Limited Enforcement and the Organization of Production

Erwan Quintin*
Federal Reserve Bank of Dallas

April 4, 2003

Abstract

This paper describes a dynamic, general equilibrium model designed to assess whether contractual imperfections in the form of limited enforcement can account for international differences in the organization of production. In the model, limited enforcement constrains agents to operate establishments below their optimal scale. As a result, economies where contracts are enforced more efficiently tend to be richer and emphasize large scale production. Calibrated simulations of the model reveal that these effects can be large and account for a sizeable part of the observed differences in the size distribution of manufacturing establishments between the United States, Mexico and Argentina.

Keywords: Limited enforcement; Organization of production; Economic development.

JEL classification: L23; O11.

*I would like to thank Hugo Hopenhayn, Pat Kehoe, Tim Kehoe, Narayana Kocherlakota, Ed Prescott, Manuel Santos, Vincenzo Quadrini, as well as seminar participants at the Universidad Torcuato di Tella, Stanford University, Boston University, Columbia University and the Federal Reserve Bank of Minneapolis for their valuable comments. I am also grateful to Kathryn Cook for her research assistance. The views expressed are those of the author and do not necessarily reflect the position of the Federal Reserve Bank of Dallas or the Federal Reserve System.
1 Introduction

Property rights are not effectively enforced in developing nations. For instance, Djankov et al. (2002) calculate that it takes about 300 days on average to collect on a bad check in Argentina or Mexico, compared to 50 days in the U.S. In this paper, I assess the importance of the imperfect enforcement of financing contracts for documented differences in the organization of production between developing economies and the U.S.

I consider a dynamic version of the industrial organization model proposed by Lucas (1978). Agents differ in terms of managerial talent and the bequest they receive. Each period, they can either work as unskilled workers or manage a strictly concave technology that transforms unskilled labor and physical capital into the unique consumption good. Managers need to finance both their physical capital and payroll. They can write long-term financing contracts with an intermediary, but the contractual framework is imperfect. Specifically, agents can decide to default on the payment stipulated by the contract in return for the advance of funds they receive. The intermediary can seek to enforce this payment and are successful with a fixed, exogenous probability. This probability is my proxy for the efficiency of enforcement institutions in the economy under consideration.

Because enforcement is limited, some managers are borrowing constrained and operate establishments at a sub-optimal scale. I measure the impact of the resulting distortions by way of calibrated numerical simulations. Quite intuitively, rises in the probability of enforcement result in higher steady state output and labor productivity. Furthermore, production establishments are larger on average and a larger share of employment is found in small establishments
in more productive economies, which is consistent with the well-documented fact that developing nations emphasize small scale production (see e.g. Tybout, 2001). In fact, I argue that the model can generate a distribution of establishments and employment across size categories that closely resembles its empirical counterparts in Mexico or Argentina. In addition, I find that wealth and income are more equally distributed in economies where contracts are better enforced. These findings indicate that a model of development that incorporates contractual imperfections can account for several distinguishing features of developing economies.

This work builds on a number of studies of environments with limited commitment. Kehoe and Levine (1993) study the impact of limited commitment on asset trading while Kocherlakota (1996) considers efficient consumption allocations. Marcet and Marimon (1992) measure the effect of limited commitment in a stochastic growth model. Monge (1999) studies the impact of interest rate volatility on firm dynamics in an environment very similar to the one developed in this paper. Cooley et al. (2000) study the implications of limited enforceability for the business cycle and the investment policy and dynamics of firms. This paper extends this work by gauging the quantitative impact of contractual frictions on the organization of production, and builds upon the work of Banerjee and Newman (1993) and Quadrini (2000) on the impact of financing constraints on entrepreneurship and wealth accumulation. My work also relates to the literature on the link between financial intermediation and economic development. McKinnon (1973) and Shaw (1973) provide some empirical evidence of this link for several nations. King and Levine (1993) report a correlation between financial and economic development with cross-country data. More generally, the model is consistent with the correlation between the quality
of a nation’s institutions and its level of development, as documented for instance by Hall and Jones (1998) or Barro (1991).

2 The organization of production in developing economies

Tables 1 shows the distribution of manufacturing establishments and employment in the U.S., Mexico and Argentina in 1988.\(^1\) Nearly 84 percent of establishments count 10 or fewer employees in Mexico while fewer than half of establishments have between 1 and 9 employees in the U.S. Establishments with 50 or more employees account for over 80 percent of all employment in the U.S., compared to 75 percent in Mexico, and less than half of all employment in Argentina.

Tybout (2000) presents similar data for a larger cross-section of countries. In many developing countries, including large countries such as India or Indonesia, the majority of manufacturing employment is found in establishments with fewer than ten employees. Tybout also presents a survey of existing explanations for this phenomenon. For instance, developing nations tend to emphasize items that can be produced efficiently on a small scale.\(^2\) However, table 2 presents some disaggregated data for the U.S. and Mexico that indicate that differences

\(^1\)The data for Argentina come from the country’s household survey and allow one to estimate the distribution of employment, but not the size distribution of establishments. Also note that throughout this paper, I concentrate on data from the manufacturing sector because in nations with large informal sectors like Mexico or Argentina, economic censuses of non-manufacturing establishments are unreliable. Manufacturing censuses are not immune to this weakness, but because manufacturing establishments tend to be less mobile than other establishments, they are counted with more precision.

\(^2\)To the extent that financing constraints contribute to this feature of the composition of output, the model I present in this paper could be viewed as a related explanation.
Table 1: Size distribution of manufacturing establishments and employment

<table>
<thead>
<tr>
<th>Employment size</th>
<th>0 to 9</th>
<th>10 to 49</th>
<th>50 to 249</th>
<th>250+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of establishments</td>
<td>48.1</td>
<td>33.0</td>
<td>15.0</td>
<td>3.9</td>
</tr>
<tr>
<td>Percent of employment</td>
<td>3.3</td>
<td>14.0</td>
<td>30.1</td>
<td>52.6</td>
</tr>
</tbody>
</table>

Mexico, 1988

<table>
<thead>
<tr>
<th>Employment size</th>
<th>0 to 10</th>
<th>11 to 50</th>
<th>51 to 250</th>
<th>251+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of establishments</td>
<td>83.6</td>
<td>10.7</td>
<td>4.3</td>
<td>1.4</td>
</tr>
<tr>
<td>Percent of employment</td>
<td>11.9</td>
<td>13.0</td>
<td>24.7</td>
<td>50.4</td>
</tr>
</tbody>
</table>

Argentina, 1988

<table>
<thead>
<tr>
<th>Employment size</th>
<th>0 to 15</th>
<th>16 to 50</th>
<th>51+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of employment</td>
<td>37.9</td>
<td>20.7</td>
<td>41.4</td>
</tr>
</tbody>
</table>

Source: County business pattern survey, Census Bureau (U.S.), Economic census, INEGI (Mexico), Permanent household survey, INDEC (Argentina).

in the composition of manufacturing output are unlikely to fully explain observed differences in the size distribution of establishments.³

Rauch (1991) proposes another explanation for the importance of small scale production in developing nations. In those countries, many firms do not comply with government regulations and choose to operate a small scale to avoid detection. The model I describe in this paper suggests another channel via which the importance of informal economic activities matters for the organization of production. It is difficult for informal, unregistered firms to write legally enforceable contracts. Not surprisingly then, informal firms have little access to outside

³Further disaggregation continues to show that in most sectors establishments are much larger on average in the U.S. than in Mexico.
Table 2: Average employment size of establishments in manufacturing sub-sectors, 1988

<table>
<thead>
<tr>
<th>Sub-sector</th>
<th>U.S.</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and kindred products</td>
<td>70.79</td>
<td>10.78</td>
</tr>
<tr>
<td>Tobacco products</td>
<td>323.74</td>
<td>243.84</td>
</tr>
<tr>
<td>Textile mill products</td>
<td>107.93</td>
<td>25.47</td>
</tr>
<tr>
<td>Apparel and other textile products</td>
<td>48.37</td>
<td>14.56</td>
</tr>
<tr>
<td>Lumber and wood products</td>
<td>21.68</td>
<td>8.49</td>
</tr>
<tr>
<td>Paper and allied products</td>
<td>98.80</td>
<td>18.22</td>
</tr>
<tr>
<td>Printing and publishing</td>
<td>25.23</td>
<td>12.47</td>
</tr>
<tr>
<td>Chemicals and allied products</td>
<td>69.15</td>
<td>71.73</td>
</tr>
<tr>
<td>Petroleum and coal products</td>
<td>53.34</td>
<td>44.98</td>
</tr>
<tr>
<td>Stone, clay and glass products</td>
<td>32.69</td>
<td>10.52</td>
</tr>
<tr>
<td>Primary metal industries</td>
<td>108.00</td>
<td>115.30</td>
</tr>
<tr>
<td>Fabricated metal products</td>
<td>41.73</td>
<td>28.75</td>
</tr>
<tr>
<td>Industrial machinery and equipment</td>
<td>37.66</td>
<td>28.52</td>
</tr>
<tr>
<td>Transportation equipment</td>
<td>177.87</td>
<td>160.69</td>
</tr>
<tr>
<td>Instruments and related products</td>
<td>98.33</td>
<td>38.49</td>
</tr>
</tbody>
</table>

Sources: INEGI and Census Bureau

financing, as documented for instance by Thomas (1992) and Mansell Carstens (1995). These firms are therefore constrained to operate a suboptimal scale.

3 The model

3.1 Physical environment

Each period a cohort of measure one of agents are born. They live $T$ periods, where $T > 2$. In the first $T - 1$ periods of their life, agents are endowed with a unit of productive time which they devote to managing productive resources or, instead, to delivering labor. They are also endowed with an innate level $z \in (0, 1]$ of managerial ability. An agent’s managerial ability
is public information and remains unchanged as he ages. In addition, the distribution $\mu$ of managerial ability has final support and is constant across cohorts.

An agent of managerial ability $z \in (0, 1]$ who, in a given period, manages amounts $k \geq 0$ of physical capital and $n \geq 0$ of unskilled labor produces quantity

$$F(k, n; z) = Azk^{\alpha_k}n^{\alpha_n} + (1 - \delta)k$$

of the unique consumption good, where $A, \alpha_k, \alpha_n > 0$ and $\alpha_k + \alpha_n < 1$. Throughout this paper, I will think of a production unit consisting of a manager together with some physical capital and unskilled labor as an establishment.

At the beginning of each period, agents choose an occupation. Agents who choose to become managers employ fixed quantities of unskilled labor and physical capital for the duration of the period. Output is delivered at the end of the period but labor must be paid before production can start.

All agents have linear preferences and discount consumption flows at a rate of $\beta$. What’s more, a fixed proportion $\phi < 1$ of each cohort leaves a non-negative bequest $g > 0$ to a single offspring. The preferences of these altruistic agents can be represented by the following utility function:

$$U(\{c_t\}_{t=1}^T, g) = \left( \sum_{t=1}^T \beta c_t \right)^{1-\gamma} g^\gamma$$

where $c_t$ is the agent’s consumption at age $t$. Newly born agents receive their bequest at the beginning of the first period of their life. Both the altruistic and managerial type of each agent
are independent of their parent’s. I impose the following assumption on the degree $\gamma$ of altruism to place a bound on each lineage’s wealth:

**Assumption 1.** $\gamma \beta^{-T} < 1$

The economy also comprises a financial intermediary with access to perfect, outside capital markets where a risk-free security earns return $r \equiv \beta^{-1} - 1$. One should therefore think of this economy as a small, open economy. Given the assumed risk free rate and the linearity of preferences, agents seek to maximize the discounted value of their lifetime income, and altruistic agents bequeath fraction $\gamma \beta^{-(T-1)}$ of that value to their offspring.

### 3.2 Contracts

Given the timing of production, managers need to finance both the quantity of physical capital they choose to employ and their payroll before production can start. For this purpose, the intermediary and newly born agents write financing contracts. A financing contract is a sequence $\{d_t, \rho_t\}_{t=1}^T$ that stipulates for every period $t$ of the agent’s life a quantity $d_t \geq 0$ of consumption good advanced to the agent at the start of the period, and a net transfer $\rho_t$ from the agent to the intermediary at the end of the period. (The total stipulated transfer from the agent to the intermediary at the end of period $t$ is $d_t(1 + r) + \rho_t$.)

The intermediary can commit fully to the terms of the contract, but agents have the option to renege on the stipulated transfer at the end of any period. When an agent exercises

---

4One can interpret the financing contract as a debt contract between a bank and a manager. Alternatively, the intermediary can be interpreted as a corporation that hires the agent to provide unskilled labor or manage resources. In this interpretation, the corporation is the proprietor of all output produced and the manager’s net income sequence is a compensation scheme.
this option, they are caught with probability $\theta \in [0,1]$ in which case the transfer is enforced. This probability is my proxy for the efficiency with which contracts are enforced in a given economy and constitutes the basis of the comparative statics exercise I carry out in this paper.

### 3.2.1 Self-financing

It will prove useful to first consider the problem agents solve when they must self-finance all production. Let $w > 0$ be the wage, i.e. the price of labor. (Throughout, I concentrate my attention on equilibria in which the wage is constant.) An agent of talent $z$ and age $t$ who in a given period operates with quantity $a \geq 0$ of the consumption good$^5$ solves:

$$
\max_{n,k \geq 0} F(k,n;z) - k(1+r) - nw(1+r)
$$

subject to:

$$
nw + k \leq a
$$

Let $\pi(a;z,w)$ be the corresponding net income while $k(a;z,w)$ and $n(a;z,w)$ are the associated policies. When $\delta < 1$ the cost of physical capital relative to labor decreases when the financing constraint is relaxed. This leads to$^6$:

**Lemma 2.** For any $(z,w) \in (0,1] \times \mathbb{R}^+\times \mathbb{R}^+$, $k(a;z,w)$ is increasing.

Lemma 2 says that all else equal, managers who are more financing constrained choose to operate their establishment at a lower physical capital to unskilled labor ratio, an observation

---

$^5$For notational simplicity, assume that managers can costlessly transform the consumption good into the capital good.

$^6$The proof of this result, as all proofs in this paper, is in the appendix.
which will help me interpret some of the numerical results of section 4. Now define:

\[ V_t^S(a; z, w) = a(1 + r) + \max_{\{a_{t+i}\}_{i=0}^{T-1-t}} \sum_{i=0}^{T-1-t} \beta^i \max\{ w(1 + r), \pi(a_{t+i}; z, w) \} \]

subject to:

\[ a_t = a \]
\[ a_{t+i} \leq a_{t+i-1}(1 + r) + \max\{ w(1 + r), \pi(a_{t+i-1}; z, w) \} \; \forall i \geq 1 \]

With this notation, \( V_t^S(b; z, w) \) is the agent’s maximum discounted lifetime income when all production must be self-financed and his initial bequest is \( b \geq 0 \). How much they can improve upon this lower bound depends on what financing contracts can be written.

### 3.2.2 Competitive contracts

Define, for all \( t < T \), \( d \geq 0 \), \( \rho \in \mathbb{R} \) and \( \theta \in [0, 1] \) the following value of default function:

\[ V_t^D(d, \rho; z, w, \theta) = \beta[\theta V_{t+1}^S(\pi(d; z, w) - \rho; z, w) + (1 - \theta)V_{t+1}^S(\pi(d; z, w) + d\beta^{-1}; z, w)] \]

This function gives the expected remaining lifetime utility of a manager of age \( t \) and ability \( z \) who defaults on a payment \( \rho \) due for an advance \( d \) of the consumption good, assuming they are unable to borrow after defaulting. With probability \( \theta \), they are caught, in which case they make the stipulated payment and start the next period with a quantity \( \pi(d; z, w) - \rho \) of the consumption good. With probability \( (1 - \theta) \), they are not caught, and start period \( t + 1 \) with
Consider now an agent of managerial ability $z \in [0, 1]$ born with a bequest $b \geq 0$. I will call a financing contract $\{d_t, \rho_t\}_{t=1}^T$ feasible when:

$$b \beta^{-1} + \max\{w \beta^{-1}, \pi(d_1; z, w)\} - \rho_1 \geq 0$$  \hspace{1cm} (1)$$

$$\max\{w \beta^{-1}, \pi(d_t; z, w)\} - \rho_t \geq 0 \quad \forall 1 < t < T$$  \hspace{1cm} (2)$$

$$\sum_{t=1}^{T} \beta^{t-1} \rho_t \geq 0$$  \hspace{1cm} (3)$$

and, for all $t < T$,

$$\sum_{i=t}^{T} \beta^{i-t} \left[ \max\{w \beta^{-1}, \pi(d_i; z, w)\} - \rho_i \right] \geq V^D_t(d_t, \rho_t; z, w, \theta)$$  \hspace{1cm} (4)$$

The first two conditions state that the agent’s net payment to the intermediary cannot exceed his net income in any period of his life. The third equation says that the contract must be actuarially fair for the intermediary: the expected present value of the payments he receives must exceed the present value of the quantities of the consumption good advanced. The last set of conditions is the requirement that in any period of the contract, the agent must be better off following the terms of the contract than defaulting. The left-hand side of these expressions is the opportunity cost of defaulting, namely the present value of the remaining income flows stipulated by the contract. The right-hand side is the expected present value of income after default, provided agents are excluded from future borrowing.
This notion of feasibility can be justified by standard contract-theoretic arguments (see Kocherlakota, 1996). In particular, punishing default with complete exclusion from borrowing weakens incentive compatibility problems as much as possible, and is therefore optimal.

I assume that the intermediary behaves competitively: among the set of feasible contracts, the contract most favorable to the agent prevails. In other words, contracts solve:

\[
\max -\beta^{T-1} \rho_T + \sum_{t=1}^{T-1} \beta^{t-1} (\max \{ w; \beta^{-1}, \pi(d_t; z, w) \} - \rho_t)
\]

subject to (1-4). I will refer to solutions to this problem as competitive contracts. Since net income is bounded above given the agent’s characteristics, a competitive contract exists for all agent types. In fact, competitive contracts can be computed by dynamic programming techniques as in Spear and Srivastava (1987) or Albuquerque and Hopenhayn (1997).

### 3.2.3 Properties of competitive contracts

Let \( d^*(w, z) = \arg \max_d \pi(d; w, z) \) be the optimal advance for an agent of ability \( z \) when there are no contractual imperfections (\( \theta = 1 \)). Absent contractual imperfections, agents become managers when \( \pi(d^*(w, z); w, z) \geq w \) and always operate their technology at its unique optimal scale. This subsection describes the impact of limited enforcement on occupational choices and the allocation of resources. The following result will prove useful.\(^8\)

---

\(^7\)A complete argument is available upon request.

\(^8\)This is a version of corollary 1 in Albuquerque and Hopenhayn, 1997.
Lemma 3. There exists a competitive contract \( \{d_t, \rho_t\}_{t=1}^T \) such that
\[
\rho_1 = b\beta^{-1} + \max \{w\beta^{-1}, \pi(d_1; z, w)\} \quad \text{and},
\]
\[
\rho_t = \max \{w\beta^{-1}, \pi(d_t; z, w)\} \quad \forall 1 < t < T
\]

This intuition for this result is simple: postponing consumption weakens incentive compatibility constraints. Note that the consumption strategy implied by this result may exhibit superfluous patience. It is enough to postpone consumption up to the point where the non-default constraint no longer binds, which may occur strictly before the last period. But for simplicity and without any loss of generality, I restrict my attention to contracts that satisfy the two properties of lemma 3. Under this convention, the value of default in a given period \( t < T \) only depends on the advance \( d \) of consumption good and is given by:
\[
V_t^D(d; z, w, \theta) = \beta[\theta V_{t+1}^S(0; z, w) + (1 - \theta)V_{t+1}^S(\pi(d; z, w) + d\beta^{-1}; z, w)]
\]

Observe that the value of default for a given advance \( d \) decreases with \( t \), since \( V_{t+1}^S(\pi(d; z, w) + d\beta^{-1}; z, w) \) does. As a result, agents are more likely to become managers as they become older. The following result records this observation, and points out that agents born with higher bequests are also more likely to become managers.

Proposition 4. Assume that \( \{d_t, \rho_t\} \) is a competitive contract for agents of type \((b, z)\). Then,

1. \( d_{t+1} > 0 \implies d_{t+1} \geq d_t \) for all \( t \in \{1, T-2\} \);

2. \( \{d_t, \rho_t\} \) is feasible for agents of type \((b', z)\) for all \( b' \geq b \).
In the first part of the proposition, the premise $d_{t+1} > 0$ is necessary because it is possible for an agent of type $z$ to be indifferent between the two occupations for two consecutive periods. It should be obvious that this may only happen when $\pi(d^*(w, z); w, z) = w$. In that case, an agent could choose to become a manager in a period, but a worker in the next, which would yield a decreasing advance sequence. But in the generic case, proposition 4 implies that the optimal occupational profile is characterized by an age threshold. Once agents become managers, they remain managers for the rest of their life. It also implies that managers run larger establishments as they grow older. Therefore, establishment growth takes the form of convergence towards their optimal scale. The second result says that, \textit{ceteris paribus}, agents born with larger bequests will be advanced more funds in every period, become managers earlier and run larger plants at every age. Put another way, proposition 4 says that limited enforcement disrupts the allocation of resources in two ways. First, talented agents may have to become unskilled workers because they are unable to borrow enough funds while less talented but older or wealthier agents become managers. In addition, managers are generally constrained to operate an establishment at a sub-optimal scale.

I will now characterize the impact of changes in $\theta$ on the set of competitive contracts. Let $V^*(b, z; w, \theta)$ denote the maximum discounted lifetime income for an agent of type $(b, z)$ given $w$ and $\theta$, while, as before, $V^*_1(b; z, w)$ is the lifetime income an agent can obtain without outside financing. Clearly, $V^*(b, z; w, \theta) \geq V^*_1(b; z, w)$, and $V^*(b, z; w, \theta) - V^*_1(b; z, w)$ can be thought of as measuring the extent to which agents rely on outside financing. When $\theta = 0$, agents who decide to default are excluded from future borrowing but face no other cost. The next result says that under those circumstances no outside lending can be supported, as in Bulow and
Rugoff (1989). It also makes note of the fact that as $\theta$ rises, the availability of outside financing rises monotonically.

**Proposition 5.** For all $(b, z; w) \in \mathbb{R}_+ \times [0, 1] \times \mathbb{R}_{++},$

1. $V^*(b, z; w, 0) = V^S_1(b; z, w),$

2. $V^*(b, z; w, \theta)$ rises with $\theta.$

I now turn to characterizing steady state equilibria.

### 3.3 Steady state equilibria

#### 3.3.1 Definition

A steady state equilibrium is a value $w > 0$ for the unskilled wage, a distribution $\nu$ of initial bequests, and financing contracts for each agent such that almost all financing contracts are competitive, the labor market clears, and the distribution of bequest is invariant. To make this more precise, for $t \geq 0$ denote by $\eta_t(d; b, z)$ the mass of agents of age $t,$ initial bequest $b$ and managerial ability $z$ whose contract stipulates an advance $d \geq 0$ of the consumption good. In equilibrium, labor markets must clear i.e.:

$$\sum_{t=1}^{T-1} \int n(d; b, z) d(\eta_t \times \nu \times \mu) - \sum_{t=1}^{T-1} \int \{(d, b, z): d=0\} d(\eta_t \times \nu \times \mu) = 0$$

(5)
The first term in equation (5) is the aggregate demand for labor, while the second term gives the aggregate supply of unskilled labor. In addition, the distribution of bequests must be constant, i.e., for any Borel subset $B \subset \mathbb{R}_+$, the following condition holds:

$$\mathbf{(1-\phi)1}_B\{0\} + \phi \int_{\{\gamma \beta^{T-1}V^*(b,z,w,\theta) \in B\}} d(\nu \times \mu) = \nu\{B\}$$

(6)

### 3.3.2 Comparative statics

The remainder of this paper is devoted to comparing steady state equilibria in economies that differ only in the degree $\theta$ to which financing contracts can be enforced. For a given $\theta$, the mapping from unskilled wages to the excess demand for unskilled labor can be decomposed into two separate mappings. The first maps the wage $w$ to the unique distribution $\nu_w$ of bequests that satisfies (6). The second operator maps this distribution into the set of corresponding values for the aggregate excess demand for unskilled labor. In appendix A.2, I show that $\nu_w$ is the unique fixed point of strongly convergent Markov process. After guessing $w$ and solving for competitive contracts, both integrals in (5) can then be computed by applying the law of large numbers.

It seems natural to conjecture that an increase in the efficiency with which contracts are enforced has a positive impact on labor productivity. Indeed, let $w$ be the steady state

---

9 The labor demand function $n$ given a agent’s type and the financing with which they operate is defined in section 3.2.1.

10 Here, $1_B$ denotes the indicator function corresponding to set $B$.

11 The basic algorithm I use in my quantitative exercises consists of updating $w$ until this excess demand is approximately zero. I increase $w$ when there is an excess demand for labor, decrease it otherwise. In all cases, I also carry out a global grid search over the relevant range of prices to verify that the steady state equilibria I report are unique.
unskilled wage when the probability of enforcement is $\theta \in [0, 1]$. Assume now that $\theta$ rises to $\theta'$. The set of competitive contracts corresponding to $\theta$ and $w$ remains feasible when $\theta$ rises. Therefore, the unique stationary invariant bequest distribution under $(w, \theta')$ first order stochastically dominates the equilibrium distribution of bequests under $(w, \theta)$. By proposition 4, it now follows that there must be an excess demand for labor at $w$ and one would expect the unskilled wage to increase. In the special case where $\phi = 0$, one can show analytically that economies where contracts are better enforced are indeed more productive:

**Proposition 6.** Assume $\phi = 0$.

1. A steady state exists for all $\theta \in [0, 1]$

2. Assume that $w$ is a steady state wage given $\theta \in [0, 1]$. Then for any $\theta' > \theta$ a steady wage $w'$ exists with $w' \geq w$.

The opportunity cost of becoming a manager thus tends to rise when $\theta$ rises and, consequently, one would expect the equilibrium proportion of managers to decrease and the average size of establishments to rise. However, changes in $\theta$ and the unskilled wage have non-trivial effects on the quantity of funds managers are able to borrow, both directly and via their impact on the equilibrium distribution of bequests. Furthermore, the optimal employment-size of establishments decreases with the unskilled wage. The next section describes the result of these potentially conflicting forces via numerical methods.
4 Quantitative results

4.1 Parameters

I begin this section by selecting exogenous parameters so that the economy described in this paper generates steady state statistics that match the relevant U.S. statistics. I set $T = 6$ so that assuming a productive life of 40 years, each period corresponds to 8 years.\footnote{Recall that agents do not work in the last period of their life. The computational complexity of my exercise rises quickly as $T$ rises. While raising $T$ up to 6 periods yields noticeably different outcomes, raising $T$ further appears to affect my results only marginally.} I assume $\beta = 0.79$ which implies a yearly real interest rate of roughly 3 percent. As for altruism parameters, Leitner and Ohlson (2001) find that 35 percent of respondents both of whose parents are dead report receipt of a positive inheritance in the 1984 Panel Study of Income Dynamics (PSID). I therefore set $\phi = 0.35$. In that year, the PSID included data on cumulative inheritances. In present value terms (with a yearly discount rate of 3 percent), the average per capita amount inherited conditional on receiving a positive inheritance, represents approximately 5 percent of individuals’ mean lifetime earnings in the PSID sample. When $\gamma = 0.04$, my economy matches this observation.

Turning now to technological parameters, assume first that there are no contractual imperfections ($\theta = 1$). Then $\alpha_k$ is the share of capital income in GDP, approximately a third in the U.S. Similarly, when $\theta = 1$, the managerial share of GDP is $1 - \alpha_n - \alpha_k$. This managerial share could be set to match the ratio of proprietor’s income to national income, approximately 9% for the 1960-85 period in the U.S. But this ratio is a poor measure of what the managerial share represents in my model since corporations constitute the leading form of ownership in
the U.S., and corporate data does not enable one to measure the share of value added that accrues to fixed managerial inputs. I choose instead to use sole proprietorship data for all years for which data is available from the IRS in the 1979-1992 period. To measure $1 - \alpha_n - \alpha_k$, I assume that $\alpha_k$ is the share of net income that remunerates the sole proprietor’s own capital, while $1 - \alpha_k$ is the share remunerating their managerial input. I find that for manufacturing sole proprietorships in the U.S., the ratio \( \frac{(1-\alpha_k)\text{Net income}}{\text{Value added}} \) averages to 14.8 percent, where value added is business receipts minus cost of sales and operation, excluding cost of labor. I thus set $\alpha_k = 0.33$ and $1 - \alpha_k - \alpha_n = 0.85$.\(^{13}\)

Naturally, $\theta = 1$ is not an adequate assumption for the U.S., and below I will argue for a value $\theta$ strictly smaller than 1. After making that change however, I find that the capital and managerial shares of income in steady state do not change much, and, correspondingly, leave technological parameters unchanged.\(^{14}\) As for $\delta$, the depreciation rate, I set it to 0.44 (that is, 7 percent yearly) which yields a capital to (yearly) GDP ratio of roughly 3.20, near standard estimates of this ratio in the U.S.

There remains to specify the distribution of managerial talent, and the degree $\theta$ to which contracts can be enforced. Given other exogenous parameters, I choose those two parameters to jointly match the U.S. distribution of manufacturing establishments across size categories, and the average rate of growth of manufacturing establishments. The specific procedure is described in appendix A.3 and the calibrated distribution of talent is shown in figure 1. Setting

---

\(^{13}\)Interestingly, Atkeson and Ohanian (1996), using very different calibration arguments, arrive at the same degree of strict concavity of the production function. In my calibration approach, an implicit assumption is that different ownership types are not associated with systematically different technological opportunities.

\(^{14}\)Furthermore, the results I present in this paper are not sensitive to even large changes in technological parameters.
\( \theta = 0.72 \), together with the distribution of managerial talent shown in figure 1, approximately yields the desired average growth and size distribution of establishments. In the U.S., large manufacturing establishments represent a small fraction of the total number of establishments, but account for most employment. For my economy to be consistent with this feature, \( \mu \) must assign positive mass to an outlying set of highly talented managers, as shown in figure 1.

I will now ask whether \( \theta \) can be found so that instead of an economy where the size distribution of establishments resembles the U.S., the model economy generates a distribution similar to the distribution one observes in Argentina, Mexico or other developing nations. Obviously, developing economies differ from the U.S. economy in more than one respect. The goal of the exercise, however, is to focus on the quantitative importance of contractual imperfections.

### 4.2 Impact of limited enforcement

Figure 2 plots various steady state statistics as a function of the degree \( \theta \) to which contracts can be enforced. Economies with better enforcement technologies are richer (aggregate output rises with \( \theta \), panel A) and more productive (\( w \), the marginal product of labor also rises with \( \theta \), panel B.)

As discussed before proposition 5, limited enforcement disrupts the allocation of productive resources by limiting the quantity of funds available to managers. To see this quantitatively, define the proportion of self-financing for a given contract by \( s_1 = \frac{b}{d_1} \) and, for \( t > 1 \),

\[
s_t = \sum_{i=1}^{t-1} \frac{(1 + r)^{t-1-i} \rho_i}{d_t}
\]
This gives the proportion of the funds employed by managers that could be financed with past income. Let $\bar{s}$ be the economy wide average of this ratio, in steady state and define the average proportion of outside financing by $1 - \bar{s}$. This measure of average outside financing is plotted in panel C. Managers operate with more outside financing and, therefore, operate close to their optimal scale of operation as $\theta$ rises. In fact, the average size of establishments rises with $\theta$ (panel D) while, at the same time, the optimal scale of operation of a given establishment falls since the wage rises. Richer economies, in this model like in the data, emphasize large scale production. They also operate at a higher capital to output ratio (panel E.) This is largely because the wage rate rises with $\theta$, but is also due to the force described in lemma 2. Even at a constant wage, managers who are less financing constraint employ more capital because the relative price of capital falls with the shadow price of financing.

Note that the average growth rate of plants between their first and second period of existence (panel F) is relatively stable until roughly $\theta = 0.7$ but falls sharply past that point. In fact, past $\theta = 0.9$ all establishments are operated at their optimal scale in all periods, and therefore do not grow. The sharp fall in growth rates around $\theta = 0.7$ coincides with the tapering off of the average size of establishments. To understand why average size stops rising, note that when $\theta$ goes up the fraction of agent who choose to become managers (which is inversely related to the average size of establishments) is affected by two forces. The wage goes up, which all else equal means that fewer agent can profitably operate an establishment, but all agents have a better access to outside financing. For low values of $\theta$ large rises in the wage are the dominating effect. When $\theta$ approaches one, the wage effect becomes small and the average size of establishments begins to fall, albeit slightly.
The bottom two panels of chart 2 describe the impact of limited enforcement on the distribution of bequests and lifetime income. The degree of bequest inequality tends to decrease with \( \theta \) as does inequality in lifetime earnings. Intuitively, when the wage is low, the income difference between workers and managers is large. But the relationship between limited enforcement and inequality is not monotonic: inequality initially rises with \( \theta \). When \( \theta = 0 \), all production is self-financed, and even managers of high talent are unable to generate much income, despite the fact that wages are low. But the availability of outside financing rising steeply with \( \theta \), and the gini coefficient begins to fall very early.

4.3 Size distribution of manufacturing establishments

The model is broadly consistent with the fact that poor economies emphasize low scale production, but I now subject the model to a more demanding quantitative test. When \( \theta \) is selected so that the average scale of operation resembles what one observes in Argentina and Mexico, does the distribution of employment also resemble what one observes in those countries? The answer turns out to be positive.

When \( \theta = 0.2 \) the average size of establishments matches the average size of manufacturing establishments in Mexico in 1988 (approximately 19 employees per establishment.) Table 3 compares the simulated distribution of establishments and employment across size categories when \( \theta = 0.2 \) to its empirical counterpart shown in table 1. The model, given its parsimony, generates distributions that are remarkably close to Mexican data. When \( \theta = 0.1 \), the average size of establishments is now about 10 employees per establishment, which is near its empirical
Table 3: Size distribution of establishments

\[ \theta = 0.72 \]

<table>
<thead>
<tr>
<th>Employment size</th>
<th>[0,10]</th>
<th>(10,50]</th>
<th>(50,250]</th>
<th>&gt;250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of establishments</td>
<td>47.4</td>
<td>33.3</td>
<td>15.5</td>
<td>3.8</td>
</tr>
<tr>
<td>Percent of employment</td>
<td>3.6</td>
<td>14.6</td>
<td>31.0</td>
<td>50.8</td>
</tr>
</tbody>
</table>

\[ \theta = 0.20 \]

<table>
<thead>
<tr>
<th>Employment size</th>
<th>[0,10]</th>
<th>(10,50]</th>
<th>(50,250]</th>
<th>&gt;250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of establishments</td>
<td>85.6</td>
<td>9.6</td>
<td>3.6</td>
<td>1.2</td>
</tr>
<tr>
<td>Percent of employment</td>
<td>18.1</td>
<td>14.1</td>
<td>24.4</td>
<td>43.4</td>
</tr>
</tbody>
</table>

\[ \theta = 0.10 \]

<table>
<thead>
<tr>
<th>Employment size</th>
<th>[0,15]</th>
<th>(15,50]</th>
<th>&gt;50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of establishments</td>
<td>93.7</td>
<td>4.1</td>
<td>2.2</td>
</tr>
<tr>
<td>Percent of employment</td>
<td>32.3</td>
<td>12.3</td>
<td>55.4</td>
</tr>
</tbody>
</table>

counterpart for Argentina in the mid-80’s. The simulated distribution now resembles the data from Argentina’s 1988 household survey shown in table 1 in that about a third of employment is found in establishments with 15 employees or fewer.

4.4 Does wealth redistribution alleviate contractual imperfections?

The previous experiments suggest that improving the enforcement of property rights in nations where they are poorly enforced has the potential to markedly raise income per capita. But improving legal institutions is costly in nations where the bureaucracy is inefficient, and fiscal resources are limited. A question of practical interest therefore, is whether wealth redistribution policies can serve as a partial substitute for investments in property rights enforcement.

\[ ^{15} \] That average was 10.7 employees in 1985, 8.5 employees in 1993. Argentina carries out an economic census once a decade.
Table 4: Impact of bequest inequality, $\theta = 0.2$

<table>
<thead>
<tr>
<th></th>
<th>No wealth redistribution</th>
<th>Wealth redistribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>0.79</td>
<td>0.80</td>
</tr>
<tr>
<td>Output</td>
<td>8.10</td>
<td>8.11</td>
</tr>
<tr>
<td>Average size of plants</td>
<td>19.26</td>
<td>18.91</td>
</tr>
<tr>
<td>Plants with fewer than 10 emp. (%)</td>
<td>87.83</td>
<td>87.32</td>
</tr>
</tbody>
</table>

Intuitively, since bequest size is assumed independent of managerial talent, bequest inequality potentially amplifies the disruption in the allocation of resources that occurs when enforcement is limited. Wealthy agents become managers while managers with less inherited funds, but more talent, are constrained to become unskilled workers. Consider therefore the following bequest redistribution experiment when $\theta = 0.2$. Assume that bequests are pooled and redistributed evenly. Assume further (for simplicity and the to give this experiment the greatest chance to have a large aggregate effect) that this does not affect the bequest policy of altruistic agents. The results are summarized in the second column of table 4. While the impact on output is slightly positive, it is very small. That is, even drastic wealth redistribution policies do very little to alleviate the impact of limited enforcement.\(^{16}\)

5 Conclusion

This paper describes and measures the impact of the imperfect enforceability of financing contracts in a dynamic, general equilibrium model. Limited enforcement distorts the allocation

\(^{16}\)Even a policy such that only agents who choose to become managers receive a bequest raises output and labor productivity very little. Those results are available upon request.
of productive resources across establishments by imposing borrowing constraints on managers. As a result, labor productivity, output, and the average size of establishments all rise when the enforcement of contracts improves, while wealth and income inequality fall.

Calibrated numerical simulations indicate that limited enforcement can account for much of the documented differences in the size distribution of establishments between Latin America and the U.S. Many developing nations lack a well-functioning, effective judicial system and property rights are not adequately enforced. The results presented here indicate that these imperfections have an large impact, particularly on the organization of production. In addition, I find that wealth redistribution schemes are not a good substitute for improving the degree to which contracts are enforced.

In more general terms, my results suggest that contractual imperfections are important for economic development. All the economies I consider operate with the same amounts of human and physical resources. But in economies where contracts are poorly enforced, the misallocation of this resources reduces aggregate income by as much as 50 percent. The magnitude of these effects makes contractual imperfections a promising building block for a theory of total factor productivity.
A Appendix

A.1 Proofs

A.1.1 Proof of lemma 2

Proof. First order conditions for profit maximization imply
\[ \frac{\alpha_k n}{\alpha_n k} = \frac{\lambda + \delta + r}{w(\lambda + 1 + r)} \]
where \( \lambda \geq 0 \) is the Lagrange multiplier associated with the financing constraint. One easily shows that \( \lambda \) decreases with \( a \), i.e. that \( \pi \) is concave in \( a \). Since \( \delta < 1 \), the equality above now implies that \( \frac{k}{n} \) decreases with \( a \) as claimed.

A.1.2 Proof of lemma 3

Proof. Consider a competitive contract such that, say, feasibility condition (1) does not bind. Increase \( \rho_1 \) until the condition binds and decrease \( \rho_T \) until feasibility condition (3) binds. The advance profile stipulated by the competitive contract remains feasible since the left-hand side of every equation in feasibility condition (4) rises. Furthermore, since the rates of interest and time preference coincide, this leaves the agent’s lifetime income unchanged so that the altered contract is a competitive contract as well.

A.1.3 Proof of proposition 4

Proof. To see the first item, consider without loss of generality the payment profile described in lemma 3. Since all consumption is postponed until the last period, the opportunity cost of default grows at the rate of interest \( \beta^{-1} - 1 \). On the other hand, the value of defaulting with a given advance of funds decreases as the agent ages. Therefore, the maximum advance compatible with the no-default constraint is non-decreasing. As for the second item, note that any contract feasible for a given pair \((b, \theta)\) remains feasible when \(b\) or \(\theta\) rises.

A.1.4 Proof of proposition 5

Proof. Let \( \theta = 0 \) and consider, without loss of generality, contracts that satisfy the conditions of lemma 3. Note that feasibility condition (3) must bind at any competitive contract so that \( V^*(b, z; w, 0) = \sum_{i=1}^{T-1} \beta^{-(T-i)} \rho_i \). Assume by way of contradiction that \( d_{T-1} \)
\[
\sum_{i=1}^{T-2} \beta^{-(T-2-i)} \rho_i.
\]
Then,
\[
V_{T-1}^D(d_{T-1}; z, w, 0) = \pi(d_{T-1}, z, w) + d_{T-1}\beta^{-1} \\
> \rho_{T-1} + \sum_{i=1}^{T-2} \beta^{-(T-2-i)} \rho_i \\
= \beta^{T-2}V^*(b, z; w, 0)
\]
so that feasibility condition (4) is violated in period \(T - 1\). One can then proceed by induction to show that no net lending can take place in any period of the contract. The second item owes to the fact that any feasible contract remains feasible when \(\theta\) rises.

\[\square\]

**A.1.5 Proof of proposition 6**

Proof. Since \(\phi = 0\) we have \(\nu\{0\} = 1\), and an agent’s type is summarized by their managerial talent. By the theorem of the maximum, the set of competitive contract for each type is upper-hemi continuous (u.h.c) in \(w\). Because different agents of a given type can be offered different contracts, and \(\mu\) has finite support, the left-hand side of equation (5) is the sum of a finite number of u.h.c, non-empty and convex correspondences, and so is also u.h.c. in \(w\), non-empty and convex. As \(w\) becomes large, this left-hand side can be made negative.

On the other hand, this left-hand side becomes positive when \(w\) is small. To see this, note that in the second period of their life, agents of age 2 can operate with at least financing \(w(1 + r)\). With financing \(w(1 + r)\), managers can finance inputs \(\frac{1 + r}{2}\) of labor and \(\frac{w(1 + r)}{2}\) of capital. Therefore,
\[
\pi(w; z; w) > F\left(\frac{w(1 + r)}{2}, \frac{1 + r}{2}, z\right) - w(1 + r) \\
> Az\left(\frac{1 + r}{2}\right)^{\alpha_k + \alpha_n} - w(1 + r)
\]
Because \(\alpha_k < 1\) it now follows that \(\pi(w; z; w) > w(1 + r)\) for \(w\) small enough. Furthermore, it should be clear (and simple manipulations of first order conditions show) that as \(w \downarrow 0\), \(n \rightarrow w(1 + r)\) so that for \(w\) small enough, \(n > 1 + \frac{r}{2}\). But this implies that for \(w\) small enough, all agents of age 2 or more become managers, and they all hire more than one unit of labor, so that the excess demand for labor must be positive.

A standard application of Kakutani’s fixed-point theorem now shows that a value \(w\) can be found such that labor markets clear. Since \(\phi = 0\), the distribution of bequests puts all mass at zero and is trivially invariant. Therefore, we have obtained the desired steady state.

Now consider raising \(\theta\) to \(\theta'\). Given \(\theta\) and \(w\), I have shown that optimal contracts can be found so as to satisfy (5). When \(\theta\) rises to \(\theta'\), holding \(w\) fixed, agents become managers earlier, and employ more workers in every period. Therefore, at \(w\), optimal contracts can be found that make the left-hand side of equation (5) positive. The same continuity argument as above now implies that a new steady state wage \(w'\) must exist with \(w' > w\).  

\[\square\]
A.2 Steady state bequest distribution

The excess demand for labor correspondence can be defined as the composition of two mappings. The first mapping takes the wage \( w \) into the unique distribution of bequests satisfying the third condition of my equilibrium definition. The second maps this distribution into the set of corresponding values for the aggregate excess demand of labor.

Consider the first mapping. For each \( w > 0 \), \( V^* - b \) is bounded above uniformly across individual types since \( \mu \) has compact support. Let \( \bar{V} \) be such an uniform bound and define \( g(b, z; w, \theta) \equiv \gamma \beta^T V^*(b, z; w\theta) \). Then, for all \((b, z) \in \mathbb{R}^+ \times [0, 1],\)

\[
g(b, z; w, \theta) < \gamma \beta^{-(T-1)}(\bar{V} + b\beta^{-1}).
\]

Since \( \gamma \beta^{-T} < 1 \) by assumption 1, there exists \( \bar{b} \in \mathbb{R}^+ \) such that for all \( b \in [0, \bar{b}] \), \( g(b, z; w, \theta) \in [0, \bar{b}] \). Now let \( B[0, \bar{b}] \) be the set of all Borel measures on \([0, \bar{b}]\) and define \( T_w : B[0, \bar{b}] \to B[0, \bar{b}] \) by,

\[
(T_w \lambda)(B) = (1 - \phi) \lambda(0) + \phi \int_{0 < z < w} d(\nu \times \mu).
\]

The operator \( T_w \) describes the evolution of the distribution of bequests in this economy under the assumption that the unskilled wage is fixed at \( w \). Now observe the following:

**Remark 7.** For all Borel subset \( B \in [0, \bar{b}] \), and \( b \in [0, \bar{b}] \), \( (T_w \chi_b)(B) > 1 - \phi \) or \( (T_w \chi_b)(B^c) > 1 - \phi \)

where for all \( b \in \mathbb{R} \), \( \chi_b \) denotes the point-mass distribution that puts all mass at \( b \) while \( B^c \) denotes the complement of \( B \) in \([0, \bar{b}]\). To see this, note that from all \( b \in [0, \bar{b}] \) the Markov process goes to zero with probability \( 1 - \phi \). Since \( 0 \in [0, \bar{b}] = B \cup B^c \) the result trivially holds.

Theorem 11.12 in Stokey et al. (1989) now implies:

**Lemma 8.** For all \( w > 0 \), the Markov chain defined by \( T_w \) is geometrically ergodic. That is, there exist a distribution \( \nu_w \in B[0, \bar{b}] \) and a constant \( 0 < \epsilon < 1 \) such that for all \( b \in [0, \bar{b}] \),

\[
\| T^n \chi_b - \nu_w \| \leq \epsilon^n \| \chi_b - \nu_w \|.
\]

In the statement of the lemma, \( \| . \| \) denotes the total variation norm. Finally, the Markov process defined by \( T_w \) satisfies a law of large numbers.

**Proposition 9.** For \( w > 0 \) let \( \{b_n\}_{n=0}^{\infty} \) be any run of the Markov process defined by \( T_w \). Let \( f \) be a \( \nu_w \)-integrable function. Then

\[
\frac{1}{n} \sum_{i=0}^{n} f(b_n) \longrightarrow a.s. \int f(b) d\nu_w
\]

**Proof.** Pick any \( B \in [0, \bar{b}] \) such that \( \nu_w \{ B \} > 0 \). For all \( b \), let \( P \) be the transition probability function implied by \( T_w \) and let \( P^n \) be the n-fold composition of this function. Since \( \nu_w \{ B \} > 0 \), lemma 8 and remark 7 imply that for some integer \( n \), \( P^n(0, B) > 0 \). It must then be the case that \( b_n \in B \) infinitely often with probability one. The result now follows from Theorem 3 in Tierney (1994). \( \square \)
A.3 Calibration of the distribution of managerial talent

In this appendix, we select $\theta$ and specify $\mu$ so that, given other exogenous parameters, the average growth rate and the size distribution of establishments match their empirical counterparts in the U.S. manufacturing sector.

Assume first that $\theta = 1$. In that case, a manager’s talent and the employment-size of the establishment he operates are monotonically related. Indeed, let $w$ be the equilibrium wage. First order conditions for unconstrained net income maximization for a manager of talent $z$ imply:

$$zn^{\alpha_k + \alpha_n - 1} \propto w(1 + r),$$

so that employment is linear in $z^{1-\alpha_k-\alpha_n}$. Now observe that the model implies a lower bound on establishment size in this economy. Indeed, first order conditions for profit maximization imply that the manager’s net income is given by:

$$(1 - \alpha_k - \alpha_n)Az^{\alpha_k n^{\alpha_n}} = \frac{1 - \alpha_k - \alpha_n}{\alpha_n}nw(1 + r)$$

An agent will become a manager only provided this income exceeds the unskilled wage, that is:

$$\frac{1 - \alpha_k - \alpha_n}{\alpha_n}nw(1 + r) \geq w(1 + r)$$

or equivalently,

$$n \geq \frac{\alpha_n}{1 - \alpha_k - \alpha_n}$$

Thus the distribution of establishments implied by my model given the distribution of talent will be truncated at $\frac{\alpha_n}{1 - \alpha_k - \alpha_n}$ approximately 3.5 employees given the values I set for technological shares.

In the Census Bureau’s County Business Patterns survey, manufacturing establishments are classified in nine employment size categories: 1 to 4 employees, 5 to 9, 10 to 19, 20 to 49, 50 to 99, 100 to 249, 250 to 499, 500 to 999, and 1000 or more employees. Because of the lower bound implied by my model, I combine the first two size categories. An initial guess for $\mu$ is then obtained as follows. Assume that $z^{1-\alpha_k-\alpha_n}$ is log-normally distributed with location and dispersion parameters $\lambda_1$ and $\lambda_2$, respectively. Under the assumption that $\theta = 1$, the distribution of establishments implied by my model is then log-normally distributed and left-truncated at $\frac{\alpha_n}{1 - \alpha_k - \alpha_n}$. The two parameters of the distribution can then be selected by maximum-likelihood so that the implied distribution approximates its empirical counterpart in the 1988 County Business Pattern Survey. This gives me a continuous guess $\tilde{\mu}$ for the distribution of managerial talent, and a truncation point $z^*$ such that agents become managers only if $z > z^*$. To obtain a discrete guess, I assume that the support of $\mu$ consists of 4 points to the right of $z^*$ and forty equally spaced points to the left of $z^*$. Then letting $\{z_i\}_{i=1}^{44}$ be the support of $\mu$, I let $\mu\{z_i\} = \tilde{\mu}(\frac{z_i+z_{i+1}}{2}) - \tilde{\mu}(\frac{z_i+z_{i-1}}{2})$ for $i \in \{1, 44\}$, where $z_0 = 0$ ans $z_{45} = 1$. 

29
The four points to the right of \( z^* \) are chosen to match the average size of manufacturing establishments in the U.S. in each of the 4 categories shown in table 1.

This gives me a starting guess for \( \theta \) and \( \mu \). That guess gives an approximately correct distribution of establishments for the U.S., but, counterfactually, no establishment growth since \( \theta = 1 \). Evans (1987) estimates with data from 100 manufacturing industries between 1976 and 1980 that manufacturing firms with 5 years or less of existence grow at yearly rate of roughly 2.6%. Based on this estimate, I arrive at final guess for \( \theta \) and \( \mu \) by following the following iterative method:

1. Given all other parameters, update \( \theta \) until establishment growth between their first and second period of existence is \((1.026)^8 - 1\) percent on average in steady state.

2. Update \( z_{41} \) until, in steady state, the fraction of establishments with 1 to 9 employees matches the fraction shown in table 1 in the U.S.

3. Similarly update \( z_{42}, z_{43} \) and \( z_{44} \).

I repeat steps (1-3) until \( \theta \) and \( \{z_i\}_{i \geq 41} \) become approximately invariant. With this method, I arrive at \( \theta = 0.72 \) and the distribution of managerial talent shown in figure 1.
Figure 1: Distribution of managerial talent
Figure 2: Impact of $\theta$ on steady state statistics

A: Aggregate Output

B: Wage

C: Average share of outside financing

D: Average employment size of establishments

E: Capital labor ratio

F: Average growth rate of young establishments

G: Lifetime income gini coefficient

H: Bequest gini coefficient
References


Thomas, J.J., “Informal Economic Activity” (LSE handbooks in economics).
