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Time-Limited Subsidies: Optimal Taxation with Implications for Renewable Energy Subsidies

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Time-Limited Subsidies: Optimal Taxation with Implications for Renewable Energy Subsidies[*]

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Abstract

Pigouvian subsidies are efficient, but output subsidies with uncertain or limited durations are not Pigouvian. We show that optimal "time-limited" policies must also subsidize investment to correct externalities generated after the output subsidy ends. Furthermore, an output subsidy's optimal duration is characterized by the change in production when it ends. In the wind-energy industry, we find that power generation decreases by 5-10% after the end of facilities' ten-year eligibility for the Renewable Energy Production Tax Credit. This behavioral response has implications for energy transitions and highlights how time limits could cause larger distortions in more elastic industries.

JEL Classification: H23, H21, Q48

Keywords: energy taxes and subsidies, renewable energy, optimal taxation, policy uncertainty

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1. Introduction

As growing subsidy programs across the world usher in a new era in industrial and energy policy, a key question is how to efficiently correct economic externalities. The theoretical answer is simple—whether the externality is innovation, offsetting emissions, guaranteeing supply chain resilience, or maintaining a strong working class. In each case, the optimal "Pigouvian" correction is to directly subsidize every externality-generating unit by its marginal external benefit. In practice, however, because output subsidies typically have finite or uncertain durations, they do not subsidize all externality-generating units. We call these subsidies "time-limited" output subsidies and study them in this paper.

Time-limited output subsidies are prevalent across the world. For example, in the United States, the Advanced Manufacturing Production Tax Credit lasts for seven years (White House, 2022), and the Renewable Energy Production Tax Credit and Clean Vehicle Credit both create ten-year subsidies (TREAS, 2021). In Germany, feed-in tariffs for renewable energy last for twenty years (OECD, 2022) while similar Chinese tax cuts last for six (Nyberg et al., 2020). Many agricultural policies have short subsidy durations, including annual Chinese subsidies for oilseeds (Mcdonald, 2022) and market price supports for dairy in Canada and the United States (CRS, 2014). Even taxes and subsidies without explicit time limits can have uncertain durations due to changes in the political environment. For example, many policies are successively proposed and repealed, such as alcohol taxes in the US (CRS, 1999; Blanchette et al., 2020); carbon taxes in Australia and Alberta, Canada (Dayton, 2014; Raymond, 2020); and soda, sugar, or fat taxes in Denmark and many US cities and states (Schmacker and Smed, 2023; The Economist, 2017; Urban Institute, 2023).[1] Given the ubiquity of time-limited policies, this paper aims to characterize how time limits affect optimal policy considerations, including subsidy rates and the optimal choice of subsidy instruments.

To study the implications of time limits, we present a partial-equilibrium model where potential subsidies are characterized by the duration of their "subsidy period."[2] In this model, firm behavior and optimal policy will be affected by time limits whenever the subsidy period is shorter than the capital life of fixed inputs. Subsidizing output for only part of firms' capital life will reduce their initial incentive to invest and cause production reductions after the subsidy period. These behavioral responses, combined with the imperfect targeting introduced by time limits, affect optimal policy in three important ways.

First, we demonstrate that time limits change the set of policies a social planner should use to correct production externalities. Rather than only subsidizing output as the canon-

[1] See Appendix Table A.1 for more examples of corrective policies with uncertain or limited durations.
[2] We use this generalization of the phrase "credit period" used for tax credits (e.g., TREAS, 2021).

ical Pigouvian policy does, the optimal policy combines output and investment subsidies. This result diverges from production-efficiency intuition about only subsidizing output (see Diamond and Mirrlees, 1971) because time-limited output subsidies create an incentive to under-invest, and investment subsidies efficiently counteract this incentive. The optimal subsidy for both investment and output is strictly positive whenever the (expected) output subsidy duration is less than the life of the fixed inputs. As such, policymakers may want to consider output and investment subsidies as complements rather than substitutes for corrective policy.

Second, if the social planner can optimally subsidize investment, time limits do not affect the optimal output subsidy rate—*no matter* the subsidy duration. Although a social planner who can only subsidize output will set the rate higher than the externality when there are binding time limits, subsidizing investment as well returns the optimal output subsidy rate to the externality value. This is because investment subsidies can influence production after the subsidy period more effectively than output-only subsidies. Interestingly, the two subsidies are fully separable: The optimal output subsidy equals the marginal externality, and only the investment subsidy changes with the (expected) duration of the output subsidy. Policymakers should therefore subsidize investment more when output subsidies have shorter durations, all else equal, and set output subsidies equal to the externality regardless of the subsidy duration.

Third, we characterize a sufficient statistic for the optimal output subsidy duration given social costs associated with a longer subsidy period. Such costs could include statutory regulations on subsidy length, policy uncertainty from incomplete contracting, or administrative and compliance costs. Whatever the source of the costs, the efficient subsidy duration trades off the marginal cost of extending the subsidy duration against the marginal external value of increased production during a longer subsidy period.[3] Because this behavioral response captures the marginal social benefit of a longer duration, the change in production after the subsidy period is a sufficient statistic for the optimal subsidy duration (see similar result in Costa and Gerard (2021) for evaluating temporary corrective policies in the presence of hysteresis). As a result, policymakers should establish longer subsidy periods in industries with larger expected changes in production, all else equal.

Although our baseline model presents a stylized static economic environment, our findings are robust to relaxing many of our initial simplifications. For example, generalizing to a continuous time setting with discounting of future profits and depreciation of fixed inputs

[3]This decision mirrors trade-offs in the optimal tax systems literature weighing the economic benefits of expanding the tax base against increased administrative cost (Dharmapala et al., 2011; Keen and Slemrod, 2017) and decisions in the politically feasible optimal tax literature trading-off the efficiency of proposed tax reforms against their political support (Scheuer and Wolitzky, 2016; Bierbrauer et al., 2021).

does not change the key insights stated above (though the size and relative importance of the investment subsidy will depend on the discounting and depreciation rates). Similarly, if the value of the externality changes over time (but the subsidy cannot), our main results hold, but the output subsidy rate is equal to the average externality during the subsidy period and the investment subsidy rate is related to the average externality afterward. As such, time-limited output-only subsidies with unadjusted Pigouvian rates may still be approximately optimal in cases with steep discounting, depreciation, or shrinking externality values. We also present extensions with firm heterogeneity, fiscal costs, and non-production externalities. Although our model does account for subsidy-driven substitution between fixed and variable inputs (as in Ganapati et al., 2020) it does not extend to production technologies in which fixed inputs can be repurposed to produce other goods after the subsidy period. Our results also do not extend to cases presenting strategic dynamics (Langer and Lemoine, 2022), output price uncertainty (Yi et al., 2018), or market power (Cardoso, 2024). As such, our general results may need to be adapted in market where these features are first-order considerations.

Our empirical application studies the US wind industry and the Renewable Energy Production Tax Credit (PTC). As one of the largest output subsidies in the world, the PTC plays a key role in the global energy transition. The PTC subsidy period is also less than half of a wind turbine's lifespan.[4] Furthermore, wind energy is a theoretically interesting setting. Given the production technology, changes in power production should be relatively small because turbines are essential, wind is free, and after the subsidy period there are relatively few margins for response (e.g., improved maintenance, forecasting, optimization, etc.). Because time-limited subsidies are only optimal when the change in production is small, the wind industry provides a limiting case in which we test the model. If firm behavior changes in the wind industry, time limits will likely cause larger distortions in more elastic industries.

We estimate the change in electricity generation after the PTC's ten-year subsidy period, showing that wind facilities reduce their output by 5-10%. This response may seem surprising given the production technology, but it highlights the importance of subsidy duration when considering optimal policy. This response also has broader market implications. Each month, PTC ineligibility results in over 500 Gigawatt hours (GWh) of forgone production and externality benefits, amounts that will increase as additional turbines age out of the subsidy period.

Our paper makes four main contributions. First, our results highlight additional theoretical justifications for taxing and subsidizing inputs. Although production efficiency suggests it is only efficient to subsidize output (Diamond and Mirrlees, 1971; Parish and McLaren, 1982; Ganapati et al., 2020), we show this is no longer true when output subsidies have

[4]The PTC subsidy period is 10 years, but wind turbines last for 20-30 years (Wiser and Bolinger, 2021).

time limits. This reversal relates to findings in the behavioral optimal tax and tax systems literatures where production efficiency must be weighed against behavioral biases (Farhi and Gabaix, 2019) or differential evasion opportunities (Emran and Stiglitz, 2005; Best et al., 2015). Subsidizing investment may be a particularly important response to policy uncertainty given its large effects on investment decisions (e.g., Kellogg, 2014; Baker et al., 2016; Handley and Li, 2020; Chen, 2023; Wang et al., 2023).[5]

Second, we document a core complementarity between output and investment subsidies. Empirical research has generally considered output and investment subsidies as substitutes. Case studies have found that while investment subsidies do distort production efficiency (e.g., Burr, 2016; Aldy et al., 2023), other considerations can justify their cost effectiveness (see Parish and McLaren, 1982; Dunne et al., 2013; Yi et al., 2018; De Groote and Verboven, 2019). Our efficiency justification for investment subsidies reveals that combining output and investment subsidies may work better than using either in isolation. Of course, subsidizing both output and investment is also efficient when investment subsidies directly correct a second externality (as in Acemoglu et al., 2012, 2023), but in our setting we show how investment subsidies efficiently correct a production externality when output subsidies are time-limited.

Third, our results expand our understanding of optimal policy under imperfect externality targeting. According to the targeting principle, whenever externality-generating commodities are taxable, the optimal policy is separable between a Pigouvian correction and any other taxes (Sandmo, 1975; Kopczuk, 2003).[6] Although this logic is often used to calibrate output subsidies, time limits disrupt targeting in the real world. We show that using an investment subsidy restores a targeting-like result—even when not all units are targeted. In more general settings, the efficient policy may also choose to target fewer units in order to avoid administrative or fiscal costs. These results build on renewed interest in corrective taxation with imperfect targeting. But whereas most settings feature an inability to tax the externality-generating margin (e.g., Rothschild and Scheuer, 2016; Griffith et al., 2019; Dubois et al., 2020; Jacobsen et al., 2020),[7] our setting features an inability to tax all

[5]While not related to time limits, we also find that subsidizing both investment and output can be an optimal response to firm heterogeneity, budget concerns, and network externalities—other quantitatively large concerns. For example, Farrell and Klemperer (2007), Seto et al. (2016), and Acemoglu et al. (2023) consider network effects and lock-in—particularly in the case of carbon intensive technologies.

[6]As shown in case studies of commodity taxation (Sandmo, 1975), international tax policy (Dixit, 1985), public good provision (Bovenberg and van der Ploeg, 1994), and joint income and commodity taxation (Cremer et al., 1998), all generalized by Kopczuk (2003).

[7]Empirical examples include taxing fuel efficiency rather than emissions (Langer et al., 2017; Jacobsen et al., 2020), taxing beverage volume rather than sugar or alcohol content (e.g., Grummon et al., 2019; Dubois et al., 2020; Miravete et al., 2020; O'Connell and Smith, 2021), or using attribute-based regulation (Ito and Sallee, 2018; Kellogg, 2020).

externality-generating units.[8]

Finally, our empirical results extend conversations about renewable energy subsidies by showing how time limits affect power production. Of the many papers studying subsidies for wind energy,[9] two document differences in production between firms that receive output versus investment subsidies (Aldy et al., 2023; Petersen et al., 2024), and only Hamilton et al. (2020) consider the time-limited nature of the PTC. Although mainly focused on turbine degradation over time, Hamilton et al. (2020) also document an immediate drop in output after the PTC subsidy period. Our empirical approach builds on these results to estimate long-run (rather than contemporaneous) impacts and does so using an empirical approach robust to both confounding intertemporal policy changes and cross-cohort differences in effects (Sun and Abraham, 2021)—resulting in effects that are roughly twice as large.[10]

Our empirical results also have implications for larger discussions about optimal industrial and energy policy. In the PTC context, production responses after the subsidy period will lead the current fleet of wind turbines to under-produce over 220,000 GWh over the next two decades: enough renewable energy to power every household in the United States for over 18 months. If subsidies are part of a proposed energy transition, accounting for the effect of time limits on production is critical for designing optimal policy. Similarly, industries with expensive inputs like agriculture and manufacturing may have even more elastic changes in production, suggesting that subsidy periods (and investment subsidies) may have even larger welfare implications for industrial policy.

The remainder of the paper is organized as follows: Section 2 presents our model of time-limited subsidies and optimal tax results, Section 3 discusses how our model can (and cannot) be extended, Section 4 contains our empirical application to the wind industry, and Section 5 concludes.

2. Optimal Time-Limited Subsidies

This section builds an intuitive optimal tax framework where firms' investments may outlast the duration of a subsidy period. We characterize the optimal rates for output and investment subsidies given the duration of the output subsidy and derive the socially optimal duration given institutional frictions. Although we use subsidy-oriented language, corresponding arguments hold for taxation.

[8] In addition to time limits, other empirical examples could include taxing formal markets but not informal markets and only having corrective taxes in some geographical jurisdictions.

[9] See for example Schmalensee (2012); Johnston (2019); Abrell et al. (2019); Helm and Mier (2021).

[10] Quantitatively similar results can be derived from Hamilton et al. (2020) for some earlier cohorts by interpreting changes in degradation rates as part of the dynamic effect.

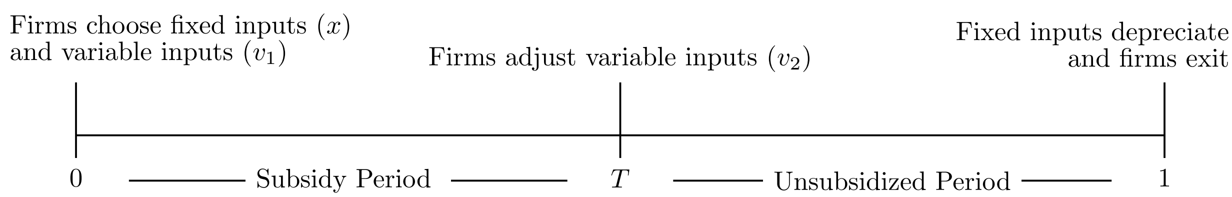
2.1 Model Setup and Intuition

We begin with a model where perfectly-competitive, price-taking firms produce two outputs, a homogeneous externality-generating output, q, and a numeraire good, z. Market demand is characterized by a representative consumer with quasi-linear utility over the two goods and income from firm profits and common factor markets. The social planner attempts to maximize welfare given the social costs and benefits of subsidization. Other economic environments to which this model can (and cannot) be extended are discussed in Section 3.

2.1.1 Firm Problem

Let a representative externality-generating firm make an irreversible investment in fixed inputs, x, and choose contemporaneous variable inputs v_t. Market supply is characterized by profit maximization given a production technology, $q(x, v)$; prices; and a policy vector, $\theta = (\tau^i, \tau^o, T)$. The policies include an investment subsidy rate for the fixed input, τ^i; an output subsidy rate, τ^o; and the duration of the output subsidy T (as a fraction of the capital life).[11] We assume that the firm faces certain input and output prices, does not discount future profits, and experiences one-hoss-shay depreciation in the fixed input. We also assume that $q(x, v)$ satisfies standard regularity conditions (for details see Online Appendix B.1).

Figure 1: Firm Decisions in Response to Time-Limited Subsidies



Note: This figure displays the economic problem presented when output subsidies have time limits. Firms make investment and input decisions with the recognition that only units produced during the subsidy period (before T), will receive the output subsidy. As such firms produce at a level of $q_1 = q(x, v_1)$ during the subsidy period and $q_2 = q(x, v_2)$ afterward.

Under these conditions the dynamic problem simplifies to a two-period model, visually depicted in Figure 1. During the subsidy period (duration T) every unit the firm produces qualifies for the output subsidy, but units produced in the second period (duration $1-T$) are unsubsidized. Knowing this, firms choose fixed inputs, x; an initial level of variable inputs, v_1; and an adjusted level of variable inputs, v_2. Fixed inputs are used to produce both subsidized

[11] As show in in Appendix Table A.1, subsidies with limited durations are ubiquitous, and these limits are often much shorter than the expected capital life.

and unsubsidized units of output, but variable inputs correspond only to production during the subsidy period, or after the subsidy period. Thus the level of production during the subsidy period is $q_1 = q(x, v_1)$ and is $q_2 = q(x, v_2)$ after the subsidy period.

The firm's problem in this two-period model is

$$\max_{x,v_1,v_2} \pi(x, v_1, v_2; \theta) = T[(p_1 + \tau^o)q(x, v_1) - m_1 v_1] + (1-T)[p_2 q(x, v_2) - m_2 v_2] - x(c - \tau^i) \quad (1)$$

During the subsidy period, the firm buys variable inputs at price m_1 and sells output at $p_1 + \tau^o$. After the subsidy period, the firm buys variable inputs at price m_2 and sells output at p_2. The after-subsidy price of fixed inputs is $c - \tau^i$.

The set of subsidy policies (θ) has important implications for firms. The first-order conditions in Appendix B.1 show that time-limited subsidies in particular can create two problematic incentives. We call the first problem *under-investment*. Because the same fixed inputs produce both subsidized and unsubsidized units, firms will invest in fixed inputs less than is socially optimal. We call the second problem *under-utilization*. Because the output subsidy ends, production after the subsidy period will be inefficiently low (given production capacity).[12] Addressing these problematic incentives is at the heart of optimal policy.

2.1.2 Social Planner Problem

Consider a social planner trading off consumer surplus, producer surplus, and the external benefits of production with the costs of a subsidy program. We assume that each unit of q produces a constant marginal externality, γ, and that the marginal cost of public funds, λ, is equal to one. As such, the first-best policy sets $T = 1$, $\tau^o = \gamma$, and $\tau^i = 0$ (Pigou, 1920). In a second-best world, however, this type of Pigouvian policy is efficient only if there are no other frictions. By revealed preference, the ubiquity of time-limited policies suggests that other frictions may often compromise ideal Pigouvian policies.

With this in mind, we include an additional social cost in our model, $\phi(T)$. We assume $\phi(T)$ is increasing and convex in the subsidy duration, T. Online Appendix Cdiscusses and compares possible sources for these costs including direct costs of administration or compliance (e.g., Dharmapala et al., 2011), statutes that encourage time limits,[13] *ex post* costs from incomplete contracting (e.g., Persson and Svensson, 1989; Alesina and Tabellini,

[12] Note that under-utilization also violates the production-efficiency rationale for subsidizing output rather than investment (see Diamond and Mirrlees, 1971) because after the subsidy period output will be produced with more than the cost minimizing amount of capital.

[13] For example, federal mandates in Germany require time limits on subsidies (German Federal Ministry of Finance, 2022). Similarly, in the United States, only outlays meeting time-specific objectives can pass via reconciliation (see discussion in Wessel, 2021).

1990; Battaglini and Harstad, 2020), and policymaker career concerns.

Equation 2 depicts the costs and benefits of each policy, given the equilibrium responses by firms and consumers $(x^f, v_1^f, v_2^f, q_1^f, q_2^f)$.

$$\max_{\tau^i,\tau^o,T} \mathcal{W}(\tau^i, \tau^o, T) \equiv \max_{\tau^i,\tau^o,T} \Pi + U + \gamma Q - TC - \phi(T) \tag{2}$$

The first three terms represent the benefits from each policy: the profits of the representative firm, the surplus of the representative consumer, and the external benefit of total production, where $Q = Tq_1^f + (1-T)q_2^f$. The fourth and fifth terms reflect the costs: the fiscal cost of funding the subsidy, $TC = T\tau^o q_1^f + \tau^i x^f$,[14] and the social cost of choosing a policy with duration T, $\phi(T)$. In this setup, the social planner trades off these social costs of a longer subsidy duration against the costs of imperfect targeting and production inefficiency created by a shorter subsidy duration.

2.2 Optimal Subsidy Policies

To build intuition, this section solves four optimal policy problems in order of increasing complexity. First, we describe the optimal investment-only subsidy and the optimal output-only subsidy. After considering these policies separately, we then describe the optimal combined subsidy. Finally, we characterize the optimal subsidy duration given the social costs that generate time limits and derive a sufficient statistic for subsidy duration.

2.2.1 Optimal Investment-Only Subsidy

A social planner trying to maximize welfare with only an investment subsidy will face a tradeoff between increasing the quantity of externality-generating units and raising costs by distorting production efficiency. Proposition 1 characterizes the optimal subsidy.

Proposition 1. Optimal Investment-Only Subsidy. Under the assumptions in Section 2.1, if $\tau^o = 0$, then

$$\tau^{i*} = \gamma \frac{\mathrm{d}q_2^f}{\mathrm{d}x^f}$$

Proof in Online Appendix E.

Proposition 1 illustrates how an investment subsidy can be used to correct a production externality. The key response is $\frac{\mathrm{d}q_2^f}{\mathrm{d}x^f}$, how much production responds to additional

[14]Because q_1^f reflects the equilibrium *level* of production, the total *amount* produced during the subsidy period is Tq_1^f.

investment.[15] The optimal investment subsidy is equal to the marginal external benefit of additional investment—just as a Pigouvian output subsidy rate captures the marginal external benefit of additional production. Note that this is a total derivative, not a partial derivative, so the change in quantity includes the direct effect of additional x, as well as any endogenous change in v_2 caused by increasing x.

This characterization also captures the production-efficiency trade-off created by subsidizing investment. To see this, note that $\frac{\mathrm{d}q_2^f}{\mathrm{d}x^f}$ can be rewritten as $\frac{\mathrm{d}q_2^f}{\mathrm{d}\tau^i}/\frac{\partial x^f}{\partial \tau^i}$. Therefore, the optimal investment subsidy will be larger in settings with large externalities or production responses ($\frac{\mathrm{d}q_2^f}{\mathrm{d}\tau^i}$), all else equal. The optimal investment subsidy will be smaller in settings with larger marginal investment distortions ($\frac{\partial x^f}{\partial \tau^i}$), all else equal.

2.2.2 Optimal Output-Only Subsidy with a Given Duration

On the other extreme, a social planner trying to maximize welfare with only a time-limited output subsidy will face a tradeoff between increasing the quantity of externality-generating units and creating over-utilization during the subsidy period. Proposition 2 characterizes the optimal subsidy.

Proposition 2. Optimal Output-Only Subsidy. Under the assumptions in Section 2.1, if $\tau^i = 0$ and T is fixed, then

$$\tau^{o*} = \gamma + \gamma \frac{1-T}{T} \frac{\frac{\mathrm{d}q_2^f}{\mathrm{d}\tau^o}}{\frac{\mathrm{d}q_1^f}{\mathrm{d}\tau^o}}$$

Proof in Online Appendix E.

This characterization of τ^o as a function of the subsidy duration captures the trade-offs time limits induce. The first term reflects the base subsidy rate, targeted to the marginal external value of production. The second term adjusts the subsidy rate up to compensate for under-investment. If $T = 1$, all units are subsidized, and policy simplifies to the Pigouvian first best, $\tau^o = \gamma$. However, as T approaches 0, τ^o diverges to infinity. Intuitively, this is because τ^o can only change production after the subsidy period by incentivizing investment, and if the subsidy period is shorter, only a larger subsidy rate can change investment.

The extent to which τ^o should be adjusted in response to a time limit depends on how effectively the subsidy affects production after the subsidy period. This efficacy is described by the ratio $\frac{\mathrm{d}q_2^f}{\mathrm{d}\tau^o}/\frac{\mathrm{d}q_1^f}{\mathrm{d}\tau^o}$. This ratio will be small if investment is unresponsive to the output

[15] This includes the direct effect of investment as well as the indirect effect as firms adjust variable inputs to respond to changes in the level of fixed inputs and the price of output.

subsidy, if the marginal product of capital is small, or if fixed and variable inputs are more substitutable. In this case, raising the output subsidy rate will not increase production after the subsidy period, so τ^{o*} remains close to γ. On the other hand, when the ratio is close to 1, τ^{o*} may be quite large with shorter durations.

One special, policy-relevant case is when production is based entirely on fixed inputs. In this case, the ratio $\frac{\mathrm{d}q_2^f}{\mathrm{d}\tau^o}/\frac{\mathrm{d}q_1^f}{\mathrm{d}\tau^o} = 1$, and the optimal subsidy will be $\frac{\gamma}{T}$. Because variable inputs are irrelevant, the resulting allocation is welfare-equivalent to both the Pigouvian subsidy (with no time limit) and the investment subsidy.[16] This insight may reflect the policy intuition behind time-limited subsidies in industries with large fixed costs and relatively small variable costs.

2.2.3 Optimal Combined Subsidy with a Given Duration

Given the shortcomings of investment-only and time-limited output-only subsidies, we now assess the benefits of combining both policy instruments. Whether there are gains from having multiple instruments is not *ex ante* obvious. For example, without time limits ($T = 1$), it is well known that it is optimal to only subsidize output even when an investment subsidy is available (Diamond and Mirrlees, 1971). At the same time, there are intuitive arguments for gains from using both instruments. Investment subsidies can target x but cannot directly affect q_1 or q_2 (resulting in the breakdown of production efficiency) whereas output-only subsidies can target q_1, but cannot directly affect x or q_2 (resulting in under-investment and under-utilization). Proposition 3 shows that there are gains from targeting both x and q_1.

Proposition 3. Optimal Combined Subsidy. Under the assumptions in Section 2.1, if T is fixed, then

$$\tau^{i*} = (1-T)\gamma \frac{\mathrm{d}q_2^f}{\mathrm{d}x^f}$$
$$\tau^{o*} = \gamma$$

Proof in Online Appendix E.

This result has two major implications. First, it shows that an investment subsidy can correct the under-investment problem created by time limits. When $T = 1$, $\tau^{i*} = 0$ because there is no need to subsidize investment (consistent with Diamond and Mirrlees, 1971). Otherwise the optimal investment subsidy is positive—even though this creates production

[16]Although these policies create the same amount of production, the subsidizing output transfers more money to firms (see Parish and McLaren, 1982). This is welfare-irrelevant because we assumed $\lambda = 1$.

inefficiency. Investment subsidies are worth this cost because they address under-investment (and increase production after the subsidy period) more effectively than increasing the output subsidy rate. When $T = 0$, and all output is unsubsidized, τ^{i*} takes the same form as in Proposition 1.

Second, combining subsidy instruments restores a targeting-like calibration for the output subsidy even though targeting is imperfect. Whereas the τ^{o*} in Proposition 2 increased above the marginal externality whenever $T < 1$, it now remains constant. The optimal output subsidy is $\tau^{o*} = \gamma$ for all values of T—whether all or almost none of the production is targeted. The optimal response to changes in the subsidy duration is captured in τ^{i*} and is fully separable from τ^{o*}. This separability is reminiscent of many other results in the targeting literature (e.g., Sandmo, 1975; Dixit, 1985; Bovenberg and van der Ploeg, 1994; Cremer et al., 1998; Kopczuk, 2003), but in our setting the appropriate tax instruments can restore a targeting-like result *even without perfect targeting.*[17]

Figure 2 depicts a comparison of the optimal policies presented in Propositions 1-3. It depicts the investment subsidy rate and the output subsidy rate as functions of T for cases where the social planner is restricted to only one of the instruments or has both available. Although the degree of curvature will depend on the production technology, $q(\cdot)$, the intercepts and limits reflect the optimal policies in general.

2.2.4 Optimal Subsidy Duration

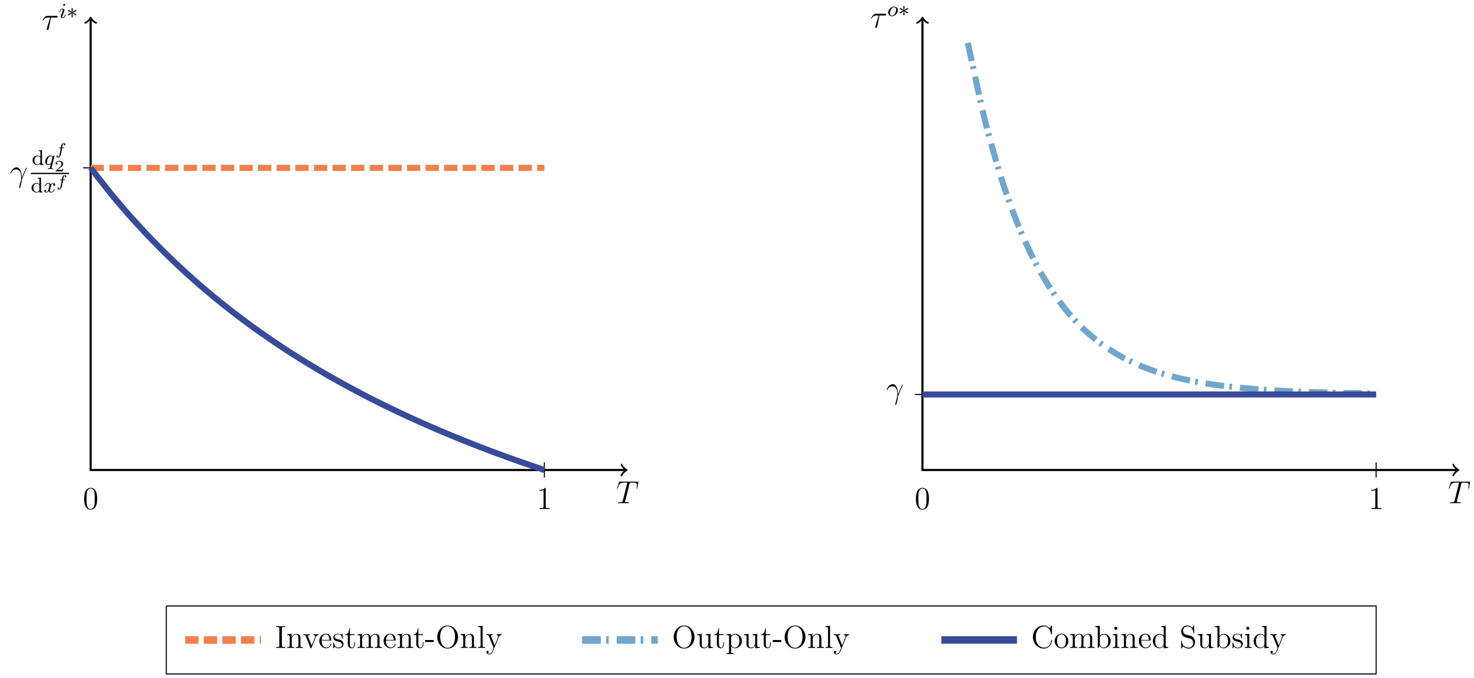
Now consider the optimal subsidy duration. Recall that although the first best policy chooses $T = 1$ and targets perfectly, revealed preference suggests constraints that make this infeasible. As such, the optimal duration for a given subsidy weighs the welfare from better targeting against the social costs of a longer duration, $\phi(T)$. We use a second-best interpretation of the results that follow, where institutional frictions make the first-best, Pigouvian policy too costly, and the social planner re-optimizes accordingly.[18] Alternatively, the derivative, $\phi'(T)$, can be interpreted as the shadow-value of relaxing an arbitrarily imposed time-limit T without requiring $\phi(T)$ to capture any specific cost or shape restriction.

Proposition 4. Optimal Subsidy Duration. Under the assumptions in Section 2.1 and a first-order Taylor approximation where q_v and u are locally linear, if there are no exogenous demand or input-price shocks at T, then the optimal subsidy duration satisfies the following

[17] This restoration of targeting through the application of additional (if not directly related) tax instruments reflects similar insights about the power of multiple instruments from elsewhere in the optimal tax literature (e.g., Rothschild and Scheuer, 2016; Scheuer and Werning, 2016, etc.).

[18] The choice of subsidy duration is technically a "first-best" problem if $\phi(T)$ is considered to be a real social cost as in the optimal tax systems literature (e.g. Keen and Slemrod, 2017).

Figure 2: Comparing Optimal Time-Limited Subsidies



Note: This figure shows the optimal rates for τ^i and τ^o as functions of the subsidy duration, T. Three policies are represented; the optimal investment-only subsidy, the optimal output-only subsidy, and the optimal combined subsidy. For the combined subsidy, note that whereas the total value of the investment subsidy is decreasing in T, the total value of the output subsidy is increasing—even though the per-unit rate is constant.

at interior solutions:

$$\phi'(T^*) = -\gamma\big[q_2^f - q_1^f\big] \equiv -\gamma\Delta q(\theta^*)$$

with corner solutions characterized by

$$T^* = 1 \quad \text{if } \phi'(1) \leq -\gamma\Delta q(\theta^*|_{T=1})$$
$$T^* = 0 \quad \text{if } \phi'(0) \geq -\gamma\Delta q(\theta^*|_{T=0})$$

Furthermore, sufficiently elastic demand guarantees uniqueness.
Proof in Online Appendix E.

This characterization shows that the optimal T trades off the costs and benefits of a longer subsidy duration. A longer subsidy period will increase the amount of the externality good produced. At the same time, it will also cost more to implement. The optimal duration equates these marginal costs and benefits.

Proposition 4 reveals a sufficient statistic for the optimal subsidy duration. For a social planner who values production at γ, the change in production after the subsidy period (Δq) is a sufficient statistic for T^*. In interior solutions, the external benefits of marginal units are exactly equal to the marginal duration cost. In corner solutions the externality benefits are greater than all costs ($T^* = 1$) or less than any cost ($T^* = 0$). This result suggests an elasticity rule for optimal subsidy durations: policymakers should choose a longer subsidy duration in industries with more elastic changes in production (and, thus, externality generation) and a shorter subsidy duration in markets with less elastic production.

This elasticity rule can deepen our understanding of empirical studies of subsidization. For example, Lohawala (2023) studies the US Electric Vehicle Tax Credit which remitted \$7,500 to buyers of new electric vehicles, but which phased out after manufactures hit predetermined quotas.[19] Proposition 4 suggests that this time limit could have large welfare effects if the market for electric vehicles is highly elastic—and that is exactly what Lohawala (2023) finds. For example, after the subsidy period, sales of the Chevy Volt drop by 50% (See Panel (a) of Appendix Figure A.1[20]). The change in Chevy Volt production after the subsidy period—and its eventual exit—highlight the potential costs of subsidies with short durations in industrial policy settings.

[19] In perfect competition with no learning spillovers, an endogenous quota should function similarly to an exogenous time-limit. If not, our maintained assumption that the end of subsidization causes the changes in production may not hold. For example, Lohawala (2023) show some evidence that firms slow production to avoid hitting the quota but then accelerate production during the phase out. This type of behavior could inflate the estimated change in production after the subsidy period.

[20] Appendix Figure A.1 in our paper is adapted directly from Appendix Figure 7 of Lohawala (2023), with output reported in percent changes.

Note that the equilibrium Δq reflects the elasticity of both supply and demand. In markets with inelastic demand, subsidy pass-through to consumers will restrict the change in production after the subsidy period. Therefore, the optimal subsidy duration will be shorter (and the optimal investment subsidy larger) when either demand or supply is less elastic. In our empirical application, wind generated electricity is highly substitutable with other electricity sources, so Δq should be entirely driven by the supply-side responses. But in industries with less elastic demand (e.g., manufactured goods), the case for shorter time limits and more investment subsidies may be stronger.

From a policy-making perspective, the fact that this change in quantity depends on the price elasticities may be particularly useful. The dependence on primitives means that engineering-based estimates of supply and model-based estimates of demand can inform the optimal policy—before implementing a policy experiment to identify the sufficient statistic. This approach may be particularly useful because firms who know they are participating in a policy experiment have an incentive to manipulate their elasticity to secure larger subsidy transfers.

2.3 Time Limits and Policy Uncertainty

In this subsection we consider whether the results in Propositions 3 and 4 apply when subsidies have a limited *expected* duration rather than a predetermined time limit. Many tax and subsidy policies are repealed, rather than ending with a predetermined deadline. We show that the optimal output and investment subsidies in response to uncertain durations are *ex ante* equivalent to statutory time limits on duration.

To illustrate this connection, consider a slight variation to our original model. A risk-neutral representative firm still makes an irreversible investment decision and a series of production decisions over the lifetime of their initial capital. But rather than knowing the expiration date of the output subsidy (T), the firm faces probability (characterized by ρ) that the output subsidy will be repealed by the end of the capital life, where the timing of the repeal is uniformly distributed. If the subsidy is repealed, the firm will be able to adjust their variable input, v_t, but not the fixed input, x.

Corollary 1. Under these new assumptions, then

$$\tau^i = \gamma\rho\frac{\mathrm{d}q_2^f}{\mathrm{d}x^f}$$
$$\tau^o = \gamma$$

Proof in Online Appendix E.

Corollary 1 shows that both uncertain and time-limited subsidies feature the same economic incentives. The optimal subsidies are identical with $\frac{\rho}{2}$ replacing $(1-T)$ from Proposition 3. As such, any output subsidy with positive probability of repeal should be accompanied by an investment subsidy. Because uncertainty reduces the expected returns to investment in the same way as a statutory time limit, the (*ex ante*) optimal subsidy corrects this under-investment with an investment subsidy. The higher the probability of repeal, ρ, the larger the optimal investment subsidy. The same is true for taxes. A production tax that could be repealed should be accompanied by an investment tax on the relevant fixed inputs.

This application to policy uncertainty deepens our understanding of empirical results about repealed tax policy. For example, Schmacker and Smed (2020) and Schmacker and Smed (2023) study the repeal of Danish taxes on soft drinks, finding that sales increase by about 25% after the tax is fully repealed (see Panel (b) of Appendix Figure A.1[21]). Seen through the lens of Corollary 1, this response highlights the risk of only taxing production (or consumption) when the policy may change before the end of firms' capital life. Producers who expect a sin tax to be repealed will over-invest in "sin making" capital. Corollary 1 shows that in this case, social planners who cannot fully commit to a permanent tax on a negative externality good should tax *both* investment and output. For example, Denmark could have taxed the capital used to produce sugary drinks (although at a lower rate if fixed inputs play a smaller role in the production technology).

The idea of taxing both production and investment could be applied to environmental policy as well. As climate and energy policy become increasingly politicized, policies aimed at addressing climate change risk repeal as the tides of political power change. For example, the Australian carbon tax introduced in 2011 was repealed in 2014 (Dayton, 2014). In this context, a policymaker that wants to address climate change, but who is concerned that a carbon tax could be repealed, might also tax investments that extract or burn fossil fuels to prevent over-investment.

3. Model Extensions, Implications, and Limitations

Although the results in Section 2 build strong intuition for time-limited subsidies, there are other economic considerations that do not map neatly into this simplified setting. This section summarizes additional extensions from a more general model. We focus our discussion on the intuitive implications of these generalizations and on the limitations of our modeling framework—providing market examples of both cases. We present a detailed setup of the

[21] This figure is adapted from Appendix Figure 2 of Schmacker and Smed (2020) and Figure 2 (a) of Schmacker and Smed (2023), reporting percentage changes.

general model in Appendix B.2 and formally present additional mathematical results in Appendix D.

3.1 Expanding Firm Dynamics

The first extensions we consider enrich the model dynamics, i.e., they change how firms and the social planner consider the future and make decisions over time. In light of this, we relax the assumptions of infinite patience, one-hoss-shay depreciation, and constant marginal externalities from Section 2. We do this by allowing for capital depreciation, discounting of the future, continuously changing input prices, and changing externalities over time and across (otherwise identical) firms. This subsection discusses three implications of the formal result stated in Proposition 5 of Appendix D.

3.1.1 Depreciation and Discounting

Including depreciation and discounting changes the investment subsidy rate but does not affect separability. Depreciation reduces the social value of subsidizing investment because it lowers the marginal external product of investment after the output subsidy period. Discounting reduces the social value of subsidizing investment because firms and the social planner care relatively less about production after the subsidy period.[22] If firms' fixed inputs are fully depreciated by the end of the subsidy period, or if the social planner does not care about the distant future, subsidizing investment may not affect welfare at all. In this sense, there may be limiting cases when a naive Pigouvian subsidy (i.e., a time-limited subsidy equal to the average externality) might be approximately optimal.

Although this insight is not likely to be relevant for wind energy (wind turbines often last for 20-30 years, much longer than the 10-year PTC time limit), it could be applicable in industries with more rapid depreciation. For example, these extensions can explain why a government wanting to incentivize the production of cutting-edge microchips or semiconductors might only subsidize output even in the presence of a relatively short time limit—the rapid depreciation of fixed inputs through obsolescence essentially eliminates the optimal investment subsidy.[23] In a sense, the capital depreciates so fast that the time-limits no longer bind, and the naive Pigouvian subsidy is close to the first best.

[22]Note that if there is *differential* discounting between firms and the social planner, then the social planner may want to subsidize investment more to correct for their impatience.

[23]Interestingly, in the US, the main subsidies for these industries are investment-only subsidies (e.g., CHIPS and Science Act and FAB Act).

3.1.2 Changing Prices

Allowing input and output prices to vary during and after the subsidy period does not change the analytical expressions for optimal subsidies beyond introducing expectations, but it will affect the optimal investment subsidy rate. Changing prices alter what level of variable inputs satisfy the first-order conditions. For example, falling variable input prices tend to reduce the optimal investment subsidy as long as fixed and variable inputs are complementary. This suggests that if the variable costs are declining fast enough, there may be no reason to subsidize investment—a practice often observed, for example, with agricultural price supports and subsidies.

3.1.3 Heterogeneous Externalities

When externalities change over time or vary across firms, it affects the optimal rates of both output and investment subsidies. Interestingly, these differences reinforce the core separability result from Section 2. When externalities vary, the optimal uniform output subsidy is equal to the average externality during the subsidy period, and the optimal investment subsidy is related to the average externality after the subsidy period. This differentiation underlies the economic intuition behind separability: the optimal policy uses the output subsidy to correct for externality generation during the subsidy period and uses the investment subsidy to correct for externality generation after the subsidy period (or equivalently to correct for the suboptimal subsidy duration).[24]

In practice, there are many settings where externality benefits are heterogeneous and could change the optimal uniform subsidies. For example, wind and solar generation offset (dirtier) coal in some times and places relative to (cleaner) natural gas in others (e.g., Cullen, 2013; Fell et al., 2021; Holland et al., 2022; EIA, 2023). If the key dimension of heterogeneity is intertemporal, and if policymakers expect wind to compete with hydropower, nuclear, and solar after the subsidy period, then there would be no reason to subsidize investment since the marginal externality is zero. In this case, a naive Pigouvian subsidy without an investment subsidy is approximately optimal. On the other hand, if externalities are growing, optimal investment subsidy rates may be dramatically higher. For example, because the external value of electric vehicle use depends on the energy mix of local electricity markets (Holland et al., 2016, 2020), externalities will increase with wind and solar penetration, and so will the optimal investment subsidies for capital to build electric vehicles.

[24] Note that the interaction of heterogeneous externalities and depreciation produces an additional term in the optimal tax formulas related to the correlation between the marginal behavioral response to each subsidy at each time period and the marginal external value at that time. This is a special case of the results discussed in Section 3.3.

3.2 Expanding Policy Considerations

The second set of extensions expand the social planner's optimal policy considerations by relaxing the assumptions that production causes the externality, that $\lambda = 1$, and that the policy space is limited to $\theta = (\tau^o, \tau^i, T)$. We do so by extending our original model to allow for more flexible externalities, revenue-raising frictions, and additional policy instruments (such as non-uniform subsidy rates, subsidies on variable inputs, and policy sunsetting). This subsection discusses Corollaries 2-4 and three implications of the formal result stated in Proposition 6 (all in Appendix D).

3.2.1 Addressing Non-Production Externalities

Policymakers often justify time-limited policies as transitional policies intended to make a "big push" toward a better equilibrium—with a subsidy period that ends once the transition is guaranteed. Rather than correcting a production externality, this logic assumes the presence of a network externality or learning spillover that leads competitive markets to stay in bad equilibria for too long.

When we extend the model to allow for a network externality in addition to a production externality, we find that the optimal policy will separably correct each. We consider network externalities where the stock of previously-constructed capital can affect contemporaneous input costs, technology, demand, or marginal externalities. In such a model, an additive term in the optimal output and investment subsidy rates corrects the network externality. This argument follows similar intuition to Acemoglu et al. (2012), where optimal policy involves both carbon taxes to correct the production externality and research subsidies to account for endogenous technological change. As such, if the main externality is a "big push" rather than a direct production externality, the social planner should subsidize the margin of the network effect—likely at the margin of investment.

In many industries, total investments do impact costs, technology, consumer demand, or externalities through network effects. For example, learning spillovers between wind turbine manufactures have generated significant cost reductions and have advanced the production technology available to the industry (Covert and Sweeney, 2022). Similarly, there is evidence that consumers who see neighbors' rooftop solar investments, invest in more solar panels of their own (Bollinger et al., 2022). In some industrial policy settings, the production capacity itself is even the differentiating criterion between multiple equilibria—as with arguments about supply chain resilience.

3.2.2 Dealing with Revenue Costs

The optimal subsidies characterized up to this point assumed away revenue costs of subsidization. This simplification is common in the literature on corrective taxation (e.g. Griffith et al., 2019) and may hold if the tax and transfer system is optimally calibrated (Jacobs, 2018; Kaplow, 2024) or if revenue is raised using non-distortionary lump-sum taxes. In practice, however, subsidy policies often go to great lengths to avoid inframarginal transfers. This consideration has implications for optimal subsidy rates and durations.

Allowing $\lambda \geq 1$ introduces two changes to the optimal subsidy rates. First, the marginal production externality, $\gamma_{j,t}$, should be divided by λ to account for the welfare costs of raising additional revenue. Second, the optimal subsidy rates have a new additive term reflecting the welfare cost of transfers to firms. These changes demonstrate that the production externality, the time limit, and fiscal considerations can all be corrected separably—extending the results of Kopczuk (2003) to settings with imperfect targeting. As the literature on optimal corrective taxation often relies on such separability to focus on production externalities, it is useful to know such separability can hold even when targeting does not. From a policy perspective, this result also means that it can sometimes be efficient to subsidize output while taxing investment to recoup revenue.

Budget concerns can also shorten the optimal subsidy duration, with interesting implications for targeting. If it is more cost-effective to subsidize investment than output, then the social planner might choose a shorter duration for the output subsidy—with the investment subsidy doing (relatively) more work. This result arises because under decreasing returns to scale, investments affect production more on average than at the margin (see Parish and McLaren, 1982). In the context of targeting, these budget considerations mean that a social planner may choose policies that reduce their ability to target all units in order to reduce budgetary costs. In this sense, the optimal extent of targeting is a policy instrument to be weighed against other costs, similar to the tax base in Keen and Slemrod (2017) and constituents' tax knowledge in Craig and Slemrod (2022).[25]

3.2.3 Adding Additional Policy Instruments

Although the set of possible time-limited subsidies is more expansive than traditional comparisons between either the optimal uniform investment subsidy or the optimal uniform output subsidy, we discuss four additional considerations regarding the choice of policy in-

[25]This insight builds on previous optimal tax research by considering the extent of targeting as a choice variable rather than considering the ideal policy taking perfect targeting as given (e.g., Diamond and Mirrlees, 1971; Sandmo, 1975; Kopczuk, 2003) or examining ways to quantify the welfare losses from imperfect targeting (e.g., Rothschild and Scheuer, 2016; Griffith et al., 2019; Jacobsen et al., 2020; Dubois et al., 2020).

struments.

Adding Variable-Input Subsidies. Given the optimality of combining output and investment subsidies, we consider the efficiency of also subsidizing variable inputs and find that the optimal variable-input subsidy is zero at the optima characterized by Proposition 3. Therefore, the exclusion of a variable-input subsidy in the baseline model is without loss of generality. Of course, this intuition would be complicated if output and variable input subsidies had different time limits, if firms are heterogeneous (leading to gains from targeting), or if there are cost effectiveness concerns.

Allowing Subsidies to Vary Between Firms and Over Time. Since uniform subsidization does not effectively target firm-level or time-period-specific marginal externalities, we also consider non-uniform output subsidies differentiated among firms and time periods. We find that subsidy rates targeting marginal externalities satisfy the necessary conditions for optimality.[26] This is unsurprising, as a differentiated subsidy is a more efficient solution than a uniform one, but varying the subsidy amount by firm and over time may be infeasible in many situations.

Subsidy Phase-Out. Subsidies sometimes end in more complicated ways than expiring after a set number of years. In practice, it is common to reduce subsidy rates over time. For example, the original subsidies for electric vehicles phased down from $7,500 to $3,750, and after 2016, each cohort of wind facilities claiming the PTC received 20% smaller output subsidies.[27] We find that this type of policy is not justified by discounting or depreciation, but can be optimal when the externality itself is changing over time.

Subsidy Sunsetting. In practice the subsidy period for multiple firms may end on a shared date—a policy approach often called subsidy "sunsetting." For example, until it was made permanent in 2015, the Research and Experimentation Tax Credit had been set to expire for all firms, regardless of the year in which they began production. This practice differs from the case studied in our main model because sharing an end date creates variation in the duration of subsidy period between firms from different cohorts. When we compare the optimal time-limited subsidy rates between these two cases, we find the analytical expressions are the same; however, economic primitives like the nature of technological progress will have greater quantitative importance when a policy sunsets.

Reducing Frictions. Given the second-best nature of Propositions 3 and 4, policymakers may want to explore options to facilitate first-best policies. Our analytical results take the administrative and institutional duration costs as given, but systematic changes reducing

[26]They also eliminate the adjusted covariance terms mentioned earlier and discussed in more detail in subsection 3.3.

[27]Some of the phaseout was retroactively reversed in 2020 and 2021.

these frictions would also generate welfare gains. The size of those gains is shown in Proposition 4, where $\phi'(T)$ can be interpreted as the social benefit of relaxing an arbitrarily imposed time limit.[28] Reducing or eliminating the barriers that necessitate time-limited subsidies could move policy toward the first-best allocation. Since the same institutional frictions could affect policy decisions in many markets, the benefits of reducing frictions (and thus implementing better policies) may be quite large.

3.3 A Note on Firm Heterogeneity and Targeting

While the unidimensional extensions above are fairly intuitive, the optimal tax formulas become more complex when they interact with firm heterogeneity. To relax the assumption of a representative firm entering in one time period, we consider a general model with overlapping cohorts of heterogeneous firms, in addition to the extensions above. Firm heterogeneity is captured by production technology, but firms still produce a homogeneous good in perfectly competitive markets. To discipline entry, we assume that each period has a set of short-lived potential entrants that must pay an entry cost, and that the set of entrants is not affected by the subsidy policy (see details in Appendix B.2).

In the presence of these complications, the optimal subsidy rates must be adjusted based on the covariance between the marginal externality and the marginal effects of subsidies, similar to the adjustments in Diamond (1973). Intuitively, the ideal policy accounts for how much behavioral change each tax instrument induces among high-externality versus low-externality firms and time periods. Because there are two policy instruments, however, the optimal policy must account for cross-policy effects between subsidies (analogous to those in Griffith et al. (2019))[29] and for the possibility of improved targeting among heterogeneous firms (see details in Proposition 6). These adjustments will be zero if the marginal externality is constant or if it varies for reasons unrelated to the production function.

Interestingly, this result also implies that a social planner will choose to subsidize (or tax) investment for targeting reasons, even when time limits extend beyond the capital life. Whether these adjustments are quantitatively important will depend on the policy setting. For example, in industries like wind energy where technologies are fairly similar across space, the heterogeneity will likely be small relative to the average externality and may not meaningfully affect the optimal policy. On the other hand, in settings like alcohol

[28] In the case that the subsidy duration was not optimally determined or $\mathcal{W}$ is not globally convex, we can still characterize this shadow value on extending T even when we can no longer characterize an optimal duration.

[29] Increasing one subsidy rate will reduce the optimal rate of the other, and the cross-policy adjustment accounts for this second change. Our cross-policy effects depend on the responsiveness of production and investment to each policy instrument, whereas those in Griffith et al. (2019) depend on cross-price elasticities.

taxation with more heterogeneity, adjustments may significantly alter optimal subsidy rates. That said, when covariances are large, the social returns from implementing non-uniform or differentiated subsidies (as in Griffith et al., 2019) may dominate those from adjusting uniform subsidies to account for heterogeneity.

3.4 Limitations of Modeling Choices

While the extended results cover a broad range of possible economic scenarios, the more fundamental aspects of the model setup still imply some restrictions worth discussing. This subsection addresses what we see as the most relevant restrictions.

3.4.1 Firm-Based Restrictions

First, while the regularity conditions imposed on the production technology are standard, assuming $q = q(x, v)$ also requires that inputs only create one output and both inputs and output can be directly measured and subsidized. This makes sense in a setting like the wind industry where turbines only have one purpose and where measuring energy produced is relatively straightforward, but the usefulness of investment subsidies may be undercut by evasion or shifting responses. For example, if automakers' capital can be used in producing multiple goods (with different externalities), output shifting may eliminate the external value of additional investment. Input shifting can likewise reduce the social value of subsidizing investment. For example, if wind turbines are subsidized but the rental costs of land are not, turbines may produce less because of wake effects (over-crowding). These concerns might be exacerbated by evasion or imperfect compliance with one or more of the tax or subsidy instruments (as in Emran and Stiglitz, 2005; Best et al., 2015).

Second, in our model there are no pricing complications from market power or price uncertainty. Assuming that no individual firm has market power in output markets allows us to focus on externality correction. Market power would add an additional source of under-production (as in O'Connell and Smith, 2021). Input markets are also competitive, with exogenous prices. This poses a possible complication to the wind-energy setting because although output markets are highly competitive, there are very few firms that manufacture turbines. Finally, although prices may vary, there is no price uncertainty. This simplification highlights our efficiency justifications for non-Pigouvian corrective policy even given perfect certainty,[30] but it may limit the applicability of our results to some markets with very unpredictable prices, like agricultural commodities.

[30] Other arguments to deviate from Pigouvian policy in response to price uncertainty do exist (if not for efficiency)—for example, to improve cost effectiveness (as in Yi et al., 2018).

The last key assumptions have to do with entry. Even in our general model, we make strong assumptions about entry to focus on the economic distortions caused by time-limits. For instance, we assume there are no dynamic entry considerations. This temporal independence can occur if potential entrants can either enter in a specific period or not at all.[31] There are settings in practice, however, in which firms strategically wait to enter later. Addressing the incentive to strategically delay will be more important in settings where entry costs are low and the economic environment is changing quickly, like solar power (see Langer and Lemoine, 2022), and less intense in setting where entry opportunities are more scarce.

We also assume that subsidy rates do not affect the composition of entrants. In markets with very rich heterogeneity, subsidy rates may induce potential entrants with capital intensive technology to enter instead of those who rely more on variable inputs (or vice versa). If differences in production technology are correlated with marginal externalities, the optimal policies we present could induce too much entry by certain types of firms. This is likely a second-order consideration in most settings but would require policy adaptions.[32]

3.4.2 Welfare Restrictions

Our formulation of the welfare function implies a handful of small restrictions previously discussed as well as one additional assumption worth noting. These restrictions include using quasi-linear utility (which eliminates income effects from policy changes), including a representative consumer (which removes redistributive considerations), and the assumed convexity of $\phi(T)$ (which we discuss in Appendix C). Although these assumptions could all be relaxed, they allow us to maintain our focus on the role of time limits in corrective policy.

Additionally, our formulation of welfare implicitly assumes that the marginal externalities are produced by the targeted good. While this sounds trivial, in the alternative energy setting, corrective subsidy policy is technically a second best solution. Because energy is generated by multiple sources and demanded inelastically, changes in energy production from one (clean) source offset production from another (dirtier) source.[33] Empirically, we consider subsidies for wind energy because they are prevalent policy tools, but technically it would be optimal to tax emissions instead. In the energy market, the marginal externality, $\gamma_{j,t}$, could therefore be interpreted as the relative externality between $q_{j,t}$ and the (additively separable) numeraire good.

[31] This assumption is sometimes called "short-lived" potential entrants in IO models (e.g., Doraszelski and Satterthwaite, 2010).

[32] From an optimal policy perspective, abstracting from entry complications could be justified if the policymaker can provide lump-sum subsidies to different types of firms in order to correct either distortion.

[33] Cullen (2013) and Fell et al. (2021) show that wind energy does, in fact, offset dirty energy in the status quo.

4. Empirical Application: Wind Energy and the 10-Year PTC

Given the theoretical results in Sections 2 and 3, the welfare implications of time-limited subsidies hinge on an empirical question: How big are changes in production after the subsidy period? Section 4 answers this question in the context of the US wind energy industry, describing the relevant features of the industry, estimating the change in production, then exploring the implications for energy markets.

4.1 A Brief Introduction to the US Wind Industry

4.1.1 Background and Motivation

Wind developers make investment and production decisions, deciding how many turbines to build and how to operate them. In the United States, investment costs average \$0.8-1.5 million per megawatt (MW) of capacity and are paid at the outset of the project (Wiser and Bolinger, 2021). These costs include turbine purchase and installation, interconnection costs, and balance of plant. For the years in our sample, the average ratio of production to capacity, called the capacity factor, was between 30-36% (Wiser and Bolinger, 2021). Wind farms sell electricity they produce for around \$40 per MWh on average and receive an additional \$25-40 per MWh in subsidies. Production mainly depends on wind speed, but operation and maintenance costs average \$7-10 per MWh (Wiser and Bolinger, 2021).[34] The main margins affecting production, conditional on investment, are maintenance decisions, speed of repairs, and subscribing to expensive forecasting and optimization programs.

This capital-intensive production technology makes wind a theoretically interesting application for our model. The welfare losses from time-limits are proportional to the change in production after the subsidy period. Because wind energy generation is fixed-input intensive and wind is free, the change in production should be relatively small. In this sense, the wind industry is a limiting case in which to test the model. If wind facilities respond to the end of subsidization, the social burden of time-limited subsidies may be much more costly in more elastic markets.

The wind industry has been widely subsidized as a central part of the worldwide energy transition. In the US (as in many other countries across the world), the industry receives many subsidies for both output and investment. The largest subsidy is the Renewable Energy Production Tax Credit (PTC), which since 1992 has awarded tax credits for every MWh of electricity a turbine produces from starting operation through a 10-year subsidy period. The

[34] Wiser and Bolinger (2021) report that in 2020 average costs are \$25 per kW-year—so a capacity factor between 0.3 and 0.4 implies average costs of \$7-\$9.5 per MWh. They note that these include both fixed and variable O&M costs like wages, materials for maintenance, and rent.

credit is nonrefundable (Grobman and Carey, 2002; Johnston, 2019), indexed to inflation, and was $25 per MWh in 2020. Investment is typically subsidized by accelerated or bonus depreciation, worth roughly 10% of investment costs,[35] but from 2009 to 2012 new turbines could claim an investment grant (called a Section 1603 grant) worth an additional 30% of the investment costs in cash in lieu of the PTC.[36] Sub-national policies also subsidize both output and investment, such as Renewable Energy Credits in states with Renewable Portfolio Standards (see Lyon, 2016) and tax abatements on land and turbine sales.

4.1.2 Data and Sample Construction

We use administrative data about wind facilities and their investment, production, and subsidy receipt. Data on investment and production are available from a census of all utility-scale wind facilities in the United States through the Energy Information Administration (EIA). The annual EIA-860 form contains information on first date of operation, location, and investment information like the nameplate capacity (EIA, 2021a). Realized production data come from the monthly EIA-906 -920 and -923 forms, which reports monthly (net) generation at the facility level (EIA, 2001-2021b).[37] We calculate monthly capacity factors by dividing realized generation by the potential generation implied by capacity.[38]

Empirically, we are interested in the change in production after the PTC subsidy period. Because the administrative data do not include tax filings, receiving the PTC is not directly observable. Instead we use the policy rule to determine eligibility. Specifically, we identify the first month each facility reports positive net-generation in the EIA-923 and impute subsidization from that first month until the end of the 10-year subsidy period (through the 120th month).

We also make four sample restrictions. First, we exclude firms who received the 1603 investment grant instead of the PTC from our baseline analysis (using the list from the replication data of Aldy et al., 2021). Second, because the EIA data cover production in 2001-2021, we keep firms who began producing in or after 2002 to observe their first month of production. Third, we drop facilities who renovated their turbines, called "repowering," during the sample period because we cannot determine their new capacity from the EIA data. To do this, we exclude firms that report repowering in the American Clean Power Association's CleanPowerIQ data (American Clean Power Association, 2020). Finally, we

[35]Although policies like accelerated and bonus depreciation do have large effects (Zwick and Mahon, 2017; Ohrn, 2018; Liu and Mao, 2019; Maffini et al., 2019), they are rarely discussed in terms of corrective policy.

[36]Aldy et al. (2023) describe the history and implications of this policy and compare it to the PTC.

[37]EIA-923 form replaced the EIA 906/920 forms in 2008.

[38]This drops 41 facilities with missing capacity information. We truncate all capacity factors above at 100, and impute 0 for periods with no generation data. Results are not sensitive to these specifications.

drop firm-month observations from the first 24 months of production because the staggered construction of turbines within a facility means that not all capacity is online in the first months of facility operation.

4.2 Measuring Production Responses after the PTC Subsidy Period

We now consider whether and how much energy generation changes at wind facilities after the PTC subsidy period. In theory, reducing the after-tax revenue per MWh will incentivize less production, but it is an empirical question whether wind facilities actually respond to this incentive. On the one hand, investment decisions are made only once, and firms have no control over how much the wind blows. On the other hand, firms may still be able to respond by optimizing, maintaining, or repairing their capital less effectively, or by choosing to exit. In this subsection, we present our empirical strategy and demonstrate that facilities do decrease production after the PTC subsidy period. Appendix Table A.2 presents evidence that the effect is not driven by exit or curtailment.

4.2.1 Event Study Design

To estimate the effect of output subsidies on generation, we estimate an event study of production around the end of the PTC subsidy period. Our main outcome of interest is the capacity factor, but in the appendix, we show results with generation, capacity, and exit. We estimate the following specification:

$$\begin{aligned}\text{Capacity Factor}_{j,t} &= \theta_j + \psi_{s_j,t} \\ &+ \sum_{v' \in \mathcal{V}} \sum_{m' \in \mathcal{M}} \beta_{m',v'} \mathbb{1}[\text{First Month}_j \in v'] \mathbb{1}[m' = t - \text{First Month}_j] + \varepsilon_{j,t}\end{aligned} \tag{3}$$

where capacity factor is indexed by firm j producing in state s_j during (monthly) time period t. The event study sums over vintage v (in which each v' is the calendar year the facility started operation) and the month of operation m. The set $\mathcal{V}$ is partitioned into years, and the set of included event indicators is $\mathcal{M} = [\underline{m}, 60, 61, ..., 119, 121, ..., 180, \bar{m}]$. We exclude $m = 120$ (the last month of the subsidy period) as a reference period. Because there are no units that will never be treated, we bin $m < 60$ and $m > 180$ together for a second normalization (see details in Sun and Abraham, 2021). We also include facility fixed effects, θ_j, and state-by-month-by-year fixed effects $\psi_{s_j,t}$. Note that the data are not a balanced panel because we only observe firms after their first month of production (i.e., $t >$ First Month$_j$).

There are four empirical considerations motivating this design. First, wind speeds vary across time and space. There are seasonal patterns (windy and slow months), annual patterns (windy and slow years), and geographic patterns (windy and slow locations). As these three dimensions of variation are correlated, naive time-period fixed effects or controls for seasonality will not capture the true heterogeneity and could leave spurious residual correlations between the event indicators and the error term. To account for this, we estimate the model with state-by-month-by-year fixed effects.[39]

Second, heterogeneity in the effects by vintage may bias the effects of a naive event study estimator. A rich literature on event study estimation has documented the importance of allowing for heterogeneous effects by treatment cohort (see Callaway and Sant'Anna, 2021; Sun and Abraham, 2021; Wooldridge, 2021). As wind production technology has been improving over time (Covert and Sweeney, 2022) and performance degradation varies by vintage (Hamilton et al., 2020), this is potentially a first-order concern. We use the estimator proposed by Sun and Abraham (2021) and estimate each event coefficient separately by vintage. Following Sun and Abraham (2021), we report vintage-weighted averages of the heterogeneous effects:

$$\beta_m = \sum_{v'} \omega_{m,v'} \beta_{m,v'}$$

where $\omega_{m,v}$ is the share of firms that entered in v' among those who produce for m months.

Third, PTC eligibility is not directly observed. PTC eligibility occurs at the turbine level, but our data are only available at the facility level, introducing "fuzziness" in the defined treatment. This could happen in two ways. Because of staggered construction, turbines that are completed after the first month of facility generation will still be subsidized after the 120th month of facility production. Furthermore, when tax filing dates do not line up with the month of first reported generation, some observations before the 120th observed month may be unsubsidized and some observations after may be subsidized.

Because each of these measurement imperfections will attenuate our estimated effects near the end of the subsidy period, we report both short- and long-term effects:

$$\beta_{\text{short}} = \sum_{m'=121}^{144} \sum_{v'} \omega_{m',v'} \beta_{m,v'} \qquad \beta_{\text{long}} = \sum_{m'=145}^{180} \sum_{v'} \omega_{m',v'} \beta_{m',v'}$$

where the $\omega_{m,v}$ weights are now the unconditional share of firms with (m', v') among the short- or long-run period. Here β_{short} represents the average change in production in the two years immediately after the subsidy period. To the extent to which staggered turbine

[39] Using facility-by-month and month-by-year fixed effects produces similar results, but ignoring this heterogeneity produces noisier estimates.

completion attenuates the effects, this will be limited to the short-run effect since all turbines seem to be completed by the end of year two. Our estimate of β_{long} will not be biased from the staggered turbine completion, but may still be attenuated if some facilities enter the unsubsidized period before their 120th month of reported production because of tax filing reasons.

The fourth concern is that event study estimates will capture any acceleration in depreciation over time. Interestingly, other research has shown that the end of the PTC subsidy period is actually a determinate of heterogeneous degradation in wind turbine generation (Hamilton et al., 2020). In the presence of accelerating depreciation, we would expect to see a downward sloping pretrend that accelerates approaching the end of the subsidy period. Interestingly, whereas this pattern is visible in other countries that don't have the PTC, it is absent in the US (see discussion in Hamilton et al., 2020). This insight suggests that we should interpret changes in depreciation as part of the long-term treatment effect on production after the PTC subsidy period.[40]

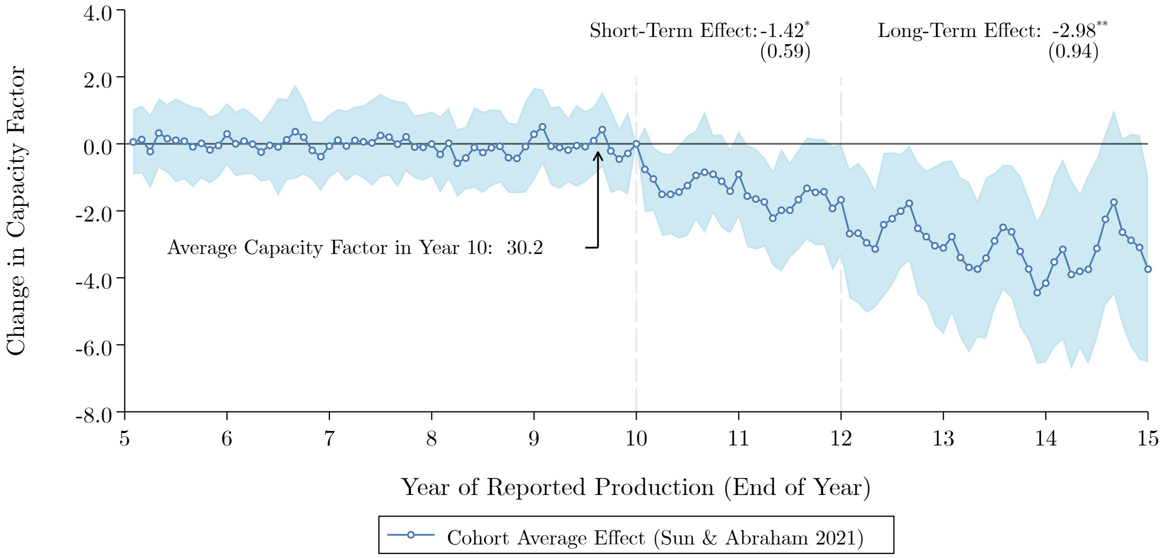
4.2.2 Wind Facilities Reduce Production After the PTC Subsidy Period

Figure 3 presents our event-study results. This figure presents the vintage-weighted average event estimates for each month of production, β_m, relative to the end of the subsidy period. The shaded area behind the series of estimates are two-way clustered, 95% pointwise confidence intervals computed using the delta method, clustering by facility and month-of-year. Looking at the patterns in these estimates, we see that the capacity factor essentially remains constant up through the end of the subsidy period. The average capacity factor in year 10 is 30.2%. After the subsidy period, production jumps down by 1 percentage point and begins sloping downward. In the first two years after the end of the PTC, the average decrease in the capacity factor is 1.4 percentage points (4.7%); this effect grows and stabilizes to 3.0 percentage points (9.9%) in the years thereafter.

The first identifying assumption required to interpret these estimates as causal effects is parallel trends in baseline outcomes (Sun and Abraham, 2021). In our setting, this means subsidized production should evolve in parallel across firms over time absent the PTC time-limit. This assumption is met if the state-by-month-by-year fixed effects reflect the counterfactual changes in production had facilities still been subsidized. With the identifying variation for the event indicators 121-180 coming from relatively older-vintage facilities, the younger facilities act as "control" units to identify the geo-temporal fixed effects. The possibility of heterogeneous responses to seasonality by vintage is why our preferred measure

[40]Technically, our results could still be driven by accelerating depreciation if the depreciation process happens to be nonlinear or discontinuous at the 10-year mark. This seems unlikely.

Figure 3: Production Decreases After Production Tax Credit Subsidization Ends



Note: This graph shows event-study estimates of the change in production after the PTC subsidy period. The sample are 65,861 non-singleton, facility-month observations from 2002-2021, including 763 firms, 307 of which produced for more than 10 years. The series present the vintage-weighted average of the event coefficients from Equation 3, with the first and long-run effects reported as well. Standard errors and pointwise 95% confidence intervals are computed with the delta method with two-way clustering by facility and month-of-year. Among firms who produce for at least eleven years, the average capacity factor in the tenth year of production is 30.2.
$^{*}p = 0.05^{**}p = 0.01$

of output is capacity factor and not MWh.[41] Although we cannot test this parallel trends assumption directly, Appendix Table A.2 presents additional confirmatory evidence from a placebo exercise showing that production does not decrease in years 11-12 for 1603-Grant firms who did not receive the PTC.

The second identifying assumption is that there is no treatment effect in pre-treatment periods (Sun and Abraham, 2021). As discussed, tax filing issues could end subsidization for some turbines before month 120, violating this assumption. As the end of the subsidy period nears, firms that make strategic maintenance decisions or change their capacity may also create anticipatory treatment effects.[42] Fortunately, the average level in the pre-treatment periods is very close to zero and does not drop until the subsidy period ends, suggesting that neither noise between tax filing and reported generation, nor anticipation significantly

[41] Models with facility-by-month-of-year fixed effects are noisier but also suggest that this is not a problem. As discussed above, other evidence suggests that subsidized production is indeed parallel despite the potential for degradation (Hamilton et al., 2020).

[42] Under the 80/20 rule, investments that are updated at a cost of more than 80% of the original investment cost re-qualify for another 10 year of the PTC. This is another reason why we drop firms that report repowering.

biases our effects.

In addition to concerns about identification and internal validity, consider three important points about external validity. First, production reductions after the subsidy period are an intensive- rather than an extensive-margin response. Appendix Table A.2 shows that although both capacity factor and generation decrease after the PTC subsidy period, the change in the probability of exit, measured by zero-generation, is almost zero and statistically insignificant [$p = 0.83$].

Second, our short-run effects are much smaller than the long-run effects. Because the event-study estimates are weighted by cohorts, the effects in periods farther from the end of the subsidy period (e.g., months 168-180) are identified off of firms from earlier vintages (e.g., 2002-2007).[43] In this case, the estimated effect in month 180 may not generalize to firms from later vintages, and the difference in long-term and short-term effects may be driven by composition rather than dynamics. To assess this concern, we estimate our event-study separately for three terciles of vintage: 2002-2006, 2007-2008, and 2009-2011. When we compare the short-term effects, the effects on the oldest and newest vintages are almost the same and there are no statistical differences between any group (see Appendix Table A.2).[44]

A third concern about external validity is that the reduction in production occurs because energy markets occasionally face negative prices. In very windy hours, turbines may generate more energy than needed but will want to still produce in order to capture the PTC, driving prices below zero. Although Aldy et al. (2023) document that curtailment accounts for at most one third of the difference in production between facilities that receive the PTC and 1603 investment grant, we also consider it in our data. We estimate our event-study separately for facilities that sold electricity at above- and below-median average wholesale prices[45] in their tenth year of operation to account for the possibility that firms selling to lower-price markets may be more likely to face negative prices. We find similar reductions in production for both groups (see Appendix Table A.2), suggesting that negative prices and curtailment do not limit the interpretation of our results.

Considering this evidence, we conclude that wind facilities reduce production by 5-10% after the PTC subsidy period and that this represents a causal response to marginal incentives to produce. Given the important role of fixed inputs like turbines, some readers may find it striking that there is any response at all. It is important to note that the end of the

[43] This is also why the standard errors grow larger in Figure 3 as the series progresses to the right.

[44] The fact that the effect in the 2007-2008 tercile is smaller seems to be driven by a handful of firms that began production in 2008 that begin producing more after the subsidy period. We conjecture that this is due to one or two repowering decision not observed in our data.

[45] Because not all firms sell in wholesale markets, we impute missing prices for the 26.9% of firm-year observations with the average prices in their state.

subsidy period reduces prices by 30%, so the implied elasticity is still quite small (about 0.1-0.25). Our event-study estimates are also smaller than the differences in Aldy et al. (2023) who show that in a subset of large facilities, those receiving the PTC produce 10-12% more than those receiving the 1603 investment grant. They point out and discuss the important margins of endogenous decisions about maintenance, repairs, forecasting, and optimization, concluding that effects even larger than ours could be very realistic.[46]

4.3 Discussion of Empirical Results

Given the changes we find in the production of wind energy, this subsection assesses the implication of time-limited subsidies for energy markets. To connect these empirical results back to the original questions about optimal taxation, we conclude with a discussion of the welfare implications of the production responses we identify in the context of our model from section 2.

4.3.1 Implications for Energy Markets

We now turn to the implications of the PTC time limit for energy markets. Wind facilities are a quickly growing feature of US energy markets, and by 2025 over 71,000 MW of wind capacity will have aged out of the PTC subsidy period. We first quantify how production changes from the PTC time limit will affect total wind energy production.

We quantify the dynamic production response attributable to PTC ineligibility with a simple extrapolation exercise using the event study estimates. For each month that firms produce after the subsidy period, we assume that their average generation would have been lower by 766 MWh in the first two years after the subsidy period and by 1429 MWh in subsequent years.[47] If loss of the PTC leads to continued degradation beyond the 5-year window (as suggested by Hamilton et al., 2020), using 1429 MWh will underestimate the total effect. We compute the cumulative effect of PTC ineligibility to date and also project the effect on the existing fleet forward in time through the year 2045, assuming a capital life of 25 years.[48]

We compare the effects of the existing policy with two counterfactual policies extending the duration of the PTC subsidy period. For these counterfactuals, we estimate the energy production that would be forgone, if in January 2022 the United States had extended the

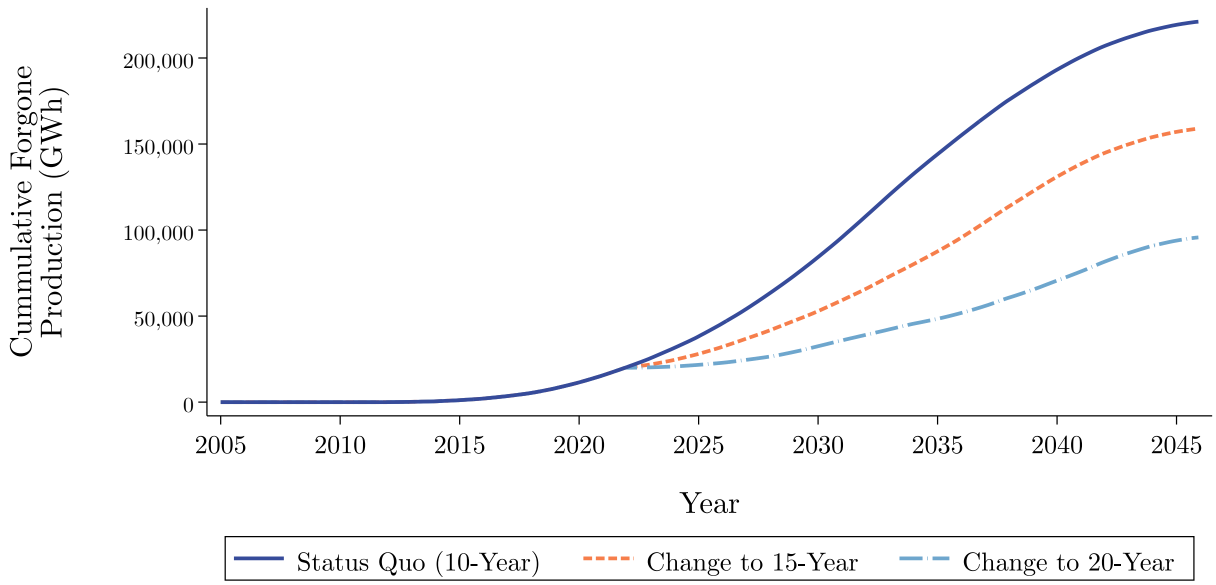
[46]Another endogenous mechanism could be strategically choosing cut-in speeds (the wind speed at which turbines begin to operate) because, as one reader pointed out, wear and tear may be more closely related to hours of operation than to MWh produced.

[47]We use MWh rather than capacity factor $\times$ capacity because newer firms tend to have larger nameplate capacity, and we want to be conservative.

[48]Which, if too short, would also lead us to understate the total forgone energy.

PTC to either 15 or 20 years. We assume that firms who "requalify" for the PTC after this policy return to full production and experience the short-term effects again when the policy expires rather than resuming where they had been in the dynamics. If there are persistent effects from the degradation that firms allow to occur after the end of subsidy eligibility, it will also lead us to underestimate the quantity changes from extending the subsidized period.

Figure 4: Forgone Clean Energy Production from PTC Ineligibility



Note: This figure shows the energy production that was lost from PTC ineligibility and projections for the amount of forgone energy resulting from different possible changes to the PTC for the existing fleet of wind facilities. To calculate these estimates, we apply the short- and long-term effects on generation to each month and sum up the total effects. For the counter-factual policies, we assume the same responses as estimated at the ten-year time limit, even though this is likely an underestimate of the true effect. Note that these estimates only capture the production lost along the intensive margin for the existing fleet, not for firm entry and investment decisions as new capacity comes online.

Figure 4 shows that the amount of forgone energy is increasing rapidly and will continue to do so. By December 2021, energy markets were forgoing over 420,000 MWh/month of energy produced by wind. This corresponds to the power used by over 470,000 homes[49]—an amount that will more than double by 2030 under the current policy. By the end of 2045, when the last of the current fleet will retire, the energy market will have forgone over 220,000 GWh of clean energy from wind—enough energy to power every home in the US for over 18 months.

Figure 4 also shows how extending the PTC subsidy duration could reduce the amount of forgone energy. Lengthening the PTC subsidy duration would reduce the amount of forgone

[49]This calculation uses the EIA's estimate that the average household uses 0.893 MWh EIA (2022).

energy and would strongly reduce the rate at which that amount is increasing. Our estimates suggest that increasing the PTC time limit by 40% (20%) of the capital life could cut forgone production by more than 55% (25%).

4.3.2 Implications for Welfare

While the PTC time limit has significant implications for electricity markets, to connect the effect on wind production to changes in social welfare, we return to the optimal tax model. Recall that under the assumptions of Proposition 4, the marginal benefit of increasing the subsidy duration is the external value for marginal units of production ($-\gamma\Delta q$ in MWh). Using estimates of the external value from the literature between \$30 and \$130 (see details in Online Appendix F), the benefit would be equal to a social value between \$12 and \$55 million per month today—with total gains between \$5 and \$25 billion over the next twenty years. If time limits arise from arbitrarily imposed constraints, rather than real costs, these estimates of the marginal benefit from extending the PTC deadline would equal the total change in welfare from the PTC extension (combined with corresponding reductions of the optimal investment subsidy).[50]

For the marginal benefit of extending the PTC to be equal to $-\gamma\Delta q$, as given in Proposition 4, both the output and investment subsidies must be optimally calibrated. To assess this assumption, we conduct an inverse optimum exercise to determine what economic primitives would justify the current subsidy policy as optimal. These analyses are in Online Appendix F and show that the current subsidy regime could be optimal under two somewhat restrictive conditions. First, to justify a subsidy rate as low as the \$25 PTC, the social cost of carbon must be \$53 per ton, lower than estimates from recent research (40-70% lower), but very close to the EPA's stated cost in 2022. Second, to justify only subsidizing investment with bonus depreciation, either the externality must shrink by a factor of 4 after the subsidy period or the total product of fixed inputs must be very small.

If subsidies are miscalibrated, we can still bound the welfare gains from extending the PTC. If the investment subsidy or the output subsidy are set too low, then the marginal benefit of extending the PTC deadline is weakly greater than $-\gamma\Delta q$. In this case, the \$12 to \$55 million per month is a lower bound on the welfare costs. If the subsidies are too large, then this amount is an upper bound.

This mapping between empirical results to welfare should be interpreted with three notes of caution. First, although the marginal externality per MWh is likely bigger than the current

[50]This analysis is based on interpreting $\phi'(T)$ in Proposition 4 as the shadow value on relaxing an arbitrarily imposed constraint on T. This interpretation is consistent with an interpretation of time limits as arising from institutional frictions or other constraints rather than a welfare maximizing social planner balancing benefits from a longer subsidy against real costs associated with a longer subsidy duration.

cost of the PTC, a longer subsidy duration would transfer surplus to firms because at least 90% of their production is inframarginal. If there are concerns about the marginal cost of public funds, then extending the PTC may be a poor use of tax revenue. Second, although we can quantify the externality today and the amount of production forgone in the next 20 years, we consider it unlikely that the external value of a MWh of wind will stay constant over this period. As the US transitions to cleaner energy, the pollution offset by wind energy should decrease. As a result, the social cost of a forgone MWh of production will also decrease. Finally, if a subsidy duration of 10 years is optimally set, then there are not welfare gains from extending the PTC. This, however, would require that extending the PTC by one year must impose real social costs of at least \$350 million in total administrative costs or other institutional frictions ($\phi'(T)$).

5. Conclusion

This paper characterizes the optimal policy implications of limited or uncertain subsidy duration in corrective taxation. We show the importance of investment subsidies when a ("Pigouvian") output subsidy with no time limit is infeasible and demonstrate that changes in production after the subsidy period inform the optimal subsidy duration. We also document a 5-10% decrease in wind energy production after the PTC subsidy period, quantifying the implications of the PTC duration on energy markets, and discuss the implications of time-limited policy for energy transition goals, sin taxation, and industrial policy. We now conclude by considering our results in the context of future research.

Our research documents new complementarities between subsidizing investment and output that arise due to time limits. Because of the dynamic frictions binding time limits create, the efficient policy subsidizes output to correct the externality during the subsidy period and subsidizes investment to correct the externality afterward. We hope future research will continue to explore other settings where complementary policy instruments can correct frictions created by common policies. Furthermore, as policy uncertainty, network externalities, budget concerns, and firm heterogeneity can also justify the use of multiple subsidy instruments, we hope that future research will also continue to explore other complementarities between policy instruments for designing optimal policy in real-world, second-best settings.

Our results also reveal how separability and targeting-like results apply even under imperfect targeting. Even though time-limited output subsidies impede externality targeting, investment subsidies separably correct for the subsidy duration, restoring a Pigouvian-like output subsidy. Perhaps even more strikingly, administrative costs, firm heterogeneity, or budget concerns may make it optimal to choose a policy with less perfect targeting to address these other concerns. These insights could be extended to other settings such as multidimensional tagging, tax-systems aware income taxation, and behavioral public finance.

Empirically, we demonstrate the real-world, quantitative importance of time-limited subsidization. Due to the ubiquity of time-limited and uncertain policies, we hope many more empirical papers explore the causal effects of subsidy ineligibility or repeal. Applying such insights can have major policy implications. For example, our results suggest that a full transition to clean energy would require at least 10% more capacity than expected because of production reductions. Missing a transition target could even backfire if the remaining (rampable) units have higher marginal pollution externalities than the units displaced on average (e.g., see example in Holland et al., 2022). As such, it may be fruitful to expand structural models of markets, dynamic climate and economic models, and optimal tax calibrations to include our insights about time limits and investment subsidies (or taxes) in response to time limits and policy uncertainty.

From a policy perspective, our results suggest that there may be large welfare costs from suboptimal policies. Optimal policies must account for production reductions after a subsidy period and should consider extending time limits (if feasible given frictions) or subsidizing investment in addition to output (if extending time limits is infeasible). Furthermore, the complementarity of investment and output subsidies suggests that requiring firms to choose between output and investment subsidies (as in the Production and Investment Tax Credits) may be less effective than allowing firms to claim both (as done for low-income housing, healthcare, and research and development subsidies). Policy evaluations also hinge on production responses. In future work, our results could inform the marginal value of public funds of renewable energy subsidies (e.g., Hahn et al., 2024) or related cost-benefit analyses.

Although our results apply under a wide array of settings, other considerations could enrich them. For example, in addition to policy uncertainty, price uncertainty could generate additional complications (e.g., see Yi et al., 2018). Since a key benefit of price-controlling policies (such as feed in tariffs) is eliminating price uncertainty, extending our analyses to non-traditional corrective policy could also be fascinating. Similarly, as real-world subsidies likely subsidize non-price-taking firms, the optimal policies might change when considered in tandem with correcting market power (e.g., see Dubois et al., 2020; O'Connell and Smith, 2021). Finally, research exploring the dynamic implications of subsidization for both investment and entry (as in Langer and Lemoine, 2022) may also be valuable.

While not related to future research, our results also rationalize many real-world policies. Although certain policies may seem inefficient compared to the Pigouvian ideal—like the presence of time-limited subsidies and the coexistence of output and investment subsidies in the same industry—we show time limits and investment subsidies are rational responses to institutional frictions, budget concerns, and policy uncertainty. In this light, our paper also suggests a hopeful message that policymakers may be making much more efficient decisions than a naive economic criticism would suggest.

On the whole, this paper underscores the real stakes of implementing time-limited subsidies. By connecting output and investment subsidies in a framework of time-limited subsidies, we articulate the crucial role investment subsidies play—despite intuition to the contrary inherited from the first-best. These insights can inform subsidy design in an era of increasing attention to industrial and energy policy and of increasing policy uncertainty.

Data Availability

Data and code replicating the tables and figures in this article can be found in Ricks ⓡ Kay (2025) in the HarvardDataverse, *Replication Data for: "Time-Limited Subsidies: Optimal Taxation with Implications for Renewable Energy Subsidies,"* `https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/AIETDX&version=DRAFT`. Instructions are also included for obtaining all raw proprietary data.

A. Appendix Tables and Figures

Table A.1: US Policies with Limited and Uncertain Durations

Panel A: Policies with Limited Durations

Policy	Industry	Duration
Renewable Energy Production Tax Credit*	Energy	10 years/firm
Advanced Manufacturing Production Tax Credit	Energy	7 years
Residential Clean Energy Credit	Energy	13 years
Carbon Oxide Sequestration Credit	Energy	12 years
Clean Vehicle Credit	Transportation	10 years
Sustainable Aviation Fuel Credit	Transportation	3 years
Credit for New Energy-Efficient Homes	Construction	10 years

Panel B: Policies with Uncertain Durations

Policy	Industry	Duration Before Repeal
Excise Whiskey Tax of 1791†	Commercial	11 years
West Virginia Soda Tax	Commercial	73 years
Chicago Soda Tax	Commercial	4 months
Marihuana [sic] Tax	Agriculture	32 years
Margin Protection Program - Dairy‡	Agriculture	4 years

Note: This table gives some examples of current and former US tax and subsidy programs that have time limits or were repealed. Note that the capital life for most of these investments is between 20 and 40 years. Consider wind turbines (20-30 years), manufacturing plants (5-15 years), furnaces or water heaters (15-30 years), new homes (50-70) years, etc. for examples.
* Superseded in Clean Electricity Production Credit.
† Later raised, and lowered regularly by other policies.
‡ Rolled into Dairy Margin Coverage

Table A.2: Estimates of Changes in Production after the Subsidy Period

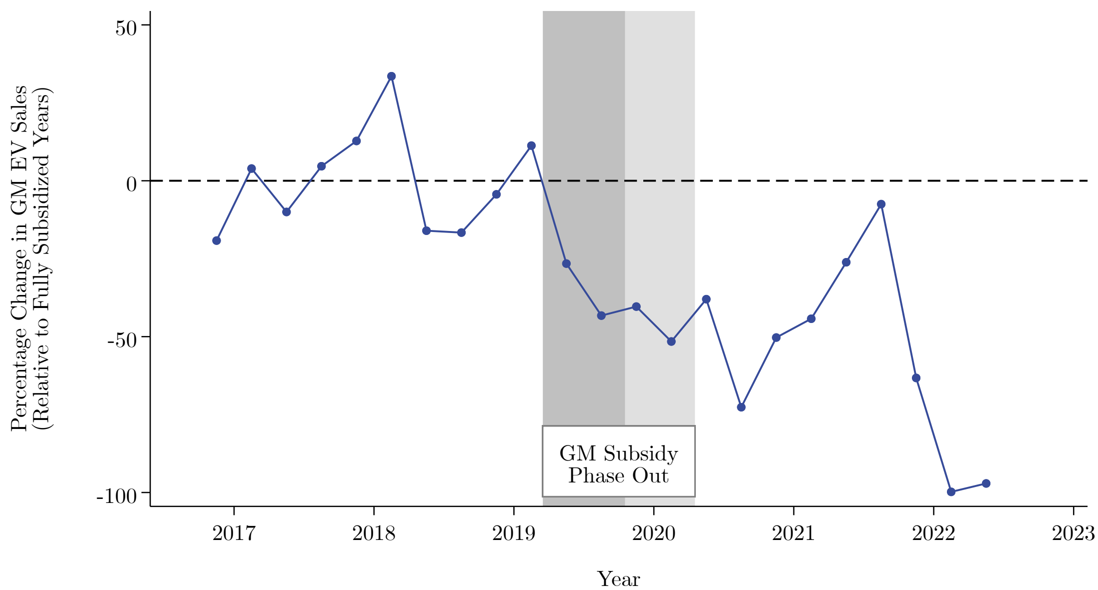
Panel A: Main Effects	Capacity Factor	Net Generation (MWh)	Exit: $\mathbb{1}$(Net Generation = 0)
Overall Effect	-2.21	-1101	-0.00
	(0.74)	(400)	(0.01)
Short-Term (Years 11-12)	-1.42	-766	0.00
	(0.59)	(365)	(0.01)
Long-Term (Years 13-15)	-2.98	-1429	-0.00
	(0.94)	(505)	(0.01)
Average in Year 10	30.2	15,701	0.02

Panel B: Heterogeneity by Vintage	2002-2006	2007-2008	2009-2010
Short-Term (Years 11-12)	-1.56	-0.54	-1.21
	(1.21)	(0.65)	(0.64)
Average in Year 10	30.0	31.0	26.6

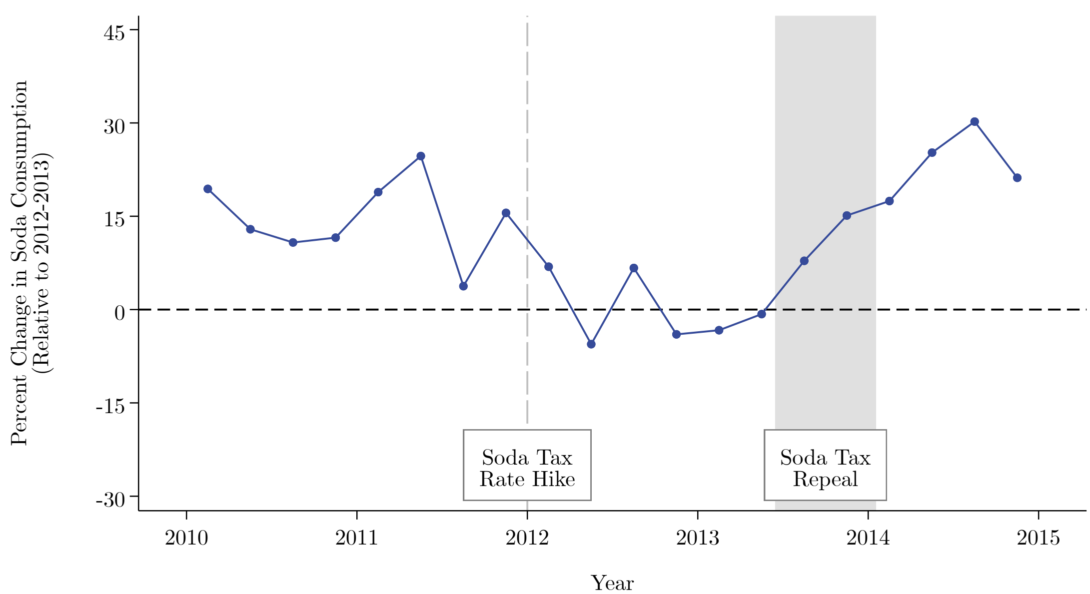
Panel C: Effect Heterogeneity	A		
Overall Effect	-	-2.22	-2.26
		(0.95)	(0.59)
Short-Term (Years 11-12)	-0.33	-1.78	-0.99
	(0.45)	(0.80)	(0.48)
Long-Term (Years 13-15)	-	-2.63	-3.63
		(1.15)	(0.86)
Average in Year 10	28.1	30.3	30.0

Note: This table reports event-study estimates of the change in production after the ten-year PTC subsidy period. All estimates are weighted averages of event-coefficients relative to the end of the subsidy period. Panel A reports the main results across three different measures of production, the capacity factor, generation, and an indicator for whether there was zero production in a given month (a measure of exit). Panel B reports the differences in short-term effects between older and newer facilities (those that began production in 2002-2006 versus 2007-2008 versus 2009-2011). Panel C reports placebo and heterogeneity tests, including wind firms that elected to receive the 1603 investment grant and were therefore not eligible for a PTC, and separately by firms who receive lower and higher average wholesale prices. For all regressions, standard errors are two-way cluster corrected for arbitrary variance-covariance structure at the facility level and month-of-year level. All regressions control for facility and state-by-month-by-year fixed effects.

Figure A.1: Replications with Other Tax and Subsidy Time Limits



(a) Electric Vehicle Production in United States



(b) Sweet Beverage Consumption in Denmark

Notes: This figure shows the changes in production and consumption in two additional industries, recast in percent changes for comparability. Following Lohawala (2023), Panel (a) looks at the market for electric vehicles when the subsidy period for General Motors ended in 2019. Percent changes in monthly sales are plotted relative to the months before subsidization ended. Following Schmacker and Smed (2023), Panel (b) looks at the market for sweetened beverages in Denmark where sin taxes were hiked in 2012 and then gradually repealed in 2013. Percent changes in quarterly consumption are plotted relative to the 18 months after the rate hike (and before the phase out and repeal).

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Online Appendix

B. Model Setup - For Online Publication

This appendix presents the detailed setup and assumptions used in the optimal tax model. Subsection B.1 presents the baseline model of Section 2, while Subsection B.2 presents the generalized economic environment discussed in Section 3. For completeness some content in the text is duplicated in this Appendix.

B.1 Setup of the Baseline Model

Consider a simple model with a representative firm where the output subsidy duration is (weakly) shorter than the lifetime of the capital input.

B.1.1 Market Supply

Market supply of the externality producing good is characterized by a representative firm that produces output using fixed inputs, x, and variable inputs, v_t, given a set of subsidy policies, θ; market prices; and a production technology, $q(x, v)$.

Assumption 1. Assume that $q(x, v)$ is twice continuously differentiable, satisfies the Inada conditions in x and v,[51] and that $\frac{\partial^2 q}{\partial x^2}\frac{\partial^2 q}{\partial v^2} - \frac{\partial^2 q}{\partial x \partial v} > 0$.

In addition, the baseline model makes the following assumption in order to express the firm's production decisions as a two period problem.

Assumption 2. Assume that a representative firm produces an externality producing good and faces certain input and output prices that are constant within sub-periods, does not discount future profits, and experiences one-hoss-shay depreciation in the fixed input, $x = x_0 \cdot \mathbb{1}[t < 1]$, where x_0 is the initial investment.

Under Assumptions 1 and 2, we can write the firm's profit maximization problem as the following two period problem:

$$\max_{x,v_1,v_2} \pi(x, v_1, v_2; \theta) = T[(p_1 + \tau^o)q(x, v_1) - m_1 v_1] + (1-T)[p_2 q(x, v_2) - m_2 v_2] - x(c - \tau^i) \quad (4)$$

Fixed and variable inputs are purchased at input prices c and m_t. Output is produced with a production technology $q(x, v_t)$ and is sold at price p_t. The policy vector $\theta = (\tau^i, \tau^o, T)$ includes the investment subsidy rate for the fixed input, τ^i; the output subsidy rate, τ^o; and the duration of the output subsidy T (as a fraction of the capital life). Because of the

[51]Specifically we assume $q(0,0) = 0$, $\frac{\partial q}{\partial x} > 0$, $\frac{\partial q}{\partial v} > 0$, $\frac{\partial^2 q}{\partial x^2} < 0$, $\frac{\partial^2 q}{\partial v^2} < 0$, $\lim_{x\to 0} q(x, v) = \infty$, $\lim_{v\to 0} q(x, v) = \infty$, $\lim_{x\to\infty} q(x, v) = 0$, and $\lim_{v\to\infty} q(x, v) = 0$.

subsidies, firm revenue in the subsidized period is $(p_1 + \tau^o)$ per unit, and investment only costs $(c - \tau^i)$ per unit of capital. Supply in period 1 and 2 is given by $q_1 = q(x^f, v_1^f)$ and $q_2 = q(x^f, v_2^f)$ respectively where $(x^f, v_1^f, v_2^f) \in \arg\max_{(x,v_1,v_2)} \pi(x, v_1, v_2; \theta)$.

The firm's first order conditions are

$$
\begin{aligned}
&(p_1 + \tau^o) q_v(x^f, v_1^f) = m_1 \\
&p_2 q_v(x^f, v_2^f) = m_2 \\
&T(p_1 + \tau_o) q_x(x^f, v_1^f) + (1 - T) p_2 q_x(x^f, v_2^f) = c(1 - \tau^i)
\end{aligned}
$$

B.1.2 Numeraire Good Production

A separate numeraire good, z, that does not produce an externality can be produced using either fixed or variable inputs. A numeraire-good-producing representative firm has a production function with constant marginal products of fixed inputs, variable inputs in period 1, and variable inputs in period 2 as c, m_1, and m_2 respectively. This assumption underpins the treatment of input prices as exogenous to production of the externality producing good and is consistent with a partial equilibrium analysis where the externality producing good is sufficiently small relative to the economy to not affect factor prices.[52]

B.1.3 Consumer Problem

The representative consumer is endowed with X, V_1, and V_2 units of the fixed input and the variable input in period 1 and 2. It has income y derived from supplying factors for the production of the externality producing good and numeraire good at rates c, m_1, and m_2, receives profits from the externality good producing firm π, but must pay taxes TC to fund subsidies.[53] The representative consumer uses income y to maximize utility by purchasing the externality producing good q and the numeraire good z.

Assumption 3. Utility is quasi-linear over q and z: $U = z + T u_1(q_1) + (1 - T) u_2(q_2)$, where u_1 and u_2 and concave and differentiable.

Consumers maximize U subject to $y + \pi - TC \geq z + T p_1 q_1 + (1 - T) p_2 q_2$. The quasi-linear utility assumes there are no income effects to consumers from changes in the subsidy rates. Given this assumption, demand in each period $t = 1, 2$ can be written as a function of the output price in that period $D_t(p_t)$.

[52] If input factors are capital and labor, and the externality producing good is wind energy, it is entirely reasonable that changing wind production will have only a negligible effect on economy wide wages and rental rates of capital.

[53] The consumer's income is therefore $y = c \cdot X + m_1 \cdot V_1 + m_2 \cdot V_2$. The numeraire good producing firm earns no profits.

B.1.4 Social Planner Problem

The social planner wants to design a subsidy system to maximize welfare given the consumer's and the firm's responses to policy, (x^f, v_1^f, v_2^f). In the simple model we make the following assumptions about welfare:

Assumption 4. Assume each unit of q produces a constant marginal externality, γ, that the marginal cost of public funds is $\lambda = 1$, and that there is an increasing and convex social cost of longer output subsidy duration $\phi(T)$.

Under Assumptions 1-4 we write the following maximization problem:

$$\max_{\tau^i, \tau^o, T} \mathcal{W}(\tau^i, \tau^o, T) \equiv \max_{\tau^i, \tau^o, T} \Pi + U + \gamma Q - TC - \phi(T) \tag{5}$$

Social welfare, $\mathcal{W}$, consists of five terms, each with implications for the economic environment. The first two terms represent the profits of the representative firm $\left(\Pi = \pi(x^f, v_1^f, v_2^f; \theta)\right)$ and the surplus of the representative consumer.

The remaining terms are standard social costs and benefits not considered by consumers or firms. The third term captures the external benefit of total production, $Q = Tq(x^f, v_1^f) + (1-T)q(x^f, v_2^f)$. The fourth term reflects the fiscal cost of funding the subsidy, $TC = T\tau^o q(x^f, v_1^f) + \tau^i x^f$. Finally, we include a social cost associated with the subsidy duration, $\phi(T)$.

B.2 Setup of the General Model

In this section, we present a more general model. The general model allows for overlapping cohorts of firms entering at different point in (continuous) time. Firms each make investment decisions when they enter and production decisions until their initial capital fully depreciates.

B.2.1 Market Demand

Consider a market for output q indexed by time t. Demand at time t is given by a time t representative consumer with quasi-linear utility for q_t and numeraire good z_t. The representative consumer has exogenous income y_t and owns all the firms and receives all profits earned at time t, π_t. Demand is given by standard consumer utility maximization problem

$$\begin{aligned} \max_{q_t, z_t} \quad & U_t(q_t, z_t; y_t, \pi_t) = u_t(q_t) + z_t \\ \text{s.t.} \quad & y_t + \pi_t = p_t q_t + z_t \end{aligned} \tag{6}$$

where u_t is differentiable and concave.

B.2.2 Market Supply

Output q is produced by heterogeneous firms indexed by j that use capital inputs $x_{j,t}$ and variable inputs $v_{j,t}$ to produce at time t accordingly to their (firm specific) production function $q_j(x_{j,t}, v_{j,t})$. Each firm's production function satisfies the regularity conditions in Assumption 1.

B.2.3 Production

If firm j enters at time $t = s_j$ and makes an initial investment decision X_j, their initial capital depreciates according to a depreciation function $\delta(t - s_j)$ (with $\delta(\cdot) \in [0, 1]$ and decreasing) such that the remaining capital at time t is given by $x_{j,t} = X_j \cdot \delta(t - s_j)$. For every $t \geq s_j$, firm j additionally chooses a level of a variable input $v_{j,t}$ and produces $q_j(x_{j,t}, v_{j,t})$.

Conditional on entry, firm j makes production decisions to maximize lifetime profits taking output prices (p_t) and input prices (m_t, c_{s_j}) as given. For subsidy policy $\theta = (\tau^o, \tau^i, T)$, firm j operating profits (discounted to time $t = s_j$) are defined as $\pi_{j|\theta}$.

Definition 1.

$$\begin{aligned}\pi_{j|\theta} &:= \max_{X_j, \{v_{j,t}\}} \int_{s_j}^{\infty} e^{-\beta(t-s_j)} \pi_{j,t} \, \mathrm{d}\, t \\ &= \max_{X_j, \{v_{j,t}\}} \int_{s_j}^{T+\kappa} e^{-\beta(t-s_j)} \left[(p_t + \tau^o) q_j(x_{j,t}, v_{j,t}) - m_t v_{j,t}\right] \mathrm{d}\, t \\ &\qquad + \int_{T+\kappa}^{\infty} e^{-\beta(t-s_j)} \left[p_t q_j(x_{j,t}, v_{j,t}) - m_t v_{j,t}\right] \mathrm{d}\, t - X_j(c_{s_j} - \tau^i)\end{aligned}$$

This setup generalizes a number of features from the simple model of Section 2. First, output and input prices are allowed to change over time. Second, producers exponentially discount future profits at the discount rate β. Third, firms have heterogeneous production functions. Fourth, fixed inputs depreciate gradually according to $\delta(\cdot)$, instead of all at once. Finally, the new term $\kappa \in \{0, s_j\}$ reflects whether the subsidy period ends at the same time for all firms, $\kappa = 0$, (often called policy "sunsetting"—e.g., Tax Cuts and Jobs Act provisions that expire in 2025) or has the same duration for firms in every cohort, $\kappa = s_j$, (e.g., the 10-year PTC).

B.2.4 Entry Decision

Let the set of potential entrants be $\mathcal{K}$ with distribution function $G(k)$. In each time period, t, there is a unique set of potential entrants $\mathcal{K}_t$. To enter, we assume that potential entrant $k \in \mathcal{K}_t$ must purchase a unitary entry slot. The supply of entry slots in time t is given by

$E_t(P_t^E(\theta))$ where $P_t^E(\theta)$ is the price of entry. Potential entrant k enters if and only if their lifetime profits are greater than the price of entry, $\pi_{k|\theta} \geq P_t^E(\theta)$. Therefore, for any entry cost at time t, there will be $\int_{\mathcal{K}_t} \mathbb{1}(\pi_{k|\theta} \geq P_t^E(\theta))\,\mathrm{d}\,G(k)$ entrants. We define $J_{t|\theta} \subset \mathcal{K}_t$ as the set of potential entrants that endogenously enter at time t when subsidy policy θ is in effect.

Definition 2.

$$J_{t|\theta} = \{k \in \mathcal{K}_t | \pi_{k|\theta} \geq P_t^E\}$$

Entry slots are limited, and the number are supplied according to the supply curve $E_t(P_t^E)$. Market clearing implies that P_t^E is set such that the total number of entrants equals the total number of entry slots.

$$\int_{\mathcal{K}_t} \mathbb{1}(\pi_{k|\theta} \geq P_t^E)\,\mathrm{d}\,G(k) = E_t(P_t^E)$$

Entry slots can be thought of as any one time purchases a firm needs to make in order to enter, such as hiring lawyers to incorporate as a company, buying land, or (in the case of wind energy) paying costs associated with interconnection studies and connecting to the grid. Because the entry decision for firms depends on their total lifetime profits (rather than the timing of production decisions), it is separable from any frictions created by time limits. As such, we abstract away from entry complications to focus our model on the role of time-limits. This abstraction could be justified by allowing the social planner to provide a firm-specific entry subsidy/tax that corrects any inefficiencies on the entry margin separately from the inefficiencies created by time limits. The following two assumptions allow for this abstraction:

Assumption 5. No Dynamic Entry Considerations The set of potential entrants at time t, $\mathcal{K}_t$, is exogenous and independent from the entry decisions made by potential entrants in $t' \neq t$. Similarly the supply of entry slots in time t, $E_t(P_t^E)$, is exogenous and independent from the entry decisions in time $t' \neq t$.

Assuming no dynamic entry considerations rules out strategic entry timing. For example, it prevents firms from buying prime land but waiting to build while technology progresses or prices increase. Although these dynamic considerations are potentially interesting, they are beyond the scope of this paper and do not directly impact the frictions created by the time limits we study.

Assumption 6. No Change in Entry The set of firms that enter in period t is unaffected by the subsidy policy $\theta = (\tau^o, \tau^i, T)$.

$$J_{t|\theta} \perp \theta$$

Assuming no change in entry maintains the same composition of firms for any subsidy design. This assumption also focuses the model on the frictions created by time limits. Although changes in subsidy rates could affect both the number of entrants and composition of entrants, these effects are not directly related to the effects of time limits and could be separately corrected for with firm specific taxes/subsidies on entry. Otherwise, this assumption could be satisfied by (1) a perfectly inelastic supply of entry slots and (2) unidimensional heterogeneity in firms (i.e., the ranking of firm profitability is the same for any subsidy policy).

B.2.5 Social Planner Problem

Assumptions 5 and 6 allow us to focus exclusively on the firms that enter. We define the set of firms to be operating at time t as $\mathcal{J}_t = \bigcup_{t'=0}^{t} J_{t'}$ which is made up of all firms that have entered in any time period $t' \leq t$. We further define the set of all firms that ever enter as $\mathcal{J} = \lim_{t\to\infty} \mathcal{J}_t$ and the distribution of $\mathcal{J}$ as $F(j)$.

The social planner chooses $\theta = (\tau^o, \tau^i, T)$ to maximize the sum of five terms: consumer utility, firm profits, the production externality, the total fiscal costs of the subsidy, and the duration costs. Welfare is given by

$$\max_{\tau^o,\tau^i,T} \mathcal{W}(\tau^o, \tau^i, T) = \max_{\tau^o,\tau^i,T} \int_0^\infty e^{-\beta t} \left[U_t + \int_{\mathcal{J}_t} \pi_{j,t} + \gamma_{j,t} q_{j,t} + \lambda TC_{j,t} \, \mathrm{d}\, F(j) \right] \mathrm{d}\, t - \phi(T)$$

where the set of firms producing in period t is $\mathcal{J}_t$; where $F(j)$ is the distribution of firms;[54] where the fiscal costs of subsidizing firm j at time t is $TC_{j,t}$;[55] and where the consumer utility, U_t, is quasi-linear as defined in Equation 6.

[54] Without loss of generality, this notation requires that each cohort produces unique "types" of firms. As such, changes in entry over time are captured by the changing measure of $\mathcal{J}_t$.

[55] The total fiscal cost includes the cost of the investment subsidy in s_j and the output subsidy for each $t \in [s_j, T + \kappa]$.

C. The Social Cost of Longer Subsidy Periods - For Online Publication

There are many possible frictions that could produce time limits in practice. This appendix details a number of these frictions and connects them to the $\phi(T)$ duration cost term. We also present conditions necessary for the convexity of $\phi(T)$.

C.1 Administrative and Compliance Costs

One cost associated with a longer subsidy duration is a firm-level administrative or compliance cost. Dharmapala et al. (2011) propose a model where the government faces fixed administrative costs of tax collection for each firm in each time period. They also suggest that this type of cost may be prevalent enough to explain common phenomena such as the empirical distribution of firm sizes in countries with size-based tax exemptions. These costs are intuitive as real-world subsidy receipt requires firms to learn new statutes, keep records, fill out and submit paperwork, and work with the possibility of audits and responses. Similarly, the government needs to process paperwork, monitor firms, and administer payments. Fixed administrative costs per firm create an increasing $\phi(T)$ that is convex under the assumptions of Lemma 1. The costs would be fixed if there are not returns to scale in performing these administrative and compliance tasks. On the other hand, if the marginal compliance costs are decreasing over time, $\phi(T)$ may still be convex, but only if the number of firms increases faster than the marginal compliance costs fall.

Lemma 1. If Assumption 1 holds for all firms, if there is a constant, positive administrative or compliance cost ϕ_0 for each firm in each time period during the subsidy period, and $\kappa = 0$, then $\phi(T)$ is increasing. Furthermore, if the number of firms is exponentially growing at a rate faster than β, $\phi(T)$ will be convex. Proof in Online Appendix E.

Although this explanation is intuitive, the presence of administrative costs from running an output subsidy would suggest that there also should be administrative costs from running an investment subsidy. Adding a second administrative cost $\phi^i = \phi \cdot \mathbb{1}[\tau^i > 0]$ into the model would slightly alter the theoretical results. There would be a duration $\hat{T}$ that would partition the optimal subsidies. For durations less than $\hat{T}$, Proposition 3 would hold, but for durations longer than $\hat{T}$ it would be better to not subsidize investment at all and use the optimal output-only subsidy instead. Alternatively, ignoring the social costs of administering the investment subsidy is analogous to arguing that those costs are small relative to the social benefits of reducing the time limit.

C.2 *Ex Post* Uncertainty Induced Duration Costs

An uncertain subsidy duration could induce *ex post* welfare costs if the firm expects the subsidy duration to be different than the actual realized duration. Intuitively, if the firm expected a subsidy duration that differs from the realized duration, their initial investment decision will not be optimal *ex post*. Lemma 2 shows that the welfare costs of *ex post* uncertainty are quadratic in the difference between the actual investment level made under uncertain beliefs, x^u, and the optimal investment decision if the subsidy duration was known with certainty, x^*. These welfare costs will be increasing and convex in (actual) subsidy duration when the expected duration ends up being shorter than the realized duration. In this sense, greater *ex ante* uncertainty implies greater *ex post* welfare loss (conditional on the eventual realized duration).

Lemma 2. Uncertainty creates *ex post* welfare loss when the realized duration differs from firms' expectations. The welfare loss is quadratic in the difference between the *ex ante* optimal investment level, x^u (i.e. given the expected duration) and the *ex post* optimal investment, x^* (i.e, given the realized duration). When the realized subsidy duration, T, exceeds the expected duration, the *ex post* investment wedge, $x^u - x^*$, increases in T. Proof in Online Appendix E.

C.3 Statutory Difficulties and Policy Processes

There are statutory aspects of the policy-making process in many countries making time-limited policies a political expediency (often at a "round" number like 10 or 20 years after the policy is passed). These policies suggest a discontinuous $\phi(T)$ where the global costs (i.e., across all firms) are constant or slightly increasing before some threshold $\tilde{t}$, after which there is a discrete jump. A globally piecewise $\phi(T)$ would be increasing, but because it is discontinuous, the optima would be a corner solution $T^* \in \{0, \tilde{t}, \infty\}$.[56]

C.4 Policymaker Career Concerns

In the political economy of policy making, policymakers seeking reelection may not value future social benefits at the same rate as the social planner. If any policy has to pass a budget vote, policymakers may discount future gains more quickly than they do present-value costs—because not all gains improve election prospects. In this case, $\phi(T)$ would be increasing but not globally convex, suggesting a corner solution of subsidizing only investment if the career

[56]Note that an unknown $\tilde{t}$ can generate a convex $\phi(T)$ if we allow $\phi(T)$ to instead represent the expected costs given the uncertain political constraints. In this case $\phi(T)$ would be convex wherever the PDF of priors over $\tilde{t}$ is increasing.

concern is relatively strong or only output if the social value of the policy to constituents is relatively strong.

D. Additional Theoretical Results - For Online Publication

D.1 Helpful Definitions

Before presenting the optimal subsidy expressions from the general model, it is helpful to define (present-value weighted) expectation and covariance operators across three domains (all firms, all subsidized production space, and all unsubsidized production space) as follows:

Definition 3.

$$\mathbb{E}_0[g(\cdot)] = \int_{\mathcal{J}} \frac{e^{-\beta s_j}}{N} g(\cdot)\,\mathrm{d}\,F(j) \qquad \mathrm{Cov}_0(X,Y) \quad = \mathbb{E}_0[(X-\mathbb{E}_0[X])(Y-\mathbb{E}_0[Y])]$$

$$\mathbb{E}_1[g(\cdot)] = \int_{\mathcal{J}} \int_{s_j}^{T+\kappa} \frac{e^{-\beta t} g(\cdot)}{N_1}\,\mathrm{d}\,t\,\mathrm{d}\,F(j) \quad \mathrm{Cov}_1(X,Y) \quad = \mathbb{E}_1[(X-\mathbb{E}_1[X])(Y-\mathbb{E}_1[Y])]$$

$$\mathbb{E}_2[g(\cdot)] = \int_{\mathcal{J}} \int_{T+\kappa}^{\infty} \frac{e^{-\beta t} g(\cdot)}{N_2}\,\mathrm{d}\,t\,\mathrm{d}\,F(j) \quad \mathrm{Cov}_2(X,Y) \quad = \mathbb{E}_2[(X-\mathbb{E}_2[X])(Y-\mathbb{E}_2[Y])]$$

where $N = \int_{\mathcal{J}} e^{-\beta s_j} dF(j)$ is related to the total number of firm-period units over all time, $N_1 = N\mathbb{E}_0[\frac{1-e^{-\beta(T+\kappa-s_j)}}{\beta}]$ is related to the share during the subsidy period, and $N_2 = N\mathbb{E}_0[\frac{e^{-\beta(T+\kappa-s_j)}}{\beta}]$ the share after.

D.2 Proposition 5 - Subsidy Rates with Dynamics

First, to focus on firm dynamics, we consider the environment described in Appendix B.2 with only one cohort of homogeneous firms with identical production technologies and schedules of prices, depreciation, and externality values.

Proposition 5. If Assumptions 1, 5, 6 hold for a set of homogeneous firms that all enter at $s_j = 0$, and either the marginal externality is constant or there are constant input prices and one-hoss shay depreciation, then for a given subsidy duration, T, the optimal subsidy rates are given by

$$\begin{aligned}
\tau^{i*} &= \frac{(1-\widetilde{T}_0)}{\beta}\,\overline{\gamma_2}\;\mathbb{E}_2\left[q_x(x_{j,t},v_{j,t}) + q_v(x_{j,t},v_{j,t})\frac{\partial v_{j,t}}{\partial X_j} + q_v(x_{j,t},v_{j,t})\frac{\partial v_{j,t}}{\partial p_t}\frac{\partial p_t}{\partial X_j}\right] \\
&\equiv \frac{(1-\widetilde{T}_0)}{\beta}\,\overline{\gamma_2}\;\mathbb{E}_2\left[\frac{\mathrm{d}q(x_t^f,v_t^f)}{\mathrm{d}X_j^f}\right] \\
\tau^{o*} &= \overline{\gamma_1}
\end{aligned}$$

where $\widetilde{T}_0 = 1 - e^{-\beta T}$ and $\overline{\gamma_n} = \mathbb{E}_n \gamma_t$. Proof in Online Appendix E.

D.3 Proposition 6 - Subsidy Rates in General

We assume Assumptions 1, 5, 6 hold for all firms and that $\lambda \geq 1$. Note that λ can be interpreted generally as capturing the social value of \$1 of government revenue relative to \$1 of profits.

Proposition 6. If $T + \kappa$ is fixed, then the optimal subsidy rates are given as

$$\begin{aligned}
\tau^{i*} = & \frac{1-\tilde{T}_\kappa}{\beta}\left(\frac{\overline{\gamma_2}}{\lambda}\,\mathbb{E}_2\left[\frac{\partial q2_{j,t}}{\partial X_j}\right] + \zeta_{\frac{\partial p2}{\partial \tau^i}}\right) \\
& + \frac{\tilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^i}} + \frac{1-\tilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda}q2,\tau^i} + \frac{1-\lambda}{\lambda}\Psi_{\tau^i} \\
\tau^{o*} = & \frac{\overline{\gamma_1}}{\lambda} + \frac{1-\tilde{T}_\kappa}{\tilde{T}_\kappa}\zeta_{\frac{\partial p2}{\partial \tau^o}} \\
& + \Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^o}} + \frac{1-\tilde{T}_\kappa}{\tilde{T}_\kappa}\Omega_{\frac{\gamma}{\lambda}q_2,\tau^o} + \frac{1-\lambda}{\lambda}\Psi_{\tau^o}
\end{aligned}$$

where

$$\begin{aligned}
\zeta_{\frac{\partial p2}{\partial \tau^i}} = & \mathbb{E}_2\Big[\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2_{j,t}}{\partial p_t}\Big]\left(\frac{\mathbb{E}_2[\frac{\partial p_t}{\partial \tau^i}] - \eta_q \mathbb{E}_2[\frac{\partial p_t}{\partial \tau^o}]}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]}\right) \\
\zeta_{\frac{\partial p2}{\partial \tau^o}} = & \mathbb{E}_2\Big[\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2_{j,t}}{\partial p_t}\Big]\left(\frac{\mathbb{E}_2[\frac{\partial p_t}{\partial \tau^o}] - \eta_X \mathbb{E}_2[\frac{\partial p_t}{\partial \tau^i}]}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]}\right) \\
\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^i}} = & \frac{Cov_1(\frac{\gamma_{j,t}}{\lambda},\frac{\partial q_{j,t}}{\partial \tau^i}) - \eta_{q1}Cov_1(\frac{\gamma_{j,t}}{\lambda},\frac{\partial q_{j,t}}{\partial \tau^o})}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^o}} = & \frac{Cov_1(\frac{\gamma_{j,t}}{\lambda},\frac{\partial q_{j,t}}{\partial \tau^o}) - \eta_X Cov_1(\frac{\gamma_{j,t}}{\lambda},\frac{\partial q_{j,t}}{\partial \tau^i})}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\Omega_{\frac{\gamma}{\lambda}q2,\tau^i} = & Cov_2(\frac{\gamma_{j,t}}{\lambda},\frac{\partial q2_{j,t}}{\partial X_j}) + \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial X_j},\frac{\partial X_j}{\partial \tau^i}) - \eta_{q1}Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial X_j},\frac{\partial X_j}{\partial \tau^o})}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
& + \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t},\frac{\partial p_t}{\partial \tau^i}) - \eta_{q1}Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t},\frac{\partial p_t}{\partial \tau^o})}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]}
\end{aligned}$$

$$
\begin{aligned}
\Omega_{\frac{\gamma}{\lambda}q_2,\tau^o} =& \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial X_j}, \frac{\partial X_j}{\partial \tau^o}) - \eta_X Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial X_j}, \frac{\partial X_j}{\partial \tau^i})}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X \frac{\partial q_{j,t}}{\partial \tau^i}]} \\
& + \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial \tau^o}) - \eta_X Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial \tau^i})}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X \frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\Psi_{\tau^i} =& \frac{N\mathbb{E}_0[X_j] - \eta_{q1} N_1 \mathbb{E}[q_{j,t}]}{N\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
\Psi_{\tau^o} =& \frac{N_1\mathbb{E}_1[q_{j,t}] - \eta_X N \mathbb{E}_0[X_j]}{N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\frac{\partial q2_{j,t}}{\partial X_j} =& \delta(t-s)\Big(\frac{\partial q_j(x_{j,t}, v_{j,t})}{\partial x} + \frac{\partial q_j(x_{j,t}, v_{j,t})}{\partial v}\frac{\partial \tilde{v}_{j,t}}{\partial X_j}\Big) \\
\frac{\mathrm{d}q2_{j,t}}{\mathrm{d}p_t} =& \frac{\partial q_j(x_{j,t}, v_{j,t})}{\partial v}\frac{\partial \tilde{v}_{j,t}}{\partial p_t} \\
\eta_{q1} =& \frac{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^i}]}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}]} \\
\eta_X =& \frac{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^o}]}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i}]} \\
\tilde{T}_\kappa =& 1 - \mathbb{E}_0[e^{-\beta(T+\kappa-s)}]
\end{aligned}
$$

Remark. Proposition 6 highlights that the output subsidy is used to correct for the externality during the subsidy period, and is therefore proportional to $\overline{\gamma_1}$, while the investment subsidy is used to correct for the externality after the subsidy period, and is proportional to $\overline{\gamma_2}$. In addition, with heterogeneous firms and marginal externalities, the subsidy rates are adjusted by the Ω terms that capture the covariance between the behavioral response induced by the subsidy and the marginal externality. These terms are analogous to the covariance term in Diamond (1973), but are more complicated due to the fact we jointly optimize over multiple instruments. With multiple instruments, the standard covariance term requires a cross-policy adjustment because changing one subsidy rate affects the marginal economic distortion, and therefore the optimal rate, of the other subsidy instrument. The η terms capture the relative importance of this interaction effect compared to the traditional "direct" fiscal externality. The Ψ terms, which capture the costs of raising revenue, are similarly modified by these interaction effects.

D.4 Proposition 7 - Subsidy Duration in General

We now consider the optimal subsidy duration, which trades-off the economic benefits from targeting with the duration costs captured by $\phi(T)$.

First, it is useful to define the present-value expectation across firms, at the moment that the firm's subsidy period ends:

Definition 4.

$$\mathbb{E}_{T^*+\kappa}[f_j(\cdot)] = \mathbb{E}_0[e^{-\beta(T+\kappa-s_j)} f_{j,T+\kappa}(\cdot)]$$

It is also necessary to define the instantaneous change in output and the variable input for firm j at the end of their subsidy period:

Definition 5.

$$\begin{aligned}\Delta q_j =& q_j(x_{j,T+\kappa}, v_{j,T+\kappa+\varepsilon}) - q_j(x_{j,T+\kappa}, v_{j,T+\kappa-\varepsilon})\\ \Delta v_j =& v_{j,T+\kappa+\varepsilon} - v_{j,T+\kappa-\varepsilon}.\end{aligned}$$

Finally, the optimal subsidy duration in the general model is characterized by Proposition 7

Proposition 7. Let Δq_j and Δv_j denote the instantaneous change in firm j's output and the variable input at the end of the subsidy period ($t = T+\kappa$). Then, under Assumption 2, a first-order Taylor approximation of q_{v_j} in Δv_j, continuous variable costs m, no exogenous demand shocks, and increasing and convex $\phi(T)$, an interior optimal subsidy duration satisfies

$$\begin{aligned}\phi'(T^*) =& - \mathbb{E}_{T^*+\kappa}[\Delta q_j \gamma_{j,T+\kappa}] + \zeta_{\frac{\partial p2}{\partial T}}\\ &+ \Omega_{\gamma, \frac{\partial q_1}{\partial T}} + \Omega_{\gamma \frac{\mathrm{d}q_2}{\mathrm{d}X}, \frac{\partial X}{\partial T}} + (1-\lambda)\Psi_T.\end{aligned}$$

where

$$\begin{aligned}\zeta_{\frac{\partial p2}{\partial T}} &= N_2 \mathbb{E}_2[\gamma_{j,t}\frac{\partial q2}{\partial p_t}]\mathbb{E}_2[\frac{\partial p_t}{\partial T}] - N\lambda \mathbb{E}_0[\frac{\partial X_j}{\partial T}]\frac{1-\widetilde{T}_\kappa}{\beta}\zeta_{\frac{\partial p2}{\partial \tau^i}} - \lambda N_1 \mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\frac{1-\widetilde{T}_\kappa}{\widetilde{T}_\kappa}\zeta_{\frac{\partial p2}{\partial \tau^o}}\\ \Omega_{\gamma q1,T} &= N_1 Cov_1(\gamma_{j,t}, \frac{\partial q_{j,t}}{\partial T}) - \lambda N_1 \mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Omega_{\frac{\gamma}{\lambda}, \frac{\partial q1}{\partial \tau^o}} - \lambda \frac{N\widetilde{T}_\kappa}{\beta}\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Omega_{\frac{\gamma}{\lambda}, \frac{\partial q1}{\partial \tau^i}}\\ \Omega_{\gamma q2,T} &= N_2 \mathbb{E}_0[\frac{\partial X_j}{\partial T}]Cov_2(\gamma_{j,t}, \frac{\partial q2}{\partial X}) + N_2 Cov_2(\gamma_{j,t}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial T})\\ &\quad - \lambda \frac{N(1-\widetilde{T}_\kappa)}{\beta}\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Omega_{\frac{\gamma}{\lambda}q2,\tau^i} - \lambda \frac{N_1(1-\widetilde{T}_\kappa)}{\widetilde{T}_\kappa}\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Omega_{\frac{\gamma}{\lambda}q2,\tau^o}\\ \Psi_T &= \tau^o N \mathbb{E}_{T+\kappa^*}[q_j] - N\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Psi_{\tau^i} - N_1 \mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Psi_{\tau^o}.\end{aligned}$$

Proof in Online Appendix E.

Remark. Here, the benefit of marginally extending the subsidy duration comes from the externality value of the additional output that occurs right at the end of subsidy eligibility for each firm, captured by the $\mathbb{E}_{T^*+\kappa}$ term. This expression is complicated by the presence of heterogeneity and the cross-policy adjustments for both τ^i and τ^o. It is worth noting that the presence of the Ψ term implies that λ affects the optimal duration.

D.5 Corollary 2 - Network Effects

In order to examine the interaction between time limited policies and network effects or knowledge spillovers, we extend the model from Appendix Section D.3. Specifically, we allow total quantity of prior investment to effect the economic environment at time t.

Definition 6. Let the total prior investment in period t be $\mathcal{X}_t = \int_0^t \int_{\mathcal{J}_t} X_j \,\mathrm{d}\, F(j)$.

We include these totals as determinants of investment costs $c_t(\mathcal{X}_t)$, production technology $q_j(\cdot; \mathcal{X}_t)$, utility of consumption $U_t(\mathcal{X}_t)$, and marginal externalities $\gamma_{j,t}(\mathcal{X}_t)$. This addition to the model allows there to be a "big push" such that substantial investments could move the market to a new equilibrium.

Corollary 2. If Assumptions 1, 5, and 6 hold, and cost, production, and externalities are allowed to be functions of $\mathcal{X}_t$, then

$$\tau^{i*} = \tau_6^i + \omega_{\mathcal{X}}^i \overline{\gamma_{\mathcal{X}}} + \Omega_{\gamma_{\mathcal{X}}, \frac{\partial \mathcal{X}-}{\partial \tau^i}}$$
$$\tau^{o*} = \tau_6^o + \omega_{\mathcal{X}}^o \overline{\gamma_{\mathcal{X}}} + \Omega_{\gamma_{\mathcal{X}}, \frac{\partial \mathcal{X}}{\partial \tau^o}}$$

where τ_6 are analytically identical to the optimal subsidies from Proposition 6, and $\overline{\gamma_{\mathcal{X}}}$ is the average network externality, and $\omega_{\mathcal{X}}^i$ and $\omega_{\mathcal{X}}^o$ characterize the relative effectiveness of increasing the capital stock in early periods with investment and output subsidies. Proof in Online Appendix E.

D.6 Corollary 3 - Variable Input Subsidy

Consider the economic environment of Proposition 5 but with an expanded policy space where during the subsidy period, the policy maker is now able to subsidize the variable input, v_t with a variable input subsidy denoted by τ^n. The firm's profit maximization problem is now given by

$$\max_{X,\{v_t\}} \int_s^{T+\kappa} e^{-\beta t} \left[(p_t + \tau^o)q(x_t, v_t) - (m_t - \tau^n)v_t\right] \mathrm{d}\, t + \int_{T+\kappa}^{\infty} e^{-\beta t} \left[p_t q(x_t, v_t) - m_t v_t\right] \mathrm{d}\, t - X(c_s - \tau^i).$$

The total fiscal costs of subsidizing firm j are now $TC_j = \tau^i X_j + \int_s^{T+\kappa} e^{-\beta t}\tau^o q_{j,t} + \tau^n v_{j,t} \,\mathrm{d}\, t$ but the welfare function is otherwise unchanged.

Corollary 3. For a cohort of homogeneous firms (described in Proposition 5) with a constant externality value, the optimal variable input subsidy will be zero if the output and investment subsidies are set optimally. Proof in Online Appendix E.

D.7 Corollary 4 - Time and Firm Specific Output Subsidy

Consider the economic environment of Proposition 5 but allowing the social planner to differentiate the output subsidy rate between firms and over time.

Corollary 4. An output subsidy that can be differentiated across firms and over time (before the end of the subsidy period) should equal the marginal externality value plus a constant value to account for the relatively effectiveness of the output subsidy to target the period 2 production. Proof in Online Appendix E.

E. Proofs for the Optimal Tax Model - For Online Publication

E.1 Helpful Lemmas

In order to prove the main results in Propositions 1-3, we first prove a helpful Lemma.

Lemma 3.

$$\frac{\frac{\partial v_2^f}{\partial \tau^i}}{\frac{\partial x^f}{\partial \tau^i}} = \frac{\frac{\partial v_2^f}{\partial \tau^o}}{\frac{\partial x^f}{\partial \tau^o}} = \frac{\frac{\partial v_2^f}{\partial T}}{\frac{\partial x^f}{\partial T}} = -\frac{p_2 q_{xv}(x^f, v_2^f) + \frac{q_x(x^f, v_2^f) q_v(x^f, v_2^f)}{D_2'(p_2)}}{p_2 q_{vv}(x^f, v_2^f) + \frac{\left(q_v(x^f, v_2^f)\right)^2}{D_2'(p_2)}}$$

Proof. The firm's first order condition for v_2 is given by

$$p_2(\theta) q_v(x^f(\theta), v_2^f(\theta)) = m_2$$

Differentiating with respect to τ^i gives

$$p_2(\theta)\left(q_{xv}(x^f(\theta), v_2^f(\theta))\frac{\partial x^f}{\partial \tau^i} + q_{vv}(x^f(\theta), v_2^f(\theta))\frac{\partial v_2^f}{\partial \tau^i}\right) + \frac{\partial p_2}{\partial \tau^i} q_v(x^f(\theta), v_2^f(\theta)) = 0$$

Noting that p_2 is implicitly defined by the market clearing condition $D_2(p_2(\theta)) = q(x^f(\theta), v_2^f(\theta))$, $\frac{\partial p_2}{\partial \tau^i}$ can be found by differentiating the market clearing condition with respect to τ_i and solving for $\frac{\partial p_2}{\partial \tau^i}$.

$$\frac{\partial p_2}{\partial \tau^i} = \frac{q_x(x^f, v_2^f)\frac{\partial x^f}{\partial \tau^i} + q_v(x^f, v_2^f)\frac{\partial v_2^f}{\partial \tau^i}}{D_2'(p_2)}$$

Substituting in leads to

$$p_2\left(q_{xv}(x^f, v_2^f)\frac{\partial x^f}{\partial \tau^i} + q_{vv}(x^f, v_2^f)\frac{\partial v_2^f}{\partial \tau^i}\right) + \frac{q_x(x^f, v_2^f) q_v(x^f, v_2^f)}{D_2'(p_2)}\frac{\partial x^f}{\partial \tau^i} + \frac{\left(q_v(x^f, v_2^f)\right)^2}{D_2'(p_2)}\frac{\partial v_2^f}{\partial \tau^i} = 0.$$

Rearranging for $\frac{\frac{\partial v_2^f}{\partial \tau^i}}{\frac{\partial x^f}{\partial \tau^i}}$ proves the first part of the Lemma. Similarly, differentiating the period 2 firm's first order condition and market clearing conditions by τ^o and T proves the Lemma. □

Lemma 4. Defining $\frac{\mathrm{d}q_2}{\mathrm{d}x}$ as the additional output in the unsubsidized period from a one unit increase in the fixed input, x:

$$\frac{\mathrm{d}q_2}{\mathrm{d}x} \equiv q_x(x, v_2^f(x,\theta)) + q_v(x, v_2^f(x,\theta))\frac{\partial v_2^f(x,\theta)}{\partial x}$$

Then under Assumptions 1-3

$$\frac{\mathrm{d}q_2}{\mathrm{d}x} = q_x(x^f(\theta), v_2^f(\theta)) + q_v(x^f(\theta), v_2^f(\theta))\frac{\frac{\partial v_2^f}{\partial \tau}}{\frac{\partial x^f}{\partial \tau}}$$

for any $\tau \in \theta$.

Proof. Consider x to be a parameter, rather than an endogenous variable. Doing so means that v_1, v_2, and p_t are functions of θ and x, implicitly defined by market clearing and the firm's first order conditions with respect to v_1 and v_2.

Therefore, $\frac{\mathrm{d}q_2}{\mathrm{d}x}$ is given as

$$\frac{\mathrm{d}q_2}{\mathrm{d}x} = q_x(x, v_2^f(x,\theta)) + q_v(x, v_2^f(x,\theta))\frac{\partial v_2^f(x,\theta)}{\partial x}.$$

The Lemma therefore follows if $\frac{\partial v_2^f(x,\theta)}{\partial x} = \frac{\partial v_2^f(\theta)}{\partial \tau} / \frac{\partial x^f(\theta)}{\partial \tau}$. Note that v_2^f is implicitly defined by $p_2 q_v(x^f, v_2^f) = m$. Let the function $\tilde{v}(m/p, x)$ define the level of v_2 for any value of x and m/p_2. Differentiating with respect to x therefore gives

$$\frac{\partial v_2^f}{\partial x} = \frac{\partial \tilde{v}(x,\theta)}{\partial x} = \tilde{v}_x + \tilde{v}_{(m/p)} \cdot \left(\frac{-m}{p(x,\theta)^2}\right)\frac{\partial p_2(x,\theta)}{\partial x}.$$

Next differentiating the market clearing condition by x gives

$$\frac{\partial p_2(x,\theta)}{\partial x} = \frac{q_x(x, v_2^f(x,\theta)) + q_v(x, v_2^f(x,\theta))\frac{\partial v_2^f}{\partial x}}{D_2'(p_2)}.$$

Combining and solving for $\frac{\partial p_2}{\partial x}$ gives

$$\frac{\partial p_2}{\partial x} = \frac{\left(q_x(x, v_2^f(x,\theta)) + q_v(x, v_2^f(x,\theta))\tilde{v}_x\right)/D_2'(p_2)}{\left(1 + q_v(x, v_2^f(x,\theta))m\tilde{v}_{(m/p)}\right) / \left(p_2(x,\theta)^2 D_2'(p_2)\right)}.$$

Instead treating x as endogenous and differentiating v_2 with respect to $\tau \in \theta$ therefore gives

$$\frac{\partial \tilde{v}(\theta)}{\partial \tau} = \tilde{v}_x \frac{\partial x(\theta)}{\partial \tau} + \tilde{v}_{(m/p)} \left(\frac{-m}{p(\theta)^2} \right) \frac{\partial p_2(\theta)}{\partial \tau}.$$

Combining with the derivative of the market clearing condition and solving for $\frac{\partial p_2}{\partial \tau}$ gives

$$\frac{\partial p_2}{\partial \tau} = \frac{\partial x}{\partial \tau} \frac{\left(q_x(x^f(\theta), v_2^f(\theta)) + q_v(x^f(\theta), v_2^f(\theta))\tilde{v}_x \right) / D_2'(p_2)}{\left(1 + q_v(x^f(\theta), v_2^f(\theta)) m \tilde{v}_{(m/p)} \right) / \left(p_2(\theta)^2 D_2'(p_2) \right)}.$$

Therefore, $\frac{\partial p_2}{\partial x} = \frac{\partial p_2}{\partial \tau} / \frac{\partial x}{\partial \tau}$, which implies that $\frac{\partial v_2^f(x,\theta)}{\partial x} = \frac{\partial v_2^f(\theta)}{\partial \tau} / \frac{\partial x^f(\theta)}{\partial \tau}$, proving the lemma.

□

E.2 Proof of Proposition 1, 2, and 3 - Optimal Subsidy Rates

Proof. Proof of Proposition 1, 2, and 3

The optimal investment and output subsidies for a given T are derived from the first order conditions from Equation 5. Taking the derivative of Equation 5 with respect to τ^o and τ^i, setting $\lambda = 1$, and canceling the counteracting effects of changing prices on consumer and producer surplus, the optimal τ^{o*} and τ^{i*} are defined by the following equations:

$$\tau^{o*} = \frac{\gamma \frac{\mathrm{d}Q}{\mathrm{d}\tau^o}}{T \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^o}} - \tau^i \frac{\frac{\partial x^f}{\partial \tau^o}}{T \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^o}} \tag{7}$$

$$\tau^{i*} = \frac{\gamma \frac{\mathrm{d}Q}{\mathrm{d}\tau^i}}{\frac{\partial x^f}{\partial \tau^i}} - \tau^o \frac{T \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^i}}{\frac{\partial x^f}{\partial \tau^i}} \tag{8}$$

Setting $\tau^o = 0$ and $T = 0$ in Equation 8 and simplifying proves Proposition 1. Setting $\tau^i = 0$ in Equation 7 and simplifying proves Proposition 2.

To prove Proposition 3, substitute 7 into 8 and rearrange for τ^i:

$$\tau^{i*} = \gamma \left(\frac{\frac{\mathrm{d}Q}{\mathrm{d}\tau^i} \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^o} - \frac{\mathrm{d}Q}{\mathrm{d}\tau^o} \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^i}}{\frac{\partial x^f}{\partial \tau^i} \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^o} - \frac{\partial x^f}{\partial \tau^o} \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^i}} \right)$$

Expanding and simplifying leads to

$$\tau^{i*} = \gamma(1-T)\left(q_x(x^f, v_2^f) + q_v(x^f, v_2^f)\frac{\frac{\partial v_2^f}{\partial \tau^i}}{\frac{\partial x^f}{\partial \tau^i}} \frac{\frac{\mathrm{d}q1}{\mathrm{d}\tau^o} - \frac{\mathrm{d}q1}{\mathrm{d}\tau^i}\frac{\frac{\partial v_2^f}{\partial \tau^o}}{\frac{\partial v_2^f}{\partial \tau^i}}}{\frac{\mathrm{d}q1}{\mathrm{d}\tau^o} - \frac{\mathrm{d}q1}{\mathrm{d}\tau^i}\frac{\frac{\partial x^f}{\partial \tau^o}}{\frac{\partial x^f}{\partial \tau^i}}} \right)$$

From Lemma 3, $\frac{\partial v_2^f}{\partial \tau^o} / \frac{\partial v_2^f}{\partial \tau^i} = \frac{\partial x^f}{\partial \tau^o} / \frac{\partial x^f}{\partial \tau^i}$ so the above simplifies to

$$\tau^{i*} = \gamma(1-T)\left(q_x(x^f, v_2^f) + q_v(x^f, v_2^f)\frac{\frac{\partial v_2^f}{\partial \tau^i}}{\frac{\partial x^f}{\partial \tau^i}} \right)$$

From Lemma 4, $q_x(x^f, v_2^f) + q_v(x^f, v_2^f)\frac{\frac{\partial v_2^f}{\partial \tau^i}}{\frac{\partial x^f}{\partial \tau^i}}$ is equal to $\frac{\mathrm{d}q_2}{\mathrm{d}x}$, noting that this captures the additional output produced by the firm in period 2 for each additional unit of x, accounting for the firm's endogenous change in v_2^f. Doing so gives the final expression for τ^{i*}.

$$\tau^{i*} = \gamma(1-T)\frac{\mathrm{d}q_2}{\mathrm{d}x} \tag{9}$$

To solve for τ^{o*}, substitute Equation 9 into Equation 7:

$$\tau^{o*} = \frac{\gamma}{T}\frac{\frac{\mathrm{d}Q}{\mathrm{d}\tau^o} - (1-T)\frac{\partial x^f}{\partial \tau^o}\frac{\mathrm{d}q_2}{\mathrm{d}x}}{\frac{\mathrm{d}q_1}{\mathrm{d}\tau^o}}$$

Again expanding, using Lemma 3 and canceling terms gives

$$\tau^{o*} = \gamma\frac{T\frac{\mathrm{d}q_1}{\mathrm{d}\tau^o} + (1-T)\frac{\mathrm{d}q_2}{\mathrm{d}\tau^o} - (1-T)\frac{\mathrm{d}q_2}{\mathrm{d}\tau^o}}{T\frac{\mathrm{d}q_1}{\mathrm{d}\tau^o}} = \gamma$$

□

E.3 Proof of Proposition 4 - Optimal Subsidy Duration

Proof. Proof of Proposition 4 Interior Solution

The optimal subsidy duration, T, is found by differentiating Equation 5 with respect to

T. Setting $\lambda = 1$, $m_1 = m_2 = m$, and $u = u_1 = u_2$, the first order condition is

$$\begin{aligned}\frac{\partial W}{\partial T} &= \big(u(q_1) - u(q_2)\big) - (p_1 q_1 - p_2 q_2) + p_1 q(x^f, v_1^f) - p_2 q(x^f, v_2^f) - (m v_1 - m v_2) \\ &\quad + \gamma\Big(q(x^f, v_1^f) - q(x^f, v_2^f)\Big) + \gamma\Big(T\frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}T} + (1-T)\frac{\mathrm{d}q(x^f, v_2^f)}{\mathrm{d}T}\Big) \\ &\quad - \frac{\partial x^f}{\partial T}\tau^{i*} - T\tau^{o*}\frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}T} - \phi'(T)\end{aligned}$$

Using the expressions for τ^{i*} and τ^{o*} from Proposition 3 and the result from Lemma 3 to simplify $\frac{\partial x^f}{\partial T}\tau^{i*}$ the first order condition becomes

$$\frac{\partial W}{\partial T} = u(q_1) - u(q_2) + m\Delta v - \gamma\Delta q - \phi'(T)$$

Here $\Delta q = q(x^f, v_2^f) - q(x^f, v_1^f)$ is used to denote the change in output at the end of the output subsidy resulting from a change in the variable input $\Delta v = v_2 - v_1$.

For small changes in v and q, we can Taylor expand $u(q_1)$ around $u(q_2)$ and $q(x, v_1^f)$ around $q(x, v_2^f)$.[57] Making this approximation leads to

$$\frac{\partial W}{\partial T} = u'(q_2)(v_1 - v_2)q_v(x^f, v_2^f) + m\Delta v - \gamma\Delta q - \phi'(T)$$

Plugging in the equilibrium conditions $u'(q_2) = p_2$ and $q_v(x^f, v_2^f) = m/p_2$ leads to the following characterization for the optimal T

$$\phi'(T) = -\gamma\Delta q$$

□

Proof. Sufficient Conditions for Uniqueness and Corner Solutions for Proposition 4

Proposition 4 provides a unique solution if $\frac{\partial^2 W}{\partial T^2} < 0 \ \forall T \in [0, 1]$.

$$\frac{\partial^2 W}{\partial T^2} = -\gamma\frac{\partial \Delta q}{\partial T} - \phi''(T)$$

By assumption, ϕ is convex so $\frac{\partial \Delta q}{\partial T} \geq 0$ is a sufficient condition for a unique solution.

[57]If the change in v is large or if q_v is not locally linear, then $\Delta q > \Delta v q_v(x^f, v_2^f) = m\Delta v$. In this case, the benefit of extending the subsidy is slightly larger, and the unapproximated optimal policy will be slightly longer than the approximated one.

$$
\begin{aligned}
\frac{\partial \Delta q}{\partial T} =& \frac{\partial q(x,v_2)}{\partial T} - \frac{\partial q(x,v_1)}{\partial T} = q_x(x,v_2)\frac{\partial x}{\partial T} + q_v(x,v_2)\frac{\partial v_2}{\partial T} - \Big(q_x(x,v_1)\frac{\partial x}{\partial T} + q_v(x,v_1)\frac{\partial v_1}{\partial T}\Big) \\
=& \Big(q_x(x,v_2) - q_x(x,v_1)\Big)\frac{\partial x}{\partial T} + q_v(x,v_2)\frac{\partial v_2}{\partial T} - q_v(x,v_1)\frac{\partial v_1}{\partial T}
\end{aligned}
$$

For small changes in v, we can use the following Taylor expansions of $q_v(x^f, v_1^f)$ and $q_x(x^f, v_1^f)$ around (x^f, v_2^f):

$$
\begin{aligned}
q_v(x^f, v_1^f) &\approx q_v(x^f, v_2^f) - \Delta v \, q_{vv}(x^f, v_2^f) \\
q_x(x^f, v_1^f) &\approx q_x(x^f, v_2^f) - \Delta v \, q_{xv}(x^f, v_2^f)
\end{aligned}
$$

Therefore,

$$
\frac{\partial \Delta q}{\partial T} = \Delta v \left(q_{xv}(x,v_2)\frac{\partial x}{\partial T} + q_{vv}(x,v_2)\frac{\partial v_2}{\partial T}\right) - q_v(x,v_1)\frac{\partial \Delta v}{\partial T}
$$

Using the firm's first order conditions and the Taylor expansion of q_v, we find that

$$
\Delta v = \frac{m}{q_{vv}(x^f, v_2^f)} \left(\frac{1}{p_2} - \frac{1}{p_1 + \tau^o} \right).
$$

Using Lemma 3 to substitute in for $q_{xv}(x,v_2)\frac{\partial x}{\partial T} + q_{vv}(x,v_2)\frac{\partial v_2}{\partial T}$ and evaluating $\frac{\partial \Delta v}{\partial T}$ we find

$$
\begin{aligned}
\frac{\partial \Delta q}{\partial T} =& \frac{-\Delta v q_v(x,v_2)}{p_2 D'(p_2)} \left(q_v(x,v_2)\frac{\partial v_2}{\partial T} + q_x(x,v_2)\frac{\partial x}{\partial T} \right) \\
& - q_v(x,v_1)\frac{m}{q_{vv}(x,v_2)} \left[\frac{1}{(p_1+\tau^o)^2} \left(\frac{q_x(x,v_1)\frac{\partial x}{\partial T} + q_v(x,v_1)\frac{\partial v_1}{\partial T}}{D'(p_1)} \right) \right. \\
& \left. - \frac{1}{(p_2)^2} \left(\frac{q_x(x,v_2)\frac{\partial x}{\partial T} + q_v(x,v_2)\frac{\partial v_2}{\partial T}}{D'(p_2)} \right) \right]
\end{aligned}
$$

Therefore $\frac{\partial \Delta q}{\partial T} \propto \frac{1}{D'(\cdot)}$. For sufficiently elastic demand, $\frac{\partial \Delta q}{\partial T}$ will be small relative to $\phi''(\cdot)$. Therefore, for convex ϕ and sufficiently elastic demand, $\frac{\partial^2 W}{\partial T^2} < 0$ and the solution in Proposition 4 is unique. □

E.4 Proof of Corollary 1 - Policy Uncertainty

In order to prove the corollary, it is useful to prove the following lemma.

Lemma 5. For an output subsidy with probability 2ρ of being overturned before $t = 1$ and a uniformly distributed timing of repeal, the optimal investment decision x^f, and variable inputs when the subsidy is (v_1) and is not (v_2) in place are defined by

$$\begin{aligned}
&(p_1 + \tau^o) q_v(x^f, v_1^f) - m = 0 \\
&p_2 q_v(x^f, v_2^f) - m = 0 \\
&(1-\rho)(p_1 + \tau^o) q_x(x^f, v_1^f) + \rho p_2 q_x(x^f, v_2^f) - (c - \tau^i) = 0.
\end{aligned}$$

Proof. Treat τ^o and p_t as random variables that depend on if the subsidy has been repealed. τ^o equals τ^o at time t with probability $1 - 2\rho t$ and zero otherwise. The firm's expected profits are

$$\begin{aligned}
&\mathbb{E}\left[\int_0^1 [q(x, v_t)(p_t + \tau^o) - m v_t] dt - x(c - \tau^i)\right] \\
&= \int_0^1 [(p_1 + \tau^o) q(x, v_{1t}) - m v_{1t}](1 - 2\rho t)\, dt \\
&\quad + \int_0^1 [p_2 q(x, v_{2t}) - m v_{2t}] 2\rho t\, dt - x(c - \tau^i).
\end{aligned}$$

At each time t, the firm chooses a contingent level of variable input in the event that the subsidy remains in place, v_{1t}, and in the event that the subsidy has been repealed, v_{2t}. The first order conditions for v_1 and v_2 give the first two equations of the lemma. Differentiating expected profits with respect to x and noting that v_1 and v_2 are constant proves the lemma. □

Proof. Proof of Corollary 1

The policymaker maximizes welfare, taking ρ as given, and treating v_t as a random variable equal to v_1 at time t with probability $1 - 2\rho t$ and equal to v_2 otherwise. The policymaker therefore maximizes

$$
\begin{aligned}
&\mathbb{E}\Bigg[\pi + U + \gamma Q - TC\Bigg] \\
=&\mathbb{E}\Bigg[\int_0^1 [q(x,v_t)(p_t+\tau^o) - mv_t]\,\mathrm{d}\,t - x(c-\tau^i) + \int_0^1 u(q_t,z_t;p_t)\,\mathrm{d}\,t \\
&+\gamma\int_0^1 q(x,v_t)\,\mathrm{d}\,t - x\tau^i - \int_0^1 \tau^o q(x,v_t)\,\mathrm{d}\,t\Bigg] \\
=&\int_0^1 [(p_1+\tau^o)q(x,v_{1t}) - mv_{1t}](1-2\rho t)\,dt + \int_0^1 [p_2 q(x,v_{2t}) - mv_{2t}]2\rho t\,dt - x(c-\tau^i) \\
&+\int_0^1 u(q_1)(1-2\rho t)\,\mathrm{d}\,t + \int_0^1 u(q_2)(2\rho t)\,\mathrm{d}\,t + \gamma\int_0^1 q(x,v_1)(1-2\rho t)\,\mathrm{d}\,t + \gamma\int_0^1 q(x,v_2)(2\rho t)\,\mathrm{d}\,t \\
&-\int_0^q \tau^o q(x,v_1)(1-2\rho t)\,\mathrm{d}\,t - x\tau^i
\end{aligned}
$$

Noting from Lemma 5 that v_1 and v_2 are constant and the only terms that depend on t are the $2\rho t$ and $(1-2\rho t)$ terms, the policymaker's objective function can be written as

$$
\begin{aligned}
&(1-\rho)\Big[(p_1+\tau^o)q(x^f,v_1^f) - mv_1^f\Big] + \rho\Big[p_2 q(x^f,v_2^f) - mv_2^f\Big] - x^f(c-\tau^i) \\
&+(1-\rho)\Big[u(q_1)\Big] + \rho\Big[u(q_2)\Big] + \gamma\Big((1-\rho)q(x^f,v_1^f) + \rho q(x^f,v_2^f)\Big) \\
&-\lambda\Big(\tau^i x^f + (1-\rho)\tau^o q(x^f,v_1^f)\Big).
\end{aligned}
$$

This model is therefore isomorphic to the model defined in Section 2, if $\rho = (1-T)$ — thus proving Corollary 1. □

E.5 Proof of Propositions 5 and 6 - Subsidy Rates in General

Lemma 6. Firm's Problem Firm j's initial capital X_j and variable input at time t $(v_{j,t})$ are defined by

$$
\begin{aligned}
c_{s_j} - \tau^i = &\int_{s_j}^{T+\kappa} e^{-\beta(t-s_j)}\left[(p_t+\tau^o)\frac{\partial q_{j,t}(x_{j,t},v_{j,t})}{\partial x}\delta(t-s_j)\right]\mathrm{d}\,t + \\
&\int_{T+\kappa}^{\infty} e^{-\beta(t-s_j)}\left[p_t\frac{\partial q_{j,t}(x_{j,t},v_{j,t})}{\partial x}\delta(t-s_j)\right]\mathrm{d}\,t
\end{aligned}
$$

$$\begin{cases} (p_{t'} + \tau^o)\frac{\partial q_j(x_{t'}, v_{t'})}{\partial v} = m_{t'} & t' \in [s_j, T+\kappa] \\ p_{t'}\frac{\partial q_j(x_{t'}, v_{t'})}{\partial v} = m_{t'} & t' \in (T+\kappa, \infty). \end{cases}$$

Lemma 7. Consumer Demand In equilibrium, the representative consumer's marginal utility at time t is equal to the output price p_t and the Lagrange multiplier on their budget constraint L_t is equal to one.

Proof. The quasi-linear representative consumer's problem in period t is

$$\max_{q_t, z_t} u_t(q_t) + z_t + L_t(y_t - p_t q_t - z_t),$$

where y_t is their period t income and z_t is numeraire consumption good. The lemma follows from the standard solution. □

Proof. Proof of Propositions 5 and 6

Note that the welfare function can be rewritten as

$$\begin{aligned} &\mathcal{W}(\tau^o, \tau^i, T) = \\ &\int_{\mathcal{J}}\int_{s_j}^{\infty} e^{-\beta t}\pi_{j,t}\,\mathrm{d}\,t\,\mathrm{d}\,F + \int_0^{\infty} e^{-\beta t}U_t + L_t(y_t - p_t q_t - w_t)\,\mathrm{d}\,t \\ &+ \int_{\mathcal{J}}\int_{s_j}^{\infty} e^{-\beta t}q_{j,t}\gamma_{j,t}\,\mathrm{d}\,t\,\mathrm{d}\,F - \int_{\mathcal{J}}\left(e^{-\beta s_j}X_j\tau^i + \tau^o\int_{s_j}^{T+\kappa} e^{-\beta t}q_{j,t}\,\mathrm{d}\,t\right)\mathrm{d}\,F - \phi(T) \end{aligned}$$

Differentiating, employing the envelope theorem, substituting in $L_t = 1$ to cancel $\frac{\partial p_t}{\partial \tau}$ terms, and rewriting with expectation operators gives the following first order conditions for τ^o,

$$\underbrace{N_1\mathbb{E}_1[\tau^o\lambda\frac{\partial q_{j,t}}{\partial \tau^o}]}_{\text{Direct fiscal externality}} + \underbrace{N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^o}]}_{\text{Cross fiscal externality}}$$
$$= \underbrace{N_1\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}] + N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}]}_{\text{Environmental externality benefit}} + \underbrace{N_1\mathbb{E}_1[(1-\lambda)q_{j,t}]}_{\text{Direct transfer effect}},$$

and τ^i,

$$\underbrace{N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial\tau^i}]}_{\text{Direct fiscal externality}} + \underbrace{N_1\mathbb{E}_1[\tau^o\lambda\frac{\partial q_{j,t}}{\partial\tau^i}]}_{\text{Cross fiscal externality}} +$$

$$= \underbrace{N_1\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^i}] + N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^i}]}_{\text{Environmental externality benefit}} + \underbrace{N\mathbb{E}_0[(1-\lambda)X_j]}_{\text{Direct transfer effect}}.$$

Combining equations gives

$$\begin{aligned}\tau^{o*} =& \frac{\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^o}] - \eta_X\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^i}]}{\lambda\left(\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial\tau^o}] - \eta_X\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial\tau^i}]\right)} + \frac{N_2}{N_1}\frac{\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^o}] - \eta_X\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^i}]}{\lambda\left(\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial\tau^o}] - \eta_X\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial\tau^i}]\right)} \\ &+ \frac{(1-\lambda)\left(N_1\mathbb{E}_1[q_{j,t}] - \eta_X\mathbb{E}_0[X_j]\right)}{N_1\lambda\left(\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial\tau^o}] - \eta_X\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial\tau^i}]\right)}\end{aligned} \tag{10}$$

and

$$\begin{aligned}\tau^{i*} =& \frac{N_1}{N}\frac{\left(\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^i}] - \eta_{q_1}\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^o}]\right)}{\lambda\left(\mathbb{E}_0[\frac{\partial X_j}{\partial\tau^i}] - \eta_{q_1}\mathbb{E}_0[\frac{\partial X_j}{\partial\tau^o}]\right)} + \frac{N_2}{N}\frac{\left(\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^i}] - \eta_{q_1}\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^o}]\right)}{\lambda\left(\mathbb{E}_0[\frac{\partial X_j}{\partial\tau^i}] - \eta_{q_1}\mathbb{E}_0[\frac{\partial X_j}{\partial\tau^o}]\right)} \\ &+ \frac{(1-\lambda)\left(N\mathbb{E}_0[X_j] - \eta_{q_1}N_1\mathbb{E}_1[q_{j,t}]\right)}{\lambda N\left(\mathbb{E}_0[\frac{\partial X_j}{\partial\tau^i}] - \eta_{q_1}\mathbb{E}_0[\frac{\partial X_j}{\partial\tau^o}]\right)}\end{aligned} \tag{11}$$

Noting that for $t \in [T+\kappa, \infty)$, $v_{j,t}$ is implicitly defined by $\frac{\partial q_j(x_{j,t}, v_{j,t})}{\partial v} = m_t/p_t$ for any level of initial capital X_j and prices. We can therefore write $v_{j,t}$ as a function of X_j and p_t/m_t, $\tilde{v}_{j,t}(X_j\delta(t-s_j), m_t/p_t)$ for $t \in [T+\kappa, \infty)$. Changes in $v_{j,t}$ for $t \in [T+\kappa, \infty)$ resulting from change in $\tau \in \theta$ can therefore be decomposed into an input complementarity effect and a price effect:

$$\frac{\partial v_{j,t}}{\partial\tau} = \tilde{v}_{j,t_{(x)}} \cdot \frac{\partial X_j}{\partial\tau}\delta(t-s_j) + \tilde{v}_{j,t_{(m/p)}} \cdot \left(\frac{-m_t}{(p_t)^2}\right)\frac{\partial p_t}{\partial\tau}.$$

Expanding and simplifying equations 10 and 11, and making use of the above decomposition we find

$$\begin{aligned}
\tau^{i*} =& \frac{1-\tilde{T}_\kappa}{\beta}\mathbb{E}_2[\frac{\gamma_{j,t}}{\lambda}]\ \mathbb{E}_2\big[\frac{\partial q2_{j,t}}{\partial X_j}] + \frac{1-\tilde{T}_\kappa}{\beta}\zeta_{\frac{\partial p2}{\partial \tau^i}} \\
&+ \frac{\tilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^i}} + \frac{1-\tilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda}q2,\tau^i} + \frac{1-\lambda}{\lambda}\Psi_{\tau^i} \\
\tau^{o*} =& \mathbb{E}_1[\frac{\gamma_{j,t}}{\lambda}] + \Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^o}} + \frac{1-\tilde{T}_\kappa}{\tilde{T}_\kappa}\zeta_{\frac{\partial p2}{\partial \tau^o}} + \frac{1-\tilde{T}_\kappa}{\tilde{T}_\kappa}\Omega_{\frac{\gamma}{\lambda}q_2,\tau^o} + \frac{1-\lambda}{\lambda}\Psi_{\tau^o}
\end{aligned}$$

where

$$
\begin{aligned}
\zeta_{\frac{\partial p2}{\partial \tau^i}} =& \mathbb{E}_2[\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2_{j,t}}{\partial p_t}]\left(\frac{\mathbb{E}_2[\frac{\partial p_t}{\partial \tau^i}] - \eta_q \mathbb{E}_2[\frac{\partial p_t}{\partial \tau^o}]}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]}\right) \\
\zeta_{\frac{\partial p2}{\partial \tau^o}} =& \mathbb{E}_2[\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2_{j,t}}{\partial p_t}]\left(\frac{\mathbb{E}_2[\frac{\partial p_t}{\partial \tau^o}] - \eta_X \mathbb{E}_2[\frac{\partial p_t}{\partial \tau^i}]}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]}\right) \\
\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^i}} =& \frac{Cov_1(\frac{\gamma_{j,t}}{\lambda}, \frac{\partial q_{j,t}}{\partial \tau^i}) - \eta_{q1} Cov_1(\frac{\gamma_{j,t}}{\lambda}, \frac{\partial q_{j,t}}{\partial \tau^o})}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^o}} =& \frac{Cov_1(\frac{\gamma_{j,t}}{\lambda}, \frac{\partial q_{j,t}}{\partial \tau^o}) - \eta_X Cov_1(\frac{\gamma_{j,t}}{\lambda}, \frac{\partial q_{j,t}}{\partial \tau^i})}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\Omega_{\frac{\gamma}{\lambda}q2,\tau^i} =& Cov_2(\frac{\gamma_{j,t}}{\lambda}, \frac{\partial q2_{j,t}}{\partial X_j}) + \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial X_j}, \frac{\partial X_j}{\partial \tau^i}) - \eta_{q1} Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial X_j}, \frac{\partial X_j}{\partial \tau^o})}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
&+ \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial \tau^i}) - \eta_{q1} Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial \tau^o})}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
\Omega_{\frac{\gamma}{\lambda}q_2,\tau^o} =& \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial X_j}, \frac{\partial X_j}{\partial \tau^o}) - \eta_X Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial X_j}, \frac{\partial X_j}{\partial \tau^i})}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
&+ \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial \tau^o}) - \eta_X Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial \tau^i})}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\Psi_{\tau^i} =& \frac{N\mathbb{E}_0[X_j] - \eta_{q1} N_1 \mathbb{E}[q_{j,t}]}{N\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
\Psi_{\tau^o} =& \frac{N_1\mathbb{E}_1[q_{j,t}] - \eta_X N\mathbb{E}_0[X_j]}{N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\frac{\partial q2_{j,t}}{\partial X_j} =& \delta(t-s)\Big(\frac{\partial q_j(x_{j,t}, v_{j,t})}{\partial x} + \frac{\partial q_j(x_{j,t}, v_{j,t})}{\partial v}\frac{\partial \tilde{v}_{j,t}}{\partial X_j}\Big) \\
\frac{\mathrm{d}q2_{j,t}}{\mathrm{d}p_t} =& \frac{\partial q_j(x_{j,t}, v_{j,t})}{\partial v}\frac{\partial \tilde{v}_{j,t}}{\partial p_t} \\
\eta_{q1} =& \frac{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^i}]}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}]} \\
\eta_X =& \frac{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^o}]}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i}]} \\
\tilde{T}_\kappa =& 1 - \mathbb{E}_0[e^{-\beta(T+\kappa-s)}]
\end{aligned}
$$

To prove Proposition 5, we explain how the simplified setup simplifies the optimal subsidy expressions. When either the externality is constant or input prices and depreciation are constant, any covariance terms involving γ/λ are equal to zero ($\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^i}}$, $\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^o}}$, and the first term of $\Omega_{\frac{\gamma}{\lambda}q2,\tau^i}$). When firms are homogeneous and are subsidized for the same time, there is no variation in $\frac{\partial X_j}{\partial \tau^i}$ and $\frac{\partial X_j}{\partial \tau^o}$ making the second term in $\Omega_{\frac{\gamma}{\lambda}q2,\tau^i}$ and first term in $\Omega_{\frac{\gamma}{\lambda}q2,\tau^o}$ zero. When firms all enter at the same time, at any time t, all firms are either subsidized or all firms are unsubsidized, allowing for a version of Lemma 4 to hold. This means that $\frac{\partial p_t}{\partial \tau^o} = \frac{\partial X_j}{\partial \tau^o}\frac{\partial p_t}{\partial X_j}$. Factoring out $\frac{\partial p_t}{\partial X_j}$ implies the $\frac{\partial p_t}{\partial \tau}$ terms in $\Omega_{\frac{\gamma}{\lambda}q2,\tau^o}$, and $\zeta_{\frac{\partial p2}{\partial \tau^o}}$ will cancel for homogeneous firms. Further, the second term of $\zeta_{\frac{\partial p2}{\partial \tau^i}}$ will equal $\frac{\partial p_t}{\partial X_j}$, leaving $\zeta_{\frac{\partial p2}{\partial \tau^i}} = \mathbb{E}_2[\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2_{j,t}}{\partial p_t}]\mathbb{E}_2[\frac{\partial p_t}{\partial X_j}]$ and $\Omega_{\frac{\gamma}{\lambda}q2,\tau^i}$ will reduce to $Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial X_j})$. Combining these terms leads to $\mathbb{E}_2[\frac{\gamma}{\lambda}\frac{\partial q_{j,t}}{\partial p_t}\frac{\partial p_t}{\partial X_j}]$. Finally, when $\lambda = 1$, the Ψ terms are multiplied by zero and can be ignored, proving Proposition 5. □

E.6 Proof of Proposition 7 - Subsidy Duration in General

Proof. Proof of Proposition 7

Differentiate $\mathcal{W}$ with respect to T:

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial T} =& N\mathbb{E}_0[e^{-\beta(T+\kappa-s_j)}(-p_{T+\kappa}(\Delta q_j - m_{T+\kappa}\Delta v_j))] + (1-\lambda)\tau^o N\mathbb{E}_0[e^{-\beta(T+\kappa-s_j)}q_{j,T+\kappa}] \\ &+ N_1\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial T}] - \lambda\tau^o N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}] + N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial T}] - \lambda\tau^i N\mathbb{E}_0[\frac{\partial X_j}{\partial T}] \\ &+ N\mathbb{E}_0[-e^{-\beta(T+\kappa-s_j)}\Delta q_j\gamma_{j,T+\kappa}] - \phi'(T).\end{aligned}$$

Using a first-order Taylor Approximation for the change in output for firm j after the subsidy ends, substituting in τ^{o*}, τ^{i*}, and noting that $\frac{\partial q_{j,t}}{\partial v} = m$ for $t \geq T+\kappa$ the first order condition becomes

$$\frac{\partial \mathcal{W}}{\partial T} = -\mathbb{E}_{T^*+\kappa}[\Delta q_j\gamma_{j,T+\kappa}] + \zeta_{\frac{\partial p2}{\partial T}} + \Omega_{\gamma q1,T} + \Omega_{\gamma q2,T} + (1-\lambda)\Psi_T - \phi'(T)$$

where

$$\begin{aligned}\zeta_{\frac{\partial p2}{\partial T}} &= N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q2}{\partial p_t}]\mathbb{E}_2[\frac{\partial p_t}{\partial T}] - N\lambda\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\frac{1-\widetilde{T}_\kappa}{\beta}\zeta_{\frac{\partial p2}{\partial \tau^i}} - \lambda N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\frac{1-\widetilde{T}_\kappa}{\widetilde{T}_\kappa}\zeta_{\frac{\partial p2}{\partial \tau^o}} \\ \Omega_{\gamma q1,T} &= N_1 Cov_1(\gamma_{j,t}, \frac{\partial q_{j,t}}{\partial T}) - \lambda N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^o}} - \lambda\frac{N\widetilde{T}_\kappa}{\beta}\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^i}} \\ \Omega_{\gamma q2,T} &= N_2\mathbb{E}_0[\frac{\partial X_j}{\partial T}]Cov_2(\gamma_{j,t}, \frac{\partial q2}{\partial X}) + N_2 Cov_2(\gamma_{j,t}\frac{\partial q2}{\partial p_t}, \frac{\partial p_t}{\partial T}) \\ &\qquad - \lambda\frac{N(1-\widetilde{T}_\kappa)}{\beta}\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Omega_{\frac{\gamma}{\lambda}q2,\tau^i} - \lambda\frac{N_1(1-\widetilde{T}_\kappa)}{\widetilde{T}_\kappa}\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Omega_{\frac{\gamma}{\lambda}q2,\tau^o} \\ \Psi_T &= \tau^o N\mathbb{E}_{T+\kappa^*}[q_j] - N\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Psi_{\tau^i} - N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Psi_{\tau^o}.\end{aligned}$$

$\square$

E.7 Proof of Corollary 2 - Network Effects

Proof. Proof of Corollary 2. When investment costs, production, demand, and the externalities are allowed to be functions of $\mathcal{X}_t$, the social planner's problem is

$$\max_{\tau^o,\tau^i,T} \mathcal{W}(\tau^o,\tau^i,T) =$$
$$\max_{\tau^o,\tau^i,T} \int_0^\infty e^{-\beta t}\left[U_t(\mathcal{X}_t) + \int_{\mathcal{J}_t} \pi_{j,t} + \gamma^o_{j,t}(\mathcal{X}_t) q_{j,t}(\mathcal{X}_t) + \lambda T C_{j,t} \,\mathrm{d}\, F(j)\right] \mathrm{d}\, t - \phi(T).$$

The first order conditions are

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \tau^o} =&(1-\lambda)N_1\mathbb{E}_1[q_{j,t}] - \lambda\tau^i N\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^o}] - \lambda\tau^o N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}] + N_1\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}] + N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}]\\ &+ N_1\mathbb{E}_1\left[\frac{\partial \mathcal{X}_t}{\partial \tau^o}\left((p_t+\tau^o-\lambda\tau^o+\gamma_{j,t})\frac{\partial q_{j,t}}{\partial \mathcal{X}} + \frac{\partial \gamma_{j,t}}{\partial \mathcal{X}} q_{j,t}\right)\right]\\ &+ N_2\mathbb{E}_2\left[\frac{\partial \mathcal{X}_t}{\partial \tau^o}\left((p_t+\gamma_{j,t})\frac{\partial q_{j,t}}{\partial \mathcal{X}} + \frac{\partial \gamma_{j,t}}{\partial \mathcal{X}} q_{j,t}\right)\right] - N_0\mathbb{E}_0[X_j\frac{\partial c_{s_j}}{\partial \mathcal{X}}\frac{\partial \mathcal{X}_{s_j}}{\partial \tau^o}] + \int_0^\infty e^{-\beta t}\frac{\partial U}{\partial \mathcal{X}}\frac{\partial \mathcal{X}_t}{\partial \tau^o}\,\mathrm{d}\, t\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \tau^i} =&(1-\lambda)N_0\mathbb{E}_0[X_j] - \lambda\tau^i N\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i}] - \lambda\tau^o N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^i}] + N_1\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}] + N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}]\\ &+ N_1\mathbb{E}_1\left[\frac{\partial \mathcal{X}_t}{\partial \tau^i}\left((p_t+\tau^o-\lambda\tau^o+\gamma_{j,t})\frac{\partial q_{j,t}}{\partial \mathcal{X}} + \frac{\partial \gamma_{j,t}}{\partial \mathcal{X}} q_{j,t}\right)\right]\\ &+ N_2\mathbb{E}_2\left[\frac{\partial \mathcal{X}_t}{\partial \tau^i}\left((p_t+\gamma_{j,t})\frac{\partial q_{j,t}}{\partial \mathcal{X}} + \frac{\partial \gamma_{j,t}}{\partial \mathcal{X}} q_{j,t}\right)\right] - N_0\mathbb{E}_0[X_j\frac{\partial c_{s_j}}{\partial \mathcal{X}}\frac{\partial \mathcal{X}_{s_j}}{\partial \tau^i}] + \int_0^\infty e^{-\beta t}\frac{\partial U}{\partial \mathcal{X}}\frac{\partial \mathcal{X}_t}{\partial \tau^i}\,\mathrm{d}\, t\end{aligned}$$

Setting $\lambda = 1$, combining and solving for τ yields

$$\begin{aligned}\tau^{o*} =& \frac{\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}] - \eta_X\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}]}{\lambda\left(\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}] - \eta_X\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^i}]\right)} + \frac{N_2}{N_1}\frac{\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}] - \eta_X\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}]}{\lambda\left(\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}] - \eta_X\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^i}]\right)}\\ &+ \frac{\int_0^\infty e^{-\beta t}\gamma^{\mathcal{X}}_t(\frac{\partial \mathcal{X}_t}{\partial \tau^o} - \eta_x\frac{\partial \mathcal{X}_t}{\partial \tau^i})\,\mathrm{d}\, t}{\lambda\left(\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}] - \eta_X\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^i}]\right)}\\ \equiv& \tau^o_6 + \omega_{\mathcal{X}}\overline{\gamma_{\mathcal{X}}} + \Omega_{\gamma_{\mathcal{X}},\frac{\partial \mathcal{X}}{\partial \tau^o}}\end{aligned}$$

$$
\begin{aligned}
\tau^{i*} = & \frac{N_1}{N} \frac{\left(\mathbb{E}_1[\gamma_{j,t} \frac{\partial q_{j,t}}{\partial \tau^i}] - \eta_{q_1} \mathbb{E}_1[\gamma_{j,t} \frac{\partial q_{j,t}}{\partial \tau^o}]\right)}{\lambda \left(\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i}] - \eta_{q_1} \mathbb{E}_0[\frac{\partial X_j}{\partial \tau^o}]\right)} + \frac{N_2}{N} \frac{\left(\mathbb{E}_2[\gamma_{j,t} \frac{\partial q_{j,t}}{\partial \tau^i}] - \eta_{q_1} \mathbb{E}_2[\gamma_{j,t} \frac{\partial q_{j,t}}{\partial \tau^o}]\right)}{\lambda \left(\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i}] - \eta_{q_1} \mathbb{E}_0[\frac{\partial X_j}{\partial \tau^o}]\right)} \\
& + \frac{\int_0^\infty e^{-\beta t} \gamma_t^{\mathcal{X}} (\frac{\partial \mathcal{X}_t}{\partial \tau^i} - \eta_{q_1} \frac{\partial \mathcal{X}_t}{\partial \tau^o}) \, \mathrm{d}\, t}{\lambda \left(\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i}] - \eta_{q_1} \mathbb{E}_0[\frac{\partial X_j}{\partial \tau^o}]\right)} \\
& \equiv \tau_6^i + \omega_{\mathcal{X}}^i \overline{\gamma_{\mathcal{X}}} + \Omega_{\gamma_{\mathcal{X}}, \frac{\partial \mathcal{X}}{\partial \tau^i}}
\end{aligned}
$$

where $\gamma_t^{\mathcal{X}} = \frac{\partial U_t}{\partial \mathcal{X}} + \int_{\mathcal{J}_t} p_t \frac{\partial q_{j,t}}{\partial \mathcal{X}} + \frac{\partial \gamma^o}{\partial \mathcal{X}} q_{j,t} + \gamma_{j,t}^o \frac{\partial q_{j,t}}{\partial \mathcal{X}} - X_j \frac{\partial c}{\partial \mathcal{X}} \mathbb{1}[t = s_j] \, \mathrm{d}\, F(j)$[58] and $\omega_{\mathcal{X}}^o = \frac{\int_0^\infty e^{-\beta t} (\frac{\partial \mathcal{X}_t}{\partial \tau^o} - \eta_x \frac{\partial \mathcal{X}_t}{\partial \tau^i}) \, \mathrm{d}\, t}{\mathbb{E}[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_x \frac{\partial q_{j,t}}{\partial \tau^i}]}$ and $\omega_{\mathcal{X}}^i = \frac{\int_0^\infty e^{-\beta t} (\frac{\partial \mathcal{X}_t}{\partial \tau^i} - \eta_{q_1} \frac{\partial \mathcal{X}_t}{\partial \tau^o}) \, \mathrm{d}\, t}{\mathbb{E}[\frac{\partial X_j}{\partial \tau^i} - \eta_{q_1} \frac{\partial X_j}{\partial \tau^o}]}$ each reflect the relative effectiveness of increasing early investment with an output or investment subsidy. The Ω terms capture the adjusted covariance between $\gamma_t^{\mathcal{X}}$ and $\frac{\partial \mathcal{X}}{\partial \tau}$.

□

E.8 Proof of Corollary 3 - Variable Input Subsidy

Proof. Denote welfare evaluated at the optimal output and investment subsidy as $\mathcal{W}(\tau^{o*}, \tau^{i*})$. The optimal variable input subsidy is characterized by the following first order condition:

$$
\begin{aligned}
\frac{\partial \mathcal{W}(\tau^{o*}, \tau^{i*})}{\partial \tau^n} = & \int_0^T e^{-\beta t} v_t \, \mathrm{d}\, t + \int_0^\infty e^{-\beta t} \gamma \frac{\partial q_t}{\partial \tau^n} \, \mathrm{d}\, t \\
& - \Big[\frac{\partial X}{\partial \tau^n} \tau^{i*} + \int_0^T e^{-\beta t} \left(\tau^{o*} \frac{\partial q_t}{\partial \tau^n} + v_t + \tau^n \frac{\partial v_t}{\partial \tau^n} \right) \mathrm{d}\, t \Big] \\
= & \int_0^T e^{-\beta t} (\gamma - \tau^{o*}) \frac{\partial q_t}{\partial \tau^n} \, \mathrm{d}\, t + \int_T^\infty e^{-\beta t} \gamma \frac{\mathrm{d} q_t}{\mathrm{d} \tau^n} \, \mathrm{d}\, t - \tau^{i*} \frac{\partial X}{\partial \tau^n} + \tau^n \int_0^T e^{-\beta t} \frac{\partial v_t}{\partial \tau^n} \, \mathrm{d}\, t = 0.
\end{aligned}
$$

Noting that $\tau^{o*} = \gamma$ and $\tau^{i*} = \int_T^\infty e^{-\beta t} \gamma \frac{\mathrm{d}q}{\mathrm{d}x} \, \mathrm{d}\, t$ and applying a input-subsidy version of Lemma 3 reveals that the first order condition becomes

$$
\tau^n \int_0^T e^{-\beta t} \frac{\partial v_t}{\partial \tau^n} \, \mathrm{d}\, t = 0.
$$

Noting that $\frac{\partial v_t}{\partial \tau^n} > 0$, the first order condition is satisfied if and only if $\tau^n = 0$.

□

[58]Note that here $\frac{\partial q_{j,t}}{\partial \mathcal{X}}$ captures the increase in production for firm j at time t, holding X_j and $v_{j,t}$ constant while $\frac{\partial q_{j,t}}{\partial \tau}$ captures the changes in q from the inputs holding $\mathcal{X}$ constant.

E.9 Proof of Corollary 4 - Changing Output Subsidies

Proof. A time and firm varying output subsidy and investment subsidy together must satisfy the following first order conditions (for $\lambda = 1$):

$$N_1\mathbb{E}_1[(\tau^o_{j,t} - \gamma_{j,t})\frac{\partial q_{j,t}}{\partial \tau^o}] + N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^o}] = N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}]$$
$$N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^i}] + N_1\mathbb{E}_1[(\tau^o_{j,t} - \gamma_{j,t})\frac{\partial q_{j,t}}{\partial \tau^i}] = N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}].$$

If $\tau^o_{j,t} = \gamma_{j,t} + \bar{\tau^o}$ then the first order conditions become

$$\bar{\tau^o}N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}] + N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^o}] = N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}]$$

and

$$N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^i}] + N_1\bar{\tau^o}\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^i}] = N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}].$$

The optimal combined subsidy is therefore

$$\tau^o_{j,t} = \gamma_{j,t} + \frac{1-\tilde{T}_\kappa}{\tilde{T}_\kappa}\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial \tau^o}}$$

$$\tau^i = \frac{1-\tilde{T}_\kappa}{\beta}\frac{\overline{\gamma_2}\,\mathbb{E}_2\left[\frac{\mathrm{d}q_{j,t}}{\mathrm{d}X_j}\right]}{\lambda} + \frac{1-\tilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial \tau^i}}$$

□

E.10 Proof of Lemma 1 - $\phi(T)$ as Administrative Costs

Proof. Let the number of firms present at time period t be $J(t) = \int_{\mathcal{J}_t} dF(j)$. Note that $J'(t) > 0$. Furthermore, let $\phi(T)$ be the sum of all constant administrative costs paid in all periods by firms form each cohort receiving the subsidy.

$$\phi(T) = \int_0^T e^{-\beta t}\phi_0 J(t)\,\mathrm{d}\,t.$$

Differentiating with respect to T:

$$\phi'(T) = e^{-\beta T}\phi_0 J(t).$$

Given that $\phi_0 > 0$, then $\phi'(T) > 0$ and $\phi(T)$ is increasing in T. Differentiating again yields

$$\phi''(T) = \beta e^{-\beta T}\phi_0(J'(t) - \beta J(T)).$$

Therefore $\phi(T)$ is convex if and only if the number of firms, $J(t)$, is exponentially growing at rate faster than β $\left(\frac{J'(T)}{J(T)} > \beta\right)$. $\square$

E.11 Proof of Lemma 2 - $\phi(T)$ as *ex post* Uncertainty

Proof. Using the model from Section 2, if a firm expects the output subsidy to last for duration T^u at the time of investment, but the realized duration is T, then the *ex post* welfare loss is

$$\begin{aligned}
&\mathcal{W}^u - \mathcal{W}^* = \\
&\Big[T(q(x^u, v_1^u)(1+\gamma) - mv_1^u) + (1-T)(q(x^u, v_2^u)(1+\gamma) - mv_2^u) - c\,x^u\Big] \\
&- \Big[T(q(x^*, v_1^*)(1+\gamma) - mv_1^*) + (1-T)(q(^*, v_2^*)(1+\gamma) - mv_2^*) - c\,x^*\Big].
\end{aligned}$$

Making a second-order Taylor Approximation around $q(x^*, v_1^*)$ and $q(x^*, v_2^*)$, and substituting in the firm's first order conditions and the expressions for optimal policy from Proposition 3, the welfare wedge simplifies to

$$\begin{aligned}
&\mathcal{W}^u - \mathcal{W}^* = \\
&\gamma(1-T)\Big(q_v(x^*, v_2^*)\frac{q_{xv}(x^*, v_2^*)}{q_{vv}(x^*, v_2^*)}(x^u - x^*) + q_v(x^*, v_2^*)(v_2^u - v_2^*)\Big) \\
&+ T(1+\gamma)\Big(q_{xx}(x^*, v_1^*)(x^u - x^*)^2 + q_{vv}(x^*, v_1^*)(v_1^u - v_1^*)^2 + q_{xv}(x^*, v_1^*)(x^* - x^u)(v_1^u - v_1^*)\Big) \\
&+ (1-T)(1+\gamma)\Big(q_{xx}(x^*, v_2^*)(x^u - x^*)^2 + q_{vv}(x^*, v_2^*)(v_2^u - v_2^*)^2 + q_{xv}(x^*, v_2^*)(x^* - x^u)(v_2^u - v_2^*)\Big).
\end{aligned}$$

Note that $q_v(x^u, v_t^u) = q_v(x^*, v_t^*)$ and a first order Taylor Approximation for $q_v(x^u, v_t^u)$ implies that $q_{xv}(x^*, v_t^*)(x^u - x^*) = q_{vv}(x^*, v_t^*)(v_t^u - v_t^*)$. Substituting in this expression and simplifying, the welfare wedge becomes

$$\mathcal{W}^u - \mathcal{W}^* = (1+\gamma)(x^u - x^*)^2\Big(Tq_{xx}(x^*, v_1^*) + (1-T)q_{xx}(x^*, v_2^*)\Big).$$

As $q_{xx} < 0$, this expression shows the wedge is negative if $T^u \neq T^*$ and the loss is increasing quadratically in $(x^u - x^*)$. If the returns to scale in x are constant in v ($q_{xx}(x*, v_1^*) = q_{xx}(x^*, v_2^*)$ or equivalently $q_{xxv} = 0$ locally between v_1 and v_2) then the *ex post* welfare costs of firm's duration uncertainty is increasing in T if $(x^u - x^*)$ is increasing in T and it will be convex in T if $(x^u - x^*)$ is increasing at a rate faster than $\sqrt{T}$.

$\square$

F. The Optimality of the PTC - For Online Publication

F.1 Determining the External Value of Wind

Although there are external benefits to offsetting other pollutants, in most locations reducing CO_2 is the main benefit of wind energy (see calculations in Cullen, 2013). If 1 MWh of wind energy reduces average CO_2 emissions by 0.71 metric tons (as estimated by EPA, 2022), then the external value of wind is between \$35 (based on the EPA's social cost of carbon estimate of \$51 per ton) and \$131 (based on recent academic work like \$87 in Cai and Lontzek (2019) or \$185 in Rennert et al. (2022)). Note that computing the true external value of wind from these average figures is complicated by two considerations. First, the average CO_2 and pollution offsets reported by the EPA may not reflect the marginal offset in the short run (Cullen, 2013, although in the long run the "marginal" effect of clean energy will be closer to the average difference in pollution as dirty firms exit). Second, there is heterogeneity across time and space in the value of one MWh of wind energy (e.g., Hollingsworth and Rudik, 2019; Fell et al., 2021; Sexton et al., 2021).

F.2 Inverse Optima

To assess the optimality of existing subsidy policies, we will assume that the current policy is calibrated appropriately and will consider what model primitives would justify each policy. We only consider the PTC and bonus depreciation. For simplicity we assume that production technology and externalities do not vary across firms enough to make the Ω terms quantitatively meaningful and that corporate taxation accounts for the net Ψ and ζ terms.[59] To give the low subsidy rates the best chance of being optimal we assume a relatively large marginal cost of public funds, $\lambda = 1.5$, and a short capital life, 25 years.

First, we consider the \$25 rate of the PTC which can only be justified by a small external value of wind energy. With the assumptions above, Proposition 6 suggests that if all policy parameters are optimally chosen, the output subsidy should be $\tau^o = \frac{\gamma_1}{\lambda}$; thus, optimality requires that $\frac{\gamma}{\lambda}$ =\$25/MWh. Given our estimates of the value of wind energy, then the optimal subsidy should be $\tau^o = \frac{0.709\text{SCC}}{1.5}$, and a \$25/MWh subsidy benefit implies a social cost of carbon of \$53 per ton. Although this estimate is very similar to the EPA's estimate (\$51 per ton), it is much lower than \$87 (Cai and Lontzek, 2019) and \$185 (Rennert et al., 2022)).[60]

[59] Of course, if the rest of the tax system is not optimally set, inframarginal transfers could be a first-order consideration.

[60] Note that considering REC prices as an added output subsidy raises the SCC to \$84 in markets with renewable portfolio standards and REC markets.

Second, we consider the PTC's ten-year subsidy period and show that it could only be optimal under large institutional frictions. Recall, that at the optimum $\phi'(T) = -\mathbb{E}[\gamma\Delta q_j]$. We estimate that the average change in production is about 1100 MWh/month/firm. For context, this means that (again assuming $\frac{\gamma}{\lambda} = \$25$) extending the PTC by one year would cost society over \$377 million.[61]

Finally, we consider the policy of accelerated and bonus depreciation, and show it is only an optimal investment subsidy if the average capital share of output is very small or if the external value of production is falling quickly. This subsidy was probably worth about 7-12% of investment costs in our sample period. Recall that the optimal investment subsidy should be $\tau^i = (1-T)\frac{\gamma_2}{\lambda}\mathbb{E}\frac{\mathrm{d}q}{\mathrm{d}X}$. Given an output subsidy with a ten-year time limit, and assuming the life of a wind turbine is twenty five years, $T \approx 0.4$. If the average investment cost for producing an additional MWh over the capital life is between \$10-20,[62] and if $\frac{\gamma_2}{\lambda}$ is also \$25, then the bonus depreciation is optimally subsidizing investment only if $\frac{\mathrm{d}q}{\mathrm{d}X} < 0.16$.[63] Whereas such a small fixed-inputs share on the margin might be true in some industries, it seems implausible in the wind industry. Alternatively, if γ_2 is smaller than γ_1, then the investment subsidy rate could still be optimal with a larger investment share.

[61] That is, \$37.5 in value for about 13,200 MWh forgone in the eleventh year for all 764 firms. This linear approximation of $\phi(T)$ around $T = 0.4$ underestimates the administrative cost from a convex $\phi(T)$ (and convexity is necessary for an interior solution like $T = 0.4$ to be optimal).

[62] 1 MW of capacity operates for 8760 hours each year for 25 years with an average capacity factor of 30.2, it will produce just under 70,000 MWh, thus to produce 1 Mwh over the capital life it requires $\frac{1}{70{,}000}$ MW of capacity. Recalling that 1 MW of capacity costs about \$0.8-1.5 M, this means the cost will be \$11-20.

[63] If $\frac{\tau^i}{c} \in [0.07, 0.12]$ is optimal, then plugging in $T \approx 0.4$, $\frac{\gamma_2}{\lambda} = \$25$, and $c \in (10, 20)$ for bounds we can simplify $\frac{(1-T)\gamma}{\lambda c}\mathbb{E}[\frac{\mathrm{d}q}{\mathrm{d}X}]$ to $\mathbb{E}[\frac{\mathrm{d}q}{\mathrm{d}X} \in (0.09, 0.16)]$.