Real Business Cycle Dynamics under First-Order Risk Aversion

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Abstract

This paper incorporates preferences that display first-order risk aversion (FORA) into a standard real business cycle model. Although FORA preferences represent a sharp departure from the expected utility/constant relative risk aversion (EU/CRRA) preferences common in the business cycle literature, the change has only a negligible effect on the model's second moment implications. In fact, for what I argue is an empirically reasonable "ballpark" calibration of the FORA preferences, the moment implications are essentially identical to those under EU/CRRA, while the welfare cost of aggregate fluctuations in the model is substantially larger.

1 Introduction

Risk preferences that display first-order risk aversion (FORA), of the form studied in this paper and others, have many attractive features. In contrast to the way in which attitudes towards risk are modeled in most of the business cycle literature—*i.e.*, expected utility, generally with constant relative risk aversion—FORA preferences can be calibrated to give plausible degrees of risk aversion for both large gambles and small gambles.[1] Epstein and Zin [14], in the context of a Mehra-Prescott-type endowment economy, showed that FORA preferences can do a better job of rationalizing the observed behavior of consumption growth and asset returns than either the standard EU/CRRA specification or the recursive specifications used by Weil [40] and by

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[1] The willingness-to-pay calculations in Epstein and Zin [14] and Dolmas [11] argue by example the advantage of FORA preferences in giving intuitively, or introspectively, plausible degrees of aversion to both large and small risks. In Section 2 below, I try to demonstrate that FORA preferences can be calibrated to give degrees of risk aversion that are more *empirically* plausible over a wide range of gamble sizes.

Epstein and Zin in their earlier [13]. Bernasconi [4] showed how FORA preferences can rationalize the puzzlingly low incidence of tax evasion in the face of very small probabilities of detection (and modest penalties contingent upon detection). These results come on top of the fact that the FORA preferences employed in these papers can rationalize seeming anomalies of choice under risk, such as the Allais paradox or the common ratio effect.[2] The present paper shows that FORA preferences can be easily incorporated into a standard real business cycle model and produce second-moment predictions that are virtually identical to those obtained under standard EU/CRRA preferences.

The results in this paper, like those of Tallarini [37], have implications for how we think about the role of risk aversion in the basic neoclassical model and raise the question of what factors, if not risk preferences, give standard RBC models their ability to reproduce many of the relative volatilities and co-movements of key macroeconomic variables that we observe in data from the United States and elsewhere.[3]

The results also have implications for measuring the welfare cost of business cycles. In the years since Robert Lucas's *Models of Business Cycles* [22] was first published, the welfare calculations that form the substance of the book's third chapter have spurred a large number of papers either reinforcing or challenging Lucas's basic conclusion—namely, that the welfare cost of business cycle volatility in the post-World War II US is exceedingly small, on the order of no more than 0.1% of annual consumption. Some of these papers have emphasized individual-level consumption or income risk, in contrast to Lucas's representative agent formulation.[4] Others have looked at the interaction between business cycle volatility and growth, replacing Lucas's exogenous consumption streams with consumption paths determined in the context of fully articulated stochastic endogenous growth models.[5] Among the papers finding the largest potential costs are those that modify the risk preferences of Lucas's representative agent. Pemberton [27] and Dolmas [11], in particular, examined the implications of FORA specifications of the representative agent's risk preferences, finding welfare cost numbers that can be very large even for arguably moderate amounts of risk aversion; as shown in [11], this is particularly true the more persistent are shocks to consumption.

The motivation for the risk-preference approach was, at least in part, the failure of the standard EU/CRRA preferences to rationalize the observed behavior of consumption growth and asset returns.[6] Of course, the equity premium puzzle did not

[2] See Segal [33].

[3] Given that the basic stochastic neoclassical growth model now lies at the core of many models outside of the RBC paradigm—for example, the "new neoclassical synthesis" models described by Goodfriend and King [17] or the "new open economy macro" models typified by Erceg, *et al.* [15]—results that clarify the workings of the stochastic growth model should be of interest regardless of one's stand on the relative importance of real or nominal shocks.

[4] See, for example, Krebs [21] or Storesletten, *et al.* [35].

[5] For example, Barlevy [3] or Epaulard and Pommeret [12].

[6] Though not employing FORA preferences, Tallarini's work [37] is in this vein as well. Alvarez and Jermann [1] also focus on asset return data, but dispense with a specification of preferences

lead to a wholesale abandonment of the standard preference specification; neither had the Allais or Ellsberg paradoxes or other anomalies recognized before Mehra and Prescott's seminal paper [24]. This is natural—science is necessarily conservative, and the standard specification would not have become the standard if it did not have much to recommend it. Among the attractive features that recommend the standard EU/CRRA framework to macroeconomists are its tractability and the fact that, in the context of business cycle models, it seems to "work". One point of this paper is that FORA preferences share at least the latter of these virtues and, perhaps, also the former.

This paper both is, and is not, about "extreme" risk aversion. On the one hand, I do consider some very high degrees of risk aversion in the numerical simulations below, the point being to illustrate the robustness of the stochastic growth model's dynamics in the face of sizeable variation in risk preferences. At the same time, the results show that FORA preferences need not be calibrated to extreme levels of risk aversion in order to generate significantly larger business cycle costs than obtain under standard preferences. In section 2.3, I offer a parametrization of the FORA preferences that yields degrees of risk aversion that are "in the ballpark" of empirical plausibility across three insurance market examples where the size of the risks varies from quite small to fairly large. This is not to claim that the parametrization is "definitive", but, more modestly, that it cannot be rejected out of hand as being extremely risk averse. In the simulations, that parametrization produces model second moments essentially identical to EU/CRRA preferences (calibrated in the usual way, with risk aversion coefficients around 1), but with costs of aggregate fluctuations ranging from 1.25% to 12% of consumption, depending on the elasticity of intertemporal substitution.

This is a point that deserves emphasis. In a survey of the state of the cost of business cycles literature, *circa* 2003, Lucas [23] writes, with regard to the models that alter risk preferences in ways suggested by the equity premium puzzle:

> The risk-aversion levels needed to match the equity premium, under the assumption that asset markets are complete, ought to show up somewhere besides securities prices, but they do not seem to do so. No one has found risk-aversion parameters of 50 or 100 in the diversification of individual portfolios, in the level of insurance deductibles, in the wage premiums associated with high earnings risk, or in the revenues raised by state-operated lotteries. It would be good to have the equity premium resolved, but I think we need to look beyond high estimates of risk aversion to do it.

This statement is fair enough, if restricted to EU/CRRA preferences: in settings where the risks are large, one will not find relative risk aversion coefficients of 50 or

altogether. See Barlevy [2] for a nice survey of the entire cost of business cycles literature.

100. But, given that individuals do buy actuarially unfair insurance against small risks—given, for example, a world where Sears offers a 3-year extended warranty on a \$1000 refrigerator for \$240 and, I assume, has at least a few takers[7]—one is not likely to find risk neutrality in small-risk settings, either. And, if one insists on applying the EU/CRRA model, then one will find relative risk aversion coefficients in those settings of 50, 100 or higher.[8] EU/CRRA preferences require very large risk aversion coefficients to generate even a modest aversion to small risks, and those coefficients seem outlandish when we contemplate their use in other settings. FORA preferences can avoid this problem.

The paper most closely related to the present one is Tallarini [37]. Working in a standard RBC framework, but with recursive preferences of the sort employed by Epstein and Zin in their 1989 paper [13], Tallarini shows that when the elasticity of intertemporal substitution is equal to one, the intertemporal preferences that result are formally analogous to the type of objective functions used in the literature on risk-sensitive optimal control (see [42])—*i.e.*, the social planner's problem that results takes the form of a discounted linear exponential quadratic Gaussian control problem. In contrast to standard linear-quadratic control, risk-sensitive control does not entail certainty equivalence—decision rules, and not just value functions, may depend on the variances of the model's shocks. Tallarini uses methods developed by Hansen and Sargent [18] to calculate approximate solutions to the model. He then shows that the model's business cycle predictions—relative volatilities and correlations—are little changed when the coefficient of relative risk aversion is increased from 1 to 100, though this has a significant impact on the model's asset-pricing and welfare implications.

The analysis here differs from Tallarini's in a couple important ways. First, since I solve the model using discrete state space methods, I need not restrict attention to the case of a unitary elasticity of intertemporal substitution; as I show in section 5, variation in the elasticity of intertemporal substitution has a non-negligible effect on the model's dynamics and a potentially large effect on the model's implications for the cost of aggregate fluctuations. Most importantly, though, the preferences Tallarini uses suffer from the same difficulty of calibration for both large and small risks that is present with the standard EU/CRRA specification—indeed, for static or timeless gambles, the risk preferences Tallarini uses are EU/CRRA. The FORA preferences I employ here, as I argue in the next section, can be calibrated more sensibly for both large and small risks, and thus their welfare implications are less easily dismissed on the grounds Lucas cites in the quotation above.

The remainder of the paper is organized as follows. Section 2 gives a brief introduction to the FORA preferences used in the paper, discusses the sense in which they are 'first-order' risk averse, and shows, via a few numerical examples, the sense

[7] Sears item #04656772000, priced online at www.sears.com on May 12, 2006.
[8] Examples of this sort are given in section 2.3.

in which they can be calibrated to give empirically plausible attitudes towards risk over a wide range of gambles, while the standard EU/CRRA specification cannot. Section 3 describes the model, which, aside from the specification of preferences, is a standard RBC model. Section 4 discusses the solution method and calibration. Section 5 presents results from quantitative experiments with the model, both second moment implications and implications for the welfare cost associated with aggregate fluctuations. Section 6 concludes.

2 First-order risk aversion

2.1 Background

The risk preferences I employ in this paper are similar to those used by Epstein and Zin [14] in studying the equity premium puzzle. Epstein and Zin's specification, in turn, is based on the non-expected utility formulations of Yaari [43] and Quiggin [31]. Risk preferences of this sort are referred to in the literature variously as "rank-dependent expected utility," "expected utility with rank-dependent probabilities," or "anticipated utility" (the latter following the language of Quiggin [31]). They can be derived under various sets of axioms (see Wakker [39], and the references therein). A key feature of these preferences—like many other alternatives to expected utility—is that they are non-linear in probabilities. Among the aims of the authors who originally formulated risk preferences of this form was to elaborate models of choice under risk capable of rationalizing the apparent fact that individuals often make choices that are inconsistent with the independence axiom of expected utility—for example, the Allais paradox or the common ratio effect documented by Kahneman and Tversky.[9] As noted in the introduction, the risk preferences described below, like other alternatives to expected utility, can be parametrized to be consistent with the choices generally made by individuals in the Allais paradox and are consistent with the common ratio effect.

The fact that risk preferences of this form are non-linear in probabilities gives them another attractive feature: the ability to at least partially divorce agents' attitudes towards risk from their attitudes towards wealth.[10] Under expected utility, aversion to risk is equivalent to diminishing marginal utility of wealth, and the intimate connection between the two concepts has been shown to be problematic for the EU model. For example, Chetty [7] has shown that estimates of labor supply elasticity (and the degree of complementarity between consumption and leisure) can put sharp bounds on admissible coefficients of relative risk aversion, since both values are linked to the curvature of agents' von Neumann-Morgenstern utilities over consumption. Chetty finds that the mean coefficient of relative risk aversion implied

[9] Starmer [34] is an excellent recent survey of this literature.

[10] As Yaari [43] puts it: "At the level of fundamental principles, risk aversion and diminishing marginal utility of wealth, which are synonymous under expected utility, are horses of different colors." In Yaari's theory the divorce of the two concepts is complete.

by 33 studies of labor supply elasticity is roughly unity, which would mean that the EU model is incapable of rationalizing both observed labor supply behavior and the degrees of risk aversion observed in many risky choice settings, many of which imply double-digit coefficients of relative risk aversion.[11]

The most attractive feature of these preferences, though, for my purposes, is the fact that they can be parametrized to give a plausible amount of risk aversion for both large and small gambles. This is in contrast to the standard expected utility specification. In the CRRA class, for example, if the coefficient of risk aversion is calibrated so that an agent with those preferences gives plausible answers to questions about large gambles, the agent will be roughly risk neutral for small gambles. If, on the other hand, the coefficient of risk aversion is set sufficiently large that the agent gives plausible answers to questions about small gambles, he will appear extremely risk averse when confronted with large gambles.[12] This is because the standard expected utility specification with constant relative risk aversion is "smooth at certainty"—the agent's indifference curves between consumption in different states of nature are smooth and tangent (at the certainty point) to the indifference curves of a risk neutral agent.

The risk preferences I use introduce a kink into agents' indifference curves at the certainty point; the kink is what allows for a plausible calibration of risk aversion for small gambles.[13] A second parameter, analogous to the risk aversion coefficient in CRRA preferences, governs curvature away from the certainty point and allows for a plausible calibration of risk aversion for large gambles.

2.2 Formal description

These preferences are probably easiest to describe in a static (or timeless) setting, before embedding them in a dynamic setting. Imagine an agent whose wealth varies across a finite set of n possible states of nature.[14] Consider lotteries of the form $\{\tilde{w}; p\}$ where $\tilde{w} = (w_1, w_2, \ldots, w_n)$ are wealth realizations in the n states and $p = (p_1, p_2, \ldots, p_n)$ are probabilities associated with the n states. For any $\{\tilde{w}; p\}$ let $\{s_1, s_2, \ldots, s_n\}$ be a permutation of $\{1, 2, \ldots, n\}$ that ranks the outcomes w_s from lowest to highest in the sense that

$$w_{s_1} \leq w_{s_2} \leq \cdots \leq w_{s_n}.$$

[11] Examples of this sort are given in section 2.3.

[12] Note that my claim here and below is much more modest than that in Rabin [28]. Indeed, as Safra and Segal show in a recent paper [32], almost all common alternatives to expected utility are susceptible to a Rabin-like critique. The one exception noted by Safra and Segal is Yaari's dual theory of choice under risk, which is a special case of the preferences I employ here (see equation 3). The calculations I make below follow (albeit to a somewhat different end) the spirit of Palacios-Huerta, Serrano and Volij [26]—"[I]t is more useful not to argue whether expected utility is literally true (we know that it is not, since many violations of its underpinning axioms have been exhibited). Rather, one should insist on the identification of a useful range of empirical applications where expected utility is a useful model to approximate, explain, and predict behavior."

[13] See figure 1 in [14].

[14] Lifetime utility will take the place of wealth when we move to the dynamic setting of section 3.

Given that notation, the risk preferences I employ value lotteries over wealth according to their certainty equivalent value, $\mu(\tilde{w})$, where, for $\gamma \in (0,1]$, and $1 \neq \theta \geq 0$

$$\mu(\tilde{w}) = \left\{ \sum_{m=1}^{n} \left[\left(\sum_{h=1}^{m} p_{s_h} \right)^{\gamma} - \left(\sum_{h=1}^{m-1} p_{s_h} \right)^{\gamma} \right] w_{s_m}^{1-\theta} \right\}^{1/(1-\theta)} \tag{1}$$

or, corresponding to $\theta = 1$,

$$\mu(\tilde{w}) = \exp \left\{ \sum_{m=1}^{n} \left[\left(\sum_{h=1}^{m} p_{s_h} \right)^{\gamma} - \left(\sum_{h=1}^{m-1} p_{s_h} \right)^{\gamma} \right] \log(w_{s_m}) \right\}. \tag{2}$$

In interpreting these expressions, note that I treat $\sum_{h=1}^{m-1} p_{s_h}$ as equal to zero for $m = 1$. The terms $\left(\sum_{h=1}^{m} p_{s_h}\right)^{\gamma} - \left(\sum_{h=1}^{m-1} p_{s_h}\right)^{\gamma}$ attached to each outcome are often referred to as decision weights, in contrast to the objective probabilities which would appear in their place under EU.

Note that if $\gamma = 1$, the decision weights do become the objective probabilities: for each m we have

$$\left(\sum_{h=1}^{m} p_{s_h} \right)^{\gamma} - \left(\sum_{h=1}^{m-1} p_{s_h} \right)^{\gamma} = \sum_{h=1}^{m} p_{s_h} - \sum_{h=1}^{m-1} p_{s_h} = p_{s_m}.$$

In that case, $\mu(\tilde{w}) = \left\{ E\left(\tilde{w}^{1-\theta}\right)\right\}^{1/(1-\theta)}$ or $\mu(\tilde{w}) = \exp\{E \log(\tilde{w})\}$, and these certainty equivalents rank lotteries in a way that is ordinally equivalent to expected utility under constant relative risk aversion (with θ as the coefficient of relative risk aversion).[15]

If $\gamma < 1$ but $\theta = 0$, we have

$$\mu(\tilde{w}) = \sum_{m=1}^{n} \left[\left(\sum_{h=1}^{m} p_{s_h} \right)^{\gamma} - \left(\sum_{h=1}^{m-1} p_{s_h} \right)^{\gamma} \right] w_{s_m} \tag{3}$$

which is the form that risk preferences take in Yaari's [43] "dual theory" of choice under risk.

When $\gamma < 1$, states with worse outcomes receive greater weight (compared to expected value) than would be given by their objective probabilities. This is most readily seen in the case of a binary lottery. For example, when $n = 2$ and $w_1 < w_2$, then

$$\mu(\tilde{w}) = \left[p_1^{\gamma} w_1^{1-\theta} + (1 - p_1^{\gamma}) w_2^{1-\theta} \right]^{1/(1-\theta)}. \tag{4}$$

With $\gamma < 1$, we have $p_1^{\gamma} > p_1$ and $1 - p_1^{\gamma} < 1 - p_1 = p_2$.

We can see the impact of $\gamma < 1$ on risk aversion for small gambles in the binary

[15] In the dynamic setting below, $\gamma = 1$ and θ equal to the inverse of the elasticity of intertemporal substitution gives the standard EU/CRRA specification; the case of $\gamma = 1$ for an arbitrary value of θ corresponds to what are sometimes called Epstein-Zin (or Epstein-Zin/Weil) preferences (after their [13] and [40]).

case by replicating standard textbook approximations of risk premia. Suppose that wealth in the two states is given by

$$\tilde{w} = \begin{cases} 1 - \sigma\sqrt{\frac{1-p}{p}} & \text{with probability } p \\ \\ 1 + \sigma\sqrt{\frac{p}{1-p}} & \text{with probability } 1-p \end{cases}$$

Then, mean wealth is 1 and σ is its standard deviation. Consider the case of $\theta \neq 1$. Plugging the gamble into (4) gives the the lottery's certainty equivalent as:

$$\mu\left(\tilde{w}\right) = \left\{ p^{\gamma}\left(1 - \sigma\sqrt{\frac{1-p}{p}}\right)^{1-\theta} + \left(1 - p^{\gamma}\right)\left(1 + \sigma\sqrt{\frac{p}{1-p}}\right)^{1-\theta} \right\}^{1/(1-\theta)}. \tag{5}$$

Define the lottery's risk premium r by

$$\mu\left(1 - r\right) = \mu\left(\tilde{w}\right).$$

Note that, because of the linear homogeneity of μ—*i.e.*, $\mu\left(a\right) = a$ for any constant a—we have $\mu\left(1-r\right) = 1 - r$, so that

$$r = 1 - \mu\left(\tilde{w}\right).$$

Using second order Taylor approximations to the $\left(\ \cdot\ \right)^{1-\theta}$ terms inside the brackets in (5), and the first-order approximation $\left(1+x\right)^{1/(1-\theta)} \cong 1 + \left[1/\left(1-\theta\right)\right]x$ gives

$$r \cong \sigma A\left(p\right) + \frac{1}{2}\theta\sigma^{2}B\left(p\right),$$

where

$$A\left(p\right) \equiv p^{\gamma}\sqrt{\frac{1-p}{p}} - \left(1 - p^{\gamma}\right)\sqrt{\frac{p}{1-p}}$$

and

$$B\left(p\right) \equiv p^{\gamma}\left(\frac{1-p}{p}\right) + \left(1 - p^{\gamma}\right)\left(\frac{p}{1-p}\right).$$

Note that when $\gamma < 1$, $A\left(p\right) > 0$ and the risk premium incorporates a first-order term in σ, in contrast to what we obtain under expected utility. In that case—when $\gamma = 1$—we have $A\left(p\right) = 0$ and $B\left(p\right) = 1$, so that

$$r_{\mathrm{EU}} \cong \frac{1}{2}\theta\sigma^{2},$$

as usual.

2.3 Some calibration exercises

A few calculations can illustrate the sense in which the FORA risk preferences just described can be parametrized to give an empirically plausible degree of risk aversion over a broader range of gamble sizes than is possible with the EU specification.

Consider first a very small risk. Suppose an agent with initial wealth of \$30,000 faces a 0.00477 probability of losing \$55. This is a small risk—the standard deviation of the lottery $\{\tilde{w}; p\} = \{(29945, 30000); (0.00477, 0.99523)\}$, as a percent of mean wealth, is about 0.013%. If the agent has FORA preferences with $\gamma = .9$ and $\theta = 1$, he would be willing to pay just under 45¢ to insure against this risk. Is that a lot? Apparently not: while the initial wealth level of \$30,000 is purely hypothetical, the 0.00477 probability and \$55 loss are averages from Cicchetti and Dubin's [6] data on phone wire insurance: repair charges averaged \$55 per claim and the average probability of a claim was 0.00477 per month. The average price of phone wire insurance was 45¢ per month (nearly two times the expected loss), and 57% of the customers in their sample purchased phone wire insurance. Coaxing a willingness to pay 45¢ for this insurance out of an EU/CRRA agent with the same initial wealth would require a coefficient of relative risk aversion of 550.[16]

Now consider a more modest-sized risk. Suppose the agent with wealth equal to \$30,000 faces a 0.245 probability of losing \$182. The standard deviation of this gamble, as percent of mean wealth, is 0.26%. A FORA agent with $\gamma = .9$ and $\theta = 1$ (as in the last example), would be willing to pay about \$51 to insure against this risk. The loss and loss probability again come from an empirical study: Cohen and Einav's [10] analysis of the choice of auto insurance deductibles in a large sample of Israeli drivers. The \$51 the FORA agent would pay is in the right ballpark—in Cohen and Einav's data, the average deductible-premium menu offers savings on deductible of \$182 (in the event of claims, which have an average frequency of 0.245) at a price of \$55. About 18% of the individuals in the sample chose higher premiums in exchange for a lower deductible. Coaxing a willingness to pay \$51 for this insurance out of an EU/CRRA agent with the same initial wealth of \$30,000 would require a coefficient of relative risk aversion of about 50.[17]

Finally, consider a large risk. Suppose the agent, again with initial wealth of \$30,000, faces a 7% probability of suffering a \$5,000 loss. This represents a gamble with a standard deviation equal to 4.3% of mean wealth. A FORA agent with $\gamma = .9$ and $\theta = 1$ (as in the previous two examples) would be willing to pay \$495 to insure against this risk. The 7% probability and \$5,000 loss are roughly the US average

[16] Note that my interpretation of Cicchetti and Dubin's data differs from their own, as they conclude that the data are consistent with EU with only a modest coefficient of risk aversion. My interpretation is more akin to that of Rabin and Thaler [29].

[17] Cohen and Einav estimate a structural model taking account of adverse selection and allowing for heterogeneity in individual risk and risk aversion. Using average annual Israeli income as a proxy for wealth, they obtain an average relative risk aversion coefficient of 81 in their benchmark specification. Sydnor [36] presents a similar example using data on deductible choices in the market for homeowners insurance and finds implied relative risk aversion coefficients in the triple digits.

homeowners' multi-peril insurance claim rate and claim intensity for the period 2000–2004, according to the Insurance Information Institute.[18] $495 is low compared to the US average premium, in 2004, of over $600, but it's in the general vicinity. Coaxing a willingness to pay $495 for insurance against this risk from an EU/CRRA agent with the same initial wealth is easier here than in the smaller-risk examples—a coefficient of relative risk aversion of about 4 will work. Of course, the market for homeowners' insurance is complex—the industry is regulated, homeowners with mortgages have little choice as to whether to insure or not, and the average figures mask considerable heterogeneity. The point of this example, though, together with the two previous examples, is simply to show that a FORA agent with risk preferences that depart modestly from EU/CRRA (which is $\gamma = 1$) will be in the ballpark of empirical plausibility in all three cases. To achieve the same for the EU/CRRA specification meant re-calibrating the coefficient of relative risk aversion from 550 for the very small risk, to 50 for the modest risk and finally to 4 for the large risk. Choose only one of those numbers and apply it to all three examples, and the EU/CRRA agent will be far out of the ballpark in two out of three cases.

There is, of course, also a large literature in experimental economics that seeks to adduce individuals' attitudes towards risk, though I confess to finding the results in that literature difficult to interpret, as it is generally assumed that the consequences to the subjects of the choices they make are simply the payoffs they receive in the experiment—initial wealth or income are "checked at the door", so to speak. The recent study by Holt and Laury [19] is a good case in point, in particular because it seems so well-done. In Holt and Laury's experiments, subjects are given a choice between pairs of binary gambles, a safer gamble (call it A) that pays $2.00 with probability p and $1.60 with probability $1 - p$ and a riskier gamble (B) that pays $3.85 with probability p and $0.10 with probability $1 - p$. When the probability of the higher outcomes, p, is near zero, gamble A has a higher expected value than B; the opposite is true when p is near one. They are interested in the effects that payoff size has on their experimental results, so in addition to the gambles just described, Holt and Laury also perform experiments where the payoffs are scaled up by a factors of 20, 50 and 90. They conduct experiments where the payoffs are real (a subject can really go home with $346.50, say) and experiments where the payoffs are hypothetical.

Holt and Laury's experiments ask subject to choose between the A and B gambles at different values of p, the probability of the higher outcomes; in particular, they record the point at which subjects switch from the safer A gamble to the riskier B gamble p is increased from 1/10 to 1 in increments of 1/10. An expected income maximizer, for example, would choose A up to $p = 4/10$, then switch to B. Among the results Holt and Laury report are the proportions of subjects' choices that are consis-

[18] `http://www.iii.org/media/facts/statsbyissue/homeowners/`. [This link was active as of 4/27/2006.]

tent with maximizing $E\left[x^{1-\theta}/(1-\theta)\right]$ for a CRRA coefficient θ in various ranges.[19] While they find significantly more risk aversion than had been found previously in the experimental literature—especially when the payoffs are real and high—only a very small fraction of the choices are consistent with maximizing $E\left[x^{1-\theta}/(1-\theta)\right]$ for θ as big as even 1.37. But this is under the assumption that the relevant x in the subjects' minds is simply the payoff from the experiment. If one allows that the consequences are final wealth levels—$w+x$ where w is a subject's initial wealth and x is income earned in the experiment—then even a modest value of w will blow up the the implied most-common values of θ considerably.[20] For example, in their '$\times$20 real' treatment—when payoffs are 20 times the values given in the previous paragraph, and are real rather than hypothetical—the most common behavior is sticking with A up to $p = 6/10$, then switching to B. If an individual is indifferent between A and B at $p = 6/10$, and is an EU/CRRA maximizer who regards the consequences of the experiment as simply the payoffs from the experiment, then he must have a value of $\theta \cong 0.41$. If the subject has even \$100 in initial wealth, though, and views the consequences as being values of final wealth, he must have $\theta \cong 2$; if initial wealth is \$1,000, this value becomes $\theta \cong 15$.[21]

In this section, I've tried to argue that FORA risk preferences that depart modestly from EU/CRRA preferences have some empirical appeal. The subsequent sections will try to show that FORA preferences that even depart very sharply from EU/CRRA make little difference for the dynamics of the basic neoclassical stochastic growth model, though they do make a great deal of difference for what the model says about the cost of aggregate volatility.

3 The model economy

3.1 Preferences

The economy is inhabited by an infinitely-lived representative agent. The agent gets utility from consumption, C_t, and leisure, L_t. The agent's within-period utility function, or felicity function, defined over current consumption and leisure, is

$$u(C, L) = CL^{\psi}.$$

The agent's intertemporal preferences are defined recursively using an aggregator W which combines current felicity with a certainty equivalent value for lifetime utility

[19] This coefficient is r in Holt and Laury's notation.

[20] And if one doesn't factor in initial wealth, then the gambles they consider—in terms of their standard deviation—are extremely large, with percent standard deviations ranging from about 6 to over 200 (excluding degenerate gambles). In that case it's no surprise that the CRRA coefficients that can be ascribed to most subjects must be quite small.

[21] Cohen and Einav [10] estimate a CARA utility model for the Holt and Laury subjects who particpated in the highest-payoff experiment, assuming a lognormal distribution for the subjects' CARA parameters, and report a point estimate of the mean of this distribution equal to 0.032. Taking average US disposable income in 2002 as a proxy for wealth, they calculate an average relative risk aversion coefficient of 865.75.

from tomorrow onward (conditional on today's information) to give lifetime utility as of today:

$$U_t = W\left[u\left(C_t, L_t\right), \mu_t\left(\tilde{U}_{t+1}\right)\right]. \tag{6}$$

The aggregator W captures information about the agent's willingness to substitute over time, while the certainty equivalent operator $\mu_t(\ \cdot\ )$ captures information about the agent's attitudes towards risk.[22]

I assume a CES aggregator of the form:

$$W(u,\mu) = \begin{cases} \left[(1-\beta)\, u^{1-(1/\varepsilon)} + \beta \mu^{1-(1/\varepsilon)}\right]^{\frac{1}{1-(1/\varepsilon)}} & (0 < \varepsilon \neq 1) \\ u^{1-\beta}\mu^{\beta} & (\varepsilon = 1) \end{cases} \tag{7}$$

Here, β is the utility discount factor—*i.e.*, $(1/\beta) - 1$ is the household's pure rate of time preference along constant paths—and ε is the household's elasticity of intertemporal substitution (defined along deterministic paths). For deterministic consumption-leisure paths, the household's preferences are ordinally equivalent to the common time-additive forms

$$\sum_{t=0}^{\infty} \beta^t \frac{\left(C_t L_t^{\psi}\right)^{1-(1/\varepsilon)}}{1-(1/\varepsilon)}$$

or

$$\sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t\right) + \psi \ln\left(L_t\right)\right].$$

The certainty equivalent operator $\mu_t(\ \cdot\ )$ is analogous to $\mu(\tilde{w})$ described in equations (1) and (2) in the previous section. While FORA preferences can be defined in the case where random variables are of the continuous type—in fact, this is the setting of Yaari's original [43] "dual theory"—the computational experiments I will perform use exogenous shocks which follow finite state Markov chains. Investing in a notation which I don't plan to use seems like a waste of resources—especially those of the reader—so I will treat the agent's lifetime utility from tomorrow onward as a discrete random variable, and describe $\mu_t(\ \cdot\ )$ in those terms. In particular, suppose that, conditional on shocks realized through date t and the agent's decisions taken at date t, $\tilde{U}_{t+1}$ is a discrete random variable taking on values $U_{t+1}(s)$, $s = 1, 2, \ldots, n$, with probabilities $P_t(s)$, $s = 1, 2, \ldots, n$. Then, for $1 \neq \theta \geq 0$

$$\mu_t\left(\tilde{U}_{t+1}\right) = \left\{\sum_{m=1}^{n}\left[\left(\sum_{h=1}^{m} P_t\left(s_h\right)\right)^{\gamma} - \left(\sum_{h=1}^{m-1} P_t\left(s_h\right)\right)^{\gamma}\right] U_{t+1}\left(s_m\right)^{1-\theta}\right\}^{1/(1-\theta)}$$

[22] To be precise, intertemporal substitutability as regards deterministic paths of consumption and leisure is governed by the parameters of the aggregator, while risk aversion as regards timeless gambles is governed by the certainty equivalent operator.

while for $\theta = 1$,

$$\mu_t\left(\tilde{U}_{t+1}\right) = \exp\left\{\sum_{m=1}^{n}\left[\left(\sum_{h=1}^{m} P_t(s_h)\right)^{\gamma} - \left(\sum_{h=1}^{m-1} P_t(s_h)\right)^{\gamma}\right]\log\left[U_{t+1}(s_m)\right]\right\}.$$

In these expressions, as in the previous section, $\{s_1, s_2, \ldots, s_n\}$ is a permutation of $\{1, 2, \ldots, n\}$ that ranks the n states from worst to best, in this case according to $U_{t+1}(s)$—*i.e.*, $U_{t+1}(s_1) \leq U_{t+1}(s_2) \leq \cdots \leq U_{t+1}(s_n)$.

As before, when $\gamma = 1$, we have

$$\mu_t\left(\tilde{U}_{t+1}\right) = \left[E_t\left(\tilde{U}_{t+1}^{1-\theta}\right)\right]^{1/(1-\theta)}$$

or

$$\mu_t\left(\tilde{U}_{t+1}\right) = \exp\left\{E_t \log\left(\tilde{U}_{t+1}\right)\right\}.$$

In that case, the agent's intertemporal preferences specialize to the form employed by Epstein and Zin in [13] or Weil in [40]; if we also have $\varepsilon = 1$, then the agent's preferences specialize further to the form considered by Tallarini [37].

If $\gamma = 1$ and $\theta = 1/\varepsilon$—that is, the agent's coefficient of relative risk aversion is equal to the inverse of the agent's elasticity of intertemporal substitution—the agent's utility process would become

$$U_t = \begin{cases} \left[(1-\beta)\left(C_t(1-N_t)^{\psi}\right)^{1-(1/\varepsilon)} + \beta E_t\left(\widetilde{U}_{t+1}^{1-(1/\varepsilon)}\right)\right]^{\frac{1}{1-(1/\varepsilon)}} & (0 < \varepsilon \neq 1) \\ \left(C_t(1-N_t)^{\psi}\right)^{1-\beta}\exp\left(\beta E_t \log\left(\widetilde{U}_{t+1}\right)\right) & (\varepsilon = 1) \end{cases}.$$

These processes are ordinally equivalent to the standard EU/CRRA intertemporal specifications.[23] The model's output with these preferences will be the benchmark against which we compare results under various values of γ and θ.

3.2 Technology and resource constraints

Except for the specification of intertemporal preferences given above, the model is a garden-variety real business cycle model. The production technology is of the Cobb-Douglas form

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{8}$$

Total factor productivity A_t has both a deterministic trend and a stochastic component. In particular $A_t = (1+\eta)^t a_t$, where η is the trend growth rate of total factor productivity, and a_t is assumed to evolve according to an n_S-state Markov chain taking values in a set S, with transition probability matrix P. Output in each period

[23] Define $V_t = [1-(1/\varepsilon)]^{-1} U_t^{1-(1/\varepsilon)}$ or $V_t = \log(U_t)$ to recover their more common time-additively-separable representations.

is divided between consumption (C_t) and investment (X_t):

$$Y_t \geq C_t + X_t.$$

Capital accumulates according to

$$K_{t+1} = (1-\delta) K_t + X_t,$$

and the agent begins with some quantity $K_0 > 0$. Finally, the agent's time endowment per period is normalized to one, and leisure hours are simply time not spent working:

$$L_t = 1 - N_t.$$

Given K_0 and an initial realization of the shock a_0, the agent seeks to maximize lifetime utility as of date 0, U_0, subject to the above technology and resource constraints and the law of motion for the exogenous shocks.

3.3 The agent's dynamic program

The representative agent's problem is recursive and can be represented as a dynamic program. First, though, given the trend growth in total factor productivity, all quantities (except labor hours, N_t) need to be deflated by $(1+\eta)^t$ to yield an environment that is stationary. Lower-case letters denote variables thus deflated—$c_t = C_t / (1+\eta)^t$, for example. The descriptions of preferences, technology and resource constraints from the last section are modified only slightly by this transformation. The evolution of the transformed capital stock becomes $(1+\eta) k_{t+1} = (1-\delta) k_t + x_t$, as usual. The agent's utility process is modified as well. To economize on notation here and below, define a modified aggregator $\hat{W}(u, \mu)$ by

$$\hat{W}(u,\mu) = \begin{cases} \left[(1-\beta) u^{1-(1/\varepsilon)} + \beta (1+\eta)^{1-(1/\varepsilon)} \mu^{1-(1/\varepsilon)}\right]^{\frac{1}{1-(1/\varepsilon)}} & (0 < \varepsilon \neq 1) \\ u^{1-\beta}\mu^{\beta} & (\varepsilon = 1) \end{cases}.$$

The agent's utility process in terms of transformed consumption is then given by

$$U_t = \hat{W}\left[u(c_t, 1-N_t), \mu_t\left(\tilde{U}_{t+1}\right)\right].$$

Write $v(s,k)$ for the agent's value function—*i.e.*, the maximized value of lifetime utility from exogenous state $a(s)$ and (transformed) capital stock k. The Bellman equation is

$$v(s,k) = \max_{c,N,k'} \hat{W}\left[u(c, 1-N), \mu(v(\tilde{s}, k') : s)\right]$$

subject to

$$a(s) k^{\alpha} N^{1-\alpha} + (1-\delta) k \geq c + (1+\eta) k'$$

and

$$1 \geq N \geq 0.$$

The certainty equivalent $\mu(v(\tilde{s}, k') : s)$ on the right-hand side of the Bellman equation is given by:

$$\mu(v(\tilde{s}, k') : s) = \left\{ \sum_{m=1}^{n_S} \left[\left(\sum_{h=1}^{m} P(s, s_h) \right)^{\gamma} - \left(\sum_{h=1}^{m-1} P(s, s_h) \right)^{\gamma} \right] v(s_m, k')^{1-\theta} \right\}^{1/(1-\theta)}$$

or

$$\mu(v(\tilde{s}, k') : s) = \exp \left\{ \sum_{m=1}^{n_S} \left[\left(\sum_{h=1}^{m} P(s, s_h) \right)^{\gamma} - \left(\sum_{h=1}^{m-1} P(s, s_h) \right)^{\gamma} \right] \log v(s_m, k') \right\}$$

as either $0 \leq \theta \neq 1$ or $\theta = 1$. As before, $\{s_1, s_2, \ldots, s_{n_S}\}$ denotes the ranking of the n_S exogenous states given by $v(\ \cdot\ , k')$; that is,

$$v(s_1, k') \leq v(s_2, k') \leq \cdots \leq v(s_{n_S}, k').$$

To be precise, we should index the ranking $\{s_h\}_{h=1}^{n_S}$ by v and k'—that is, as $\left\{ s_h^{v,k'} \right\}_{h=1}^{n_S}$—since the ranking of the states obviously depends on v and can differ at different values of k'.

4 Solution method and calibration

4.1 Discretization

I solve the model using discrete state space dynamic programming, given values for the model's parameters. A reference point for both the calibration of parameters and the solution method is the economy's non-stochastic steady state, by which I mean the steady state of the deterministic model one obtains by setting the technology shocks a_t equal to their expected value for all t. The capital grid I use consists of $n_K = 1500$ uniformly-spaced grid points; the lower endpoint was set at 25% below the non-stochastic steady state capital stock, while the upper endpoint was set at 50% above the non-stochastic steady state stock.[24]

I first calculate the agent's one-period reward at each combination of exogenous state, today's capital stock and tomorrow's capital stock. The one-period reward, call it u^*, is the value of the intratemporal problem of maximizing felicity $u(c, 1-N)$ given the current state variables and planned investment:

$$u^*(s, i, j) = \max_{N \in [0,1]} \left[a(s) k(i)^{\alpha} N^{1-\alpha} + (1-\delta) k(i) - k(j) \right] (1-N)^{\psi}.$$

[24] I experimented on coarser grids to identify a range which would encompass the entire support of the long-run distribution of capital stocks. Because the agent engages in precautionary savings, the mean of the long-run distribution of capital is to the right of the non-stochastic steady state stock.

Note that the one-period reward does not depend on the parameters we are interested in varying, neither ε, the elasticity of intertemporal substitution, nor γ and θ, the parameters governing risk aversion. This means that the problem of calculating $u^*(s,i,j)$ at all $n_S \times n_K \times n_K$ grid points, while tedious, need only be done once and for all; the resulting array u^* can then be taken as data in experiments that solve the model for varying values of ε, γ and θ.

The discretized dynamic program is then

$$v(s,i) = \max\left\{\hat{W}\left[u^*(s,i,j), \mu(v(\tilde{s},j) : s)\right] : j \in \{1,2,\ldots,n_K\}\right\}$$

where, in a slight abuse of our previous notation, I've written the value function at exogenous state $a(s)$ and capital stock $k(i)$ as $v(s,i)$. The optimal policy function giving next-period's capital stock as a function of today's state variables—call it π—is defined as

$$\pi(s,i) = \arg\max\left\{\hat{W}\left[u^*(s,i,j), \mu(v(\tilde{s},j) : s)\right] : j \in \{1,2,\ldots,n_K\}\right\}.$$

Solving the model from this point is standard—given an initial guess for the value function, v_0, apply the Bellman operator to construct sequences $v_{\tau+1}$ and π_τ according to

$$\begin{aligned} v_{\tau+1}(s,i) &= \max\left\{\hat{W}\left[u^*(s,i,j), \mu(v_\tau(\tilde{s},j) : s)\right] : j \in \{1,2,\ldots,n_K\}\right\} \\ \pi_\tau(s,i) &= \arg\max\left\{\hat{W}\left[u^*(s,i,j), \mu(v_\tau(\tilde{s},j) : s)\right] : j \in \{1,2,\ldots,n_K\}\right\}. \end{aligned}$$

I terminated the iterations when the policy functions stopped changing—that is, when $\pi_{\tau+1} = \pi_\tau$. This convergence generally occurred very rapidly. To better approximate the value function, I iterated further using the converged policy functions, rather than a maximization, on the right-hand side of the Bellman equation, stopping when $\|v_{\tau+1} - v_\tau\|_\infty < 10^{-16}$.

4.2 Calibration

The parameter values I use are fairly standard in the basic RBC literature.[25] I set η, the trend growth rate of TFP, to be 1.8% per year. δ is set to give 10% depreciation per year. Capital's share in the Cobb-Douglas production function, α, is set to 0.42. The parameter ψ in cL^ψ is set, given the other parameters, so as to imply hours worked equal to 0.21 in the economy's non-stochastic steady state. Given η and a choice of the elasticity of intertemporal substitution, ε, β is chosen to give an annual return on capital equal to 6.5%. Table 1 summarizes these basic parameter choices:

[25] They are similar to those used in King, Plosser and Rebelo [20].

Table 1: Basic parameter values					
Parameter:	η	δ	α	ψ	β
Value or target:	1.8%/year	10%/year	0.42	$\bar{n} = 0.21$	6.5% return/year

I calibrate the Markov chain for a_t to approximate an AR(1) process for $\log(a_t)$:

$$\log(a_{t+1}/\bar{a}) = \rho_a \log(a_t/\bar{a}) + \xi_{a,t+1} \qquad \left(\xi_{a,t+1} \sim \text{i.i.d.}\left(0, \sigma_{\xi_a}\right)\right), \tag{9}$$

I assume values of $\rho_a = .98$ and $\sigma_{\xi_a} = .0076$. The mean parameter $\bar{a}$ is set so that $E(a_t)$, in conjunction with the other parameters, gives output equal to unity in the model's non-stochastic steady state. I approximate (9) using a five-state Markov chain calibrated via Tauchen's [38] method, assuming that the disturbances $\xi_{a,t+1}$ in (9) are normally distributed. I set the parameter that Tauchen calls m equal to 1.5; this gives the set of states for the Markov chain, S, equal to $\left\{-\frac{3}{2}\sigma_a, -\frac{3}{4}\sigma_a, 0, +\frac{3}{4}\sigma_a, +\frac{3}{2}\sigma_a\right\}$, where σ_a is the unconditional standard deviation of $\log(a_t/\bar{a})$ implied by (9) given $\rho_a = 0.98$ and $\sigma_{\xi_a} = 0.0076$. I also ran some experiments using a nine-state Markov chain; the results were little affected by the addition of more states.[26]

The remaining parameters are ε, γ and θ. As indicated above, these are parameters which I will be varying in the experiments that follow, so no attempt was made to calibrate them.

5 Quantitative results

5.1 Model dynamics

In the experiments of this section, I consider three values of ε, the elasticity of intertemporal substitution, $\varepsilon = 1, 2/3$ and $1/3$. The former is a standard choice in the business cycle literature, while the latter is closer to the values found in most microeconomic studies of intertemporal substitution. Note that the three EIS specifications imply three EU/CRRA specifications, $\theta = 1, 3/2$ and 3, since $\theta = 1/\varepsilon$ for EU/CRRA.

For each value of ε, I examine three FORA specifications: $(\gamma, \theta) = (.9, 1)$, $(.8, 1)$ and $(.7, 1)$. The degree of risk aversion implied by $(\gamma, \theta) = (.9, 1)$ was discussed extensively in section 2, and argued to be empirically plausible over a wide range of risk sizes. The degrees of risk aversion implied by the smaller values of γ are more extreme. To see how extreme, consider the insurance examples from section 2. A FORA agent with $(\gamma, \theta) = (.8, 1)$, and wealth of \$30,000 as in section 2, would be willing to pay about \$0.76 per month per month in the phone wire insurance example, while an agent with $(\gamma, \theta) = (.7, 1)$ would be willing to pay about \$1.31 per month—nearly 3 times the willingness to pay of a FORA agent with $(\gamma, \theta) = (.9, 1)$.

In addition to the three FORA specifications, I also consider an Epstein-Zin/Weil

[26] In applying Tauchen's method to the $n_S = 9$ case, I set Tauchen's m parameter equal to 3.

specification that will allow some comparison between the FORA results and the results obtained by Tallarini [37]. For this specification, denoted EZ/W in the tables, I set $\gamma = 1$ and $\theta = 100$.

One could consider more cases, but these seem sufficient to establish the paper's points. In particular, given the small standard deviation of the volatility in the model, the choice of $\theta = 1$ in all of the FORA specifications is innocuous—θ in those specifications governs attitudes towards very large risks; all the action really lies with γ. As for the EIS, ε, I know of no work suggesting it should be bigger than 1, and taking it down to 1/3 gives, I think, enough of an indication of its impact on the model's dynamics (which, in contrast to the risk parameters, is non-negligible).

In order to calculate the model's second-moment predictions, I use the decision rules obtained from solving the representative agent's problem, together with the transition matrix from the Markov chain for a_t, to simulate a time series of the model variables of length 10,000 periods.[27] The data were then logged and detrended using a Hodrick-Prescott filter and their moments calculated.

Table 2 shows results for the case $\varepsilon = 1$. The moments reported are the standard ones in the RBC literature—100 times the standard deviations of the logged and detrended variables (Panel A), their standard deviations relative to output (Panel B) and their contemporaneous correlations with output (Panel C). Four of the five variables reported in Table 2 have been defined previously; the other—W—is the real wage rate, defined simply as the marginal product of labor according to the production technology (8).

The first row in each Panel gives results for the EU/CRRA case; with $\varepsilon = 1$ and $\theta = 1/\varepsilon$, this corresponds to the standard case of logarithmic preferences considered in much of the RBC literature. The entries in these parts of the table are about what one would expect, suggesting that the discretization of the capital grid and the five-state Markov chain assumption on a_t are both reasonable approximations.

The key fact recorded in Table 2, though, is how little the results change as we move to the FORA specifications, and as the value of γ is lowered. In particular, the relative volatility of consumption falls very slightly, while the relative volatilities of investment and hours worked rise very slightly. Consumption's contemporaneous correlation with output falls slightly, but the other correlations are essentially unchanged. Note that the results under FORA with $\gamma = .90$—the parametrization which delivered an empirically plausible aversion to risk in the three insurance examples of section 2.3—are essentially identical to the EU/CRRA results. Across all the panels, the FORA preferences record the same successes—the relative volatilities of output, consumption and investment, for example—and the same failures—the high correlation between the real wage and output—as the standard EU/CRRA specification.

[27] I use the same 10,000-period realization of the exogenous shock process in all of the experiments.

Table 2: Simulated second moments, $\varepsilon = 1$					
	$\log(Y_t)$	$\log(C_t)$	$\log(X_t)$	$\log(N_t)$	$\log(W_t)$
			Panel A: $100\sigma(z)$		
EU/CRRA	1.27	0.53	3.10	0.64	0.67
FORA: $\gamma = .9$, $\theta = 1$	1.28	0.52	3.16	0.66	0.66
FORA: $\gamma = .8$, $\theta = 1$	1.30	0.50	3.26	0.70	0.65
FORA: $\gamma = .7$, $\theta = 1$	1.33	0.48	3.42	0.74	0.63
EZ/W: $\gamma = 1$, $\theta = 100$	1.35	0.46	3.53	0.78	0.62
			Panel B: $\sigma(z)/\sigma[\log(Y_t)]$		
EU/CRRA	1.00	0.42	2.44	0.51	0.53
FORA: $\gamma = .9$, $\theta = 1$	1.00	0.40	2.47	0.52	0.52
FORA, $\gamma = .8$, $\theta = 1$	1.00	0.38	2.51	0.54	0.50
FORA, $\gamma = .7$, $\theta = 1$	1.00	0.36	2.58	0.56	0.48
EZ/W, $\gamma = 1$, $\theta = 100$	1.00	0.34	2.61	0.58	0.46
			Panel C: $\mathrm{corr}[z, \log(Y_t)]$		
EU/CRRA	1.00	0.91	0.99	0.97	0.97
FORA: $\gamma = .9$, $\theta = 1$	1.00	0.91	0.99	0.97	0.97
FORA, $\gamma = .8$, $\theta = 1$	1.00	0.89	0.99	0.97	0.96
FORA, $\gamma = .7$, $\theta = 1$	1.00	0.88	0.99	0.97	0.96
EZ/W, $\gamma = 1$, $\theta = 100$	1.00	0.86	0.99	0.97	0.96

Table 3 shows the first- through fourth-order autocorrelations in output that result under the four different risk preference specifications. The similarity of results across specifications is even more striking than in table 2. This is perhaps not surprising, as it is often argued that the basic RBC model has only a very weak internal propagation mechanism, so that the persistence properties of the model's output simply reflect the persistence properties of the technology shocks being fed into it.

Table 3: $\mathrm{corr}[\ln(Y_t), \ln(Y_{t-k})]$, $\varepsilon = 1$				
		k		
	1	2	3	4
EU/CRRA	0.71	0.46	0.26	0.10
FORA, $\gamma = .9, \theta = 1$	0.71	0.46	0.26	0.10
FORA, $\gamma = .8, \theta = 1$	0.71	0.46	0.26	0.10
FORA, $\gamma = .7, \theta = 1$	0.71	0.46	0.26	0.11
EZ/W, $\gamma = 1, \theta = 100$	0.71	0.46	0.26	0.11

If changing the form of the representative agent's risk preferences—and varying widely the agent's attitude towards risk—has only a negligible effect on the RBC model's dynamics, do other aspects of the agent's preferences make a noticeable difference? As a comparison of Table 2 and with Tables 4 and 5 shows, variation in the agent's elasticity of intertemporal substitution, ε, can have a non-negligible effect on the model's output, though it is still the case that for a given value of ε, wide variation in risk preference parameters has very little effect on the model's output. Tables 4 and 5 present the same set of statistics as in Table 2, but for the cases of $\varepsilon = 2/3$ and $1/3$:

Table 4: Simulated second moments, $\varepsilon = 2/3$

	$\log(Y_t)$	$\log(C_t)$	$\log(X_t)$	$\log(N_t)$	$\log(W_t)$
			Panel A: $100\sigma(z)$		
EU/CRRA	1.14	0.64	2.36	0.43	0.74
FORA: $\gamma = .9$, $\theta = 1$	1.15	0.62	2.42	0.45	0.73
FORA: $\gamma = .8$, $\theta = 1$	1.17	0.60	2.52	0.48	0.72
FORA: $\gamma = .7$, $\theta = 1$	1.20	0.57	2.66	0.53	0.70
EZ/W: $\gamma = 1$, $\theta = 100$	1.20	0.57	2.69	0.54	0.69
			Panel B: $\sigma(z)/\sigma[\log(Y_t)]$		
EU/CRRA	1.00	0.56	2.07	0.38	0.65
FORA: $\gamma = .9$, $\theta = 1$	1.00	0.54	2.10	0.39	0.63
FORA, $\gamma = .8$, $\theta = 1$	1.00	0.51	2.15	0.41	0.61
FORA, $\gamma = .7$, $\theta = 1$	1.00	0.48	2.22	0.44	0.58
EZ/W, $\gamma = 1$, $\theta = 100$	1.00	0.47	2.23	0.45	0.58
			Panel C: $\text{corr}[z, \log(Y_t)]$		
EU/CRRA	1.00	0.97	0.99	0.96	0.99
FORA: $\gamma = .9$, $\theta = 1$	1.00	0.97	0.99	0.96	0.98
FORA, $\gamma = .8$, $\theta = 1$	1.00	0.96	0.99	0.96	0.98
FORA, $\gamma = .7$, $\theta = 1$	1.00	0.95	0.99	0.97	0.98
EZ/W, $\gamma = 1$, $\theta = 100$	1.00	0.96	0.99	0.97	0.98

Table 5: Simulated second moments, $\varepsilon = 1/3$					
	$\log(Y_t)$	$\log(C_t)$	$\log(X_t)$	$\log(N_t)$	$\log(W_t)$
			Panel A: $100\sigma(z)$		
EU/CRRA	1.06	0.71	1.91	0.31	0.78
FORA: $\gamma = .9$, $\theta = 1$	1.07	0.70	1.97	0.33	0.77
FORA: $\gamma = .8$, $\theta = 1$	1.10	0.67	2.09	0.37	0.76
FORA: $\gamma = .7$, $\theta = 1$	1.13	0.64	2.22	0.41	0.74
EZ/W: $\gamma = 1$, $\theta = 100$	1.12	0.64	2.18	0.40	0.74
			Panel B: $\sigma(z)/\sigma[\log(Y_t)]$		
EU/CRRA	1.00	0.67	1.80	0.29	0.74
FORA: $\gamma = .9$, $\theta = 1$	1.00	0.65	1.84	0.31	0.72
FORA, $\gamma = .8$, $\theta = 1$	1.00	0.61	1.90	0.33	0.69
FORA, $\gamma = .7$, $\theta = 1$	1.00	0.57	1.97	0.37	0.65
EZ/W, $\gamma = 1$, $\theta = 100$	1.00	0.58	1.95	0.36	0.66
			Panel C: $\text{corr}[z, \log(Y_t)]$		
EU/CRRA	1.00	0.98	0.99	0.93	0.99
FORA: $\gamma = .9$, $\theta = 1$	1.00	0.98	0.99	0.94	0.99
FORA, $\gamma = .8$, $\theta = 1$	1.00	0.98	0.99	0.95	0.99
FORA, $\gamma = .7$, $\theta = 1$	1.00	0.97	0.99	0.96	0.99
EZ/W, $\gamma = 1$, $\theta = 100$	1.00	0.98	0.99	0.96	0.99

Note in particular, that compared to Panel B of Table 2, the relative volatilities of investment and hours are lower at the lower values of ε, while the relative volatility of consumption is higher. In comparing Panel A of Table 2 to Panel A of Tables 4 and 5, it may seem odd that the absolute volatility of consumption is higher at the lower values of ε, since conventional wisdom has it that a lower EIS leads to smoother consumption. It's important to remember, though, from (6) and (7), that what the agent seeks to stabilize is felicity, $u(C_t, 1 - N_t) = C_t (1 - N_t)^{\psi}$, and in fact the absolute volatility of hours falls substantially when ε is reduced.[28] In any event, the changes in the various moments engendered by varying the risk aversion parameters are negligible compared to the effects of varying ε.

5.2 The cost of fluctuations

While the technology shock-driven RBC model lacks any role for policy in mitigating fluctuations, it is still a well-posed question to ask how much better off the rep-

[28] One can calculate the absolute volatility of $C_t (1 - N_t)^{\psi}$ and show that it does, in fact, fall when ε is reduced.

resentative agent would be in a deterministic, but otherwise identical, economy.[29] The answer to this question reveals a dimension along which FORA and EU/CRRA preferences do differ, significantly.

The deterministic economy I consider is identical to the economy described above, except that the technology shocks a_t are replaced by their unconditional expectation $E(a_t)$. Since the lifetime utility of an agent in the stochastic economy differs depending on the initial exogenous state and the initial capital stock, the experiments I consider envision the agent evaluating the stochastic environment using his value function, the long-run distribution of the states and his risk preferences, embodied in the certainty equivalent operator μ. Let $f : \{1, 2, \ldots, n_S\} \times \{1, 2, \ldots, n_K\} \rightarrow [0, 1]$ denote the long-run distribution of the states; it is the limit of f_τ where

$$f_{\tau+1}(s', i') = \sum_{(s,i) \in \pi^{-1}(i')} P(s, s') f_\tau(s, i)$$

and where $\pi^{-1}(i') = \{(s, i) : \pi(s, i) = i'\}$. The value the agent assigns to living in the stochastic environment is then defined as

$$V = \mu(\tilde{v})$$

where μ operates on $\tilde{v} \equiv \{v(s, i)\}$ in the (by now) obvious way, using the probabilities $f = \{f(s, i)\}$.

The value to the agent of living in the deterministic environment depends on the agent's initial capital. For each combination of γ, θ and ε, I assume the agent enters the deterministic economy with initial capital equal to mean capital in the stochastic economy.[30] Call the resulting value of lifetime utility $\bar{v}$.

All the preference specifications considered so far have the feature that the agent's lifetime utility is homogeneous of degree one in consumption—*e.g.*, increasing consumption at all dates and states by 1% raises the agent's lifetime utility by 1%.[31] Therefore, the percentage increase in consumption at all dates and states that would make the agent indifferent between living in the stochastic environment and living in the deterministic environment is the value of λ that solves

$$\left(1 + \frac{\lambda}{100}\right) V = \bar{v}$$

[29] One could also note that estimated Solow residuals have been shown to be correlated with other sorts of shocks, particularly shocks that might be thought of as "demand shocks". See, for example, Evans [16] or Burnside, Eichenbaum and Rebelo [5].

[30] I also performed a couple variants on these calculations. In one, I assumed that initial capital in the deterministic environment was equal to the deterministic steady state stock. In another, I had the agent evaluate the stochastic environment at *each value* of k, using the invariant distribution over the exogenous shocks, and compared these valuations to the agent's lifetime utility at the corresponding values of k in the deterministic environment. Neither of these variations substantially altered the results.

[31] This follows from the facts that $W(u, \mu)$ is homogeneous of degree one in (u, μ); $u(c, l)$ is homogeneous of degree one in c; and $\mu(v)$ is homogeneous of degree one in v.

—*i.e.*,

$$\lambda = 100\left(\frac{\bar{v}}{V} - 1\right).$$

Table 6 records values of λ for three values of the elasticity of intertemporal substitution $\left(\varepsilon = 1, \frac{2}{3}, \frac{1}{3}\right)$ and for the same risk preference specifications considered in the experiments above—EU/CRRA, FORA with $\theta = 1$ and $\gamma = .9, .8, .7$ and EZ/W with $\theta = 100$.[32]

Table 6: The cost of fluctuations (λ)

	ε		
	1	2/3	1/3
EU/CRRA	−0.85	0.16	0.32
FORA, $\gamma = .9, \theta = 1$	1.24	6.80	12.56
FORA, $\gamma = .8, \theta = 1$	3.24	8.90	15.63
FORA, $\gamma = .7, \theta = 1$	4.85	11.83	17.74
EZ/W, $\gamma = 1, \theta = 100$	6.56	6.67	7.15

Several things are noteworthy about the numbers in this table, apart from the large cost magnitudes for the non-EU/CRRA specifications. First, the reader will no doubt have noticed the negative cost in the case of $\varepsilon = 1$ with EU/CRRA risk preferences—the agent with these preferences is actually better off in the stochastic economy. This surprising result seems to be a manifestation of the phenomenon documented by Cho and Cooley [8]: many standard business cycle models with EU/CRRA preferences have the property that multiplicative uncertainty is welfare-enhancing, provided that the coefficient of relative risk aversion is not too large.[33]

Also of interest is the impact of variation in ε on the various cost numbers, in particular the contrast between ε's effect on costs under the EZ/W specification and under FORA. The choice of ε makes relatively little difference for the EZ/W specification. For the FORA specifications reducing ε from 1 to 1/3 increases the costs significantly, though the impact falls as risk aversion is increased. Correspondingly, the increase in welfare cost as risk aversion increases, for a given ε, is smaller at the lower values of ε.

Why variation in the elasticity of intertemporal substitution has this differential impact on the welfare cost numbers, depending on whether preferences are FORA or EZ/W, is unclear. Reis [30], showed in the context of a simpler model—one without labor supply, in which the representative agent faces a sequence of constraints $K_{t+1} + C_t = R_t K_t$, with $\{R_t\}$ an i.i.d. process—that the welfare gain from replacing R_t with its expected value is independent of the elasticity of intertemporal substitution

[32] Note that as ε varies the value of the discount factor β varies so that the economies corresponding to different values of ε all have the same steady state return to capital.

[33] The intuition given by Cho and Cooley for this result is that the level of output in the reduced forms of these models is a convex function of the level of the technology shock when certain parameter restrictions—such as the restriction that risk aversion not be too large—are satisfied.

when preferences have the EZ/W form.[34] So, perhaps, while the model here is more complicated, the relative insensitivity of the welfare costs in the EZ/W case to variation in ε is not surprising. However, Reis's argument can be generalized to the FORA case as well, so the difference here must owe to some features of our model—endogenous labor supply, diminishing returns to capital, *etc.*—that cause intertemporal substitution and first-order risk aversion to interact in ways they do not in Reis's simpler setting. Understanding the nature and source of this interaction is an important avenue for future work.

Finally, note that one need not employ "extreme" degrees of risk aversion to generate significant costs: for the $\gamma = .90$ case the costs, depending on the value of ε, are 12–120 times as big as the 1/10% cost which Lucas suggested as an upper bound in [22].

6 Conclusions and directions for further work

In this paper, I've tried to show that a particular form of preferences displaying first-order risk aversion can be incorporated into the basic stochastic neoclassical growth model with only a negligible impact on the dynamics implied by the model. This is true for both modest and more extreme departures from EU/CRRA, as such preferences are typically calibrated in the business cycle literature.

At the same time, making the switch from standard to FORA preferences can have a significant effect on the model's implications for the welfare cost of aggregate fluctuations. This is the case not just for hypothetically extreme degrees of risk aversion, but also for a parametrization of preferences which—given observed behavior of individuals in several insurance markets—I argue has some empirical plausibility as a "ballpark" parametrization.

There are a number of potentially interesting directions for future work, including evaluating the asset-pricing implications of the model with FORA preferences, incorporating additional shocks, and extending the model in a way that gives policy a role in mitigating aggregate fluctuations.

Also, while the exercizes above show that the basic RBC model's dynamics are robust to the introduction of FORA preferences, the basic RBC model has well-known deficiencies as a model of business cycles. An important next step is to assess the impact of FORA preferences in models which can generate more realistic business cycle behavior, in particular models with stronger internal propagation mechanisms.[35] Otrok [25], for example, considers a two-sector RBC model with variable capital utilization, time-to-build, dynamic complementarities in production and habit persistence in preferences. He estimates the model with a Bayesian procedure, using Markov Chain Monte Carlo methods, and finds a negligible cost of business cycles

[34] ...and when, as here, the agent's discount factor is adjusted so that economies with different elasticities of intertemporal substitution support identical steady state rates of return on capital.

[35] See Cogley and Nason [9] or Wen [41], among others, for critiques of the dynamic properties of the standard RBC model.

within the estimated model. While the preferences Otrok considers incorporate time-non-separability, they still conform to EU/CRRA for timeless gambles. What effect FORA risk preferences might have on both the dynamics and welfare cost implications of such a model—and, more generally, how FORA preferences might interact with habit persistence—are interesting open questions.

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