Online Appendix to Monetary Policy Interactions: The Policy Rate, Asset Purchases and Optimal Policy with an Interest Rate Peg

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ONLINE APPENDIX

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A Summary of Federal Reserve and ECB Balance Sheet Policies: Objectives and Timing

A.1 Federal Reserve

Date	Statement	Objective§	Policy Rate Level	Source
11/25/2008	"The Federal Reserve announced . . . a program to purchase the direct obligations of housing-related government-sponsored enterprises (GSEs) . . . and mortgage-backed securities (MBS) . . . Spreads of rates on GSE debt and on GSE-guaranteed mortgages have widened appreciably of late. This action is being taken to reduce the cost and increase the availability of credit for the purchase of houses, which in turn should support housing markets and foster improved conditions in financial markets more generally. . . ."	Financial Market	1.000	Press Release
12/01/2008	"The second arrow in the Federal Reserve's quiver–the provision of liquidity–remains effective. Indeed, there are several means by which the Fed could influence financial conditions through the use of its balance sheet [:] . . . First, the Fed could purchase longer-term Treasury or agency securities on the open market in substantial quantities. . . . Second, the Federal Reserve can provide backstop liquidity not only to financial institutions but also directly to certain financial markets, as we have recently done for the commercial paper market. . . . Each of these approaches has the potential to improve the functioning of financial markets and to stimulate the economy. . . ."	Financial Market	1.000	Chair Speech

Date	Statement	Objective§	Policy Rate Level	Source
3/18/2009	"The Federal Reserve will employ all available tools to promote economic recovery and to preserve price stability. . . . To provide greater support to mortgage lending and housing markets, the Committee decided today to increase the size of the Federal Reserve's balance sheet further by purchasing up to an additional $750 billion of agency mortgage-backed securities . . . and to increase its purchases of agency debt this year by up to $100 billion . . . Moreover, to help improve conditions in private credit markets, the Committee decided to purchase up to $300 billion of longer-term Treasury securities over the next six months. . . ."	Financial Market, Inflation	0.125	Press Release
11/03/2010	"Consistent with its statutory mandate, the Committee seeks to foster maximum employment and price stability. . . . To promote a stronger pace of economic recovery and to help ensure that inflation, over time, is at levels consistent with its mandate, the Committee decided today to expand its holdings of securities. The Committee will maintain its existing policy of reinvesting principal payments from its securities holdings. In addition, the Committee intends to purchase a further $600 billion of longer-term Treasury securities by the end of the second quarter of 2011, a pace of about $75 billion per month. . . ."	Inflation, Unemployment	0.125	Press Release
9/13/2012	"To support a stronger economic recovery and to help ensure that inflation, over time, is at the rate most consistent with its dual mandate, the Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. . . . These actions . . . should put downward pressure on longer-term interest rates, support mortgage markets, and help to make broader financial conditions more accommodative. . . ."	Financial Market, Inflation, Unemployment	0.125	Press Release

Date	Statement	Objective§	Policy Rate Level	Source
12/12/2012	"To support a stronger economic recovery and to help ensure that inflation, over time, is at the rate most consistent with its dual mandate, the Committee will continue purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will purchase longer-term Treasury securities after its program to extend the average maturity of its holdings of Treasury securities is completed at the end of the year, initially at a pace of $45 billion per month. . . . Taken together, these actions should maintain downward pressure on longer-term interest rates, support mortgage markets, and help to make broader financial conditions more accommodative. . . ."	Financial Market, Inflation, Unemployment	0.125	Press Release
9/17/2019	"The Federal Reserve Bank of New York will conduct an overnight repurchase agreement (repo) operation . . . to help maintain the federal funds rate within the target range of 2 to 2-1/4 percent. . . . Securities eligible as collateral in the repo include Treasury, agency debt, and agency mortgage-backed securities. . . ."	Financial Market	2.125	Desk Statement
3/23/2020	"The Federal Reserve will continue to purchase Treasury securities and agency mortgage-backed securities in the amounts needed to support smooth market functioning and effective transmission of monetary policy to broader financial conditions. . . ."	Financial Market	0.125	Press Release

§ For the Fed, the policy rate level is the federal funds target rate through 2008, after which it adopted a target range for the federal funds rate, [DFEDTARL, DFEDTARU], and the level reported here is the midpoint of this range.

A.2 European Central Bank

Date	Statement	Objective§	Policy Rate Level	Source
5/05/2011	"The non-standard measures have a clear purpose: ensuring that the standard measures themselves are transmitted as effectively as possible despite the otherwise abnormal functioning of some markets. . . . I refer to this duality as the "separation principle". The non-standard measures have to be commensurate with what we are observing on the market, namely to help the transmission of our monetary policy to function better again. The standard measures are there to deliver price stability in the medium term. . . ."	Financial Market	1.250	President Speech
2/05/2015	"At its meeting on 22 January 2015 the Governing Council of the ECB decided to launch an expanded asset purchase programme (APP) . . . Under this expanded programme, the combined monthly purchases of public and private sector securities will amount to €60 billion. . . . and will . . . be conducted until the Governing Council sees a sustained adjustment in the path of inflation . . . At the lower bound for policy interest rates, the adoption of further quantitative measures that can expand the size and change the composition of the Eurosystem's balance sheet constitutes the only effective tool to provide further monetary policy accommodation. . . ." (pp. 15-18)	Financial Market, Inflation	0.050	Economic Bulletin
9/09/2015	"The ECB's expanded asset purchase programme . . . has strongly signalled the ECB's commitment to deliver its medium-term price stability objective, which has in turn been reflected in an upward shift in inflation expectations at all horizons. . . . The expanded APP, together with our credit easing package launched in summer 2014, has also had a positive impact on both market and bank credit dynamics. . . . The credit easing package and expanded APP has been effective too in reducing wholesale funding costs for banks, allowing banks to pass on better funding conditions to their customers. . . ."	Financial Market, Inflation	0.050	Executive Board Member Speech

Date	Statement	Objective§	Policy Rate Level	Source
11/20/2015	"The ECB's monetary policy measures . . . have been instrumental in arresting and reversing the deflationary pressures that hit the euro a year ago. . . . For asset purchases to boost activity and inflation . . . improvements in financial markets need to be passed through into credit conditions for the real economy. . . . [T]he power of transmission through the banking system has been rising through the life of our programme. . . ."	Financial Market, Inflation	0.050	President Speech
12/11/2015	"Since June 2014 the ECB has adopted a series of new monetary policy measures, with the aim to both enhance the transmission of policy and to reinforce the accommodative policy stance to counter growing risks of a too prolonged period of low inflation. Those measures have included reducing key policy rates to levels below zero, introducing a credit easing package – specifically our Targeted Long-Term Refinancing Operations (TLTROs) – and expanding our Asset Purchase Programme (APP). They were decided against the backdrop of [weak] credit dynamics . . . , heightened downside risks to the inflation outlook and a concrete threat to the stability of inflation expectations. . . ."	Financial Market, Inflation	0.050	Executive Board Member Speech
3/18/2020	"The Governing Council decided the following: (1) To launch a new temporary asset purchase programme of private and public sector securities to counter the serious risks to the monetary policy transmission mechanism and the outlook for the euro area posed by the outbreak and escalating diffusion of the coronavirus, COVID-19. This new Pandemic Emergency Purchase Programme (PEPP) will have an overall envelope of €750 billion. . . ."	Financial Market	0.000	Press Release

§ For the ECB, the reported policy rate is the main refinancing operations interest rate.

B Equilibrium Definitions

The nonlinear equilibrium system is given below with 20 endogenous variables, $\{C_p, C_b, N, \Omega, \Lambda^N, Y, RE, \Xi, \mathrm{MC}, \Pi, \Pi_{\#}, G, H, \Delta, Q, R^L, B, \Phi, Y_*, Gap\}$, 2 exogenous variables, $\{A, \Theta\}$ and a need to specify two policy strategies for the policy rate, R_t, and the size of the central bank balance sheet, RE_t.

$$\Xi_t = \Gamma \frac{C_{p,t}}{C_{b,t}} \tag{NL.1}$$

$$\mathrm{MC}_t A_t = \psi \left((1-\Omega_t)N_t\right)^\eta C_{p,t} \tag{NL.2}$$

$$\Xi_t = \Gamma \left(\frac{\Omega_t}{1-\Omega_t}\right)^\eta \tag{NL.3}$$

$$\Lambda^N_{t-1,t} = \beta \frac{C_{p,t-1}}{C_{p,t}} \Pi_t^{-1} \tag{NL.4}$$

$$1 = \mathbb{E}_t \Lambda^N_{t,t+1} R_t \tag{NL.5}$$

$$1 = \zeta \mathbb{E}_t \Lambda^N_{t,t+1} \frac{\Xi_{t+1}}{\Xi_t} R^L_{t+1} \tag{NL.6}$$

$$C_{b,t} = \Omega_t \mathrm{MC}_t A_t N_t + Q_t B_t - \frac{R^L_t}{\Pi_t} Q_{t-1} B_{t-1} \tag{NL.7}$$

$$R^L_t = \frac{1+\kappa Q_t}{Q_{t-1}} \tag{NL.8}$$

$$1 = \Phi_t \left[\Theta_t - \mathbb{E}_t \Lambda^N_{t,t+1} \left(R^L_{t+1} - R_t\right)\right] \tag{NL.9}$$

$$Q_t B_t = \Phi_t X^s + RE_t \tag{NL.10}$$

$$\Pi_{\#,t} = \frac{\varepsilon}{\varepsilon-1} \Pi_t \frac{G_t}{H_t} \tag{NL.11}$$

$$G_t = C_{p,t}^{-1} \mathrm{MC}_t Y_t + \beta \phi_p \mathbb{E}_t \Pi^\varepsilon_{t+1} G_{t+1} \tag{NL.12}$$

$$H_t = C_{p,t}^{-1} Y_t + \beta \phi_p \mathbb{E}_t \Pi^{\varepsilon-1}_{t+1} H_{t+1} \tag{NL.13}$$

$$\Pi_t^{1-\varepsilon} = (1-\phi_p)\Pi_{\#,t}^{1-\varepsilon} + \phi_p \tag{NL.14}$$

$$\Delta_t = (1-\phi_p)\left(\frac{\Pi_t}{\Pi_{\#,t}}\right)^\varepsilon + \phi_p \Pi_t^\varepsilon \Delta_{t-1} \tag{NL.15}$$

$$Y_t \Delta_t = A_t N_t \tag{NL.16}$$

$$Y_t = C_{p,t} + C_{b,t} \tag{NL.17}$$

$$QE_t = \frac{RE_t}{Y_t} \tag{NL.18}$$

$$Y^*_t = \left[\frac{1}{\psi} \frac{1+\Gamma}{(1-\Omega)^\eta} \frac{\varepsilon-1}{\varepsilon}\right]^{\frac{1}{1+\eta}} A_t \tag{NL.19}$$

(NL.20) $$GAP_t = \frac{Y_t}{Y_t^*}\frac{Y^*}{Y}$$

(NL.21) $$\ln A_t = (1-\rho_a)\ln A + \rho_a \ln A_{t-1} + \sigma_a \epsilon_t^a$$

(NL.22) $$\ln \Theta_t = (1-\rho_\theta)\ln \Theta + \rho_\theta \ln \Theta_{t-1} + \sigma_\theta \epsilon_t^\theta$$

Steady state calculations: Given values for $\{\beta, \eta, \phi_p, \varepsilon\}$, and the calibration targets: $R^L/R = 1.015^{0.25}$; $\Pi = 1$; $QE = 0.05$; $Y = 1$; $N = 1$; $(1-\kappa)^{-1} = 40$; $Qb/Y = 14$; $\Phi = 4$; $\Gamma = 1$; solve the following:

$$R = \frac{\Pi}{\beta} \leftarrow \text{implies } R^L$$

$$Q = \frac{1}{R^L - \kappa}$$

$$\zeta = \frac{R}{R^L}$$

$$\Pi_\# = \left(\frac{\Pi^{1-\varepsilon} - \phi_p}{1-\phi_p}\right)^{\frac{1}{1-\varepsilon}}$$

$$\Delta = \frac{\Pi^\varepsilon}{\Pi_\#^\varepsilon}\frac{1-\phi_p}{1-\phi_p\overline{\Pi}^\varepsilon}$$

$$A = \frac{Y\Delta}{N}$$

$$\text{MC} = \frac{\varepsilon-1}{\varepsilon}\frac{\Pi_\#}{\Pi}\frac{\left(1-\beta\phi_p\Pi^\varepsilon\right)}{\left(1-\beta\phi_p\Pi^{\varepsilon-1}\right)}$$

jointly solve for C_b and Ω,

$$C_b = \left[1 + \left(\frac{\Omega}{1-\Omega}\right)^\eta\right]^{-1}$$

$$\frac{Y - C_b}{C_b} = \left(\frac{\Omega}{1-\Omega}\right)^\eta$$

$$G = \frac{\text{MC}Y}{C_p\left(1-\beta\phi_p\Pi^\varepsilon\right)}$$

$$H = \frac{Y}{C_p\left(1-\beta\phi\overline{\Pi}^{\varepsilon-1}\right)}$$

$$B^{FI} = B - \frac{QE}{Q}Y$$

$$\psi = \frac{\text{MC}}{\left((1-\Omega)N\right)^\eta C_p}$$

$$\Xi = \frac{C_p}{C_b}$$

$$Y^* = \left[\frac{1}{\psi}\frac{1+\Gamma}{(1-\Omega)^\eta}\frac{\varepsilon-1}{\varepsilon}\right]^{\frac{1}{1+\eta}} A$$

$$\Theta = \Phi^{-1} + \frac{1-\zeta}{\zeta}$$

$$X^s = \frac{QB-RE}{\Phi}$$

B.1 Log-linear Equilibrium System

Let $c_t = \ln C_t - \ln C$. Note, for the interest and inflation rates, we make use of the approximation $\ln R_t - \ln R \approx R_t - 1 - (R-1) = R_t - R$ allowing us to interpret these variables as net rate deviations from steady state.

(L.1) $$\xi_t = c_{p,t} - c_{b,t}$$

(L.2) $$\mathrm{mc}_t + a_t = \eta n_t + c_{p,t} - \eta\frac{\Omega}{1-\Omega}\omega_t$$

(L.3) $$\xi_t = \frac{\eta}{1-\Omega}\omega_t$$

(L.4) $$\lambda^N_{t-1,t} = c_{p,t-1} - c_{p,t} - \pi_t$$

(L.5) $$0 = \mathbb{E}_t\lambda^N_{t,t+1} + r_t$$

(L.6) $$0 = \mathbb{E}_t\xi_{t+1} - \xi_t + \mathbb{E}_t r^L_{t+1} - r_t$$

(L.7) $$\overline{C}_b c_{b,t} = \Omega \mathrm{MC}\Delta\left(\omega_t + \mathrm{mc}_t + a_t + n_t\right) + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + r^L_t - \pi_t\right)\right)$$

(L.8) $$r^L_t = \frac{\kappa\beta\zeta}{\Pi}q_t - q_{t-1}$$

(L.9) $$\phi_t = \frac{\Phi}{\zeta}\left(\mathbb{E}_t r^L_{t+1} - r_t\right) - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t$$

(L.10) $$q_t + b_t = \left(1 - \overline{RE}\right)\phi_t + \overline{RE}re_t$$

(L.11) $$\pi_{\#,t} = \pi_t + g_t - h_t$$

(L.12) $$g_t = \left(1 - \beta\phi_p\Pi^\varepsilon\right)\left(\mathrm{mc}_t + y_t - c_{p,t}\right) + \beta\phi_p\Pi^\varepsilon\left(\varepsilon\mathbb{E}_t\pi_{t+1} + \mathbb{E}_t g_{t+1}\right)$$

(L.13) $$h_t = \left(1 - \beta\phi_p\Pi^{\varepsilon-1}\right)\left(y_t - c_{p,t}\right) + \beta\phi_p\Pi^{\varepsilon-1}\left((\varepsilon-1)\mathbb{E}_t\pi_{t+1} + \mathbb{E}_t h_{t+1}\right)$$

(L.14) $$\pi_t = (1-\phi_p)\left(\frac{\Pi}{\Pi_*}\right)^{\varepsilon-1}\pi_{*,t}$$

(L.15) $$\delta_t = \varepsilon\pi_t - \varepsilon\left(1 - \phi_p\Pi^\varepsilon\right)\pi_{*,t} + \phi_p\Pi^\varepsilon\delta_{t-1}$$

(L.16) $$y_t + \delta_t = a_t + n_t$$

(L.17) $$y_t = \left(1 - \overline{C}_b\right)c_{p,t} + \overline{C}_b c_{b,t}$$

(L.18) $qe_t = re_t - y_t$

(L.19) $y_t^* = a_t$

(L.20) $gap_t = y_t - y_t^*$

(L.21) $a_t = \rho_a a_{t-1} + \sigma_a \epsilon_t^a$

(L.22) $\theta_t = \rho_\theta \theta_{t-1} + \sigma_\theta \epsilon_t^\theta$

where $\overline{C}_b$ is the steady state debt-financed expenditure share of aggregate expenditures and $\overline{RE}$ is the steady state share of long-term debt held by the central bank.

Consider the above log-linear approximation of the model around a zero net inflation steady-state. The structural system consists of IS and Phillips curves, augmented by the interest rate spread and liquidity premium where the liquidity premium is equal to the forward-looking path of the interest rate spread:

$$\begin{aligned}
\pi_t &= \gamma gap_t + \beta \mathbb{E}_t \pi_{t+1} + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \\
gap_t &= \mathbb{E}_t gap_{t+1} - \left(\left(1-\overline{C}_b\right) r_t + \overline{C}_b \mathbb{E}_t r_{t+1}^L - \mathbb{E}_t \pi_{t+1} - r_t^n\right) \\
r_t^n &= -(1-\rho_a) a_t \\
r_t^L &= \kappa \beta \zeta q_t - q_{t-1} \\
\xi_t &= \mathbb{E}_t \xi_{t+1} + \mathbb{E}_t r_{t+1}^L - r_t
\end{aligned}$$

where $\gamma = {}^{(1-\phi_p)(1-\phi_p\beta)(1+\eta)}/_{\phi_p}$. The model also includes a robust financial block:

$$\begin{aligned}
c_{b,t} &= gap_t + a_t - \left(1-\overline{C}_b\right)\xi_t \\
\overline{C}_b c_{b,t} &= \Omega \frac{\varepsilon - 1}{\varepsilon}\left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\right)\xi_t + (2+\eta) gap_t + a_t\right) \\
&\qquad + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + r_t^L - \pi_t\right)\right) \\
\phi_t &= \frac{\Phi}{\zeta}\left(\mathbb{E}_t r_{t+1}^L - r_t\right) - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t \\
q_t + b_t &= \left(1 - \overline{RE}\right)\phi_t + \overline{RE} re_t
\end{aligned}$$

Where exogenous shocks follow:

$$\begin{aligned}
a_t &= \rho_a a_{t-1} + \sigma_a \epsilon_t^a \\
\theta_t &= \rho_\theta \theta_{t-1} + \sigma_\theta \epsilon_t^\theta
\end{aligned}$$

Simplify further by writing the productivity shock, a_t, in terms of the natural rate.

(LZ.1) $\pi_t = \gamma gap_t + \beta \mathbb{E}_t \pi_{t+1} + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t$

(LZ.2) $gap_t = \mathbb{E}_t gap_{t+1} - \left(\left(1-\overline{C}_b\right) r_t + \overline{C}_b \mathbb{E}_t r_{t+1}^L - \mathbb{E}_t \pi_{t+1} - r_t^n\right)$

$$r_t^L = \kappa\beta\zeta q_t - q_{t-1} \tag{LZ.3}$$

$$\xi_t = \mathbb{E}_t\xi_{t+1} + \mathbb{E}_t r_{t+1}^L - r_t \tag{LZ.4}$$

where $\gamma = {}^{(1-\phi_p)(1-\phi_p\beta)(1+\eta)}/_{\phi_p}$. The model also includes a robust financial block:

$$c_{b,t} = gap_t - \left(1-\overline{C}_b\right)\xi_t - \frac{1}{1-\rho_n}r_t^n \tag{LZ.5}$$

$$\begin{aligned} \overline{C}_b c_{b,t} = \Omega\frac{\varepsilon-1}{\varepsilon}\Bigg(&\left(\frac{1-\Omega}{\eta}+\overline{C}_b-\Omega\right)\xi_t + (2+\eta)gap_t - \frac{1}{1-\rho_n}r_t^n\right) \\ &+\frac{QB}{Y}\left(q_t+b_t-\frac{1}{\beta\zeta}\left(q_{t-1}+b_{t-1}+r_t^L-\pi_t\right)\right)\Bigg) \end{aligned} \tag{LZ.6}$$

$$q_t + b_t = \left(1-\overline{RE}\right)\left[\frac{\Phi}{\zeta}\left(\mathbb{E}_t r_{t+1}^L - r_t\right) - \left(1+\Phi\frac{1-\zeta}{\zeta}\right)\theta_t\right] + \overline{RE}re_t \tag{LZ.7}$$

Where exogenous shocks follow:

$$r_t^n = \rho_n r_{t-1}^n + \sigma_n\epsilon_t^n \tag{LZ.8}$$

$$\theta_t = \rho_\theta\theta_{t-1} + \sigma_\theta\epsilon_t^\theta \tag{LZ.9}$$

Equations (LZ.1) – (LZ.7) correspond to equations (3.14) – (3.20) in the text. To derive equation (3.21), start by writing the output gap in terms of output, y_t:

$$gap_t = y_t - a_t = y_t + \frac{r_t^n}{1-\rho_n}$$

and substitute this from the IS and Phillips curves:

$$\begin{aligned} \pi_t &= \widetilde{\gamma}(1+\eta)\left(y_t+\frac{r_t^n}{1-\rho_n}\right) + \beta\mathbb{E}_t\pi_{t+1} + \widetilde{\gamma}\left(\overline{C}_b-\Omega\right)\xi_t \\ y_t &= \mathbb{E}_t y_{t+1} - \left(\left(1-\overline{C}_b\right)r_t + \overline{C}_b\mathbb{E}_t r_{t+1}^L - \mathbb{E}_t\pi_{t+1}\right) \end{aligned}$$

where $\widetilde{\gamma}$ is the marginal cost semi-elasticity of inflation, $\gamma = \widetilde{\gamma}(1+\eta)$. Iterating forward the IS curve defines output in terms of the forward paths of the ex-ante real short- and long-term rates:

$$y_t = -\mathbb{E}_t\sum_{j=0}^{\infty}\left\{(1-\overline{C}_b)r_{t+j} + \overline{C}_b r_{t+j+1}^L - \pi_{t+j+1}\right\}$$

Rearrange the Phillips curve:

$$y_t = -\frac{r_t^n}{1-\rho_n} + \frac{1}{\eta}\left[-\frac{r_t^n}{1-\rho_n} + \frac{1}{\widetilde{\gamma}}\left(\pi_t - \beta\mathbb{E}_t\pi_{t+1}\right) - y_t + \left(\Omega-\overline{C}_b\right)\xi_t\right]$$

and substitute out the forward-looking definitions of output and the liquidity premium from the right-hand side:

$$y_t = -\frac{r_t^n}{1-\rho_n} + \frac{1}{\eta}\left[-\frac{r_t^n}{1-\rho_n} + \frac{1}{\widetilde{\gamma}}\left(\pi_t - \beta\mathbb{E}_t\pi_{t+1}\right)\right.$$
$$\left. + \mathbb{E}_t\sum_{j=0}^{\infty}\left\{(1-\overline{C}_b)r_{t+j} + \overline{C}_b r^L_{t+j+1} - \pi_{t+j+1}\right\} + \left(\Omega - \overline{C}_b\right)\mathbb{E}_t\sum_{j=0}^{\infty}\left\{r^L_{t+j+1} - r_{t+j}\right\}\right]$$

which simplifies to equation (3.21):

$$y_t = -\frac{r_t^n}{1-\rho_n} + \frac{1}{\eta}\left[-\frac{r_t^n}{1-\rho_n} + \frac{1}{\widetilde{\gamma}}\left(\pi_t - \beta\mathbb{E}_t\pi_{t+1}\right) + \mathbb{E}_t\sum_{j=0}^{\infty}\left\{(1-\Omega)r_{t+j} + \Omega r^L_{t+j+1} - \pi_{t+j+1}\right\}\right]$$

C The Banking Model

Banks indexed by j operate under perfect competition. Banks are financial intermediaries that originate bonds for financing debt-financed expenditures to households and hold reserves with funding from deposits and bank equity. Banks survive each period with probability σ and pay accumulated equity to the household upon exit. Consider a bank balance sheet with nominal private debt, $Q_t\widetilde{B}^{FI}_{jt}$, paying the interest rate R^L_{t+1} in the subsequent period and nominal reserves, $\widetilde{RE}_{jt}$, paying the interest rate R^{re}_t, backed by deposits, $\widetilde{S}_{jt}$, requiring interest payments, R_t, and accumulated bank equity, $\widetilde{X}_{jt}$:[23]

$$Q_t\widetilde{B}^{FI}_{jt} + \widetilde{RE}_{jt} = \widetilde{S}_{jt} + \widetilde{X}_{jt}$$

Implying bank equity accumulation follows:

$$\begin{aligned}\widetilde{X}_{jt+1} &= R^L_{t+1}Q_t\widetilde{B}^{FI}_{jt} + R^{RE}_{jt}\widetilde{RE}_{jt} - R_t\widetilde{S}_{jt}\\ &= \left(R^L_{t+1} - R_t\right)Q_t\widetilde{B}^{FI}_{jt} + \left(R^{re}_t - R_t\right)\widetilde{RE}_{jt} + R_t\widetilde{X}_{jt}\end{aligned}$$

with bank j's value function:

$$\begin{aligned}\widetilde{V}_{jt} &= \mathbb{E}_t\sum_{i=1}^{\infty}(1-\sigma)\sigma^{i-1}\Lambda^N_{t,t+i}\widetilde{X}_{jt+i} = (1-\sigma)\mathbb{E}_t\Lambda^N_{t,t+1}\widetilde{X}_{jt+1} + \sigma\mathbb{E}_t\Lambda^N_{t,t+1}V_{jt+1}\\ &= (1-\sigma)\mathbb{E}_t\Lambda^N_{t,t+1}\left[\left(R^L_{t+1} - R_t\right)Q_t\widetilde{B}^{FI}_{jt} + \left(R^{re}_t - R_t\right)\widetilde{RE}_{jt} + R_t\widetilde{X}_{jt}\right] + \sigma\mathbb{E}_t\Lambda^N_{t,t+1}\widetilde{V}_{jt+1}\end{aligned}$$

In the event of bank default, banks can walk away with 100% of private debt holdings, $Q_t\widetilde{B}^{FI}_t$, with probability Θ_t.[24] To prevent this, depositors impose the following limited enforcement constraint on banks to ensure bank continuation:

$$\text{Expected value of default} = \Theta_t Q_t\widetilde{B}^{FI}_{jt} \leq \widetilde{V}_{jt} = \text{Continuation value}$$

Banks maximize the expected sum of future profits subject to the limited enforcement constraint. A Lagrangian, with $\varkappa_{jt}$ as the multiplier on the limited enforcement constraint, is given by:

$$\begin{aligned}\mathcal{L}_t = &(1+\varkappa_{jt})(1-\sigma)\mathbb{E}_t\Lambda^N_{t,t+1}\left[\left(R^L_{t+1} - R_t\right)Q_t\widetilde{B}^{FI}_{jt} + \left(R^{re}_t - R_t\right)\widetilde{RE}_{jt} + R_t\widetilde{X}_{jt}\right]\\ &+ (1+\varkappa_{jt})\sigma\mathbb{E}_t\Lambda^N_{t,t+1}\widetilde{V}_{jt+1} - \varkappa_{jt}\Theta_t\left(Q_t\widetilde{B}^{FI}_{jt}\right)\end{aligned}$$

23. Variables, Z_t, with a tilde, $\widetilde{Z}_t$, reflect nominal quantities.
24. Reserves are fully recoverable in the case of bank default.

Implying the first-order conditions:

$$\widetilde{B}_{jt}: \qquad \Theta_t \frac{\varkappa_{jt}}{1+\varkappa_{1,jt}} = \mathbb{E}_t \Lambda^N_{t,t+1} \left[1-\sigma+\sigma \frac{\partial \widetilde{V}_{jt+1}}{\partial \widetilde{X}_{jt+1}} \right] \left(R^L_{t+1} - R_t \right)$$

$$\widetilde{RE}_{jt}: \qquad 0 = \mathbb{E}_t \Lambda^N_{t,t+1} \left[1-\sigma+\sigma \frac{\partial \widetilde{V}_{jt+1}}{\partial \widetilde{X}_{jt+1}} \right] \left(R^{re}_{t} - R_t \right)$$

From the optimality condition for reserves, it follows that the reserve and deposit rates are equal. To solve for the envelope condition with respect to bank equity, $\partial \widetilde{V}_{jt+1}/\partial \widetilde{X}_{jt+1}$, start with expressing the continuation value of bank j as:

$$\widetilde{V}_{jt} = \chi_{b,jt} Q_t \widetilde{B}^{FI}_{jt} + \chi_{re,jt} \widetilde{RE}_{jt} + \chi_{x,jt} \widetilde{X}_{jt}$$

with:

$$\chi_{b,jt} = (1-\sigma)\mathbb{E}_t \Lambda^N_{t,t+1} \left(R^L_{t+1} - R_t \right) + \sigma \mathbb{E}_t \Lambda^N_{t,t+1} \frac{Q_{t+1} \widetilde{B}^{FI}_{jt+1}}{Q_t \widetilde{B}^{FI}_{jt}} \chi_{b,jt+1}$$

$$\chi_{re,jt} = (1-\sigma)\mathbb{E}_t \Lambda^N_{t,t+1} \left(R^{re}_{t} - R_t \right) + \sigma \mathbb{E}_t \Lambda^N_{t,t+1} \frac{\widetilde{RE}_{jt+1}}{\widetilde{RE}_{jt}} \chi_{re,jt+1}$$

$$\chi_{x,jt} = 1-\sigma + \sigma \mathbb{E}_t \Lambda^N_{t,t+1} \frac{\widetilde{X}_{jt+1}}{\widetilde{X}_{jt}} \chi_{x,jt+1}$$

Imposing the optimality conditions, $\chi_{RE,jt} \equiv 0$, implies that the limited enforcement constraint can be written as:

$$\Theta_t Q_t \widetilde{B}^{FI}_{jt} \leq \overbrace{\chi_{b,jt} Q_t \widetilde{B}^{FI}_{jt} + \chi_{x,jt} \widetilde{X}_{jt}}^{\widetilde{V}_{jt}}$$

(C.1) $$\Rightarrow Q_t \widetilde{B}^{FI}_{jt} \leq \frac{\chi_{x,jt}}{\Theta_t - \chi_{b,jt}} \widetilde{X}_{jt} = \Phi_{jt} \widetilde{X}_{jt} = \frac{\widetilde{V}_{jt}}{\Theta_t}$$

Assuming that the modified leverage constraint binds on average, $\varkappa_{jt} > 0$, and imposing that the reserve and deposit rates are equal, the bank capital accumulation can be written as follows:[25]

(C.2) $$\Phi_{jt} = \frac{Q_t \widetilde{B}^{FI}_{jt}}{\widetilde{X}_{jt}} \Rightarrow \frac{\widetilde{X}_{jt+1}}{\widetilde{X}_{jt}} = \left(R^L_{t+1} - R_t \right) \frac{Q_t \widetilde{B}^{FI}_{jt}}{\widetilde{X}_{jt}} + R_t = \left(R^L_{t+1} - R_t \right) \Phi_{jt} + R_t$$

25. The time-varying long- to short-term interest rate spread in the data suggests that the constraint binds on average.

Rewrite the bank's continuation value, $\widetilde{V}_{jt}$, in terms of bank equity using the relationship in equation (C.1):

$$\Theta_t \Phi_{jt} \widetilde{X}_{jt} = \mathbb{E}_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda^N_{t,t+i} \widetilde{X}_{jt+i}$$
$$= (1-\sigma)\mathbb{E}_t \Lambda^N_{t,t+1} \widetilde{X}_{jt+1} + \sigma \mathbb{E}_t \Lambda^N_{t,t+1} \Theta_{t+1} \Phi_{jt+1} \widetilde{X}_{jt+1}$$

which, with a binding leverage constraint and exploiting equation (C.2), can be written as:

$$= \mathbb{E}_t \Lambda^N_{t,t+1} \left[1-\sigma+\sigma\Theta_{t+1}\Phi_{jt+1}\right] \left[\left(R^L_{t+1} - R_t\right)\Phi_{jt} + R_{jt}\right] \widetilde{X}_{jt}$$
$$\Rightarrow \Theta_t \Phi_{jt} = \mathbb{E}_t \Lambda^N_{t,t+1} \left[1-\sigma+\sigma\Theta_{t+1}\Phi_{jt+1}\right] \left[\left(R^L_{t+1} - R_t\right)\Phi_{jt} + R_{jt}\right]$$

Since no term on the right-hand side is bank-specific besides allowed modified leverage, the modified leverage ratio is the same across all banks and follows:

$$\Phi_{jt} = \Phi_t = \frac{\mathbb{E}_t \Lambda^N_{t,t+1} \left[1-\sigma+\sigma\Theta_{t+1}\Phi_{t+1}\right] R_t}{\Theta_t - \mathbb{E}_t \Lambda^N_{t,t+1} \left[1-\sigma+\sigma\Theta_{t+1}\Phi_{t+1}\right] \left(R^L_{t+1} - R_t\right)}$$

as in Gertler and Karadi (2011). With a binding leverage constraint, allowed modified leverage is constant across all banks and independent of the level of bank equity. This implies the envelope condition: $\partial \widetilde{V}_{jt} / \partial \widetilde{X}_{jt} = \Theta_t \Phi_t$; with the first-order condition over bond holdings given by:

$$\frac{\varkappa_{jt}}{1+\varkappa_{jt}} \Theta_t = \mathbb{E}_t \Lambda^N_{t,t+1} \left[1-\sigma+\sigma\Theta_{t+1}\Phi_{t+1}\right] \left(R^L_{t+1} - R_t\right)$$

Furthermore, bank equity and asset holdings integrate across all banks:

$$Q_t \widetilde{B}^{FI}_t = \Phi_t \widetilde{X}_t$$

and the law of motion for survivor bank equity can be written as:

$$\widetilde{X}_t = \left[\left(R^L_t - R_{t-1}\right)\Phi_{t-1} + R_{t-1}\right] \widetilde{X}_{t-1}$$

The law of motion for aggregate bank net worth includes the evolution of survivors' net worth and the net worth of new entrants. A fraction σ of bankers at $t-1$ survive until t with net worth evolution as described above. A fraction $1-\sigma$ of bankers at $t-1$ exit with the market value of end-of-life long-term assets $(1-\sigma)\kappa Q_t \widetilde{B}^{FI}_{t-1}$. Assume that each period, the household transfers $(1-\sigma)^{-1} P_t X^s$ to each new entrant to maintain the scale of the assets managed by the financial sector. Aggregate real net worth evolves according to:

$$X_t = \sigma \Pi_t^{-1} \left[\left(R^L_t - R_{t-1}\right)\Phi_{t-1} + R_{t-1}\right] X_{t-1} + X^s$$

with the real modified leverage constraint:

$$Q_t B_t^{FI} = \Phi_t X_t$$

Consider the relevant equations to the model equilibrium conditions under the deterministic exit limit, $\sigma = 0$, imposing $\mathbb{E}_t \Lambda_{t,t+1}^N R_t = 1$ from the household problem:

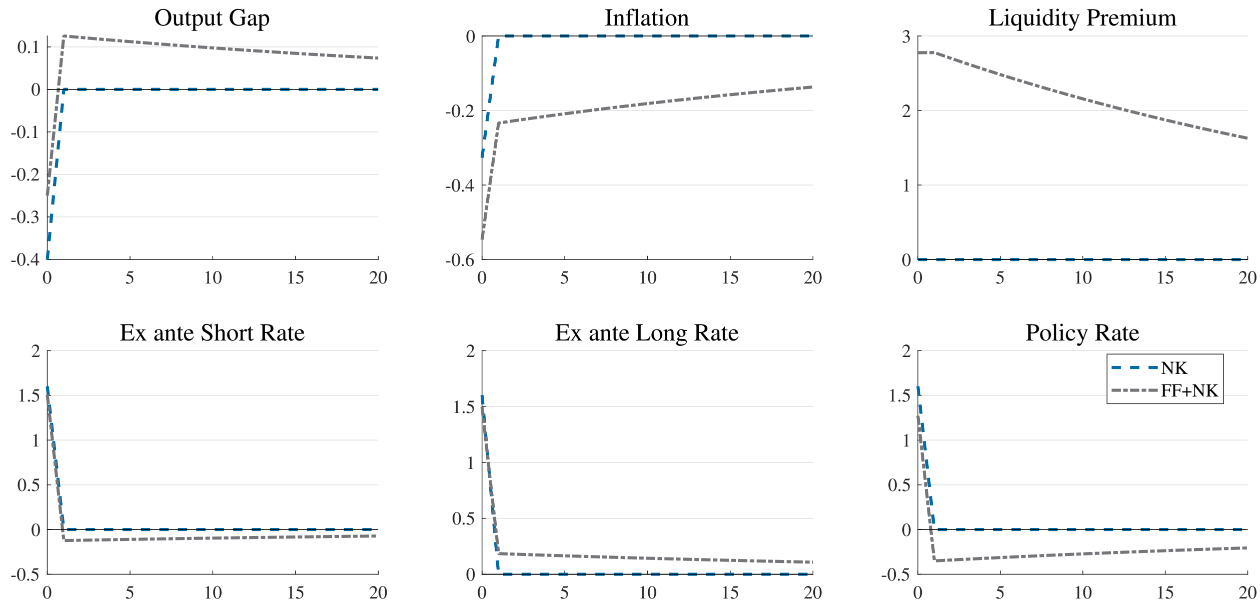
$$\begin{aligned}
1 + \frac{\varkappa_t}{1+\varkappa_t}\Theta_t &= \mathbb{E}_t \Lambda_{t,t+1}^N R_{t+1}^L \\
Q_t B_t^{FI} &\leq \Phi_t X^s \\
1 &= \Phi_t \left[\Theta_t - \left(\mathbb{E}_t \Lambda_{t,t+1}^N R_{t+1}^L - 1\right)\right]
\end{aligned}$$

The modified leverage ratio applies over the entire net worth and is endogenous, differing from Sims et al. (2023) who assume an exogenous leverage ratio. The net worth accumulation and long-term rate definitions are identical to Sims et al. (2023). Modified leverage rises for a given value of the financial shock as the long-term interest rate increases.

D Additional Results

D.1 Responses to a Monetary Shock

Figure A.1: Impulse Responses to a Monetary Shock

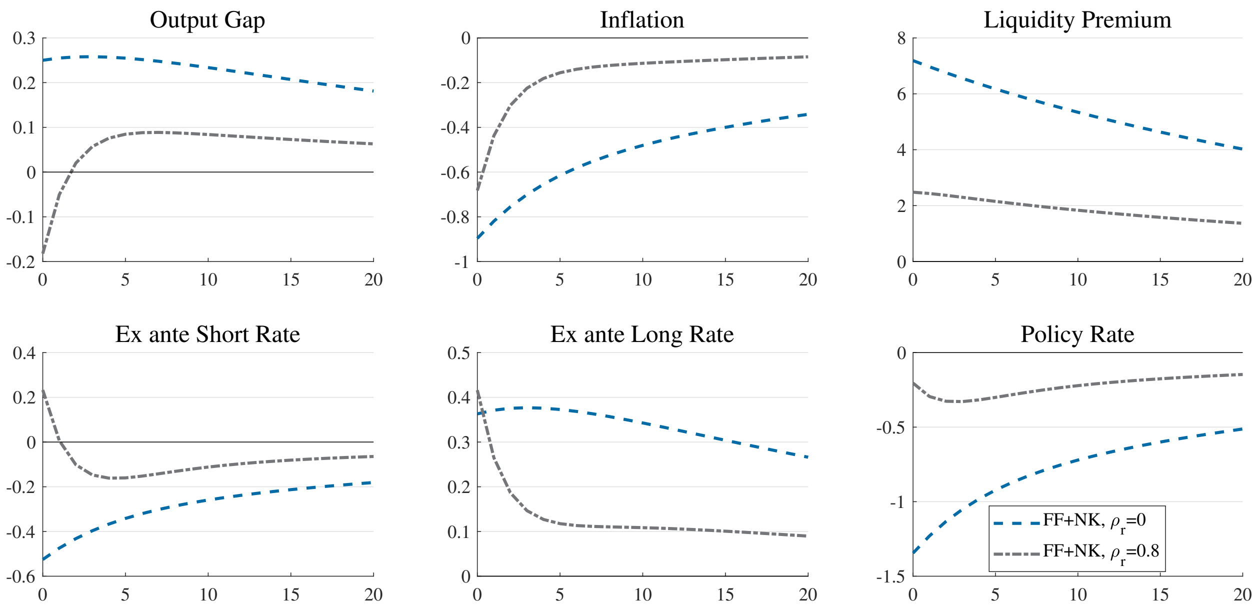


Notes: Dashed lines: NK model, $\gamma = 0.204$ and equations (3.18)–3.20 replaced by $\xi_t \equiv 0$; dashed-dotted lines: the complete model described by equations (3.14)–3.20 with nominal price rigidity (FF+NK), $\gamma = 0.204$. The monetary shock is scaled such that the output gap response in the FF+NK model is 0.25%. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

D.2 *Robustness With Interest Rate Smoothing*

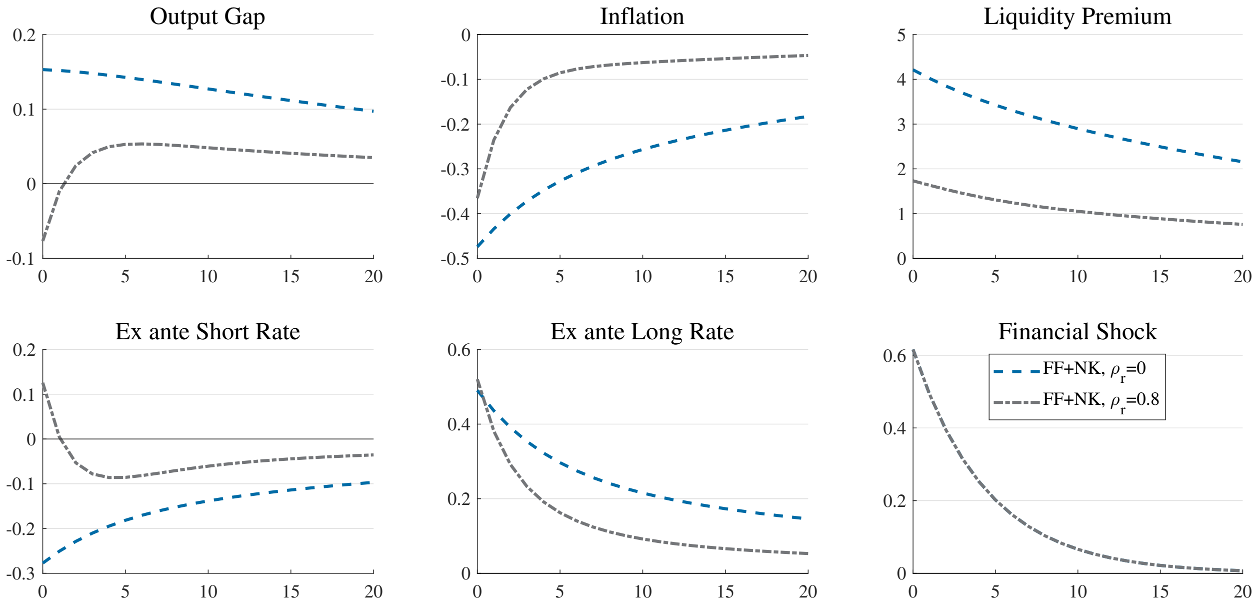
We modify the Taylor rule allowing for interest rate smoothing: $r_t = \rho_r r_{t-1} + (1 - \rho_r)\phi_\pi \pi_t + \sigma_r \epsilon_t^r$, where ρ_r is set to 0.8. Figures A.2, A.3 and A.4 contrast the responses to natural rate, financial, and monetary shocks with and without interest rate smoothing.

Figure A.2: Impulse Responses to a Natural Rate Shock



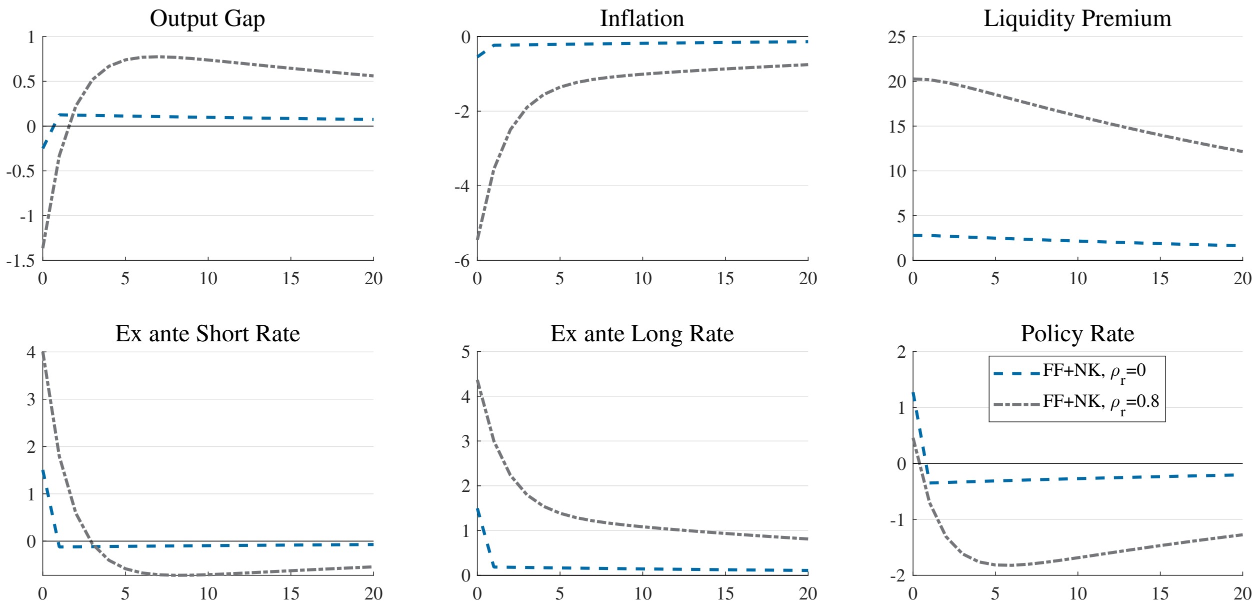
Notes: Dashed lines: the complete model without interest rate smoothing (ρ_r=0); dashed-dotted lines: the complete model with interest rate smoothing (ρ_r=0.8). The natural rate shock is scaled such that the output gap response in the FF+NK model (ρ_r=0) is 0.25%. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

Figure A.3: Impulse Responses to a Financial Shock



Notes: Dashed lines: the complete model without interest rate smoothing (ρ_r=0); dashed-dotted lines: the complete model with interest rate smoothing (ρ_r=0.8). The financial shock is scaled to explain 20% of the variability in the output gap in the FF+NK model. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

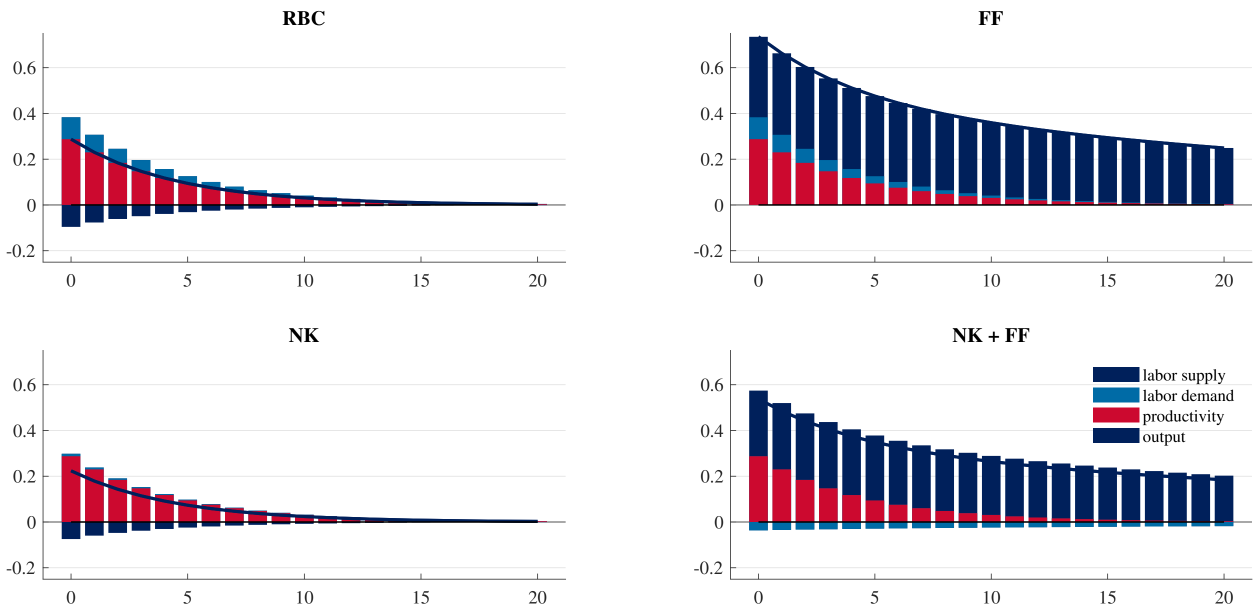
Figure A.4: Impulse Responses to a Monetary Shock



Notes: Dashed lines: the complete model without interest rate smoothing (ρ_r=0); dashed-dotted lines: the complete model with interest rate smoothing (ρ_r=0.8). The monetary shock is scaled such that the output gap response in the FF+NK model is 0.25%. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

D.3 Decomposition of Output Response

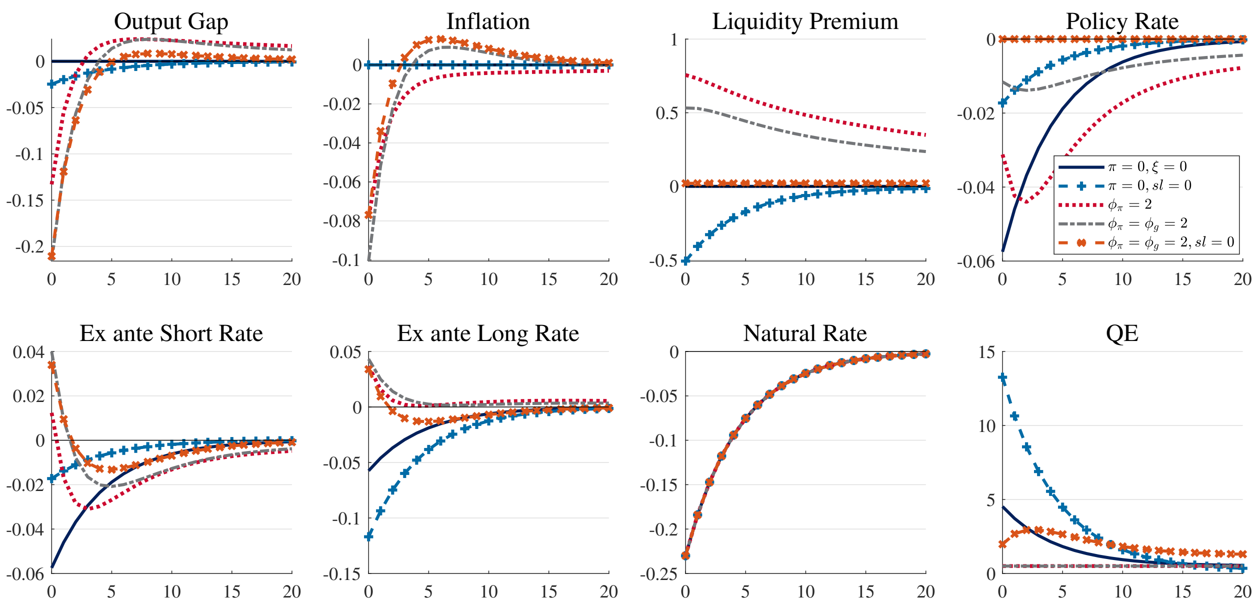
FIGURE A.5: OUTPUT IMPULSE RESPONSE TO A NATURAL RATE SHOCK: LABOR SUPPLY AND LABOR DEMAND CHANNELS



Notes: Solid line: Output response to natural rate shock. Dark blue bars: output response attributed to the labor supply channel; medium blue bars: output response attributed to the labor demand channel; red bars: output response attributed to the productivity channel. The natural rate shock is scaled such that the output gap response in the FF+NK model is 0.25%. Output is in terms of percentage deviations from steady state.

D.4 Implementable Policy Strategies

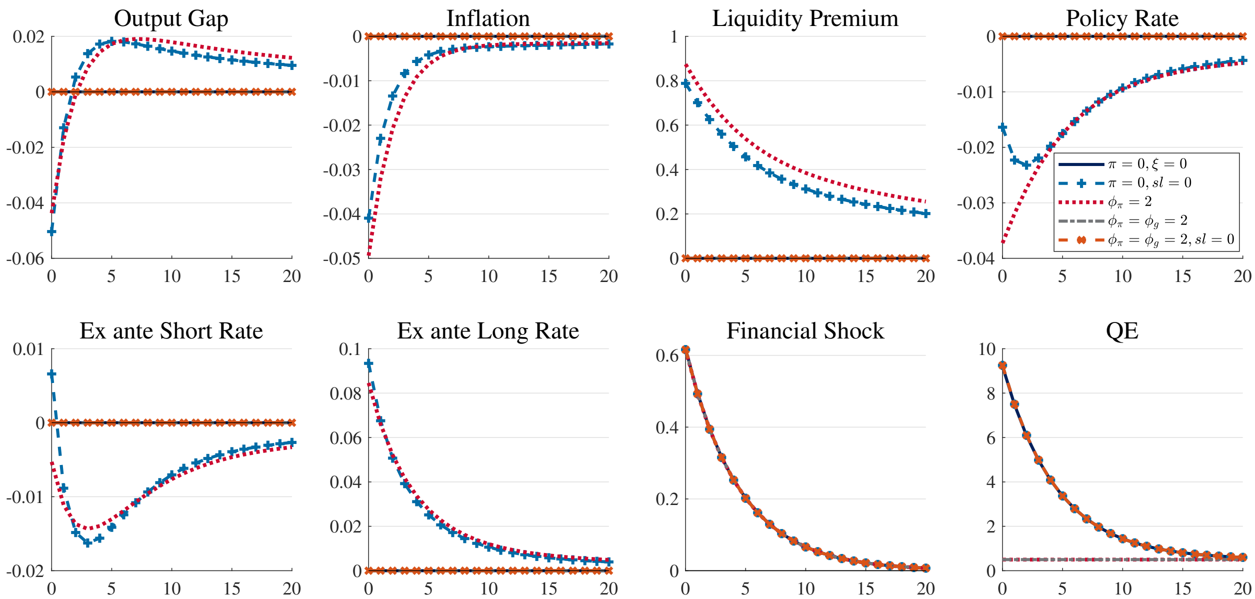
Figure A.6: Impulse Responses to a Natural Rate Shock: Varying Implementable Policy Strategies



Notes: Solid lines: inflation- ($\pi_t = 0$) and liquidity premium-targeting ($\xi_t = 0$) dual-instrument monetary policy; dashed-crossed lines: inflation- ($\pi_t = 0$) and yield curve slope-targeting ($sl_t = 0$) dual-instrument monetary policy (r_t and re_t time-varying); dotted lines: single-instrument interest rate policy responding to inflation only; dashed-dotted lines: single-instrument interest rate policy responding to inflation and output growth; dashed-crossed lines: to interest rate policy responding to inflation and output growth and balance sheet policy to yield curve-targeting ($sl_t = 0$). The natural rate shock is scaled such that the output gap response in the FF+NK model is 0.25%. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

In response to a natural rate shock, Figure A.6, under a dual-instrument policy with inflation ($\pi_t = 0$) and yield curve slope targeting, there is no effect on inflation and a relatively small effect on the output gap. Thus, targeting only observable variables largely neutralizes the macroeconomic effects and stabilizes the economy to a natural rate shock. The central bank responds by lowering the policy rate and engaging in quantitative easing. In contrast, conventional interest rate policy simply leads to a contraction of the output gap and inflation in response to a natural rate shock. Under conventional interest rate policy with a strict yield curve slope-targeting balance sheet policy, a natural rate shock has similar effects on macroeconomic variables, but the central bank responds with quantitative easing instead of interest rate policy.

FIGURE A.7: IMPULSE RESPONSES TO A FINANCIAL SHOCK: VARYING IMPLEMENTABLE POLICY STRATEGIES



Notes: Solid lines: inflation- ($\pi_t = 0$) and liquidity premium-targeting ($\xi_t = 0$) dual-instrument monetary policy; dashed-crossed lines: inflation- ($\pi_t = 0$) and yield curve slope-targeting ($sl_t = 0$) dual-instrument monetary policy (r_t and re_t time-varying); dotted lines: single-instrument interest rate policy responding to inflation only; dashed-dotted lines: single-instrument interest rate policy responding to inflation and output growth; dashed-crossed lines: to interest rate policy responding to inflation and output growth and balance sheet policy to yield curve-targeting ($sl_t = 0$). The financial shock is scaled to explain 20% of the variability in the output gap in the FF+NK model. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

Looking at the responses to the financial shock, we observe a complete stabilization of the output gap and inflation when using balance sheet policy. The responses are similar for the dual-instrument policy strategies—optimal dual policy, dual policy targeting only observable variables, and conventional interest rate policy combined with a strict yield-curve slope targeting balance sheet policy. In the case of only interest rate policy, the central bank lowers the policy rate, but not by enough so that the output gap and inflation still contract.

E Properties of Endogenous Balance Sheet Policy

E.1 Dual Instrument Policy Ensures that the Divine Coincidence Holds

Proposition 2. *Absent endogenous balance sheet policy, the divine coincidence fails due to liquidity premium variability.*

Let r_t^π be the policy rate that supports an inflation target. The IS and Phillips curves can be written as:

$$
\begin{aligned}
0 &= \gamma gap_t + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \\
gap_t &= \mathbb{E}_t gap_{t+1} - \left(r_t^\pi - \overline{C}_b \mathbb{E}_t\left(\xi_{t+1} - \xi_t\right) - r_t^n\right)
\end{aligned}
$$

From the Phillips curve, it is clear that under inflation-targeting interest rate policy the output gap co-varies with the liquidity premium. If the liquidity premium is not stabilized, the divine coincidence does not hold. The IS curve implies:

$$
\mathbb{E}_t \xi_{t+1} - \xi_t = \frac{1+\eta}{\eta\overline{C}_b + \Omega}\left(r_t^\pi - r_t^n\right)
$$

allowing the long-term rate to be written in terms of the inflation-targeting policy rate and the natural rate:

$$
\mathbb{E}_t r_{t+1}^L = \left(1 - \frac{1+\eta}{\eta\overline{C}_b + \Omega}\right) r_t^\pi + \frac{1+\eta}{\eta\overline{C}_b + \Omega} r_t^n
$$

Consider the financial block in the economy, substituting the output gap from the problem and instituting fixed balance sheet policy, $re_t \equiv 0$:

$$
c_{b,t} = \underbrace{-\frac{\overline{C}_b - \Omega + \left(1 - \overline{C}_b\right)(1+\eta)}{1+\eta}}_{\mathbb{A}} \xi_t - \frac{1}{1-\rho_n} r_t^n
$$

$$
\begin{aligned}
\overline{C}_b c_{b,t} = \Omega \frac{\varepsilon - 1}{\varepsilon} &\left(\underbrace{\left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\right) - (2+\eta)\frac{\overline{C}_b - \Omega}{1+\eta}\right)}_{\mathbb{B}} \xi_t - \frac{1}{1-\rho_n} r_t^n \right) \\
&+ \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + \left(1 - \underbrace{\frac{1+\eta}{\eta\overline{C}_b + \Omega}}_{\mathbb{C}}\right) r_{t-1}^\pi + \underbrace{\frac{1+\eta}{\eta\overline{C}_b + \Omega}}_{\mathbb{C}} r_{t-1}^n \right)\right)\Bigg)
\end{aligned}
$$

$$q_t + b_t = \left(1 - \overline{RE}\right) \left(\frac{\Phi}{\zeta} \underbrace{\frac{1+\eta}{\eta\overline{C}_b + \Omega}}_{\mathbb{C}} \left(r_t^{\pi} - r_t^n\right) - \underbrace{\left(1 + \Phi\frac{1-\zeta}{\zeta}\right)}_{\mathbb{D}} \theta_t \right)$$

Combining the equations above and collecting terms, this simplifies to:

$$\left(\overline{C}_b \mathbb{A} - \Omega\frac{\varepsilon-1}{\varepsilon}\mathbb{B}\right)\xi_t = \left(\overline{C}_b - \Omega\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{1-\rho_n}r_t^n + \frac{QB}{Y}\left(\left(1-\overline{RE}\right)\frac{\Phi}{\zeta}\mathbb{C}\left(r_t^{\pi} - r_t^n\right)\right) -$$
$$\frac{QB}{Y}\frac{1}{\beta\zeta}\left(\left(1-\overline{RE}\right)\frac{\Phi}{\zeta}\mathbb{C} + (1-\mathbb{C})\right)r_{t-1}^{\pi} + \frac{QB}{Y}\frac{1}{\beta\zeta}\left(\left(1-\overline{RE}\right)\frac{\Phi}{\zeta}\mathbb{C} - \mathbb{C}\right)r_{t-1}^n$$
$$-\frac{QB}{Y}\mathbb{D}\left(\theta_t - \frac{1}{\beta\zeta}\theta_{t-1}\right)$$

The liquidity premium varies with natural rate and financial shocks, deviations of the policy rate from the natural rate, and lagged shocks and deviations.

Proposition 3. *There exists endogenous balance sheet policy, re_t^*, that stabilizes the output gap, inflation, and the liquidity premium, the equivalent of the "divine coincidence" in this economy.*

Corollary 3.1. *The policy rate, r_t, equals the natural rate when balance sheet policy supports liquidity premium stabilization.*

As shown above, output gap variability is proportional to liquidity premium variability when the central bank targets inflation. If both are equal to zero in all periods, the Phillips curve holds and the IS curve holds for the policy rate equal to the natural rate, $r_t = r_t^n$. Conjecture that there exists a balance sheet specification, re_t^*, for which this is true. The financial block in the economy under the divine coincidence simplifies to:

$$\left(\Omega\frac{\varepsilon-1}{\varepsilon} - \overline{C}_b\right)\frac{r_t^n}{1-\rho_n} = \frac{QB}{Y}\left((1-\kappa)q_t + b_t - \frac{1}{\beta\zeta}b_{t-1}\right)$$
$$q_t + b_t = -\left(1-\overline{RE}\right)\left(1+\Phi\frac{1-\zeta}{\zeta}\right)\theta_t + \overline{RE}re_t^*$$

With the liquidity premium fixed, the ex-ante nominal long-term rate equals the natural rate. Using the definition of the long-term rate in terms of the bond price allows us to define the current bond price as a function of the natural rate:

$$\mathbb{E}_t r_{t+1}^L = \kappa\beta\zeta\mathbb{E}_t q_{t+1} - q_t = r_t^n \Rightarrow q_t = -\frac{r_t^n}{1-\kappa\beta\zeta\rho_n}$$

implying that the equilibrium debt level is given by:

$$b_t = \frac{r_t^n}{1-\kappa\beta\zeta\rho_n} - \left(1-\overline{RE}\right)\left(1+\Phi\frac{1-\zeta}{\zeta}\right)\theta_t + \overline{RE}re_t^*$$

Modified leverage and the bond price are fully exogenous in this case. Thus, the equilibrium debt level only depends on exogenous terms and the policy variable, re_t^*. The equilibrium level of debt-financed expenditure is also fully exogenous and only varies with natural rate shocks. The financial account constraint defines the balance sheet policy that instills the divine coincidence:

$$\begin{aligned} re_t^* = {} & \frac{1}{\beta\zeta} re_{t-1}^* + \frac{1-\overline{RE}}{\overline{RE}} \left(1 + \Phi \frac{1-\zeta}{\zeta}\right) \left(\theta_t - \frac{1}{\beta\zeta}\theta_{t-1}\right) \\ & - \frac{1}{\overline{RE}} \left(\frac{\kappa(1-\rho_n)}{1-\kappa\beta\zeta\rho_n} - \frac{Y}{QB} \left(\Omega \frac{\varepsilon - 1}{\varepsilon} - \overline{C}_b \right) \right) \frac{r_t^n}{1-\rho_n} + \frac{1}{\overline{RE}} \frac{1}{\beta\zeta} \frac{r_{t-1}^n}{1-\kappa\beta\zeta\rho_n} \end{aligned} \tag{DC}$$

E.2 *Balance Sheet Policy Supports Model Determinacy with a Policy Rate Peg*

Proposition 4. *Endogenous balance sheet policy can provide a determinate rational expectations equilibrium with a permanent policy rate peg.*

Consider the log-linear model and suppose the monetary authority credibly targets the liquidity premium, $\xi_t \equiv 0$ for all t, via reserve management, re_t^{ξ}, under a permanent policy rate peg, $r_t = 0$, implying:

$$\begin{aligned} & \pi_t = \gamma gap_t + \beta \mathbb{E}_t \pi_{t+1} \\ & gap_t = \mathbb{E}_t gap_{t+1} + \mathbb{E}_t \pi_{t+1} + r_t^n \\ & \left[\frac{\overline{C}_b}{\Omega} \frac{\varepsilon}{\varepsilon - 1} - 1 \right] \left(gap_t - \frac{1}{1-\rho_n} r_t^n \right) = (1+\eta) gap_t + \frac{1}{\Omega} \frac{\varepsilon}{\varepsilon - 1} \frac{QB}{Y} \left(b_t - \frac{1}{\beta\zeta} \left(b_{t-1} - \pi_t \right) \right) \\ & b_t = -\left(1 - \overline{RE}\right) \left(1 + \Phi \frac{1-\zeta}{\zeta}\right) \theta_t + \overline{RE} re_t^{\xi} \end{aligned}$$

Abstracting from shocks, the system can be written as:

$$\begin{bmatrix} \pi_t \\ gap_t \\ re_{t-1}^{\xi} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \chi_\pi & \chi_{gap} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \gamma + \beta & \gamma & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \zeta\beta \end{bmatrix}}_{\mathbf{D}} \begin{bmatrix} \pi_{t+1} \\ gap_{t+1} \\ re_t^{\xi} \end{bmatrix}$$

which is stable so long as the matrix $\mathbf{D}$ has two out of three eigenvalues inside the unit circle. The characteristic equation for matrix $\mathbf{D}$ is given by:

$$(\zeta\beta - e)\left[(\beta + \gamma - e)(1 - e) - \gamma\right] = 0$$

The eigenvalues are:

$$\begin{aligned} 0 < & e_1 = \zeta\beta < 1 \\ -1 < & e_2 = \frac{1 + \beta + \gamma - \sqrt{(1+\beta+\gamma)^2 - 4\beta}}{2} < 1 \end{aligned}$$

$$1 < e_3 = \frac{1+\beta+\gamma+\sqrt{(1+\beta+\gamma)^2-4\beta}}{2}$$

Corollary 4.1. *Inflation-targeting balance sheet policy, re_t^π, provides a determinate linear rational expectations equilibrium with a permanent policy rate peg.*

Consider the log-linear model and suppose the monetary authority credibly targets inflation, $\pi_t \equiv 0$ for all t, via reserve management, re_t^π, under a permanent policy rate peg, $r_t = 0$:

$$\begin{aligned}
0 &= \gamma gap_t + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \\
gap_t &= \mathbb{E}_t gap_{t+1} + \overline{C}_b \mathbb{E}_t \Delta\xi_{t+1} + r_t^n \\
r_t^L &= \kappa\beta\zeta q_t - q_{t-1} \\
\xi_t &= \mathbb{E}_t \xi_{t+1} + \mathbb{E}_t r_{t+1}^L \\
c_{b,t} &= gap_t - \left(1-\overline{C}_b\right)\xi_t - \frac{1}{1-\rho_n} r_t^n \\
\left(\overline{C}_b - \Omega\frac{\varepsilon-1}{\varepsilon}\right) c_{b,t} &= \Omega\frac{\varepsilon-1}{\varepsilon}\left(\left(\frac{1-\Omega}{\eta}+1-\Omega\right)\xi_t + (1+\eta)gap_t\right) \\
&\quad + \frac{QB}{Y}\left((1-\kappa)q_t + b_t - \frac{1}{\beta\zeta}b_{t-1}\right) \\
\phi_t &= \frac{\Phi}{\zeta}\mathbb{E}_t r_{t+1}^L - \left(1+\Phi\frac{1-\zeta}{\zeta}\right)\theta_t \\
q_t + b_t &= \left(1-\overline{RE}\right)\phi_t + \overline{RE}re_t^\pi
\end{aligned}$$

The Phillips curve implies that the output gap is proportional to the liquidity premium:

$$gap_t = \frac{\Omega - \overline{C}_b}{1+\eta}\xi_t$$

Substituting the output gap from the IS curve defines a first-order difference equation for liquidity premium dynamics:

$$\xi_t = \mathbb{E}_t \xi_{t+1} + \frac{1+\eta}{\Omega+\eta\overline{C}_b} r_t^n \tag{E.1}$$

This implies from the liquidity premium definition that the expected future long-term rate is proportional to the natural rate:

$$\mathbb{E}_t r_{t+1}^L = \frac{1+\eta}{\Omega+\eta\overline{C}_b} r_t^n$$

and, from the allowed modified leverage definition, allowed modified leverage is fully exogenous:

$$\phi_t = \frac{\Phi}{\zeta}\frac{1+\eta}{\Omega+\eta\overline{C}_b}r_t^n - \left(1+\Phi\frac{1-\zeta}{\zeta}\right)\theta_t$$

Rearranging the definition of the expected long-term rate defines the current bond price as a linear function of the natural rate:

$$\kappa\zeta\beta\mathbb{E}_t q_{t+1} - q_t = \frac{1+\eta}{\Omega+\eta\overline{C}_b}r_t^n \Rightarrow q_t = \kappa\zeta\beta\mathbb{E}_t q_{t+1} - \frac{1+\eta}{\Omega+\eta\overline{C}_b}r_t^n = -\frac{1}{1-\zeta\beta\kappa\rho_n}\frac{1+\eta}{\Omega+\eta\overline{C}_b}r_t^n$$

The equilibrium debt level is a linear function of the natural rate, financial shock, and balance sheet policy:

$$b_t = (1-\overline{RE})\left[\frac{\Phi}{\zeta}\frac{1+\eta}{\Omega+\eta\overline{C}_b}r_t^n - \left(1+\Phi\frac{1-\zeta}{\zeta}\right)\theta_t\right] + \overline{RE}re_t^{\pi} + \frac{1}{1-\zeta\beta\kappa\rho_n}\frac{1+\eta}{\Omega+\eta\overline{C}_b}r_t^n$$

Given that the output gap is proportional to the liquidity premium, the two equations defining debt-financed expenditure can be written as:

(E.2)

$$c_{b,t} = -\left[1-\frac{\Omega+\eta\overline{C}_b}{1+\eta}\right]\xi_t - \frac{1}{1-\rho_n}r_t^n$$

(E.3)

$$\left(\frac{\overline{C}_b}{\Omega}\frac{\varepsilon}{\varepsilon-1}-1\right)c_{b,t} = \left(\frac{1-\Omega}{\eta}+1-\overline{C}_b\right)\xi_t + \frac{1}{\Omega}\frac{\varepsilon}{\varepsilon-1}\frac{QB}{Y}\left((1-\kappa)q_t + b_t - \frac{1}{\beta\zeta}b_{t-1}\right)$$

Equations (E.2) and (E.3) consolidate to define the current liquidity premium. We have shown that current bond price is a linear function of the current natural rate and that the equilibrium debt level is a linear function of the current natural rate, financial shock, and balance sheet policy. Given this, the current liquidity premium is a linear function of current and lagged values of the exogenous variables, $\{r_t^n, \theta_t\}$, and balance sheet policy, re_t^{π}:

(E.4)
$$\xi_t = \omega_1 r_t^n + \omega_2 r_{t-1}^n + \omega_3\theta_t + \omega_4\theta_{t-1} + \omega_5 re_t^{\pi} - \omega_6 re_{t-1}^{\pi}$$

Substituting equation (E.4) into equation (E.1) defines a first-order difference equation for balance sheet policy log-differences, $\Delta re_t^{\pi} = re_t^{\pi} - re_{t-1}^{\pi}$, as a function of exogenous terms:

$$\Delta re_t^{\pi} = \frac{\omega_5}{\omega_6}\mathbb{E}_t\Delta re_{t+1}^{\pi} + \left[\omega_2 - \omega_1(1-\rho_n) + \frac{1+\eta}{\Omega+\eta\overline{C}_b}\right]\frac{r_t^n}{\omega_6} + [\omega_4 - \omega_3(1-\rho_\theta)]\frac{\theta_t}{\omega_6} - \omega_2\frac{r_{t-1}^n}{\omega_6} - \omega_4\frac{\theta_{t-1}}{\omega_6}$$

This equation has a solution, and therefore completes the proof of model determinacy, if $|\omega_5| < |\omega_6|$. From equations (E.2) and (E.3) along with the definitions of the bond price,

q_t, and debt level, b_t, in terms of the exogenous variables and the balance sheet policy:

$$\frac{\omega_5}{\omega_6} = \beta\zeta < 1 \Rightarrow \omega_5 < \omega_6$$

ω_5 and ω_6 have the same sign implying that the solution is non-oscillatory. Note, the inflation-targeting balance sheet policy under and interest rate peg is stationary in first-differences. This implies that the balance sheet level is non-stationary under this policy. Following shock innovations, the model converges to a new steady state over time.

Corollary 4.2. *Balance sheet policy that targets the output gap, re_t^{gap}, with a permanent policy rate peg results in model indeterminacy.*

Consider the log-linear model and suppose the monetary authority credibly targets the output gap, $gap_t \equiv 0$ for all t, via reserve management, re_t^{gap}, under a permanent policy rate peg, $r_t = 0$:

$$\begin{aligned}
\pi_t &= \beta\mathbb{E}_t\pi_{t+1} + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \\
0 &= \overline{C}_b\mathbb{E}_t\Delta\xi_{t+1} + \mathbb{E}_t\pi_{t+1} + r_t^n \\
r_t^L &= \kappa\beta\zeta q_t - q_{t-1} \\
\xi_t &= \mathbb{E}_t\xi_{t+1} + \mathbb{E}_t r_{t+1}^L - r_t \\
c_{b,t} &= -\left(1-\overline{C}_b\right)\xi_t - \frac{1}{1-\rho_n}r_t^n \\
\overline{C}_b c_{b,t} &= \Omega\frac{\varepsilon-1}{\varepsilon}\left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\right)\xi_t - \frac{1}{1-\rho_n}r_t^n\right) \\
&\qquad\qquad + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + r_t^L - \pi_t\right)\right) \\
\phi_t &= \frac{\Phi}{\zeta}\mathbb{E}_t r_{t+1}^L - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t \\
q_t + b_t &= \left(1-\overline{RE}\right)\phi_t + \overline{RE}re_t^{gap}
\end{aligned}$$

The IS curve defines a relationship between inflation expectations and forward-looking changes in the liquidity premium:

$$\mathbb{E}_t\pi_{t+1} = -\left(\overline{C}_b\mathbb{E}_t\Delta\xi_{t+1} + r_t^n\right)$$

Substituting inflation expectations from the Phillips curve defines the current inflation rate in terms of the liquidity premium and the natural rate:

$$\pi_t = -\beta\left(\overline{C}_b\mathbb{E}_t\Delta\xi_{t+1} + r_t^n\right) + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t$$

With output gap-targeting balance sheet policy under a permanent policy rate peg, the non-financial block of the model defines the current inflation rate in terms of the liquidity premium path and the natural rate. The financial block of the economy in this case is given by:

$$
\begin{aligned}
c_{b,t} &= -\frac{1}{1-\rho_n} r_t^n - \left(1 - \overline{C}_b\right) \xi_t \\
\overline{C}_b c_{b,t} &= \Omega \frac{\varepsilon - 1}{\varepsilon} \left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega \right) \xi_t - \frac{1}{1-\rho_n} r_t^n \right) \\
&\quad + \frac{QB}{Y} \left(q_t + b_t - \frac{1}{\beta\zeta} \left(q_{t-1} + b_{t-1} - \Delta\xi_t - \pi_t \right) \right) \\
q_t + b_t &= -\left(1 - \overline{RE}\right) \left(\frac{\Phi}{\zeta} \mathbb{E}_t \Delta\xi_{t+1} + \left(1 + \Phi \frac{1-\zeta}{\zeta} \right) \theta_t \right) + \overline{RE} re_t^{gap}
\end{aligned}
$$

Given the relationship between inflation and the liquidity premium from the non-financial model block, the financial block consolidates into a single equation relating the liquidity premium and endogenous balance sheet balance sheet policy to exogenous variables. Given that both the liquidity premium and balance sheet policy are unknown, this results in model indeterminacy.

F Calculation of the Yield Curve Slope

The yield curve slope measures the relative level of the long-term yield-to-maturity (YTM) to the short-term YTM. Correspondingly, we define YCC as a policy commitment to holding the yield curve slope fixed via balance sheet policy.

To relate YCC to liquidity premium-targeting, first define the long-term YTM and the yield curve slope

$$
\begin{aligned}
R_t^{long} &= \frac{1}{Q_t} + \kappa, \\
SLOPE_t &= \frac{R_t^{long}}{R_t}.
\end{aligned}
$$

R_t^{long} is the long-term yield-to-maturity given the bond price, Q_t, and coupon decay rate. $SLOPE_t$ is the gross yield curve slope, relating R_t^{long} to R_t, the short-term yield-to-maturity.

A useful asset pricing benchmark to consider is the expectations hypothesis. The expectations hypothesis states that the gross return to holding a long-term asset equals the return to rolling over short-term asset holdings of appropriate size from period to period to match the payout structure of the long-term asset—or no arbitrage. This provides a theory of the term structure of interest rates in which long-term interest rates are the forward-looking average of expected short-term rates. The long-term bond price and long-term YTM for the perpetual coupon bond under the expectations hypothesis (denoted by superscript EH) are given by

$$
\begin{aligned}
Q_t^{EH} &= \frac{1 + \kappa \mathbb{E}_t Q_{t+1}^{EH}}{R_t}, \\
R_t^{long,EH} &= \frac{1}{Q_t^{EH}} + \kappa.
\end{aligned}
$$

Note, the introduction of the expectations hypothesis benchmark allows decomposing the yield curve slope into the product of two terms—TP_t and $Slope_t^{EH}$,

$$
Slope_t = \underbrace{\frac{R_t^{long}}{R_t^{long,EH}}}_{TP_t} \times \underbrace{\frac{R_t^{long,EH}}{R_t}}_{Slope_t^{EH}}.
$$

The first term, TP_t, is the term premium, that is, the excess return to holding long-term debt versus rolling over short-term debt of appropriate size to match the payout structure of the asset. The second term, $Slope_t^{EH}$, is the yield curve slope under the expectations hypothesis. Variability in the latter reflects changes in the forward-looking policy rate path relative to the current policy rate level. An expected future monetary policy tightening, respectively increasing future short-term policy rates, correspondingly implies a positive slope. The liquidity premium relates to the yield curve slope via the term premium.

To see this, consider a log-linear approximation of these new variables around the zero net inflation steady state with some simplifications given (3.16) and approximating $\zeta = 1$ as in Carlstrom et al. (2017) for simplicity,

$$\begin{aligned}
r_t^{long} &= -(1-\kappa\beta\zeta)q_t = (1-\kappa\beta)\mathbb{E}_t \sum_{j=0}^{\infty} (\kappa\beta)^j \, r_{t+1+j}^L, \\
q_t^{EH} &= \kappa\beta\mathbb{E}_t q_{t+1}^{EH} - r_t = -\mathbb{E}_t \sum_{j=0}^{\infty} (\kappa\beta)^j \, r_{t+j}, \\
r_t^{long,EH} &= -(1-\kappa\beta)q_t^{EH} = (1-\kappa\beta)\mathbb{E}_t \sum_{j=0}^{\infty} (\kappa\beta)^j \, r_{t+j}, \\
slope_t &= \underbrace{r_t^{long} - r_t^{long,EH}}_{tp_t} + \underbrace{r_t^{long} - r_t}_{slope_t^{EH}}.
\end{aligned}$$

Combining the equations above equations and using (3.17) allows the term premium to be written in terms of the liquidity premium and future policy rate expectations,

$$\frac{slope_t}{1-\kappa\beta} = \underbrace{\xi_t - \mathbb{E}_t \sum_{j=0}^{\infty} \left[1-(\kappa\beta)^j\right] \left(r_{t+1+j}^L - r_{t+j}\right)}_{\text{term premium, } tp_t/(1-\kappa\beta)} + \underbrace{\mathbb{E}_t \sum_{j=0}^{\infty} (\kappa\beta)^j \, r_{t+j} - \frac{r_t}{1-\kappa\beta}}_{\text{future policy rate expectations}}$$

Note, the yield curve slope convolutes term premium movements, which are directly tied to the liquidity premium, and changes to the expected path of future policy. In so much that a central bank can reasonably estimate the term premium, term-premium-targeting balance sheet policy would more closely resemble targeting the liquidity premium.