

Nonparametric Estimation of the Impact of Taxes on Female Labor Supply

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Abstract

Econometric models with nonlinear budget sets frequently arise in the study of impact of taxation on labor supply. Blomquist and Newey (2002) have suggested a nonparametric method to estimate the uncompensated wage and income effects when the budget set is nonlinear. This paper extends their nonparametric estimation method to censored dependent variables. The modified method is applied to estimate female wage and income elasticities using the 1985 and 1989 waves of PSID exploiting the drastic change in the complete budget set caused by TRA 1986 as a source of identification. I find evidence of downward bias in estimated elasticities if the nonlinearity in the budget set is ignored. The estimated wage elasticities range from 0.6-0.74 for total hours and from 0.26-0.29 on the intensive margin. The income elasticity estimates range from -0.4 to -0.67 overall and from -0.12 to -0.15 on the intensive margin.

Keywords: Taxes and Female Labor supply, Kinked Budget Set, Nonparametric Estimation

JEL Numbers: J22, H24, C14, C24

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1. Introduction:

Nonlinear-budget-set techniques have been used widely in the analysis of tax and transfer policies on labor market outcomes.¹ It is well known that not accounting for the nonlinearity in the budget set generally leads to biased estimates of the impact of the after-tax wage on labor supply. Most theoretical predictions become ambiguous when the budget set is nonlinear (Moffitt, 1990); in particular, comparative statics of consumer demand may not conform to usual textbook predictions (Moffitt, 1986). Another source of bias in the estimated behavioral elasticities is the tendency for individuals to “bunch up” at such points, as location at kink points is consistent with a multiplicity of budget-set slopes. These individuals will not respond to moderate changes in the tax rate simply because they are located at the kink points, which have little to do with their underlying preferences. Failure to account for this phenomenon will tend to bias the wage effect downward.²

In a survey paper on the econometrics of nonlinear budget constraints, Moffitt (1990) reviewed existing methods for dealing with kinks in the budget set. Two of the most widely used estimation methods are Maximum Likelihood Estimation (MLE) and instrumental-variable estimation of marginal labor supply functions. The MLE method proposed by Burtless and Hausman (1978) comprehensively takes into account the entire budget set, with each segment and kink contributing to the likelihood function. The budget set generated by the tax system can be treated as exogenous and does not require instrumenting for the endogenous after-tax price or slope. While MLE provides the most efficient estimates if the distributional assumptions underlying the model are correct, the estimates will be inconsistent if the functional form and

¹ For example see Burtless and Hausman (1978), Hausman (1980), Hausman (1981), Hausman (1985), Burtless and Moffitt (1980), Triest (1990), Friedberg (1996), MaCurdy, Green and Paarsch (1990), among others.

² Blundell, Duncan and Meghir (1998) explicitly control for such selection bias caused due to location at the kink points without modeling the entire budget constraint.

distributional assumptions are false. Most papers applying maximum-likelihood techniques have estimated a linear index function.³ Thus, there may be two possible sources of bias in the MLE approach: (1) the bias due to parametric assumptions about the regression function; and, (2) the bias due to distributional assumption about the error term. Limited dependent variable models based on normality and homoscedasticity perform very poorly if these assumptions are violated (Hausman, 1985; Wooldridge, 2002). Many papers have found significant biases due to misspecification of the error distribution in such models (Chay and Powell, 1996; Gerfin, 1996; Martins, 2001).

Due to the complications surrounding MLE in this setting, several papers have instead chosen to estimate labor supply parameters using instrumental-variable methods. To address the issue of the slope and virtual income at kink points, some papers use a smooth and differentiable approximation of the budget set and estimate the model using instrumental variables. MaCurdy, Green and Paarsch (1990) have suggested combining the differentiable budget constraint methodology with MLE to weaken the strong restrictions imposed by MLE at kink points. Of course, the instrumental variable method will produce biased results if the instruments for the endogenous observed after-tax wage are not valid, while it uses only local information on the budget set and lacks the efficiency of the MLE approach. This method also does not allow for optimization error in the econometric specification of labor supply and constrains the utility-maximizing choice to be on the observed segment. The presence of optimization error in the model invalidates the assumption in the instrumental variable method, that the observed labor supply is indeed the utility maximizing one. Heim and Meyer (2004) show that Slutsky positivity

³ For example see Hausman (1980), Hausman (1981), Hausman (1985), Hausman and Ruud (1984), Burtless and Moffitt (1980), Triest (1990), and Friedberg (1996).

restrictions imposed by Hausman's MLE approach is tantamount to assuming convex preferences and propose a direct utility approach.

All of the approaches discussed above rely on either a parametric specification of the labor supply function or a known distribution for the stochastic specification. Blomquist and Newey (2002) have proposed an estimator that relaxes many of the restrictive parametric assumptions which form the basis of the MLE approach and that overcomes many of the limitations of the instrumental-variable approach. Their estimator models the labor supply equation as a function of the entire budget set (i.e., all the segments and kinks). They derive an expression for the expected labor supply function that effectively isolates a selection-bias-type term which arises due to the nonlinearity of the budget set and yields an additive specification.⁴ Their proposed method does not require any functional form assumptions for labor supply and can be conveniently estimated using methods that allow one to impose additivity, such as power series, splines or generalized additive models (Hastie and Tibishirani, 1998). An added advantage of employing their approach is the convenience for testing the linearity of the labor supply function or nonlinear budget sets or both, as these cases are nested within their derived specification for the expected labor supply function. However, Blomquist and Newey (2002) derived this expression assuming that the probability of not working is zero.⁵

I make the following three contributions. First, I extend the nonparametric estimation method proposed by Blomquist and Newey (2002) to incorporate cases in which the dependent variable is censored. The derived expression has a form that is similar to Blomquist and Newey

⁴ This selection bias type term is not due to wages that are endogenously missing.

⁵ Blomquist and Newey (2002) studied male labor supply in Sweden where censoring of hours of work was not an issue. The only other paper that has applied this method is Wu (2003), who does it for estimating the labor supply effects of the Earned Income Tax Credit (EITC). This paper accounts for mild censoring in the data by estimating a Heckman-type selection-bias-corrected model on a subsample of workers; including an inverse mills ratio from a first stage probit; and relying on nonlinearity of the inverse Mills ratio for identification.

(2002), with one additional term to be accounted for in estimation, and is empirically tractable. This extension can be exploited to handle censoring in many instances involving nonlinear budget sets e.g. taxation and labor supply, social security earnings test, charitable contributions, demand for goods with nonlinear pricing structure, 401(k) contributions. Second, I nonparametrically estimate female uncompensated wage and income elasticities, taking into account the entire U.S. federal income tax structure, using the waves of Panel Study of Income Dynamics (PSID) before and after Tax Reform Act of 1986 (TRA-1986). Third, I estimate a conditional hours equation with nonparametric selection correction (Das, Newey and Vella, 2003).

There are four primary findings. First, I find statistically and economically significant evidence of bias induced when the nonlinearity in the budget set is ignored. Second, the uncompensated wage elasticity is declining in the hourly wage and turns negative at higher wage rates. This suggests a backward-bending shape for the labor supply curve. Third, I obtain estimates of income elasticity that are somewhat larger (in absolute value) than the previous literature and statistically significant. Fourth, response on intensive margin accounts for about 40 percent of the overall response of female labor supply due to change in wages and less than 30 percent of total response due to income changes. I estimate an uncompensated wage elasticity of 0.6-0.74 overall and 0.26-0.29 on the intensive margin. My estimates of income elasticity are somewhat higher than the previous literature and range from -0.4 to -0.67 for total hours and -0.12 to -0.15. In particular, estimates of the income elasticity from the nonlinear budget set specification are about 25-30 percent higher than those from assuming a linear budget set. These estimates imply a compensated elasticity of about 0.61-0.81 overall and close to 0.30 on the intensive margin.

This paper is organized as follows. Section 2 presents a short review of the previous literature on nonlinear-budget-set and nonparametric estimation of labor supply functions. Section 3 forms the core of the paper, where I derive the expected hours function when labor supply is censored at zero. I suggest econometric strategies to estimate the new expected hours function. The econometric specification is discussed in section 4. Section 5 provides a description of the data; construction of the important budget set variables; and describes the identification strategy. Section 6 explains the estimation approach. The findings are discussed in section 7 while section 8 concludes.

2. Background and Literature Review

2.1 The Static Model with Taxes

Let $U(C_t, h_t, Z_t)$ be a strictly quasi-concave utility function in which C_t is consumption in period t , h_t is hours worked, and Z_t is a vector of exogenous taste shifters. In a standard static labor supply model with taxes, the consumer maximizes the utility function in period t , $U(C_t, h_t, Z_t)$, subject to the budget constraint:

$$C_t = W_t h_t + y_t - T(I_t, D_t, E_t) \quad (1)$$

where W_t is the gross wage, y_t is the unearned income, I_t , the taxable income of the individual, D_t , the tax deductions, E_t , exemptions and $T(\cdot)$ is a function determining tax liability.⁶ A graduated tax rate and bracket structure creates a piecewise linear budget set with kinks at the points where the marginal tax rate changes. Figure 1 presents the budget set for a typical individual under a hypothetical progressive income tax with two tax brackets. The only kink in the budget set is located at h hours. Earnings up to Wh are taxed at a marginal tax rate of τ_1 ,

⁶ This is deterministic model in which wages, other income and taxes are known with certainty.

where Wh is the dollar amount of labor earnings for working h number of hours. Earnings above Wh are taxed at the marginal tax rate of τ_2 . This tax system creates two budget set segments with slopes $W(1-\tau_1)$, and $W(1-\tau_2)$, respectively, and one kink at h . Virtual income is the income associated with zero hours on each budget segment. In Figure 1, for the first budget segment with a marginal tax rate of τ_1 , the virtual income is y_1^v . However, the second segment yields virtual income of y_2^v . The intuition behind using virtual income is that the consumer's last-dollar marginal tax rate (in the case of progressive taxation) is higher than marginal tax rate she faces on other parts of her budget set. Virtual income appropriately adds a lump-sum transfer to the consumer's actual unearned income to account for the nonlinear taxation of labor income.

Figure 1 also presents a simple labor supply example from a decline in the marginal tax rate on the first segment. Individual A and B both have a convex two-segment budget set with one kink. Individual A, who is located on the first segment, will work more if the substitution effect dominates. But individual B, who is located on the second segment, will face only an income effect, even though her marginal tax rate stays the same, and if leisure is a normal good, will work less. Clearly, the effect of this tax decrease in this case will be ambiguous.

2.2 Literature Review

In his survey paper on kinked budget sets, Moffitt (1990) outlined four methods to deal econometrically with nonlinear budget constraints: (1) estimate the complete demand function; (2) estimate the marginal demand function; (3) instrumental-variable estimation of the marginal demand function; and, (4) the MLE approach. While the latter three approaches have been used widely in the labor supply literature, the complete-demand-function approach is less widely used,

due, in part, to its complexity. Specifically, Hanoch and Honig (1978) are the only ones to use this approach by solving out for complete demand.

The primary problem that researchers have faced when estimating marginal demand functions is which slope and virtual income to include in the regression specification. Hall (1973) was the first to propose the idea of linearizing around the observed point on the budget set. However, this method failed to address two important problems: (1) what slope should be used if the individuals are bunched at kink points; and (2), how to deal with endogeneity of the observed after-tax wage and virtual income. Instrumental-variable methods proposed by Hausman and Wise (1976) and Rosen (1976) addressed the endogeneity issue by using an exogenously predicted marginal tax rate as an instrument. However, they left the issue of kink points unexamined.

In their seminal paper, Burtless and Hausman (1978) proposed the MLE approach to effectively account for the kinks and segments on the budget set, with each segment and kink contributing to the likelihood function. They also solved the problem of the endogeneity of the marginal tax rate. The likelihood-function approach was further elaborated by Hausman (1981) and Hausman (1985). Moffitt (1986) provided a comprehensive exposition of the maximum likelihood technique, and Moffitt (1990) presented an excellent survey of the problems and solutions available in the presence of kinked budget constraints.

However, the MLE approach has been criticized for three reasons. First, it imposes strong parametric assumptions, and limited dependent variable models have long been known to be sensitive to the assumption of normality. Second, while it allows for measurement error in hours worked, it breaks down when the budget set variables are measured with error for other

reasons (Heckman and MaCurdy, 1982; Blundell and MaCurdy, 1999).⁷ Heckman and MaCurdy (1981) proposed a selection-bias-oriented approach to account for nonlinear budget sets in a way less sensitive to measurement error in the budget set. They suggested estimating an ordered-choice model of location at segments and kinks and, in the second stage, using the inverse Mills ratio to account for selection bias. Third, it imposes strong theoretical restrictions on economic behavior. In particular, MaCurdy (1992) has argued that the MLE approach imposes the Slutsky restrictions at the kink points in order for the probability of observing individuals at the kinks to be positive and for the likelihood function to be well defined.⁸ MaCurdy, Green, and Paarsch (1990) found the imposition of this constraint to be the reason why Hausman estimated higher substitution and lower income effects using the MLE approach. Other papers have found these restrictions to be relatively mild.⁹ Another nontrivial problem with the MLE approach is that the likelihood function can be very complicated and fail to possess a global maximum. To make matters worse, the likelihood function also may have points of nondifferentiability (Wales and Woodland, 1979).

As an alternative estimation procedure, MaCurdy, Green, and Paarsch (1990) proposed a smooth and differentiable budget-constraint methodology to deal with the problems posed by kink points. Although originally applied with MLE, this method is also attractive for instrumental-variable type estimation and has been used by several papers to account for

⁷ Of course, measurement error of the budget set variables poses a problem even in the nonparametric estimation framework proposed by Blomquist and Newey (2002). The nonparametric instrumental variable approach of Blundell and Powell (2004) can be used to correct for endogeneity. Combination of the two methods will be explored in future work.

⁸ Heim and Meyer (2004) show that this is tantamount to assuming convex preferences.

⁹ Eklof and Sacklen (1999) sought to resolve the Hausman-MaCurdy controversy by showing that estimates of labor supply functions are so sensitive to division bias in the wage measure that this type of measurement error might well have contributed to the result found in MaCurdy, Green, and Paarsch (1990). The division bias caused by the wage measure also has been found by Ziliak and Kniesner (1999).

nonlinear budget sets (Ziliak and Kniesner, 1999; Aaronson and French, 2002; Engelhardt and Kumar, 2003) to effectively account for kinked budget sets and endogenous variables.

However, this approach is not without problems of its own. First, because this method relies on valid instrumental variables (IV) for the after-tax wage for identification, the choice of instruments is critical. It does not use all the information available on the entire budget set and lacks the efficiency of the MLE approach. Second, by smoothing the budget set around the kink points, it essentially imputes a slope at the kink points, which is somewhat arbitrary. Third, this method assumes that the observed location is the utility maximizing location, and, hence, there is no scope for optimization error (Moffitt, 1990).¹⁰

2.3 Nonparametric Estimation and Female Labor Supply

In most papers, the labor-supply function is chosen to be linear, even though the few papers that have done nonparametric estimation find evidence of misspecification for such models. One reason why the labor supply function is chosen to be linear is the difficulty in conducting exact welfare analysis using the parameters of a well-defined utility function that may not exist for a nonparametric labor-supply function.¹¹ Blundell and Meghir (1986) found the fit of the simple models “unacceptably poor.” Using data from U.K. family expenditure survey, they rejected the linear specification in favor of a more flexible specification of labor supply and found that the estimated wage elasticities exhibited significant evidence of backward-bending behavior. Kniesner and Li (2001) proposed estimating a male labor-supply function based on local linear kernel methods and found evidence of heterogeneity in wage effects using panel data from the SIPP. Jang (1998) used a multivariate local linear regression approach to

¹⁰ The estimates from this model will be inconsistent if the individual is observed on a segment only due to optimization error in which case her observed price and the virtual income may not be the utility maximizing one.

adapt to nonlinearities in the labor-supply curve and found that the elasticities obtained from parametric methods could be misleading. van Soest and Gong (1998) adopted nonparametric modeling of the direct utility function and discrete choice methods to estimate female labor supply elasticities on Dutch data. By approximating the budget set via a finite number of points, they found an own uncompensated wage elasticity of 1-1.2.

Blomquist and Newey (2002) provided the most comprehensive methods for the nonparametric estimation of labor supply with a nonlinear budget set. Applying their model to Swedish data, they found that parametric (MLE) estimates of the effect of tax reforms are upward biased while nonparametric estimates perform well. Although this method is based on the assumption of a globally convex budget set, Blomquist and Newey (2002) found that it is robust to the presence of mild nonconvexity in the data. In their male labor supply application, censoring of hours of work was not a major issue. However, in the context of female labor supply I need to explicitly account for a corner solution in hours of work, because roughly 27 percent of married women in the sample are not in the labor force.

3. Derivation of the Expected Labor Supply Function in the Presence of Censoring:

Maximization of the utility function $U(C_t, H_t, Z_t)$, subject to the budget constraint (1) yields a solution for labor supply that is a function of the individual's after-tax wage, ω_t , and virtual income, y_t^v :

$$h_t = f(\omega_t(h_t), y_t^v(h_t), Z_t) \quad (2)$$

¹¹ However, recent advances in deadweight loss estimation (Hausman and Newey, 1995) now allow deadweight loss to be estimated nonparametrically.

Using the notation in Blomquist and Newey (2002), suppressing the time subscript in a static framework, and ignoring the exogenous taste shifters, desired hours function can be written as

$$h_i^* = \pi(y^v, \omega, \eta), \quad (3)$$

where y^v is the virtual income, ω is the after-tax wage, and η is an error term representing heterogeneity in preferences.¹² In a world with multi-segment budget set, the desired hours function of the i_{th} individual on the j_{th} segment is given by

$$h_{ij}^* = \pi(y_{ij}^v, \omega_{ij}, \eta_{ij}) = \pi_{ij}(\eta). \quad (4)$$

I will drop the subscript i as the following discussion applies to all the individuals.

Theorem 3.1: Under the assumptions of a convex budget constraint,

$\int |\pi(y_j^v, \omega_j, \eta)| g(\eta) d\eta < \infty$, and $\pi(y_j^v, \omega_j, \eta)$ strictly increasing in η , the expected hours function for an individual with J segments and $J-1$ kinks, in the presence of censoring can be written as,

$$E(h^*) = \bar{\pi}(y_J^v, \omega_J) - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta) g(\eta) d\eta + \sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, l_j) - \mu(y_{j+1}^v, \omega_{j+1}, l_j)], \quad (5)$$

where $\pi_j^{-1}(y_j^v, \omega_j, l_j) = \pi_j^{-1}(l_j)$ is the inverse function of $\pi_j(y_j^v, \omega_j, \eta)$ as a function of η , $g(\eta)$ is the probability density function of η and l_j is the j_{th} kink and

$$\mu(y_k^v, \omega_k, l_k) = \int_{-\infty}^{\pi_k^{-1}(l_k)} \pi_k(\eta) g(\eta) d\eta - l_k \int_{-\infty}^{\pi_k^{-1}(l_k)} g(\eta) d\eta. \quad (6)$$

¹² Although the utility maximization model assumed here is deterministic, the error specification is consistent with random utility hypothesis. There are two sources of stochastic variation in the labor supply function: heterogeneity error and measurement error. The heterogeneity error is not assumed to be additive. The measurement error is

Proof: See Appendix 1.

The first term in (5),

$$\bar{\pi}(y_j^v, \omega_j) = \int_{-\infty}^{\infty} \pi_j(\eta) g(\eta) d\eta \quad (7)$$

is the expected labor supply on the J_{th} (last) segment. Blomquist and Newey (2002) assumed that

$$\Pr(h = 0 | x) = 0, \quad (8)$$

so that the second term in (5) disappears, i.e.,

$$\int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta) g(\eta) d\eta = 0, \quad (9)$$

and the expected hours function reduces to,

$$E(h^*) = \bar{\pi}(y_j^v, \omega_j) + \sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, l_j) - \mu(y_{j+1}^v, \omega_{j+1}, l_j)]. \quad (10)$$

The third term in (5),

$$\sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, l_j) - \mu(y_{j+1}^v, \omega_{j+1}, l_j)], \quad (11)$$

represents the bias due to nonlinearity of the budget set.

Blomquist and Newey (2002) modeled the first term in (10) nonparametrically as a function of wage and virtual income on the last segment. The second term in (10) is the bias term that arises due to nonlinearity and is additive in all the segments and kinks. In the presence of censoring in the dependent variable, the probability of not working is positive and one needs to model (9) as it is not non-zero. Theorem 3.2 below shows that this additional term can be accounted for in a straightforward manner.

assumed to be additive but the estimation method is valid in the presence of heteroscdastic error term that might

Theorem 3.2: Under the assumptions of convex budget constraint,

$\int |\pi(y_j^v, \omega_j, \eta)| g(\eta) d\eta < \infty$, and $\pi(y_j^v, \omega_j, \eta)$ strictly increasing in η , the expected hours

function in the presence of censoring can be written as,

$$E(h^*) = \int_{\pi_1^{-1}(0)}^{\infty} \pi_1(\eta) g(\eta) d\eta + [\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)] + \sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, l_j) - \mu(y_{j+1}^v, \omega_{j+1}, l_j)] \quad (12)$$

Proof: Without imposing the assumption that $\Pr(h = 0 | x) = 0$, the expression for expected labor supply is given by (5), i.e.

$$E(h^*) = \bar{\pi}(y_J^v, \omega_J) - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta) g(\eta) d\eta + \sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, l_j) - \mu(y_{j+1}^v, \omega_{j+1}, l_j)] \quad (13)$$

$$= \int_{-\infty}^{\infty} \pi_J(\eta) g(\eta) d\eta - \left[\int_{-\infty}^{\infty} \pi_1(\eta) g(\eta) d\eta - \int_{\pi_1^{-1}(0)}^{\infty} \pi_1(\eta) g(\eta) d\eta \right] + \sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, l_j) - \mu(y_{j+1}^v, \omega_{j+1}, l_j)] \quad (14)$$

$$= \int_{\pi_1^{-1}(0)}^{\infty} \pi_1(\eta) g(\eta) d\eta - \left[\int_{-\infty}^{\infty} \pi_1(\eta) g(\eta) d\eta - \int_{-\infty}^{\infty} \pi_J(\eta) g(\eta) d\eta \right] + \sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, l_j) - \mu(y_{j+1}^v, \omega_{j+1}, l_j)] \quad (15)$$

$$= \int_{\pi_1^{-1}(0)}^{\infty} \pi_1(\eta) g(\eta) d\eta + [\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)] + \sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, l_j) - \mu(y_{j+1}^v, \omega_{j+1}, l_j)]. \quad (16)$$

QED

In (12), the new nonlinearity bias term consists of

$$[\bar{\pi}(y_J, w_J) - \bar{\pi}(y_1, w_1)] + \sum_{j=1}^{J-1} [\mu(y_j, w_j, \eta_j) - \mu(y_{j+1}, w_{j+1}, \eta_j)] \quad (17)$$

Each term in (12) has an intuitive interpretation. The first term is the expectation of the hours function if the individual is maximizing utility on the first segment of the budget set and faces an

result in a random utility model (Brown and Walker, 1989).

after-tax wage based on the first-dollar marginal tax rate and, therefore, represents average hours if the individual has a linear budget set and appropriately accounts for censoring in choice of leisure.¹³ The derivatives of this term are comparable to uncompensated wage and income effects assuming a linear budget set. The second term and third term in (12) capture the nonlinearity of the budget set and will disappear if there is no nonlinearity in the budget set. They can be interpreted as an analogue of the inverse Mill's ratio correcting for potential bias due to the nonlinearity of the budget set. If unobservables in the underlying labor supply function are correlated with the unobservables determining the individual's location on different segments and kink points, these terms will not be zero and the estimates of the wage and income effects will be biased.

The expected hours function in (13) has a familiar structure if we assume that the underlying labor supply function on the j_{th} segment is linear so that,

$$h^* = x_j\beta, \quad (18)$$

and, assuming without loss of generality that the budget set consists of two segments and one kink l , then the expression for expected labor supply can be written as:

$$\begin{aligned} E(h^* | x) = & x_2\beta - F(-x_1\beta)[x_1\beta + E(\eta | \eta < -x_1\beta)] \\ & + F(l - x_1\beta)[x_1\beta + E(\eta | \eta < l - x_1\beta) - l] \\ & - F(l - x_2\beta)[x_2\beta + E(\eta | \eta < l - x_2\beta) - l], \end{aligned} \quad (19)$$

where $F(z) = \Pr(\eta < z)$. Heckman and MaCurdy (1982) suggested estimating the parameters of this function by assuming that η is normally distributed, estimating $F(z)$ and

¹³ To see this more clearly note that the second and third terms disappear if the budget set is linear. Then there is just one tax rate and to estimate the parameters of the labor supply function, we would use usual methods to deal with censored dependent variables e.g. Tobit or semiparametric estimators depending upon what we want to assume about the error distribution.

$E(\eta | \eta < l - x_j \beta)$ using an ordered probit, and then estimating (19) by least squares in the second stage.

If the assumptions of linearity of the labor supply function, additivity of η , and normality are imposed in (13), then the expected hours function can be written as

$$\begin{aligned} E(h | X) &= P(h > 0 | x) * E(h | x, h > 0) \\ &= \Phi(x\beta / \sigma) \{x\beta + \sigma\lambda(x\beta / \sigma)\} \end{aligned} \quad (20)$$

where $\Phi(x\beta / \sigma)$ is the CDF of the normal standard normal distribution and $\lambda(x\beta / \sigma)$, the inverse Mills ratio.¹⁴

The following four cases summarize the different possibilities when estimating the expected labor supply function:

Case 1: There is no censoring and the budget set is nonlinear. In this case the expected labor supply function is given by (10) as derived in Blomquist and Newey (2002).

Case 2: There is no censoring and the budget set is linear. In this case bias due to nonlinearity is zero and the expected labor supply function is given by (7). This can be estimated using nonparametric estimation methods such as kernel regression or series estimation (Hausman and Newey, 1995).

Case 3: There is censoring and the budget set is nonlinear. Then the expected labor supply function is given by (12). This expression is additive in several nonparametric functions. It can be estimated by approximating the different terms with a power series or spline. Semiparametric methods such as Symmetrically Censored Least Squares (SCLS) (Powell, 1986) and Censored Least Absolute Deviations (CLAD) (Powell, 1984) can be used to econometrically handle the

¹⁴ This expression is derived in most econometrics textbooks. One can even use least squares to estimate the parameters of this function if Φ and λ are estimated using a Probit (Amemiya, 1985).

censoring in hours of work.¹⁵ Alternatively, as suggested in Blomquist and Newey (2002), the expected labor supply function (12) can be estimated conditional on participation in the labor force along with a labor force participation equation.

Case 4: There is censoring and the budget set is linear. In this case again bias due to nonlinearity is zero and the expected labor supply function collapses to a nonparametric function in net wage evaluated at the first dollar and virtual income, censored at zero. It can be estimated using nonparametric methods for censored variables (e.g., Lewbel and Linton, 2003).

4. Econometric Specification

In the empirical analysis of female labor supply, I use the method from Case 3 above, where the labor supply function is given by

$$h_{it} = \int_{\pi_1^{-1}(0)}^{\infty} \pi_1(\eta_{it}) g(\eta_{it}) d\eta_{it} + \left[\bar{\pi}(y_{itJ}^v, \omega_{itJ}) - \bar{\pi}(y_{it1}^v, \omega_{it1}) \right] + \sum_{j=1}^{J-1} \left[\mu(y_{itj}^v, \omega_{itj}, \text{kink}_{itj}) - \mu(y_{itj+1}^v, \omega_{itj+1}, \text{kink}_j) \right] + \sum_{j=1}^m \gamma_j z_{it} + \varepsilon_{it}, \quad (21)$$

where η_{it} represents heterogeneity error and ε_{it} represents measurement error in hours of work,

y_{it}^v is the virtual income, ω_{it} , the after-tax wage and z_{it} represents other demographic variables,

i.e., age, number of children in different age ranges, and self reported health status. Thus there

are two sources of stochastic variation in the model in (21): individual level heterogeneity and

measurement error in hours.¹⁶ Let x_{it} be the vector of all explanatory variables

$(\omega_{it1} \dots \omega_{itJ}, y_{it1}^v \dots y_{itJ}^v, l_{it1} \dots l_{itJ-1}, z_{it})$. I allow the effect of regressors other than after-tax wage and

¹⁵ For a motivation on the idea to use semiparametric estimators such as SCLS and CLAD to estimate flexible regression functions, see Lewbel and Linton (2003).

¹⁶ Most papers using MLE approach estimate at most a dual error term model, one error capturing heterogeneity and the other denoting either measurement or optimization error.

virtual income to enter the hours function linearly as done in (Blomquist and Newey, 2002; Wu, 2002). I follow Blomquist and Newey (2002) in assuming that $E(\varepsilon_{it} | x_{it}) = 0$ and $Cov(\eta_{it}, \varepsilon_{it}) = 0$. In the remainder of the paper, I will call the expected labor supply function (21), the “censored specification” as it is estimated with nonworking females in the data with zero hours.

An attractive feature of (21) is that the linear budget set specification is nested within the nonlinear budget set specification. I can test for the exclusion of the nonlinearity bias term in (21). Similarly, the widely used linear labor supply specification is also nested within (21) as one can test the null hypothesis whether the coefficients on higher powers and interactions of ω and y^v enter significantly.

5. Data

I use data on female labor supply from the 1985 and 1989 waves of the PSID. The PSID began in 1968, and is a longitudinal study of a representative sample of U.S. individuals and the family units in which they reside. The sample consists of married women. Women belonging to Survey of Economic Opportunity (SEO) subsample were excluded from the analysis sample as they were nonrandomly selected. The final sample consists of 1771 married women. Table 1 provides the descriptive statistics on selected variables. Figure A1 in Appendix 1 presents a distribution of the annual hours worked in the data.¹⁷ Table A1 in Appendix 1 provides and accounting for observations excluded from the sample.

5.1 Identification Strategy

The data used in the paper satisfies the conditions for identification derived in Blomquist and Newey (2002). A primary condition for identification is that we should have individuals with

observations on both the first dollar and the last dollar wage and virtual income. We have this information for everyone in the sample. I use both cross-sectional as well as time-series variation in budget set variables to identify the wage and income effect. The dataset used in this paper spans one of the most comprehensive tax reforms- Tax Reform Act of 1986 (TRA 1986). In particular, drastic change in the budget set of individuals brought about by TRA 1986 due to reduction in the number of federal tax brackets, aids in identification. I also use the state level variation in tax rates to identify the labor supply effects of tax reforms. The calculation of kinks and slopes takes into account this state level variation in tax rules. The expected labor supply function derived in the paper can be thought of consisting of two components: the average labor supply function when the budget set is linear; and a term that corrects for the nonlinearity of the budget set. The former depends on the first dollar tax rate, which in a secondary earner married females is the just the last dollar tax rate of the husband. Thus there is sufficient variation in the first dollar budget set slopes and virtual incomes which help in identification.¹⁸

I must mention one important caveat to the identification strategy in this paper. In line with a voluminous literature in labor economics I treat gross wage as exogenous. This assumption will clearly be violated if wages are correlated with unobserved taste for leisure or if wages and hours are jointly determined. However, the nonlinear budget set approach of Blomquist and Newey (2002) fully takes into account the endogeneity of the marginal tax rate

¹⁷ The observed distribution of the hours in the data should, to some extent, mitigate the concern that there is insufficient variation in the number of hours worked.

¹⁸I did not include a fixed effect in the specification presented in equation (21) as it became collinear with the cross sectional variation. It is important to use the cross-sectional variation as the time series variation may not suffice. Handling fixed effects in a nonparametric context presents additional challenges that are beyond the scope of this paper and will be explored in future research. Although the inclusion of a fixed effect would help in dealing with possible endogeneity of after tax wage and virtual income, in the estimation framework of Blomquist and Newey (2002), variation in the entire budget set of the individuals mitigates that concern somewhat. Another reason why, fixed effects may not be very useful is that an important source of stochastic variation is heterogeneity error which has been modeled nonparametrically.

and hence the net wage and virtual income by considering the complete budget set of the individual.¹⁹

5.2 Construction of the budget set variables

The tax information for all the individuals in the dataset was obtained using the NBER TAXSIM calculator (Feenberg and Coutts, 1993). To determine all of the slopes and kinks, I ran a grid of adjusted gross income (AGI) levels from 0 to \$200,000 at increments of \$1000, through TAXSIM.²⁰ The federal marginal tax rate information for every income level was used to calculate the slopes and kinks for every individual. I assumed that females are secondary earners. So, while laying out the budget set, I assumed that a married female's budget set would start after the husband's last-dollar tax rate.²¹ This further helped in reducing the dimensionality of the budget set by reducing the number of kinks and segments. The payroll tax was taken into account in making the budget set of the individual. To further reduce the dimensionality of the budget set, I assumed an upper limit of 5000 on number of hours worked.²²

¹⁹Exogeneity of gross wage is a standard assumption in the literature on taxation and labor supply. In this context the most important issue is the endogeneity of the marginal tax rate due to nonlinear nature of the budget set. The endogeneity of taxes contaminates the net wage and virtual income variables and is solved by an instrumental variable approach if the budget set is linearized (Ziliak and Kniesner, 1999; Eissa, 1995; Blundell, Duncan and Meghir, 1998) or by using the global information on the complete budget set (Burtless and Hausman, 1978; Triest, 1990, Heim and Meyer, 2004; Blomquist and Newey, 2002;). For a discussion of how fully accounting for nonlinearity solves the endogeneity of taxes, also see Heckman and MaCurdy (1982).

²⁰ More specifically, I ran data on every individual through taxsim to obtain the marginal tax rate at every point of the grid of labor income. This allowed me to calculate slopes and kinks for every individual conditional on other characteristics i.e. tax filing status, marital status and other variable required by taxsim.

²¹The secondary earner model is a standard assumption in much of literature on taxes and female labor supply (e.g., Hausman, 1985; Triest, 1990; Eissa, 1995). In this model the labor supply decisions within the family are sequential with the husband choosing his labor supply under the assumption of no other labor income and then wife chooses her labor supply conditional on husband's labor income. Of course, if labor supply decisions are made at the family level, the individual labor supply will also depend on spouse's wages and the pooled income and the estimates presented here will be biased. The direction of the bias is generally not known and depends on the relationship between labor supply and wages of the spouse (whether they are substitutes or complements). There is no consensus on this relationship. Further the unitary model of family labor supply has been found to be restrictive as the assumptions of Slutsky symmetry and income pooling do not stand up to econometric testing. Collective models of family labor supply relax this assumption and assume that labor supply decisions are Pareto-efficient (Chiappori, 1988) or they are Nash-bargained solutions (McElroy, 1981). In these models assumption of household production is a problem (Blundell and MaCurdy, 1999). Combining the estimation approach presented here in the framework of collective labor supply is something that I will pursue in future research.

²² There were inconsequentially small number of individuals in the sample who worked more than 5000 hours.

I do not compute the gross wage by dividing annual earnings by number of hours, because this induces division bias. The self reported measure of wage that I use has been found to be a more robust measure of wage (Ziliak and Kniesner, 1999). In PSID, the information on wife's hourly wage was collected from the head. First, the head was asked whether the wife was salaried or paid by the hour. The follow up question was: "How much is her salary?" The values for this variable represent dollars and cents per hour; if salary is given as an annual figure, it is divided by 2000 hours per year; if weekly, by 40 hours per week.

Figure 2 presents the tax structure before and after TRA 1986 for a person filing married jointly, claiming two dependent exemptions and under age 65. As the figure suggests TRA 1986 resulted in a remarkable simplification of the budget set. Figure 3 shows the distribution of number of kinks before and after TRA 1986. After TRA 1986 most individuals ended up with three or less kinks in their budget set.²³ The budget set before the tax reform was highly nonlinear. This dramatic simplification in the tax code and the budget set is a crucial source of identification in the estimation framework suggested in Blomquist and Newey (2002).

There are three potential sources of nonconvexity in the budget set: the Earned Income Tax Credit (EITC), the payroll tax and fixed costs. Because, I assume that married females are secondary earners, very few individuals face the nonconvex portion due to the EITC.²⁴ The nonconvexity caused by the cap on the payroll tax, occurs at an extremely high number of hours.

²³ While looking at the number of kinks it is important to keep in mind that in a secondary earner model, the budget set of wives starts with the marginal tax rate of the husband as the tax rate on the first segment.

²⁴ The first dollar tax rate for married women in a secondary earner's model is the last dollar tax rate on her husband. Thus, wife's budget set only begins at husband's marginal tax rate. Nonconvexities can be addressed specifically by including additional terms in the estimating equation (13). However they will raise the dimensionality in nonparametric estimation. Therefore I account for nonconvexities in the budget set by taking a convex hull (MaCurdy, Greene and Paarsch, 1990). This method does not offer a tractable solution to nonconvexities caused due to fixed costs of work. Incorporating fixed costs of work in the estimation framework derived in this paper is a potential area of future research. Nevertheless, one of the attractive features of the nonparametric estimation approach based on estimator proposed in Blomquist and Newey (2002) is that the integration over individual heterogeneity term η helps mitigate the problem caused by nonconvex budget sets.

6. Estimation

6.1 Estimation of Labor Supply Function with Censored Hours

The resulting model for the expected hours of work, as derived in Theorem 3.1, results directly from utility maximization over a globally convex budget constraint. It is useful to consider nonparametric estimators that facilitate imposing additivity implied by the model. Series estimation is particularly useful in estimating models like this (Stone, 1985).²⁵ To account for censoring, I have estimated the model using the Symmetrically Censored Least Squares (SCLS) (Powell, 1986)²⁶

The econometric specification (21) is additive in nonparametric components that depend on slope, virtual income and variables representing the nonlinearity of the budget set. The demographic variables enter linearly. Hausman and Newey (1995) showed that using power-series approximations to model the nonparametric components conveniently results in partialling out the linear component of (21) in the sense of Robinson (1988).

6.2 Nonparametric Estimation of the Reduced Form Participation Equation

I estimated the following reduced form labor force participation equation using locally weighted regression methods.

$$P(dLFP_{it} = 1 | age_{it}, education_{it}) = m(age_{it}, education_{it}, children_{it} < 6yrs) + e_{it} \quad (22)$$

$dLFP_{it}$ is a dummy variable for labor force participation of individual i at time t . Letting the vector x_{it} contain $(age_{it}, education_{it}, children < 6yrs)$, the nonparametric regression function is

²⁵ One can in principle do nonparametric kernel estimation of the regression function and the derivatives but there is no obvious way to impose additivity in kernel regressions.

²⁶ SCLS is the least square counterpart of estimating regression functions with censored dependent variables. The estimation proceeds in a recensoring step and then a regression step using least squares. The Censored Least Absolute Deviation (CLAD) estimator uses median regression instead of least squares. I estimated all the specifications using CLAD and found that the results were similar to SCLS. I report only the SCLS results in the paper. For an excellent intuitive discussion of SCLS and CLAD, see Chay and Powell (2001).

estimated by local polynomial smoothing that minimizes the following criterion function (Cleveland, Devlin and Grosse, 1988):

$$\sum_{i=1}^N K_h(x_{it} - x)[dLFP_{it} - b_0 - b_1(x_{it} - x) - b_2(x_{it} - x)^2]^2 \quad (23)$$

where $K_h(u) = \frac{1}{h} K\left(\frac{u}{h}\right)$ is a nonnegative weight function with bandwidth h .²⁷ The weight function varies inversely with the distance between x and its i_{th} neighbor x_i . Local polynomial regression has many desirable properties including bias reduction feature at the boundary of the distribution. The choice of bandwidth h determines the rate of decrease in weight with increase in distance between x and x_i . The regression function is estimated as $\hat{m}(age_{it}, education_{it}) = \hat{b}_0$. I selected the bandwidth using generalized cross-validation (Craven and Wahba, 1979).

6.3 Nonparametric Estimation of the Wage Equation

The specification derived in (21) requires wage data for everyone in the sample- workers and non-workers. Due to the non-availability of gross wages for non-workers, the imputed wage was used for non-workers, as has been done by previous researchers (e.g. Hausman and Ruud, 1986; Van Soest, Woittiez and Kapteyn, 1990). To be consistent with the nonparametric estimation strategy, I estimated a nonparametric wage equation with nonparametric selection correction using the estimator proposed in Das, Newey and Vella (2003). The procedure involves estimating the propensity score from a first stage nonparametric regression of the selection equation and then entering them nonparametrically (e.g. power series, spline, kernel) in the primary equation. The following wage equation was estimated:

²⁷ I used the tricube weight function $K(u) = (1 - u^3)^3, 0 \leq u \leq 1$

$$E(\text{Wage}_{it} | \text{age}_{it}, \text{education}_{it}) = g_1(\text{age}_{it}, \text{education}_{it}) + g_2(\hat{\psi}_{it}) \quad (24)$$

where $\hat{\psi}_{it}$ is the estimated propensity score from the nonparametric estimation of the labor force participation equation. The wage equation was modeled as an additive function of age and education and the estimated propensity score from the first step. I used Generalized Additive Model (Hastie and Tibshirani, 1990), to estimate the wage equation.²⁸ The nonparametrically estimated wage equation was used to impute wages for non-workers, to estimate the censored specification.

6.4 Hours Elasticities Conditional on Working

One of the important characteristics of female labor supply is its tendency to be elastic on both participation and intensive margins. While estimates from equation (21) give us the overall labor supply elasticity, it is more informative to be able to calculate the labor supply response both on the participation and the intensive margin. Moreover recent work shows that female labor supply is more responsive on the participation margin (cite Heckman, 1994; Heim, 2004).²⁹

Blomquist and Newey (2002) suggest that when the probability of nonparticipation is positive, a simultaneous estimation of the labor force participation decision and the hours of work conditional on participation may be considered. Accordingly, I estimated equation (21) only on the sample that worked positive hours by including a selection bias term estimated from

²⁸ In this estimation method the underlying assumption is that mean of wage depends on age and education through a nonlinear link function. For example if mean of wage is ϖ then it is linked to age, education and the selection correction term by the function $f(\varpi) = g_1(\text{age}_{it}, \text{education}_{it}) + g_2(\psi_{it})$. I used a gaussian link function. The unknown functions $g_1(\text{age}_{it}, \text{education}_{it})$ and $g_2(\psi_{it})$ are estimated flexibly by smoothing. I used a cubic smoothing spline to estimate the $g_i(\cdot)$ functions. The smoothing parameter or the degrees of freedom was chosen using generalized cross-validation.

²⁹ The typical censored estimator in the Tobit framework also constrains the parameters on the extensive and the intensive margin to be the same and imposes a continuous labor supply restriction. Estimating the selection corrected hours equation mitigates this concern somewhat.

a reduced form nonparametric regression of the decision to work on age and education.³⁰ More specifically, I estimated the following specification,

$$E(h_{it} | h_{it} > 0) = \int_{\pi_1^{-1}(0)}^{\infty} \pi_1(\eta_{it}) g(\eta_{it}) d\eta_{it} + \left[\bar{\pi}(y_{itJ}^v, \omega_{itJ}) - \bar{\pi}(y_{it1}^v, \omega_{it1}) \right] + \sum_{j=1}^{J-1} \left[\mu(y_{ij}^v, \omega_{ij}, \text{kink}_{ij}) - \mu(y_{ij+1}^v, \omega_{ij+1}, \text{kink}_j) \right] + \sum_{j=1}^m \gamma_j z_{it} + f(\hat{\psi}_{it}) + v_{it}, \quad (25)$$

where $f(\hat{\psi}_{it})$ is a function of the estimated propensity score from estimating the labor force participation equation (22). Since (25) has age and higher order terms in it, education and its higher orders serve as the exclusion restrictions to identify the selection bias correction term in estimating (21) only on workers. This strategy for selection correction closely follows Das, Newey and Vella (2003). The difference in the estimated elasticities from estimating (21) on the entire sample and equation (25) on the sample of workers gives us an indication of the relative importance of extensive or the intensive margin.³¹

6.5 Estimation of Nonparametric Labor Supply Function with Nonlinear Budget Sets

The first three terms of the specification in (21) and (25) are approximated nonparametrically using power series, as in Blomquist and Newey (2002) and Hausman and Newey (1995). For ease of exposition, in this section I drop the subscripts i and t for individuals and time respectively and just keep the subscripts for segment j . The first term in (21) is approximated as

$$\varphi^K(\omega_1, y^v) = (\varphi_{1K}(\omega_1, y_1^v), \dots, \varphi_{KK}(\omega_1, y_1^v))', \quad (26)$$

³⁰ I also estimated the labor participation equation using the single index model estimator proposed in Klein and Spady (1993). The results were similar.

³¹ Estimation of (21) on the entire sample using censored regression methods yields an estimate of the total elasticity. Estimation of the labor supply function derived in (25), in turn, provides an estimate of the elasticity on the intensive margin. Using the decomposition suggested in McDonalds and Moffitt(1980) then allows me to recover the participation elasticity as the difference between the total and intensive margin. Triest (1990) use a similar insight. I did not estimate a structural labor force participation analogue of (22) as nonlinearities in the budget set play a less important role in the participation decision unless there are fixed costs of work. Accounting for fixed costs of work is beyond the scope of this paper. If fixed costs are ignored, the labor force participation decision will be based on the wage on the first segment of the budget set.

where

$$\varphi_{kK}(\omega_1, y^v) = \omega_1^{a(k)} y_1^{v b(k)} \quad (27)$$

are approximating functions.³² The second term $[\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)]$ is approximated as

$$\xi^K(\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)) = (\xi_{1K}(\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)), \dots, \xi_{KK}(\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)))' \quad (1)$$

where

$$\xi_{kK}(\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)) = \omega_J^{c(k)} y_J^{v d(k)} - \omega_1^{c(k)} y_1^{v d(k)}. \quad (28)$$

Finally, the third term is approximated exactly as in Blomquist and Newey (2002), i.e.,³³

$$\begin{aligned} & \zeta^K \left(\sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, \text{kink}_j) - \mu(y_{j+1}^v, \omega_{j+1}, \text{kink}_j)] \right) = \\ & \left(\zeta_{1K} \left(\sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, \text{kink}_j) - \mu(y_{j+1}^v, \omega_{j+1}, \text{kink}_j)] \right), \dots, \zeta_{KK} \left(\sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, \text{kink}_j) - \mu(y_{j+1}^v, \omega_{j+1}, \text{kink}_j)] \right) \right)' \end{aligned} \quad (29)$$

$$\zeta_{kK} = \sum_{j=1}^{J-1} l_j^{m(k)} (y_j^{vp(k)} \omega_j^{q(k)} - y_{j+1}^{vp(k)} \omega_{j+1}^{q(k)}). \quad (30)$$

³² I modeled $\varphi^K(\omega_1, y^v)$ as a bivariate polynomial approximation of order θ . I chose θ using SBIC and AIC criteria. More specifically $\varphi^K(\omega_1, y^v) = \sum_{i,j:i+j \leq \theta} \beta_{ij} \omega_1^i y^j$. For example if $\theta=3$, $\varphi^K(\omega_1, y^v)$ was estimated as

$\beta_{00} + \beta_{10}\omega_1 + \beta_{20}\omega_1^2 + \beta_{30}\omega_1^3 + \beta_{01}y^v + \beta_{02}y^{v2} + \beta_{03}y^{v3} + \beta_{11}\omega_1 y^v + \beta_{21}\omega_1^2 y^v + \beta_{12}\omega_1 y^{v2}$

³³ $\xi^K(\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)), \zeta^K \left(\sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, \text{kink}_j) - \mu(y_{j+1}^v, \omega_{j+1}, \text{kink}_j)] \right)$ were also modeled analogously to the first

term. However θ was set to 2 as the terms became collinear. Also sufficient coefficient restrictions were imposed to prevent perfect collinearity in $\left(\varphi^K(\omega_1, y^v), \xi^K(\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)), \zeta^K \left(\sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, \text{kink}_j) - \mu(y_{j+1}^v, \omega_{j+1}, \text{kink}_j)] \right) \right)$. For

example $\xi^K(\bar{\pi}(y_J^v, \omega_J) - \bar{\pi}(y_1^v, \omega_1)) = \phi_{10}\Delta\omega_1 + \phi_{20}\Delta\omega_1^2 + \phi_{01}\Delta y^v + \phi_{02}\Delta y^{v2} + \phi_{11}\Delta\omega_1 y^v$ where Δ is the difference operator between segment 1 and segment J. Analogously $\zeta^K \left(\sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, \text{kink}_j) - \mu(y_{j+1}^v, \omega_{j+1}, \text{kink}_j)] \right)$ contained terms

like $\kappa_{10}\sum_j l_j \Delta\omega_j + \kappa_{20}\sum_j l_j \Delta\omega_j^2 + \kappa_{01}\sum_j l_j \Delta y_j^v + \kappa_{02}\sum_j l_j \Delta y_j^{v2} + \kappa_{11}\sum_j l_j \Delta\omega_j y^v$ where the summation is over all segments in the individual's budget set where Δ is the difference operator between segment j and $j-1$.

where l_j is the j^{th} kink.

Let the vector of approximating functions be written as

$$\varpi^K(x) = \left(\varphi^K(\omega_1, y^v), \xi^K(\bar{\pi}(y_j^v, \omega_j) - \bar{\pi}(y_1^v, \omega_1)), \left(\sum_{j=1}^{J-1} [\mu(y_j^v, \omega_j, kink_j) - \mu(y_{j+1}^v, \omega_{j+1}, kink_j)] \right) \right) \quad (31)$$

For observations $i = 1, \dots, n$, let $\Gamma = [\varpi^K(x_1), \dots, \varpi^K(x_n)]'$ be a matrix of all the approximating functions for all the individuals. For the vector of coefficients on the approximating functions, a series estimator for hours of work is defined as

$$\hat{h}(x) = \varpi^K(x)' \hat{\beta}, \quad (32)$$

and

$$\hat{\beta} = (\Gamma' \Gamma)^{-1} \Gamma' h \quad (33)$$

respectively.

Selection of the number of terms in the power series, K , is an important part of estimation. This is analogous to choosing the bandwidth for nonparametric kernel regression. I use well known Akaike Information Criterion (AIC) and Schwarz, Bayesian. Information Criterion (SBIC) to select the best model. It is well-known from lag-length selection in time series that AIC overestimates the order of the model (Shibata, 1976). So I put more emphasis on SBIC to select the best model.³⁴

7. Estimation Results

The estimated wage and income elasticities from five models are presented in Table 2-7. The number and description of terms included are presented in the note to the tables. Each of

³⁴ Both these criteria penalize the model fit for number of included regressors. The SBIC imposes a heavier penalty and so leans towards a more parsimonious model (Greene, 2003).

these tables present the estimated marginal effects and elasticities with respect to wage and income, the compensated elasticity and the estimated effect of change in the budget set due to TRA 1986 on Labor supply. Tables 2, 3 and 6 present the results for the censored specification (21) while Tables 4, 5 and 7 show analogous results for the conditional hours specification (25).

Tables 2-5 present the results from approximating $\int_{\pi_1^{-1}(0)}^{\infty} \pi_1(\eta)g(\eta)d\eta$ in (21) and (25) as a 1st to a

5th order polynomial in wage and income to nonparametrically model the expected labor supply function. Tables 6 and 7 present results from robustness check for the censored hours specification and the conditional hours specification respectively. Robustness results are presented for the preferred specification for the nonparametric term with a fourth order polynomial. For Tables 2-5, column 1 presents results for a 1st order polynomial, column 2 for 1st order polynomial and so on. 1st order polynomial collapses to a linear labor supply function and so the first column of Tables 2-5 contains the results for a linear specification of labor supply.³⁵ The results are presented for specifications controlling for the *nonlinearity bias* i.e. the second the third terms in (21) and without including these terms i.e. imposing the assumption that the budget set is linear. Table 3 and 4 present results for the censored hours specification while Tables 5 and 6 show analogous estimates for the conditional hours specifications.

7.1 Censored Hours Specification

In Table 2 i.e. for censored hours specification with nonlinear budget set, the estimate of the wage elasticity increases from 0.31 for a linear specification in column 1 to 0.60 for the specification with a 4th order polynomial in column 4. The wage elasticity estimate in column 1

³⁵This specification is comparable to reduced form labor supply specifications in many studies that proxy the observed net wage with the net wage evaluated at the first dollar or at a synthetic marginal tax rate computed by assuming a constant number of hours for each individual in the sample. This strategy has been adopted many papers to circumvent the problem with the endogeneity of the observed after-tax-wage, e.g., Rosen (1976), Hausman and Wise (1976), Heim (2005).

is akin to a semiparametric counterpart of the typical linear labor supply estimation estimated with nonlinear budget set method using maximum likelihood (e.g. Triest, 1990).³⁶ The estimates in column 1 can also be compared with linear labor supply specifications using the after-tax-wage evaluated at the first-dollar marginal tax rate as an instrument for the endogenously observed after-tax-wage.³⁷ The income elasticity increases (in absolute value) from -0.54 to -0.71.³⁸ The compensated elasticity increases from 0.40 to 0.71. Thus Table 2 indicates that including higher powers of wage and virtual income and their interactions increases the labor supply elasticities.³⁹ Comparison across columns 1-5 in Table 2 and Table 3 gives a sense of the sensitivity of estimated effects and elasticities to flexible specifications of the average labor supply function in net wage and virtual income. Comparing Table 2 with the corresponding columns in Table 3 shows the effect of including the terms due to nonlinearity of the budget set. In general, the elasticities increase with more flexible specifications and when *nonlinearity bias*

³⁶The semiparametric nature stems from not assuming any distributions for the error term by estimating it using Symmetrically Censored Least Squares. This estimator just imposes symmetry of the error distribution. Estimating a labor supply function with the terms included to control for the nonlinearity of the budget set using a Tobit likelihood function should yield estimates comparable to Hausman's maximum likelihood method albeit with less efficiency as the expected labor supply function in Blomquist and Newey (2002) approach uses only information on the first moment. I estimated the censored hours specification using Tobit and found that estimated elasticities were twice as large as the ones obtained in column (1) of table 4. The estimates were comparable with maximum likelihood estimates in Triest (1990) who used 1983 wave of the PSID and estimated both a censored and a truncated estimator. The results from a truncated specification for a linear labor supply in Triest (1990) are comparable to the estimates reported in column (1) of table 5.

³⁷ For example, Eissa (1995) who estimates an elasticity of around 0.6-1 for females married to high income men.

³⁸These estimates are comparable within the range of estimates surveyed in Heckman and Killingsworth (1983) and Mroz (1987). However, the income elasticity estimated here is on the higher side of the literature on estimation of female labor supply tax effects in a nonlinear budget set environment, using the PSID. Hausman (1981) estimated income elasticity for females working full time of -0.5. Triest (1990) estimated income elasticity between -0.15 to -0.31. Rosen (1976) estimated an income elasticity of -0.42. Hausman and Ruud (1986) estimated -0.36. Thus, my estimates of income elasticity are qualitatively similar to the previous literature.

³⁹The elasticities are calculated at the sample mean of after-tax wage and virtual income on the first segment of \$5.21 and \$34,700 respectively and at mean annual hours of 1140. These means are over both workers and nonworkers. This implied a share of earnings relative to non-labor income of 0.17, used to compute the compensated elasticity using the formula $E_c = E_w - E_y b$, where E_c , E_w , E_y are the compensated, wage and income elasticities and b the budget share.

terms are included.⁴⁰ I performed a Wald test to test the significance of the *nonlinearity bias* terms and P-values indicate that these terms cannot be excluded from the specification. Both AIC and SBIC criterion are minimized for the cubic polynomial specification.

7.2 Hours Elasticities Conditional on Participation

Tables 4 and 5 present the estimates of marginal effects and elasticities on the intensive margin from estimating equation (25) i.e. restricting the sample to workers while correcting nonparametrically for selection bias. The estimated intensive margin elasticities are 30-40 percent of the magnitude of the estimates of total hours elasticities in Tables 2 and 3, suggesting that estimated labor supply is more responsive on the participation margin both with respect to wages and non-labor income. The wage elasticity ranges from 0.18 to 0.29 from column 1 to 5. The income elasticity varies from -0.11 to -0.15.⁴¹ The results mirror the findings in the censored hours case as the elasticities are higher in specifications with higher order terms. However, controlling for the *nonlinearity bias* term does not seem to matter. This suggests that modeling the nonlinearity of the budget set is even more important when there is nonparticipation.⁴²

7.3 Robustness Check

Tables 6 and 7 indicate that the estimated marginal effects and elasticities are fairly robust to inclusion of other regressors. These results are from modeling the first term in (21) and (25) as a 4th order polynomial. Column 1 contains the results from the baseline specification with children below six years, quartic in age and poor health as controls, additional regressors included are race, union (column 2), a dummy if years equals 1985 (column 3). Column 4, 5 and 6 include

⁴⁰ Of course the confidence intervals around the estimates obtained from specifications including the bias term and those obtained without including the bias term overlap, indicating that the difference is not statistically significant. But the terms representing the bias term in (21) were jointly statistically significant.

⁴¹ The elasticities are calculated at the sample mean of after-tax wage and virtual income on the first segment of \$5.35 and \$32,960 respectively and at mean annual hours of 1616, for working individuals.

⁴² This may also mean that fixed costs are important as they alter the budget set of the individuals in such a way that accounting for nonlinear budget set becomes important even for modelling participation decision (Hausman, 1980).

occupation fixed effects, state fixed effects and occupation and state interactions respectively. The estimated wage elasticities range from 0.5 to 0.7 for the censored hours specification and are remarkably stable at around 0.27 for the hours regression conditional on working. The income elasticities range from -0.4 to -0.67 for the censored specification and about -0.14 for the conditional hours specification.⁴³ The AIC and SBIC criteria suggest picking the baseline model.

Figures 6-11 present a graphical illustration of the downward bias in magnitudes of the estimated elasticities when the nonlinearity bias term is omitted from the specification in (21) or (25). The downward bias is larger for the less flexible specifications and is the largest for the linear labor supply specification. This suggests that inclusion of higher powers and interactions of net wage and virtual income partially captures the misspecification due to omission of the nonlinearity bias terms. Figures 8 and 9 indicate that the downward bias in estimated income elasticities is more pronounced for the censored hours specification than for the conditional hours specification.

7.4 Effects of TRA 1986

TRA 1986 resulted in changes in the budget set of individuals belonging to different groups. The variation in the budget sets can be used to identify the average effect of change in tax structure on labor supply. After estimating the parameters of the labor supply function, it is straightforward to calculate nonparametrically, the effect of the tax change from the difference between estimated average labor supply responses based on pre and post-TRA1986 budget set. Let the budget set before and after TRA1986 be $Z^{pre-TRA86}$ and $Z^{post-TRA86}$ respectively, then as

⁴³ The wage elasticity estimates found in this paper are within the range of estimates presented in the previous literature. Blundell and MaCurdy (1999) tabulate the results from several studies. The estimates are clustered around 0.7-1, although they are highly sensitive to estimation method.

suggested in Blomquist and Newey (2002), the estimated tax change effect of the tax reform can be written as⁴⁴

$$\hat{M} = \frac{\sum_{i=1}^N \hat{h}(Z_i^{post-TRA86}) - \sum_{i=1}^N \hat{h}(Z_i^{pre-TRA86})}{\sum_{i=1}^N \hat{h}(Z_i^{pre-TRA86})} \quad (34)$$

I evaluated $\hat{h}(Z_i^{post-TRA86})$ and $\hat{h}(Z_i^{pre-TRA86})$ at the mean of individual characteristics. The estimates measure the impact of the experiment of changing the budget set of an individual- with mean net wage, virtual income and other characteristics- from pre-TRA1986 to post-TRA1986 while holding all the variables constant at the pre-1986 level. The results are presented in the row labeled “Tax Change Effect”. Even though the effects were imprecisely estimated, the point estimates lend support to the claim that accounting for nonlinearity matters in measuring the tax change effects. The estimated TRA 1986 effect calculated from nonlinear budget set estimation exceeds that obtained from imposing a linear budget set. Using the censored specification, TRA-1986 increased labor supply by about 4-6% at the sample mean. On the intensive margin, the response was lower at about 1%. This suggests that most of the labor supply effects of TRA 1986 may have been on the participation margin.⁴⁵

7.5 Behavior of the Labor Supply Curve at different Wage Levels

Nonparametric modeling of labor supply allows one to robustly quantify elasticities at different points in the distributions of the wage and virtual income. The most widely-used

⁴⁴ The estimation of average tax effects is based on individuals who are observed in both time-periods. Specifically, I calculated the effect of changing the pre-TRA 1986 budget set to post-TRA 1986 budget set while holding constant the pre-TRA 1986 characteristics at the mean. Of course, the main contribution of this paper in estimating tax change effects of TRA 1986 is that they are based on nonparametric estimates using Blomquist and Newey (2002).

⁴⁵The estimated response of TRA 1986 in this paper is much lower than the estimated in Eissa (1995) who estimated a response of 12.3% and 14.6% for women in the 75th percentile and 90th percentile of the income distribution respectively. The corresponding participation response was 4.3% and 11.4% respectively. However, given that estimates in this paper are for average married women, the differences between the two sets of estimates can be explained by the difference in points of the income distribution.

models for labor supply constrain the marginal effects to be constant by estimating a linear labor supply curve. With a flexible labor supply function estimated in this paper, I can examine the nonlinear behavior of the labor supply curve with respect to wage levels.⁴⁶ The marginal effects are calculated at the mean non-labor income while allowing the wage to vary. Figures 4 and 5 plot the estimates of marginal effects of wage and income against a grid of wages under two scenarios, (1) controlling for the bias term in (21), and, (2) omitting the bias term due to nonlinearity of the budget set in (21) and imposing a linear budget set. The effect of the bias is apparent from the vertical distance between the solid line and the dashed line. In Figure 4, the response to changes in the wage declines with increases in the wage rate. At higher wages, of say above \$12, the uncompensated wage elasticity from the nonlinear budget set specification turns negative, which suggests that the labor supply curve exhibits a backward-bending behavior. Not accounting for bias due to nonlinearity of the budget set results in underestimation of the wage effect at lower wages, while for higher wages, there is an upward bias. Figure 5 shows how the income effect varies with wage. The income effect is also decreasing in wages. For most parts of the wage distribution, the effect of not accounting for a nonlinear budget set leads to an underestimation of the income effect. Figure 4 indicates that labor supply curve is backward-bending. Even though the income effect is declining in wage, at higher wages income effect is still large enough to dominate a steeply falling uncompensated wage effect. It is interesting to note that the linear budget set specification does not capture the backward-bending nature of the labor supply curve.

⁴⁶In a flexible labor supply specification, the wage effect will vary with wage and income levels due to the interaction terms between wage and income included in the specification.

8. Conclusions and Extensions

Estimating demand models assuming that the agent's budget set is linear, when, in fact, there are kinks in the budget set, can produce estimates of behavioral parameters that are biased and inconsistent. Most existing studies on taxes and labor supply in the presence of a nonlinear budget set have used either Hausman's Maximum Likelihood approach or instrumental-variables estimation of the marginal labor supply function. While the Maximum Likelihood approach relies on strong parametric assumptions for consistency, the instrumental-variables requires a valid instrument for the marginal tax rate. Recently, Blomquist and Newey (2002) have proposed an estimation strategy that does not rely on arbitrary distributional or functional form assumptions and effectively accounts for the bias due to nonlinearities in the budget set.

In this paper, I extend their estimator to the case where the dependent variable is censored. I augment the sparse literature on nonparametric estimation of labor supply models and apply the newly-suggested method to estimate female labor supply elasticities. I investigate both possible sources of bias in estimates of wage and income elasticities for female labor supply: first, due to restrictive functional forms and, second, by ignoring nonlinearity of the budget set.

The wage and income elasticities are underestimated if the nonlinearity is ignored. I estimate an uncompensated wage elasticity of 0.6-0.74 overall and 0.26-0.29 on the intensive margin. My estimates of income elasticity are somewhat higher than the previous literature and range from -0.4 to -0.67 for total hours and -0.12 to -0.15 on the intensive margin. In particular, estimates of the income elasticity from the nonlinear budget set specification are about 25-30 percent higher than those from assuming a linear budget set. These estimates imply a compensated elasticity of about 0.61-0.81 overall and close to 0.30 on the intensive margin. All

the elasticities are statistically significant. I also find evidence that the wage elasticity is declining in the wage, and the labor supply curve is backward-bending. The nonparametric estimates imply that TRA 1986 was associated with an increase in labor supply of 4-5% at the sample mean overall and 1% on the intensive margin although the estimates are imprecisely estimated. Most of the response of TRA 1986 was concentrated on the extensive margin.

This paper can be extended in several directions. First, the nonparametric estimates obtained here can be used to get new measures of the deadweight loss arising from taxation of labor supply by using the nonparametric approach proposed in Hausman and Newey (1995). The identification strategy calls for more exogenous variation in the budget set of individuals. Extending the econometric strategy proposed here to panel data spanning several years will be a fruitful area of future research. There are many instances where economic behavior results in corner solutions. These can only be modeled in a limited dependent variable framework. Examples include female labor supply, labor supply effects of social security earnings test, charitable contributions behavior, 401(k) contributions, and labor supply of the elderly.⁴⁷ The econometric strategy adopted here can be readily applied to these settings.

⁴⁷ Nonlinear budget techniques have also been employed to analyze the following issues: effect of tax deductibility on charitable contribution behavior (Rees and Zieshang (1995)); disability applications (Hausman 1985); employer matching on saving behavior (Engelhardt and Kumar (2003)); water demand; capital gains taxation; and housing demand. Nonlinear budget constraints can also arise in the private sector in case of goods which have a block-pricing structure or quantity discounts.

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Table 1: Summary Statistics

	Nonworking			Working			Overall		
	Mean	Std Dev	Median	Mean	Std Dev	Median	Mean	Std Dev	Median
Hours	0.00	0.00	0.00	1616.65	592.05	1752.00	1140.25	889.21	1356.00
Gross Wage	7.38	2.44	6.97	7.98	4.70	7.00	7.81	4.18	7.00
Marginal Tax Rate	33.23	10.38	34.62	35.73	5.97	34.75	35.00	7.63	34.70
First-Dollar Tax Rate	11.65	11.52	7.10	8.71	8.37	4.10	9.58	9.50	4.10
Adjusted Gross Income ('000)	31.68	24.96	29.00	43.75	22.43	40.00	40.19	23.84	37.31
Slope on the First Segment	4.88	1.71	4.59	5.35	3.25	4.62	5.21	2.89	4.61
Slope on the Last Segment	4.55	1.71	4.12	5.11	2.97	4.35	4.94	2.67	4.26
Virtual Income on the First Segment ('000)	38.84	24.55	34.76	32.96	18.93	29.50	34.70	20.91	30.45
Virtual Income on the Last Segment ('000)	39.39	24.10	34.50	33.50	18.89	29.96	35.24	20.73	30.92
Total Number of children	1.27	1.30	1.00	1.25	1.15	1.00	1.26	1.19	1.00

Family Size	3.57	1.36	4.00	3.52	1.17	4.00	3.53	1.23	4.00
Belongs to Union	0.02	0.12	0.00	0.13	0.33	0.00	0.09	0.29	0.00
Owens Home	0.84	0.36	1.00	0.85	0.36	1.00	0.85	0.36	1.00
Age	42.59	10.96	41.00	39.14	9.10	38.00	40.16	9.81	38.00
White	0.92	0.27	1.00	0.93	0.26	1.00	0.93	0.26	1.00
Education	12.42	2.49	12.00	13.12	2.25	12.00	12.91	2.35	12.00

Note: Nonparametrically predicted wages used as wages for nonworkers. After-tax wage, virtual income, and assets are expressed in 1989 dollars.

Table 2: Estimated Elasticities from Censored Specification with Nonlinear Budget Set

	(1)	(2)	(3)	(4)	(5)
Wage Effect	66.91 (16.22)	92.82 (15.32)	99.16 (18.60)	130.48 (19.78)	123.26 (30.62)
Wage Elasticity	0.31 (0.07)	0.42 (0.07)	0.45 (0.08)	0.60 (0.09)	0.56 (0.14)
Income Effect	-17.65 (2.62)	-19.04 (2.27)	-19.08 (2.98)	-22.09 (3.87)	-23.49 (5.42)
Income Elasticity	-0.54 (0.08)	-0.58 (0.07)	-0.58 (0.09)	-0.67 (0.12)	-0.71 (0.17)
Compensated Elasticity	0.40 (0.08)	0.52 (0.07)	0.55 (0.09)	0.71 (0.10)	0.69 (0.15)
Tax change effect	0.03	0.04	0.05	0.07	0.03
P-value on bias correction term	0.00	0.00	0.00	0.00	0.00
AIC	34567.62	34759.38	34275.21	34503.97	34406.47
SBIC	34719.58	34927.79	34464.87	34720.98	34655.93
Observations	1681.00	1690.00	1667.00	1678.00	1674.00
R-squared	0.30	0.31	0.30	0.31	0.32

Note: Dependent variable in all the regressions was annual hours of work. The results in this table are from estimating equation (21) in the text. This equation has four terms excluding the error term. The first term is the nonparametric labor supply function for a linear budget set and is modeled as a power series. In column (1) the order of the power series is one, in column (2) the order is two and so on. See footnote 32 for further description. The second and third terms represent the bias due to budget set nonlinearity. See footnote 33 for an explanation of how this term was modeled. The results in this table control for the bias terms. Other individual characteristics entered in the regression additively (i.e. the 4th term) were number of children, a quartic in age, self reported health status. The results presented in the table are from Symmetrically Censored Least Square (SCLS) estimation (Powell, 1985). Bootstrapped standard errors reported in parentheses are based on 99 replications. The wages for nonworkers were obtained from a nonparametric selection-corrected regression of wage on age and education by estimating equation (24). The elasticities are calculated at the sample mean of after-tax wage and virtual income on the first segment of \$5.21 and \$34,700 respectively and at mean annual hours of 1140. These means are over both workers and nonworkers.

Table 3: Estimated Elasticities from Censored Specification with Linear Budget Set

	(1)	(2)	(3)	(4)	(5)
Wage Effect	51.09 (12.47)	77.73 (12.33)	84.73 (17.51)	112.06 (16.37)	117.94 (30.92)
Wage Elasticity	0.23 (0.06)	0.36 (0.06)	0.39 (0.08)	0.51 (0.08)	0.54 (0.14)
Income Effect	-11.36 (2.08)	-13.28 (2.15)	-13.96 (2.66)	-15.77 (3.65)	-19.23 (4.31)
Income Elasticity	-0.35 (0.07)	-0.40 (0.07)	-0.42 (0.08)	-0.48 (0.11)	-0.59 (0.13)
Compensated Elasticity	0.29 (0.06)	0.42 (0.06)	0.46 (0.08)	0.59 (0.08)	0.64 (0.15)
Tax change effect	0.02	0.02	0.03	0.05	0.05
AIC	35451.24	35150.91	34660.89	34787.87	34669.29
SBIC	35500.28	35216.22	34747.75	34901.96	34815.88
Observations	1719.00	1706.00	1683.00	1690.00	1685.00
R-squared	0.26	0.28	0.27	0.29	0.29

Note: Dependent variable in all the regressions was annual hours of work. The results in this table are from estimating equation (21) in the text. This equation has four terms excluding the error term. The first term is the nonparametric labor supply function for a linear budget set and is modeled as a power series. In column (1) the order of the power series is one, in column (2) the order is two and so on. See footnote 32 for further description. The second and third terms represent the bias due to budget set nonlinearity. See footnote 33 for an explanation of how this term was modeled. The results in this table do not control for the bias terms. Other individual characteristics entered in the regression additively (i.e. the 4th term) were number of children, a quartic in age, self reported health status. The results presented in the table are from Symmetrically Censored Least Square (SCLS) estimation (Powell, 1985). Bootstrapped standard errors reported in parentheses are based on 99 replications. The wages for nonworkers were obtained from a nonparametric selection-corrected regression of wage on age and education by estimating equation (24). The elasticities are calculated at the sample mean of after-tax wage and virtual income on the first segment of \$5.21 and \$34,700 respectively and at mean annual hours of 1140. These means are over both workers and nonworkers.

Table 4: Selection Corrected Hours Elasticities Conditional on Working with Nonlinear Budget Set

	(1)	(2)	(3)	(4)	(5)
Wage Effect	54.11 (10.52)	67.95 (11.73)	82.54 (11.53)	84.53 (13.12)	88.19 (21.02)
Wage Elasticity	0.18 (0.05)	0.22 (0.05)	0.27 (0.05)	0.28 (0.06)	0.29 (0.10)
Income Effect	-5.41 (1.03)	-6.40 (1.27)	-7.70 (1.45)	-7.10 (2.18)	-7.50 (2.80)
Income Elasticity	-0.11 (0.03)	-0.13 (0.04)	-0.16 (0.04)	-0.14 (0.07)	-0.15 (0.09)
Compensated Elasticity	0.21 (0.05)	0.26 (0.05)	0.31 (0.05)	0.32 (0.06)	0.33 (0.10)
Tax change effect	0.01	0.01	0.02	0.02	0.01
P-value on bias correction term	0.00	0.00	0.00	0.01	0.01
P-value on selection bias	0.01	0.00	0.00	0.00	0.00
AIC	24535.00	24531.89	24488.34	24493.41	24504.47
SBIC	24698.91	24711.16	24688.10	24718.78	24760.58
Observations	1239.00	1239.00	1239.00	1239.00	1239.00
R-squared	0.13	0.14	0.17	0.17	0.18

Note: Dependent variable in all the regressions was annual hours of work. The results in this table are from estimating equation (25) in the text. This equation has five terms excluding the error term. The first term is the nonparametric labor supply function for a linear budget set and is modeled as a power series. In column (1) the order of the power series is one, in column (2) the order is two and so on. See footnote 32 for further description. The second and third terms represent the bias due to budget set nonlinearity. See footnote 33 for an explanation of how this term was modeled. The results in this table control for the bias terms. The fourth term is the nonparametric analogue of the inverse mills ratio obtained from a first stage nonparametric labor force participation equation (22). Other individual characteristics entered in the regression additively (i.e. the 5th term) were number of children, a quartic in age, self reported health status. The results presented in the table are from nonparametric selection-corrected hours regression using power series, conditional on participation in the labor force. Bootstrapped standard errors reported in parentheses are based on 99 replications. The elasticities are calculated at the sample mean of after-tax wage and virtual income on the first segment of \$5.35 and \$32,960 respectively and at mean annual hours of 1616, for working individuals.

Table 5: Selection Corrected Hours Elasticities Conditional on Working with Linear Budget Set

	(1)	(2)	(3)	(4)	(5)
Wage Effect	30.87 (8.00)	55.75 (11.82)	74.17 (11.34)	85.27 (12.43)	86.12 (18.28)
Wage Elasticity	0.10 (0.04)	0.18 (0.05)	0.25 (0.05)	0.28 (0.06)	0.29 (0.08)
Income Effect	-5.46 (0.90)	-6.60 (1.09)	-7.15 (1.17)	-6.79 (1.93)	-6.24 (2.58)
Income Elasticity	-0.11 (0.03)	-0.13 (0.03)	-0.15 (0.04)	-0.14 (0.06)	-0.13 (0.08)
Compensated Elasticity	0.13 (0.04)	0.22 (0.05)	0.28 (0.05)	0.32 (0.06)	0.32 (0.09)
Tax change effect	0.01	0.01	0.01	0.01	0.01
P-value on selection bias	0.01	0.00	0.00	0.00	0.00
AIC	24561.28	24539.18	24509.29	24497.01	24507.77
SBIC	24627.87	24621.13	24611.73	24625.06	24666.55
Observations	1239.00	1239.00	1239.00	1239.00	1239.00
R-squared	0.08	0.10	0.13	0.15	0.15

Note: Dependent variable in all the regressions was annual hours of work. The results in this table are from estimating equation (25) in the text. This equation has five terms excluding the error term. The first term is the nonparametric labor supply function for a linear budget set and is modeled as a power series. In column (1) the order of the power series is one, in column (2) the order is two and so on. See footnote 32 for further description. The second and third terms represent the bias due to budget set nonlinearity. See footnote 33 for an explanation of how this term was modeled. The results in this table do not control for the bias terms. The fourth term is the nonparametric analogue of the inverse mills ratio obtained from a first stage nonparametric labor force participation equation (22). Other individual characteristics entered in the regression additively (i.e. the 5th term) were number of children, a quartic in age, self reported health status. The results presented in the table are from nonparametric selection-corrected hours regression using power series, conditional on participation in the labor force. Bootstrapped standard errors reported in parentheses are based on 99 replications. The elasticities are calculated at the sample mean of after-tax wage and virtual income on the first segment of \$5.35 and \$32,960 respectively and at mean annual hours of 1616, for working individuals.

Table 6: Robustness of Censored Regression Results to Inclusion of other Controls

	(1)	(2)	(3)	(4)	(5)	(6)
Wage Effect	130.48 (23.93)	116.21 (24.77)	114.84 (25.04)	161.12 (23.75)	108.50 (25.73)	154.58 (24.19)
Wage Elasticity	0.60 (0.11)	0.53 (0.11)	0.52 (0.11)	0.74 (0.11)	0.50 (0.12)	0.70 (0.11)
Income Effect	-22.09 (4.02)	-21.91 (3.78)	-21.59 (3.80)	-13.41 (2.79)	-21.36 (4.00)	-13.11 (2.74)
Income Elasticity	-0.67 (0.13)	-0.67 (0.12)	-0.66 (0.12)	-0.41 (0.09)	-0.65 (0.12)	-0.40 (0.09)
Compensated Elasticity	0.71 (0.11)	0.65 (0.12)	0.64 (0.12)	0.81 (0.11)	0.61 (0.12)	0.77 (0.11)
Tax change effect	0.07	0.04	0.04	0.06	0.05	0.59
P-value on bias correction term	0.00	0.00	0.00	0.00	0.00	0.00
AIC	34503.97	34545.43	34570.44	35547.02	34589.80	35740.98
SBIC	34720.98	34773.37	34803.84	35820.36	35072.92	37233.73
Observations	1678.00	1681.00	1682.00	1749.00	1683.00	1751.00
R-squared	0.31	0.32	0.32	0.49	0.35	0.56
Race and Union Dummies	No	Yes	Yes	Yes	Yes	Yes
Year Dummy	No	No	Yes	Yes	Yes	Yes
Occupation Fixed Effects	No	No	No	Yes	Yes	Yes
State Fixed Effects	No	No	No	No	Yes	Yes
Occupation and State Fixed Effects	No	No	No	No	No	Yes

Note: Dependent variable in all the regressions was annual hours of work. The results in this table are from estimating equation (21) in the text. This equation has four terms excluding the error term. The first term is the nonparametric labor supply function for a linear budget set and is modeled as a power series. All columns present results for power series of order four. See footnote 32 for further description. The second and third terms represent the bias due to budget set nonlinearity. See footnote 33 for an explanation of how this term was modeled. The results in this table control for the bias terms. In the baseline model, other individual characteristics entered in the regression additively (i.e. the 4th term) were number of children, a quartic in age, self reported health status. Column (1) is for the baseline model. The results presented in the table are from Symmetrically Censored Least Square (SCLS) estimation (Powell, 1985). Bootstrapped standard errors reported in parentheses are based on 99 replications. The wages for nonworkers were obtained from a nonparametric selection-corrected regression of wage on age and education by estimating equation (24). The elasticities are calculated at the sample mean of after-tax wage and virtual income on the first segment of \$5.21 and \$34,700 respectively and at mean annual hours of 1140. These means are over both workers and nonworkers.

Table 7: Robustness of Conditional Regression Results to Inclusion of other Controls

	(1)	(2)	(3)	(4)	(5)	(6)
Wage Effect	84.53 (12.35)	87.58 (12.36)	86.69 (12.28)	81.88 (13.29)	86.38 (13.04)	80.03 (14.23)
Wage Elasticity	0.28 (0.06)	0.29 (0.06)	0.29 (0.06)	0.27 (0.06)	0.29 (0.06)	0.26 (0.07)
Income Effect	-7.10 (2.26)	-7.00 (2.20)	-6.58 (2.21)	-6.63 (2.19)	-5.86 (2.49)	-7.56 (2.60)
Income Elasticity	-0.14 (0.07)	-0.14 (0.07)	-0.13 (0.07)	-0.14 (0.07)	-0.12 (0.08)	-0.15 (0.08)
Compensated Elasticity	0.32 (0.06)	0.33 (0.06)	0.32 (0.06)	0.31 (0.06)	0.32 (0.06)	0.31 (0.07)
Tax change effect	0.02	0.02	0.01	0.01	0.01	0.01
P-value on bias correction term	0.01	0.01	0.01	0.04	0.01	0.05
P-value on selection bias	0.00	0.00	0.00	0.00	0.00	0.00
AIC	24493.41	24470.92	24468.10	24398.90	24394.66	24585.92
SBIC	24718.78	24706.50	24708.80	24634.44	24630.24	25958.20
Observations	1239.00	1238.00	1238.00	1237.00	1238.00	1227.00
R-squared	0.17	0.18	0.18	0.21	0.23	0.36
Race and Union Dummies	No	Yes	Yes	Yes	Yes	Yes
Year Dummy	No	No	Yes	Yes	Yes	Yes
Occupation Fixed Effects	No	No	No	Yes	Yes	Yes
State Fixed Effects	No	No	No	No	Yes	Yes
Occupation and State Fixed Effects	No	No	No	No	No	Yes

Note: Dependent variable in all the regressions was annual hours of work. The results in this table are from estimating equation (25) in the text. This equation has five terms excluding the error term. The first term is the nonparametric labor supply function for a linear budget set and is modeled as a power series. All columns present results for power series of order four. See footnote 32 for further description. The second and third terms represent the bias due to budget set nonlinearity. See footnote 33 for an explanation of how this term was modeled. The results in this table control for the bias terms. The fourth term is the nonparametric analogue of the inverse mills ratio obtained from a first stage nonparametric labor force participation equation (22). In the baseline model, other individual characteristics entered in the regression additively (i.e. the 5th term) were number of children, a quartic in age, self reported health status. Column (1) is for the baseline model. The results presented in the table are from nonparametric selection-corrected hours regression using power series, conditional on participation in the labor force. Bootstrapped standard errors reported in parentheses are based on 99 replications. The elasticities are calculated at the sample mean of after-tax wage and virtual income on the first segment of \$5.35 and \$32,960 respectively and at mean annual hours of 1616, for working individuals.

Figure1: Kinked Budget Sets and Ambiguous Effects

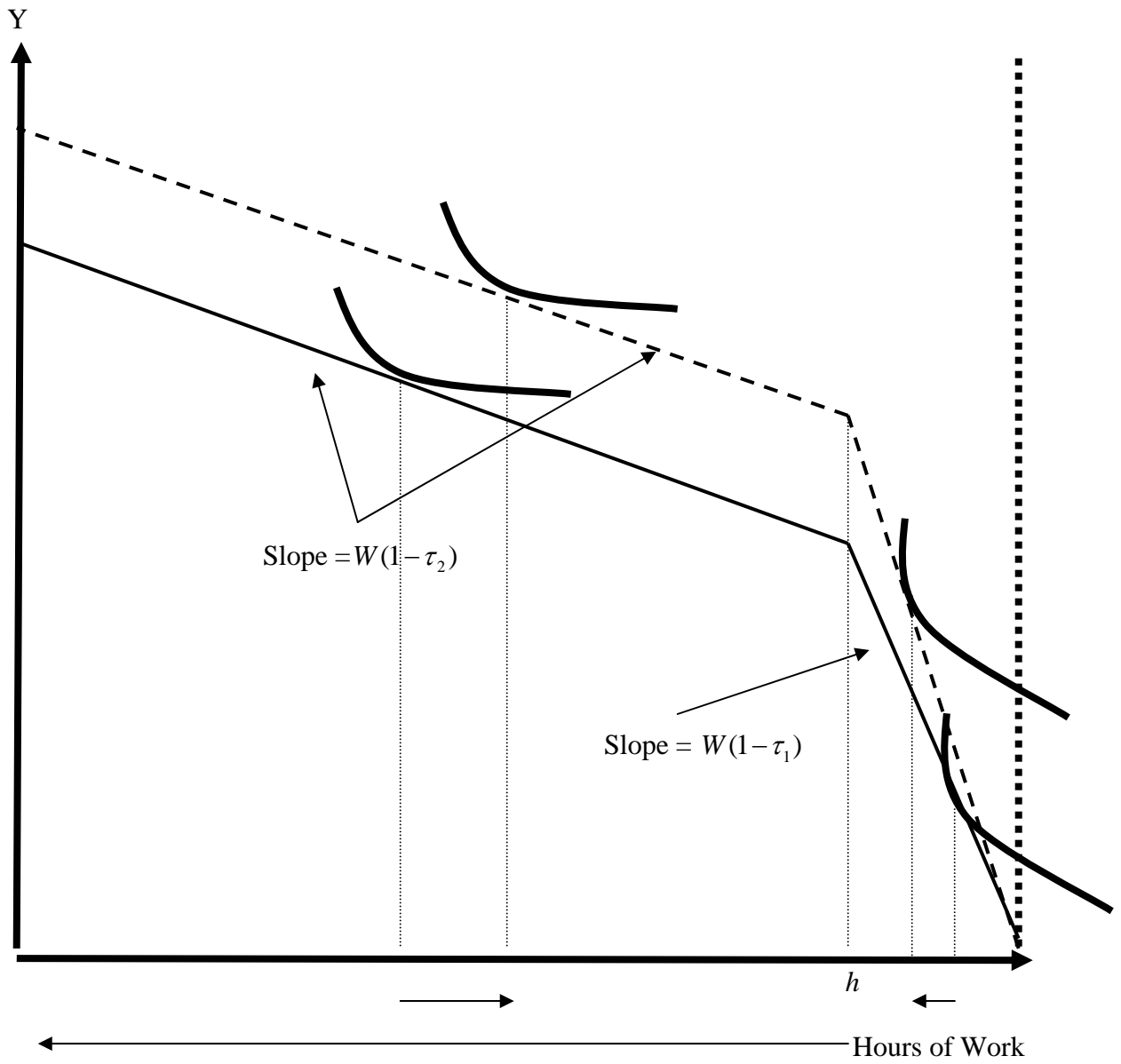


Figure 1

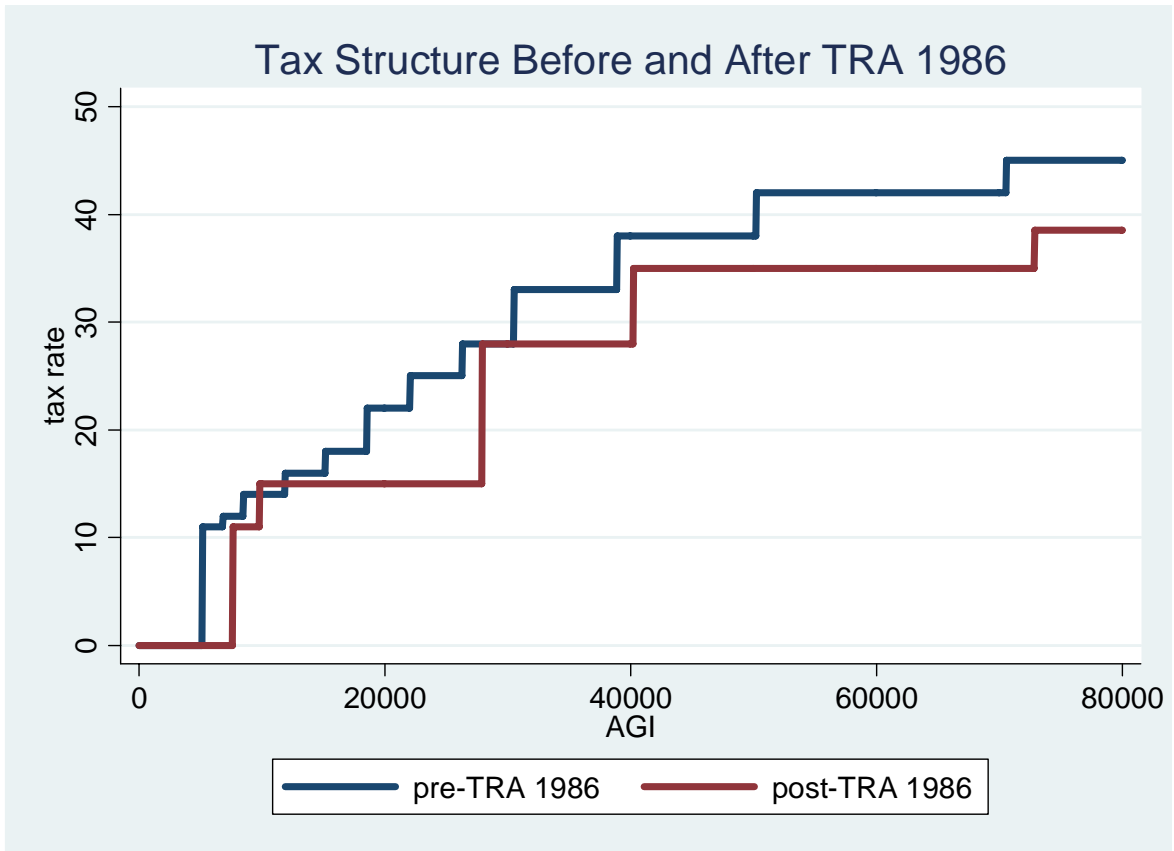


Figure 2

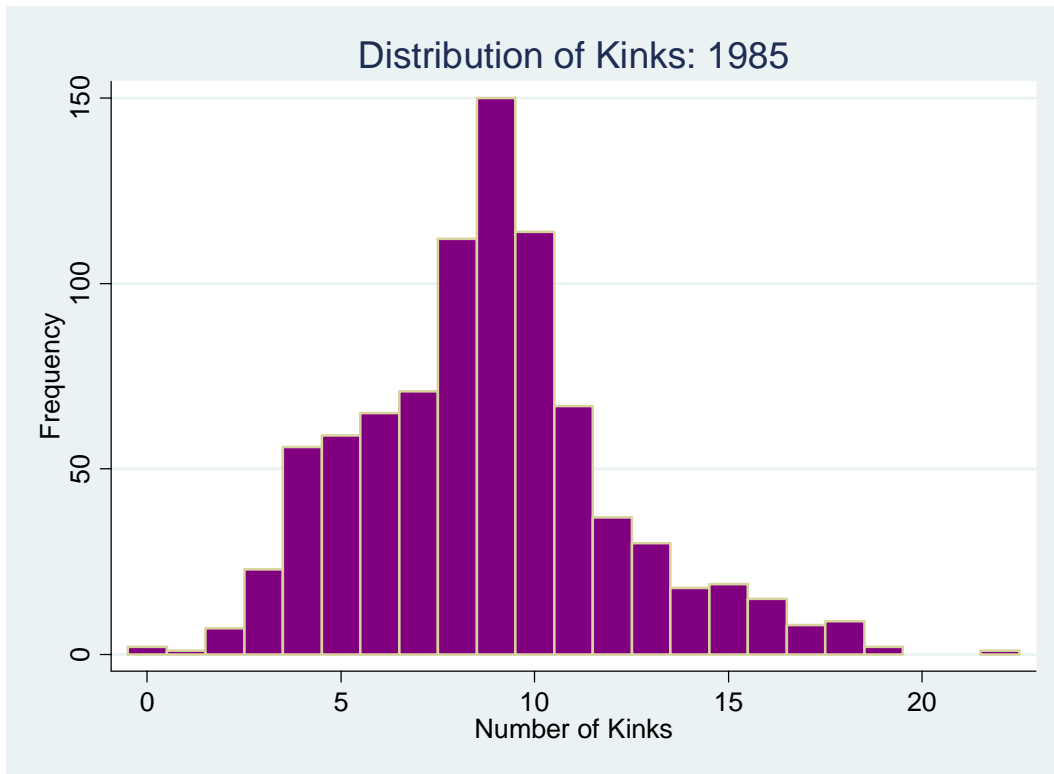


Figure 3

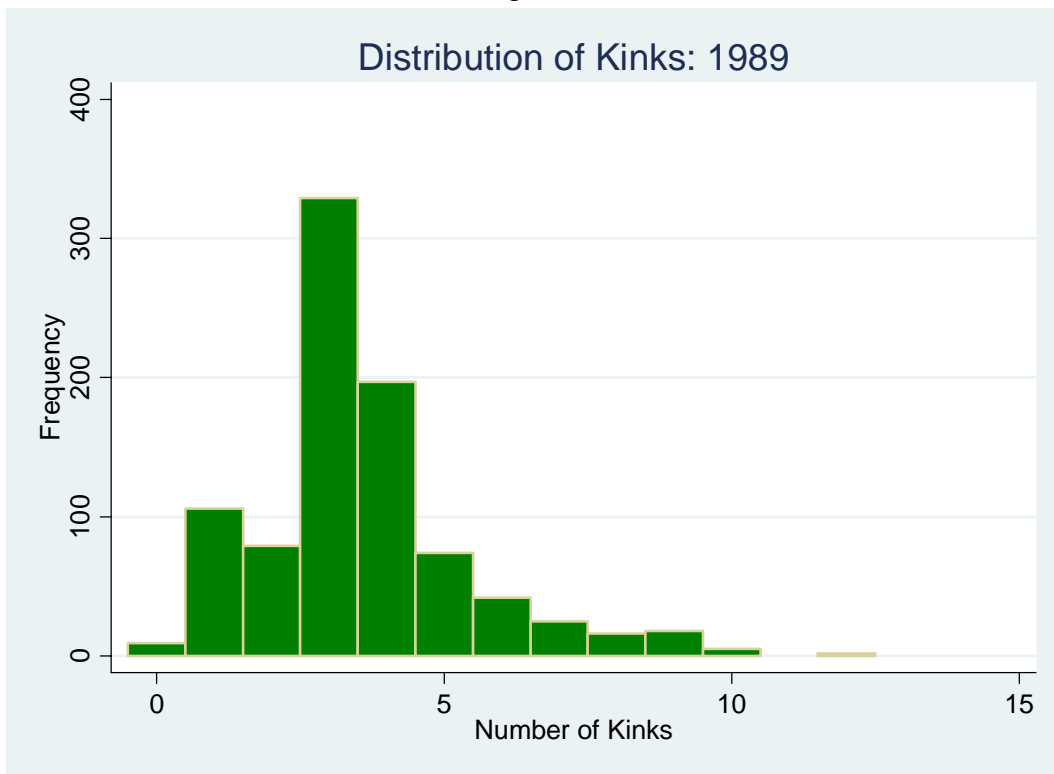


Figure 4

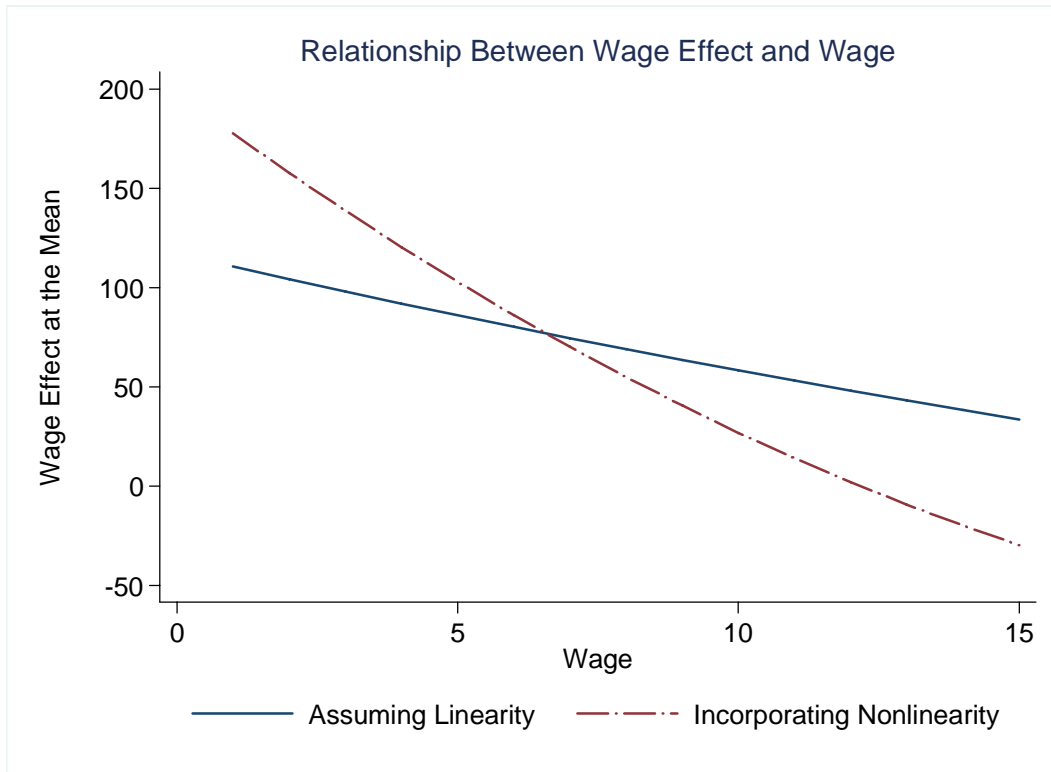


Figure 5

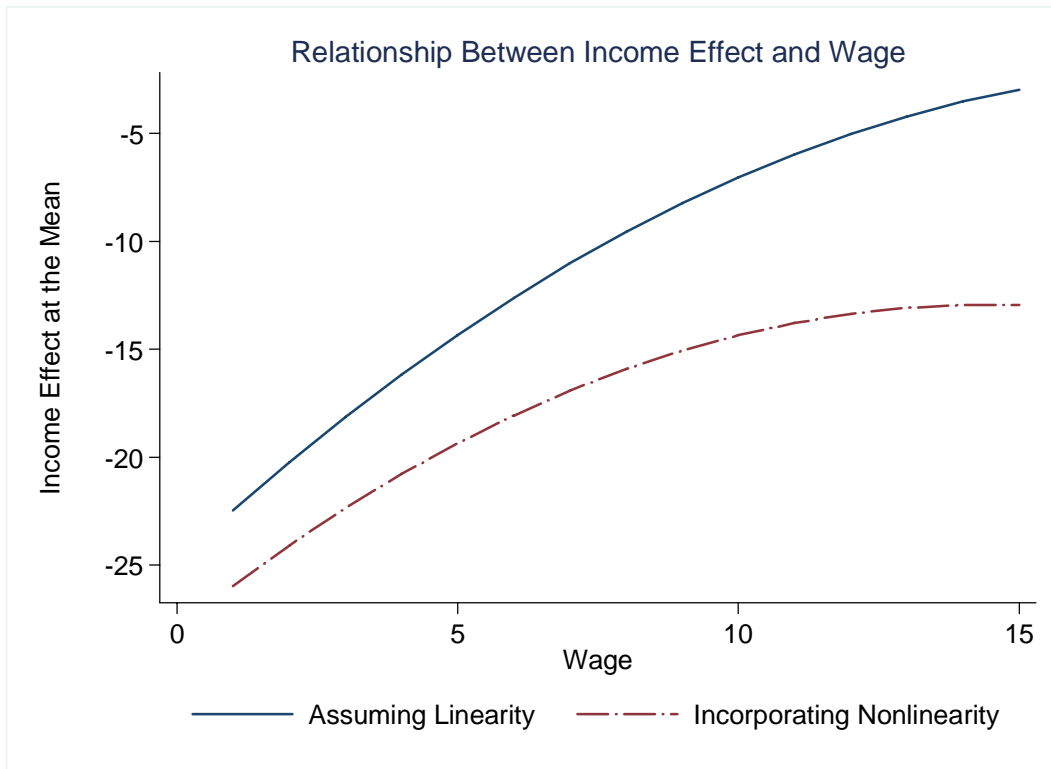


Figure 6



Figure 7

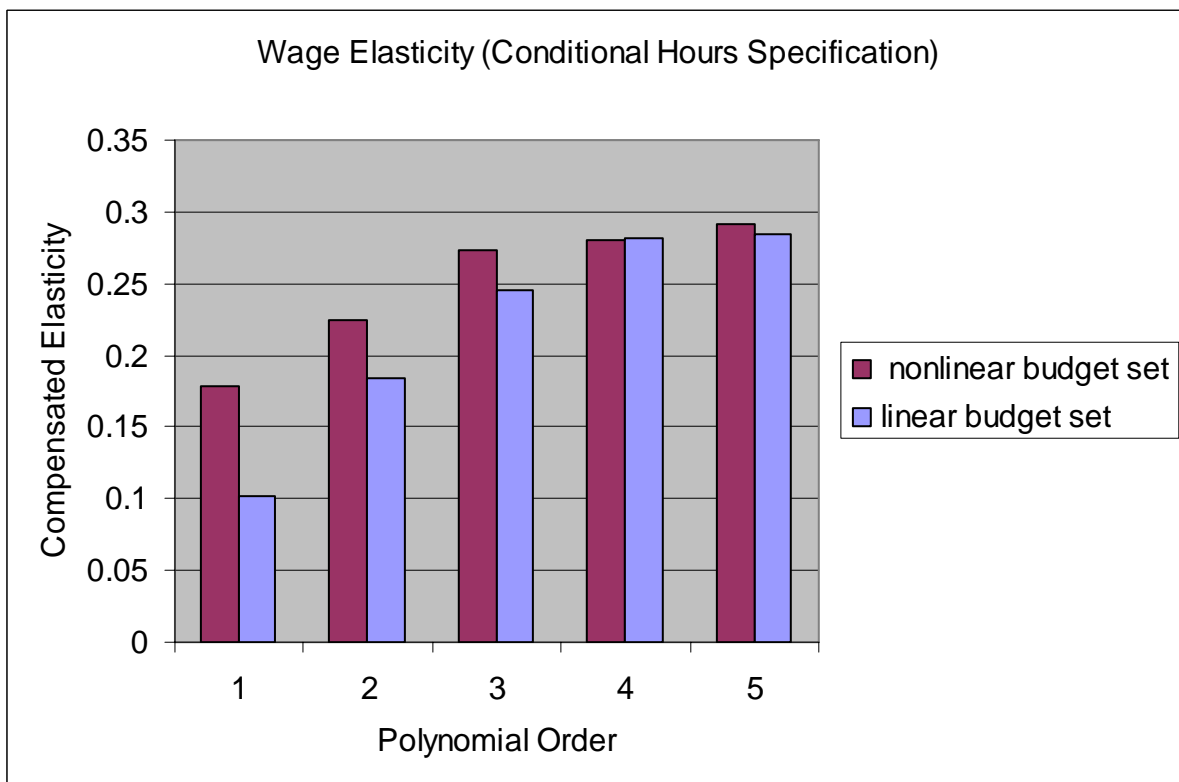


Figure 8

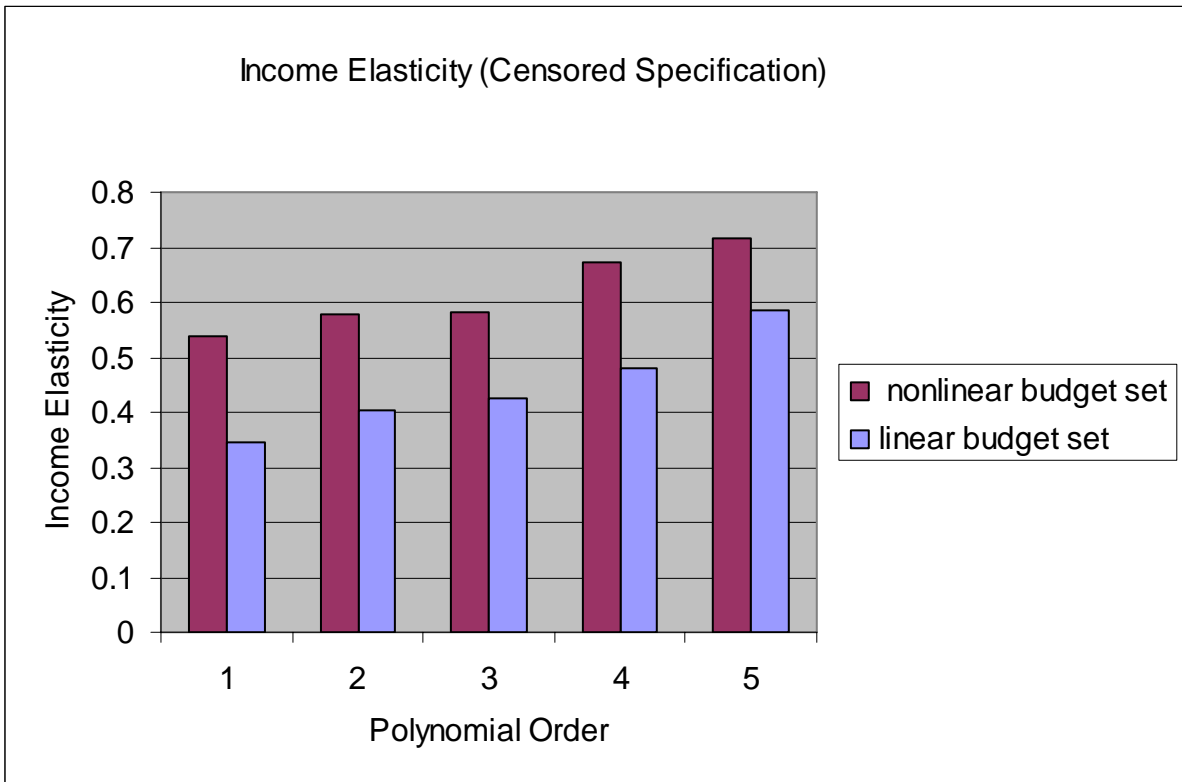


Figure 9

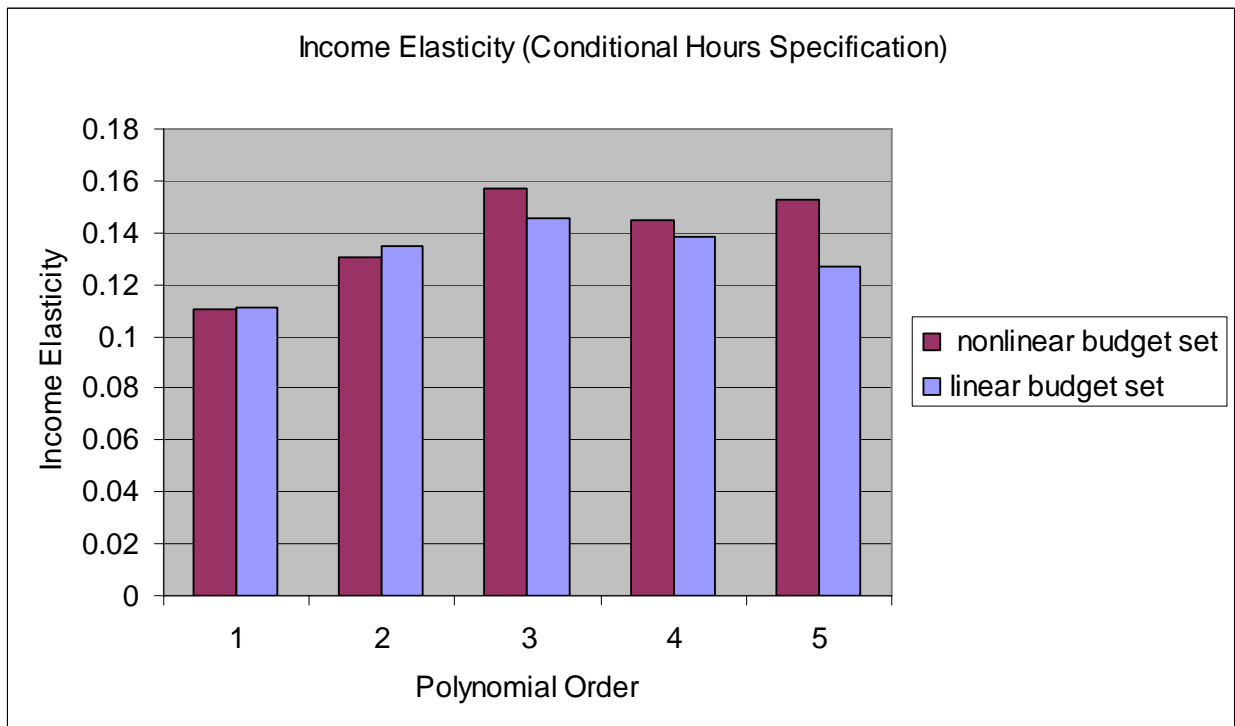


Figure 10

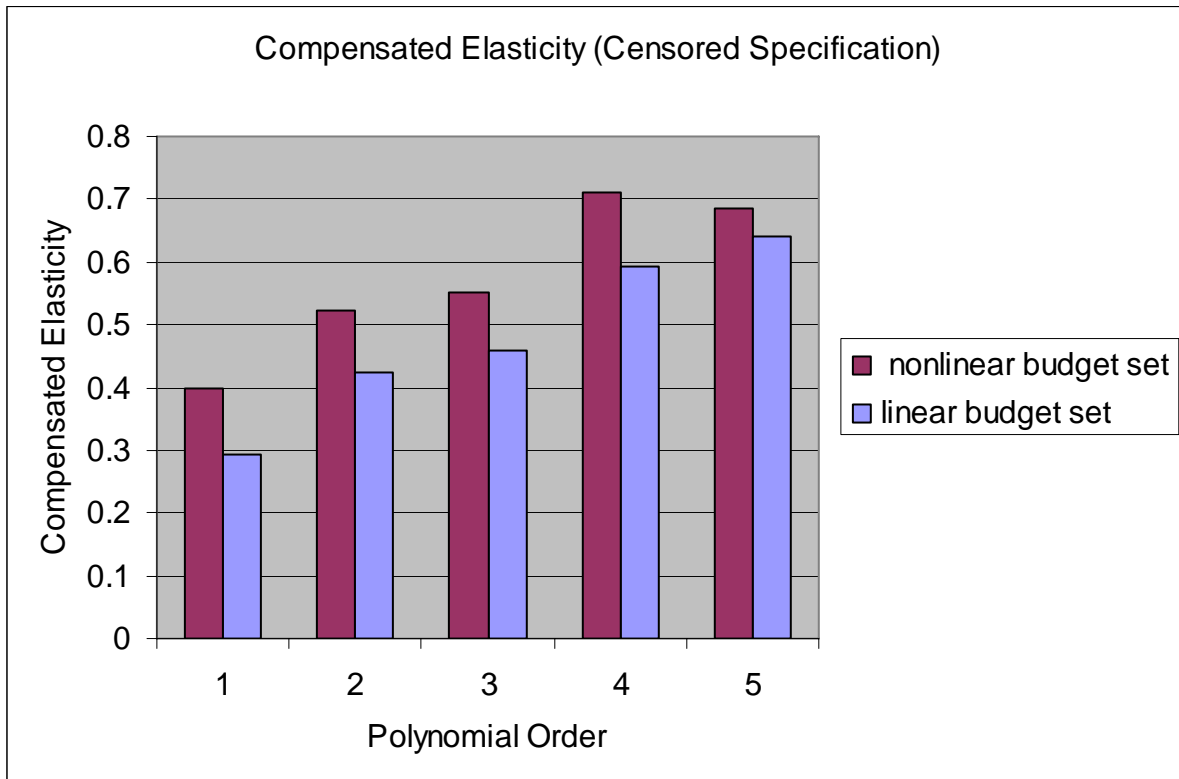
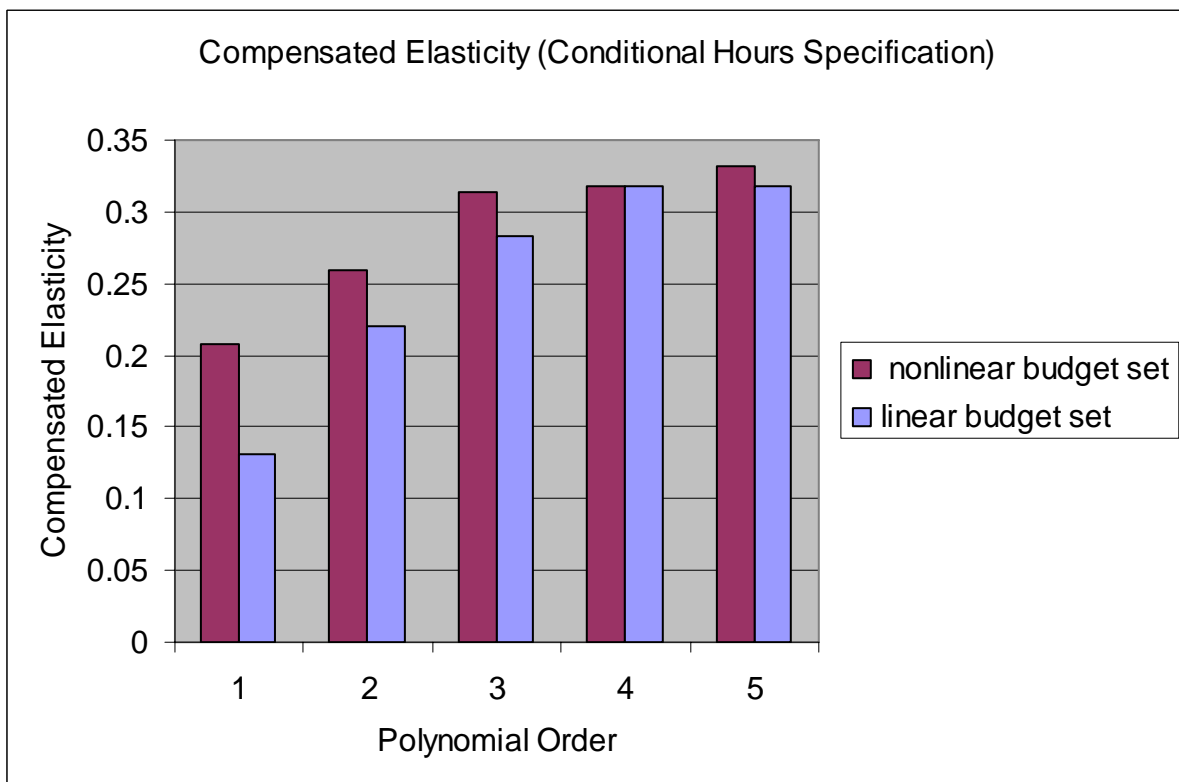


Figure 11



APPENDIX 1

Figure A1

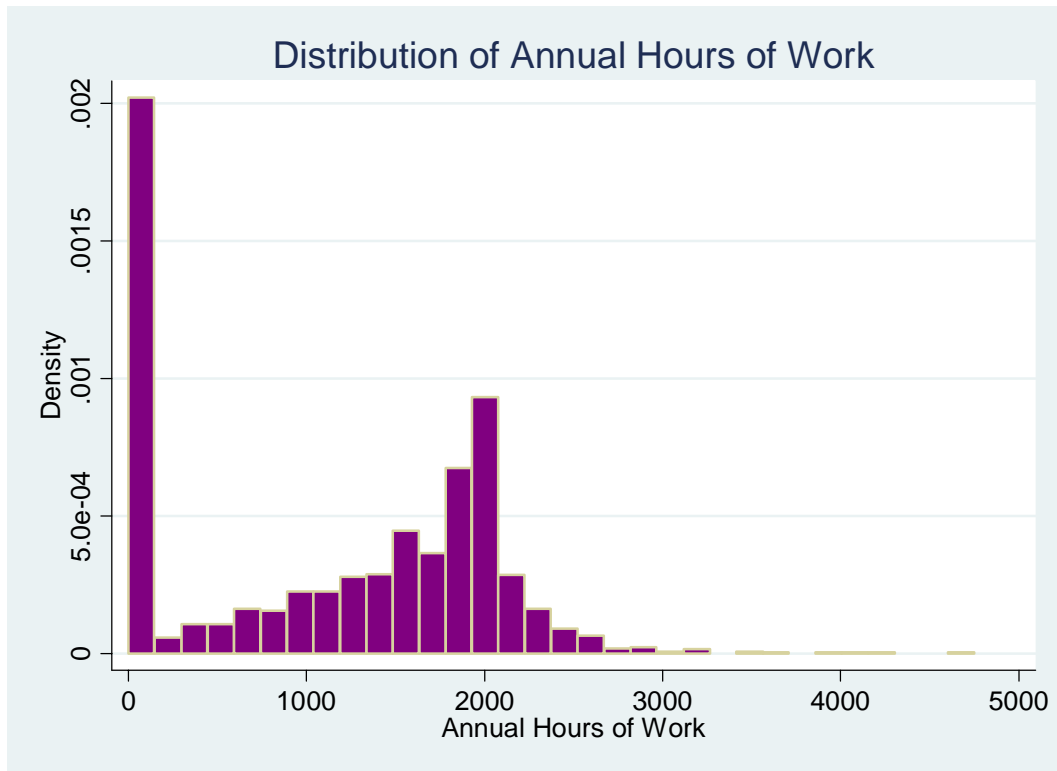


Table A1 Sample Exclusions from PSID 1985 and PSID 1989

Reason	Observations Excluded	Observations Remaining
Total number of observations	0	38360
Only wives	32025	6335
SEO	2233	4102
25>Age>60	1427	2675
Head Self Employed	576	2099
Self Employed	316	1783
Missing Variables	12	1771

Appendix 2

Proof of Theorem 3.1:

Suppose there are two segments. Let the hours function be given by

$$h_i = h(x_i, \eta_i) + \varepsilon_i \quad (\text{A.1})$$

where

$$x_i = (y_1^v, \dots, y_J^v, \omega_1, \dots, \omega_J, l_1, \dots, l_{J-1}), \quad (\text{A.2})$$

is a vector J virtual incomes and slopes for J segments and $J - 1$ kinks.

For example, in a world with two segments

$$x_i = (y_1^v, y_2^v, \omega_1, \omega_2, l_1) \quad (\text{A.3})$$

Write the desired hours function as

$$h_i^* = \pi(y^v, \omega, \eta) \quad (\text{A.4})$$

The desired hours function of the i_{th} individual on the j_{th} segment is given by

$$h_{ij}^* = \pi(y_j^v, \omega_j, \eta) = \pi_j(\eta) \quad (\text{A.5})$$

Suppressing the individual subscript and without loss of generality assuming that the budget set consists of two segments and one kink, the desired hours can expressed as:

$$h^* = 0 \text{ if } \pi_1(\eta) \leq 0 \Rightarrow -\infty \leq \eta \leq \pi_1^{-1}(0) \quad (\text{A.6})$$

if individual does not work;

$$h^* = \pi_j(\eta) \text{ if } l_{j-1} \leq \pi_j(\eta) \leq l_j \Rightarrow \pi_j^{-1}(l_{j-1}) \leq \eta \leq \pi_j^{-1}(l_j) \text{ for } j=1, \dots, J-1 \quad (\text{A.7})$$

if the individual locates on the j_{th} of the first $J - 1$ segments;

$$\begin{aligned} h^* &= l_j \text{ if } l_j \leq \pi_j(\eta) \text{ and } \pi_{j+1}(\eta) \leq l_j \\ &\Rightarrow \pi_j^{-1}(l_j) \leq \eta \leq \pi_{j+1}^{-1}(l_j) \text{ for } j=1, \dots, J-1 \end{aligned} \quad (\text{A.8})$$

if the individual locates on the j_{th} of the $J - 1$ kinks;

and

$$h^* = \pi_J(\eta) \text{ if } l_{J-1} \leq \pi_J(\eta) \Rightarrow \pi_J^{-1}(l_{J-1}) \leq \eta \leq \infty \quad (\text{A.9})$$

if the individual locates on the J_{th} segment;

For the probability density function $g(\eta)$ of η , the expected value of desired hours can be written as⁴⁸

$$E(h^*) = \sum_{j=1}^{J-1} \int_{\pi_j^{-1}(l_{j-1})}^{\pi_j^{-1}(l_j)} \pi_j(\eta) g(\eta) d\eta + l_j \int_{\pi_j^{-1}(l_j)}^{\pi_{j+1}^{-1}(l_j)} g(\eta) d\eta + \int_{\pi_J^{-1}(l_{J-1})}^{\infty} \pi_J(\eta) g(\eta) d\eta \quad (\text{A.10})$$

⁴⁸ The proof presented here closely follows Blomquist and Newey (2002). (A.10) has been shown in Blomquist and Newey (2002). However, subsequently they assume that $\Pr(h = 0) = 0$, while this assumption is dropped here.

$$= \int_{\pi_1^{-1}(0)}^{\pi_1^{-1}(l_1)} \pi_j(\eta)g(\eta)d\eta + \sum_{j=2}^{J-1} \int_{\pi_j^{-1}(l_{j-1})}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta + l_j \int_{\pi_j^{-1}(l_j)}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta \quad (\text{A.11})$$

$$+ \int_{\pi_J^{-1}(l_{J-1})}^{\infty} \pi_J(\eta)g(\eta)d\eta$$

$$= \int_{-\infty}^{\pi_1^{-1}(l_1)} \pi_1(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta + \sum_{j=2}^{J-1} \int_{\pi_j^{-1}(l_{j-1})}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta \quad (\text{A.12})$$

$$+ \sum_{j=1}^{J-1} l_j \int_{\pi_j^{-1}(l_j)}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta + \int_{\pi_J^{-1}(l_{J-1})}^{\infty} \pi_J(\eta)g(\eta)d\eta$$

$$= \int_{-\infty}^{\pi_1^{-1}(l_1)} \pi_1(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta + \sum_{j=2}^{J-1} \int_{\pi_j^{-1}(l_{j-1})}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta \quad (\text{A.13})$$

$$+ \sum_{j=1}^{J-1} l_j \int_{\pi_j^{-1}(l_j)}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta + \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta - \int_{-\infty}^{l_{J-1}} \pi_J(\eta)g(\eta)d\eta$$

$$= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)dv + \int_{-\infty}^{\pi_1^{-1}(l_1)} \pi_1(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta \quad (\text{A.14})$$

$$+ \sum_{j=2}^{J-1} \int_{\pi_j^{-1}(l_{j-1})}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta + \sum_{j=1}^{J-1} l_j \int_{\pi_j^{-1}(l_j)}^{\pi_{j+1}^{-1}(l_j)} g(\eta)dv - \int_{-\infty}^{l_{J-1}} \pi_J(\eta)g(\eta)d\eta$$

$$= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta + \int_{-\infty}^{\pi_1^{-1}(l_1)} \pi_1(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta$$

$$+ \sum_{j=2}^{J-1} \int_{-\infty}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_j^{-1}(l_{j-1})} \pi_j(\eta)g(\eta)d\eta \quad (\text{A.15})$$

$$+ \sum_{j=1}^{J-1} l_j \int_{\pi_j^{-1}(l_j)}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta - \int_{-\infty}^{l_{J-1}} \pi_J(\eta)g(\eta)d\eta$$

$$\begin{aligned}
&= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta + \int_{-\infty}^{\pi_1^{-1}(l_1)} \pi_1(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta \\
&+ \sum_{j=2}^{J-1} \left[\int_{-\infty}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_j^{-1}(l_{j-1})} \pi_j(\eta)g(\eta)d\eta \right] \tag{A.16}
\end{aligned}$$

$$\begin{aligned}
&+ \sum_{j=1}^{J-1} l_j \left[\int_{-\infty}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta - \int_{-\infty}^{\pi_j^{-1}(l_j)} g(\eta)d\eta \right] - \int_{-\infty}^{\pi_J^{-1}(l_{J-1})} \pi_J(\eta)g(\eta)d\eta \\
&= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta + \int_{-\infty}^{\pi_1^{-1}(l_1)} \pi_1(\eta)g(\eta)d\eta \\
&+ \sum_{j=2}^{J-1} \int_{-\infty}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta + \sum_{j=2}^{J-1} \left[- \int_{-\infty}^{\pi_j^{-1}(l_{j-1})} \pi_j(\eta)g(\eta)d\eta \right] \tag{A.17}
\end{aligned}$$

$$\begin{aligned}
&+ \sum_{j=1}^{J-1} l_j \int_{-\infty}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta + \sum_{j=1}^{J-1} l_j \left[- \int_{-\infty}^{\pi_j^{-1}(l_j)} g(\eta)d\eta \right] - \int_{-\infty}^{\pi_J^{-1}(l_{J-1})} \pi_J(\eta)g(\eta)d\eta \\
&= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta + \sum_{j=1}^{J-1} \int_{-\infty}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta \\
&+ \sum_{j=2}^{J-1} \left[- \int_{-\infty}^{\pi_j^{-1}(l_{j-1})} \pi_j(\eta)g(\eta)d\eta \right] - \int_{-\infty}^{\pi_J^{-1}(l_{J-1})} \pi_J(\eta)g(\eta)d\eta \tag{A.18} \\
&+ \sum_{j=1}^{J-1} l_j \int_{-\infty}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta + \sum_{j=1}^{J-1} l_j \left[- \int_{-\infty}^{\pi_j^{-1}(l_j)} g(\eta)d\eta \right]
\end{aligned}$$

$$\begin{aligned}
&= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta \\
&+ \left\{ \sum_{j=1}^{J-1} \int_{-\infty}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta - \sum_{j=1}^{J-1} l_j \left[\int_{-\infty}^{\pi_j^{-1}(l_j)} g(\eta)d\eta \right] \right\} \tag{A.19} \\
&- \left\{ \sum_{j=2}^{J-1} \left[\int_{-\infty}^{\pi_j^{-1}(l_{j-1})} \pi_j(\eta)g(\eta)d\eta \right] + \int_{-\infty}^{\pi_J^{-1}(l_{J-1})} \pi_J(\eta)g(\eta)d\eta - \sum_{j=1}^{J-1} l_j \int_{-\infty}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta \right\}
\end{aligned}$$

$$\begin{aligned}
&= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta + \\
&\left\{ \sum_{j=1}^{J-1} \int_{-\infty}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta - \sum_{j=1}^{J-1} l_j \left[\int_{-\infty}^{\pi_j^{-1}(l_j)} g(\eta)d\eta \right] \right\} \\
&- \left\{ \sum_{j=1}^{J-1} \left[\int_{-\infty}^{\pi_{j+1}^{-1}(l_j)} \pi_{j+1}(\eta)g(\eta)d\eta \right] - \sum_{j=1}^{J-1} l_j \int_{-\infty}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta \right\}
\end{aligned} \tag{A.20}$$

$$\begin{aligned}
&= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta \\
&+ \left\{ \sum_{j=1}^{J-1} \int_{-\infty}^{\pi_j^{-1}(l_j)} \pi_j(\eta)g(\eta)d\eta - l_j \int_{-\infty}^{\pi_j^{-1}(l_j)} g(\eta)d\eta \right\} \\
&- \left\{ \sum_{j=1}^{J-1} \int_{-\infty}^{\pi_{j+1}^{-1}(l_j)} \pi_{j+1}(\eta)g(\eta)d\eta - l_j \int_{-\infty}^{\pi_{j+1}^{-1}(l_j)} g(\eta)d\eta \right\}
\end{aligned} \tag{A.21}$$

Let

$$\mu(y_k^v, w_k, l_k) = \int_{-\infty}^{\pi_k^{-1}(l_k)} \pi_k(\eta)g(\eta)dv - l_k \int_{-\infty}^{\pi_k^{-1}(l_k)} g(\eta)d\eta \tag{A.22}$$

Then (A.21) can be written as

$$\begin{aligned}
&= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta \\
&+ \left\{ \sum_{j=1}^{J-1} \mu(y_j^v, w_j, l_j) \right\} - \left\{ \sum_{j=1}^{J-1} \mu(y_{j+1}^v, w_{j+1}, l_j) \right\}
\end{aligned} \tag{A.23}$$

$$\begin{aligned}
&= \int_{-\infty}^{\infty} \pi_J(\eta)g(\eta)d\eta - \int_{-\infty}^{\pi_1^{-1}(0)} \pi_1(\eta)g(\eta)d\eta \\
&+ \left\{ \sum_{j=1}^{J-1} \mu(y_j^v, w_j, l_j) - \mu(y_{j+1}^v, w_{j+1}, l_j) \right\}
\end{aligned} \tag{A.24}$$

QED