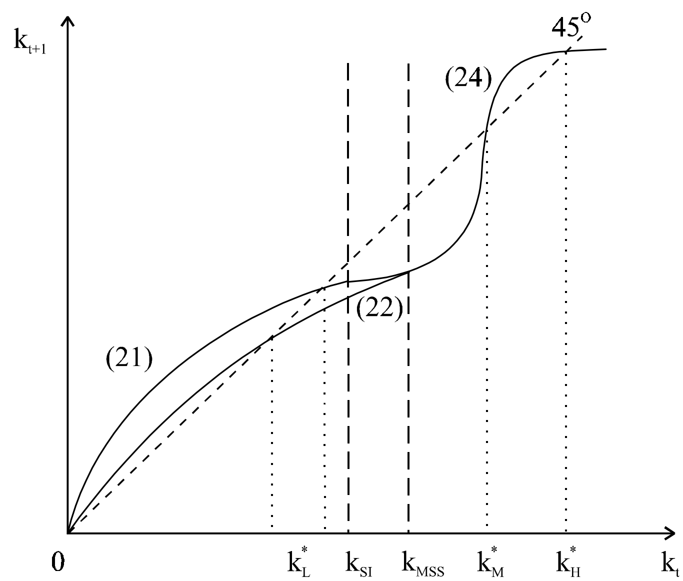

Figure 4: Monopoly Bank: Multiple Steady State Equilibria

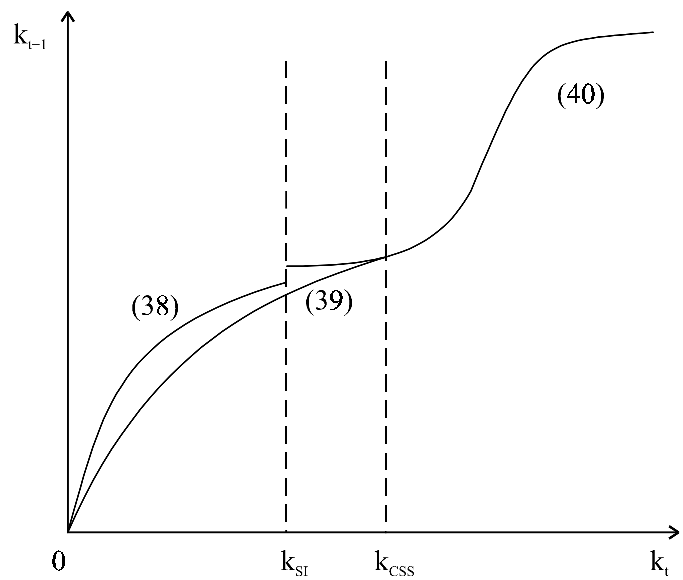

Figure 5: Competitive Banking System: Equilibrium Laws of Motion

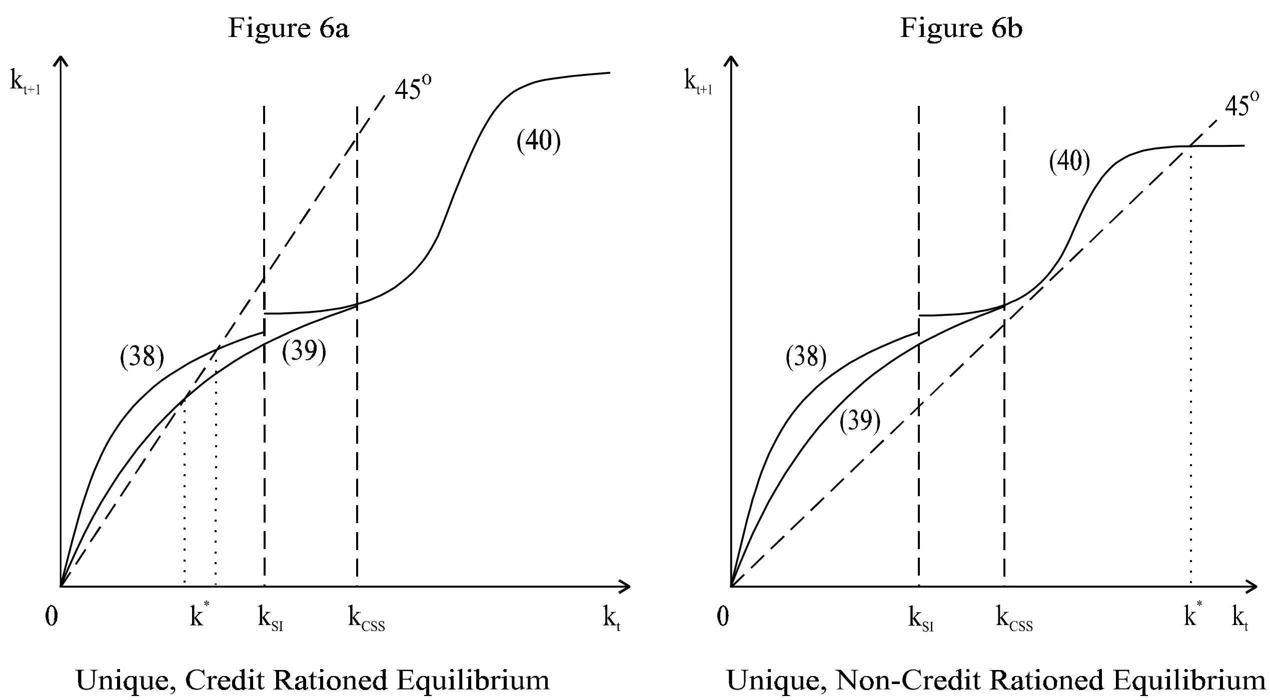

Figure 6: Competitive Bank: Unique Steady State Equilibria

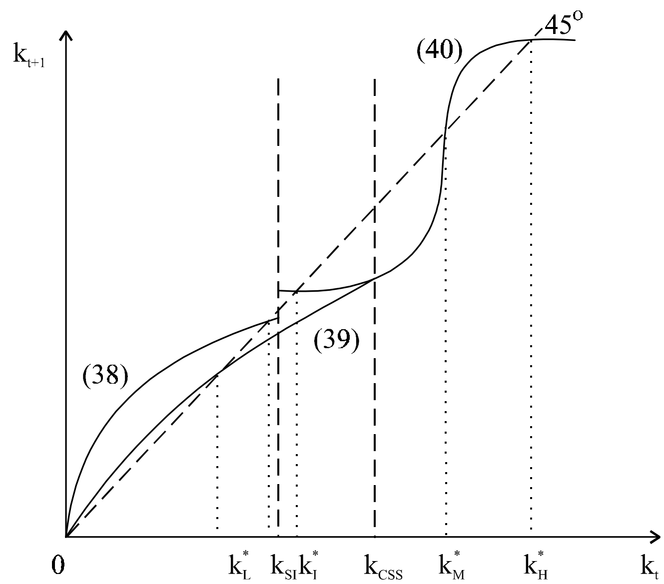

Figure 7: Competitive Bank: Multiple Steady State Equilibria

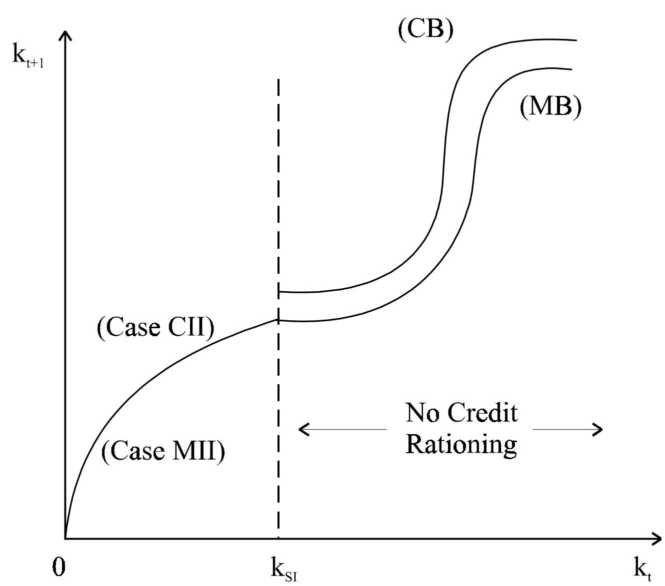

Figure 8: Credit Rationing in CB and MB due to Insufficient Income

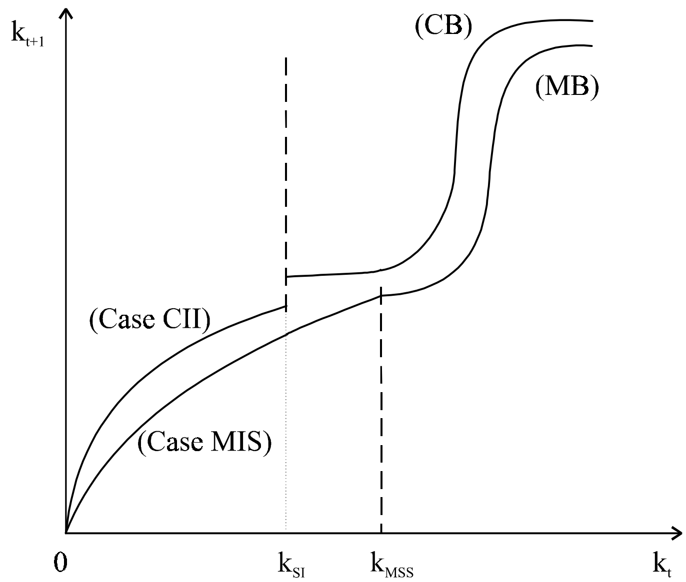

Figure 9: Credit Rationing for CB (MB) due to Insufficient Income (Insufficient Saving)

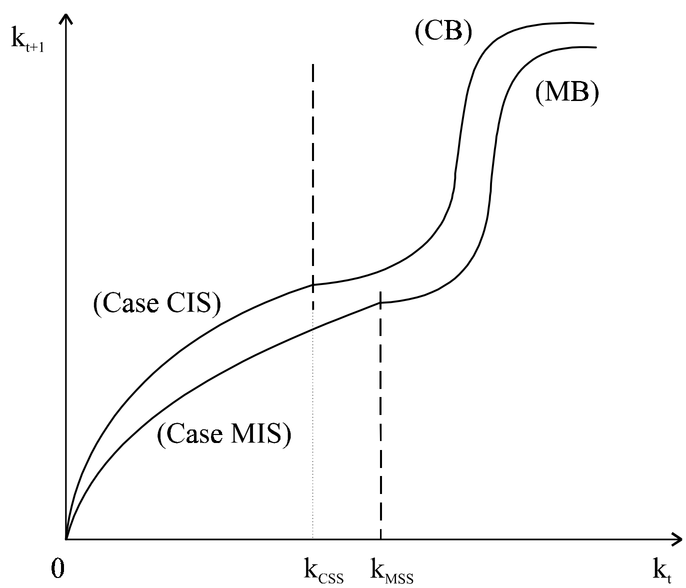

Figure 10: Credit Rationing in CB and MB due to Insufficient Saving