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Debt Maturity and Commitment on Firm Policies*

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Abstract

When firms can trade debt only at discrete dates, debt maturity becomes an effective tool to address the commitment problem related to debt and investment policies. In the absence of other market frictions, single-period debt restores first-best investment. With market freezes, underinvestment worsens the leverage ratchet effect, which in turn increases investment distortions for long debt maturities. A calibrated model shows that choosing the right maturity can reduce the cost of commitment problems and market frictions by up to 4% of firm value. A decomposition of the equilibrium credit spread reveals that the portion driven by the commitment problem on future debt issuance is significant when leverage and default risk are low, and is smaller for short maturities.

JEL Classification: G12, G31, G32, E22.

Keywords: credit risk, debt-equity agency conflicts, leverage ratchet effect, financial contracting, debt maturity.

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1 Introduction

Maturity is a key feature of corporate debt, as shown by a large theoretical and empirical literature. It affects future refinancing costs, credit risk, the interest tax shield, and conflicts between debt and equity over future investments. However, DeMarzo and He (2021) show that even when debt is beneficial under the static tradeoff theory, shareholders repeated attempts to profit from trading it weaken their credibility in limiting future debt issuances. As a result, future debt financing and thus debt maturity may become irrelevant for firm value.

Analyzing whether debt maturity is relevant to financial contracting then requires that some commitment power is reinstated by deviating from the model considered by DeMarzo and He (2021). There are now several contributions that analyze the role of debt maturity under no commitment: DeMarzo (2019) (and DeMarzo et al. (2023) in the context of sovereign debt models) shows that debt is beneficial if debt trades occurs only at discrete (even if random) dates. In this case, debt maturity has an impact on the value of the firm. Dangl and Zechner (2021), reinstate shareholders' commitment to future debt policies by adding a debt covenant and find that debt maturity is relevant because a shorter maturity reduces credit risk and increases the benefit of debt financing. Benzoni et al. (2022) add debt issuance costs to DeMarzo and He (2021), and find that in equilibrium the choice of debt maturity trades off tax benefits against issuance costs. Malenko and Tsoy (2021) consider a time-invariant equilibrium concept based on a trigger strategy, which is different from the one in DeMarzo and He (2021), and show that short maturity creates value by committing the shareholders to repay the debt when the firm, after a negative shock, is in financial distress and there is no other credible actions the shareholders can take.

To date, while there has been extensive research on how debt maturity can help mitigate shareholders' lack of commitment to future debt policies, its role in addressing the commitment problem related to investment and payout policies has received less attention. To study this, we develop a simple dynamic discrete-time model of a firm subject to productivity shocks. Over time, the firm invests in depreciating capital, facing adjustment costs, and raises funds through debt and equity in an efficient and frictionless financial market. Debt financing is attractive due to the tax shield on interest payments. Debt takes the form of an unsecured long-term contract, with a given (average) maturity determined by a contractual amortization parameter, ξ. The firm may face credit market freezes that limit its ability to refinance maturing debt, which makes longer-maturity debt desirable.

As in previous studies, we assume two commitment problems: shareholders cannot commit to repay debt, and they cannot commit to future debt policies. Regarding the first, if shareholders default, debt holders can claim the firm's assets net of bankruptcy costs. Regarding the second, debt trades at current market prices, which reflect both current actions and expectations of future actions by shareholders. Relative to existing literature, we introduce a third commitment problem: *shareholders cannot credibly commit to the investment/liquidation policy of the firm's assets.* Debt trades and investment or liquidation of assets can occur only at discrete dates, only once per period, and aim at maximizing the current equity value. In line with other models, the solution of the game between the shareholders and the debt holders is based on the Markov Perfect Equilibrium (MPE), in which debt price depends only on fundamentals.

The model allows for several novel contributions. First, in the absence of credit market freezes, single-period debt not only eliminates the leverage ratchet effect (LRE), but also restores firm-value maximizing investment. This effectively achieves the (constrained) first best value for the firm, even when shareholders act in their own interest. However, the possibility of being excluded from the credit market makes short maturity costly for the firm, resulting in a hump-shaped relationship between firm value and debt maturity.

When maturity extends beyond one period, the interaction between the commitment problems on leverage and investment becomes non-trivial. In our model, leverage affects investment through marginal q in two distinctive ways. First, because of the lack of commitment on debt repayment, the component of marginal q determined by future after-tax cash flows and interest tax shields is negatively affected by current leverage, which is the "usual" debt overhang channel. Second, because of shareholders' incentive to dilute debt in the future, there is a component of marginal q that increases in leverage. Because it disappears with single-period debt, this component of marginal q is solely due to the LRE.

At the same time, investment plays a key role in shaping the LRE. Greater underinvestment worsens the dilution of creditors' claims, as it leads to higher debt issuance compared to a model with fixed investment, all else equal. This effect is more pronounced with longer debt maturity, everything else equal, since marginal q rises with shareholders' ability to dilute future debt holders (as discussed above), although the marginal value of issuing new debt declines with longer maturity. Hence, choosing shorter maturity debt reduces underinvestment and mitigates this channel of the LRE.

The interaction between the commitment problem on investment and the commitment problem on debt issuance determines a non-trivial firm's leverage dynamics. Shorter debt maturity not only speeds up the downward adjustment of leverage through higher amortization, but also accelerates the upward adjustment, as the marginal value of debt issuance is higher when short-term debt commands a higher price. Conversely, when debt maturity is sufficiently long, there can be equilibria in which leverage does not converge and becomes non-stationary, depending on initial conditions. This behavior does not arise in LRE models with constant investment.

Overall, the net effect of the three commitment problems is not simply a value transfer from debt to equity, but a reduction in total firm value. To quantify these effects, we calibrate the model at a quarterly frequency using empirical moments from non-financial firms in the S&P 500 index, including investment behavior, financing decisions, risk measures, credit spreads, and default frequency. Based on the calibrated model, the trade-off between the benefits of short maturity, which mitigates commitment problems, and the benefits of long maturity, which reduces the impact of credit market freezes, yields a net contribution of debt financing to firm value of about 6%, consistent with Korteweg (2010) and van Binsbergen et al. (2010). The value loss due to the combined effects of commitment problems and credit market freezes ranges from 3% to 7% of firm value, so that optimal maturity can reduce this loss by up to 4%. Consistent with the non-monotonic effect of debt maturity on firm value, long-run leverage is also hump-shaped, although it is less sensitive to debt maturity and the likelihood of market freezes.

We use the calibrated model to study the firm's investment, leverage, and payouts over time. Notably, when debt maturity is long and current leverage is above the long-run level, the firm actively repurchases debt using equity proceeds which reduces the negative effects of the commitment problem on debt policy and investment. This behavior is a clear departure from the LRE in DeMarzo and He (2021). On the other hand, when debt maturity is long and leverage is non-stationary, shareholders worsen the LRE by selling assets to fund self-serving payouts.

The analysis of a simulated economy based on the calibrated model shows that, all else equal, debt maturity has a real effect by influencing the three commitment problems in our model. Specifically, when maturity is shorter, investments (and Tobin's Q) are higher, more persistent, and less sensitive to shocks. The correlation between investment and productivity shocks increases with debt maturity: with short maturity, investment fully responds to positive shocks in productivity, while with long maturity, the correlation with negative productivity shocks rises due to more severe underinvestment.

The cost of debt financing is higher for longer debt maturity, which shows that short debt maturity plays a role in alleviating the commitment problem on debt repayments and the commitment problem on future debt issuances and investments. Given this dual role, we separate the component of credit spread that is solely due to shareholders' lack of commitment to debt repayment (default spread), from the component that is solely driven by time-inconsistent investment and debt policies (agency spread). This allows us to determine under what circumstances one component is bigger than the other, and how and if debt maturity is a more or less effective device against either commitment problems. We show that, everything else equal, the agency component of credit spread is higher when the leverage is low, and that a shorter debt maturity can significantly reduce it both in absolute and relative terms.

Literature review. There is an extensive theoretical and empirical literature on corporate debt maturity.[1] Differently from this literature, our analysis focusses on the effect of debt maturity on shareholders' commitment on future debt and investment decisions.

[1] As for the theoretical contributions, for capital structure models excluding investment, Leland (1994) and Leland and Toft (1996) show that longer debt maturity better allows to capture the interest tax shield. When considering investment, Myers (1977) predicts that debt overhang makes shorter maturity preferable for firms with valuable growth options. However, Diamond and He (2014) show that a shorter maturity may also lead to more severe debt overhang. Diamond (1991, 1993) argues that a shorter maturity allows firms with valuable future projects to benefit from the expected improvement of their credit standing, but exposes them to higher refinancing risk and higher sensitivity of financing costs to new information. Refinancing risk in relation to debt maturity is also the focus of He and Xiong (2012) and He and Milbradt (2014). As for the empirical literature on corporate debt maturity, Barclay and Smith (1995), Guedes and Opler (1996), and Stohs and Mauer (1996) study the determinants of debt maturity choices and find that larger firms with more growth opportunities tend to have shorter debt maturity to alleviate the debt overhang issues described by Myers (1977).

Relative to the recent theoretical literature on leverage policies without commitment, we depart from DeMarzo and He (2021) by assuming that debt trades occur only in predetermined discrete dates, to make sure some commitment power is possible so that debt maturity becomes ex ante relevant, as recognized by DeMarzo (2019) and DeMarzo et al. (2023), and by adding a commitment problem on the firm investment policy.[2]

Different from Benzoni et al. (2022) and Malenko and Tsoy (2021), our solution concept is MPE, which is the equilibrium giving the lowest possible equity value. This choice suites us, because if we show that debt maturity matters in our setting, it will do so also under equilibrium concepts in which the ex ante value of equity is higher.

We analyze debt maturity as a device against commitment problems on future debt issuances. Such a role in our model is in addition to the role of debt maturity, already analyzed by Dangl and Zechner (2021), played against shareholders' lack of commitment to debt repayment, whereby a shorter debt maturity increases the likelihood of debt repayments. Because Dangl and Zechner (2021) remove the shareholders' commitment problem to debt policies by introducing a covenant that forces shareholders to repay all outstanding debt when they want to increase the debt, the question of whether the debt maturity can control the commitment problem to the debt policy cannot be addressed in their model.

In Dangl and Zechner (2021) and Benzoni et al. (2022), an interior solution to the optimal debt maturity problem emerges because they assume an offsetting force, non-proportional debt issuance costs, which makes short debt maturity costly. Differently from them, to obtain an interior value maximizing debt maturity, we introduce exogenous credit market shocks which disadvantage short debt maturity.

Finally, Benzoni et al. (2022) study the role played by debt issuance costs as a commitment device in the setting by DeMarzo and He (2021). As we show that the choice of debt maturity in such a setting is per se an important device against shareholders' lack of commitment to future policies, our analysis is in a way complementary to the

[2]There is a vast literature that extends the baseline *sovereign* debt model under no commitment by Eaton and Gersovitz (1981) to long-term debt, with similar modeling challenges to the ones of long-term *corporate* debt under no commitment. In the sovereign debt literature, the more directly related contributions to ours are by Chatterjee and Eyigungor (2012) and DeMarzo et al. (2023), who extend the results by DeMarzo and He (2021) to sovereign debt and show among other results that discrete-time debt trading can restore commitment power and make debt maturity relevant. However, endogenous investment is typically excluded in sovereign debt models.

one by Benzoni et al. (2022). In Malenko and Tsoy (2021), the role of debt maturity is to discipline debt reductions when the firm is in financial distress. In our model the mitigating effect of debt maturity is always present, also when there is an upturn and shareholders issue new debt.

The rest of the paper has the following structure: In Section 2 the model is introduced. In Section 3 we analyze the MPE and how debt maturity affects equilibrium investment and financing policies of the firm. In Section 4 we calibrate the model and provide a quantitative analysis of the effect of debt maturity on the value of corporate securities, leverage and payout dynamics, and the cost of debt financing. Section 5 presents the concluding remarks. All derivations and proofs are in Appendix, where we offer also additional benchmark models and additional discussions.

2 The model

The model of the firm is in discrete time, and the firm's shareholders and creditors are assumed risk neutral. The firm has capital stock k, with law of motion $k' = I + (1-\delta)k$, where $\delta > 0$ is the depreciation rate and I is the amount invested, which has unrestricted sign. When the firm invests, the adjustment cost is $\Psi(k, k') = (k'/k - 1 + \delta)^2 \varphi k/2$, with $\varphi > 0$. The after tax cash flow is $(1-\tau)(y+x)k$, where τ is a tax wedge which captures the effect of corporate and personal taxed, x is an idiosyncratic i.i.d. shock with density $\phi(x)$, support $[\underline{x}, \overline{x}]$ with $\underline{x} < 0 < \overline{x}$ such that $\mu_x = \int_{\underline{x}}^{\overline{x}} x\phi(x)dx = 0$, and $y > 0$ is a persistent shock, with dynamics $\log y' = (1-\nu)\log \overline{y} + \nu \log y + \sigma\varepsilon$, where $\varepsilon \sim \mathcal{N}(0, 1)$.

We assume the firm can issue long-term debt with face value b and coupon rate $r = 1/\beta - 1$, where β is the risk-free discount factor, which ensures that the debt trades at the face value when it is risk free. Trading takes place in a competitive financial market, where investors make zero profits when buying corporate debt. The use of debt financing by the firm is incentivized by tax-deduction of coupon payments. Each period, the firm commits to repay a fraction ξ of the debt at face value. The parameter ξ can therefore be interpreted as the inverse of the average debt maturity (if the debt remains unchanged and there is no default). The model is analogous to one where the firm holds multiple vintages of legacy debt, each decaying geometrically at the same rate $(1-\xi)$, as shown for instance by Arellano and Ramanarayanan (2012).

The equity holders optimally decide when to default on the debt payments by maximizing the equity value. At default, the absolute priority rule applies and equity holders get nothing, while creditors recover $(1-\tau)(y+x)k+R(b,k,y)$, conditional on this being positive, where $R(b,k,y) = \min\{(1-\delta-\alpha)k,\ \tau rb + V(b,k,y)\}$, in which $V(b,k,y)$ is the value of equity defined later, and α is a liquidation cost parameter. For tractability, we will assume the debt is *pari passu* at issuance.

Given the current state (b,k), equity holders maximize their value by deciding to change the debt level to b' (i.e., issue new debt or repurchase outstanding debt) *only once per period*. This is a key assumption in our discrete-time setting, as if multiple debt trades were possible in each period, debt maturity would be ineffective as a commitment device, as shown by Bizer and DeMarzo (1992). Debt trades at the market price, $p(b,k,y)$, which reflects the current state of the firm.

All debt issuances in the model use the same debt contract. Except for debt rollover risk that we discuss later, there are no frictions on debt and equity trades to raise (disburse) capital from (to) the financial market. Therefore, similar to Chatterjee and Eyigungor (2012), Aguiar et al. (2019), and DeMarzo et al. (2023) for sovereign debt and DeMarzo and He (2021) for corporate debt, our model has two main frictions: lack of commitment to repay the debt and lack of commitment to future debt issuances. Differently from these contributions, we add shareholders' lack of commitment on investment policies by assuming that shareholders make self-serving investment/disinvestment decisions (to maximize equity value, rather than total firm value). This third friction allows us to investigate how debt maturity influences the commitment problem on investment.

Our model includes debt rollover risk. Specifically, in each period, the firm may be excluded from credit markets ($\omega = 1$) with probability π, which limits equity holders' ability to refinance maturing debt and thus lowers their value. Conversely, with probability $1-\pi$, no credit market freeze occurs ($\omega = 0$) and there is no constraint on rolling over maturing debt. In practice, the negative impact of this friction is more pronounced if more debt is expiring and the firm has higher leverage, both of which are (directly and indirectly) affected by the debt amortization parameter ξ.

Given V the value of equity under normal credit market conditions, we define V^0 as the value of equity in the event of a credit market freeze during the current period, with $V^0 \leq V$. The precise definition of V^0 will be provided later. At the end of the period, if

$0 < (1-\tau)\left[(y'+x')k' - rb'\right] - \xi b' + V^0(b',k',y')$, the firm remains solvent even if a market freeze occurs, since $V^0 \leq V$. Conversely, if $(1-\tau)\left[(y'+x')k' - rb'\right] - \xi b' + V(b',k',y') < 0$, the firm defaults regardless of whether a credit market shock occurs. In these two scenarios, the occurrence of a market shock is irrelevant. However, in the intermediate case where $V^0(b',k',y') \leq \xi b' - (1-\tau)\left[(y'+x')k' - rb'\right] \leq V(b',k',y')$, if $\omega = 0$, the firm makes an unconstrained debt decision, resulting in a value of $V(b',k',y')$, under which it is solvent; but if $\omega = 1$, the firm's equity value is $V^0(b',k',y')$, leading to default. Ultimately, debt rollover risk increases the probability of firm default.

To conclude, the timeline for each period is as follows: given (b,k), shareholders observe (y,x) at the beginning of the period and decide whether to service the debt or default on their obligations, which may be determined also by a credit market shock. Conditional on remaining solvent, they make the investment and financing decisions (b',k') for the rest of the period.

We can now derive the value of the securities issued by the firm. Conditional on the firm being solvent at (b,k,y), the levered equity value is

$$V(b,k,y) = \max_{(b',k')} -\left[k' - (1-\delta)k\right] + \tau\delta k - \Psi(k,k') + \left[b' - (1-\xi)b\right] p(b',k',y) + \beta\mathcal{V}(b',k',y), \tag{1}$$

where for convenience we define[3]

$$\begin{aligned}\mathcal{V}(b',k',y) = \mathbb{E}_y \Bigg[&\int_{x_d(b',k',y')}^{\overline{x}} \left\{(1-\tau)(y'+x')k' - \left[(1-\tau)r + \xi\right] b' + V(b',k',y')\right\} \phi(x')dx' \\ &-\pi \int_{x_d(b',k',y')}^{x_d^0(b',k',y')} \left\{(1-\tau)(y'+x')k' - \left[(1-\tau)r + \xi\right] b' + V(b',k',y')\right\} \phi(x')dx' \Bigg],\end{aligned}$$

in which the second line is the expected loss due to a credit market freeze, and the default threshold is

$$x_d(b',k',y') = r\frac{b'}{k'} + \frac{\xi b' - V(b',k',y')}{(1-\tau)k'} - y'. \tag{2}$$

[3]Here and thereafter, the lower limit of integration is actually $x_d(b',k',y') \vee \underline{x}$, which we shorten for the convenience of exposition.

The threshold $x_d^0(b', k', y')$ has the same expression as (2), except with V^0 in place of V. It is possible to show that if $V^0 \leq V$ then $x_d \leq x_d^0$. The program in (1) reflects the fact that the equity holders cannot credibly commit to future decisions on debt and capital stock, rather at each date these are made to maximize the current equity value.

The price of debt, consistent with zero profit for investors, is

$$\begin{aligned} p(b,k,y) = \beta \mathbb{E}_y \Bigg[& \{r + \xi + (1-\xi) p(b',k',y')\} \int_{x_d(b,k,y')}^{\overline{x}} \phi(x') dx' \\ & + \int_{x_f(b,k,y')}^{x_d(b,k,y')} \frac{(1-\tau)(y'+x')k + R(b,k,y')}{b} \phi(x') dx' \\ & + \pi \Bigg\{ \int_{x_d(b,k,y')}^{x_d^0(b,k,y')} \frac{(1-\tau)(y'+x')k + R(b,k,y')}{b} \phi(x') dx' \\ & - \{r + \xi + (1-\xi) p(b',k',y')\} \int_{x_d(b,k,y')}^{x_d^0(b,k,y')} \phi(x') dx' \Bigg\} \Bigg] \end{aligned} \quad (3)$$

where $(b', k') = G(b, k, y')$ is the optimal policy of the equity program in (1),

$$x_f(b,k,y') = -\frac{R(b,k,y')}{(1-\tau)k} - y' \quad (4)$$

is the threshold for x' below which debt holders payoff at default is negative. It is easy to show that $x_f(b,k,y') \leq x_d(b,k,y')$. In (3), the first line represents the debt service if the firm is solvent, the second line denotes creditors' recovery in the event of default, considering the limited liability of the debt contract, and the last two lines capture the replacement of the continuation value with the debt recovery in the case of default due to a market freeze, $x_d(b,k,y') < x' < x_d^0(b,k,y')$.

To close the model, we define $V^0(b,k,y)$, representing equity value due to suboptimal debt issuance if a credit market freeze occurs. Let $b^* = b'(k')$ denote the optimal debt in (1) as a function of k'. Then,

$$\begin{aligned} V^0(b,k,y) = \max_{k'} \; & -[k' - (1-\delta)k] + \tau \delta k - \Psi(k,k') \\ & + \left(1 - \frac{\xi b}{k'}\right) \{[b^* - (1-\xi)b] \, p(b^*,k',y) + \beta \mathcal{V}(b^*,k',y)\}. \end{aligned} \quad (5)$$

The impact of a market freeze appears in the second line, where it reduces the value of equity derived from the optimal b^*. We assume that this reduction is proportional to $\xi b/k'$, meaning the reduction is greater the larger the amortized debt, ξb, and the smaller the new capital stock, k'. In the numerical solution of the model, we make sure that $\xi b/k' < 1$.

The model in (1)-(5), which is based on state variables (b, k), is non-stationary. Because we must calculate the Markov perfect equilibrium of the model, we need a *stationary* program. Given the linearity of the production function and the assumption made on capital adjustment cost, the equity price, V, is homogeneous of degree one and the debt price, p, of degree zero in (b, k), and stationarity is achieved by using book leverage as state variable. The following proposition presents a stationary version of the model, which is used in the subsequent analysis. The proof is in Appendix A. With a small abuse of notation, we denote respectively by V and p the equity and debt value for $k = 1$, which solve the stationary program.

Proposition 1. *For given* (b, k, y)*, the equilibium solution for* (1)-(5)*, and defining* $\ell = \frac{b}{k}$ *and* $\kappa = \frac{k'}{k}$*, is equivalent to solving the program*

$$V(\ell, y) = \max_{\kappa} \; -[\kappa - 1 + (1-\tau)\delta] - \frac{\varphi}{2}(\kappa - 1 + \delta)^2 + \kappa \; v\left(\frac{\ell}{\kappa}, y\right), \tag{6}$$

where $\kappa^* = h(\ell, y)$ *is the optimal investment policy. In* (6)*, the equity value per unit of capital stock,* $v(\cdot, y)$*, solves the* leverage program*,*

$$v\left(\frac{\ell}{\kappa}, y\right) = \max_{\ell'} \; \left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right] p(\ell', y) + \beta \mathcal{V}(\ell', y), \tag{7}$$

where $\frac{\ell}{\kappa} = \frac{b}{k'}$*,* $\ell' = \frac{b'}{k'}$*, and* $\ell^* = g(\ell/\kappa, y)$ *is the related optimal leverage policy. In* (7)*, the continuation value of equity is*

$$\begin{aligned} \mathcal{V}(\ell', y) = \mathbb{E}_y \Bigg[& \int_{x_d(\ell',y')}^{\overline{x}} \left\{(1-\tau)(y + x' - r\ell') - \xi\ell' + V(\ell', y')\right\} \phi(x') dx' \\ & - \pi \int_{x_d(\ell',y')}^{x_d^0(\ell',y')} \left\{(1-\tau)(y + x' - r\ell') - \xi\ell' + V(\ell', y')\right\} \phi(x') dx' \Bigg], \end{aligned} \tag{8}$$

where $x_d(\ell,y) = r\ell + \frac{\xi\ell - V(\ell,y)}{1-\tau} - y$ *and* $x_d^0(\ell,y) = r\ell + \frac{\xi\ell - V^0(\ell,y)}{1-\tau} - y$*. We define* $V^0(\ell,y)$ *as in* (6)*, except it is based on* $v^0(\ell/\kappa,y) = v(\ell/\kappa,y)\,(1-\xi\ell/\kappa)$*. The debt price for given leverage* ℓ *is*

$$
\begin{aligned}
p(\ell,y) = \beta\mathbb{E}_y\Bigg[&(r+\xi+(1-\xi)p(\ell',y'))\int_{x_d(\ell,y')}^{\overline{x}}\phi(x')dx' \\
&+\int_{x_f(\ell,y')}^{x_d(\ell,y')}\frac{(1-\tau)(y'+x')+R(\ell,y')}{\ell}\phi(x')dx' \\
&+\pi\Bigg\{\int_{x_d(\ell,y')}^{x_d^0(\ell,y')}\frac{(1-\tau)(y'+x')+R(\ell,y')}{\ell}\phi(x')dx' \\
&-(r+\xi+(1-\xi)p(\ell',y'))\int_{x_d(\ell,y')}^{x_d^0(\ell,y')}\phi(x')dx'\Bigg\}\Bigg] \qquad (9)
\end{aligned}
$$

where $x_f(\ell,y') = -R(\ell,y')/(1-\tau) - y$*,* $R(\ell,y') = \min\{1-\delta-\alpha, \tau r\ell + V(\ell,y')\}$*, based on the next-period leverage decision* $\ell' = g(\ell/\kappa,y')$*, where* $\kappa = h(\ell,y')$*.*

Inspection of expression (8) shows that, for given probability of a credit market shock, the value loss due to a shock is larger the bigger the difference between $x_d^0(\ell',y')$ and $x_d(\ell',y')$. The difference is proportional to the loss due to market freeze, $V(\ell',y') - V^0(\ell',y')$, which is more severe the bigger the contractual reduction in leverage, $\xi\ell$.

From Proposition 1, given V and h, we solve (7)-(9) simultaneously to determine the debt price, p, the optimal leverage policy function, g, and v. Once v is determined, we find V and h from (6). The procedure is iterated until the solution is found. From the equilibrium of the problem in Proposition 1 we recover the equilibrium of the main model in (1)-(5).

3 Markov perfect equilibrium

We focus on an equilibrium concept, Markov perfect equilibrium (MPE), whereby the firm policy depends only on the payoff-relevant state variables, and the policy is time-consistent, in that it is a fixed point in which future policies are the same as this period's policy, and depends only on the state. Specifically, the MPE of the firm model is defined by security prices, $V(b,k,y) = kV(b/k,y)$ and $p(b,k,y) = p(b/k,y)$, and optimal policy

$(b', k') = G(b, k, y')$, such that $k' = kh(b/k, y)$ and $b' = k'g(b/k', y)$, where the security prices reflect the expectations regarding the optimal policy, and given the security prices the equity holders will not deviate from said policy.

The MPE cannot be found analytically and is calculated by solving equations (7)-(9) simultaneously, using a discrete-state value function iteration approach. At a given step of the iterative procedure, based on $V(\ell, y)$, where $\ell = b/k$, and on $\kappa = k'/k$ from the previous step, we find $v(\ell/\kappa, y)$ and $\ell' = g(\ell/\kappa, y)$ from (7), and we calculate $p(\ell, y)$ in (9). Then, in (6) we solve the first-order condition for optimal investment, which requires a numerical approach, as κ is defined implicitly, and because $v(\cdot, y)$ is defined numerically. This is done by using a spline interpolation of $v(\ell/\kappa, y)$, which allows the calculation of its first derivative. This step finds $\kappa = h(\ell, y)$ and $V(\ell, y)$, from which a new iteration is started. The iterative procedure is halted when the maximum between the improvement of value function and the improvement of debt price over an iteration is lower than a given tolerance. More details of the numerical algorithm are in Appendix H.

In order to characterizes the MPE and derive the main theoretical results, we consider the case without roll over risk ($\pi = 0$). The proof of the following proposition is in Appendix E.

Proposition 2. *1. For a given y, $\ell = b/k$, and $\kappa = k'/k$, $v(\ell/\kappa, y)$ is convex in ℓ/κ and $V(\ell, y)$ is convex in ℓ.*

2. For one-period debt, $\xi = 1$, the equilibrium investment and leverage policy in the MPE maximizes the value of the firm (equity plus debt). Because the optimal policies are independent of current ℓ, there is no commitment problem on the debt policy and on the investment policy.

3. If $\xi < 1$,

(a) assuming $v(\cdot, y)$ is twice differentiable, then $v(\ell/\kappa, y)$ is decreasing in ℓ/κ and $V(\ell, y)$ is decreasing in ℓ; $p(\ell, y)$ is a decreasing function of ℓ, that is $\partial_1 p < 0$;[4]

[4]The notation $\partial_2 f(\hat{x}, \hat{y})$ indicates the first partial derivative of $f(x, y)$ with respect to y, evaluated at $(\hat{x}, \hat{y})$.

(b) *if the objective function in* (7) *is concave with respect to* ℓ'*,*[5]*, there is a unique equilibrium policy function,* $\ell' = g(\ell/\kappa, y)$ *that satisfies condition*

$$
\begin{aligned}
&-\left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right]\partial_1 p(\ell', y) \\
&= \beta\mathbb{E}_y\left[\tau r \int_{x_d(\ell',y')}^{\overline{x}} \phi(x')dx' + \int_{x_f(\ell',y')}^{x_d(\ell',y')} \frac{(1-\tau)(y'+x') + R(\ell',y')}{\ell'}\phi(x')dx'\right].
\end{aligned}
\tag{10}
$$

Under the same conditions, $g(\ell/\kappa, y)$ *is strictly increasing in* ℓ/κ*;*

(c) *if the objective function in* (6) *is concave with respect to* κ*, which is true if and only if proportional adjustment cost,* φ*, is large enough, the equilibrium investment policy,* $\kappa = h(\ell, y)$*, satisfies condition*

$$
v\left(\frac{\ell}{\kappa}, y\right) + (1-\xi)\frac{\ell}{\kappa}p(\ell', y) = 1 + \varphi\left[\kappa - (1-\delta)\right], \tag{11}
$$

where the left-hand side of (11) *is marginal* q*, with* $\ell' = g(\ell/\kappa, y)$*. Under the same conditions,* $h(\ell, y)$ *is strictly decreasing in* ℓ*.*

Part 1 confirms in our setup with investment know properties of the value function following from optimality of leverage policy. The result in Part 2 of Proposition 2 is striking: debt maturity can achieve (constrained) first-best investment if debt is single-period. The intuition is that the price of new debt fully reflects potential conflicts over future leverage policies, and with $\xi = 1$ shareholders bear the full cost of these distortions. Hence, they have no incentive to deviate from the value-maximizing leverage policy. Shutting down the LRE channel, investment decisions focus solely on increasing the value of installed capital, as in (11) the marginal q term related to debt dilution is absent. Therefore, with $\xi = 1$, although shareholders are self-motivate, they make investment and financing decisions to maximize firm value. Hence, if there was no other frictions, debt maturity would be a powerful tool to mitigate the effects of commitment problems on firm policies.

[5] In the case of zero recovery at default and no credit market shocks, this condition is satisfied if $p(\ell, y)$ is a concave function of ℓ. However, it is not generally true that the price of debt is concave in ℓ. See e.g. DeMarzo et al. (2023), Section 5.2.2., for a case in which the debt price is convex.

As for Part 3, (a) confirm properties of the value of securities already shown in similar models with no commitment on the leverage policy. Point (b) shows that $\partial_1 g > 0$, a characteristic of the equilibrium leverage policy with limited commitment, defined by Admati et al. (2018) as the leverage ratchet effect (LRE). The first-order condition (10) balances the marginal cost and benefit of increasing leverage. On the right-hand side, there are two benefits. The first is the value of interest tax shield in future solvent states, which diminishes as leverage increases due to the rising likelihood of default (higher x_d). The second benefit is the debt recovery value at default. Despite the absolute priority rule, this recovery value is captured by shareholders through debt dilution. Thus, compared to a model with no debt recovery at default, the incentive to issue debt is higher.

On the left-hand side of (10), the marginal cost for shareholders of increasing leverage is the reduction in the proceeds for each unit of newly issued debt. This cost is greatest when $\xi = 1$, as it is fully borne by shareholders, and is lower when $\xi < 1$, as it is shared with current creditors. Unlike other LRE models, underinvestment plays a central role here. Specifically, a lower $\kappa = k'/k$ increases the part of the marginal cost shifted onto current creditors through the dilution of their claims. As a result, underinvestment makes ℓ' higher than in a model with fixed investment, all else being equal, thus exacerbating the LRE. Later on, we will show that underinvestment becomes more severe with longer debt maturity, which in turn affects the leverage policy. However, the dependence of $g(\ell, y)$ on ξ is not straightforward, as it is non-monotonic in $\ell = b/k$. We will discuss this in more detail based on the numerical solution.

As for Point (c), the first-order condition for investment, (11), follows the usual neoclassical structure, with a positive marginal q on the left-hand side and the cost of newly installed capital on the right-hand side. As $v(\ell/\kappa, y) + (1-\xi)\frac{\ell}{\kappa}p(\ell', y) = \beta\mathcal{V}(\ell', y) + \ell' p(\ell', y)$, the marginal q can be written as

$$v(\ell/\kappa, y) + (1-\xi)\frac{\ell}{\kappa}p(\ell', y) = \mathbb{E}_y\left[\int_{x_f(\ell', y')}^{\overline{x}} (1-\tau)(y' + x')\phi(x')dx' \right.$$
$$+ \{\tau r\ell' + V(\ell', y')\}\int_{x_d(\ell', y')}^{\overline{x}} \phi(x')dx' + R(\ell', y')\int_{x_f(\ell', y')}^{x_d(\ell', y')} \phi(x')dx'$$
$$\left. + (1-\xi)\ell' p(\ell'', y')\int_{x_d(\ell', y')}^{\overline{x}} \phi(x')dx'\right],$$

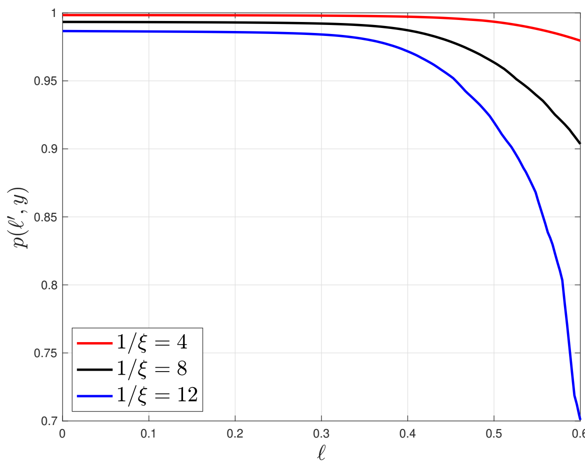
where $\ell'' = g(\ell'/\kappa', y')$. This expression shows the two main components of marginal q in our model, which features commitment problems related to debt repayment, debt policy, and investment policy.

The first component of q (the first two lines of the expression above) comes from the fact that, given ℓ', an additional unit of capital increases firm value through future after-tax cash flows and interest tax shields. This increase in value happens both when the firm is solvent, $x' \geq x_d(\ell', y')$, and when the firm is insolvent, $x_f(\ell', y') \leq x' < x_d(\ell', y')$, through the recovery value of defaulted debt, which is captured by current shareholders via debt dilution, as shown in Point (c). Because g is strictly increasing in its first argument, higher current leverage implies a higher ℓ'. Together with the fact that $V(\ell', y')$ decreases with ℓ', this leads to a higher default threshold $x_d(\ell', y')$. To the extent that $V(\ell', y')$ decreases in ℓ' faster than $-\tau r \ell'$, the recovery value $R(\ell', y')$ is lower, and the limited liability threshold for debt, $x_f(\ell', y')$, is higher. Altogether, higher current leverage lowers the first component of q. This is the debt overhang problem described by Myers (1977), where higher leverage shifts investment gains toward creditors. This effect arises from shareholders' inability to commit to repay debt, as it results from the truncation of the equity horizon due to default (see Hennessy (2004)).

Marginal q has a second positive component (last line of the above expression), which is due to shareholders' inability to commit to the debt policy and reflects the value captured by shareholders via future debt dilutions. Intuitively, one more unit of physical capital today increases the firm's debt capacity, allowing shareholders to issue, in future states where the firm is solvent, more debt at price $p(\ell'', y')$, which multiplies the residual leverage given the investment, $(1-\xi)\ell'$. In other words, one more unit of capital stock today increases future shareholders' value transfer from creditors. Unlike the first component of marginal q, through the LRE this component *increases*, rather than decreases, with current leverage ℓ, though its marginal effect diminishes with higher leverage since p decreases as ℓ' is high. This second component of marginal q shows that the underinvestment mechanism in a world with lack of commitment on the debt policy differs from the Myers (1977) debt overhang, which is reflected in the first component of marginal q and is driven solely by the lack of commitment on debt repayment. The second component of marginal q is due to the LRE: indeed, if debt is single-period ($\xi = 1$), debt dilution is eliminated, and this component of marginal q

Figure 1: Debt maturity and debt price

The figure plots the debt price, $p(\ell', y)$, for maturities 1, 2, and 3 years (4, 8, and 12 quarters) at a given y against leverage, $\ell = b/k$. The debt price is at the optimal leverage policy, $\ell' = g(\ell/\kappa, y)$, where $\kappa = h(\ell, y)$ is optimal investment.



disappears. In general, however, the dependence of this component of marginal q on ξ is not straightforward, and we will evaluate it numerically later on.

Overall, for $\xi < 1$ and if the proportional cost of installing capital, φ, is sufficiently large, investment is negatively affected by current leverage ($\partial_1 h < 0$). That is, shareholders' concerns about the reduced benefits from investment due to lack of commitment to repay the debt (first component of q) outweigh the incentives to increase the value of installed capital to exploit future debt dilution (second component of q), resulting in underinvestment. As a result of the interaction between the effect of underinvestment on the LRE from (10) and the effect of the LRE on underinvestment from (11), the debt policy for the case with two commitment problems (debt repayment and debt issuance) is qualitatively different from the one with three commitment problems (also investment) Later on, we will shown this effect numerically.

To conclude, Figure 1 shows that the debt price at the optimal $\ell' = g(\ell/\kappa, y)$ for optimal $\kappa = h(\ell, y)$ is decreasing with respect to ℓ, and is decreasing with respect to the debt maturity. This has important implications for the effect of debt maturity on equilibrium leverage and investment policy, analyzed here below.

3.1 Debt maturity and leverage policy

While investment and leverage policies are jointly determined and interdependent, we simplify the analysis by first focusing on the leverage policy given investment, and then turning to the investment policy. Figure 2 confirms that under optimal investment, with $\kappa = k'/k$ at ℓ, the function $\ell' = g(\ell/\kappa, y)$ is increasing in $\ell/\kappa = b/k'$. However, it also shows that the relationship between leverage policy and debt maturity is non-monotonic: when ℓ is low, new leverage can be higher for short maturity than for long maturity, and the opposite holds when ℓ is high.

By plotting where $g(\cdot)$ crosses the 45 degree line, in Figure 2 we single out the equilibrium long-run leverage of the firm, denoted by $\hat{\ell}$, as the fixed point of $g(\cdot)$, that is the solution of equation $\ell = g(\ell/\kappa, y)$. Depending on $1/\xi$, there can be more than one solution. Indeed, while for short debt maturity there is only one solution, for long maturity $1/\xi$ there are two fixed points, $\hat{\ell}_1 < \hat{\ell}_2$, in the interval $[0, 1]$:[6] at the lower one the function $g(\ell/\kappa, y)$ is convex in ℓ/κ, whereas at the higher one g is concave, which has implications for the leverage dynamics. Since the discussion in this section is qualitative, we will distinguish between short and long debt maturity based on whether there is a single fixed point or two.

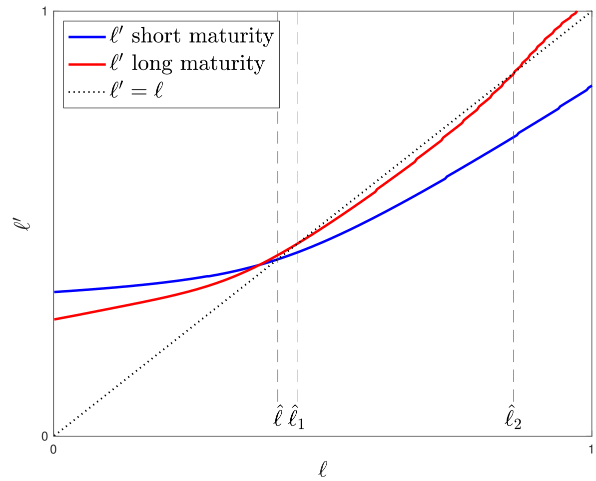
The second fixed point arises from the interaction of the three commitment problems in our setup. As shown in Proposition 2, on the one hand higher leverage exacerbates underinvestment, and on the other hand, underinvestment amplifies the LRE. Moreover, as we show later, underinvestment becomes more severe with longer debt maturity. Together, the combination of the LRE and underinvestment under long debt maturity makes the incentive to increase debt strong enough to generate a second fixed point. This distinctive qualitative outcome in our model is specific to the commitment problem in the investment policy. Indeed, in Section 3.2 we show that this behavior disappears in a model with fixed investment, even under long debt maturity.

Depending on whether there is one or more such fixed points (or none), and depending on current leverage, the leverage dynamics of the firm (that is, the relationship between ℓ' and ℓ) can be of three types: (i) for $\ell < \hat{\ell}$, the leverage adjusts upwards towards $\hat{\ell}$; (ii) for short debt maturity, if $\ell > \hat{\ell}$ the leverage adjusts downward towards $\hat{\ell}$; (iii) for long

[6] For high $1/\xi$, there can be more fixed points when extending the interval of ℓ above 1. We do not consider those cases, to limit our analysis to plausible levels of leverage.

Figure 2: Debt maturity, leverage policy, and long-run leverage

The figure plots the optimal leverage policy, $\ell' = g(\ell/\kappa, y)$ where $\kappa = h(\ell, y)$, at a given y for short (blue) and long (red) debt maturity, $1/\xi$, against $\ell = b/k$. We plot the 45 degree (dotted) line to determine the long-run leverage, $\hat{\ell}$, as the fixed point of g, that is as the solution of $\ell = g(\ell/\kappa, y)$.

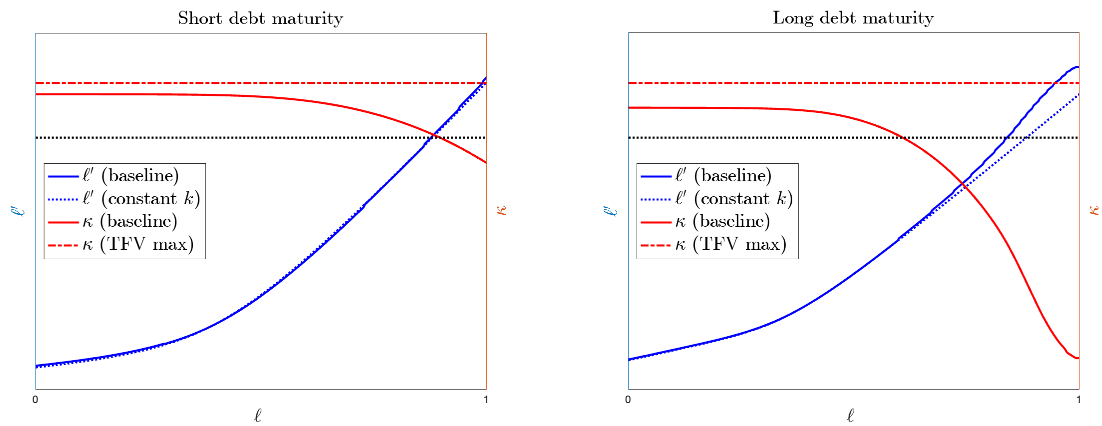


debt maturity, if $\ell < \hat{\ell}_2$, the leverage adjusts downward towards $\hat{\ell}_1$, whereas if $\ell > \hat{\ell}_2$, the leverage diverges to 100%.[7] The behavior in (i) and (ii) has been described also by DeMarzo et al. (2023), where the fixed point is always an attracting point, meaning that long-term leverage converges to it. However, the one in (iii), whereby $\hat{\ell}_2$ is not an attracting point, is specific to a model with lack of commitment on investment.

Figure 2 also reveals that, when maturity is short, the speed of mean reversion towards $\hat{\ell}$ is higher (i.e., the slope of $g(\ell/\kappa, y)$ is lower). To appreciate the importance of the result, some comments are in order. It is well understood that the incentives that give rise to leverage ratcheting are *asymmetric*: shareholders actively increase leverage when it is lower than $\hat{\ell}$, but debt reductions towards $\hat{\ell}$ occur only via debt amortization, which is mechanically faster for shorter debt maturities, as it is the case for $\ell > \hat{\ell}$. Surprisingly, Figure 2 shows that also the speed of *upward* adjustment is increasing in ξ. This is a consequence of the higher marginal value of new debt issuances when

[7]In unreported results, for very high debt maturity (in the calibration used in Section 4, $1/\xi > 40$) the equilibrium leverage policy is such that $g(\ell/\kappa, y) > \ell$ for all $\ell \in [0, 1]$, and there is no fixed point. Also in this case, the leverage diverges to 100%.

Figure 3: Debt maturity and interaction of investment and leverage policy
We plot investment (red lines, right axis) and leverage (blue lines, left axis) policies, at a given y for short and long debt maturity, against $\ell = b/k$. Three cases are presented in this figure: the baseline model with no commitment (solid lines), the case with constant investment ($k' = k = 1$) (dotted), and the total firm value (TFV) maximizing case (dashed). We also show the case with permanently no debt (black dotted).



commitment increases for higher ξ. While also Dangl and Zechner (2021) highlight the effect of maturity on downward debt adjustments, the positive effect of shorter maturity on upward adjustments, which is a consequence of the maturity's role as a commitment device against time-inconsistent debt policies, emerges clearly only in our setting.[8]

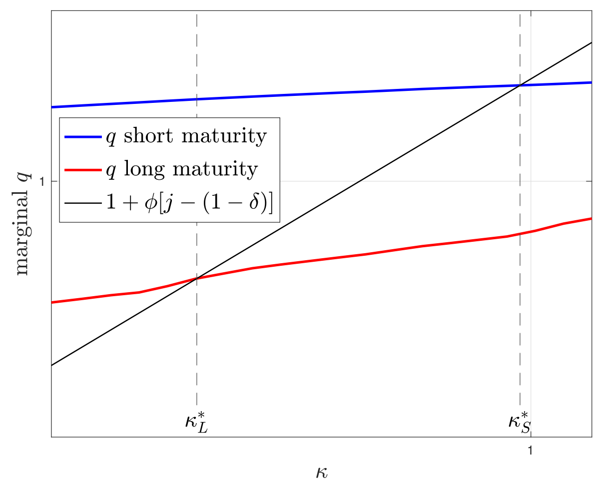
3.2 Debt maturity and investment policy

To illustrate the interaction between the commitment problem on investment and the LRE, Figure 3 plots the investment policy against current leverage, for short and long debt maturity for the baseline case with no commitment, and for two different benchmark models. In the first, the firm has a fixed investment policy so that $k' = k = 1$ at all dates, and the second is the case with full commitment, with total firm value (TFV) maximizing policy. The equilibria for these benchmark cases are described in Appendix C and D,

[8]Dangl and Zechner (2021) exclude time-inconsistent debt policies from their setting by forcing the firm to repurchase all existing debt before any leverage increase.

Figure 4: Debt maturity, marginal q, and investment

The figure plots marginal $q = v\left(\frac{\ell}{\kappa}, y\right) + (1-\xi)\frac{\ell}{\kappa}p\left(\ell', y\right)$, with $\ell' = g(\ell/\kappa, y)$ for short and long maturity, at given ℓ and y, and marginal cost, $1 + \phi[j - (1-\delta)]$, against $\kappa = k'/k$. The respective optimal investment are denoted by κ^*_S and κ^*_L.



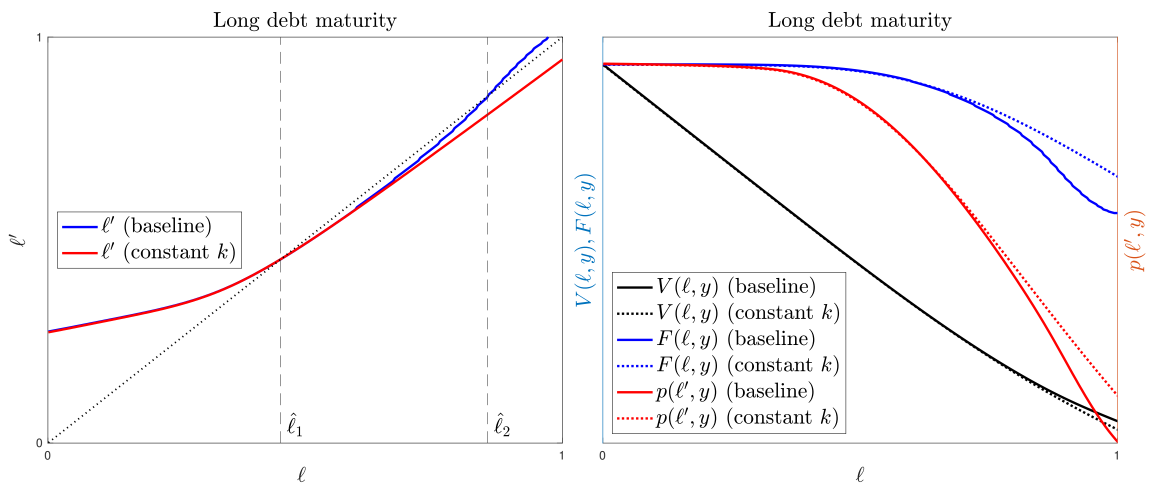
respectively. To single out the effect of debt financing, we also show optimal investment if the firm remains permanently unlevered, as derived in Appendix B.

Figure 3 shows that limited commitment on debt repayment, debt policy, and investment policy (red solid line) leads to underinvestment compared to the firm value maximizing equilibrium (red dashed line), where these commitment problems are absent and the full value of the tax shield can be realized at all ℓ. When current leverage is low, debt financing—even under limited commitment—supports higher investment relative to the permanently unlevered firm (black dotted line). However, when leverage is high, limited commitment reduces investment to the point where the firm would be better off without debt. As shown in the previous section, this is due to debt overhang: high leverage reduces $v(\ell/\kappa, y)$, which in turn lowers marginal q in (11).

Figure 3 shows also that, for a given ℓ, investment declines more with longer debt maturity, due to a more severe LRE. To explain this effect, Figure 4 shows how optimal investment is determined by equating marginal q to marginal cost, as stated in equation (11), for different debt maturities. Holding all else constant, including ℓ, longer debt maturity reduces marginal q, which in turn leads to lower optimal investment in

Figure 5: Underinvestment, leverage policy, and firm value

In left panel, the figure plots the leverage policy for the baseline model (blue line) and the model with constant investment (red line), for a firm with long debt maturity at a given y, against $\ell = b/k$. In the right panel, we plot the equilibrium value of equity, $V(\ell, y)$, debt price at the new debt policy, $p(\ell', y)$, and total firm value, $F(\ell, y)$ for the baseline case with no commitment (solid lines), and the constant investment case ($k' = k = 1$) (dotted lines), also against $\ell = b/k$.



equilibrium. The effect of ξ on investment arises from the limited commitment to future debt policy. As noted earlier, when $\xi = 1$, this effect disappears. For $\xi < 1$, there are two parts: one depends on $(1-\xi)$, which decreases with ξ, and the other on $p(\ell')$, which increases with ξ, as shown in Figure 1. In our setting, the second effect dominates, so longer debt maturity leads to lower marginal q and, consequently, lower investment.

The commitment problem on leverage policy is more severe when coupled with underinvestment, as seen in Figure 3 by comparing the baseline equilibrium to the one with fixed investment: the blue solid and the blue dotted lines almost overlap for short debt maturity, but for long debt maturity and large leverage they diverge, with new leverage being higher due to underinvestment. To focus on this interaction, the left panel of Figure 5 shows, for long debt maturity, that while the baseline case has two solutions to equation $\ell = g(\ell, y)$ as shown before, the case with fixed investment has only one solution, $\hat{\ell}_1$. In other words, models with constant investment policy *underestimate* the LRE, because they exclude the underinvestment channel to the debt dilution problem.

In the end, limited commitment models with constant investment policy produce leverage dynamics that are qualitatively (as opposed to just quantitatively) different from the ones in models in which there is also a commitment problem on investment.

Figure 5 left panel shows also that the effect of the investment commitment problem on the LRE is *asymmetric*: for low ℓ, the speed of *upward* adjustment towards long-run leverage is unchanged, but for high ℓ, underinvestment accelerates the debt issuance and reduces the speed of *downward* adjustment.[9]

Underinvestment and leverage ratcheting transfer value from debt to equity holders, and therefore *reduce* the price of debt and *increase* the equity value in the baseline model, relative to the case with fixed investment, and this effect is mostly visible for high current leverage, as showed in the right panel of Figure 5. The effect is present but less visible also for short debt. Overall, firm value is lower due to the combined and compounding effects of a reduced interest tax shield and underinvestment.

4 Quantitative analysis

In this section, we examine the quantitative impact of the commitment problems introduced in the model on firm value and policy dynamics. We also provide a quantitative analysis of how debt maturity helps address these frictions.

To do so, we calibrate the model on a quarterly frequency and choose the baseline parameters described in Table 1. To discuss the calibration, the discount factor is $\beta = 0.987$, in line with similar discrete-time settings (e.g., Cooley and Quadrini (2001)). The corresponding risk-free rate is $r = 1/\beta - 1 = 1.25\%$. The depreciation rate, δ, is 0.025, as in Gomes et al. (2016), Jungherr and Schott (2021), and Xiang (2024). The tax wedge, τ, is set at 32%, which is midway between 40% in Gomes et al. (2016) and Jungherr and Schott (2021), 35% in Xiang (2024), and 20% in Benzoni et al. (2022).

[9] Chaderina et al. (2022) show that debt maturity affects equity returns via the effect of debt maturity on debt dynamics in the presence of systematic risk: for longer debt maturity, debt adjustment are slower, as predicted by DeMarzo and He (2021), and exposure to economic downturns last longer, which in equilibrium requires a higher risk premium. Although in this paper we are not studying the asset pricing implications of debt maturity, we complement the finding in Chaderina et al. (2022) by showing how debt maturity affects not only the leverage dynamics, but also the investment dynamics, providing an additional channel through which systematic shocks can impact equity returns.

Table 1: Base case parameters

This table reports the baseline parameter values. The parameters $\overline{y}$, σ, τ, and α are calibrated to match the observed leverage, credit spread, and default probability. The remaining parameters are taken from the literature. The model is calibrated so that one period corresponds to a quarter in calendar time.

symbol	description	value
β	risk-free discount factor	0.987
$[\underline{x}, \overline{x}]$	support iid profitability	$[-1.15, 1.15]$
$(\overline{y}, \nu, \sigma)$	persistent profitability	$(4.35\%, 0.9, 0.07)$
τ	tax wedge	32%
φ	capital adjustment cost	2.4
δ	depreciation rate	0.025
α	bankruptcy costs	0.23
ξ	debt amortization rate	1/20
π	probability of credit market freeze	0.1

As for the investment side of the model, given δ, τ, and r, the capital adjustment cost parameter, φ, and the average of the persistent component of profitability, $\overline{y}$ are jointly chosen to satisfy condition (25), which ensures that a solution for the unlevered version of the model exists.[10] Hence, we set $\varphi = 2.4$, as in Xiang (2024). Also the parameters ν and σ of the persistent component of profitability are chosen in line with Xiang (2024). The debt maturity parameter is $\xi = 1/20$, corresponding to 5-year average maturity, in line with Gomes et al. (2016), Jungherr and Schott (2021), and Xiang (2024). The idiosyncratic component of profitability, x has the continuous distribution specified in Appendix H with support $[\underline{x}, \overline{x}] = [-1.15, 1.15]$, which is the same as in Xiang (2024). Proportional bankruptcy costs, α, is set to 0.23, which is in line with Xiang (2024), and within the range of values considered by Jungherr and Schott (2021), who chose 0.1, and Gomes et al. (2016), who chose 0.29. Out of these parameters, we used $\overline{y}$, σ, τ, and α to produce jointly a sample average of leverage, credit spread, and default probability in line with the data. The parameter π is the frequency of credit market freezes for the firm and affects the optimal debt maturity, as a tradeoff between the benefits of (a shorter)

[10] As explained in Appendix H, for an MPE to exist, also the investment program in (6) must have a solution. A sufficient condition for this is that the unlevered firm program, which can be solved analytically, has a solution, as shown in Appendix B.

Table 2: Moments

This table compares sample averages of the moments of the simulated economy from the model, obtained following the procedure described in Appendix H, using the baseline parameters in Table 1, to the corresponding moments for the sample of S&P 500 non-financial firms described in Appendix I. Credit spreads in the data are for 1-year CDS and 5-year CDS, respectively. All moments, from the data and from the simulated economy, are annualized for ease of comparison. The data sample spans the years from 2001 to 2014.

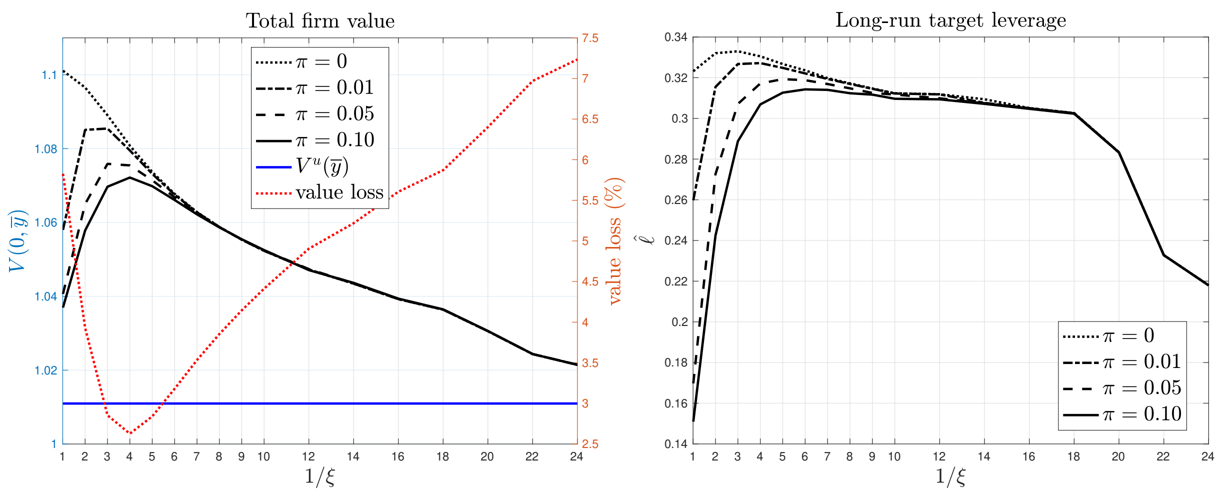
	model	data
debt/asset	0.28	0.30
market leverage	0.26	0.20
market-to-book	1.04	2.02
investment/asset	0.07	0.08
cash flow/asset	0.12	0.12
credit spread (bps)	120	[110 169]
default probability (pct)	0.17	0.20

maturity deriving from a mitigation of debt dilution incentives and the cost of increased exposure to credit market freezes. We choose as baseline case a market freeze frequency of 10 quarters. Since these freezes may be of idiosyncratic and systematic nature, this parameter is hard to calibrate, and therefore, we provide a sensitivity analysis to show how our results depend on it including the case in which there is never a market freeze.

Our calibration produces a realistic model, as shown in Table 2. The table compares average annualized moments from the simulated economy to the corresponding moments from a sample of non-financial firms in the S&P 500 index. Appendix H details numerical calculations. Appendix I describes the data. Table 2 shows that the model closely matches firm leverage in the sample, both in book and market terms. As is common in this class of models, matching the average Q is more challenging, as we do not model a stochastic discount factor. However, the model closely matches average investment and profitability. It also performs well in generating realistic credit spreads and default probabilities, which are roughly in line with those of BBB-rated corporate debt in the US. While the CDS bracket $[110, 169]$ for 1-year and 5-year maturities reflects the effect of market risk premia, which are not included in our model, the average credit spread produced by the model remains consistent with that observed in the sample.

Figure 6: Debt maturity, firm value, and long-run leverage

The left panel shows the value of unlevered equity (black lines, left axis), $V(0, \overline{y}; \xi)$, which coincides with total firm value, as a function of debt maturity, $1/\xi$. We benchmark against the value of a firm with permanently no debt, $V^u(\overline{y})$ (blue line). We show the firm value shortfall (red dotted line, right axis) relative the case with $1/\xi = 1$, as $1 - V(0, \overline{y}; 1)/V(0, \overline{y}; \xi)$ for $\pi = 0.1$. The right panel shows the long-run target leverage, $\hat{\ell}(\xi)$, for a firm starting at zero leverage, as a function of debt maturity. In both panels, we also show the effect of variations on π, from the baseline value to zero. All remaining parameters are as in Table 1.



4.1 Debt maturity, firm value, and long-run leverage

DeMarzo and He (2021) show that in continuous time with no commitment—where it is only optimal to follow a smooth leverage policy—the MPE value of levered equity is independent of future debt policy. It equals the equity value for a firm with the same current debt that is subsequently excluded from the debt market. Since debt policy does not affect the ex-ante value of equity, debt maturity is also irrelevant ex ante. As DeMarzo and He (2021) point out, this debt irrelevance result is a consequence of the continuous-time setting.

A discrete-time setting reintroduces some commitment, as the borrower cannot default, undertake equity-maximizing investment, or issue new debt before the next trading date. As a result, there is a positive gain from debt trade, as shown by DeMarzo (2019). In our discrete-time setup, we can assess the role that maturity plays on the ex ante value of the firm, before any debt is issued.[11] By setting $y = \overline{y}$, we aim to provide an average valuation. In the left panel of Figure 6, we show that, given the three commitment problems considered in our model, the value of equity (and of the firm) currently at zero leverage, $V(0, \overline{y})$, is hump-shaped with respect to average debt maturity, with the highest value for $1/\xi = 4$. This shape is due to the role of (longer) debt maturity as a hedge against credit market freezes. Indeed, a shorter debt maturity implies that in each period a significant amount of debt matures and needs to be refinanced, exposing the firm to the severity of credit market freezes.

When we exclude credit market freezes, that is $\pi = 0$, the value of equity and of the firm is strictly decreasing in $1/\xi$, as the LRE increases with maturity, without the offsetting effect from potential market freezes. The highest firm value is found for $1/\xi = 1$, the limiting case in which full commitment is restored, as stated in Proposition 2, with the same value as under total firm value maximization.[12] With $\pi = 0$, a corner solution with respect to debt maturity emerges because in our model no other frictions, such as refinancing costs, offset the positive effect of shorter debt maturity.[13] Also, for longer debt maturity the effect of market freeze becomes insignificant, as the value of the firm is independent of π.

To gauge the value created by debt policy in the presence of the three commitment problems, in the left panel of Figure 6 we report (blue line) the value of the firm that remains *permanently* unlevered, $V^u(\overline{y})$.[14] The figure shows that shareholders of an unlevered firm benefit from issuing debt, as it increases firm value between 1% and 6%, in line with the empirical quantifications of 5.5% by Korteweg (2010) and of 3.5% by van

[11]Since the firm will issue debt in equilibrium, the unlevered state provides the best benchmark to fully capture the value created by debt, and thus the effect of debt maturity. This value also depends on y.

[12]The case $\xi = 1$ has full commitment because we assume that only one debt issuance can take place in each period. The solution for the limiting case $\xi = 1$ is based on a modified algorithm based on the version of the model presented in Appendix E in the proof of Proposition 2. The case $\xi = 1$ in our setting is equivalent to the case of the debt amortization parameter going to infinity (i.e., zero maturity) in continuous time, as they both correspond to complete repayment of the outstanding debt.

[13]Arellano and Ramanarayanan (2012) argue that longer-maturity debt provides a hedge against negative systematic shocks. To the extent that y captures systematic risk, the fact that, for $\pi = 0$, the firm's value is strictly decreasing in $1/\xi$ (across all calibrations we considered, including higher σ; results are available upon request) suggests that this hedging effect is negligible in our model relative to effect of the commitment problems on investment and debt policies.

[14]This value is derived analytically in Appendix B, and is different from $V(0, \overline{y})$, which is the value of a firm that is at the moment unlevered, but has access to the debt market, and therefore will issue debt immediately right after.

Binsbergen et al. (2010), in the baseline case with $\pi = 0.1$, with maximum at $1/\xi = 4$. To appreciate this result, recall that DeMarzo and He (2021) find two MPEs, the permanently unlevered equilibrium for a currently unlevered firm, and the leverage ratcheting equilibrium for a firm that has already debt in place, and that the switch between the first and the second equilibrium cannot occur. In contrast, in our model a current unlevered firm starts issuing debt in equilibrium. Figure 2 shows that this firm does not jump to the long-run leverage at once, but will gradually build up debt, the upward adjustment being slower the higher $1/\xi$. Along this path, while increasing the leverage net of the contractual debt repayment, the shareholders will capture the incremental tax shield and the benefits of investment, but will not endogenize the increased bankruptcy costs for legacy debt. The present value of the benefits captured by the shareholders following this policy is the difference between $V(0, \overline{y})$ and $V^u(\overline{y})$. Figure 6 shows that such benefits are lower the longer the maturity because the bigger the commitment problem, and for higher $1/\xi$ the accrual of these benefits is slower. Yet, the benefits are strictly positive.[15]

However, the value of debt financing is lower than it could be due to limited commitment and credit market freezes. Specifically, the impact of these frictions is measured by the shortfall in firm value between the case with full commitment and no market freeze, corresponding to $1/\xi = 1$ and $\pi = 0$, and the baseline case. In the left panel of Figure 6 this shortfall (red dotted line) ranges from 3% to 7%, depending on debt maturity.

In the right panel of Figure 6, we plot the long-run target leverage, $\hat{\ell}$, against debt maturity, $1/\xi$. Also in this case a hump shape emerge, showing a non-trivial relation between debt maturity and leverage. When maturity is short, debt refinancing risk dominates the benefit that can be extracted from debt financing, and therefore the firm keeps the leverage low. When maturity is long, the negative effect of credit market freeze is almost absent, which would suggest to increase leverage, but the benefit that can be extracted from each unit of issued debt is small due to the prevalence of the LRE. Hence, also in this case the firm keeps leverage low. It is when debt maturity is intermediate, in the case of the figure between 6 and 16 quarters, that the negative effect of credit market

[15] When these benefits become small, as it happens with very long debt maturity, they can be easily offset by a small one-time initial debt issuance cost, which we do not model. This prediction is different from the one in Benzoni et al. (2022), where transaction costs are incurred at any time the firm refinances, as their model shows that a firm optimally remains unlevered for combinations of high issuance costs and relatively short debt maturity.

freeze and of debt dilution are kept low, and therefore each unit of debt contributes a higher value to the firm, which therefore pushes the leverage high. It is remarkable that the leverage range is between 0.14 and 0.45 only because of the effect of debt maturity.

A lower likelihood of credit market freezes affects long-run target leverage only when debt maturity is short. In the limiting case with $\pi = 0$, shorter maturity strengthens the ability of leverage to increase firm value by reducing the LRE. As a result, long-run leverage is higher.

4.2 Firm dynamics

We use the calibrated model to derive implications of debt maturity for the dynamics of the firm policies and security values. In particular, we will focus on the dynamics of the investment and debt policy, and the related dynamics of cash flow and of the payout policy through the flow of funds equation.[16]

Figure 7, based on the calibration in Table 1, shows the evolution of these variables for different initial levels of b_0. Since the model is homogeneous of degree one in k, without loss of generality we normalize $k_0 = 1$, and so $\ell_0 = b_0$. To isolate the effects of limited commitment on debt policy from the dynamics of y, we fix y at $\overline{y}$. The figure shows that the firm's dynamics are driven by its initial leverage. In the top row of Figure 7, with $b_0 = 0.2$, the firm converges to the long-run leverage level $\hat{\ell} = 0.28$ (or $\hat{\ell}_1$, if there are multiple fixed points in the leverage policy). To do so, capital adjusts downward while

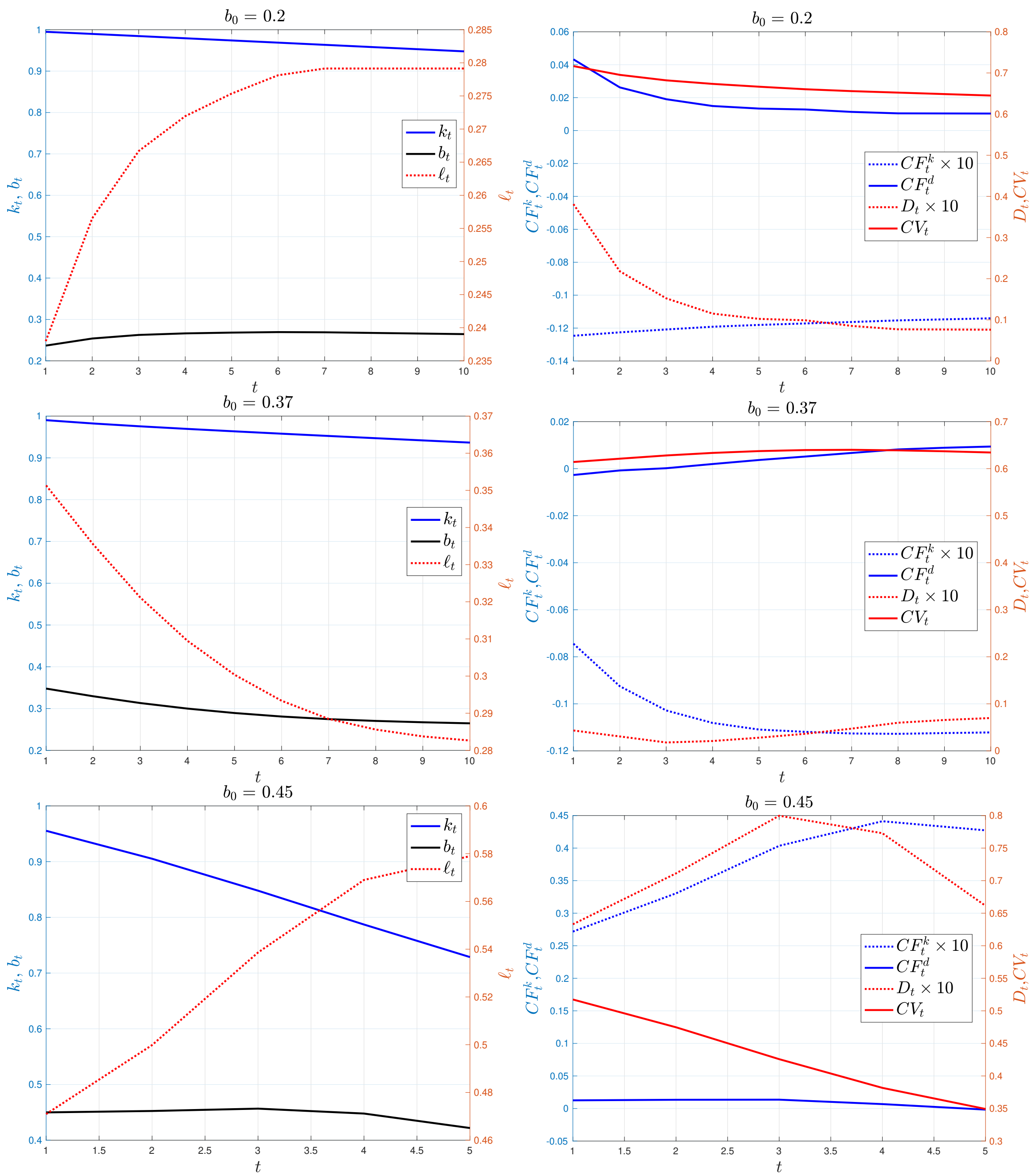
[16] Defining the state at t as (b_t, k_t, y_t), we have $(b_t, k_t, y_t) = k_t(\ell, 1, y)$, and the related optimal policy (κ, ℓ'), where $\kappa = h(\ell, y)$ and $\ell' = g(\ell/\kappa, y)$, the cash flow from investment policy for $k_t = 1$ is $cf^k(\ell, y) = -(\kappa - 1 + \delta) + \tau\delta - \frac{1}{2}\varphi(\kappa - 1 + \delta)^2$ and the cash flow from the debt policy for $k_t = 1$ is $cf^d(\ell, y) = \kappa\left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right] p(\ell', y)$. From these, given the capital stock at t, k_t, we derive the respective dollar values of the two cash flows at t as $CF_t^k = k_t\ cf^k(\ell, y)$ and $CF_t^d = k_t\ cf^d(\ell, y)$. The (expected) dividend for $k_t = 1$ over the period $[t, t+1]$ is

$$d(\ell, y) = -(\kappa - 1 + \delta) + \tau\delta - \frac{1}{2}\varphi(\kappa - 1 + \delta)^2 + \kappa\left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right] p(\ell', y)$$
$$+ \kappa\beta\mathbb{E}_y\left[\int_{x_d(\ell', y')}^{\overline{x}} \{(1-\tau)(y' + x') - ((1-\tau)r + \xi)\,\ell'\}\,\phi(x')dx'\right],$$

from which we calculate the dollar dividend $D_t = k_t\ d(\ell, y)$ at t. Finally, the continuation value of the firm at t for $k_t = 1$ is $cv(\ell, y) = \kappa\beta\mathbb{E}_y\left[\int_{x_d(\ell', y')}^{\overline{x}} V(\ell', y')\phi(x')dx'\right]$, from which we determine the dollar continuation value $CV_t = k_t\ cv(\ell, y)$.

Figure 7: Asset and debt dynamics

The figure displays the simulation paths of k_t, b_t, and $\ell_t = b_t/k_t$ (in the left column), and the simulation paths of CF_t^k, CF_t^d, D_t, and CV_t (in the right column). For ease of comparison, the values of CF_t^k and D_t are multiplied by 10. In the simulations, the firm starts with $k_0 = 1$ and initial debt levels $b_0 = 0.2$ (first row), $b_0 = 0.37$ (second row), and $b_0 = 0.45$ (third row). The simulation paths are conditional on the firm being solvent at the beginning of each period. The simulations are conducted assuming that y is non-stochastic and constant at $\overline{y}$, using the parameters in Table 1.



debt slightly increases. The right panel of the first row reports the corresponding cash flow and payout dynamics. With low leverage, the commitment problem on debt policy is limited. Hence, the debt price remains relatively high, allowing the firm to raise enough funds through debt issuance to maintain a positive, though declining, dollar dividend and investment. Overall, the firm's (dollar) value, measured by the continuation value, slightly declines over time.

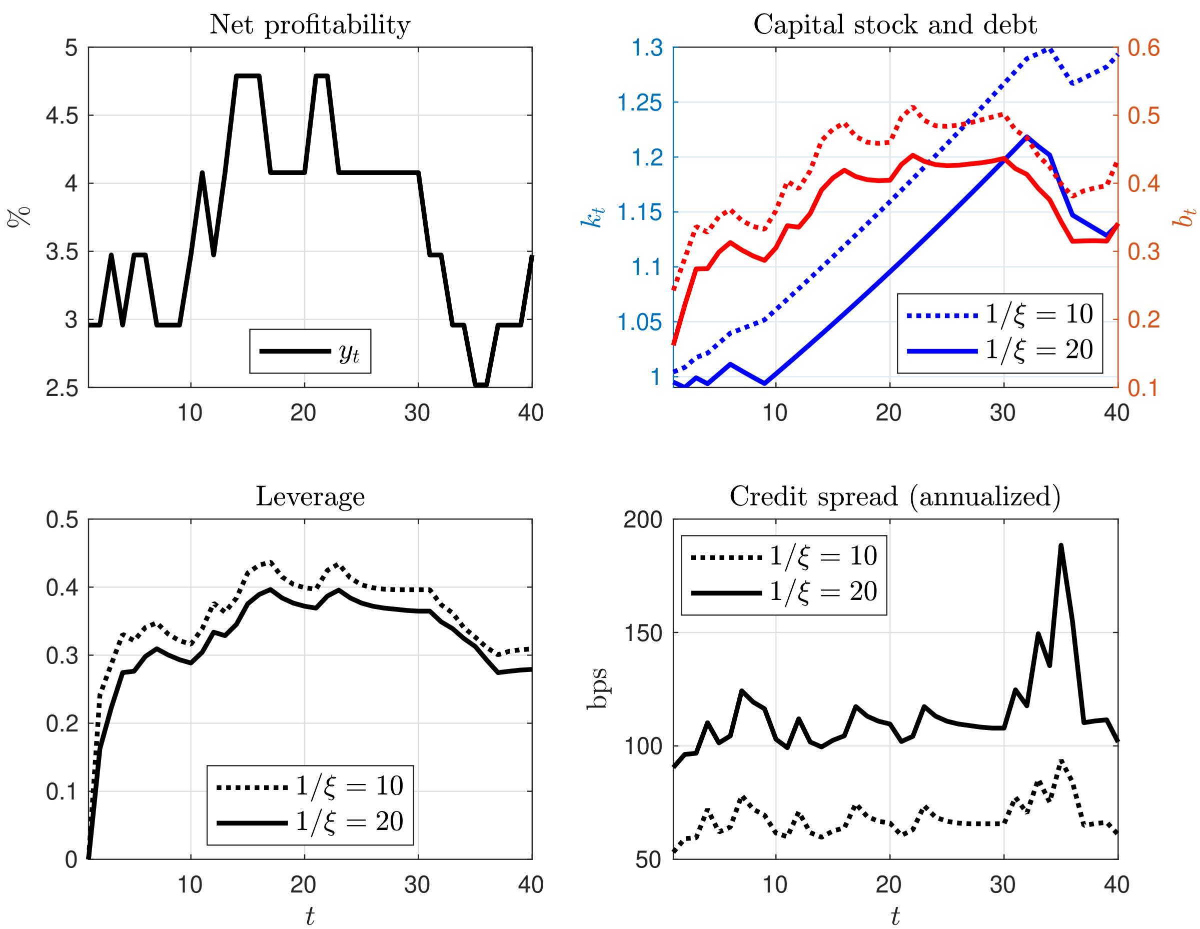
If $b_0 = 0.37$, as shown in the second row of Figure 7, investment is positive and grows over time. Notably, not only is the initially high debt gradually reduced through amortization, but there is also an early phase in which net proceeds from debt issuance are negative. This indicates that shareholders actively reduce debt beyond the scheduled amortization, deviating from the LRE equilibrium. In our model, the LRE amplifies underinvestment, and to restore a positive NPV of capital investment, shareholders reduce debt more aggressively. Later, debt issuance becomes positive again as leverage approaches the long-run level $\hat{\ell} = 0.28$. The initial debt reduction is mainly financed through payout reduction, although dividends remain positive over the observed period. Overall, the firm's value remains relatively stable, even though the capital stock declines. This suggests that, when initial leverage is not too high, debt maturity serves as an effective commitment device to mitigate debt overhang.

The firm's trajectory is quite different when $b_0 = 0.45$, as shown in the third row of Figure 7.[17] Indeed, the firm's path depends on whether the initial leverage ℓ_0 is below or above $\hat{\ell}_2$. If $\ell_0 < \hat{\ell}_2$, leverage converges to $\hat{\ell}_1$, as in the previous cases. However, if $\ell_0 > \hat{\ell}_2$, as in the third row of Figure 7, leverage increases rapidly. This happens because the capital stock k_t shrinks sharply due to asset sales, while debt remains constant or declines only slightly. This outcome fully reflects the effects of limited commitment on debt policy and on investment: shareholders extract value early by paying themselves large dividends funded by asset sales. Meanwhile, the (dollar) value of the firm declines steadily. In other words, the LRE of debt policy leads shareholders to gradually cash out reducing the value of the firm.

[17] For illustration, we choose the base case ξ so that equation $\ell = g(\ell, y)$ has multiple solutions for $\ell \in [0, 1]$ and $y = \overline{y}$, as seen in Figure 2. Specifically, for the parameters in Table 1 we find $\hat{\ell}_1 = 0.28$ and $\hat{\ell}_2 = 0.42$. In unreported results (available upon request) we show that, for lower $1/\xi$, the dynamic is similar (with the main difference that capital stock is generally increasing over time), but because there is only one fixed point for leverage, we do not observe leverage divergence as in the third row of Figure 7.

Figure 8: Firm policies and credit risk dynamics with time-varying profitability

The figure shows one path of $(1-\tau)y_t$ and the corresponding evolution of k_t and b_t, starting from $y_0 = \overline{y}$, $b_0 = 0$ and $k_0 = 1$, and using the optimal policy in the MPE. This produces new leverage, $\ell_t = b_t/k_t$ and credit spread, cs_t. These paths are conditional on the firm being solvent at the beginning of each period. We present with solid lines the case with maturity $1/\xi = 10$ (two and half years), and with dotted lines the case with maturity $1/\xi = 20$ (five years). The equilibrium solution is based on the parameters in Table 1.



Next, we analyze how debt maturity affects the leverage and credit risk dynamics in response to the evolution of the persistent component of productivity, y_t. We first analyze one path of y_t, starting from $y_0 = \overline{y}$. Because of the homogeneity property of the model, it suffices to show the case of a firm starting with unit capital, $k_0 = 1$, as the dynamics would be the same for a different $k_0 \neq 1$. In Figure 8, we illustrate the evolution of capital stock, debt, leverage, and credit spread

$$cs(b', k', y) = cs(\ell', y) = (r + \xi) \left(\frac{1}{p(\ell', y)} - 1 \right), \tag{12}$$

for a firm currently unlevered ($b_0 = 0$), and then implementing the optimal policy from the MPE. We show the evolution for two possible debt maturities: $1/\xi = 10$ and $1/\xi = 20$. Clearly, although y_t follows a stationary process, (k_t, b_t) do not, due to capital accumulation. However, stationarity is recovered when considering ℓ_t as state variable.

By construction, given our assumption of zero adjustment costs for debt and positive adjustment costs for capital, debt policy is more responsive than investment to changes in profitability, as shown in Figure 8. In fact, only during a significant downturn in y_t—such as the one toward the end of the sample period—does the firm's capital stock deviate from its growth path and contract. In contrast, the firm's debt policy reacts immediately to productivity shocks. The speed of this adjustment depends on debt maturity: firms with shorter maturity debt adjust more quickly, as discussed in Section 3.1.

A few new facts emerge from Figure 8. First, the response of investment to a negative shock is faster if the debt maturity is longer, as underinvestment is more severe in this case. The resulting leverage dynamics shows that with shorter debt maturity the correlation of new leverage and profitability is higher, whereas for longer maturity leverage is more persistent.

Secondly, Figure 8 shows that the credit spread is affected by the leverage dynamics, and it is lower when the debt has shorter maturity. This is the result of two effects. First, a lower $1/\xi$ reduces the shareholders' commitment problem to debt repayment, as noted also by Dangl and Zechner (2021), and therefore the credit spread for short maturity debt is lower and the spike of the credit spread during the period of economic decline of the firm is contained.[18] The second effect, unique to our setting, is that a

[18] In unreported results, we show that 1-period default probability follows a similar path to the credit spread.

lower $1/\xi$ increases the shareholders' commitment to debt and investment policy, and therefore it will reduce the incentive to increase the leverage in those future states in which the firm is solvent. This has a positive ex ante effect on the debt price which reduces the credit spread even further.[19]

Thirdly, capital accumulation in Figure 8 is slower for longer debt maturity, a consequence of the fact that for higher $1/\xi$ the issues with limited commitment are more severe. Indeed, as clarified in Section 3.2, the more severe the commitment problems, the lower the ex ante value of the tax shield, and therefore of the marginal q. This has the effect of reducing investment in each period, and ultimately of reducing the real size of the firm in the long run.

A more detailed description of the dynamics of the firm's policies and security prices can be seen in Table 3, in which we report several summary statistics of interesting quantities from a simulated economy under different scenarios for the debt maturity, $1/\xi$. The paths for y and the initial states are the same for all ξ. Within each maturity scenario, given the MPE based on the parameters in Table 1, simulation is done as described in Appendix H. While a firm may default over each quarter, we record the relevant beginning-of-period quantities conditional on a firm being solvent.[20] The statistics presented in Table 3 are calculated as time-series average of cross-sectional averages of the specific statistics.

Several remarks from Table 3 are in order. The first and most important is that different debt maturities have real effects because they induce different leverage and investment policies. Tobin's Q is higher (and more persistent, and less sensitive to shocks) when the commitment problem is less severe for shorter debt maturities, which mitigate the commitment problems on debt policy and on investment. Indeed, correlation of investment with y is increasing in debt maturity. This can be explained by considering that, for low $1/\xi$, investment fully reacts to positive shocks on y, but with high $1/\xi$, the

[19]In Section 4.3 we separate the effect of ξ on the credit spread due to the first commitment problem, as analyzed already by Dangl and Zechner (2021), from the one due to the second commitment problem, which is unique to this setting.

[20]In particular, for each (b, k, y) visited with positive probability, given the equilibrium policy (b', k'), where $k' = kh(b/k, y)$ and $b' = k'g(b/k', y)$, we calculate new book leverage as b'/k', new market leverage as $b'p(b', k', y)/F(b, k, y)$, investment as $i = k'/k - (1 - \delta)$, Tobin's Q as $F(b, k, y)/k'$, where $F(b, k, y) = V(b, k, y) + (1 - \xi)bp(b', k', y)$ is total firm value and $V(b, k, y) = kV(\ell, y)$ with $V(\ell, y)$ from (6), and $p(b', k', y) = p(\ell', y)$ from (9).

Table 3: Debt maturity and firm dynamics

The table presents summary statistics of quantities calculated from simulated economies constructed as described in Appendix H, based on the parameters from Table 1, but varying the debt maturity from 1 year (4 quarters) to 5 years (20 quarters). $\mu(\cdot)$ indicates average, $ac(\cdot)$ autocorrelation, $\rho(\cdot,\cdot)$ correlation, bl new book leverage, ml new market leverage, Q Tobin's q, i investment, cs credit spread (in bps), and dp default probability (in %).

$1/\xi$	4	8	12	16	20
$\mu(bl)$	0.31	0.32	0.32	0.31	0.28
$ac(bl)$	0.92	0.95	0.96	0.97	0.97
$\rho(bl, y)$	0.99	0.98	0.96	0.93	0.89
$\mu(ml)$	0.29	0.30	0.29	0.29	0.26
$\mu(Q)$	1.07	1.06	1.05	1.05	1.04
$\mu(i)$	0.11	0.10	0.09	0.08	0.07
$\rho(i, y)$	0.76	0.81	0.88	0.91	0.90
$\mu(cs)$	33.03	59.94	81.36	99.11	119.43
$\mu(dp)$	0.24	0.39	0.49	0.43	0.17

correlation of investment with negative productivity shocks increases due to more severe underinvestment, as illustrated in our previous analysis.

The second main point is that longer debt maturity makes leverage more persistent. This aligns with the intuition from Figure 8, which shows that shorter maturity leads to faster adjustment of leverage toward its long-run level after profitability shocks. This is further confirmed by the fact that the correlation between changes in leverage and profitability shocks is nearly perfect for short maturities but declines significantly as maturity increases.

The final point is that, when there is limited commitment on debt and investment policies, debt maturity significantly affects credit risk and the cost of debt. While default frequency is hump-shaped, in line with average (book and market) leverage, the credit spread increases four-fold when the maturity is 5 years compared to 1 year. We further analyze this point in the next section.

4.3 Limited commitment and the cost of debt

The previous analysis shows that commitment problems related to debt repayment and future firm policies significantly affect leverage, investment, and ultimately the cost of debt. Both types of commitment issues are influenced by debt maturity, $1/\xi$. On one hand, a higher ξ means that equity holders must repay a larger fraction of the outstanding debt at face value, which increases default risk and the credit spread. On the other hand, in Section 4 we showed that a lower ξ leads to more underinvestment and debt dilution, which lower the price of debt and increase the credit spread. To assess whether (and under what conditions) the commitment problem in debt policy matters more for the cost of debt than the commitment problem in investment—and, relatedly, whether debt maturity is a more effective commitment device for one problem or the other—we decompose the cost of debt financing into two parts: a *default component*, reflecting the commitment problem in debt repayment, and an *agency component*, reflecting underinvestment and debt dilution.

Given the MPE, the credit spread in (12) can be interpreted as the difference between the yield on the firm's debt and the yield on an otherwise identical contract—same maturity and face value—that is not affected by default. Denoting by $p^d(\cdot)$ the price of this default-free contract, evaluated under the equilibrium policy (b', k') in state (b, k, y), equation (12) becomes $(r + \xi)\left(1/p(b', k', y) - 1/p^d(b', k', y)\right)$. To separate the agency and default components of this spread, we would need to isolate the price decline due to commitment problems—on investment and debt issuance—in future states where the firm remains solvent. Let $p^a(\cdot)$ denote the price of a contract identical to the original one, except it is not affected by any commitment problems. Then the agency spread would be $(r+\xi)/p^d(b', k', y) - (r+\xi)/p^a(b', k', y)$. However, we have the following proposition.

Proposition 3. *If the equilibrium leverage dynamics is such that the debt is risk free (i.e., it is always $x_d < \underline{x}$), the equilibrium investment and leverage policy in the MPE maximizes the value of the firm (equity plus debt), and there is no agency conflict.*

Because under the MPE eliminating default also removes commitment problems on future debt and investment policies, so that $p^d \equiv p^a \equiv 1$, the default and agency components cannot be separated in (12). Indeed, over an infinite horizon, the accumulation of agency conflicts ultimately leads to the firm's default, meaning the credit spread reflects only the default component.

We propose a workable decomposition of credit spreads into a default and an agency component that is based on the MPE and is conditional on a finite horizon H. The default component is due to the cost of default (according to the equilibrium policy) up to H. The agency component derives from the agency conflicts in *non-default states*, at all dates before H. Because the equilibrium policy is expressed in a dynamic programming setting as one-period decisions, and prices are for infinite-lived securities (with finite average maturity $1/\xi$), we can only define the decomposition in a recursive fashion.[21]

We begin with the one-period horizon case. We define the price at time t of the same debt contract as in the baseline equation (3), but assuming *no default* at $t+1$, as

$$p_1^d(b,k,y) = \beta \mathbb{E}_y \left[r + \xi + (1-\xi)\, p(b',k',y') \right], \tag{13}$$

where (b',k') is the equilibrium policy at $t+1$. The one-period *default spread*, capturing the increase in the cost of debt due to the possibility of default at $t+1$, is defined as

$$ds_1(b',k',y) = (r+\xi)\left(\frac{1}{p(b',k',y)} - \frac{1}{p_1^d(b',k',y)} \right).$$

This spread corresponds to the yield difference between the original debt and an otherwise identical contract without default risk in the first period. It isolates the cost of default occurring specifically at $t+1$. Next, we define the price at t of the same debt contract but without the effects of default and of *underinvestment and debt dilution* on the continuation value at $(t+1)$ as

$$p_1^a(b,k,y) = \beta \mathbb{E}_y \left[r + \xi + (1-\xi) \max\{p(b',k',y'), p(b,k,y')\} \right]. \tag{14}$$

The modified continuation value in (14), excludes the *negative effects* on the debt price from changing the state from (b,k) to (b',k') at $t+1$, i.e., when $p(b',k',y') < p(b,k,y')$, while any positive effects on the debt price from such changes are retained. From $t+2$ onward, debt holders will be fully exposed to the commitment problems—affecting debt repayment and firm policies—associated with the equilibrium policy, as captured by the

[21] In Appendix J, we discuss our decomposition of the credit spread vis-à-vis alternative approaches to decompose the credit spread into a pure default component vs an agency component. Our approach has the key property of being consistent with the MPE, whereas other approaches result in an off-equilibrium decomposition.

equilibrium debt price $p(b', k', y')$. Similarly to what done for the default spread, the one-period *agency spread* is

$$as_1(b', k', y) = (r + \xi) \left(\frac{1}{p_1^d(b', k', y)} - \frac{1}{p_1^a(b', k', y)} \right). \tag{15}$$

Once the default and the agency spreads have been defined, the *credit spread* for one-period exposure to all commitment problems is $cs_1(b', k', y) = ds_1(b', k', y) + as_1(b', k', y)$.

We recursively derive the same decomposition of the credit spread for a predetermined H-period horizon, $H > 1$ and generate the term-structure of the credit spreads and of their two constituents. Specifically, the debt price without the effects of default for n periods, $n = 2, \ldots, H$, is

$$p_n^d(b, k, y) = \beta \mathbb{E}_y \left[r + \xi + (1 - \xi) p_{n-1}^d(b', k', y') \right], \tag{16}$$

where $p_1^d(\cdot)$ is from (13). Hence, the spread for a H-period exposure to default is

$$ds_H(b', k', y) = (r + \xi) \left(\frac{1}{p(b', k', y)} - \frac{1}{p_H^d(b', k', y)} \right), \tag{17}$$

which is the cost of debt due to the chance that the firm defaults in any of the following H periods. In a similar fashion, the price of debt without the effect of commitment problems for n periods, $n = 2, \ldots, H$, is

$$p_n^a(b, k, y) = \beta \mathbb{E}_y \left[r + \xi + (1 - \xi) \max\{ p_{n-1}^a(b', k', y'), p_{n-1}^a(b, k, y') \} \right]. \tag{18}$$

where $p_1^a(\cdot)$ is from (14). Then, the H-period agency spread is

$$as_H(b', k', y) = (r + \xi) \left(\frac{1}{p_H^d(b', k', y)} - \frac{1}{p_H^a(b', k', y)} \right), \tag{19}$$

which is solely due to commitment problems on debt issuance and investment in non-default states in any period from 1 to H. Finally, the credit spread due to exposure to all commitment problems for H periods is $cs_H(b', k', y) = ds_H(b', k', y) + as_H(b', k', y)$.

Proposition 4. *Given the MPE, we have*

1. *for all $H \geq 1$, $ds_H \geq 0$, $as_H \geq 0$, and $cs_H \geq 0$;*

2. for all $H \geq 1$, $cs_{H-1} \leq cs_H \leq cs$, *and for* $H \to \infty$, $cs_H \to cs$;

3. for $H \to \infty$, $ds_H \to cs$ *and* $as_H \to 0$.

The proof is in Appendix G. The intuition is that the longer the horizon, H, over which the creditors are exposed to the two commitment problems, the closer cs_H is to the credit spread in (12), and asymptotically the two are the same. Also, at longer horizons the agency spread becomes negligible and the default spread dominant.

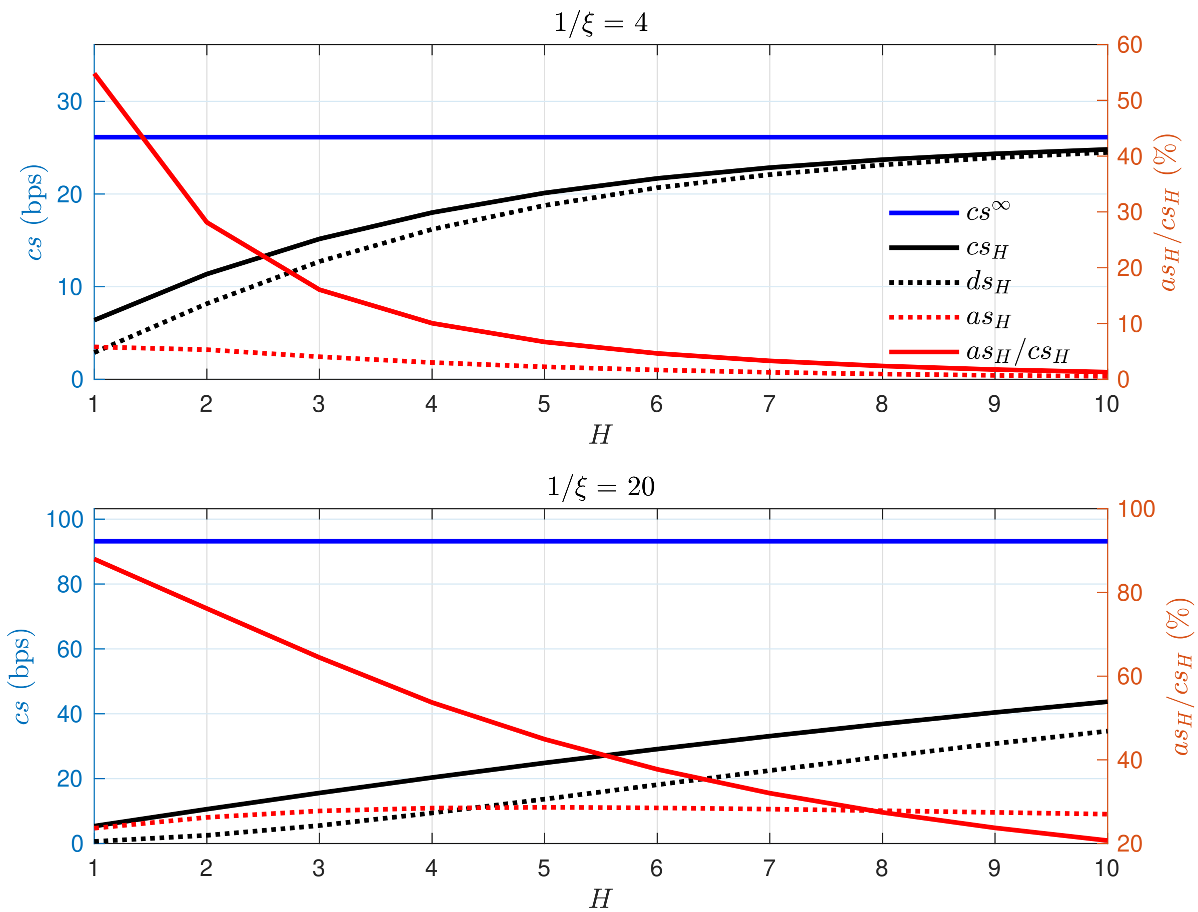
Figure 9 shows how the credit spread at a given horizon H is decomposed into the above defined default and agency components, for debt maturities of 1 and 5 years, current leverage $\ell = 0.1$, and current y at $\overline{y}$. As a benchmark, we report also the credit spread defined in (12), denoted by c^{∞}. A few remarks are in order. First, c^{∞} is higher for higher $1/\xi$. This is intuitive, as ξ controls the commitment of equity holders to future debt and investment policies. A lower ξ (i.e., longer maturity) increases debt dilution and underinvestment incentives, leading to a higher credit spread.

Second, the anticipation by debt holders to be exposed to equity-maximizing debt and investment decisions leads to a sizable agency spread of about 10-12bps at least for horizons of up 10 quarters, when debt maturity is long and current leverage is relatively low. This is because the agency spread reflects the expectation of future transfers of value from debt to equity due to the LRE. The fraction of credit spread that can be attributed solely to agency conflicts related to the leverage policy (right y-axis) is substantial at short horizons. As transfers that are far in the future weigh increasingly less, the agency spread vanishes when longer horizons are considered, and only the default spread remains. In the limit for very long horizons, as seen in Proposition 4, the credit spread only comprises the default component.

Third, the absolute and relative value of the agency spread is strongly influenced by the parameter ξ, being higher at all horizons if debt maturity is longer. As noted by DeMarzo and He (2021), a model based on lack of commitment to future debt issuances, which has negative effect on the investment policy, can generate positive yield spreads for firms even in the presence of small default risk. In the figure, for a case with low current leverage, $\ell = 0.1$, the proportion of credit spread attributable to agency conflicts for $H = 1$ goes from about 50% of the credit spread for 1 year to above 80% for 5 years. These results confirm the intuition provided by Myers (1977) that a longer debt

Figure 9: Decomposition of credit spread

The figure plots the decomposition of the credit spread into a default spread and agency spread, as defined respectively in equations (17) and (19), for different horizons H. H represents the number of quarters for which debt holders are exposed to the negative effects of equity holders' lack of commitment. The top panel is for maturity $1/\xi = 4$ (1 year) and bottom panel for maturity $1/\xi = 20$ (5 years). The MPE is based on parameters in Table 1, at a current $y = \overline{y}$, and for $\ell = 0.1$.



maturity allows for more severe debt dilution and underinvestment. However, in our case the underinvestment issue is exacerbated by lack of commitment on debt policy, which is more severe for longer maturity, as illustrated in Section 3.2. Overall, a credit risk model based on lack of commitment on future debt policies can succeed where traditional structural models have been unsuccessful (see Huang and Huang (2012)), because it allows for an additional channel of credit risk, an "agency" channel, besides the pure default one, which is sizeable in particular with low leverage firms.

5 Conclusion

Debt maturity is known to help mitigate default risk by committing a firm to repay a fixed amount at a predetermined date. We show that it also plays a key role in reducing the negative effects of shareholders' lack of commitment to future debt issuance and investment decisions.

Besides lack of commitment to debt repayment and future debt policies, *à la* DeMarzo and He (2021), our model features dynamic asset, with a commitment problem on future investment decisions, to study how the commitment problem on debt policy affects firm's investment, and how the ensuing underinvestment interacts with the incentive to dilute debt holders. To the baseline framework, we also add debt market freezes, which have a more negative effect on firm value the shorter the debt maturity.

Excluding debt market freezes, debt maturity can effectively eliminate underinvestment, as single-period debt can restore value maximizing investment levels. Even with market freezes, short debt maturity mitigates underinvestment and the leverage ratchet effect, although imperfectly.

We calibrate the model and show that debt maturity can be effective at reducing the firm value shortfall created by the three commitment problems that we analyze. Because debt maturity addresses both the commitment problem to debt repayment and the commitment problem to future policies, we decompose the cost of debt into an agency component and a default component. We show that the agency component is more important when the leverage is low and default far, and a lower debt maturity directly affects this part.

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Online Appendix

A Proof of Proposition 1

Given the linearity of the production function and the assumed capital adjustment cost, it can be proved that $V(b,k,y) = kV(b/k,1,y)$ (homogeneity of degree one) and $p(b,k,y) = p(b/k,1,y)$ and $x_d(b',k',y') = x_d(b'/k',1,y')$ (homogeneity of degree zero). Therefore, we can write the equity program in (1) as

$$kV\left(\frac{b}{k},1,y\right) = k\max_{(b',k')} -\left[\frac{k'}{k}-(1-\delta)\right]+\tau\delta-\frac{1}{2}\varphi\left[\frac{k'}{k}-(1-\delta)\right]^2+\left[\frac{b'}{k}-(1-\xi)\frac{b}{k}\right]p(b',k',y)$$
$$+\beta\mathbb{E}_y\left[\int_{x_d(\frac{b'}{k'},y')}^{\overline{x}}\left\{(1-\tau)(y'+x')\frac{k'}{k}-[(1-\tau)r+\xi]\frac{b'}{k}+\frac{k'}{k}V\left(\frac{b'}{k'},1,y'\right)\right\}\phi(x')dx'\right.$$
$$\left.-\pi\int_{x_d(\frac{b'}{k'},y')}^{x_d^0(\frac{b'}{k'},y')}\left\{(1-\tau)(y'+x')\frac{k'}{k}-[(1-\tau)r+\xi]\frac{b'}{k}+\frac{k'}{k}V\left(\frac{b'}{k'},1,y'\right)\right\}\phi(x')dx'\right].$$

Defining $\kappa = k'/k$, $\ell = b/k$, $\ell' = b'/k'$, and $\eta = b'/k$, simplifying k from both sides, and dropping the capital stock dependence (with a small abuse of notation), $V(\cdot,y) = V(\cdot,1,y)$ and $p(\cdot,1,y) = p(\cdot,y)$, from the expressions above we have

$$V(\ell,y) = \max_{(\eta,\kappa)} -[\kappa-(1-\delta)]+\tau\delta-\frac{1}{2}\varphi[\kappa-(1-\delta)]^2+[\eta-(1-\xi)\ell]\,p(\ell',y)$$
$$+\beta\mathbb{E}_y\left[\int_{x_d(\ell',y')}^{\overline{x}}\{(1-\tau)(y'+x')\kappa-[(1-\tau)r+\xi]\,\eta+\kappa V(\ell',y')\}\,\phi(x')dx'\right.$$
$$\left.-\pi\int_{x_d(\ell',y')}^{x_d^0(\ell',y')}\{(1-\tau)(y'+x')\kappa-[(1-\tau)r+\xi]\,\eta+\kappa V(\ell',y')\}\,\phi(x')dx'\right],$$

where $x_d(\ell,y) = r\ell+[\xi\ell-V(\ell,y)]/(1-\tau)-y$, and $x_d^0(\ell,y)$ has the same expression, but with $V^0(\ell,y)$ in place of $V(\ell,y)$. From this, collecting κ, we have

$$V(\ell,y) = \max_{(\ell',\kappa)} -[\kappa-(1-\delta)]+\tau\delta-\frac{1}{2}\varphi[\kappa-(1-\delta)]^2+\kappa\left[\frac{\eta}{\kappa}-(1-\xi)\frac{\ell}{\kappa}\right]p(\ell',y)$$
$$+\kappa\beta\mathbb{E}_y\left[\int_{x_d(\ell',y')}^{\overline{x}}\left\{(1-\tau)(y'+x')-[(1-\tau)r+\xi]\,\frac{\eta}{\kappa}+V(\ell',y')\right\}\phi(x')dx'\right.$$
$$\left.-\pi\int_{x_d(\ell',y')}^{x_d^0(\ell',y')}\left\{(1-\tau)(y'+x')-[(1-\tau)r+\xi]\,\frac{\eta}{\kappa}+V(\ell',y')\right\}\phi(x')dx'\right].$$

We note that $\eta/\kappa = \ell'$. The program above can be decomposed into two parts: the first, for given ℓ and κ (and therefore, the state variable for this program is ℓ/κ) optimizes the last two lines with respect to ℓ'; the second part optimizes with respect to κ. More explicitly, defining $\mathcal{V}(\ell', y)$ as in (8) we can write the program above as

$$V(\ell, y) = \max_{\kappa} \; -[\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi [\kappa - (1-\delta)]^2 + \kappa \max_{\ell'} \left[\ell' - (1-\xi)\frac{\ell}{\kappa}\right] p(\ell', y) + \beta\mathcal{V}(\ell', y),$$

where it is easy to recognize the first line gives (6) for the outer maximization with respect to κ, and the second line is the inner maximization with respect to ℓ'. From the second line, we define $v(\ell/\kappa, y)$, as in (7).

To determine $V^0(\ell, y)$, we start from (5) and based on the same steps as above, we derive

$$V^0(\ell, y) = \max_{\kappa} \; -[\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi [\kappa - (1-\delta)]^2 + \kappa v^0(\ell/\kappa, y)$$

where $v^0(\ell/\kappa, y) = v(\ell/\kappa, y)(1 - \xi\ell/\kappa)$.

As for debt value, we first note that debt recovery can never exceed the asset value at default, $(1-\tau)(y+x)k + \tau r b + V(b, k, y)$. Consequently, it also cannot surpass the total debt obligation, $(r+\xi)b$. Given $p(b, k, y) = p(\ell, y)$, $R(\ell, y') = \min\{1-\delta-\alpha, \tau r\ell + V(\ell, y')\}$ and $x_f(\ell, y') = -R(\ell, y')/(1-\tau) - y'$ we have (9). □

B Model of unlevered firm

This is a first benchmark model of a firm with no debt. As the firm is permanently unlevered, the credit market shock is irrelevant. The value of equity (and of the firm) is defined as the fixed point of the recursive program

$$V^u(k, y) = \max_{k'} - [k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + \beta\mathbb{E}_y \left[\int_{x_l(k,y')}^{\overline{x}} \{(1-\tau)(y'+x')k' + V^u(k', y')\}\, \phi(x')dx'\right],$$

where $x_l(k, y') = -\frac{V^u(k,y')}{(1-\tau)k} - y'$ is the threshold for x' below which the payoff to equity is negative, and therefore excluded by limited liability. The linearity of the production function and the assumed capital adjustment cost ensure that $V^u(k, y) = kV^u(1, y)$

(homogeneity of degree one). We define $\kappa = k'/k$, and (with another small abuse of notation) $V^u(y) = V^u(1, y)$. We have

$$V^u(y) = \max_{\kappa} -(\kappa - 1 + \delta) + \tau\delta - \frac{1}{2}\varphi\left(\kappa - 1 + \delta\right)^2 + \kappa\beta f(y), \tag{20}$$

where

$$f(y) = \mathbb{E}_y\left[\int_{x_l(y')}^{\overline{x}} \left\{(1-\tau)(y' + x') + V^u(y')\right\}\phi(x')dx'\right] \tag{21}$$

and $x_l(y') = -V^u(y')/(1-\tau) - y'$. The solution $V^u(\cdot)$ is found using value iteration together with the optimal κ from first-order condition on (20), $\kappa = \frac{\beta f(y)-1}{\varphi} + 1 - \delta$, from which

$$V^u(y) = \beta(1-\delta)f(y) + \tau\delta + \frac{1}{2}\frac{(\beta f(y) - 1)^2}{\varphi}. \tag{22}$$

Under the assumptions that y is non-stochastic and constant at $\overline{y}$ through time, the problem can be solved by replacing V^u from (22) in equation (21), which gives an equation in the unknown f that can be solved using Newton's method.

Under the additional assumption that $x_f(y) \leq \underline{x}$, we have

$$V^u = \max_{\kappa} -\left[\kappa - (1-\delta)\right] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 + \kappa\beta\left[(1-\tau)\overline{y} + V^u\right]. \tag{23}$$

The first-order condition is $1 + \varphi\left[\kappa - (1-\delta)\right] = \beta\left[(1-\tau)\overline{y} + V^u\right]$. Using this equation to eliminate V^u in (23), and after a few manipulations we find the optimal solution of the investment problem:[22]

$$\kappa^* = 1 + r - \sqrt{(r+\delta)^2 + \frac{2}{\varphi}\left[r - (1-\tau)(\overline{y} - \delta)\right]}, \tag{24}$$

where $r = 1/\beta - 1$, as long as

$$\delta + \frac{r}{1-\tau} < \overline{y} < \delta + \frac{r}{1-\tau} + \frac{\varphi}{2}\frac{(r+\delta)^2}{1-\tau}. \tag{25}$$

By using the optimal investment solution in the first-order condition above, we find the value function:

$$V^u = (1+r)\left[1 + \varphi(r+\delta)\right] - (1-\tau)\overline{y} - \varphi(1+r)\sqrt{(r+\delta)^2 + \frac{2}{\varphi}\left[r - (1-\tau)(\overline{y} - \delta)\right]}. \tag{26}$$

[22]Of the two roots of the quadratic equation, the solution is the one in the text because of the condition $\kappa < 1 + r$.

If condition (25) is true, the solution in (26) exists and is equal to the solution of (23) found using a value iteration algorithm. This is important, because existence of a solution to the stationary model (6)-(9) hinges upon the existence of a solution to (7)-(9) on the one hand, and on the existence of a solution to (6) on the other. We use condition (25) in the calibration of $\overline{y}$, φ, and δ to make sure that a solution to the investment problem exists.

C Model of levered firm with constant capital stock

This is a second benchmark model of a firm that can manage its leverage while the capital stock remains constant, $k' = k = 1$, and therefore investment equals depreciation. For the sake of brevity, we focus on the case without credit market shocks, as extending the analysis to include them is straightforward. We denote by $K = (1-\tau)\delta + \frac{1}{2}\varphi\delta^2$ the burden deriving from replacing depreciated asset. The following model holds for $\xi < 1$, that is debt maturity strictly higher then one period.[23] The value of equity is

$$\begin{aligned} V(b,y) = -K + \max_{b'} \; & [b' - (1-\xi)b]\, p(b',y) \\ & + \beta \mathbb{E}_y \left[\int_{x_d(b',y')}^{\overline{x}} \left\{ (1-\tau)(y' + x' - rb') - \xi b' + V(b',y') \right\} \phi(x') dx' \right], \end{aligned} \tag{27}$$

where $x_d(b',y') = rb' + [\xi b' - V(b',y')]/(1-\tau) - y'$. The debt price in the general case with non-zero recovery at default is

$$\begin{aligned} p(b,y) = \beta \mathbb{E}_y \Bigg[& \{r + \xi + (1-\xi) p(b',y')\} \int_{x_d(b,y')}^{\overline{x}} \phi(x') dx' \\ & + \int_{x_f(b,y')}^{x_d(b,y')} \frac{(1-\tau)(y'+x') + R(b',y')}{b} \phi(x') dx' \Bigg], \end{aligned} \tag{28}$$

where $R(b,y) = \min\{1-\delta-\alpha, \tau r b + V(b,y)\}$, $x_f(b,y') = -R(b,y')/(1-\tau) - y'$, with $x_f(b,y') \leq x_d(b,y')$, and $b' = g(b,y')$ is the optimal policy function from (27). The solution of the model is found using value function iteration based simultaneously on equations (27) and (28).

[23] We will show later on a general model for $\xi = 1$, of which the model with no investment is a subcase.

D Total firm value maximization

In this section, we derive a third benchmark model in which investment and debt decisions maximize the value of the firm, thus avoiding all agency problems. We focus on the case without credit market shocks ($\pi = 0$), since this benchmark is used to derive several results later on.

Lemma 5. *The default/investment/financing MPE in the case equity holders commit to maximize total firm value is found by solving the dynamic program*[24]

$$
\begin{aligned}
&F(k,y) = \\
&= \max_{(b',k')} \; -[k'-(1-\delta)k] + \tau\delta k - \Psi(k,k') + \beta\mathbb{E}_y \Bigg[\int_{x_f(b',k',y')}^{\overline{x}} (1-\tau)(y'+x')k'\phi(x')dx' \\
&\qquad + R(b',k',y') \int_{x_f(b',k',y')}^{x_d(b',k',y')} \phi(x')dx' + \{\tau r b' + F(k',y')\} \int_{x_d(b',k',y')}^{\overline{x}} \phi(x')dx' \Bigg], \quad (29)
\end{aligned}
$$

where the default threshold, $x_d(b',k',y')$, is defined in (2), $x_f(b',k',y')$ *is defined in* (4), *with $V(b',k',y') = F(k',y') - (1-\xi)b'p(b'',k'',y')$, and $p(b,k,y)$ is defined in* (3), *based on the optimal policy resulting from* (29).

Proof. At state (b,k,y) in t, total firm value, F, for an arbitrary (b',k') is

$$
\begin{aligned}
V(b,k,y) + (1-\xi)b\, p(b',k',y) &= -[k'-(1-\delta)k] + \tau\delta k - \Psi(k,k') + b'p(b',k',y) \\
&+ \beta\mathbb{E}_y \left[\int_{x_d(b',k',y')}^{\overline{x}} \{(1-\tau)(y'+x')k' - [(1-\tau)r + \xi]\, b' + V(b',k',y')\}\, \phi(x')dx' \right],
\end{aligned}
$$

[24]If there is not recovery at default for the debt, then the expression in (29) becomes

$$
\begin{aligned}
F(k,y) = \max_{(b',k')} \; & -[k'-(1-\delta)k] + \tau\delta k - \Psi(k,k') \\
& + \beta\mathbb{E}_y \left[\int_{x_d(b',k',y')}^{\overline{x}} \{(1-\tau)(y'+x')k' + \tau r b' + F(k',y')\}\, \phi(x')dx' \right].
\end{aligned}
$$

the right-hand side of which, by replacing $p(b', k', y)$ from (3) (with $\pi = 0$) and after few simplifications, becomes

$$- [k' - (1-\delta)k] + \tau\delta k - \Psi(k, k') + \beta \mathbb{E}_y \left[\int_{x_f(b',k',y')}^{\overline{x}} (1-\tau)(y'+x')k'\phi(x')dx' \right]$$

$$+ \beta \mathbb{E}_y \left[R(b', k', y') \int_{x_f(b',k',y')}^{x_d(b',k',y')} \phi(x')dx' \right.$$

$$\left. + \{\tau r b' + V(b', k', y') + (1-\xi)b'p(b'', k'', y')\} \int_{x_d(b',k',y')}^{\overline{x}} \phi(x')dx' \right],$$

where (b'', k'') denotes the choice at $t+1$. By replacing $V(b', k', y') + (1-\xi)b'p(b'', k'', y')$, with total firm value at (b', k', y') and choosing (b', k') that maximizes the expression above we define the recursive program in (29), from which we determine F and the related optimal policy. Notably, the program on the right-hand side of (29) does not depend on b, and therefore the optimal choice, $(b', k') = G(k, y)$, and the value function are independent of b. Therefore, a recursive application of (29) gives $F(k, y)$. The debt price is derived from equation (3) (with $\pi = 0$ by assumption) using the optimal policy $(b', k') = G(k, y)$, and the value of equity required to pin down the recovery value, R, and the thresholds x_d and x_f is $V(b', k', y') = F(k', y') - (1-\xi)b'p(b'', k'', y')$. □

Lemma 6. *The MPE for the case equity holders maximize total firm value is*

$$F(y) = \beta(1-\delta)f(y) + \tau\delta + \frac{1}{2}\frac{(\beta f(y) - 1)^2}{\varphi}$$

where[25]

$$f(y) = \max_{\ell'} \ \mathbb{E}_y \left[\int_{x_f(\ell',y')}^{\overline{x}} (1-\tau)(y'+x')\phi(x')dx' \right.$$

$$\left. + R(\ell', y') \int_{x_f(\ell',y)}^{x_d(\ell',y)} \phi(x')dx' + \{\tau r \ell' + F(y')\} \int_{x_d(\ell',y)}^{\overline{x}} \phi(x')dx' \right] \quad (30)$$

[25] If there is no recovery at default, the expression in (30) becomes

$$f(y) = \max_{\ell'} \ \mathbb{E}_y \left[\int_{x_d(\ell',y')}^{\overline{x}} \{(1-\tau)(y'+x') + \tau r \ell' + F(y')\} \phi(x')dx' \right].$$

If we consider credit market shocks, the expression is the same as in (30), except that the term $\pi \{\tau r \ell' + F(y') - R(\ell', y')\} \int_{x_d(\ell',y')}^{x_d^0(\ell',y')} \phi(x')dx'$ is subracted from it.

with thresholds

$$x_d(\ell', y') = r\ell' + \frac{\xi\ell' - [F(y') - (1-\xi)\ell' p(y')]}{1-\tau} - y', \quad x_f(\ell', y') = -\frac{R(\ell', y')}{1-\tau} - y',$$

recovery at default $R(\ell', y') = \min\{1-\delta-\alpha, \tau r\ell' + F(y') - (1-\xi)\ell' p(y')\}$, *and debt price*

$$p(y) = \beta\mathbb{E}_y\left[\{r + \xi + (1-\xi)p(y')\}\int_{x_d(\ell',y')}^{\overline{x}} \phi(x')dx' + \int_{x_f(\ell',y')}^{x_d(\ell',y')} \frac{(1-\tau)(y'+x') + R(\ell',y')}{\ell'}\phi(x')dx'\right]. \quad (31)$$

Proof. The stationary program to solve (29), following the approach in Appendix A, is

$$F(y) = \max_{\kappa} -[\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi[\kappa - (1-\delta)]^2 + \beta\kappa \max_{\ell'} \mathbb{E}_y\left[\int_{x_f(\ell',y')}^{\overline{x}} (1-\tau)(y'+x')\phi(x')dx' + R(\ell',y')\int_{x_f(\ell',y')}^{x_d(\ell',y')} \phi(x')dx' + \{\tau r\ell' + F(y')\}\int_{x_d(\ell',y')}^{\overline{x}} \phi(x')dx'\right],$$

where we used $F(k, y) = kF(1, y)$ and then dropped the first argument, with the usual abuse of notation, $F(y) = F(1, y)$. Also, $R(\ell', y') = \min\{1-\delta-\alpha, \tau r\ell' + V(\ell', y')\}$, $x_d(\ell', y') = r\ell' + (\xi\ell' - V(\ell', y'))/(1-\tau) - y'$, and $x_f(\ell', y') = -R(\ell', y')/(1-\tau) - y'$, where $V(\ell', y')$ is the related equity value. From this, we can define the outer investment program

$$F(y) = \max_{\kappa} -[\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi[\kappa - (1-\delta)]^2 + \beta\kappa f(y),$$

and the inner leverage program in (30). The optimal policy in the leverage program depends only on the current y, that is $\ell' = g(y)$. Therefore, the relevant price of debt at the current date is $p(y) = p(g(y), y)$, which only depends on y and not on current leverage, as shown in (31). Hence, the equity value required to determine the two thresholds and the recovery at default is $V(\ell', y') = F(y') - (1-\xi)\ell'\, p(y')$. Once $f(y)$ and the leverage policy $\ell' = g(y)$ are determined, the outer investment program has analytic solution $\kappa^* = \frac{\beta f(y) - 1}{\varphi} + 1 - \delta$, and replacing it in the investment program we have $F(y)$. □

Under the assumption that y is non-stochastic and constant at $\overline{y}$, the problem can be easily solved as follows: F is replaced in (30), and the optimal leverage, ℓ^*, is found as a function of the unknown f and p. Next, $\ell^*(f,p)$ is replaced on the right-hand side of (30). This gives an expression in f and p. Similarly, equation (31) depends on f and p. Hence, there are two non-linear equations, which can be solved simultaneously in the unknowns f and p using Newton's method.

E Proof of Proposition 2

1. Convexity of $v(\cdot,y)$ is a consequence of its optimality, as per equation (7). Indeed, for any $\ell' \neq \ell$, shareholders can adjust the leverage to ℓ with proceeds $(\ell-\ell')p$ from the issuance.[26] Given the optimality of v, the shareholders cannot get a higher value than $v(\ell'/\kappa,y)$, that is $v(\ell'/\kappa,y) \geq v(\ell/\kappa,y) - (\ell'-\ell)p/\kappa$, which means v is (weakly) convex. Exactly the same argument can be used to show the convexity of $V(\cdot,y)$, given $\kappa=\kappa^*$, the optimal investment policy.

2. For $\xi=1$ and $\pi=0$, the right-hand side of (1) becomes

$$\max_{(b',k')} -[k'-(1-\delta)k] + \tau\delta k - \Psi(k,k') + b'p(b',k',y)$$
$$+\beta\mathbb{E}_y\left[\int_{x_d(b',k',y')}^{\overline{x}} \{(1-\tau)(y'+x')k' - [1+(1-\tau)r]\,b' + V(b',k',y')\}\,\phi(x')dx'\right],$$

and $p(b',k',y)$ becomes independent of the future debt price in relation to the optimal policy, and only depends of the current state. By replacing in it $p(b',k',y)$ from equation (3) with $\xi=1$ and simplifying, we find

$$\max_{(b',k')} -[k'-(1-\delta)k]+\tau\delta k-\Psi(k,k')+\beta\mathbb{E}_y\left[\int_{x_f(b',k',y')}^{\overline{x}}(1-\tau)(y'+x')k'\phi(x')dx'\right.$$
$$\left.+R(b',k',y')\int_{x_f(b',k',y)}^{x_d(b',k',y)}\phi(x')dx' + \{\tau r b' + V(b',k',y')\}\int_{x_d(b',k',y')}^{\overline{x}}\phi(x')dx'\right].$$

This program coincides with the one in the right-hand side of (29), except for the value function. However, because current leverage is identically zero for $\xi=1$, equity and firm value coincide, $V \equiv F$, and the programs yield recursively the same optimal value and policy. Hence, the fixed point for this program is the same as the one for (29) with $\xi=1$.

[26]This is equivalent to adjusting the debt from b' to b, with proceeds $(b-b')p/k$.

To show that also investment is firm-value maximizing, from (29) and following the same steps as in Lemma 6, with $V(y)$ in place of $F(y)$, we have the outer investment program

$$V(y) = \max_{\kappa} -[\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 + \kappa\beta v(y), \tag{32}$$

the inner leverage program

$$v(y) = \max_{\ell'} \mathbb{E}_y \left[\int_{x_f(\ell',y')}^{\overline{x}} (1-\tau)(y'+x')\phi(x')dx' \right. \\ \left. + R(\ell',y') \int_{x_f(\ell',y')}^{x_d(\ell',y')} \phi(x')dx' + \{\tau r\ell' + V(y')\} \int_{\underline{x}}^{x_d(\ell',y')} \phi(x')dx' \right],$$

and the corresponding debt price

$$p(y) = \beta\mathbb{E}_y \left[(1+r) \int_{x_d(\ell',y')}^{\overline{x}} \phi(x')dx' \right. \\ \left. + \int_{x_f(\ell',y')}^{x_d(\ell',y')} \frac{(1-\tau)(y'+x') + R(\ell',y')}{\ell'} \phi(x')dx' \right],$$

where $\ell' = g(y)$ is the optimal leverage from the leverage program, and $\kappa = h(y)$ the optimal investment from (32). Given the independence of $v(y)$ and $V(y)$ from current leverage, $\ell' = g(y)$ and $\kappa = h(y)$ are independent from ℓ.

3. (a) From (6), we have[27]

$$\partial_1 V(\ell,y) = \kappa^* \frac{1}{\kappa^*} \partial_1 v(\ell/\kappa^*, y) = \partial_1 v(\ell/\kappa^*, y) = -(1-\xi)p(\ell',y), \tag{33}$$

where κ^* is the optimal solution of the program in (6), and where the second equation follows from (7). Because $p(\ell',y)$ is positive and $\xi < 1$, then both v and V are decreasing in their first argument. To show that $p(\cdot,y)$ is decreasing,

$$\partial_1 p(\ell,y) = \beta\mathbb{E}_y \left[(1-\xi)\partial_1 p(\ell,y')\partial_1 g(\ell,y') \int_{x_d(\ell,y')}^{\overline{x}} \phi(x')dx' \right. \\ \left. - \left[r + \xi + (1-\xi)p(\ell',y')\right] \phi(x_d(\ell,y'))\partial_1 x_d(\ell,y') \right]. \tag{34}$$

[27] The notation $\partial_2 f(\hat{x},\hat{y})$ denotes the first partial derivative of $f(x,y)$ with respect to y, evaluated at $(\hat{x},\hat{y})$. Similarly, $\partial_1^2 f(\hat{x},\hat{y})$ denotes the second partial derivative with respect to x, evaluated at the same point.

Under the assumption that v is twice differentiable, and denoting for convenience $\tilde{\ell} = \ell/\kappa$, we have that $\partial_1^2 v(\tilde{\ell}, y) = -(1-\xi)\partial_1 p(\ell', y)\partial_1 g(\tilde{\ell}, y)$ is positive because $v(\cdot, y)$ is convex and $\xi < 1$. Hence, $\partial_1 p \, \partial_1 g < 0$ in (34). Also, using (33),

$$\partial_1 x_d(\ell, y) = r + \frac{\xi - \partial_1 V(\ell, y)}{1-\tau} = r + \frac{\xi + (1-\xi)p(\ell, y)}{1-\tau},$$

which is positive. Hence, the right-hand side of (34) is negative.

(b) The first-order condition of (7) with $\pi = 0$ is

$$p(\ell', y) + \left[\ell' - (1-\xi)\tilde{\ell}\right] \partial_1 p(\ell', y) + \beta \mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \left\{ -(1-\tau)r + \xi + (1-\xi)\partial_1 V(\ell', y') \right\} \phi(x') dx' \right] = 0,$$

where again $\tilde{\ell} = \ell/\kappa$. Using (33), the first-order condition becomes

$$\left[\ell' - (1-\xi)\tilde{\ell}\right] \partial_1 p(\ell', y) + \beta \mathbb{E}_y \left[\tau r \int_{x_d(\ell', y')}^{\overline{x}} \phi(x') dx' + \int_{x_f(\ell', y')}^{x_d(\ell', y')} \frac{(1-\tau)(y'+x') + R(\ell', y')}{\ell'} \phi(x') dx' \right] = 0.$$

and $\ell' = g(\tilde{\ell}, y)$ is its unique solution under the assumption of concavity of the objective function. Denoting by $\Gamma(\tilde{\ell}, \ell')$ the left-hand side of the equation above, and using the implicit function theorem, the derivative of $g(\tilde{\ell}, y)$ is $\partial_1 g = -\partial_1 \Gamma / \partial_2 \Gamma$. Because for $\xi < 1$ we have $\partial_1 \Gamma = -(1-\xi)\partial_1 p(\ell', y) > 0$, then the sign of $\partial_1 g$ depends on the sign of $\partial_2 \Gamma$. As the objective function in (7) is concave, the latter is negative, and hence $\partial_1 g > 0$.[28]

(c) The derivation of (11) is straightforward using equation (33) and observing that, based on (7), we have

$$v(\ell/\kappa, y) + (1-\xi)\frac{\ell}{\kappa} p(\ell', y) = \beta \mathcal{V}(\ell', y) + \ell' p(\ell', y).$$

[28]If there is zero recovery at default, the partial derivative of Γ with respect to ℓ' becomes

$$\partial_2 \Gamma = \partial_1 p(\ell', y) + \left[\ell' - (1-\xi)\tilde{\ell}\right] \partial_1^2 p(\ell', y) - \beta \tau r \mathbb{E}_y \left[\partial_1 x_d \phi(x_d)\right],$$

which is negative if $\partial_1^2 p$, the second partial derivative of p with respect to ℓ, is negative.

To determine the sign of $\partial_1 h$, we use the implicit function theorem again, although based on the first-order condition for optimal investment in (11), which gives

$$\partial_1 h = \frac{\ell/\kappa^2 \partial_1^2 v\left(\ell/\kappa, y\right)}{\ell^2/\kappa^3 \partial_1^2 v\left(\ell/\kappa, y\right) - \varphi}.$$

Under the assumption that the objective function in (6) is concave, the denominator of the ratio is negative. Because v is convex, $\partial_1^2 v > 0$, the denominator is negative if and only if φ is large enough. For the same reason, the numerator of the ratio is positive. Hence, $\partial_1 h < 0$, that is investment decreases with leverage. □

F Proof of Proposition 3

If $x_d < \underline{x}$ for all (b, k, y'), then from (3) we have

$$p(b,k,y) = \beta \mathbb{E}_y \left[r + \xi + (1-\xi) p(b',k',y')\right]. \tag{35}$$

Because $r = 1/\beta - 1$, by conjecturing the solution $p(b', k', y') = 1$ on the right-hand side, then the debt price is $p(b, k, y) = 1$, which confirms this is a solution. It is easy to show this is the only solution of (35). Under the no-default assumption, the right-hand side of (1) becomes

$$\max_{(b',k')} \; -\left[k' - (1-\delta)k\right] + \tau\delta k - \Psi(k,k') - (1-\xi)b$$
$$+ \beta \mathbb{E}_y \left[(1-\tau)y'k' + \tau r b' + (1-\xi)b' + V(b',k',y')\right] =$$
$$-(1-\xi)b + \max_{(b',k')} \; -\left[k' - (1-\delta)k\right] + \tau\delta k - \Psi(k,k') + \beta \mathbb{E}_y \left[(1-\tau)y'k' + \tau r b' + F(b',k',y')\right],$$

where we took out of the objective function the constant term, $-(1-\xi)b$, which does not affect the optimal policy. Hence, the optimal policy, (b', k'), maximizes the value of the firm (for the no-default case). The equity value resulting from this program, $V(b, k, y)$, plus $(1-\xi)b$, gives $F(b, k, y)$. Hence, solving the fixed point of the above program gives the optimal firm value and the firm value maximizing investment and debt policies, as shown in Lemma 5. □

G Proof of Proposition 4

We first derive the following lemma. We focus on the case with no financial market freezes ($\pi = 0$).

Lemma. For all $n \geq 0$, $p^j_{n+1} \geq p^j_n$, where $p^j_0 = p$, for $j = d, a$.

Proof. The proof is by induction. We first consider the case $j = d$. For $n = 0$, the statement is $p^d_1(b,k,y) \geq p(b,k,y)$, which is true because in equation (3) in default $(x' < x_d(b,k,y'))$

$$(1-\tau)(y'+x')k + R(b,k,y') \leq (r+\xi)b.$$

Next, assuming the inequality is true for a given n, that is $p^d_n(b',k',y') \geq p^d_{n-1}(b',k',y')$, because the firm's policy is invariant with respect to n and using equation (16), from this inequality we have $p^d_{n+1}(b,k,y) \geq p^d_n(b,k,y)$, which proves the lemma.

We then consider the case $j = a$. For $n = 0$ the statement is $p^a_1(b,k,y) \geq p(b,k,y)$. We first notice that $p^a_1(b,k,y) \geq p^d_1(b,k,y)$ because using (13) and (18) this is equivalent to $\max\{p(b',k',y'), p(b,k,y')\} \geq p(b',k',y')$, which is obviously true. We have proved above that $p^d_1(b,k,y) \geq p(b,k,y)$, from which we conclude $p^a_1(b,k,y) \geq p(b,k,y)$. Next, assuming the statement is true for given n, that is $p^a_n \geq p^a_{n-1}$, this implies

$$\max\{p^a_n(b',k',y'), p^a_n(b,k,y')\} \geq \max\{p^a_{n-1}(b',k',y'), p^a_{n-1}(b,k,y')\}.$$

Because the firm's policy is invariant with respect to n and using (18), from the above inequality we have $p^a_{n+1}(b,k,y) \geq p^a_n(b,k,y)$, which proves the lemma. □

1. The proof is by induction. As for the non-negativity of the agency spread, for the case $H = 1$ the statement $as_1(b',k',y) \geq 0$ is equivalent to $p^a_1(b',k',y) \geq p^d_1(b',k',y)$. Given the specification of the price of the debt without default in (13) and without default and agency conflicts in (14), $p^a_1(b,k,y) \geq p^d_1(b,k,y)$ is equivalent to $\max\{p(b',k',y'), p(b,k,y)\} \geq p(b',k',y')$, which is obviously true. As for the induction step, assuming the statement $p^a_{H-1}(b',k',y') \geq p^d_{H-1}(b',k',y')$ is true for $H-1$, for the case $H > 1$, $as_H(b',k',y) \geq 0$ is equivalent to $p^a_H(b',k',y') \geq p^d_H(b',k',y')$. Using the same argument as above, comparing (18) to (16) we have

   $$\max\{p^a_{H-1}(b',k',y'), p^a_{H-1}(b,k,y')\} \geq p^a_{H-1}(b',k',y') \geq p^d_{H-1}(b',k',y'),$$

   which is assumed true. Hence the agency spread is always non-negative.

   To prove that $ds_H \geq 0$, which is equivalent to $p^d_H \geq p$, for all $H \geq 1$ we use the lemma above, and note that $p^d_H \geq p^d_{H-1} \geq \ldots \geq p$.

2. The statement $cs_{H-1} \leq cs_H$ is equivalent to $p^a_H \geq p^a_{H-1}$, which has been proved in the lemma above. The statement $cs \geq cs_H$ is equivalent to $p^a_H \leq 1$, which is true for all $H \geq 1$ as p^a_H is bounded by the risk-free price of \$1. Also, $cs_H \to cs$ for $H \to \infty$, because the sequence $\{p^a_H\}$ is increasing and bounded above, hence it has limit and $p^a_H \to 1$, which concludes the proof.

3. To prove that $ds_H \to cs$ for $H \to \infty$, we observe that $ds_H \leq cs_H \leq cs$ for all H. Also, $ds_H \leq cs$ is equivalent to $p_H^d \leq 1$ for all H. As before, the sequence $\{p_H^d\}$ is increasing and bounded above by 1, hence $p_H^d \to 1$.

   Point (1) above states that $as_H \geq 0$, which is equivalent to $p_H^a \geq p_H^d$, for all H. Because $p_H^d \to 1$ and $p_H^a \to 1$, this proves that $as_H \to 0$. □

H Numerical algorithm

The algorithm to solve the general model is based on a discretized version of the leverage interval, $[0, \overline{\ell}]$. Although the numerical results are presented for $\ell \in [0, 1]$, we solve the model for $\overline{\ell} > 1$ to avoid the effect of the upper bound on the numerical solution. We discretize the process for y with seven points the compact interval $\left[-3\sigma/\sqrt{1-\nu^2}, +3\sigma/\sqrt{1-\nu^2}\right]$ using Gauss-Hermite quadrature. Therefore, we solve the program (7)-(9) using a value iteration algorithm on a discrete space for (ℓ, y). As in Chatterjee and Eyigungor (2012), existence of a solution (and therefore convergence of the iterative procedure) of such a model in the presence of long-term debt is guaranteed by the addition to our model of an i.i.d. shock x with continuous cumulative probability function. As for the distribution of x, we set $[\underline{x}, \overline{x}] = [-X, X]$, with $X > 0$, and use the specification

$$\phi(x) = \frac{3}{4X}\left[1 - \left(\frac{x}{X}\right)^2\right].$$

With this specification of ϕ, the integrals involved in the calculations of the iterative algorithm have the following analytic expressions, which improve efficiency of the algorithm:

$$\int_{x_d}^{\overline{x}} \phi(x)dx = \frac{1}{2} - \frac{3x_d}{4X} + \frac{x_d^3}{4X^3}, \qquad \int_{x_d}^{\overline{x}} x\phi(x)dx = \frac{3X}{16} - \frac{3x_d^2}{8X} + \frac{3x_d^4}{16X^3},$$

$$\int_{\underline{x}}^{x_d} \phi(x)dx = \frac{1}{2} + \frac{3x_d}{4X} - \frac{x_d^3}{4X^3}, \qquad \int_{\underline{x}}^{x_d} x\phi(x)dx = -\frac{3X}{16} + \frac{3x_d^2}{8X} - \frac{3x_d^4}{16X^3},$$

$$\int_{x_d}^{x_d^0} \phi(x)dx = \frac{1}{4X^3}\left((x_d^0)^3 - x_d^3\right) - \frac{3}{4X}\left(x_d^0 - x_d\right),$$

$$\int_{x_d}^{x_d^0} x\phi(x)dx = \frac{3}{16X^3}\left((x_d^0)^4 - x_d^4\right) - \frac{3}{8X}\left((x_d^0)^2 - x_d^2\right).$$

The optimization in (7) with respect to ℓ' is done with an exhaustive search on a grid of 7500 points in $[0, \overline{\ell}]$. The tolerance set for maximum error in two successive iterations of the value function in (7) and of the debt price in (9) is 10^{-5}.[29]

To solve the program in (6) and determine the optimal investment policy, we use a cubic spline interpolation of $v(\ell/\kappa, y)$ with 100 points. As highlighted in Appendix B, to ensure existence of a solution to the stationary model (7)-(6), on the one hand a solution to (7)-(9) has to exist as discussed above, and on the other a solution to (6) must exist. This requires further restriction of model parameters, as shown in (25).

To obtain a simulated economy, we simulate 1000 firms starting from random initial states, for 200 quarters, and drop the first 50 quartes. We then calculate sample averages, as time series averages of cross-sectional averages in the simulated economy.

I Data construction

We collect data from multiple sources. Firm-level accounting and financial data are from the merged CRSP-COMPUSTAT database. To match the model structure, we scale variables, when appropriate, by the sum of the book value of equity and total financial debt (i.e., short-term liabilities plus long-term debt), instead of total assets.

To reduce issues stemming from differences in bond contract terms, tax treatment, and market liquidity, we focus on credit spreads from senior unsecured credit default swap (CDS) contracts. Specifically, we use one and five-year CDS spreads for non-financial firms in the S&P 500 index as our main credit spread measure. Availability of CDS prices restricts the sample to the period of time between 2001 and 2014. The final dataset includes 450 distinct firms and 12,718 firm-quarter observations.

In our sample of firms there are only 9 default events that can be identified by merging data from Moody's KMV, Bloomberg, Standard & Poor's, and FISD Mergent. These events include Chapter 7 and Chapter 11 filings, as well as missed payments on interest or principal, across both bank loans and publicly issued debt.

To avoid the measurement errors implied by basing an estimate on such a particular sample, we use the historical default frequency for BBB-rated firms computed by Moody's. In the most recent default study available (2023), Moody's report a default probability of 0.20% from a very long sample that spans the years between 1920 and 2022.

[29]Notably, Chatterjee and Eyigungor (2012) states that the algorithm fails if convergence is not achieved with tolerance of 10^{-5} in less than 3000 steps. In our numerical experiments, we always achieve convergence in less than 500 steps.

J Discussion on the proposed approach to decompose the credit spread in the agency vs the default component

The proposed decomposition of the credit spread into a default component and an agency component is motivated by the purpose of separating the affect of the two commitment problems, to repay the debt and to future debt and investment policies, which are both controlled by the same maturity parameter, ξ. Also, the decomposition is for a *given* investment and financing policy in the MPE. We argue that any alternative decomposition approach to the one introduced above would entail an interaction between the policy effects and the price effects of the agency issues, which would cause a deviation from the equilibrium policy and ultimately would violate the assumed equilibrium. Hence, the resulting separation in two components would depend on the extent of such an interaction and therefore would be arbitrary.

This can be better understood by thinking of two (seemingly plausible) decomposition approaches. In the first approach, the effect of agency would be neutralized because the value losses from dilution are 'rebated' to legacy bond holders by shareholders. Alternatively, the 'rebate' may be paid by an external party, who would need to be financed by the shareholders anyway. Because of this provision, the shareholders' policies would change relative to the equilibrium ones.

A second approach is to assume the shareholders are subject to a 'blanket' protective covenant that precludes them from taking any actions that lowers bond values at the respective horizons. While such a covenant would eliminate the effects of agency on the continuation value on the debt, it would also constrain the shareholders' future decisions, affecting their policy.[30] For both approaches, we would then define the agency spread as the difference between the credit spread obtained in the benchmark model and the one resulting from the approach. Either way, the resulting decomposition into an agency and a default spread would not separate the pure default component from the agency component under the MPE, exactly because this definition entails a deviation from that equilibrium.

Our decomposition of the credit spread naturally follows by applying the same approach used to define the default spread to the case of a debt price reduction due to

[30] Also Jungherr and Schott (2021) quantify the cost of corporate debt due to lack of commitment. Differently from our decomposition of the equilibrium credit spread, they calculate this cost in a counterfactual experiment, comparing the credit spread in an economy without commitment to the credit spread in an economy with commitment. This latter case is analogous to the second approach in the text, based on the elimination of the commitment problem. While this may give an idea of the increased cost of capital, in equilibrium the average leverage is lower under commitment, which in part drives the difference of credit spread. Our approach, by using the MPE policies and prices, overcomes this issue.

agency conflicts. Given the equilibrium policy, there is no other way to define the credit spread but $(r+\xi)/p-(r+\xi)/p^d$ (or $(r+\xi)/p-(r+\xi)/p^d_H$ for finite horizons), because the yield $(r+\xi)/p^d$ is for a debt which is identical to the one in the baseline model, except for excluding default. Notably, to define the latter debt we do not need to assume that the debt holders get a 'rebate' or there is a protective covenant (or collateral) that eliminates default risk. In fact, if the debt contract had this provision, we would be calculating the credit spread for a policy different from the one in the MPE.

Finally, the definition in (19) provides a *on-the-equilibrium-path* decomposition of the credit spread into a default component and an agency component exactly because we do not allow the equilibrium policy to be affected. An alternative definition, which entails a deviation from the equilibrium policy, would be off the equilibrium path at any state of the baseline model.