Accounting for Fluctuations in Social Network Usage and Migration Dynamics

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**Research Department
Working Paper 0402**

Federal Reserve Bank of Dallas

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August 4, 2004

[1]We would like to thank Jim Dolmas for his comments. The views expressed are those of the authors and do not represent those of the Federal Reserve Bank of Dallas or the Federal Reserve System.

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Abstract

In this paper, we examine network capital usage and migration patterns in a theoretical model. Networks are modeled as impacting the migration decision in many ways. When young, larger networks reduce the time lost moving from one region to another. In addition networks decrease the time spent searching for a job. Finally, when old, migrants receive transfer payments through the network. We show that the number and properties of steady state equilibria as well as the global dynamics depend crucially on whether the returns to network capital accumulation exhibit constant, increasing, or decreasing returns to scales relative to the level of network capital. With constant returns to scale, migration flows and network capital levels are characterized by either a unique steady state equilibria or by a two-period cycle. The fluctuations in network capital usage exhibited by our model are consistent with recent empirical data regarding the usage of networks by Mexican immigrants. In the case of increasing returns to scale, either there exists a unique, stable steady state equilibria or multiple equilibria which are characterized as either sinks or saddles. When the returns to scale are decreasing, there exists a unique, stable steady state equilibrium. Finally, we show that increasing barriers to migration will result in an increase in the flow of immigrants, contrary to the desired effect, in the constant and increasing returns to scale cases.

1 Introduction

Although many factors influence an individual's decision to migrate, economists have increasingly focused on the important role played by social networks.[1] As Massey *et al.* (1987) discuss, these networks play a crucial role, as potential migrants rely on social networks for information regarding issues such as migration routes, employment opportunities, housing, etc. Over the past two decades there has been a considerable amount of empirical work by Massey and many others (and some theoretical work) exploring in greater detail the interrelationships among networks, migration decisions, border deterrence, and other factors.[2]

However, these works are predicated on the assumption that network usage is increasing over time. Most recently Fussell and Massey (2004) present evidence that suggests that the importance of networks has changed and that network usage may wax and wane. Consequently, the existing theoretical literature on networks is not well suited to account for changes in network importance over time. The narrative approach that motivates much of the current theoretical line of inquiry focuses chiefly on social networks. In this approach, the stock of migrants already in the new region is the social network. Carrington *et al.* (1996) develop a model in which network capital is a perfect complement to the number of existing migrants, which never decreases.

Perfect complementarity, however, precludes an examination in which network capital takes other forms. We take an alternative approach, modeling network capital accumulation as an investment. Although this investment is related to the volume of migrants, it also takes into account the possibility that migrants may choose the quantity of resources to invest in maintaining and improving network infrastructure. Our approach lends itself to a richer dynamic structure that includes both migration flows and network capital accumulation.[3] Consequently, one of the questions we seek to address with this paper is under what conditions will cycles in network capital usage and migration arise. In addition, we are interested in understanding how changes in barriers

[1] For some of the earlier works exploring factors, other than networks, affecting migration, see Harris and Todaro (1970), for rural-to-urban migration; Myrdal (1944) for international migration; and Grubel and Scott (1966) and Berry and Soligo (1969) for across country borders migration. Theses papers, and those which have followed, have helped to shed light not only on migration issues, but also on issues such as development, income convergence, and economic growth.

[2] For recent additional work on the importance of networks in the migration process see Munshi (2003), Massey *et al.* (2002), Orrenius (2001), Helmenstein and Yegorov (2000), Massey *et al.* (1994), Massey (1987), and Massey *et al.* (1987).

[3] This is in contrast to Carrington *et al.* (1996), where the stock of migrants is the only variable that moves through time. Because migrants are infinitely lived and there is no return migration, it is equivalent to treat the stock of migrants as embodying the accumulated network capital. We break this link, separating migration from network capital and treating each as a decision variable.

to migration, for example border patrols or people smugglers, affect the propensity for potential migrants to immigrate and their reliance on networks.

We build on the existing literature in three distinct ways. First, our approach encompasses the notion of network capital forwarded by Carrington *et al.* (1996) and, more recently, in Colussi (2003). The initial state of the network in any given period is dependent on the previous generation of migrants. In addition however, we model network capital accumulation as a technology dependent upon young migrants' time and physical resources as inputs. Both current migrants efforts and the stock of (old) migrants and their other accumulated resources are important determinants to developing and maintaining social networks, which reduce migration costs by providing information about housing stock, transportation, knowledge of employment opportunities, etc. Thus, our modeling of networks is general enough to encompass the concept of networks found in Colussi and Carrington, *et al.,* but at the same time network capital is not a perfect complement to the stock of migrants.[4]

Second, we explicitly model the channels through which networks impact migration. More specifically, we model networks as affecting three distinct aspects of migration: the time spent crossing the border, time spent finding a job after crossing the border, and the quantity of funds remitted to elderly family members.[5] With respect to remittances, network capital serves as an alternative store of value, and thus as a means for executing intergenerational transfers. In short, network capital lowers the cost of migrating when young and serves as a social safety net when old.

Third, we explicitly consider a role for a broad range of activities that affect migration, which we collectively refer to as barriers to migration. We think of these as any activities that impact the costs of migration and examples would include government efforts to deter migration, whether it be from the home region or the host region, and people smugglers.[6] In doing so, our aim is to have a model that encompasses a broad range of migratory experiences, including country-to-country

[4] While we focus on migration in the context of cross-country moves, the model is general enough to address intra-country migration. For instance, the Underground Railroad operated in the U.S. during the middle part of the 19th Century. The migration of blacks during this period would be affected by the network of free blacks living in the states that abolished slavery as well as the investment in building and maintaining the Underground Railroad. Our model could potentially shed new light on flows of blacks from Slave states to Free states during that period.

[5] See Suro (2003) and Massey and Parrado (1994) for papers that examine the importance of remittances by migrants.

[6] There is ample evidence that barriers to migration (both good and bad) exist. For example, with cross-country migration there often times exist border patrols as well as people smugglers. Within a country examples of barriers would include the efforts of Southern states in the pre-Civil War United States to prevent slaves from escaping North and the efforts of the Underground Railroad to facilitate slave's journey to freedom.

Although we allow for any number of barriers to migration, we do not explicitly model any particular one. See Guzman *et al.* (2002) for an example of a cross-country migration paper where both people smugglers and government border enforcement are explicitly modeled as profit maximizing and welfare maximizing agents respectively.

and region-to-region.

In our model, network capital affects the migrant's decision via three channels. Specifically, network capital serves to a) reduce the time spent crossing the border due to border crossing frictions, b) reduce the time spent looking for a job due to job search frictions, and c) increase the remittances received from young migrants when old. The accumulation of network capital when young is aimed at reducing frictions for the next generation of migrants, so that their income from host-region (the region to which individuals migrate) employment will be larger than it otherwise would have been. The payoff to today's generation is that the larger tomorrow generation's income, the more they will remit back to today's generation when old.

A migrant's decision depends on both intratemporal and intertemporal factors. The intratemporal part reflects the migrant's trade-off between dividing time in the home region (from which the migrant is emigrating) and the host region. The intertemporal component captures the decision over how to divide the migrant worker's savings portfolio between a storage technology and network capital maintenance and creation. The return to network capital comes in the form of an old-age remittance. As such, network capital serves as the *de facto* means of executing intergenerational transfers. Young migrants invest in network capital today, receiving a transfer payment from the next generation of migrants. Meanwhile, the next generation of migrants is willing to participate in this intergenerational transfer because they benefit from a larger stock of network capital in the form of lower migration costs and because the next period's young will also participate.[7]

The results of our paper are divided along two lines. First, we investigate the properties of the dynamical equilibria in our model economy. We derive conditions under which there are cycles in both migration flows and the levels of network capital, even though factor payments are constant over time. What is crucial for our results is the scale return to network capital. More specifically, we show that if the return to capital (via remittances) exhibits constant returns to scale in the level of network capital, than equilibrium either consists of a unique steady state equilibria (similar to Carrington *et al.* (1996)) or of a two-period cycle where both the levels of migration and network capital fluctuate between a high and low state. Thus, in a simple dynamical model of migration we can generate fluctuations akin to those currently being observed in the real world. Furthermore, if the return to network capital exhibits increasing returns to scale, than either a unique, stable steady state exists, or if multiple steady state equilibria exist, then some are sinks and others are

[7] Obviously, it is important that there is an infinite sequence of overlapping generations. There are similarities between the participation in network capital and participation in monetary exchange in the overlapping generations economies.

saddles. These steady states have the property that the greater the level of network capital, the less migration which occurs, and vice versa. Finally, if the return to network capital exhibits decreasing returns to scale, then there exists a unique, stable steady state equilibrium. In both the increasing and decreasing returns-to-scale cases, the sinks exhibit monotonic convergence.

Second, we examine the effects that an increase in the barriers to migration would have on the long-run values of migration and network capital. In the case of constant returns to scale, an increase in the barriers to migration results in increased migration flows and larger networks. To compensate for more time spent crossing the border and looking for jobs, migrants spend more time migrating, which leads to an increase in host region wage income and counteracts the impact of the stringent barriers. Increased migration also goes hand-in-hand with greater network capital accumulation, which increases remittance income when old, again offsetting the extra time next generation must spend crossing the border due to increased barriers. A similar result holds in the increasing returns case under certain restrictions. Finally, in the case of decreasing returns, migration could either increase or decrease in response to an increase in the barriers to migration. Interestingly, such barriers result in a larger long-run stock of network capital, which helps to overcome the higher barriers effect on border crossing and job search frictions.

The remainder of the paper is organized as follows. Section 2 describes the economic environment while equilibrium and the laws of motion are characterized in section 3. In Section 4, we derive the dynamic properties of the model economy while section 5 examines the comparative statics for changes in policy parameters We provide a brief summary and conclusion in Section 6.

2 The Model

In our model economy, the physical environment consists of two regions: a home region, from which individuals may choose to emigrate, and a host region, to which individuals immigrate and from which there is no emigration.[8] The economies of both regions are characterized by a standard two-period lived, overlapping generations model with production. Time is discrete and indexed by $t = 0, 1, 2, ...$ In both regions, each generation is composed of a continuum of individuals having unit mass.[9] All individuals, regardless of their region of origin, are identical with respect to their preferences and endowments; they are endowed with one unit of labor when young and nothing when

[8] Our model is generic in the sense that we consider any two distinct geographic regions. As such, one can view these regions as within a given state or country and also as separate countries. Thus our model is able to encompass rural-to-urban, interregional and cross country migration.

[9] There is no loss in generality by assuming that the population of the two regions is identical.

old. Individuals derive utility only from old-age consumption. There is an initial old generation of migrants, who reside in the host region and posses an initial stock of network capital a_0.

Production in both regions is characterized by linear production technologies in which labor is the sole input. It is assumed that individuals in both regions produce regionally homogenous final goods that are not traded between regions. These goods are produced and saved in the individual's first period of life, and then consumed when old.[10] Finally, we assume that final goods produced in the home region are perfect substitutes for final goods produced in the host region. As such, the no-trade assumption is not as constraining as might first appear.

2.1 Home Region

All individuals born in the home region are potential migrants. Each generation of migrants is endowed with one unit of labor when young and nothing when old. Since only old-age consumption is valued, this labor is supplied inelastically when young. The migrant must decide what fraction of her labor time, μ_t, to spend working in the home region and what fraction, $1-\mu_t$, to spend outside the home region. For simplicity, we refer to the time spent outside the home-region, $1-\mu_t$, as migration. The time spent migrating is further divided between three activities: crossing the border, looking for work, and working in the host-region.

The acts of crossing the border and looking for work are time consuming because there exist frictions such as escaping home-region migration restrictions, avoiding host-region border patrols, applying for work permits, looking for work, etc. We let $1-\xi(\cdot)$ denote the fraction of non-home region time spent actually crossing the border, and thus, $\xi(\cdot)$, is the fraction of migration time spent both looking for work and actually working in the host region. Once a migrant successfully reaches the host region, they spend $1-p(\cdot)$ of their remaining time searching for a job and the remainder of their time, $p(\cdot)$, actually working. Thus, the actual amount of time spent working and earning wage income in the host region is given by $\xi(\cdot)\, p(\cdot)\,(1-\mu_t)$.

The fraction of migration time spent looking for work and working, $\xi(a_{t-1},\theta)$, is assumed to depend both on barriers to migration, θ, and on the level of the network to which the migrant has access, a_{t-1}.[11] As the frictions impeding the flow of migrants across regions decline, the fraction

[10] Although a linear, labor-only production function may be simplistic, often times migrants (for example illegal immigrants crossing from Mexico to the United States) are not highly skilled and consequently are employed in low skilled jobs subject to minimum wage requirements. In addition, this simplification allows us to focus specifically on the impact of networks on migration flows – abstracting from other factors which affect migrant's decisions.

[11] While not explicitly modeling the types and natures of potential barriers, we let θ denote the parameter that serves as a proxy for the level of border frictions enacted between the two regions. See Guzman *et al.* (2002) and

of time spent in the host region increases. Thus, a greater level of network capital reduce the time spent crossing the border while increased resources devoted to erecting barriers to migration increases the time spent crossing the border, i.e., $\xi_1 > 0$ and $\xi_2 < 0$. In addition, we assume that regardless of the level of either network capital or barriers, $\xi(a_{t-1}, \theta) > 0$ for all values of $a_{t-1} \geq 0$ and $\theta \geq 0$. Since $\xi(a_{t-1}, \theta)$ is the fraction of migration time spent in the host region, it must also be the case that $\xi(a_{t-1}, \theta) \leq 1$. Also, if no resources are devoted to erecting barriers, then there is no loss of time from crossing the border: $\xi(a_{t-1}, 0) = 1$. Finally, for simplicity, we assume that $0 \leq a_t \leq 1$ for all t.

Once the migrant successfully migrates to the host region, she must spend some time searching for a job. The amount of time spent searching, $1 - p(a_{t-1})$, will depend on the extent of the network to which she has access. The time lost searching for work is positively related to the stock of network capital; that is, $p'(a_{t-1}) > 0$ and we also assume that when the value of network capital equals one, then $p(1) = 1$.[12]

Although the breakdown of the time spent working in the host region into three distinct parts $((1 - \mu_t),\ \xi(a_{t-1}, \theta),$ and $p(a_{t-1}))$ may seem cumbersome, it permits a more explicit approach in terms of the channels through which network capital affects time spent away from the home region. More specifically, we are able to isolate the role that networks play in two very important, but distinct, aspects of any migration: crossing borders and finding a job. In addition, our set-up also allows us to better understand the benefits and limitations of networks vis-a-vis barriers to migration.

Migrants who work in the home region earn a fixed wage w per unit of time spent in home-region production.[13] All income earned in the home region is saved via a simple storage technology. For every unit of output saved at time t, the migrant receives x units of consumption good at date $t + 1$ for all $t \geq 1$. Migrants who successfully cross between regions and find work in the host region earn a fixed wage, w^*, per unit of time spent working. Income received in the host region is divided among three distinct items. First, migrants decide how much of their income to spend on

Guzman *et al.* (2003) for two papers where border frictions are explicitly modeled. Both of these papers allow for border enforcement initiated by the government of the host region and a home-region smuggling industry designed to circumvent enforcement. We abstract from the issued presented in those papers in order to focus explicitly on the role that networks play influencing migrant's decisions.

Finally, we think of the contribution of network capital as consisting of such things as communicating information regarding crossing points, border enforcement and smuggler efficacy.

[12] An example of this might be the case where a sufficient network exists such that previous migrants own businesses in which the new migrants can immediately begin working.

[13] Alternatively, one can think of ω as an endowment which the migrant receives continuously throughout his young period life. Thus, if the migrant choose to stay in the home country for μ fraction of his young life, then she will receive only $\omega\mu$ of the total endowment possible.

continuing and improving the network capital which helped them to migrate. Second, they remit part of their host-region income back to the old generation. Third, migrants save any remaining income via the host region's storage technology. For every unit of host region output saved at time t, the migrant receives y units of consumption good at date $t+1$ for all $t \geq 1$.[14]

We let α, where $0 \leq \alpha \leq 1$, denote the fraction of host region income which is remitted to the migrant's parents. We define N_t as the quantity of resources devoted to maintaining and updating the migrant network. The evolution of the stock of network capital is given by

$$a_t = (1-\mu_t)\left(a_{t-1} + \frac{N_t}{(1-\alpha)\,w^*}\right). \tag{1}$$

Thus, the stock of network capital depends on two factors: the previous stock plus any new investment. In addition the more time spent migrating, the larger the quantity of network capital. The rationale for this specification takes into account the generational investment. Unlike Carrington *et al.* (1996), in which agents are infinitely lived, the generational friction we introduce explicitly takes into account the endogenous depreciation of networks that is tied to the flow of young migrants. So, networks can vanish if members of one generation do not migrate. As such, network capital depends on each generation investing and maintaining the stock. There is a form of endogenous depreciation, reflecting each generation's decision. Moreover, we distinguish between network capital as an entity that is distinguishable from the stock of existing (old) migrants living in the host region. This modeling choice has two noteworthy implications. First, under general conditions we have planar as opposed to a scalar dynamic system. Second, it permits network capital to be the means of intergenerational transfers for migrants.

We can formally write the migrant's problem as

$$\max_{\mu_t, N_t} U\left(c_{t+1}\right) \tag{Migrant's Problem}$$

subject to

$$s_t = \mu_t w + \left[(1-\mu_t)\,\xi\left(a_{t-1},\theta\right) p\left(a_{t-1}\right)(1-\alpha)\,w^* - N_t\right] \tag{2}$$

[14] Remittances to family members play an important role in cross country migration, for example. It is not uncommon for illegal Mexican migrants to come to the US, work in the US for a period of time and send most of the income earned back to relatives and family in Mexico.

and

$$
\begin{aligned}
c_{t+1} &= x\mu_t w + y\left[\left(1-\mu_t\right)\xi\left(a_{t-1},\theta\right)p\left(a_{t-1}\right)\left(1-\alpha\right)w^* - N_t\right] \\
&\quad +\alpha\left(1-\mu_{t+1}\right)\xi\left(a_t,\theta\right)p\left(a_t\right)w^*
\end{aligned}
\tag{3}
$$

Equation (2) characterizes the saving by an agent born at date $t \geq 1$. The first term is the wages earned in the home region, which is simply the product of time spent working in that region, μ, and the wage rate, w. The second the term captures host-country income net of remittances and investment in network capital. Equation (3) represents the quantity of goods available for old-age consumption, including the returns to savings when young — the first two terms on the right hand side of the equation — and remittances received from date-$t+1$ migrants. We assume that $U\left(c_{t+1}\right)$ satisfies all the standard conditions necessary for an interior solution; namely $U\left(0\right) = 0$ and $U'\left(c_{t+1}\right) > 0$.

The solution to the migrant's maximization problem is characterized by the following two equations:

$$
xw = y\left(1-\alpha\right)w^*\left(\xi\left(a_{t-1},\theta\right)p\left(a_{t-1}\right) + \frac{a_t}{\left(1-\mu_t\right)^2}\right) \tag{4}
$$

$$
\frac{y\left(1-\alpha\right)}{1-\mu_t} = \alpha\left(1-\mu_{t+1}\right)\left[\xi_1\left(a_t,\theta\right)p\left(a_t\right) + \xi\left(a_t,\theta\right)p'\left(a_t\right)\right] \tag{5}
$$

Equation (4) is an intratemporal efficiency condition. For a utility maximizing individual, income from spending an additional unit of time in the home region must be equal to the additional income (net of remittances and network contributions) generated from spending additional time in the host region. Equation (5) is an intertemporal efficiency condition and represents the arbitrage condition between the two forms of old-age saving: storage and networks. The left-hand side of this equation represents the old-age income lost by investing an additional unit in network capital as opposed to storage. The right-hand side is the marginal income gained from an additional unit of network capital investment, where the return to network investment takes the form of remittances from the next generation.

2.2 Host Region

The utility maximization problem faced by individuals born in the host region is trivial. Because these individuals do not migrate and because we do not allow for the trade of goods across borders, they simply work in the host region, save any income earned, and consume the proceeds from their host region storage technology savings. As mentioned previously, the production technology in the host region is labor only and also linear in labor inputs. Thus, the host region native's maximization problem is given by

$$\max_{c^*_{t+1}} U\left(c^*_{t+1}\right) \tag{Host Problem}$$

subject to

$$c^*_{t+1} = yw^*.$$

This setup greatly simplifies our equilibrium analysis and allows us to focus on the role of networks in determining the migration dynamics of home region individuals.

3 Equilibrium Laws of Motion

Equilibrium requires that individuals in both regions make optimal choices and that the goods markets in both regions clear. Thus a competitive equilibrium for the economy must satisfy the following

Definition 1 *A competitive equilibrium consists of a sequence of levels of migration,* $\{\mu_t\}_{t=1}^{\infty}$*, network capital stocks,* $\{a_t\}_{t=1}^{\infty}$*, and prices,* $\{w, w^*, x, y\}_{t=1}^{\infty}$ *such that consumers satisfy (Migrant's Problem) and (Host Problem) and goods markets clear in both the home and host regions.*

For the remainder of our analysis, we employ the following functional forms to capture the border crossing frictions

$$\xi\left(a_{t-1}, \theta\right) = a^z_{t-1}\left(1-\theta\right),$$

and job search frictions

$$p\left(a_{t-1}\right) = a^z_{t-1},$$

where $z > 0$.[15] We further assume that the policy parameter, θ, lies in the unit interval. Although these functional forms are specific, what will be important later when discussing the dynamical properties of steady state equilibria is our ability to characterize the product, $\xi\left(a_{t-1}, \theta\right) p\left(a_{t-1}\right)$ (henceforth referred to as the return to network capital), as exhibiting either constant, increasing, or decreasing returns to scale with respect to the existing stock of network capital.

[15] Although technically the border crossing and job search frictions are denote by $1-\xi\left(a_{t-1}, \theta\right)$ and $1-p\left(a_{t-1}\right)$, respectively, we will slightly abuse the terminology and refer to $\xi\left(a_{t-1}, \theta\right)$ as the border crossing frictions and $p\left(a_{t-1}\right)$ as the job search frictions.

After substituting these functional forms into the equilibrium conditions (4) and (5), one obtains

$$xw = y(1-\alpha) w^* \left[a_{t-1}^{2z} (1-\theta) + \frac{a_t}{(1-\mu_t)^2} \right] \tag{6}$$

and

$$\frac{y(1-\alpha)}{1-\mu_t} = \alpha \left(1-\mu_{t+1}\right) 2z a_t^{2z-1} (1-\theta) . \tag{7}$$

Equations (6) and (7) serve as the basis for our analysis of migration dynamics. These equations represent a simple system of two, nonlinear difference equations in two unknowns, namely the level of migration flows, μ_t, and the level of network capital, a_t. When equating the marginal income gains from working in the two regions, equation (6), it is obvious that there is an inverse relationship between building network capital and spending additional time working in the host region, all other things equal.

When equating the cost of additional network capital with the benefits, equation (7), the relationship between building network capital and in which region to spend time depends on whether the return to network capital, $\xi(a_t, \theta) p(a_t)$, exhibits constant, increasing, or decreasing returns to scale with respect to the existing stock of network capital, a_t. Of particular interest will be the case when the return to network capital exhibits constant return to scale, i.e. $z = 1/2$. In this case, equation (7) completely describes the migration flows over time and is independent of the level of network capital. In other words, an additional unit of income invested in the network yields future remittances that are invariant to the size of the network capital stock. The complete dynamics for all returns to scale for both the network capital stock and migration flows are described in the next section.

4 Dynamic Equilibria

To ascertain the existence, number, and dynamical properties of any steady state equilibria, it will necessary to know certain properties about the two equilibrium laws of motion, equations (6) and (7). We begin by examining the case where the return to network capital exhibits constant returns to scale.

4.1 Constant Returns Case

When $\xi(a_t, \theta) p(a_t)$ exhibits constant returns to scale in a_t,equations (6) and (7) can be rewritten as

$$\frac{xw}{y(1-\alpha)w^*} = a_{t-1}(1-\theta) + \frac{a_t}{(1-\mu_t)^2} \tag{8}$$

and

$$\frac{y(1-\alpha)}{1-\mu_t} = \alpha\left(1-\mu_{t+1}\right)(1-\theta). \tag{9}$$

Note from equation (9) that the evolution of migrant flows over time is invariant to the level of, or changes in, network capital.[16] Thus, the two equations can be solved recursively by first solving equation (9) for $\{\mu_t\}_{t=1}^{\infty}$ and plugging these values into equation (8) to ascertain the equilibrium path for $\{a_t\}_{t=1}^{\infty}$.

Rearranging equation (9), one obtains

$$\mu_{t+1} = 1 - \frac{y(1-\alpha)}{\alpha(1-\theta)} \frac{1}{1-\mu_t} \tag{10}$$

In order for equilibrium to exists, it must be the case that there exists some values of μ_t that satisfy the constraint $0 \leq \mu_t \leq 1$. The following assumption guarantees that this indeed is the case.

Assumption 1 *: There exists parameter values for* α, θ, *and* y, *such that* $0 < y(1-\alpha) < \alpha(1-\theta)$.

This assumption is a necessary condition to ensure that $0 < \mu_t < 1$ for all $t \geq 1$. More specifically, if the ratio equals zero, migrants will spend all their time in the home region. In contrast, if the ratio equals one, the migrant will spent all their time in the host region.

Note that the equilibrium law of motion for migration, equation (10), depends on the initial value of migration, which is indeterminate in our model. Thus, the local dynamics governing migration are path dependent. In order to describe the possibilities regarding the dynamic evolution of migration, it will be useful to first describe the steady state equilibrium. Let $\mu_t = \mu_{t+1} = \mu^{SS}$, then the steady state value of migration is given by

$$\mu^{SS} = 1 - \left[\frac{y(1-\alpha)}{\alpha(1-\theta)}\right]^{\frac{1}{2}}.$$

As a result of Assumption 1, μ^{SS} lies within the unit circle. Furthermore, differentiating equation

[16] Recall that whenever we use the term migration or migration flows we are referring to the value of $1-\mu_t$.

(10) yields

$$\frac{d\mu_{t+1}}{d\mu_t} = -\frac{y(1-\alpha)}{\alpha(1-\theta)}\frac{1}{(1-\mu_t)^2} < 0.$$

In addition, it is straight forward to verify that $d^2\mu_{t+1}/d\mu_t^2 < 0$ and the points where equation (10) intersect the horizontal- and vertical-axes are the same and lie within the unit circle. Hence, equation (10) represents a downward sloping curve as depicted in Figure 1**.** The following proposition characterizes the steady state and the dynamics of the system.

Proposition 1 *For $z = 1/2$ and $\mu_0 \leq y(1-\alpha)/\alpha(1-\theta)$ there exists three possible equilibria: a unique steady state equilibria,*

$$\mu^* = \mu^{SS},$$

and a two-period cycle where

$$\begin{aligned} \mu_1^* &= \mu_1 \\ \mu_2^* &= 1 - \frac{y(1-\alpha)}{\alpha(1-\theta)}\left(\frac{1}{1-\mu_1}\right). \end{aligned}$$

The proof of this lemma also follows from simply evaluating the derivative of equation (10) at the steady state. In the constant-returns case,

$$\left.\frac{d\mu_{t+1}}{d\mu_t}\right|_{\mu^{SS}} = -1$$

and thus equation (10) is symmetric about the 45° line. Consequently, for an initial value of μ_1 such that

$$\mu_1 \neq 1 - \left[\frac{y(1-\alpha)}{\alpha(1-\theta)}\right]^{\frac{1}{2}},$$

equilibrium consists of a two period cycle. Finally, it must also be the case that $\mu_1 \leq y(1-\alpha)/\alpha(1-\theta)$. For an initial level of migration such that

$$\frac{y(1-\alpha)}{\alpha(1-\theta)} < \mu_1 \leq 1,$$

then the desired levels of migration for all subsequent periods would be greater than one, a condition not consistent with endowment constraints and thus the existence of equilibrium.

Given the recursive nature of the equilibrium laws of motion (8) and (9), we can now characterize the network capital dynamics, using the results of Proposition 1. Equation (8) can be rewritten as

$$a_t = \left[\frac{xw}{y(1-\alpha)w^*} - a_{t-1}(1-\theta)\right](1-\mu_t^*)^2. \tag{11}$$

In order for this equation to be well defined, it is necessary for $0 \leq a_t \leq 1$. The following assumption guarantees this restriction.

Assumption 2 *There exists parameter values for α, x, y, w, w^*, and θ, such that*

$$yw^*(1-\alpha)(1-\theta) \leq xw \leq yw^*(1-\alpha).$$

Differentiating equation (11) yields

$$\frac{da_t}{da_{t-1}} = -(1-\theta)(1-\mu_t^*)^2 < 0.$$

Thus equation (11) represents as a downward sloping straight line (with slope greater than minus one) and the steady state value of the network capital stock is given by

$$a^{SS}(\mu_t^*) = \frac{\frac{xw}{y(1-\alpha)w^*}(1-\mu_t^*)^2}{1+(1-\theta)(1-\mu_t^*)^2}. \tag{12}$$

Note that the number of steady states will depend on whether the equilibrium value(s) of migration are either a unique equilibrium or represented by a two-period cycle. When the equilibrium level of migration is given by $\mu^* = \mu^{SS}$, then equation (11) is as depicted in Figure 2. When the equilibrium level of migration is given by the two-period cycle

$$\begin{aligned}\mu_1^* &= \mu_1 \\ \mu_2^* &= 1 - \frac{y(1-\alpha)}{\alpha(1-\theta)}\left(\frac{1}{1-\mu_1}\right),\end{aligned}$$

then equation (11) is represented by two separate lines, as shown in Figure 3 (one for each value of μ^*) and the level of network capital alternates between points on each line. More formally, network capital equilibria and their dynamics are as follows.

Proposition 2 *For $z = 1/2$,*

i) if $\mu^* = \mu^{SS}$, then for any initial level of network capital, a_0, network capital converges to

$$a^* = a^{SS}\left(\mu^{SS}\right),$$

and the dynamics are described by damped oscillations.

ii) if equilibrium migration is given by the two-period cycle

$$\begin{aligned} \mu_1^* &= \mu_1 \\ \mu_2^* &= 1 - \frac{y(1-\alpha)}{\alpha(1-\theta)}\left(\frac{1}{1-\mu_1}\right), \end{aligned}$$

then for any initial level of network capital, a_0, network capital converges to a two-period cycle

$$\begin{aligned} a_1^* &= a^{SS}\left(\mu_1^*\right) \\ a_2^* &= a^{SS}\left(\mu_2^*\right). \end{aligned}$$

The dynamics in this case are also described by damped oscillations.

The proof of Proposition 2 follows from two facts. First, the slope of equation (11) satisfies $0 > -(1-\theta)(1-\mu_t^*)^2 > -1$. Since the slope is negative the dynamics will be characterized by oscillations and since the slope is greater than -1, those oscillations will be damped. Thus for any initial value, a_0, network capital will converge to one of the steady state values. Second, the fact that network capital converges either to a unique steady state or converges to a two-period cycle stems from the fact that the return to network capital per migrant is independent of the existing stock of network capital. Thus the only thing that affects the old-age remittance payments is the flow of migrants.

Remark: If the return to network capital accumulation exhibits constant returns to scale, then the remittances received by individuals when old is also invariant to movements in the stock of network capital. The implication is that the (intratemporal) migration decision does not depend on the (intertemporal) savings decision, as both the rate of return on storage and on network capital are fixed. Either migration flows will be constant over time (the steady state equilibrium) or they will be cyclical (two-period cycle equilibria).

It is important to note that the two-period cycle is not the product of an underlying cycle in wage differentials nor cycles in return differentials since both are constant over time in this model economy. Rather, the result owes chiefly to the fact that a given level of network capital can be achieved either by increasing the fraction of productive time spent in the host country *or* by increasing the investment in network capital per young migrant. Note that both are not required since an individual's time and resource investment are substitutes in the network creation process. Consequently, for a given level of network capital investment, N_t, an increase in time spent away from the home region will result in greater network capital.

The intuition regarding the potentially cyclical nature of migration comes straight from the arbitrage condition, equation (9), and specifically how the flow of migration must evolve over time so as to equate the returns to storage and network capital.[17] With an increase in the fraction of time migrating, for example, the quantity of network capital increases and, at the margin, increases the return to network capital at a linear rate tied to the additional time spent migrating. Ultimately however, the total return to network capital is measured in terms of remittances received when old. This, in turn, depends on both the extent of the network built up when young and the extent to which the next generation takes advantage of this network, i.e., whether they make use of it by migrating or not.

It is now possible to illustrate how the two-period cycle can occur in this optimizing model. At date-t, young migrants ascertain the best way to save for old age: either saving via wage income invested in a storage technology or via network capital accumulation. For both assets, accumulation depends on time spent migrating and network capital carried over from the previous generation. In addition, time spent migrating and the beginning-of-period stock of network capital are substitutes with respect to generating wage income and accumulating network capital. Consequently, if date-$t-1$ migrants spent a considerable time migrating, and thus built up a large stock of network capital during the previous period, young migrants at date-t will choose to spend less time migrating. In other words, a young migrant can achieve a given host-region income when young either by spending a larger fraction of her time migrating, or by having a sufficiently large network to overcome the border crossing and job search frictions. Both are not necessary. Similarly, to maintain and build a given size network for next period (and hence a given level of remittances when old), either a large base of network capital is needed or a high level of migration. Again, both are not needed. Thus, the cyclical nature of the equilibria owes to the fact that migration time and the level of network capital with which one begins her life are substitutes in a migrant's decision making process while

[17] As will be seen in the increasing and decreasing returns to network capital cases, the migration flows over time will not depend solely on equating rates of return over saving options.

migration time and the future level of the network capital stock are complements. Hence, both the time spent migrating and the level of network capital will synchronously fluctuate over time.

Recently, Fussell and Massey (2004) presented evidence suggesting that the importance of, and reliance on, networks by illegal immigrants from Mexico has declined slightly. When the return to network capital exhibits constant returns to scale, our model economy can account for this finding; indeed, the two-period cycle of our model predicts that from one generation to the next, we will observe switches from high to low levels of network capital usage. Although there does not exist sufficient data to assert that in the real world network capital formation exhibits a cyclical tendency at generational frequencies, it is noteworthy that our model economy exhibits cyclical properties based on the returns to scale to network capital. Because of the preliminary nature of the evidence, we proceed by considering cases in which there are increasing and decreasing returns to scale in returns to network capital.

4.2 Increasing Returns Case

When the return to network capital, $\xi(a_t, \theta)\, p(a_t)$, exhibits increasing returns to scale, i.e., $z > 1/2$, then the system of equilibrium laws of motion is no longer recursive and determination of equilibria will require equations (6) and (7) to be solved simultaneously. From equation (6), one can obtain the loci of points for which network capital is unchanging over time, and evaluate this loci in μ_t-a_{t-1}space.

Rewriting equation (6) one obtains

$$a_t - a_{t-1} = 0 = \left[\frac{xw}{(1-\alpha)\, yw^*} - a_{t-1}^{2z}(1-\theta)\right](1-\mu_t)^2 - a_{t-1}. \qquad (13)$$

In order for equation (13) to have a solution, it must be the case that

$$\frac{xw}{(1-\alpha)\, yw^*} \geq a_{t-1}^{2z}(1-\theta)$$

for all parameter values and $a_{t-1} > 0$. This condition simply requires that the marginal income gained from spending additional time working in the home region must be greater than the marginal wage income gained from additional time spent in the host region. Assumption 2 and the constraint that $a_{t-1} \leq 1$ guarantee that this condition is satisfied.

We proceed by characterizing the loci of μ_t and a_{t-1} that satisfy equation (13). Differentiating equation (13) yields

$$\frac{da_{t-1}}{d\mu_t}\left[(1-\mu_t)^2\,2za_{t-1}^{2z-1}(1-\theta)+1\right]=-2(1-\mu_t)\left[\frac{xw}{(1-\alpha)\,yw^*}-a_{t-1}^{2z}(1-\theta)\right].$$

Given Assumption 2, the right-hand side of the above equation is always negative and the coefficient on $da_{t-1}/d\mu_t$ is always positive and thus $da_{t-1}/d\mu_t < 0$. Hence, the locus of points at which network capital is constant over time is downward sloping in μ_t-a_{t-1} space (see Figure 4).[18] In addition, it is easy to verify that when $a_{t-1}=0$, equation (13) intersects the horizontal axis at the point $\mu_t = 1$. Furthermore, when a_{t-1} is held constant, then equation (13) indicates that an increase in μ_t must be accompanied by a decrease in a_t. Thus, the phase dynamics are represented by the arrows in Figure 4. Finally, note that nothing in the qualitative evaluation of equation (13) depends on the parameter z. It follows that the characterization of the loci and the phase dynamics apply to both the increasing returns case, when $z > 1/2$, and to the decreasing returns case, when $z < 1/2$.

In contrast to the constant returns case, the second equilibrium law of motion, equation (7) now depends on the stock of network capital. From equation (7), one can obtain the locus of points that satisfy a constant level of migration overtime. Rewriting equation (7) and using equation (6) yields the following equation

$$\mu_{t+1}=1-\frac{y(1-\alpha)}{\alpha(1-\theta)\,2z\left[\frac{xw}{y(1-\alpha)w^*}-a_{t-1}^{2z}(1-\theta)\right]^{2z-1}}\frac{1}{(1-\mu_t)^{4z-1}}.$$

In order for this equation to be well-defined, it is necessary that $0 \le \mu_t \le 1$ for the given parameters. The following assumption guarantees that there exist some values of migration flows that are inside the unit circle.

Assumption 3 *: For all* $0 \le a_{t-1} \le 1$, *there exists parameter values such that*

$$0<\frac{y(1-\alpha)}{\alpha(1-\theta)\,2z\left[\frac{xw}{y(1-\alpha)w^*}-a_{t-1}^{2z}(1-\theta)\right]^{2z-1}}<1.$$

This assumption is a necessary condition to ensure that $0 < \mu_t < 1$ for $t \ge 1$. If the ratio equals

[18] For ease of exposition we have represented equation (13) as a straight line. However, $d^2a_{t-1}/d\mu_t^2$ may be positive or negative depending on the parameter values and the values of μ_t and a_{t-1}. Thus, this line may have several "wiggles" in it. For the purpose of exposition and clarity, we represent equation (13) in all figures as being a linear, downward sloping curve.

zero, migrants will spend all their time in the home region.

The locus of points for which migration levels are unchanging over time is given by

$$\mu_{t+1} - \mu_t = 0 = (1 - \mu_t) - \frac{y(1-\alpha)}{\alpha(1-\theta)\,2z\left[\frac{xw}{y(1-\alpha)w^*} - a_{t-1}^{2z}(1-\theta)\right]^{2z-1}} \frac{1}{(1-\mu_t)^{4z-1}}. \qquad (14)$$

As in the case where network capital was unchanging over time, we are particularly interested in finding the slope of the loci of points that satisfy equation (14). Differentiating this equation with respect to μ_t yields

$$\frac{da_{t-1}}{d\mu_t}\left[\frac{xw}{y(1-\alpha)w^*} - a_{t-1}^{2z}(1-\theta)\right]^{2z-2}(2z-1)(1-\theta)\,z a_{t-1}^{2z-1} = \frac{-y(1-\alpha)}{\alpha(1-\theta)}\frac{1}{(1-\mu_t)^{4z-1}}.$$

The right-hand side of this equation is always negative. Given Assumption 2 and the fact that $z > 1/2$, the coefficient of the derivative on the left-hand side is always positive. Thus, $da_{t-1}/d\mu_t < 0$ and equation (14) represents a downward sloping equation in μ_t-a_{t-1} space. Furthermore, if the return to network capital, $\xi(a_t, \theta)\,p(a_t)$,exhibits low- to moderate increasing returns, (i.e., $1/2 < z \leq 1$) then $d^2 a_{t-1}/d\mu_t^2 < 0$ and equation (14) has the convex shape depicted in Figure 5.[19] In addition, equation (14) intersects the horizontal axis at the point

$$\mu_t = 1 - \left(\frac{y(1-\alpha)}{\alpha(1-\theta)\,2z\left(\frac{xw}{y(1-\alpha)w^*}\right)^{2z-1}}\right)^{\frac{1}{4z}} < 1. \qquad (15)$$

Finally, to characterize the phase dynamics associated with equation (14), consider the case in which the value of μ_t is fixed. An increase in a_{t-1} must be accompanied by a decrease in μ_t. Thus the phase dynamics are represented by the arrows in Figure 5. The following proposition characterizes steady state equilibria and the dynamics when there exist increasing returns to scale.

Proposition 3 *For $1/2 < z < 1$, if there exists any steady state equilibria, then they have the following properties:*

i) if there exists a unique steady state equilibria, then it is a sink,

ii) if there exists two steady state equilibria, then high-migration one is a saddle and the low migration one is a sink.

[19] If $z > 1$, then the sign of $d^2 a_{t-1}/d\mu_t^2$ will depend on the parameter values as well as the values of μ_t and a_{t-1}. In this case it is likely that equation (14) will exhibit "wiggles." However, for the purposes of this paper we focus on the simple case of a convex equation.

The proof of Proposition 3 follows directly from the discussions of equations (13) and (14) above and their respective phase diagrams; Figures 4 and 5.[20] Combining these two figures into a single phase diagram yields Figure 6 and 7. When there is a unique steady state equilibrium, as in Figure 6, then the steady state must be a sink. Note that if there is a unique equilibrium, then equation (14) will always intersect equation (13) from above since equation (14) intersects the horizontal-axis at a point $\mu_t < 1$ whereas equation (13) intersects the horizontal-axis at the point $\mu_t = 1$. When there are two steady state equilibria, as in Figure 7, then the steady state with low migration (high μ) and a low initial level of network capital will be a sink and the high-migration (low μ) and high initial level of capital stock steady state will be a saddle point.

There are three key differences when comparing the increasing returns to scale case to the case with constant returns to scale. First, and most obvious, remittances received when old are no longer invariant to the level of network capital. Investments in network capital by today's young generation will have a larger impact in terms of reducing next generations' border crossing and job search frictions. To the extent that tomorrow's young generation takes advantage of this network, the payoff in terms of remittances for network investment will be larger. Second, because the return to network capital is a non-linear function of the level of network capital, the arbitrage condition can be satisfied with a number of potential combinations of migration and network capital levels. Thus, there exists the possibility for multiple (and greater than two) equilibria depending on parameter values.[21] Third, we find that the equilibrium dynamics are characterized by monotonic convergence to either sinks or saddles. With increasing returns to scale, date-t migration and the beginning-of-period level of network capital are no longer substitutes and date-t migration and end-of-period network capital are not necessarily complements. However, in steady state, migration flows and the level of network capital are complements.

4.3 Decreasing Returns Case

In the case where $z < 1/2$, the return to network capital exhibits decreasing returns to scale. Recall from the analysis the intratemporal efficiency condition in the increasing returns case, equation (13),

[20] Recall that we are assuming that equation (13) is represented by a simple, linear downward sloping equation and equation (14) by a convex, downward sloping curve. If we allow for more general forms of equations (13) and (14) then whenever equation (14) intersects equation (13) from above, the resulting steady state will be a sink. Conversely when equation (14) intersects equation (13) from below, the resulting steady state will be a saddle.

[21] On a more technical level, in the constant returns case, equation (10), evaluated in steady state, would be a vertical line in μ_t-a_{t-1} space. The result of the return to network capital exhibiting increasing returns is that equation (14) becomes downward sloping. In addition, equation (11) is a straight line in the constant returns case and the equivalent line in the increasing returns case, equation (13), may have more curvature and "wiggles."

that neither the qualitative shape of the loci nor the phase dynamics depended on the value of z. As such, the phase diagram depicted in Figure 4 still characterizes equation (13) in the decreasing returns case.

Following the same methods developed in our analysis of the increasing returns case, we rewrite equation (7), using equation (6) to obtain

$$\mu_{t+1} = 1 - \frac{y\left(1-\alpha\right)}{\alpha\left(1-\theta\right)2z\left[\frac{xw}{y(1-\alpha)w^{*}} - a_{t-1}^{2z}\left(1-\theta\right)\right]^{2z-1}} \frac{1}{\left(1-\mu_t\right)^{4z-1}}.$$

The locus of points for which migration levels are unchanging over time is given by

$$\mu_{t+1} - \mu_t = 0 = \left(1-\mu_t\right) - \frac{y\left(1-\alpha\right)}{\alpha\left(1-\theta\right)2z\left[\frac{xw}{y(1-\alpha)w^{*}} - a_{t-1}^{2z}\left(1-\theta\right)\right]^{2z-1}} \frac{1}{\left(1-\mu_t\right)^{4z-1}}. \tag{16}$$

Differentiation this equation with respect to μ_t yields

$$\frac{da_{t-1}}{d\mu_t}\left[\frac{xw}{y\left(1-\alpha\right)w^{*}} - a_{t-1}^{2z}\left(1-\theta\right)\right]^{2z-2}\left(2z-1\right)\left(1-\theta\right)za_{t-1}^{2z-1} = \frac{-y\left(1-\alpha\right)}{\alpha\left(1-\theta\right)}\frac{1}{\left(1-\mu_t\right)^{4z-1}}.$$

The right-hand side is always negative and, in contrast to the increasing returns case, the left-hand side is now negative. Thus, with $z < 1/2$, the μ_t and a_{t-1} that satisfy constant migration over time results in a locus of points that is upward sloping in μ_t-a_{t-1} space. The μ-intercept is given by equation (15); however, the value of the intercept will change to reflect the change in the range of z. Note that for the migration level to be in the unit circle, the following condition must hold:

$$\frac{y\left(1-\alpha\right)}{\alpha\left(1-\theta\right)2z\left(\frac{xw}{y(1-\alpha)w^{*}}\right)^{2z-1}} \leq 1.$$

This will hold for some $z < 1/2$ but not for arbitrarily small z. Let $\bar{z}$ be defined by

$$\bar{z} = \frac{y\left(1-\alpha\right)}{2\alpha\left(1-\theta\right)}.$$

The following is a sufficient condition to guarantee that equation (15) is greater than zero.

Lemma 1 *For all z such that*

$$\bar{z} \leq z < \frac{1}{2},$$

the horizontal-intercept defined by equation (15) will lie between zero and one.

Thus, equation (16) is represented by a upward sloping line, as depicted in Figure 8.[22] Furthermore, when μ_t is held constant, then equation (16) indicates that an increase in a_{t-1} must be accompanied by an increase in μ_t. Thus the phase dynamics are represented by the arrows in Figure 9. The following proposition characterizes steady state equilibria and the global dynamics in the decreasing returns case.

Proposition 4 *For $\bar{z} \leq z < 1/2$, there exists a unique steady state equilibria that is a sink.*

The proof of Proposition 4 follows directly from the discussions of equations (13) and (16) above and their respective phase diagrams. Figure 9 combines the diagrams from Figures 4 and 8. In contrast to the increasing returns case, even if equation (16) and (13) are more "wavy," there exists a unique steady state equilibrium. Moreover, our results indicate that as long as the degree of decreasing returns to network capital is not too great, the dynamics of migration flows and network capital accumulation are characterized by a monotonic convergence to the steady state.

It is interesting to note that in the case where additional network capital decreases border crossing and job search frictions, but at a decreasing rate, the locus of points for which migration levels are unchanging over time is now upward sloping (as opposed to downward sloping in the increasing returns case and vertical in the constant returns case.) Thus in the decreasing returns case, the arbitrage condition can be satisfied in steady state migration levels only if migration and the beginning-of-period network capital stock are substitutes – consistent with constant returns case. In both cases, when migration and initial network capital stocks are substitutes, there exists a unique steady state equilibria.

Remark: Overall, the dynamics of the economic system and the levels and properties of steady state equilibria are intimately tied to the returns to scale exhibited by the returns to network capital accumulation. In particular, constant returns to scale is unique in the sense that, in general, the dynamics are characterized by two-period cycles where migration and network capital are substitutes in the migrant's decision. In contrast, convergence is monotonic in the cases in which returns to scale are increasing or decreasing, although the numbers and properties of steady state equilibria in these two cases will differ. Thus, our analysis suggests that the specifics of how networks are explicitly modeled (in our case networks alter the migration process by reducing both

[22] For ease of exposition we have represented equation (16) as a straight line. However, $d^2 a_{t-1}/d\mu_t^2$ may be positive or negative depending on the parameter values and the values of μ_t and a_{t-1}. Thus, this line may have several "wiggles" in it. For the purpose of expositional clarity, we depict equation (16) as being a linear, upward sloping curve.

border crossing and job search frictions, and this, in turn, affects migrant's decisions regarding time and income allocations) are crucial to obtaining a better understanding of migration flows and the role played by networks. In addition, by more explicitly modeling the role of networks in migrant's decision making process, we can better understand the impact of changes in policy parameters on migrants flows, which we do in the following section.

5 Barriers to Migration

We now focus on the impact that changes in the barriers to migration would have on the dynamics in our model economy and on the steady state values of migration and network capital accumulation. Our aim is to shed light on how changes in the efforts devoted to inhibiting (or fostering) migration can have unexpected effects. The parameter θ will be interpreted in a manner that is consistent with the migration process under study. For instance, in the case of cross-country migration, one can think of θ as encompassing both the level of border enforcement enacted by the host country and also the levels of people smuggler activity intended to circumvent enforcement. Similarly, in the case of the Underground Railroad in the mid-nineteenth century in the United States, it would encompass both enforcement imposed by the home region (the South) as well as people smugglers in the host region (the Underground Railroad). As with the analysis of the dynamics in section 4, three cases are considered depending on whether the return to network capital exhibits constant, increasing, or decreasing returns to scale.

5.1 Constant Returns Case

To ascertain the impact of an increases in the barriers to migration, it is sufficient to examine equations (11) and (10),

$$a_t = \left[\frac{xw}{y\,(1-\alpha)\,w^*} - a_{t-1}\,(1-\theta)\right](1-\mu_t)^2 \tag{11}$$

and

$$\mu_{t+1} = 1 - \frac{y\,(1-\alpha)}{\alpha\,(1-\theta)}\frac{1}{1-\mu_t}. \tag{10}$$

From equation (10), it is straightforward to show that $d\mu_{t+1}/d\theta < 0$. As Figure 10 indicates, the locus shifts toward the origin. While the characterization of equilibria remains unchanged (there is either a unique steady state or a two-period cycle), the level of migration rises when compared against the equilibrium outcomes where migration barriers are lower. In particular, the steady state value of migration is higher and, when equilibrium is characterized by a two-period cycle, the level of migration in the second period (and every even number period thereafter) also rises. The initial level of migration (and every odd period thereafter) is unchanged because it is chosen by the migrant and assumed to be unchanged in our comparative statics exercise. Thus, increasing barriers to entry has the undesired impact of actually increasing the number of migrants flowing into the host region

The impact on the equilibrium value of network capital is similar to that of migration flows. Differentiating equation (11) with respect to θ indicates that increases in barriers to migration causes this equation to pivot up and become flatter, as depicted in Figure 11. For the case of a unique steady state, we see that network capital increases in response to greater deterrence.[23]

The intuition for these results, both migration flows and network capital increase in response to greater barriers to migration, is straightforward. Increasing the frictions affecting border crossings increase the time spent traversing from the home to host region. As a result, both host region wage income and remittances received when old decline. To compensate, migrants spend more time migrating, which leads to an increase in host region wage income and counteracts the impact of greater deterrence. Increased migration also goes hand-in-hand with greater network capital accumulation, which increases remittance income when old, again offsetting the extra time next generation must spend crossing the border due to increased barriers.

5.2 Increasing Returns Case

When $z > 1/2$, the following equations characterize the dynamic evolution of the economy:

$$a_t - a_{t-1} = 0 = \left[\frac{xw}{(1-\alpha)\, yw^*} - a_{t-1}^{2z}\,(1-\theta)\right](1-\mu_t)^2 - a_{t-1}, \tag{13}$$

and

$$\mu_{t+1} - \mu_t = 0 = (1-\mu_t) - \frac{y\,(1-\alpha)}{\alpha\,(1-\theta)\,2z\left[\frac{xw}{y(1-\alpha)w^*} - a_{t-1}^{2z}\,(1-\theta)\right]^{2z-1}} \frac{1}{(1-\mu_t)^{4z-1}}. \tag{14}$$

As in the constant returns case, it is straightforward to verify from equation (13) that $d\,(a_t - a_{t-1})\,/d\theta = a_{t-1}^{2z}\,(1-\mu_t)^2 > 0$. As Figure 12 shows, the locus pivots in a clockwise direction. Differentiating

[23] Although Figure 11 illustrates the case where there is a unique steady state value for the level of network capital, the analysis naturally applies to the case where network capital converges to a two-period cycle.

equation (14) with respect to the barriers to migration parameter yields

$$\frac{d\left(\mu_{t+1}-\mu_t\right)}{d\theta}=\frac{-y\left(1-\alpha\right)}{\alpha 2z\left(1-\mu_t\right)^{4z-1}\left(1-\theta\right)^2\left[\frac{xw}{y(1-\alpha)w^*}-a_{t-1}^{2z}\left(1-\theta\right)\right]^{2z}}\left[\frac{xw}{y\left(1-\alpha\right)w^*}-2za_{t-1}^{2z}\left(1-\theta\right)\right] \tag{17}$$

The sign of equation (17) is ambiguous, depending on whether the second term on the right-hand side satisfies

$$\frac{xw}{y\left(1-\alpha\right)w^*}-2za_{t-1}^{2z}\left(1-\theta\right)\lesseqgtr 0.$$

For sufficiently small z, i.e., z close to $1/2$ this expression will be positive, by Assumption 3. Thus, if the return to network capital exhibits only mildly increasing returns, then equation (14) will shift left, otherwise it will shift right. We concentrate our analysis on the low-migration steady state because it is a sink.

If the degree of increasing returns are sufficiently low, then as Figure 12 shows, an increase in barriers to migration would be represented by a leftward shift in the locus corresponding to constant migration and the steady state levels of both migration and network capital would increase. Because the low-migration steady state is a sink, the economy would move to the new low-migration steady state. The intuition for this increase in migration and network capital is the same as in the constant returns case. Thus, the impact of greater deterrence would again be counterproductive; there is an increase in migration.

In addition, if the return to network capital exhibited large enough increasing returns, than equation (14) shifts right. The impact on the low-migration steady state is ambiguous, as depending on the magnitudes of the shifts of equations (13) and (14) both the levels of migration and network capital could increase or decrease. Finally, regardless of the exact impact on equilibrium values, the economy will converge to the new low-migration steady state equilibrium.[24]

[24] At the high-migration steady state, if the degree of increasing returns was sufficiently low, than new high-migration steady state would be characterized by less migration and less network capital. However, the dynamics of the system would not cause the economy to move from the old steady state to the new. In fact, the economy would be on an unsustainable equilibrium path after the increase in barriers to migration.

Conversely, if the degree of increasing returns was sufficiently high, then the levels of migration and network capital at the new high-migration steady state would be ambiguous relative to the initial high-migration steady state. However, the dynamical tendencies of the economy would move the economy towards the new low-migration steady state, with less network capital and migration.

5.3 Decreasing Returns Case

Then only difference between this case and the increasing returns case, is that when there is an increase in deterrence, we can now sign $d\left(\mu_{t+1}-\mu_t\right)/d\theta$. Recall that the sign of $d\left(\mu_{t+1}-\mu_t\right)/d\theta$ depends on

$$\frac{xw}{y\left(1-\alpha\right)w^{*}}-2za_{t-1}^{2z}\left(1-\theta\right)\lesseqqgtr 0.$$

However, in order for a problem to be well defined we know that

$$\frac{xw}{y\left(1-\alpha\right)w^{*}}-a_{t-1}^{2z}\left(1-\theta\right)>0.$$

In addition, Assumption 2 guarantees that

$$\frac{xw}{y\left(1-\alpha\right)w^{*}}-\left(1-\theta\right)>0$$

Thus, since $a_{t-1}<1$ and when $z<1/2$, then $2z<1$, it must be the case that

$$\frac{xw}{y\left(1-\alpha\right)w^{*}}-2za_{t-1}^{2z}\left(1-\theta\right)>0.$$

and hence $d\left(\mu_{t+1}-\mu_t\right)/d\theta\ <0$. Figure 13 illustrates the respective shifts in equations (13) and (14). As the figure shows, an increase in barriers to migration results in an increase in network capital accumulation, but the effect on the level of migration is indeterminate. Thus, the increase in deterrence may, or may not, have the desired affect of reducing migration flows. Greater barriers reduce the marginal income from host country wage income. One way to maintain equality between the constant marginal income in the home region and the marginal income of the host region is to increase the level of network capital (see the intratemporal efficiency condition, equation (6)).

Remarks: Overall, we show that changes in barriers to migration have long-run effects on migration and the size of network capital. Indeed, these effects are surprises under certain versions of the returns to network capital investment. The presence of network capital means that there is another way in which migrants can affect the returns to migrating. Our setup emphasizes the role that returns to network capital plays along two specific dimensions: the allocation time between home and host region and the allocation of savings between storage and network capital. Note surprisingly, if the return to network capital is constant or increasing in the size of network capital, we show that there is greater incentive to migrate. In part, the incentive stems from the direct return to network capital. Also, network capital is effectively cheaper to accumulate when migration is higher.

6 Conclusion

Social networks play an important role in facilitating migration, whether across borders or across regions. However, recent empirical work suggests that the importance of networks to the migration process may vary over time. Previous theoretical work is unable to accommodate these recent findings as a result of the manner in which networks were modeled; namely networks were equivalent to the stock of older migrants.

In this paper we break with these previous works and instead model networks as a combination of the number of migrants and the physical resources they devote to maintaining and improving the network. In addition we explicitly model network capital as being a factor which reduces the real world frictions surrounding crossing borders and looking for employment. In our model economy, the existing level of network capital affects both a potential migrant's migration decision (where to spend her time) as well as her income allocation decision (by which means to save) and thus migration flows and network capital are not one and the same, as with the previous literature.

We present two main sets results. First, we explore the global dynamics of the economy. We show that when the costs of moving into the host region are linear in the stock of network capital, the dynamical system is a recursive pair of nonlinear difference equations that, in general, can exhibit two-period cycles. Thus, we are able to demonstrate in a simple model an economy where the importance of networks will fluctuate over time. This fluctuation is the result of the fact that migrants have two means for saving for old age: a storage technology and larger networks (which result in greater old-age remittances). A given level of old-age income can be maintained either through increased migration or via larger networks, but both are not simultaneously required.

We also explore cases in which the returns to network capital exhibit either increasing or decreasing returns to scale. In the increasing returns case, there exists the possibility of multiple equilibria, which are characterized as either sinks or saddles, while in the decreasing returns case there exists a unique equilibrium. In both the increasing and decreasing cases, there exists monotonic convergence to steady states and thus fluctuations, as observed in the constant returns case, are ruled out.

Second, we study the effect that changes in barriers to migration have on steady states. The long-run impact depends on the returns to network capital relative to the level of network capital. In both the constant returns case and (the more relevant steady states of) the increasing returns case, increasing the barriers to migration has the unintended consequence of actually raising the quantity of migration. Increasing barriers makes crossing borders more difficult and time consuming and thereby reduces income and savings from working in the host region. One way for migrants to overcome these declines in income is to devote greater time (more migration) and resources (greater network capital) to the migration process. This suggests that governments that try to restrict border crossings in places where networks play an important role in the migration process may be instituting counterproductive policies.

There are several areas where our model can be fleshed out in greater detail in future work. However, there are two primary areas for further consideration. The first would be to determine the extent to which the functional forms for the returns to network capital could be generalized. The second area for further refinement would be to make more explicit the various barriers to entry which exist and their role in the migrant's decision making process. Both of these refinements would further our understanding of how migrants use and build networks aimed at enhancing the migration process.

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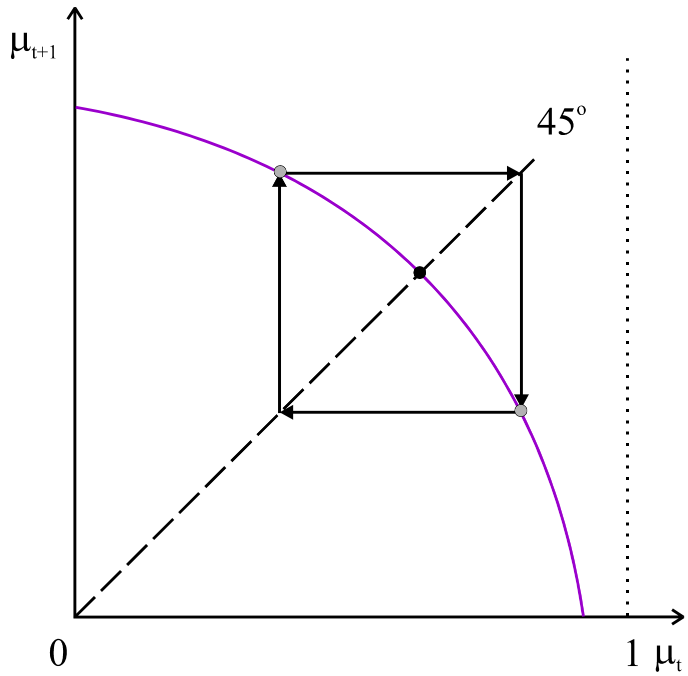

Figure 1: Migration Flows in the Linear Case: An Unique Equilibrium or a Two-Period Cycle

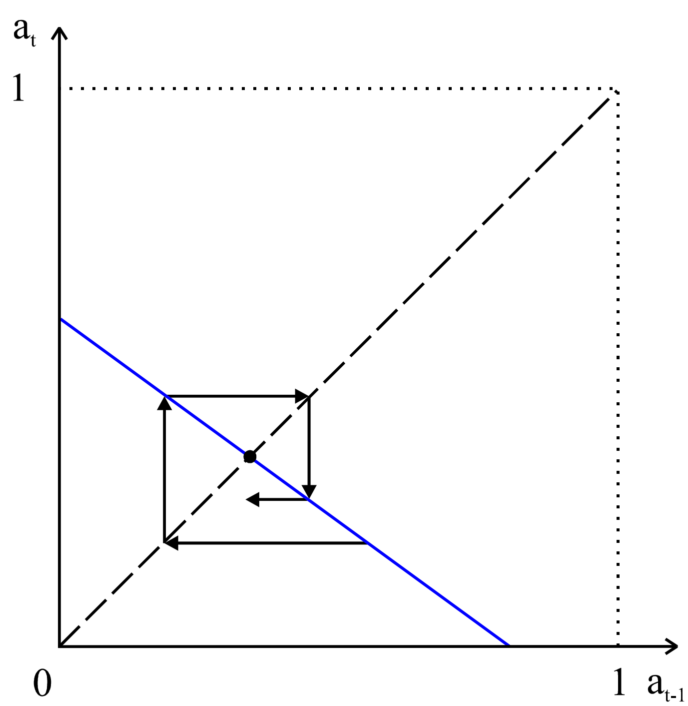

Figure 2: Network Capital in the Linear Case: An Unique Steady State Equilibrium

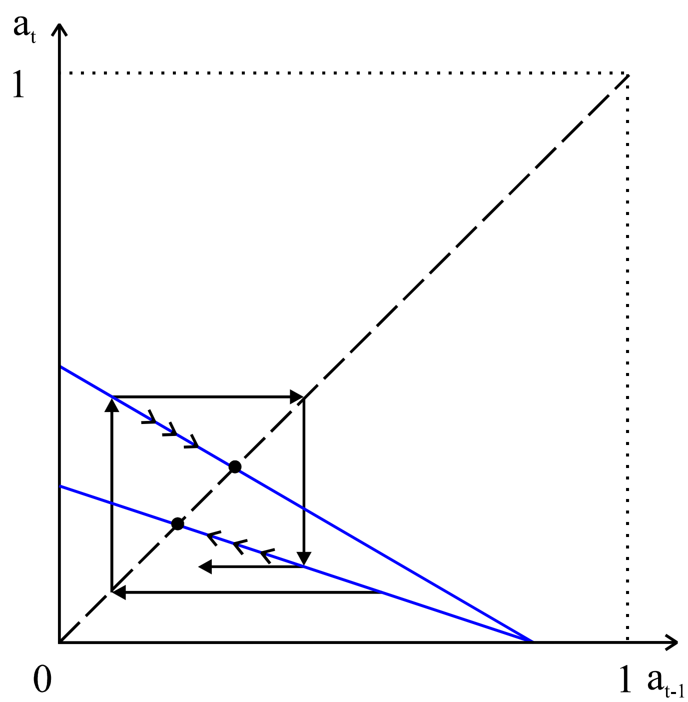

Figure 3: Network Capital in the Linear Case: A Two Period Cycle

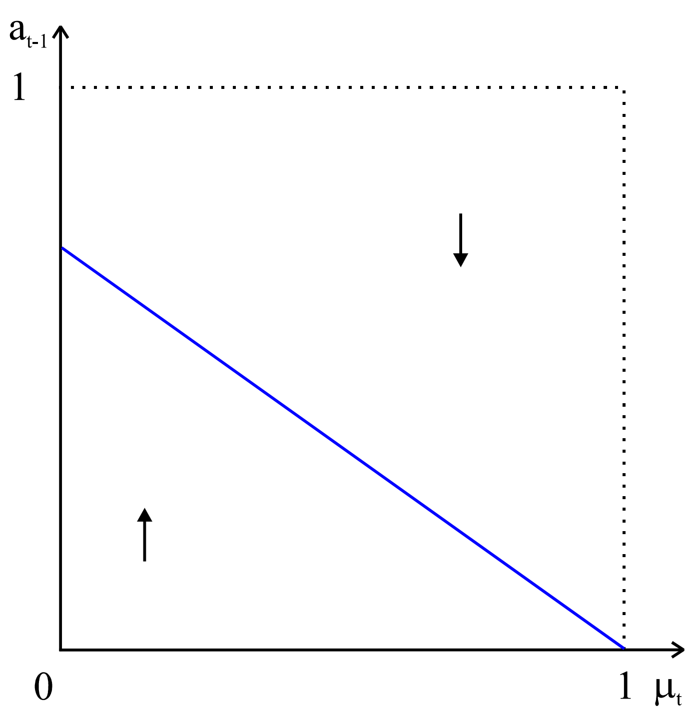

Figure 4: Steady State Network Capital in the Convex Case

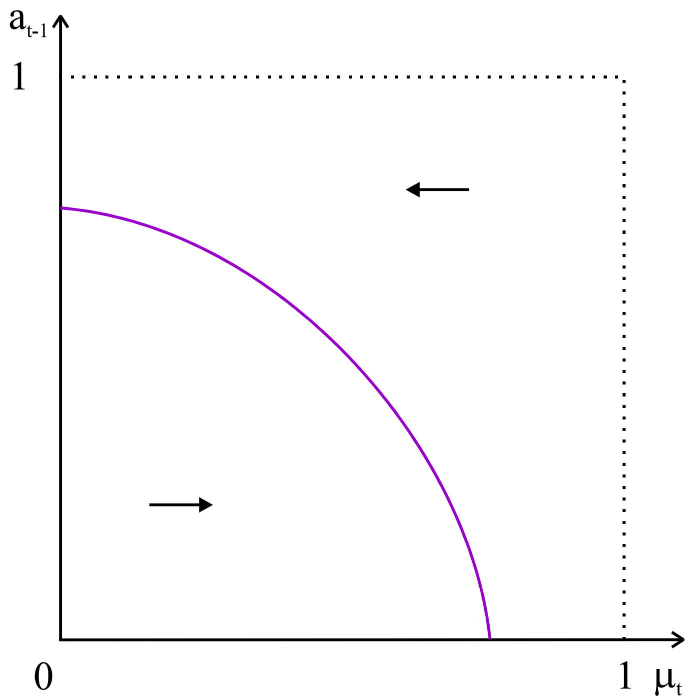

Figure 5: Steady State Migration Flows in the Convex Case

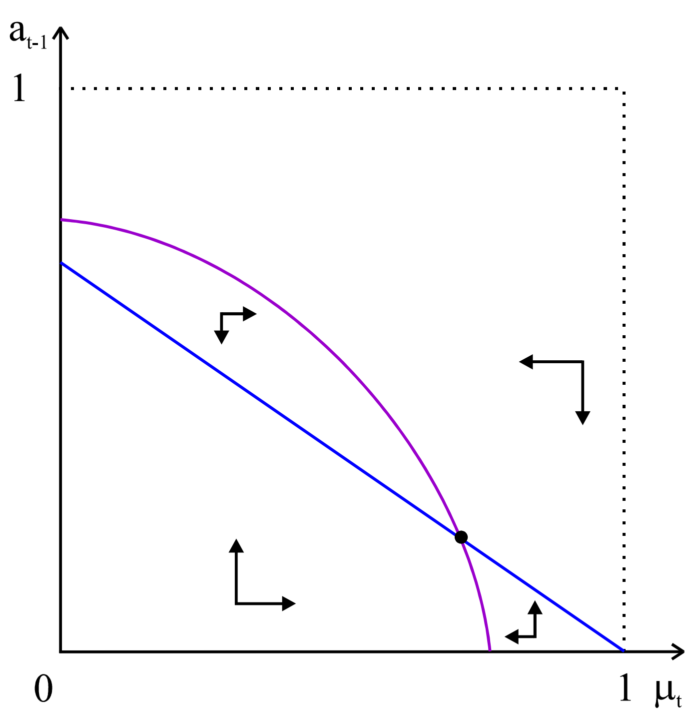

Figure 6: Unique, Stable Steady State Equilibrium in the Convex Case

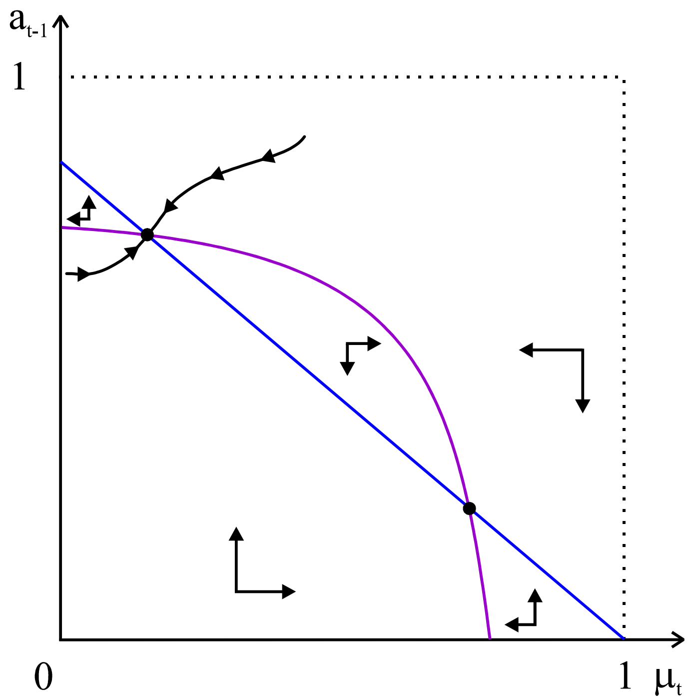

Figure 7: Multiple Steady State Equilibria in the Convex Case

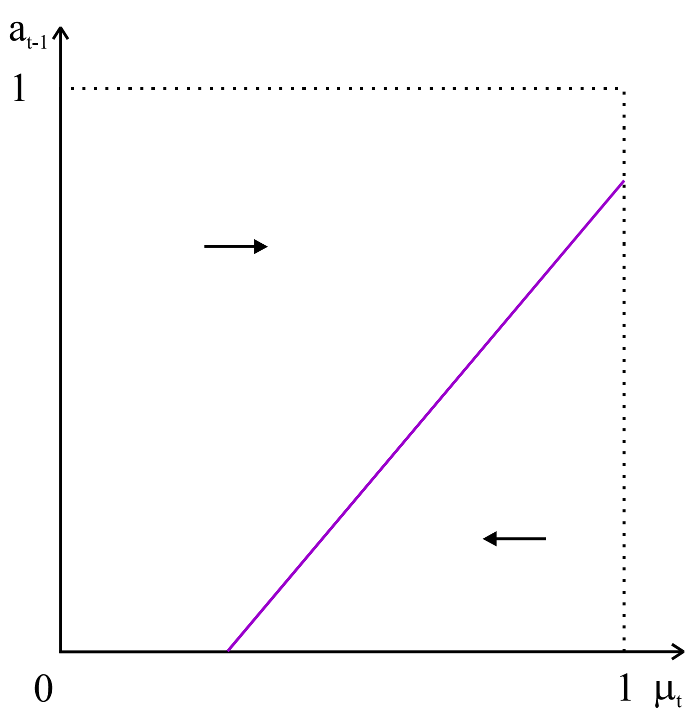

Figure 8: Steady State Migration Flows in the Concave Case

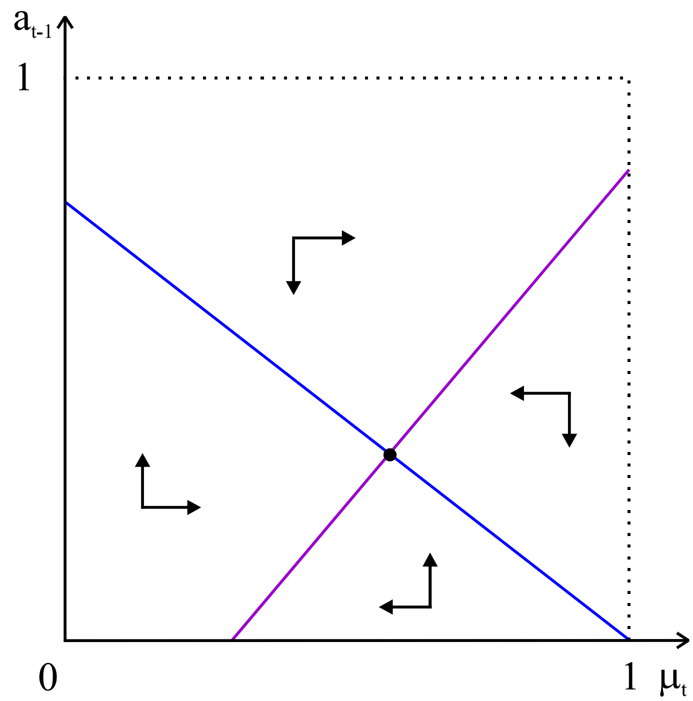

Figure 9: Unique, Steady State Equilibrium in the Concave Case

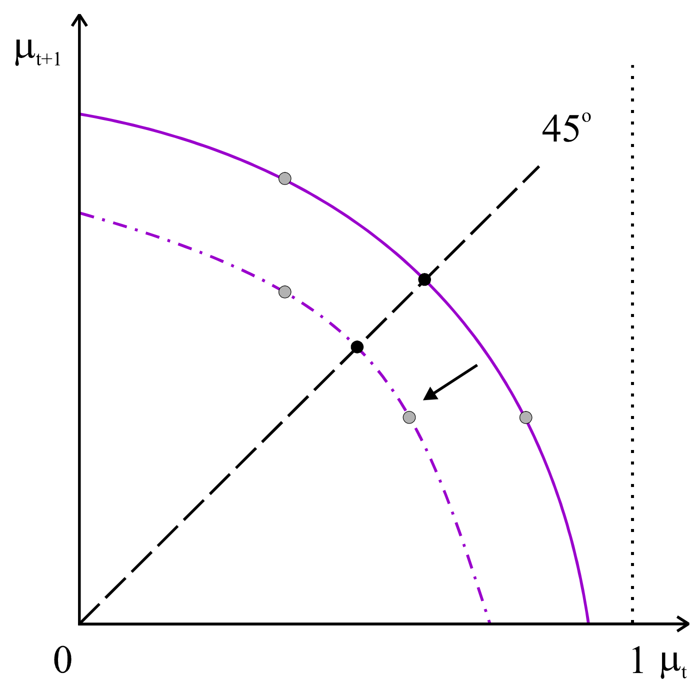

Figure 10: Increased Barriers to Migration Increases Migration Flows

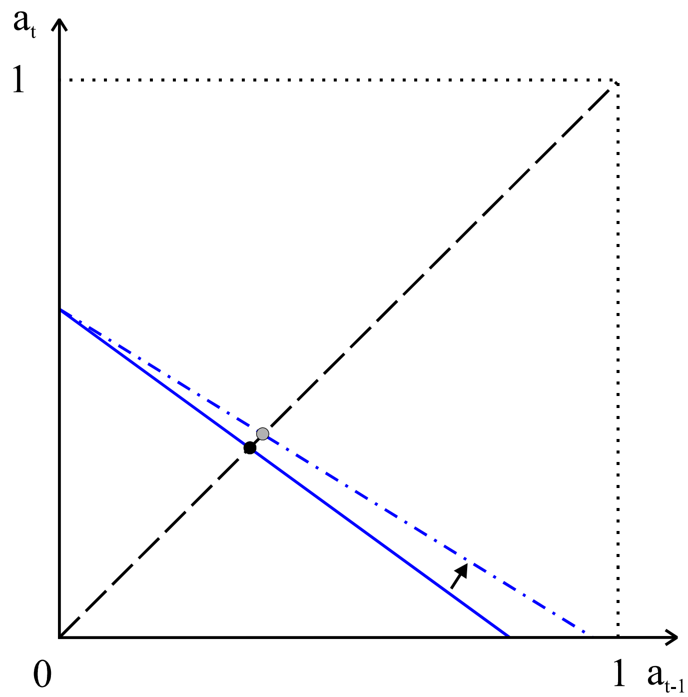

Figure 11: Increased Barriers to Migration Raises the Level of Network Capital

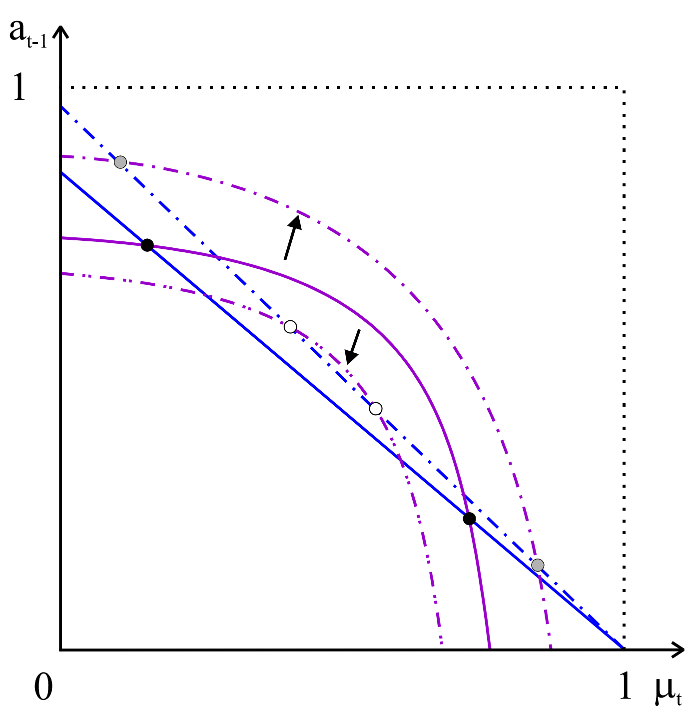

Figure 12: Impact of an Increase in Barriers to Migration on Networks and Migration

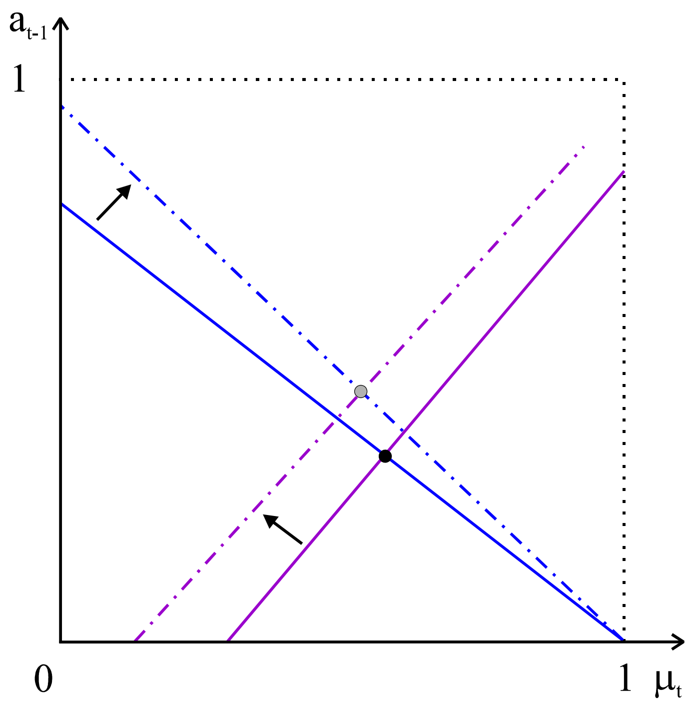

Figure 13: Increase in Barriers to Migration Raises Network Capital, but Impact on Migartion is Ambiguous