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Abstract

In open economies, the effectiveness of carbon taxes is diminished by “pollution leakage,” where some polluting activity shifts abroad because of the tax. This paper shows that the same conditions that lead to pollution leakage enhance the efficacy of clean subsidies. As a result, the optimal policy in an open economy combines a pollution tax and a clean subsidy, the balance of which depends on the leakage rate. Furthermore, efficient policy sets the sum of the tax and subsidy rates, a measure of policy ambition, equal to the marginal damages from pollution, and does not depend on the leakage rate.

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1 Introduction

Climate change is a *global* challenge. Greenhouse gases are the classic example of a global pollutant: they have the same polluting effect regardless of where they are emitted and emissions released anywhere will impact the climate everywhere. Furthermore, the emissions levels of different countries are interconnected, as the majority of greenhouse gases responsible for climate change are released by the production or use of tradable goods. This is even more true for states or provinces within a country where it can be unconstitutional to create trade barriers with other states.

For many of these emissions sources, cleaner substitute goods have been developed. Renewable electricity and electric vehicles are two prominent examples, but cleaner technologies for producing a number of agricultural commodities and industrial materials such as steel, concrete, fertilizers, aviation fuels, and hydrogen are rapidly emerging. Given the global nature of production and the consequences of greenhouse gas pollution, it is critical to study climate policy in the context of an open economy.

Governments have adopted a variety of policies aimed at reducing greenhouse gas emissions. Many jurisdictions have adopted pollution taxes that price greenhouse gas emissions directly, while others have subsidized clean substitute goods, hoping to crowd out dirty production by encouraging substitution to a cleaner alternative.[1] Furthermore, some policies set very large tax and subsidy rates on broad tax bases, while others set much lower rates on narrow bases, indicating that the climate policy ambition of different countries, as measured by the combination of the effective pollution tax rate and clean subsidy rate, can vary dramatically as well. This paper investigates the policy incentives of an individual open economy trying to correct for a global pollutant, deriving an individual policymakers's privately optimal choice of tax and subsidy instruments and level of policy ambition.

Policymakers interested in encouraging the transition to cleaner production processes and products can do so with a number of different policies. Clausing and Wolfram (2023) categorize policy instruments based on how ambitious they are and as either cost-imposing (taxes) or cost-reducing (subsidies). The European Union's emissions trading system and California's cap-and-trade law (AB32) are notable tax-like instruments. The United States' Inflation Reduction Act sets high subsidy rates on a wide variety of clean industries, which are clear examples of high-ambition clean subsidies. State-level renewable portfolio standards

[1] Pollution taxes and clean subsidies are both examples of price instruments, which directly affect the price of emissions or clean production while allowing the quantities to fluctuate. Another set of policy tools include quantity instruments, which fix quantities while allowing prices to fluctuate. Quantity instruments include cap-and-trade programs and renewable portfolio standards. With full information, pricing instruments and quantity instruments can be used interchangeably. This paper refers to price instruments throughout, but its results equally apply to quantity instruments.

for renewable electricity are additional examples of clean subsidy programs.

For a global policymaker, the targeting principle of optimal taxation provides a clear theoretical prescription for both the choice of policy instrument used and the level of policy ambition: the globally efficient policy is a pollution tax equal to the marginal global social cost of pollution, and there is no role for any additional tax or subsidy, if markets are otherwise undistorted. However, the incentives and tax instruments available to a policymaker in an individual country, state, or province differ from those of a global policymaker.

Open economy policymakers are limited in the tax instruments they can use, as they cannot directly tax pollution emitted in other jurisdictions. Thus, the targeting principle is not directly applicable. The analysis of an individual policymaker's incentives will therefore be an analysis of second-best policy choices.

Policymakers in a particular jurisdiction typically have different objectives than a global social planner. An individual policymaker may only consider domestic welfare and discount or ignore the costs and benefits of its policies that are borne elsewhere. For a global pollutant such as greenhouse gas emissions, this will lead to climate policy free riding. One ton of carbon dioxide has the same climatic warming effect regardless of where it is emitted and affects countries all over the world. However, an individual country or state that considers only the damages occurring domestically from its greenhouse gas emissions will set insufficiently ambitious climate policy.

Furthermore, in an open economy, trade creates linkages between economies, policies, and environmental outcomes. Changes in policies or economic conditions in one country impact world prices and trade flows, affecting economic activity and the effectiveness of policy abroad. In addition, openness to trade causes pollution leakage, where polluting industries move from jurisdictions with high pollution taxes to jurisdictions with low pollution taxes. Pollution leakage will impact the effectiveness of climate policies intended to reduce greenhouse gas emissions. As emissions are a global pollutant, the geographic source of the emissions is irrelevant. All that matters is the total quantity of emissions.

These linkages and considerations are especially important for state, provincial, and local governments enacting climate policy. Local economies within the same country are even more interconnected than national economies and there are often legal reasons why trade barriers cannot be created within a country. Furthermore, some high-emitting commodities that are not globally traded, notably electricity, are highly traded between states or provinces within a country. Thus, considerations about trade and pollution leakage may be even more relevant to state and local governments.

This paper presents a model of the domestically optimal tax and subsidy policy for an individual open economy attempting to correct for a global pollutant emitted through the production of an energy-intensive tradable good. Crucially, the tradable energy good can be produced using either a "clean" or a "dirty" production process, but each process produces a homogeneous output and is traded on a single global market. Local policymakers can tax or subsidize either production process domestically, but cannot directly tax foreign dirty production.[2]

The main set of results in this paper derive the optimal policy considerations for a local policymaker that takes foreign tax and subsidy policies as given. The policy choices faced by a local policymaker will be impacted by free-riding and pollution leakage. The domestic policymaker free rides by setting environmental policy proportional only to how much the local policymaker values reductions in global emissions rather than the global cost of pollution. This is true regardless of whether the economy is open to trade.

In addition, openness to trade affects the choice of policy instruments through pollution leakage. Trade rationalizes the use of a clean subsidy in addition to a pollution tax on efficiency grounds. When foreign dirty production is very responsive to changes in the world price, the marginal rate of pollution leakage will be high, making a domestic pollution tax less effective. However, these conditions also make a clean subsidy more effective. In an open economy, the clean subsidy can be used to flood the world market with the clean good, lowering the world price and crowding out dirty production abroad. The domestic policymaker in an open economy will therefore optimally use both a pollution tax and a clean subsidy.

These findings lead to two main results for climate policy in an open economy. First, the optimal level of climate policy ambition in an open economy is constant and exactly equal to the value the policymaker places on a marginal reduction in emissions. Second, the optimal choice of policy instruments, i.e. whether a policymaker should rely on pollution taxes or clean subsidies, is determined by the rate of pollution leakage. In small open economies, the marginal rate of pollution is a sufficient statistic for the optimal choice of policy instruments.

Together, these results highlight that choosing (1) how ambitious climate policy should be and (2) which policy instruments to use to implement the policy can be considered separately. The value the policymaker places on reduced global emissions will inform how ambitious policy should be, regardless of the characteristics of the country or industry in question. Similarly, whether the efficient policy should predominately be implemented with a pollution tax or a clean subsidy depends on the marginal rate of pollution leakage, regardless of the level of ambition of the overall policy.[3]

[2]Attempts to tax foreign production through import tariffs (such as a border-carbon adjustment) are discussed in Appendix A. However, these policies still do not have taxing jurisdiction to tax dirty goods that are both produced and consumed abroad.

[3]It is possible that policy ambition will impact the equilibrium marginal rate of pollution leakage, therefore

The paper presents a number of additional extensions and provides further discussion regarding the interpretation of the results of the baseline model. One important discussion highlights that the baseline results are still relevant for small open economies. Domestically optimal tax and subsidy rates depend on the *world* supply and demand elasticities rather than local elasticities. Therefore, even a very small open economy would choose to implement a positive carbon tax and clean subsidy. A small open economy should still use a clean subsidy even though doing so creates only a very small change in the world price of the tradable good and therefore sends only a small price signal to dirty producers. Intuitively, this is because the price signal creates a very small change among a very large number of foreign dirty producers, and the aggregate change in environmental benefits remains on the same order of magnitude as the domestic distortions induced by the subsidy.

An additional extension considers the possibility that domestic policy could change equilibrium policy abroad when countries set climate policy competing in a Nash game. These results characterize when there will be climate policy interactions and the nature of these interactions. In this equilibrium characterization, policy interactions occur when changing domestic environmental policy impacts the marginal effectiveness of environmental policy abroad. In the political and policy discourse surrounding the interdependence of climate change policy, clean subsidy races are sometimes framed as a mechanism to overcome policy free-riding while pollution leakage is often framed as creating a race to the bottom in carbon tax rates.[4] However, analysis of the multi-country equilibrium suggests that considering policy interdependence does not favor the use of one policy instrument over another. The conditions that make policy rates strategic complements when all countries only use a clean subsidy would also lead to carbon taxes being strategic complements were all countries to only use pollution taxes. Finally, if all countries optimally use both a pollution tax and clean subsidy, any policy interdependence will simply shift the focus from one instrument to the other rather than increasing the overall level of climate policy ambition.

This research contributes to several lines of literature at the intersection of public finance and environmental economics. First, this research contributes to a significant literature on the choice of environmental tax instruments in the closed economy. A long targeting literature has shown that if all units of the externality producing good are taxable, the externality produced can be corrected for separately with a tax on the externality producing good (Sandmo, 1975; Dixit, 1985; Kopczuk, 2003). However, recent literature has focused on other domestic distortions in the closed economy that may complicate this logic for

impacting the efficient choice of policy instruments. However, policy ambition will not separately impact policy instrument choice beyond its effect on the marginal rate of leakage.

[4]See the comments by John Podesta (Senior Advisor for International Climate Policy for President Biden) discussing a race to the bottom and race to the top in climate policy (Podesta, 2024).

real-world policy. Borenstein and Kellogg (2023) compare policy instruments in a closed economy, pointing out that if preexisting markups in electricity markets make retail electricity priced above social marginal cost (as Borenstein and Bushnell (2022) show is often the case), then a clean subsidy could be more efficient than a carbon tax. Alternatively, clean subsidies are often justified as second-best policies if Pigouvian taxation is not available (Sallee, 2025). This could be due to incomplete regulation or the politically infeasibility of carbon taxes—academic work has documented the political unpopularity of carbon taxes (Anderson, Marinescu and Shor, 2023; Dechezleprêtre et al., 2025).[5] Finally, clean subsidies can also be justified in models with endogenous growth paths and technological learning spillovers (Acemoglu et al., 2012, 2016; Arkolakis and Walsh, 2023; Xiang, 2023).

This paper shows that openness to trade creates an efficiency rationale for subsidizing clean substitute goods in addition to taxing pollution, even without other economic distortions present. This finding can be reconciled with the targeting literature by considering domestic dirty production and foreign dirty production as separate externalities. Domestic pollution can be directly taxed, and is therefore entirely corrected for with the pollution tax. Foreign dirty production cannot be directly taxed. Instead, both the pollution tax and clean subsidy influence foreign production through the world price, so both instruments are modified accordingly.

This paper also contributes to a significant literature studying pollution leakage and the interactions between environmental and trade policy. This literature examines how local pollution taxes and regulations can shift polluting production abroad to less stringently regulated jurisdictions (notably in Copeland and Taylor (1995, 2004) and more comprehensively reviewed in Copeland, Shapiro and Taylor (2022)). A number of recent papers have empirically measured pollution leakage in specific industries (Fowlie, Reguant and Ryan, 2016; Fowlie, Petersen and Reguant, 2021; Domınguez-Iino, 2023; Hsiao, 2024), systematically measured leakage across industries (Fowlie and Reguant, 2018; Fischer and Fox, 2018), or documented environmental implications of trade policies (Shapiro, 2021).

As climate policies have become more prominent, a large literature has studied how the leakage effect impacts the optimal level and design of carbon taxes in an individual open economy (Markusen, 1975; Hoel, 1994; Fischer and Fox, 2012; Böhringer, Rosendahl and Storrøsten, 2017; Weisbach et al., 2023; Böhringer et al., 2022; Kortum and Weisbach, 2022; Grubb et al., 2022; Kotchen and Maggi, 2025). This literature has focused on modifying the design of a pollution tax to include either a carbon border adjustment or an output-based

[5] A number of papers have also studied the efficiency of alternative mechanisms to correct for externalities when it is not feasible to implement the first-best Pigouvian tax directly on emissions. Kellogg (2020); Jacobsen et al. (2023); Ito and Sallee (2018); Ricks and Kay (2025) are a few examples.

subsidy to account for pollution leakage. The carbon border adjustment is an import tax and export subsidy set at the pollution tax rate intended to put domestic dirty producers on an equal footing with foreign producers. Output-based subsidies are per-unit output subsidies designed to support domestic industry. Both of these policies are designed to prevent carbon taxes from making domestic firms less competitive with foreign competition. Kortum and Weisbach (2022) also consider how openness to trade impacts domestic policy with multiple tax instruments, but focus instead on production and consumption taxes. They show that the optimal domestic policy is to include a combination of both supply-side and demand-side policies, showing that the optimal border adjustment should not fully compensate for the supply-side tax.

Most of this literature does not model subsidies for clean substitute goods. Kotchen and Maggi (2025), developed concurrently with this paper, is an exception. Kotchen and Maggi (2025) consider a model where fossil fuel energy and green energy are used as inputs for a homogeneous final consumption good and present a similar result to this paper showing that an open economy provides an efficiency rationale to subsidize green energy and that the subsidy operates through a pollution leakage channel.[6] They use this framework to analyze the impact of political lobbying and the properties of international cooperative agreements. In contrast, this paper focuses on the implications of a combined tax and subsidy policy for climate policy ambition and instrument choice in the open economy—showing that policy ambition is constant and the optimal choice of instruments is determined by the rate of pollution leakage.

In addition, this paper contributes to a literature on interdependent climate policy and climate policy ambition in the international setting. In their article, Clausing and Wolfram (2023) consider environmental policies in an international context, discussing how pollution leakage, free-riding, and subsidy races can influence the policy ambition and instrument choice of individual economies. This paper embeds many of these ideas into a formal framework, incorporating ideas both from optimal taxation and non-cooperative game theory. Doing so illustrates that pollution leakage impacts the choice of tax and subsidy instruments and that this decision is largely separable from the level of policy ambition, which is impacted by free riding.

Hoel (1994); Nordhaus (2015); Kotchen (2018); Baksi and RayChaudhuri (2024); Farrokhi and Lashkaripour (2025) all look at international coalitions of countries that agree to set policy more ambitiously than the domestic cost of carbon emission. Farrokhi and

[6] Proposition 4 and 5 in Section 2 below—developed independently from Kotchen and Maggi (2025)—carry similar implications. The analysis in Section 2 explicitly models leakage, showing how it affects both tax and subsidy rates.

Lashkaripour (2025) also look at how policy choice can impact international climate policy ambition. However, they consider the choice between a carbon tax with a border adjustment and the carbon club (which requires some cooperation as proposed by Nordhaus (2015)) rather than a choice between a pollution tax and clean subsidy. They use a general equilibrium trade model which allows them to model economy-wide taxes, but requires them to compute multi-country equilibrium numerically. In contrast, this paper models the domestic economy in a sector-specific way, focusing on one policy targeted to one polluting industry and interactions in output markets (rather than factor markets). These simplifications allow for an explicit comparison with a clean subsidy and an analytical analysis of the multi-country equilibrium.

The remainder of the paper proceeds as follows. Section 2 presents the baseline model and domestic optimal policy, showing that openness to trade rationalizes the use of a clean subsidy on efficiency grounds. Section 3 presents and discusses two main results regarding optimal policy in open economies: policy ambition is constant, and the choice of policy instruments depends on the rate of pollution leakage. Section 4 extends the baseline model to allow for the emissions intensity of dirty production to vary between countries. Section 5 discusses a number of extensions and implications of the model, including discussing connections to other leakage mitigation policies, country size, and policy spillovers. Section 6 concludes.

2 Baseline Model

This section presents the setup for the base model and solves for the policy incentives of a domestic social planner. Before presenting the main results for the open economy, I present two benchmark policy problems, one for a global policymaker and one for a closed economy. The global policymaker's problem illustrates the first-best policy that maximizes global social welfare. The closed economy policymaker's problem serves as a useful intermediate benchmark illustrating which frictions arise specifically due to openness to trade and which are simply the result from a domestic policymaker ignoring costs and benefits to other countries.

2.1 Model Setup

Consider a model with a homogeneous tradable energy-intensive final good and a separate numeraire good. The energy-intensive good can be produced using either a "dirty" or a "clean" production process, but the output from the two production processes are identical from the perspective of the consumer. Dirty production produces a global externality, where the externality depends on the total quantity of worldwide dirty production, regardless of where it occurs.

2.1.1 Consumer Demand

There are N countries and demand for the tradable energy-intensive good and numeraire good is given by a country i representative consumer. The energy-intensive good, x_i, is sold on a worldwide market and there is a single market clearing price p. Consumers have exogenous endowed income y_i and receive profits from the firm, π_i and lump-sum government transfers T_i that they spend on x_i and numeraire good z_i. Demand is given by a standard quasi-linear utility maximization problem

$$\begin{aligned} \max_{x_i, z_i} \; & u_i(x_i) + z_i \\ \text{s.t. } \; & y_i + \pi_i + T_i = p x_i + z_i \end{aligned} \tag{1}$$

where $u_i(x_i)$ is twice continuously differentiable and concave. Consumer demand for good x_i in country i is therefore simply a function of p, $x_i(p)$, and does not depend on income, profits, or transfers.

2.1.2 Supply

There are two production technologies for producing good x, a clean process and a dirty process that produces an environmental externality. Country i produces $x_{i,c}$ units using the clean process and $x_{i,d}$ units using the dirty production process. It costs $C_{i,d}(x_{i,d})$ and $C_{i,c}(x_{i,c})$ units of the numeraire good to produce $x_{i,c}$ and $x_{i,d}$ respectively. Finally, the government of country i can impose production taxes $\tau_{i,c}$ and $\tau_{i,d}$ on the two production processes (with negative values corresponding to subsidies). The representative firms produce to maximize after-tax profits taking prices as given. The profit maximizing problems are given as

$$\max_{x_{i,c}} \pi_{i,c} = \max_{x_{i,c}} (p - \tau_{i,c}) x_{i,c} - C_{i,c}(x_{i,c}) \tag{2a}$$

$$\max_{x_{i,d}} \pi_{i,d} = \max_{x_{i,d}} (p - \tau_{i,d}) x_{i,d} - C_{i,d}(x_{i,d}). \tag{2b}$$

Total costs $C_{i,c}$ and $C_{i,d}$ are increasing and convex. Markets are perfectly competitive and firms are price-takers, but strictly convex costs imply that there will be non-zero profits. Note that the cost functions depend only on the quantity produced using that production process. This is consistent with an economic environment where there is scarcity among technology specific input factors, but the industries are small enough not to impact prices of the shared input factors (capital and labor). Solving the representative firm problem shows that supply of the tradable good using either the clean or dirty production technologies are simply functions of the after-tax price, $x_{i,c}(p-\tau_{i,c})$ and $x_{i,d}(p-\tau_{i,d})$.

2.1.3 Market Clearing

The market clearing condition states that the total worldwide demand for good x has to equal the total worldwide supply from the clean and dirty production processes,

$$\sum_i x_i(p) = \sum_i x_{i,c}(p-\tau_{i,c}) + \sum_i x_{i,d}(p-\tau_{i,d}). \tag{3}$$

2.1.4 Policymaker's Problem

The policymaker in country i sets tax/subsidy rates, $\vec{\tau}_i$, to maximize country i welfare. In addition to utility, welfare is impacted by an externality related to total dirty production, $\Gamma_i(X_d)$ where $X_d = \sum_{j=1}^{N} x_{j,d}$. The function $\Gamma_i(\cdot)$ captures how much the policymaker in i values damages from the global pollutant.[7] The government transfers any tax revenue back to the consumer in a lump sum fashion $T_i = \tau_{i,c}x_{i,c} + \tau_{i,d}x_{i,d}$.

I explore the case where the policymaker is able to supplement production taxes with a tax on the demand of x_i in Appendix A. Allowing for a tax on consumption allows for a direct comparison to carbon border adjustments, as the marginal incentives of tariffs can be recreated by a combination of production and consumption taxes (e.g. Kortum and Weisbach, 2022). However, this analysis only leads to minor modifications to the optimal tax and subsidy rates. Also, throughout the paper, I maintain the assumption that the numeraire good z_i will remain untaxed.[8]

The policymaker's problem is therefore stated formally as

$$\max_{\tau_{i,c},\tau_{i,d}} U_i(x_i(\vec{\tau}_i), z_i(\vec{\tau}_i)) - \Gamma_i(X_d(\vec{\tau}_i)) + \pi_i(\vec{\tau}_i) + \tau_{i,c}x_{i,c} + \tau_{i,d}x_{i,d}. \tag{4}$$

[7]Differences in the damage function between jurisdictions can be seen as capturing either differences in climate damages or policy preferences from the local policymaker. For example, $\Gamma_i()$ could be systematically higher if jurisdiction i is more vulnerable to climate change for "natural science" reasons (i.e. is in a hotter climate or more of its cities are in flood-prone areas) or if the policymakers value reductions in global emissions more for political reasons. The model is agnostic to how the damage function is micro-founded, it instead provides optimal policy expressions given a particular damage function.

[8]If the numeraire good can be taxed or subsidized then there is an equivalence between the tax and subsidy as a tax on one good is equivalent to a subsidy on every other good in the economy (Corlett and Hauge, 1953).

The policymaker sets tax and subsidy rates accounting for consumer utility of the domestic consumer, profits of the domestic firms, the impact of global emissions on local economic activity, and tax revenues. Changing the tax rate for a production process $k \in \{c, d\}$ in country j has the effect of both creating a wedge between the price received by producers and the world price, as well as changing the world price of the tradable good, which in turn effects the quantity produced by suppliers in other industries and countries. The size of these two effects depends on the tax incidence of $\tau_{i,k}$, captured by how much the world price p changes when $\tau_{i,k}$ changes. The responsiveness of p to a change in $\tau_{i,k}$ can be calculated by differentiating Equation 3 and is given as

$$\frac{\partial p}{\partial \tau_{i,k}} = \frac{\alpha_{i,k}\eta_{i,k}}{\sum_n \alpha_{n,d}\eta_{n,d} + \sum_n \alpha_{n,c}\eta_{n,c} - \sum_n \beta_n \zeta_n}. \tag{5}$$

where $\alpha_{n,k}$ is the world market share of country n production using technology k, $\eta_{n,k} = \frac{\partial x_{j,k}(p-\tau_{j,k})}{\partial(p-\tau_{j,k})}\frac{p}{x_{j,k}}$ is the price elasticity of supply for country i technology k production, β_n is the consumption share of country n and ζ_n is country n price elasticity of demand.

Before solving the policymaker's problem in the open economy, it is helpful to present the policy problem for two benchmark cases: the global policymaker and the policymaker in the closed economy. The global policymaker will provide an efficiency benchmark, showing the first-best policy the maximizes social welfare and the closed-economy policymaker's problem distinguishes between effects from maximizing domestic (rather than global) welfare and effects coming from openness to trade.

2.2 Global Policymaker

Before deriving the optimal policy for an individual open economy, it is helpful to consider the optimal policy solutions for a global policymaker that is able to control all tax instruments in all countries. The global policymaker sets production tax and subsidy rates in every country to maximize the sum of global utility, serving as a useful illustration of the first-best globally efficient policy. It values pollution according to the damage function $\Gamma^*(\cdot)$ that captures the global social cost of pollution. Allowing the social planner to set all tax rates, the optimal tax rates are simply given by Proposition 1.

Proposition 1. The globally efficient tax and subsidy policy is to tax dirty production at the global social cost of pollution and to have no clean subsidies;

$$\tau_{i,d} = \gamma^* \quad \forall i$$
$$\tau_{i,c} = 0 \quad \forall i$$

where $\gamma^* \equiv \Gamma^{*'}(\sum_n x_{n,d})$. Proof in Appendix D.

As is well-known, the globally efficient policy is to impose a Pigouvian tax on dirty production everywhere. With a Pigouvian tax, there is no need to use a clean subsidy. Absent other frictions, clean production should not be taxed or subsidized. This well-known result is consistent with the targeting principle (Sandmo, 1975; Kopczuk, 2003).

The global policymaker chooses to use a clean subsidy only when it is prevented from using a tax on the dirty production process. Corollary 1 shows the optimal subsidy rates when the policymaker is restricted from taxing dirty production.[9]

Corollary 1. When the global policymaker is constrained to only use subsidies on the clean production process ($\tau_{i,d} = 0 \ \forall i$) the globally optimal subsidy rates are given as

$$\tau_{i,c} = -\gamma^* \left(\frac{\sum_{n=1}^{N} \alpha_{n,d}\eta_{n,d}}{\sum_{n=1}^{N} \alpha_{n,d}\eta_{n,d} - \sum_{n=1}^{N} \beta_n \zeta_n} \right). \tag{6}$$

Proof in Appendix D.

First, note that $\tau_{i,c}$ will always be (weakly) less than zero, indicating that the optimal policy is in fact a subsidy. The subsidy increases the price that clean producers receive, increasing clean output and lowering the market clearing price for the tradable good. The lower world price discourages dirty production but also encourages consumers to consume more of good x and less of the numeraire good z. The economic rationale for subsidizing clean production is to flood the market with clean production, crowding out dirty production, and reducing the negative externality associated with doing so. The increase in consumption of x_i is an (inefficient) unintended consequence of subsidizing clean production. This trade-off is reflected in the expression for $\tau_{i,c}$ in Equation 6.

If world demand is perfectly inelastic ($\sum_n \beta_n \zeta_n = 0$), then an increase in clean production will entirely crowd out dirty production and the subsidy rate will be set at $\tau_{i,c} = -\gamma^*$. Here the clean subsidy is able to perfectly recreate the marginal incentives created by a pollution tax.[10] However, as demand becomes very elastic or dirty production becomes very inelastic, the subsidy for clean production mostly encourages an inefficient increase in consumption of

[9] This case could separately be of interest if there were extra frictions (i.e. political economy reasons) that made implementing a tax more difficult than a subsidy.

[10] In this case, the only difference between a pollution tax and a clean subsidy is that the subsidy will feature a lower price for consumers and government revenue is negative by the amount of the subsidy. This results in a lump sum transfer from taxpayers to consumers, whereas the pollution tax is associated with a transfer from consumers to taxpayers. In a model with a representative consumer and lump sum taxes/subsidies, this has no effect on welfare, but if the social planner valued consumers and taxpayers separately this could be meaningful.

x, rather than discouraging dirty production, and therefore has limited externality benefits. In the case where $|\sum_n \beta_n \zeta_n| >> \sum_n \alpha_{n,d} \eta_{n,d}$ the subsidy rate will go to zero.

2.3 Closed Economy

In a closed economy, domestic supply must meet domestic demand so country i price is set such that domestic markets clear,

$$x_i(p_i) = x_{i,c}(p_i - \tau_{i,c}) + x_{i,d}(p_i - \tau_{i,d}).$$

The policymaker in the closed economy sets tax and subsidy rates to maximize domestic welfare,

$$\begin{aligned} &\max_{\tau_{i,c},\tau_{i,d}} U_i(x_i, z_i) - \Gamma_i(X_d) \\ &\quad \text{s.t. } x_i(p_i) = x_{i,c}(p_i - \tau_{i,c}) + x_{i,d}(p_i - \tau_{i,d}) \\ &\quad\quad T_i = \tau_{i,c} x_{i,c} + \tau_{i,d} x_{i,d} \end{aligned} \tag{7}$$

The domestically optimal tax and subsidy policy is given in Proposition 2.

Proposition 2. The closed economy policymaker setting tax and subsidy rates to maximize domestic welfare will tax dirty production equal to the domestic social cost of pollution and will not use clean subsidies.

$$\begin{aligned} \tau_{i,d} &= \Gamma_i'(X_d) \equiv \gamma_i \\ \tau_{i,c} &= 0 \end{aligned}$$

Proof in Appendix D.

Two things are worth noting about the privately optimal policy in the closed economy. First, the tax is proportional to only the marginal domestic damages from pollution (γ_i) rather than the marginal global damages from pollution ($\gamma^* = \sum_i \gamma_i$). This is the free riding problem in climate policy. If individual countries are only concerned with their domestic welfare, they will not consider the externality damages caused to other countries when pollution is emitted at home and will set insufficiently ambitious climate policy accordingly.

Second, moving to a closed economy and climate policy free riding does not affect the decision of which tax instrument to use. The policymaker in the closed economy only uses a tax on the dirty production process to correct for the externality, and the tax is set equal to the domestic social cost of pollution, like a Pigouvian tax calibrated only to domestic damages. There is no use for a subsidy on the clean production process and the tax rate on the dirty production process is not affected by the fact that other countries release pollutants as well. Although dirty production abroad impacts welfare at home, in a closed economy there are no economic interactions between countries meaning that there is no risk of leakage, but also no ability to crowd out foreign dirty production. Mathematically, foreign pollution affects the level of welfare but does not affect the marginal incentives. The closed economy simply takes foreign pollution as given and optimizes accordingly.

To further build intuition for how a subsidy on the clean production process could be useful, Corollary 2 shows the optimal subsidy rate when the closed economy policymaker cannot use a pollution tax.

Corollary 2. If the policymaker cannot implement a tax on the dirty production process, then the domestically optimal clean subsidy rate is

$$\tau_{i,c} = -\gamma_i \left(\frac{\alpha_{i,d}\eta_{i,d}}{\alpha_{i,d}\eta_{i,d} - \beta_i \zeta_{i,d}} \right).$$

Proof in Appendix D.

This expression for a closed economy clean subsidy is very similar to Corollary 1, except that it is calibrated only to the domestic economy. The optimal subsidy rate will be increasing as domestic dirty production becomes more elastic and decreasing as domestic demand becomes more elastic.

2.4 Open Economy

Finally, consider the domestically optimal policy for an individual open economy. In the open economy, the global market-clearing conditions hold, and the domestic policymaker sets policies to maximize domestic welfare, accounting for domestic damages from global pollution. For the domestic policymaker in country i that uses the set of taxes $\vec{\tau_i}$ and is considering a marginal increase in $\tau_{i,k} \in \vec{\tau_i}$, the first-order condition characterizing an optimal interior solution is

$$\lambda_i \cdot \underbrace{\vec{\tau_i} \cdot \frac{\partial \vec{x}}{\partial \tau_{i,k}}}_{\text{Fiscal Externality}} = \underbrace{\gamma_i \frac{\partial \sum_n x_{n,d}}{\partial \tau_{i,k}}}_{\text{Environmental Externality}} + \underbrace{\frac{\partial p}{\partial \tau_{i,k}} (x_{i,d} + x_{i,c} - x_i)}_{\text{Terms of Trade Manipulation}} + \underbrace{(\lambda_i - 1)\, x_{i,k}.}_{\text{Value of Inframarginal Transfer}} \quad (8)$$

Derivation in Appendix D.

In order to focus on the role of correcting for the environmental externality, I ignore the last two terms, implicitly assuming that the domestic policymaker cannot use environmental policy to manipulate their terms of trade and ignores inframarginal transfers. Formally, these assumptions are:

Assumption 1. For the purpose of setting domestic policy, the policymaker acts *as if* $x_{i,d} + x_{i,c} - x_i = 0$.

Assumption 2. For each country $\lambda_i = 1$.

Ignoring terms of trade manipulation can be justified by strong international trade rules that allow domestic policy to correct for environmental policy but not to manipulate trade (as WTO rules aspire to do). Note that the terms of trade effect for country i is proportional to its net exports or imports of the tradable good x. If country i produces as much of x_i as it consumes, as would happen if countries are identical, then the incentive to manipulate the terms of trade will not have first-order effects on the optimal tax or subsidy.

Ignoring inframarginal transfers, and therefore assuming $\lambda_i = 1$, is common in the literature on optimal externality correction (e.g. Griffith, O'Connell and Smith, 2019). This assumption is true in a representative consumer model if governments can use lump sum taxes and transfers to raise revenue.[11]

The primary difference between the policymaker's incentives in a closed economy and open economy relate to the potential for domestic policy to change the level of pollution emitted abroad through an integrated market. This "pollution leakage" problem has been studied widely and will play a critical role in determining the optimal domestic policy. Following Fowlie and Reguant (2022), the marginal rate of pollution leakage for country i is defined as the increase in pollution abroad divided by the decrease in domestic pollution caused by a small increase in the pollution tax. Formally, this term is defined as

Definition 1.

$$L_i := \frac{-\sum_{n \neq i} \frac{\partial x_{n,d}}{\partial \tau_{i,d}}}{\frac{\partial x_{i,d}}{\partial \tau_{i,d}}} = \frac{\sum_{n \neq i} \alpha_{n,d} \eta_{n,d}}{\sum_{n \neq i} \alpha_{n,d} \eta_{n,d} + \sum_n \alpha_{n,c} \eta_{n,c} - \sum_n \beta_n \zeta_n}. \tag{9}$$

The second equality in the expression for the marginal rate of leakage is derived in Appendix D. Pollution leakage occurs because increasing the pollution tax for country i increases the world price p, which in turn sends a price signal for foreign dirty producers to increase their

[11] Note that this assumption is also true in a model with heterogeneous consumers if the income tax system is set optimally Kaplow (2024). Crucially, if none of these explanations are true, the parameter will depend on how climate policy interacts with other policy goals such as redistribution and crucially depends on the progressivity of both the source and use of the revenue (as discussed in Goulder et al., 2019). Allowing $\lambda_i \neq 1$ will add additively separable revenue raising terms to the optimal policy expressions, but will not impact the environmental externality correcting terms.

production. The leakage rate in the baseline model will be bounded between zero and one.[12] Note that the leakage rate will be higher when dirty production abroad is more elastic or makes up a larger share of the world market. Conversely, the leakage rate will be larger when demand or clean production is relatively more elastic or when clean production has a larger market share.

2.4.1 Pollution Tax Only

I first consider the optimal policy for a country that only is able to use a pollution tax on the dirty production process. Here the policymaker must trade-off discouraging domestic dirty production against the possibility of pollution leakage. The optimal pollution-tax only rate is given in Proposition 3.

Proposition 3. If country i uses only a tax on dirty production to maximize welfare, the optimal pollution only tax is given as

$$\tau_{i,d} = \gamma_i (1 - L_i) \tag{10}$$

where L_i is the pollution leakage rate.

Proof in Appendix D.

Proposition 3 shows that in an open economy the optimal polluting tax is now smaller than the domestic social cost of pollution. Instead, the tax rate is adjusted to account for foreign leakage. When an open economy imposes a carbon tax, it puts a wedge between the price domestic producers receive and the world price, but also changes the world price. When the world price increases, foreign producers will increase production and consumers will decrease consumption. If foreign dirty producers increase production in response, then the net effect on worldwide pollution is smaller than the change in domestic dirty production. This is pollution leakage. Leakage makes the pollution tax less effective, and therefore the greater the leakage rate, the smaller the optimal pollution tax will be, when used alone.

2.4.2 Clean Subsidy Only

Instead, if the policymaker only uses a clean subsidy, they will set the subsidy rate in order to lower the worldwide price, reducing producer prices for dirty producers and decreasing

[12] Section 4 explores the case where the emissions intensity of dirty production can vary between countries. In this case, the difference in emissions intensities will contribute to the leakage rate, with higher foreign emissions intensities leading to larger leakage. In the extreme case where the emissions intensity of foreign dirty production is much larger than the domestic emissions intensity, the leakage rate can be greater than one.

production. When used alone, the domestically optimal clean subsidy rate is given by Proposition 4.

Proposition 4. If country i uses only a subsidy on clean production to maximize welfare, the optimal clean-subsidy rate is given as

$$\tau_{i,c} = -\gamma_i \left(\frac{\sum_n \alpha_{n,d}\eta_{n,d}}{\sum_n \alpha_{n,d}\eta_{n,d} + \sum_{n\neq i} \alpha_{n,c}\eta_{n,c} - \sum_n \beta_n \zeta_n} \right).$$

Proof in Appendix D.

The domestic clean subsidy now is influenced by the worldwide elasticity of dirty production, the worldwide elasticity of demand, and the elasticity of clean supply in the rest of the world. Equation 5 shows that the more elastic worldwide supply and demand is, the larger a subsidy rate is required to change the worldwide price. If it is worldwide dirty supply that is highly elastic, then it is worthwhile for an individual country to choose a large subsidy rate because even though it creates a large distortion domestically, a decrease in the worldwide price translates to a large reduction in global pollution. However, if instead world demand or clean supply in the rest of the world are very elastic, the domestic subsidy rate will be small. This is because very elastic foreign clean supply or world demand make it so that a large domestic subsidy is required to change the world price, creating large economic distortions domestically, but this change in the world price does not necessarily translate into reductions in global pollution. Therefore, if foreign clean supply or world demand are very elastic relative to world dirty supply, it will not be worthwhile for a domestic policymaker to use a large clean subsidy.

2.4.3 Combined Pollution Tax and Clean Subsidy

If the policymaker is able to use a pollution tax and a clean subsidy together, the policymaker now optimally uses both instruments as shown in Proposition 5.

Proposition 5. If country i uses both a production tax and clean subsidy, the optimal tax policy is given as

$$\tau_{i,d} = \gamma_i \left(1 - L_i \frac{1}{1-\omega_i}\right) \tag{11}$$

$$\tau_{i,c} = -\gamma_i \left(L_i \frac{1}{1-\omega_i}\right) \tag{12}$$

where L_i is the marginal rate of leakage and $\omega_i = -\frac{\partial x_{i,c}/\partial \tau_{i,d}}{\partial x_{i,d}/\partial \tau_{i,d}} = \frac{\alpha_{i,c}\eta_{i,c}}{\sum_{n\neq i}\alpha_{n,d}\eta_{n,d}+\sum_n \alpha_{n,c}\eta_{n,c}-\sum_n \beta_n\zeta_n}$.

Proof in Appendix D.

Proposition 5 shows that in an open economy, it is optimal to use both a pollution tax and a clean subsidy. This is in contrast to the global policymaker and closed economy where a tax on the polluting production process was able to fully internalize the externality and there was no need for a clean subsidy. Openness to trade impacts the instrument choice of which tax and subsidy instruments the policymaker optimally chooses to use.

In Proposition 5, the pollution tax is still modified by an "adjusted" leakage term that is identical to the expression for the clean subsidy. This suggests that in the open economy, the pollution tax and clean subsidy are substitutes and market conditions that make a tax more (less) effective make the clean subsidy less (more) effective.[13]

Equation 9 shows that the leakage rate will be higher when foreign production is more elastic or has a greater market share relative to clean production and demand. These conditions make it so that an increase in the world price resulting from the pollution tax will result in a greater increase in foreign dirty production. At the same time, when foreign dirty production is very responsive to changes in the world price, a decrease in the world price resulting from a clean subsidy will lead to larger reductions in world pollution. The leakage effect from pollution taxes and the pollution abatement effect of clean subsidies work through the same mechanism, changing world pollution through changes in the world price. A stronger leakage effect reduces the effectiveness of the pollution tax but makes the clean subsidy more effective.

At one extreme, when demand and clean supply are very inelastic, the leakage rate will approach one. This indicates that any reduction from domestic dirty production caused by the pollution tax will be almost entirely offset by an increase in foreign dirty production, and a pollution tax will not change world pollution. At the same time, any decrease in the world price from the clean subsidy will entirely crowd out dirty production, making the clean subsidy more effective.[14]

On the other hand, when foreign dirty production is very inelastic, or the foreign market is almost entirely clean, the leakage rate will be close to zero. Here the pollution tax can be set at the Pigouvian rate $\tau_{i,d} = \gamma_i$ without being undermined by pollution leakage. At the same time, setting a clean subsidy will be ineffective, as foreign pollution is very inelastic so

[13]The adjustment to the leakage term, $1/(1-\omega_i)$, captures the cross fiscal externality between the tax and subsidy rates. When both instruments are used, changing one instrument will impact the economic distortions created by the other instrument, an interaction accounted for by ω_i. As the country size gets smaller, ω_i will become small, and in the limit the adjustment to the leakage rate can be ignored.

[14]Although very inelastic clean supply makes it necessary to impose a larger subsidy to change the world price, the very limited behavioral response means that there is only a very limited economic distortion. The optimal subsidy rate ends up being independent from the elasticity of domestic clean production, just as the Pigouvian rate on a polluting good is independent of the supply elasticity.

any increase in domestic clean production will either crowd out foreign clean production or cause demand to increase. Even if domestic dirty production is elastic, this is most directly corrected for by the pollution tax rather than the clean subsidy.

More generally, the optimal policy rates in Proposition 5 are consistent with the targeting principle of optimal taxation if domestic dirty production and foreign dirty production are considered as separate externality producing goods. The generalized principle of targeting, as formulated by Kopczuk (2003), states that when externality producing goods are directly taxable, the optimal tax rates are expressed as the sum of the Pigouvian tax on the externality producing good and the optimal rates were there no externality. In the open economy context, only domestic dirty production is directly taxable, so the optimal policy rates should be the sum of the Pigouvian tax on domestic dirty production plus the optimal rates were there no externality from domestic dirty production.

If domestic "dirty" production was not dirty, the policymaker would set policy rates on all forms of domestic production to crowd-out dirty production abroad. Here the optimal policy would be to subsidize both forms of domestic production equally as both instruments can be used to lower the world price and crowd out dirty production. The optimal policy expressions in Proposition 5 show this explicitly, both instruments are modified equally to reduce the world price, reducing foreign production. The only difference comes from an additive Pigouvian term to the domestic pollution tax used to correct for the externality from domestic dirty production.

3 Policy Ambition and Instrument Choice

This section presents two important results from the baseline model. It first discusses what the expressions for the domestically optimal combined pollution-tax and clean-subsidy rates imply for the overall level of policy ambition. It then discusses results related to the optimal choice of policy instruments, highlighting when a pollution tax should be relied on more heavily than a clean subsidy. Finally, it discusses the connection between policy ambition and avoided emissions and presents a stylized example.

3.1 Optimal Level of Climate Policy Ambition

The expressions for the optimal policy in Proposition 5 have important implications for policy ambition in the open economy. The optimal policy expressions lead directly to the result that optimal policy ambition is invariant in the open economy, as stated in Proposition 6.

Proposition 6. The optimal policy ambition from the optimal combined pollution tax and clean subsidy, as measured as the sum of the tax rate and subsidy rate, is constant and always equal to the value of avoided pollution.

$$\tau_{i,d} + |\tau_{i,c}| = \gamma_i. \tag{13}$$

Proposition 6 demonstrates that in the open economy, the overall ambition of climate policy is only a function of the value that the domestic policymaker places on reductions in pollution, γ_i. This is true regardless of the values of the marginal leakage rate, market shares, or supply and demand elasticities domestically and abroad. Although all of these values will affect the optimal pollution tax rate and optimal clean subsidy rate individually, the total ambition of climate policy, as measured by the sum of the pollution tax and clean subsidy, will remain unchanged.

3.2 Optimal Choice of Tax and Subsidy Instruments

Although Proposition 6 shows that the overall optimal level of policy ambition depends only on how much the policymaker values avoided pollution, it does not provide insight into when policy should primarily use pollution taxes or clean subsidies.

To investigate the choice of policy instruments, Proposition 7 presents the expression for the optimal policy ratio, defined as the optimal pollution tax rate divided by the optimal clean subsidy rate.

Proposition 7. The optimal policy ratio in an open economy is

$$\frac{\tau_{i,d}}{\tau_{i,c}} = -\frac{\sum_{n\neq i} \alpha_{n,c}\eta_{n,c} - \sum_n \beta_n \zeta_n}{\sum_{n\neq i} \alpha_{n,d}\eta_{n,d}} = -\frac{1 - L_i - \omega_i}{L_i}. \tag{14}$$

The optimal policy ratio provides a measure of how much country i should optimally rely on a pollution tax relative to a clean subsidy to reduce pollution.

A close examination of Equation 14 shows that the policy ratio changes in a way that is consistent with the effectiveness of pollution taxes and clean subsidies. If foreign dirty supply is much more responsive to price increases than foreign clean supply or world demand, a change in prices will have a large impact on foreign dirty production. This will exacerbate leakage from the pollution tax but at the same time make a clean subsidy more effective at reducing global pollution, making country i favor a clean subsidy. On the other hand, if dirty supply is very inelastic, the leakage problem of a pollution tax is minor while the clean subsidy is ineffective, making country i favor a pollution tax.

If supply elasticities are constant, then the relative use of a tax/subsidy will be related to the market share in the rest of the world. If the foreign supply is mostly dirty, the marginal rate of leakage will be larger, making a clean subsidy relatively more attractive than a pollution tax. However, if the foreign market is already dominated by clean supply, the clean subsidy will be relatively ineffective and there will be little leakage from the pollution tax, favoring the use of a pollution tax.

The second equality in Proposition 7 shows that the policy ratio only depends on the leakage rate, L_i, and the cross-fiscal-externality adjustment, ω_i. If domestic clean supply is either very inelastic or a small share of the world market, ω_i will go to zero. In this case, the optimal policy ratio will simply be equal to $-\frac{1-L_i}{L_i}$. The marginal rate of leakage is therefore a sufficient statistic for the optimal policy ratio in a small open economy where the world market share of domestic clean production will be close to zero. Economies with lower leakage rates should rely more heavily on pollution taxes while economies with higher leakage rates should instead rely on clean subsidies.

Importantly, the optimal policy ratio does not depend on γ_i. Proposition 6 shows that γ_i is a sufficient statistic for policy ambition. The choice of policy instruments is therefore independent of the level of policy ambition, further demonstrating the separability between climate policy ambition and instrument choice. An open economy policymaker should therefore place the same relative reliance on a tax versus a subsidy regardless of how much they value reductions in global emissions (all else equal).

3.3 Avoided Emissions

It is important to note that the constant policy ambition result in Proposition 6 does not mean that emissions reductions will be the same for different industries or economies with a common level of policy ambition. The reduction in global pollution created by country i is a function of both the tax and subsidy rates, as well as the relevant elasticities.

The connection between avoided emissions and pollution leakage is explored in more detail in Appendix C. Surprisingly, the relationship between avoided emissions and the rate of pollution leakage is ambiguous. It is not necessarily the case that optimal policy in jurisdictions or industries with higher leakage rates will have a smaller effect on global emissions. If domestic dirty production is very inelastic, must pollution mitigation will occur abroad and conditions that create higher leakage rates will lead to larger reductions in global pollution. These conditions will increase the use of clean subsidies, decreasing the world price and emissions abroad, more than offsetting the smaller emissions reductions domestically from a lower pollution-tax rate.

3.4 Stylized Example

For concreteness, consider a stylized example in a two country world where home (h) and foreign (f) supply and demand curves are linear and given as follows:[15]

$$x_{h,c} = 2(p - \tau_{h,c}) \qquad x_{f,c} = p - \tau_{f,c}$$
$$x_{h,d} = p - \tau_{h,d} \qquad x_{f,d} = 2(p - \tau_{f,d})$$
$$x_h = 3 - \frac{\kappa}{2}p \qquad x_f = 3 - \frac{\kappa}{2}p.$$

Home is a lower cost and more elastic clean producer but a higher cost and less elastic dirty producer. In this stylized example with linear supply and demand, home policy will not depend on the policy choices of f.[16]

The rate of pollution leakage will be higher in h, as f has more elastic dirty production. As a result, country h will rely relatively more on the clean subsidy and relatively less on a pollution tax than country f. The parameter κ controls the slope of the world demand curve and will affect the rate of pollution leakage, and therefore the optimal policy choices. Table 1 summarizes the leakage rate, optimal policies, and avoided global pollution from country h's policy under high-leakage ($\kappa = 0$), mid-leakage ($\kappa = 0.5$) and low-leakage ($\kappa = 1$) cases when $\gamma_h = 1$.

Table 1: Home (h) Policy in a Two-Country World ($\gamma_h = 1$)

	Case 1: High-Leakage	Case 2: Mid-Leakage	Case 3: Low-Leakage
Marginal Leakage Rate	0.4	0.36	0.33
Pollution Tax Rate	0.33	0.43	0.5
Clean Subsidy Rate	0.67	0.57	0.5
Optimal Policy Ratio ($\tau_{i,d}/\tau_{i,c}$)	0.5	0.75	1
Policy Ambition	1	1	1
Avoided Global Pollution	28%	27%	28%

Note: This table summarizes the leakage rates, optimal policies, and total avoided pollution for country h under low-leakage (column 1), mid-leakage (column 2) and high-leakage (column 3) scenarios, where the difference in the leakage rate comes from making world demand more price sensitive. The marginal leakage rates are calculated according to Equation 9. The optimal pollution tax rate, $\tau_{h,d}$, and clean subsidy rate, $|\tau_{h,c}|$, are calculated according to Proposition 5 when $\gamma_h = 1$. Avoided global pollution is measured as the percent reduction in domestic and foreign dirty production from country h's optimal policy, relative to the case with no climate policy in either country.

[15] The case of linear supply given in this example comes from quadratic total cost curves.

[16] A more complete analysis of policy interaction effects is presented in Appendix B

The table shows the leakage rate, optimal policy, and avoided pollution for country h under each scenario when $\gamma_h = 1$. In the high-leakage scenario, country h has a relatively high leakage rate (0.4) and relies much more on a clean subsidy than a pollution tax (the subsidy rate is twice the tax rate). In the mid-leakage and low-leakage scenarios, the leakage rate decreases, making the pollution tax more effective and the clean subsidy less effective. As a result, the optimal tax rates increase and the optimal subsidy rates decrease, leading to an increase in the policy ratio. In all three scenarios, the sum of the tax rate and subsidy rate, the policy ambition, is equal to γ_h, demonstrating that policy ambition is invariant to the rate of leakage (Proposition 6).

Finally, the last row shows how much country h's policy reduces pollution from a baseline of no climate policy. In this example, the relationship between the rate of leakage and avoided pollution is not monotonic. In both the high and low leakage scenarios, the combined tax and subsidy policy reduces global dirty production by 28%. However, in the mid-leakage scenario, the reduction in global emissions drops to 27%. In general, the relationship between leakage and the level of emissions avoided by the optimal policy is ambiguous, and depends on the model primitives, emphasizing that the presence of pollution leakage is not necessarily a problem for effective climate policy.[17]

4 Differing Emissions Intensities

In the base model of Section 2, one unit of dirty production is assumed to produce one unit of pollution. This section relaxes that assumption, allowing for the realistic case where the rate of pollution created by dirty production differs between countries.

The emissions intensity in country i is normalized to one. This normalization means that γ_i is still interpreted as the value to country i from the reduction in pollution associated with a one unit decrease in country i dirty production. The relative emissions intensity in country $n \neq i$, e_n, can be interpreted as the amount of pollution produced for one unit of country n dirty production relative to the amount of pollution created by one unit of country i dirty production.

The rate of leakage in country i will still be defined as the increase in foreign emissions per each unit of avoided domestic emissions associated with the domestic pollution tax.

[17]This result highlights that the rate of pollution leakage is not an exogenous model primitive, but rather is determined by the same model primitives that determine how effective clean subsidies are at reducing emissions.

Accounting for differing emissions intensities, the leakage rate is given as

$$L_i := \frac{-\sum_{n\neq i} e_n \frac{\partial x_{n,d}}{\partial \tau_{i,d}}}{\frac{\partial x_{i,d}}{\partial \tau_{i,d}}} = \frac{\sum_{n\neq i} e_n x'_{n,d}}{\left[\sum_{n\neq i} x'_{n,d} + \sum_n x'_{n,c} - \sum_n x'_n\right]}. \tag{15}$$

The leakage rate is very similar to the base model, except that it now depends on the relative emissions intensity of dirty production abroad. If domestic dirty production is much dirtier than abroad ($e_n << 1$) then foreign dirty production has only a minor impact on global pollution and the leakage rate will be close to zero. On the other hand, as the emissions intensity of domestic dirty production decreases relative to the emissions intensity abroad, the leakage rate will increase and is no longer bounded above by one. When the domestic emissions intensity is very low relative to abroad, a tax on dirty domestic production could increase worldwide pollution by increasing much higher emitting dirty production abroad.

The variability in the emissions intensities will affect the domestic policymaker's optimal policy. The policymaker still values a marginal unit of avoided pollution at γ_i, which means that the environmental welfare impacts of domestic and foreign dirty production will differ. The domestically optimal policy is given in Proposition 8.

Proposition 8. The optimal tax and subsidy rates for the policymaker in country i are

$$\tau_{i,d} = \gamma_i \left(1 - L_i \frac{1}{1-\omega_i}\right)$$
$$\tau_{i,c} = -\gamma_i L_i \frac{1}{1-\omega_i}.$$

Proposition 8 looks identical to Proposition 5. The fact that emissions intensities can vary between countries is entirely captured by the effect of e_n on the marginal leakage rate, L_i. A higher foreign emissions intensity will increase the leakage rate, resulting in a lower optimal tax rate and a higher optimal subsidy rate. When the adjusted leakage rate is less than one, the qualitative results from Section 2 remain unchanged: openness to trade creates an incentive for the domestic policymaker to both subsidize clean production and tax dirty production, and the overall level of climate ambition remains constant.

4.1 Optimal Policy with Leakage Rates Greater than One

Unlike in the uniform emissions intensity case of Section 2, where $L_i \in [0, 1]$, the fact that the leakage rate can be greater than one has important implications for policy. Specifically, when the adjusted leakage rate is greater than one, it is optimal for the domestic policymaker to subsidize, rather than tax domestic dirty production.[18] Even though dirty production still produces a negative externality, a leakage rate greater than one implies that encouraging additional domestic dirty production will crowd out enough of the even dirtier foreign production to reduce global emissions in aggregate.

In this case, the optimal policy will still set a constant wedge between domestic clean and dirty producers equal to γ_i:

$$\tau_{i,d} - \tau_{i,c} = \gamma_i. \tag{16}$$

However, because both policies are subsidies, the total policy ambition will no longer be constant, when policy ambition is defined as $|\tau_{i,d}| + |\tau_{i,c}|$. As the leakage rate continue to increase above one, the policymaker will further subsidize domestic dirty and clean production. Intuitively, if domestic dirty production has a much smaller emissions intensity than abroad, dirty production at home is not very dirty and both instruments work in the same way—increasing domestic production to crowd out dirtier foreign production.

It will always remain the case that the policymaker wants to incentivize domestic clean production more than domestic dirty production by γ_i dollars per unit. The normalization of the domestic emissions intensity is important for interpreting this wedge as γ_i is interpreted as the value of reduced pollution associated with one unit of *domestic* dirty production.

This finding remains consistent with the generalized targeting principle as formulated by Kopczuk (2003). The targeting principle implies that a Pigouvian-like tax should be applied to domestic dirty production exactly equal to the externality it causes. The Pigouvian component should be additive to any other tax policy objectives, which in this case are crowding out foreign dirty production that cannot be directly taxed. When the adjusted leakage term is greater than one, the incentive to crowd out foreign production is large enough that the optimal tax on dirty production is negative (a subsidy). The constant wedge between domestic dirty and clean production follows directly from the additive correction prescribed by the targeting principle.

5 Model Implications and Discussion

This section discusses a number of extensions and implications of the baseline model. It first discusses how other policy tools that are used to mitigate pollution leakage, notably import taxes (carbon border adjustments) and output-based subsidies, connect to the analysis in the base model. It then discusses how the results from the baseline model are influenced by the size of the jurisdiction implementing the policy. Finally, it presents an extension allowing

[18] Jaqua and Schaffa (2022) show that in general, it can be optimal to subsidize harmful activities when the policymaker is constrained from implementing the first best Pigouvian tax.

for domestic policy to influence policy rates in other jurisdictions and discusses these policy spillovers.

5.1 Traditional Leakage Mitigation Policies

Pollution leakage has long been recognized as impacting optimal policy for correcting global pollutants. To date, pollution leakage has primarily been discussed as a problem for environmental and climate policy given that leakage decreases the effectiveness of the pollution tax (see Grubb et al., 2022, for example). Underappreciated in this discussion is that the same conditions that create pollution leakage make subsidizing clean substitute goods more effective at reducing global pollution.

The academic literature and real-world carbon pricing policies have traditionally discussed and employed two policy tools intended to address leakage: carbon border adjustments and output-based subsidies. This subsection discusses the connections between these two policy tools and the analysis presented in Sections 2 and 3.

5.1.1 Carbon Border Adjustments

Carbon border adjustments are one of the most common policies intended to complement carbon taxes by addressing pollution leakage. The idea of a carbon border adjustment is to supplement a domestic pollution tax with an import tax and export subsidy based on the embedded emissions associated with importing pollution-intensive goods from abroad. Border adjustments are often described as leveling the playing field for domestic producers by setting import tariffs and export subsidies equal to the difference between the domestic and foreign pollution-tax rates. The European Union has recently adopted a border-carbon adjustment to complement its emissions trading system (European Comission, 2024). However, as Weisbach et al. (2023) and Kortum and Weisbach (2022) discuss, the optimal border adjustment rate does not entirely offset the domestic pollution tax, i.e. the optimal border tax is should be set at a lower rate than the domestic tax on dirty production.

Appendix A.1 incorporates an import tariff into the model setup of Section 2 by allowing the policymaker to tax consumption of the energy composite good. When the policymaker can use a border adjustment, it is optimal to do so. However, the optimal policy still includes a subsidy on domestic clean production. The intuition is that, although the border adjustment can be used to keep domestic consumers from importing polluting energy from abroad, it cannot reduce polluting energy that is produced and consumed abroad. Subsidizing domestic clean production essentially floods the world market, crowding out dirty production abroad.

It is worth noting that in this setup, the wedge between the price received by domestic clean producers and domestic dirty producers remains constant and equal to γ_i, reflecting the constant policy ambition discussed in Proposition 6. It is also true that the wedge between what domestic dirty producers receive and what domestic consumers pay is also equal to γ_i, an insight discussed by Weisbach et al. (2023) and Kortum and Weisbach (2022). The introduction of the clean good does not undermine the rationale to tax both consumers and producers, nor does allowing for a border tariff undermine the rationale for subsidizing clean producers specifically.

5.1.2 Output-Based Subsidies

The other common policy tool used to address pollution leakage is an output-based subsidy. Output-based subsidies provide all producers of a good with a per-unit subsidy and are traditionally targeted at firms that pay emissions taxes on a pollution factor of production, typically energy. In quantity-based cap-and-trade programs, output-based subsidies are implemented as free allocations of permits. In practice, this is how two of the most prominent carbon pricing programs, the European Union's emissions trading system and California's cap-and-trade program, have traditionally addressed leakage (European Comission, 2015).

The combined pollution tax and output-based subsidy regimes are often targeted at industries "exposed to a significant risk of 'carbon leakage' to support their competitiveness" (European Comission, 2016). They are commonly justified as a necessary tool for preventing energy-intensive sectors from being put at "a competitive disadvantage" despite "limiting the urgency to reduce emissions" (European Comission, 2015).

The analysis of subsidies for clean substitute goods in Sections 2 and 3 is related to many of the more theoretically grounded rationales for using output-based subsidies in combination with carbon taxes. For example, the combined tax and subsidy policy given by Proposition 5 could be implemented as a carbon tax on dirty production equal to γ_i and an output-based subsidy on all producers, equal to $-\gamma_i L_i \frac{1}{1-\omega_i}$.

However, the economic motivation for clean subsidies in this paper differs from the rationale traditionally given for output-based subsidies. Subsidizing clean production proactively reduces global pollution and can be an effective pollution mitigation policy even without an accompanying tax on pollution. In addition, the analysis of clean subsidies demonstrates that, through effects on global prices, clean subsidies can crowd-out dirty production in foreign jurisdictions, even if there is only limited trade directly between the two jurisdictions.

Finally, although output-based subsidies are typically granted to polluting firms, clean subsidies are designed to be granted to firms even if they do not produce any pollution, as a way of promoting substitute goods. Although the baseline model considers a model where the output of dirty and clean production is homogeneous and perfectly substitutable, the economic mechanism rationalizing clean subsidies extends to differentiated but substitutable goods. Therefore, clean subsidies could be justified in open economies for a wider set of sectors and industries (e.g., electric vehicles or other tradable clean consumer goods) than for just the narrow set of industrial materials producers that traditionally received output-based subsidies.

5.2 Optimal Policy in Small Economies

Sections 2 and 3 do not explicitly discuss how country size affects the incentive to use a pollution tax or a clean subsidy. Given that small open economies have only very small effects on world prices, it is tempting to think that a small open economy policymaker need not worry about pollution leakage from a pollution tax and that a clean subsidy would be ineffective, given that subsidies change foreign pollution through changing the world price. This reasoning, however, is incomplete. This section discusses the role of climate policy in small open economies by considering an example with many identical countries. The market share of any individual country will decrease as the number of countries increases.

Consider the case with N identical countries where the slopes of the supply and demand curves in each country are identical and given by

$$x'_{i,d} = x'_d; \qquad x'_{i,c} = x'_c; \qquad x'_i = x' \quad \forall i.$$

In this case, the ratio of the pollution tax to the clean subsidy in any particular country is given as

$$\left|\frac{\tau_{i,d}}{\tau_{i,c}}\right| = \frac{(N-1)x'_c - Nx'}{(N-1)x'_d} = \frac{x'_c - x'}{x'_d} + \frac{-x'}{(N-1)x'_d}. \tag{17}$$

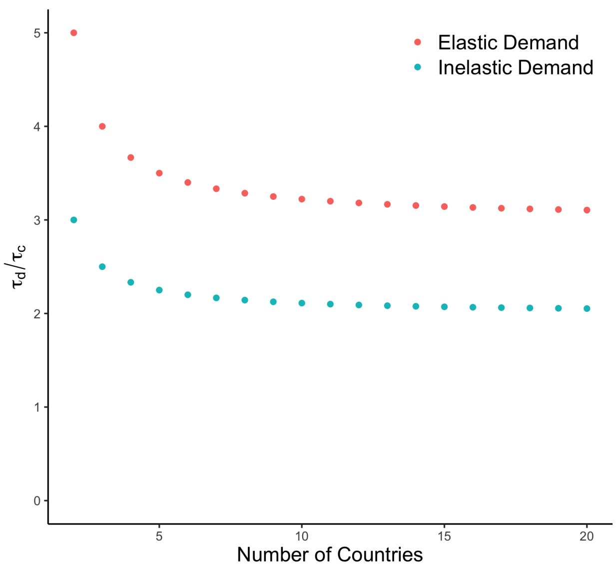
The number of countries impacts only the last term, $\frac{-x'}{(N-1)x'_d}$, which is positive. As the number of countries increases, this term becomes smaller, slightly decreasing the ratio of $\mid \tau_{i,d}/\tau_{i,c} \mid$. Therefore, as country size decreases, the open economy policymaker should still use both a pollution tax and clean subsidy. In fact, decreasing country size (by increasing the number of countries) will increase the leakage rate, giving the policymaker stronger incentives to use a clean subsidy rather than a tax.[19]

Figure 1 demonstrates this point, showing simulated values for Equation 17 as the number of countries increases for examples with relatively elastic demand (red) and relatively

[19] This observation is true in general and not specific to identical countries. An examination of the leakage rate in Equation 9 reveals that decreasing the domestic market share will decrease the denominator of L_i, resulting in a higher leakage rate.

inelastic demand (blue). As the number of countries increases, the market share for any individual country decreases, and the policy ratio for both the elastic and inelastic economy asymptote. Therefore, even for small open economies, the optimal policy is to use both policy instruments. As country size decreases, $|\tau_{i,d}/\tau_{i,c}|$ decreases, indicating that small economies should rely more on clean subsidies, not less.

Figure 1: Optimal Tax-to-Subsidy Ratio as Country Size Decreases



The policymaker setting taxes and subsidies trades off the environmental benefit of reducing worldwide pollution against the distortions caused in the domestic economy caused by the tax or subsidy. For a small open economy, a clean subsidy sends only a very small price signal but this price signal is sent to many countries. In fact, with identical countries, the decrease in worldwide production from a marginal increase in the clean subsidy in one country is the same whether the economy is open or closed. Moving to an open economy, the effect on the world price is $\frac{1}{N}$ times smaller than in the closed economy, but it is reduces pollution in N countries instead of just one, exactly offsetting the smaller change in price.

Large open economies rely less on a clean subsidy than small economies due to the larger relative significance of demand distortions in the local market. When a clean subsidy increases the production of clean goods domestically, some of this additional production crowds out foreign dirty production, but some crowds out foreign clean production or induces increased consumption. Inducing additional domestic consumption is an additional (inefficient) distortion caused by an increase in clean production. This distortion is less important in small open economies than in large open economies, making the clean subsidy slightly more effective in a small economy.

5.3 Policy Spillovers in a Multi-Country Equilibrium

The analysis in Sections 2 and 3 focuses on the unilaterally optimal policy—the optimal policy when policies in other jurisdictions are constant. However, it is possible that changing policy at home can change the effectiveness of policy abroad, thus leading to policy spillover effects.

Many of the discussions of international climate policy have focused on the potential for policy spillovers, although the nature of the spillover is theoretically ambiguous. In theory, home and foreign policy could be strategic complements, where increasing the ambition of policy at home catalyzes policymakers in other jurisdictions to increase their ambition as well. However, increasing policy ambition at home could reduce the urgency with which policymakers elsewhere feel the need to reduce emissions, leading domestic and foreign policy to be strategic substitutes.

Appendix B presents a formal analysis of policy spillovers, focusing in particular on whether the choice of using a pollution tax or a clean subsidy changes the nature of the spillover. In this context, the expressions in either Propositions 3, 4, or 5 can be viewed as the best response functions for jurisdiction i, depending on which tax instruments are available. The policy at home can change the equilibrium policy abroad if it changes the values of these expressions, either by affecting the level of γ, the leakage rate, or the correction ω.

The analysis in Appendix B derives the sign of the policy spillovers for pollution taxes and clean subsidies, focusing on whether different policy instruments create different policy spillovers. For example, if clean subsidies lead policymakers in other jurisdictions to increase policy ambition, while pollution taxes lead foreign policymakers to reduce their policy ambition, this may impact the optimal choice between a pollution tax and a clean subsidy in a way that is not considered in Section 3.

However, as detailed in Appendix B, this appears unlikely to be a concern. First, if all countries optimally use both a pollution tax and a clean subsidy, Proposition 6 shows that policy ambition abroad will be constant and equal to how much foreign policymakers value a reduction in emissions. Furthermore, if all countries are constrained to use only a pollution tax or only a clean subsidy, the nature of the policy spillovers will be the same. If policy ambition at home and abroad are strategic complements when all countries exclusively use pollution taxes, home and foreign policy ambition will also be strategic complements when all countries use clean subsidies. This analysis suggests that the potential for policy spillovers should not be used to justify one policy instrument over another.

6 Conclusion

State, provincial, and national governments setting climate policies have used both clean subsidies and pollution taxes to correct for greenhouse gas emissions, despite the widespread advocacy for carbon taxes from economists. Although the use of clean subsidies can be rationalized if other market imperfections are present, the classic "targeting principle" of optimal taxation suggests that a policymaker in a closed economy should correct for an environmental externality with a direct tax on pollution and that a clean subsidy is of no use. However, an open economy limits the taxing jurisdiction of an individual policymaker, making the targeting principle no longer directly applicable.

This paper demonstrates that openness to trade creates an economic rationale for clean subsidies to supplement pollution taxes, even when markets are perfectly competitive and no other externalities or frictions are present. The optimal environmental policy of an open economy uses both a pollution tax and a clean subsidy, with the relative importance of each policy determined by the marginal rate of pollution leakage. Furthermore, the optimal level of policy ambition is invariant to the degree of leakage. This implies that a higher pollution leakage rate should not discourage climate policy generally, but instead should shift some of the reliance from a pollution tax to a clean subsidy.

These considerations will only be more important for designing efficient climate policy in the future. As national governments in the US and abroad have backed away from ambitious climate policies, attention has shifted back to state and provincial policymakers. In subnational jurisdictions, efficient policy must consider the important role that trade and pollution leakage have on the effectiveness of potential policy tools. Critically, while pollution leakage should be analyzed to inform policymakers as to how to implement policy, it should not be used as a justification for not implement any policy at all.

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Appendix

A Model Extensions

A.1 Carbon Border Adjustments

I allow the social planner to additionally impose a tax on consumption of the tradable good. Taxing both consumption and production can be used to recreate the incentives of a border tax (as pointed out by Böhringer, Rosendahl and Storrøsten (2017) and Weisbach et al. (2023) in the context of pollution leakage). I add a consumption tax, t_i, to the baseline model and allow the policymaker to jointly optimize production taxes on clean and dirty production and consumption of the tradable good. Doing so is akin to extending the analysis of Weisbach et al. (2023) to allow a clean substitute good with a subsidy.

The consumer problem is only slightly altered to include a tax t_i on the tradable good x_i. The consumer's problem is now

$$\max_{x_i, z_i} U_i(x_i, z_i)$$
$$\text{s.t. } y_i + \pi_i = (p + t_i)x_i + z_i$$

The firms' problems are unchanged and market clearing condition is unchanged as well. The domestic policymaker jointly optimizes $\tau_{i,d}$, $\tau_{i,c}$, and t_i. The optimal tax and subsidy rates are given by Proposition 9.

Proposition 9. A domestic policymaker using a pollution tax, clean subsidy, and consumer tax will set tax and subsidy rates as follows:

$$\tau_{i,d} = \gamma_i \left(1 - L_i \frac{1}{1 - \omega_i^{cba}}\right)$$
$$\tau_{i,c} = -\gamma_i L_i \frac{1}{1 - \omega_i^{cba}}$$
$$t_i = \gamma_i L_i \frac{1}{1 - \omega_i^{cba}}$$

where $\omega_i^{cba} = \frac{\alpha_{i,c}\eta_{i,c} - \beta_i \zeta_i}{\sum_{n \neq i} \alpha_{n,d}\eta_{n,d} + \sum_n \alpha_{n,c}\eta_{n,c} - \sum_n \beta_n \zeta_n}$.

A few notes are worth making. First, even with the tax on consumption serving as a way to control for leakage (and a way to compare to a BCA), there is still a role for the clean subsidy to discourage foreign production. In fact, the expression for the clean subsidy is largely unchanged from the baseline model (with the only difference being the baseline model included a term for x_i' in the denominator).

Second, instead of only taxing dirty production at γ_i, the tax is divided between $\tau_{i,d}$ and

t_i, but it remains true that $\tau_{i,d} + t_i = \gamma_i$. This is the insight from Weisbach et al. (2023), that it is optimal to divide the Pigouvian tax between upstream production and downstream consumption. This is clearly seen if clean production is perfectly inelastic ($\eta_{n,c} = 0$), in which case these results mirror the results of Weisbach et al. (2023). This insight does not negate the use of a clean subsidy, nor does it depend on a subsidy not being present.

B Multi-Country Equilibrium

Section 2 discusses unilaterally optimal policies used to correct for a global pollutant if the policies abroad are held fixed. However, climate policies in different countries may be interdependent if one country's policy impacts the marginal effectiveness or costs of foreign policy. With a global pollutant, interdependence is especially important as environmental damages are determined by worldwide pollution. Ultimately, it is the policy rates set by each country in a multi-country equilibrium that are welfare relevant.

If domestic policy induces foreign countries to increase their policy ambition, this will produce first-order benefits through reduced world pollution. On the other hand, if domestic policy weakens the incentives for foreign countries to enact climate policy, and foreign jurisdictions reduce policy ambition in response to an increase in domestic policy, this will have first-order costs to domestic welfare. If pollution taxes and clean subsidies have different interaction effects on foreign policies, and result in different multi-country equilibrium, this will directly affect welfare.

This appendix analyzes the role of policy interdependence in the open economy policymaker's choice between a pollution tax and clean subsidy. If policy interdependence varies based on which tax instruments are used, this could meaningfully impact the policymaker's choice of policy instrument in a manner that is not captured in Section 2. In order to investigate this possibility, this section studies the properties of the multi-country equilibrium in a non-cooperative Nash policy game when each country either only uses a pollution tax, only uses a clean subsidy, or uses both a tax and subsidy.

Policy interdependence in a non-cooperative game is captured by the strategic complementarity or substitutability between climate policy at home and policy abroad. Politicians and the media have increasingly discussed the US, EU, and China as competing in a "clean subsidy war" or a "green tech race" (e.g. Podesta, 2024; Wallace-Wells, 2024; Brower, 2023). Implicit in this language is the idea that clean subsidies are strategic complements—increasing subsidy rates at home lead to increases in subsidy rates abroad, creating a "race-to-the-top" in clean subsidies.

The following analysis shows that if countries compete in a Nash game, the nature of the strategic complementarity will be the same for pollution taxes and clean subsidies. If clean subsidies are strategic complements, then pollution taxes will also be strategic complements. This suggests that policy interdependence is not a reason to prefer one particular tax or subsidy instrument over the other. Although the nature of these complementarities may be important for considering overall climate policy ambition, it should not affect the decisions between which tax instrument to use.

B.1 Setup of the Multi-Country Equilibrium

I analyze the choice of each country's tax and subsidy rates as a Nash game. Consider the market environment described in Section 2 and denote the policy instruments of country i to be $\vec{\tau_i} \in \{\{\tau_{i,d}\}, \{\tau_{i,c}\}, \{\tau_{i,d}, \tau_{i,c}\}\}$. The set of policy instruments available to all countries is given as $\vec{\tau} = \sum_i \vec{\tau_i}$ and characterize the environmental policy-setting game. Each country sets its own tax and subsidy rates $\vec{\tau_i}$ to maximize domestic welfare, taking the tax rates set by other countries as given. Although tax rates do not impact other countries' marginal incentives directly, they might do so through their effect on equilibrium prices and quantities. In this setup, the world price p and equilibrium quantities of consumption and clean and dirty production are all implicitly defined by world tax and subsidy rates $\vec{\tau}$. Country i's best response functions are given by the optimal tax expressions in Propositions 3, 4, and 5, depending on which taxing instruments are available to the policymaker.

A Nash Equilibrium is characterized by a set of tax rates, τ, such that the price of the tradable good $p(\tau)$ is set so that markets clear and $\vec{\tau_i}$ is a best response given the instruments available and the set of tax rates in the rest of the world, $\vec{\tau}_{-i}$. In the remainder of this section, I discuss the equilibrium properties under different assumptions allowing countries to optimize with different policy instruments. Specifically, I consider comparative statics to changes in the domestic cost of pollution γ_i and how this increase can affect policy rates at home and abroad. An increase in γ_i can be interpreted as the policymaker in country i valuing emissions reductions more, and will lead country i to undertake more ambitious climate policy.

Although the strategic complementarities will depend on the convexity of the best-response functions, the complementarity will be the same for the pollution-tax-only and clean-subsidy-only games. This suggests that the choice between a pollution tax and clean subsidy will not impact the policy ambition of the multi-country equilibrium policy rates. The remainder of the section first analyzes the pollution-tax-only and clean-subsidy-only games where all countries are restricted to only using a single instrument, before discussing the combined tax and subsidy game.

B.2 Single Policy Instrument Equilibrium

This subsection analyzes strategic complementarities in multi-country equilibrium where either all countries are restricted to only using a pollution tax or all countries are restricted to only using a clean subsidy. In the pollution-tax-only game, each country will set the domestic pollution tax according to Proposition 3 and will simply depend on the value of reduced emissions, γ_i, and the domestic leakage rate, L_i. In the clean-subsidy-only game, each country will set subsidy rates according to Proposition 4. Equations 18 and 19 reproduce these equations written in terms of the slopes of the demand, clean supply, and dirty supply curves.

$$\tau_{i,d}^{\text{tax only}} = \gamma_i \left(1 - \frac{\sum_{n \neq i} x'_{n,d}}{\sum_{n \neq i} x'_{n,d} + \sum_n x'_{n,c} - \sum_n x'_n} \right) \tag{18}$$

$$\tau_{i,c}^{\text{subsidy only}} = -\gamma_i \left(\frac{\sum_n x'_{n,d}}{\sum_n x'_{n,d} + \sum_{n \neq i} x'_{n,c} - \sum_n x'_n} \right) \tag{19}$$

The slopes of the supply and demand curves are functions of the producer and consumer prices and $\gamma_i := \Gamma_i(\sum_n x_{n,d})$ is simply a function of total emissions. A domestic change to climate policy will therefore only affect the equilibrium climate policy abroad through a change in the world price or a change in the total quantity of global emissions.

The climate damage function, $\Gamma(\cdot)$, is often assumed to be either linear or convex. A linear climate damage function has a constant marginal damage parameter, γ_i, and therefore would imply that there are no policy interaction effects occurring through changing the total quantity of global emissions. Under this assumption, domestic policy will only affect policy abroad through a change in the global price.

However, if the damage function is convex, then a domestic increase in climate policy ambition will make the marginal damages less severe abroad, making equilibrium policy abroad less ambitious. This effect will be the same for both the pollution-tax-only game and the clean-subsidy-only game, pushing domestic and foreign policies towards strategic substitutes in either equilibrium set-up.

The policy interaction effects that occur through the world price could push domestic and foreign policy to either be strategic complements or strategic substitutes. To isolate this mechanism, the strategic complementarity can be calculated assuming the climate damage function is linear. These strategic complementarities between domestic and foreign policies are summarized in Table B.1. The top row shows results for the pollution-tax-only game where each country optimizes using tax rates while the bottom row shows results from a clean-subsidy-only game where each country is restricted to only use a clean subsidy.

Table B.1: Strategic Complementarity Between Domestic and Foreign Climate Policy

	Case 1: Linear Supply and Demand	Case 2: Convex Demand	Case 3: Concave Demand
Pollution-tax-only game	No Interaction	Strategic Substitutes	Strategic Complements
Clean-subsidy-only game	No Interaction	Strategic Substitutes	Strategic Complements

Note: This table summarizes the strategic complementarities between domestic and foreign policy resulting from an exogenous increase in γ_i. The top row shows the case where each country only uses a pollution tax and the bottom row shows the case when each row only uses a clean subsidy. The increase in γ_i will always lead policy in country i to be more ambitious, but will have different effects on policy in country j. Policies are listed as strategic complements (substitutes) if policy j increases (decreases) in policy ambition when γ_i increases.

The three columns in table B.1 show three cases with varying assumptions on the shapes of the demand and supply curves. For each case, the strategic complementarity is calculated for each game by considering the comparative statics for a small increase in the value of avoided pollution in country i, γ_i. An increase in γ_i will always increase the policy ambition in country i, but could either increase, decease, or not affect the policy ambition in country j. Derivations of the comparative statics are in Appendix D.

In Case 1, when supply and demand curves are linear, the slopes in Equations 18 and 19 will be constant and there will be no policy interaction effect. Increasing domestic policy ambition will not impact the marginal effectiveness or the marginal cost of environmental policy abroad, and the optimal policy rate abroad will remain unchanged. This is no longer true, when supply and demand curves are no longer linear, as changing policy in country i will impact the world price, which in turn will change the marginal leakage rate in country j. The nature of the policy interaction crucially depends on how the leakage rate abroad changes, which is determined by the convexity of the underlying supply and demand curves. Cases 2 and 3 show that allowing demand to be strictly convex or strictly concave will change the nature of the strategic complementarities.[20]

[20]Note that the concave demand requires demand isn't extremely concave for the strategic interaction to be well behaved. This criteria is defined in the proof appendix and is similar but not identical for the case of pollution-taxes and clean-subsidies.

Importantly, however, the sign of these policy interaction effects are the same within each column. Therefore, if pollution-taxes are strategic complements, it will also be the case that clean subsidies are strategic complements. This suggests that, although policy interaction effects are important for analyzing the level of policy ambition, they should not affect the choice of policy instruments.

B.3 Combined Tax and Subsidy Games

Finally, if all countries use both pollution taxes and clean subsidies, any policy interaction effects will simply shift whether the pollution tax or clean subsidy is emphasized rather than changing the overall level of policy ambition. This follow directly from Proposition 6 that states that $\tau_{j,d} + |\tau_{j,c}| = \gamma_j$. Any policy spillover that increases the pollution tax rate abroad will necessarily reduce the clean subsidy rate abroad in order to hold γ_j constant.

The nature of this interaction will depend on the specific comparative statics examined. For example, considering an increase in γ_i, the effect on the world price and therefore the policy choices of countries abroad will depend on both on the comparative emphasis on taxes versus subsidies in country i, but also the supply slopes. The partial effect of an increase on γ_i on the equilibrium price is given as

$$\frac{\partial p}{\partial \gamma_i} = \frac{\partial p}{\partial \tau_{i,d}} \left(1 - \frac{\sum_{n \neq i} x'_{n,d}}{\sum_{n \neq i} x'_{n,d} + \sum_{n \neq i} x'_{n,c} - \sum_n x'_n}\right) - \frac{\partial p}{\partial \tau_{i,c}} \frac{\sum_{n \neq i} x'_{n,d}}{\sum_{n \neq i} x'_{n,d} + \sum_{n \neq i} x'_{n,c} - \sum_n x'_n}$$

It is therefore the case that increasing γ_i will increase p if and only if

$$\tau_{i,d} x'_{i,d} > \tau_{i,c} x'_{i,c}. \tag{20}$$

This effect on the price, p, is the mechanism through which there is policy interaction. In combination with the convexity of demand, the effect on p will determine if policy interactions make foreign jurisdictions shift towards tax rates or subsidy rates.

C Avoided Pollution

This appendix discusses how optimal policy impacts total pollution under different scenarios. The environmental policy ambition can be scaled by γ_i. As policy ambition increases, the change in total global pollution is given by

$$\sum_n \frac{\partial x_{n,d}}{\partial \gamma_i} = - x'_{i,d}\left(1 - \frac{L_i}{1-\omega_i}\right)\left(1 - \frac{1}{\sum_n x'_{n,d} + x'_{n,c} - x'_n}\right) - x'_{i,c}\left(\frac{L_i}{1-\omega_i}\right)\left(\frac{1}{\sum_n x'_{n,d} + x'_{n,c} - x'_n}\right). \tag{21}$$

Derivation in Appendix D. Note that the impact of leakage on the reduction in pollution is theoretically ambiguous. If domestic dirty production is much more responsive than domestic clean production, $x'_{i,d} >> x'_{i,c}$, then the reduction in worldwide pollution from increased climate policy is approximately given by

$$\sum_n \frac{\partial x_{n,d}}{\partial \gamma_i} \approx - x'_{i,d}\left(1 - \frac{L_i}{1-\omega_i}\right)\left(1 - \frac{1}{\sum_n x'_{n,d} + x'_{n,c} - x'_n}\right).$$

In this case, conditions that lead to a higher leakage rate (for example if demand or foreign clean supply is more elastic) will shrink the magnitude of $\frac{\partial X_d}{\partial \gamma_i}$ towards zero. Indicating that high leakage economies are associated with less effective climate policy.

In contrast, if domestic dirty production is very inelastic, $x'_{i,d} << x'_{i,c}$, then the reduction in worldwide pollution from increased climate policy is approximately given by

$$\frac{\partial X_d}{\partial \gamma_i} \approx -x'_{i,c}\left(\frac{L_i}{1-\omega_i}\right)\left(\frac{1}{\sum_n x'_{n,d} + x'_{n,c} - x'_n}\right).$$

Here, an increase in the leakage rate will increase the magnitude of this expression, suggesting that higher leakage economies are associated with larger reductions in global pollutants from climate policy.

D Proofs

D.1 Derivation of Equation 5

Differentiate the market clearing condition with respect to $\tau_{i,k}$ gives

$$\sum_n x_n'(p) = \sum_n x_{n,c}'(p-\tau_{n,c})\frac{\partial p}{\partial \tau_{i,k}} + \sum_n x_{n,d}'(p-\tau_{n,d})\frac{\partial p}{\partial \tau_{i,k}} - x_{i,k}'(p-\tau_{i,k}).$$

Rearranging gives

$$\frac{\partial p}{\partial \tau_{i,k}} = \frac{x_{i,k}'}{\sum_n (x_{n,c}' + x_{n,d}' - x_n')}.$$

Rewriting x_n', $x_{n,c}'$, and $x_{n,d}'$ in terms of elasticities derives the equation.

D.2 Proof of Proposition 1

The social planner sets tax rates and subsidy rates in each country to maximize the sum of world welfare.

$$\max_{\{\tau_{i,c},\tau_{i,d}\ \forall i\}} \sum_{n=1}^{N}\Bigg(\max_{x_n,z_n}\left[u_n(x_n,z_n)+\mu_n\left(y_n+T_n-px_n-z_n\right)\right]+$$
$$\mu_n\left(\max_{x_{n,c}}\left[(p-\tau_{n,c})x_{n,c}-C_{n,c}(x_{n,c})\right]+\max_{x_{n,d}}\left[(p-\tau_{n,d})x_{n,d}-C_{n,d}(x_{n,d})\right]\right)+$$
$$\lambda_n(x_{n,c}\tau_{n,c}+x_{i,d}\tau_{i,d})\Bigg)-\Gamma^*(\sum_{m=1}^{N}x_{m,d})\Bigg)$$

Differentiating with respect to $\tau_{i,c}$ and $\tau_{i,d}$, the social planner's first order conditions are

$$
\begin{aligned}
\frac{\partial W}{\partial \tau_{i,c}} =& \frac{\partial p}{\partial \tau_{i,c}} \sum_{n=1}^{N} \left[\mu_n \left(x_{n,c} + x_{n,d} - x_n \right) \right] + \sum_{n=1}^{N} \left[\lambda_n \left(\tau_{n,c} \frac{\partial x_{n,c}}{\partial \tau_{i,c}} + \tau_{n,d} \frac{\partial x_{n,d}}{\partial \tau_{i,c}} \right) \right] + \\
& (\lambda_i - \mu_i) x_{i,c} - \Gamma^{*'} (\sum_{m=1}^{N} x_{m,d}) \sum_{m=1}^{N} \frac{\partial x_{m,d}}{\partial \tau_{i,c}} = 0 \\
\frac{\partial W}{\partial \tau_{i,d}} =& \frac{\partial p}{\partial \tau_{i,d}} \sum_{n=1}^{N} \left[\mu_n \left(x_{n,c} + x_{n,d} - x_n \right) \right] + \sum_{n=1}^{N} \left[\lambda_n \left(\tau_{n,c} \frac{\partial x_{n,c}}{\partial \tau_{i,d}} + \tau_{n,d} \frac{\partial x_{n,d}}{\partial \tau_{i,d}} \right) \right] + \\
& (\lambda_i - \mu_i) x_{i,d} - \Gamma^{*'} (\sum_{m=1}^{N} x_{m,d}) \sum_{m=1}^{N} \frac{\partial x_{m,d}}{\partial \tau_{i,d}} = 0
\end{aligned}
$$

If the social planner can provide lump sum transfers between countries or values welfare equally between countries $\mu_n = \mu_m$ and $\sum_{n=1}^{N} \left[\mu_n \left(x_{n,c} + x_{n,d} - x_n \right) \right] = 0$. Similarly, if the social planner ignores within country distribution consequences (or has separate instruments available to correct these distortions), $\lambda_i = \mu_i = 1$. Under these conditions, the social planner sets optimal tax and subsidy rates to trade-off the marginal *global* benefits of reduced environmental pollution against the marginal *global* inefficiency caused by each tax/subsidy rate (captured by marginal fiscal externalitites).

The optimal tax and subsidy rates are characterized by

$$
\begin{aligned}
\sum_{n=1}^{N} \left[\lambda_n \tau_{n,c} \frac{\partial x_{n,c}}{\partial \tau_{i,d}} + (\lambda_n \tau_{n,d} - \gamma^*) \frac{\partial x_{n,d}}{\partial \tau_{i,d}} \right] = 0 \\
\sum_{n=1}^{N} \left[\lambda_n \tau_{n,c} \frac{\partial x_{n,c}}{\partial \tau_{i,c}} + (\lambda_n \tau_{n,d} - \gamma^*) \frac{\partial x_{n,d}}{\partial \tau_{i,c}} \right] = 0
\end{aligned}
$$

for all countries i. These first order conditions are satisfied for all i if $\tau_{i,d} = \frac{\gamma^*}{\lambda_i}$ and $\tau_{i,c} = 0$, a direct application of the targeting principle.

D.3 Proof of Corollary 1

Proof. Following the Proof of Proposition 1, when $\tau_{i,d} = 0$ for all countries i, the optimal subsidy rates are characterized by

$$
\frac{\partial W}{\partial \tau_{i,c}} = \sum_{n=1}^{N} \left(\lambda_n \tau_{n,c} \frac{\partial x_{n,c}}{\partial \tau_{i,c}} \right) - \Gamma^{*'} (\sum_{m=1}^{N} x_{m,d}) \sum_{m=1}^{N} \frac{\partial x_{m,d}}{\partial \tau_{i,c}} = 0.
$$

The optimal subsidy rates are therefore defined by

$$\underbrace{\begin{pmatrix} \frac{\partial x_{1,c}}{\partial \tau_{1,c}} & ... & \frac{\partial x_{N,c}}{\partial \tau_{1,c}} \\ ... & ... & ... \\ \frac{\partial x_{1,c}}{\partial \tau_{N,c}} & ... & \frac{\partial x_{N,c}}{\partial \tau_{N,c}} \end{pmatrix}}_{\text{Behavioral Response (B)}} \begin{pmatrix} \lambda_1 \tau_{1,c} \\ ... \\ \lambda_N \tau_{N,c} \end{pmatrix} = \gamma^* \begin{pmatrix} \sum_n \frac{\partial x_{n,d}}{\partial \tau_{1,c}} \\ ... \\ \sum_n \frac{\partial x_{n,d}}{\partial \tau_{N,c}} \end{pmatrix} \tag{22}$$

Inverting matrix B therefore solves for the optimal subsidy rates. To invert B, note that $\frac{\partial x_{i,c}}{\partial \tau_{j,c}} = x'_{i,c} \cdot \left(\frac{\partial p}{\partial \tau_{j,c}} - \mathbb{1}_{i=j} \right)$ and B can therefore be written as

$$B = \underbrace{\begin{pmatrix} -x'_{1,c} & 0 & ... & 0 \\ 0 & -x'_{2,c} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & -x'_{N,c} \end{pmatrix}}_{\text{A}} + \underbrace{\begin{pmatrix} \frac{\partial p}{\partial \tau_{1,c}} \\ ... \\ \frac{\partial p}{\partial \tau_{N,c}} \end{pmatrix}}_{u} \underbrace{\begin{pmatrix} x'_{1,c} \\ ... \\ x'_{N,c} \end{pmatrix}^T}_{v}$$

From the Sherman-Morrison formula (Bartlett, 1951),

$$B^{-1} = A^{-1} - \frac{A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}$$

Note that

$$A^{-1} u v^T A^{-1} = \frac{1}{\sum_n \left(x'_{n,d} + x'_{n,c} - x'_n \right)} J_N$$

$$1 + v^T A^{-1} u = 1 - \sum_n \frac{\partial p}{\partial \tau_{n,c}}$$

where J_N is the N x N matrix of ones.

Therefore solving for the τ vector and evaluating B^{-1} in Equation 22, the optimal subsidy rate for country i, $\tau_{i,c}$ is given by

$$\begin{aligned}
\tau_{i,c}\lambda_i &= -\gamma^* \left(\frac{\sum_n \frac{\partial x_{n,d}}{\partial \tau_{i,c}}}{x'_{i,c}} + \frac{1}{1-\sum_n \frac{\partial p}{\partial \tau_{n,c}}} \frac{\sum_j \sum_n \frac{\partial x_{n,d}}{\partial \tau_{j,c}}}{\sum_n \left(x'_{n,d} + x'_{n,c} - x'_n\right)} \right) \\
&= -\gamma^* \left(\frac{\sum_n x'_{n,d} \frac{\partial p}{\partial \tau_{i,c}}}{x'_{i,c}} + \frac{1}{1-\sum_n \frac{\partial p}{\partial \tau_{n,c}}} \frac{\sum_j \left[\sum_n x'_{n,d} \frac{\partial p}{\partial \tau_{j,c}}\right]}{\sum_n \left(x'_{n,d} + x'_{n,c} - x'_n\right)} \right) \\
&= -\gamma^* \left(\frac{\sum_n x'_{n,d}}{\sum_n \left(x'_{n,d} + x'_{n,c} - x'_n\right)} \left(1 + \frac{\sum_j \frac{\partial p}{\partial \tau_{j,c}}}{1-\sum_n \frac{\partial p}{\partial \tau_{n,c}}}\right)\right) \\
&= -\gamma^* \left(\frac{\sum_n x'_{n,d}}{\sum_n \left(x'_{n,d} + x'_{n,c} - x'_n\right)} \left(\frac{1}{1-\sum_n \frac{\partial p}{\partial \tau_{n,c}}}\right)\right) \\
&= -\gamma^* \left(\frac{\sum_n x'_{n,d}}{\sum_n \left(x'_{n,d} + x'_{n,c} - x'_n\right) - \sum_n x'_{n,c}} \right) \\
&= -\gamma^* \left(\frac{\sum_n x'_{n,d}}{\sum_n \left(x'_{n,d} - x'_n\right)} \right)
\end{aligned}$$

□

D.4 Proof of Proposition 2

In a closed economy, when $\lambda_i = 1$, the policymakers' first order conditions for the combined tax and subsidy rates are given by

$$\underbrace{\begin{pmatrix} \frac{\partial x_{i,d}}{\partial \tau_{i,d}} & \frac{\partial x_{i,c}}{\partial \tau_{i,d}} \\ \frac{\partial x_{i,d}}{\partial \tau_{i,c}} & \frac{\partial x_{i,c}}{\partial \tau_{i,c}} \end{pmatrix}}_{\text{Behavioral Response (B)}} \begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \end{pmatrix} = \gamma_i \begin{pmatrix} \sum_{n=1}^N \frac{\partial x_{n,d}}{\partial \tau_{i,d}} \\ \sum_{n=1}^N \frac{\partial x_{n,d}}{\partial \tau_{i,c}} \end{pmatrix}$$

As there is no trade, the domestic market will not affect prices abroad, and changes in $\tau_{i,d}$ and $\tau_{i,c}$ will only impact $x_{i,d}$ and $x_{i,c}$. Dirty production abroad is therefore taken as given and $\sum_{n=1}^N \frac{\partial x_{n,d}}{\partial \tau_{i,d}} = \frac{\partial x_{i,d}}{\partial \tau_{i,d}}$. The first order equations are therefore given as

$$\underbrace{\begin{pmatrix} \frac{\partial x_{i,d}}{\partial \tau_{i,d}} & \frac{\partial x_{i,c}}{\partial \tau_{i,d}} \\ \frac{\partial x_{i,d}}{\partial \tau_{i,c}} & \frac{\partial x_{i,c}}{\partial \tau_{i,c}} \end{pmatrix}}_{\text{Behavioral Response (B)}} \begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \end{pmatrix} = \gamma_i \begin{pmatrix} \frac{\partial x_{i,d}}{\partial \tau_{i,d}} \\ \frac{\partial x_{i,d}}{\partial \tau_{i,c}} \end{pmatrix}$$

This condition is satisfied if $\tau_{i,d} = \gamma_i$ and $\tau_{i,c} = 0$.

D.5 Proof of Corollary 2

When the pollution tax is constrained to be zero, the first order condition for the subsidy is given by

$$\tau_{i,c} \frac{\partial x_{i,c}}{\partial \tau_{i,c}} = \gamma_i \frac{\partial x_{i,d}}{\partial \tau_{i,c}}.$$

In the closed economy, markets clear domestically so $\frac{\partial p}{\partial \tau_{i,c}} = \frac{x'_{i,c}}{x'_{i,c}+x'_{i,d}-x'_i}$. Substituting in $\frac{\partial x_{i,c}}{\partial \tau_{i,c}} = x'_{i,c} \cdot \left(\frac{\partial p}{\partial \tau_{i,c}} - 1\right)$, $\frac{\partial x_{i,d}}{\partial \tau_{i,c}} = x'_{i,d} \cdot \frac{\partial p}{\partial \tau_{i,c}}$, the expression for $\frac{\partial p}{\partial \tau_{i,c}}$, and rewriting as elasticities proves the Corollary.

D.6 Derivation of Equation 8

In order to derive the open economy policymakers' first order condition for tax instrument $\tau_{i,k}$ on quantity $x_{i,k}$ consider the policymaker's objective function. If the policymaker uses the set of tax instruments $\vec{\tau}_i$, their maximization problem is

$$\max_{\vec{\tau}} \Bigg\{ \max_{x_i, z_i} \left[U_i(x_i, z_i) + \mu_i(y_i + \pi_i - px_i - z_i)\right] - \Gamma_i\left(\sum_{n=1}^{N} x_{n,d}\right) \\ + \nu_i\left(\pi_i - \max_{x_{i,c}}\left[(p - \tau_{i,c})x_{i,c} - C_{i,c}(x_{i,c})\right] - \max_{x_{i,d}}\left[(p - \tau_{i,d})x_{i,d} - C_{i,d}(x_{i,d})\right]\right) \\ + \lambda_i\left(\vec{x} \cdot \vec{\tau}\right) \Bigg\}$$

Differentiating with respect to $\tau_{i,k}$ and making use of the envelope theorem, leads to $-\nu_i = \mu_i = 1$ and therefore to

$$\lambda_i \vec{\tau}_i \cdot \frac{\partial \vec{x}}{\partial \tau_{i,k}} = \gamma_i \frac{\partial \sum_n x_{n,d}}{\partial \tau_{i,k}} + \frac{\partial p}{\partial \tau_{i,k}}\left(x_{i,d} + x_{i,c} - x_i\right) + \left(\lambda_i - 1\right) x_{i,k}$$

D.7 Derivation of Equation 9

By definition, the marginal leakage rate is given as

$$L_i := \frac{-\sum_{n\neq i}\frac{\partial x_{n,d}}{\partial \tau_{i,d}}}{\frac{\partial x_{i,d}}{\partial \tau_{i,d}}}$$

Noting that $\tau_{i,d}$ only affects pollution abroad through its effect on the output price, this equation can be rewritten as

$$L_i = \frac{-\sum_{n\neq i} x'_{n,d}\frac{\partial p}{\partial \tau_{i,d}}}{x'_{i,d}(\frac{\partial p}{\partial \tau_{i,d}}-1)} = \frac{-\sum_{n\neq i} x'_{n,d}\cdot\frac{x'_{i,d}}{\sum_n\left(x'_{n,d}+x'_{n,c}-x'_n\right)}}{x'_{i,d}\frac{-\left(\sum_{n\neq i}x'_{n,d}+\sum_n x'_{n,c}-\sum_n x'_n\right)}{\sum_n\left(x'_{n,d}+x'_{n,c}-x'_n\right)}} = \frac{\sum_{n\neq i}x'_{n,d}}{\sum_{n\neq i}x'_{n,d}+\sum_n x'_{n,c}-\sum_n x'_n}.$$

Rewriting the slopes of the supply and demand curves in terms or elasticities yields equation 9.

D.8 Proof of Proposition 3

Under Assumptions 1 and 2 the optimal tax rate is given by

$$\tau_{i,d}\frac{\partial x_{i,d}}{\partial \tau_{i,d}} = \gamma_i\frac{\partial\sum_n x_{n,d}}{\partial \tau_{i,d}}$$

Noting that $\frac{\partial x_{i,d}}{\partial \tau_{i,d}} = x'_{i,d}\cdot(\frac{\partial p}{\partial \tau_{i,d}}-1)$ and $\frac{\partial x_{n,d}}{\partial \tau_{i,d}} = x'_{n,d}\cdot\frac{\partial p}{\partial \tau_{i,d}}$ for $n\neq i$, the optimal tax rate is given by

$$\tau_{i,d} = \gamma_i\left(1-\frac{\frac{\partial p}{\partial \tau_{i,d}}}{1-\frac{\partial p}{\partial \tau_{i,d}}}\frac{\sum_{n\neq i}x'_{n,d}}{x'_{i,d}}\right).$$

Substituting in the expression for $\frac{\partial p}{\partial \tau_{i,d}}$ from Equation 5 proves the Proposition.

D.9 Proof of Proposition 4

Under Assumptions 1 and 2 the optimal tax/subsidy rate on the clean good is given by

$$\tau_{i,c}\frac{\partial x_{i,c}}{\partial \tau_{i,c}} = \gamma_i\frac{\partial\sum_n x_{n,d}}{\partial \tau_{i,c}}.$$

Noting that $\frac{\partial x_{i,c}}{\partial \tau_{i,c}} = x'_{i,c}\cdot(\frac{\partial p}{\partial \tau_{i,c}}-1)$ and $\frac{\partial x_{n,d}}{\partial \tau_{i,c}} = x'_{n,d}\cdot\frac{\partial p}{\partial \tau_{i,c}}$, the optimal (subsidy) rate is given by

$$\tau_{i,c} = -\gamma_i \left(\frac{\frac{\partial p}{\partial \tau_{i,c}}}{1 - \frac{\partial p}{\partial \tau_{i,c}}} \frac{\sum_n x'_{n,d}}{x'_{i,c}} \right).$$

Substituting in the expression for $\frac{\partial p}{\partial \tau_{i,c}}$ from Equation 5 proves the Proposition.

D.10 Proof of Proposition 5

When $\vec{\tau_i} = \{\tau_{i,d}, \tau_{i,c}\}$, and Assumptions 1 and 2 hold, the first order conditions from Equation 8 imply an interior solution satisfies

$$\underbrace{\begin{pmatrix} \frac{\partial x_{i,d}}{\partial \tau_{i,d}} & \frac{\partial x_{i,c}}{\partial \tau_{i,d}} \\ \frac{\partial x_{i,d}}{\partial \tau_{i,c}} & \frac{\partial x_{i,c}}{\partial \tau_{i,c}} \end{pmatrix}}_{\text{Behavioral Response (B)}} \begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \end{pmatrix} = \gamma_i \begin{pmatrix} \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial \tau_{i,d}} \\ \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial \tau_{i,c}} \end{pmatrix} \tag{23}$$

The optimal tax and subsidy rates are therefore

$$\begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \end{pmatrix} = \gamma_i B^{-1} \begin{pmatrix} \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial \tau_{i,d}} \\ \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial \tau_{i,c}} \end{pmatrix} \tag{24}$$

To analytically invert matrix B, note that each element in B can be decomposed into the wedge between the producer price and world price and changes to the world price. B can be written as

$$B = \begin{pmatrix} x'_{i,d} \left(\frac{\partial p}{\partial \tau_{i,d}} - 1 \right) & x'_{i,c} \frac{\partial p}{\partial \tau_{i,d}} \\ x'_{i,d} \frac{\partial p}{\partial \tau_{i,c}} & x'_{i,c} \left(\frac{\partial p}{\partial \tau_{i,c}} - 1 \right) \end{pmatrix} = \begin{pmatrix} -x'_{i,d} & 0 \\ 0 & -x'_{i,c} \end{pmatrix} + \begin{pmatrix} \frac{\partial p}{\partial \tau_{i,d}} \\ \frac{\partial p}{\partial \tau_{i,c}} \end{pmatrix} \begin{pmatrix} x'_{i,d} \\ x'_{i,c} \end{pmatrix}^T$$

With this decomposition of B, B^{-1} can be calculated with the Sherman-Morrison formula as

$$B^{-1} = A^{-1} - \frac{A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}$$

where

$$A = \begin{pmatrix} -x'_{i,d} & 0 \\ 0 & -x'_{i,c} \end{pmatrix}; \qquad u = \begin{pmatrix} \frac{\partial p}{\partial \tau_{i,d}} \\ \frac{\partial p}{\partial \tau_{i,c}} \end{pmatrix}; \qquad v = \begin{pmatrix} x'_{i,d} \\ x'_{i,c} \end{pmatrix}.$$

The optimal tax rates are therefore

$$\begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \end{pmatrix} = \gamma_i \begin{pmatrix} -\frac{1}{x'_{i,d}} & 0 \\ 0 & -\frac{1}{x'_{i,c}} \end{pmatrix} \begin{pmatrix} -x'_{i,d} + \sum_{n=1}^{N} x'_{n,d} \frac{\partial p}{\partial \tau_{i,d}} \\ \sum_{n=1}^{N} x'_{n,d} \frac{\partial p}{\partial \tau_{i,c}} \end{pmatrix}$$
$$- \gamma_i \left(\frac{1}{\sum_{n \neq i} x'_{n,d} + \sum_{n \neq i} x'_{n,c} - \sum_n x'_n} \right) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -x'_{i,d} + \sum_n (x'_{n,d} \frac{\partial p}{\partial \tau_{i,d}}) \\ \sum_n (x'_{n,d} \frac{\partial p}{\partial \tau_{i,c}}). \end{pmatrix}$$

Substituting in the expression for the change in the price level given by Equation 5 and simplifying proves Proposition 5.

D.11 Proof of Proposition 6

This proposition follows directly from adding Equations 11 and 12 from Proposition 5.

D.12 Proof of Proposition 7

Evaluating the expression gives

$$\frac{\tau_{i,d}}{\tau_{i,c}} = \frac{\gamma_i \left(1 - L_i \frac{1}{1-\omega_i}\right)}{-\gamma_i L_i \frac{1}{1-\omega_i}} = -\frac{1 - L_i - \omega_i}{L_i}.$$

Substituting in for L_i and ω_i gives

$$\frac{\tau_{i,d}}{\tau_{i,c}} = -\left(\frac{1 - \frac{x'_{i,c}}{\sum_{n \neq i} x'_{n,d} + \sum_n \left(x'_{n,c} - x'_n\right)} - \frac{\sum_{n \neq i} x'_{n,d}}{\sum_{n \neq i} x'_{n,d} + \sum_n \left(x'_{n,c} - x'_n\right)}}{\frac{\sum_{n \neq i} x'_{n,d}}{\sum_{n \neq i} x'_{n,d} + \sum_n \left(x'_{n,c} - x'_n\right)}} \right)$$
$$= -\frac{\sum_{n \neq i} x'_{n,c} - \sum_n x'_n}{\sum_{n \neq i} x'_{n,d}}$$

D.13 Derivation of Equation 15

When the emissions intensity for country i is normalized to one, and foreign emissions intensities, e_n, are written relative to that in country i, the leakage rate is defined as the increase in foreign emissions for each unit of avoided domestic emissions from a pollution tax. Mathematically, it is defined as

$$L_i = \frac{-\sum_{n\neq i} e_n \frac{\partial x_{n,d}}{\partial \tau_{i,d}}}{\frac{\partial x_{i,d}}{\partial \tau_{i,d}}} = \frac{-\sum_{n\neq i} e_n x'_{n,d} \frac{\partial p}{\partial \tau_{i,d}}}{x'_{i,d}\left(\frac{\partial p}{\partial \tau_{i,d}} - 1\right)} = \frac{\sum_{n\neq i} e_n x'_{n,d}}{\sum_{n\neq i} x'_{n,d} + \sum_n (x'_{n,c} - x'_n)}.$$

D.14 Proof of Proposition 8

The optimal tax and subsidy rates are defined by

$$\begin{pmatrix} \frac{\partial x_{i,d}}{\partial \tau_{i,d}} & \frac{\partial x_{i,c}}{\partial \tau_{i,d}} \\ \frac{\partial x_{i,d}}{\partial \tau_{i,c}} & \frac{\partial x_{i,c}}{\partial \tau_{i,c}} \end{pmatrix} \begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \end{pmatrix} = \gamma_i \begin{pmatrix} \sum_{n=1}^N e_n \frac{\partial x_{n,d}}{\partial \tau_{i,d}} \\ \sum_{n=1}^N e_n \frac{\partial x_{n,d}}{\partial \tau_{i,c}} \end{pmatrix} \tag{25}$$

Following the same steps as the proof to Proposition 5, the tax and subsidy rates are

$$\begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \end{pmatrix} = \gamma_i \begin{pmatrix} -\frac{1}{x'_{i,d}} & 0 \\ 0 & -\frac{1}{x'_{i,c}} \end{pmatrix} \begin{pmatrix} -x'_{i,d} + \sum_{n=1}^N e_n x'_{n,d} \frac{\partial p}{\partial \tau_{i,d}} \\ \sum_{n=1}^N e_n x'_{n,d} \frac{\partial p}{\partial \tau_{i,c}} \end{pmatrix}$$
$$- \gamma_i \left(\frac{1}{\sum_{n\neq i} x'_{n,d} + \sum_{n\neq i} x'_{n,c} - \sum_n x'_n} \right) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -x'_{i,d} + \sum_n (e_n x'_{n,d} \frac{\partial p}{\partial \tau_{i,d}}) \\ \sum_n (e_n x'_{n,d} \frac{\partial p}{\partial \tau_{i,c}}). \end{pmatrix}$$

Combining terms and simplifying proves the proposition.

D.15 Proof of Proposition 9

First note that by differentiating the market clearing condition, the effect of t_i on the world price is given by

$$\frac{\partial p}{\partial t_i} = \frac{x'_n}{\sum_n (x'_{n,d} + x'_{n,c} - x'_n)}.$$

Moving to the optimal tax and subsidy regime, an interior optimal solution will be defined by

$$\underbrace{\begin{pmatrix} \frac{\partial x_{i,d}}{\partial \tau_{i,d}} & \frac{\partial x_{i,c}}{\partial \tau_{i,d}} & \frac{\partial x_{i}}{\partial \tau_{i,d}} \\ \frac{\partial x_{i,d}}{\partial \tau_{i,c}} & \frac{\partial x_{i,c}}{\partial \tau_{i,c}} & \frac{\partial x_{i}}{\partial \tau_{i,c}} \\ \frac{\partial x_{i,d}}{\partial t_{i}} & \frac{\partial x_{i,c}}{\partial t_{i}} & \frac{\partial x_{i}}{\partial t_{i}} \end{pmatrix}}_{\text{Behavioral Response (B)}} \begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \\ t_i \end{pmatrix} = \gamma_i \begin{pmatrix} \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial \tau_{i,d}} \\ \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial \tau_{i,c}} \\ \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial t_{i}} \end{pmatrix}$$

The inverse of B is again found from the Sherman-Morrison formula and is given by

$$\begin{pmatrix} -1/x'_{i,d} & 0 & 0 \\ 0 & -1/x'_{i,c} & 0 \\ 0 & 0 & 1/x'_i \end{pmatrix} - \frac{1}{\sum_{n\neq i}(x'_{n,d} + x'_{n,c} - x'_n)} \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

Therefore, the optimal policy rates are given by

$$\begin{pmatrix} \tau_{i,d} \\ \tau_{i,c} \\ t_i \end{pmatrix} = \gamma_i \left(\begin{pmatrix} -1/x'_{i,d} & 0 & 0 \\ 0 & -1/x'_{i,c} & 0 \\ 0 & 0 & 1/x'_i \end{pmatrix} - \frac{1}{\sum_{n\neq i}(x'_{n,d} + x'_{n,c} - x'_n)} \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} \right) \begin{pmatrix} \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial \tau_{i,d}} \\ \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial \tau_{i,c}} \\ \sum_{n=1}^{N} \frac{\partial x_{n,d}}{\partial t_{i}} \end{pmatrix}$$

Evaluating and simplifying proves the proposition.

D.16 Proof of Multi-Country Comparative Statics

Pollution Tax Only

Consider a pollution tax game where countries use pollution taxes to correct for the externality. The Nash Equilibrium is a set of tax rates $\vec{\tau}$ such that market clearing holds and each country's tax rate represents their optimal tax rate, given the tax rates (and therefore subsequent equilibrium prices and quantities and elasticities) set by foreign countries. Nash Equilibirum is therefore characterize by the following set of $f_1, f_2, ...f_n$ where

$$f_i: \ \gamma_i \left(1 - \frac{\sum_{j\neq i} x'_{j,d}(p(\vec{\tau}) - \tau_{j,d}))}{\sum_{j\neq i} x'_{j,d}(p(\vec{\tau}) - \tau_{j,d})) + \sum_j x'_{j,c}(p(\vec{\tau}) - \tau_{j,c})) - \sum_j x'_j(p(\vec{\tau}))} \right) - \tau_{i,d}$$

Consider the comparative statics for if country i ups their ambition of their climate policy γ_i, perhaps because new science reveals that the global pollutant will impose larger damages. The comparative statics are given by the implicit function theorem

$$\frac{\partial \vec{\tau}}{\partial \gamma_i} = -[J_{f,\tau}]^{-1} J_{f,\gamma_i}$$

where $J_{f,\tau}$ is an n x n matrix where element i,j is $\frac{\partial f_i}{\partial \tau_{j,d}}$ and J_{f,γ_i} is an n x 1 vector. Under linear supply and convex demand,

$$\frac{\partial f_i}{\partial \tau_{j,d}} = -\gamma_i \frac{\sum_{j \neq i} x'_{j,d}}{(\sum_{j \neq i} x'_{j,d} + \sum_j x'_{j,c} - \sum_j x'_j)^2} \left(\sum_j x''_j \right) \frac{\partial p}{\partial \tau_{j,d}} - \mathbb{1}_{i=j} \tag{26}$$

Therefore

$$J_{f,\tau} = u\ v^T - I_N \tag{27}$$

where I_n is the n x n identity matrix and u and v are N dimensional vectors with elements n given by

$$u_n = \gamma_n \frac{-\sum_{j \neq n} x'_{j,d} \sum_j x''_j}{(\sum_{j \neq n} x'_{j,d} + \sum_j x'_{j,c} - \sum_j x'_j)^2} \tag{28}$$

$$v_n = \frac{\partial p}{\partial \tau_{n,d}} \tag{29}$$

The inverse of $J_{f,\tau}$ can therefore be computed using the Sherman-Morrison formula

$$[J_{f,\tau}]^{-1} = (-I_N)^{-1} - \frac{(-I_N)^{-1} u v^t (-I_N)^{-1}}{1 + v^t (-I_N)^{-1} u} = (-I_N) - \frac{u v^t}{1 + v^t (-I_N) u} \tag{30}$$

Looking first at the denominator, $1 + v^t(-I_N)u$. We know that $-I_N u$ is an N x 1 vector with n^{th} element given by

$$\gamma_n \frac{\sum_{j \neq n} x'_{j,d} \sum_j x''_j}{(\sum_{j \neq n} x'_{j,d} + \sum_j x'_{j,c} - \sum_j x'_j)^2}$$

so

$$
\begin{aligned}
D :=& 1 + v^t(-I_N)u \\
=& 1 + \sum_{n=1}^{N} \left(\frac{\gamma_n \sum_{j \neq n}(x'_{j,d})x'_{n,d}}{(\sum_{j \neq n} x'_{j,d} + \sum_j x'_{j,c} - \sum_j x'_j)^2} \right) \frac{\sum_j x''_j}{\sum_j (x_{j,d} + x'_{j,c} - x'_j)}
\end{aligned}
\tag{31}
$$

The (i, j) element of $[J_{f,\tau}]^{-1}$ is therefore

$$
\frac{\gamma_n}{D} \frac{x'_{j,d} \sum_{n \neq i} x'_{n,d} \sum_n x''_n}{(\sum_{n \neq i} x'_{n,d} + \sum_n x'_{n,c} - \sum_n x'_n)^2 (\sum_n x'_{n,d} + \sum_n x'_{n,c} - \sum_n x'_n)} - \mathbb{1}_{i=j}
$$

Now consider the comparative statics for an increase in γ_i. $[J_{f,\gamma_i}]$ is an n dimension where the i^{th} element is given by $\frac{\partial f_i}{\partial \gamma_i}$.

This increase will impact the Nash equilibrium tax rates as follows:

$$
\begin{aligned}
\frac{\partial \tau_{j,d}}{\partial \gamma_i} = & \left(\mathbb{1}_{i=j} - \frac{\gamma_n}{D} \frac{x'_{j,d} \sum_{n \neq i} x'_{n,d} \sum_n x''_n}{(\sum_{n \neq i} x'_{n,d} + \sum_n x'_{n,c} - \sum_n x'_n)^2 (\sum_n x'_{n,d} + \sum_n x'_{n,c} - \sum_n x'_n)} \right) \\
& \left(1 - \frac{\sum_{j \neq i} x'_{j,d}}{\sum_{j \neq i x'_{j,d}} + \sum_j x'_{j,c} - \sum_j x'_j} \right)
\end{aligned}
$$

Therefore, for $\sum_n x''_n > 0$, then $\frac{\partial \tau_{j,d}}{\partial \gamma_i} \leq 0$ for $j \neq i$ and tax rates are strategic complements. If $\sum_n x''_n < 0$, then $\frac{\partial \tau_{j,d}}{\partial \gamma_i} \geq 0$ for $j \neq i$ assuming demand isn't too concave (D is still greater than 0). For very concave demand,

$$
-\sum_n x''_{n,d} > \frac{1}{\sum_m \gamma_m \frac{\sum_{n \neq m} x'_{n,d}}{\sum_{n \neq m} x'_{n,d} + \sum_n (x'_{n,c} - x'_n)} \frac{\partial p}{\partial \tau_{m,d}} \frac{1}{\sum_{n \neq m} x'_{n,d} + \sum_n x'_{n,c} - x'_n}}
$$

then $1 + v^T A^{-1} u$ will be negative and the sign of the comparative static will flip again.

Clean Subsidy Only

If each country only uses a clean subsidy, equilibrium is defined by

$$
f_i : -\gamma_i \left(\frac{\sum_n x'_{n,d}}{\sum_n x'_{n,d} + \sum_{n \neq i} x'_{n,c} - \sum_n x'_n} \right) - \tau_{i,c} = 0
$$

Again, the comparative statics can be found from the implicit function theorem. The matrix $J_{f,\tau}$ can similarly be defined as $-I_N + uv^T$ where

$$u_n = \gamma_n \frac{-\sum_j x'_{j,d} \sum_j x''_j}{(\sum_{j \neq n} x'_{j,c} + \sum_j x'_{j,d} - \sum_j x'_j)^2} \tag{32}$$

$$v_n = \frac{\partial p}{\partial \tau_{n,c}} \tag{33}$$

The quantity $D = 1 + v^T A^{-1} u$ is

$$1 + \sum_n \sum_m \left(\frac{\gamma_i \sum_n (x'_{n,d}) \sum_n x''_n}{(\sum_{j \neq n} x'_{j,c} + \sum_j x'_{j,d} - \sum_j x'_j)^2} \frac{\partial p}{\partial \tau_{m,c}} \right).$$

The i, j element of uv^T is

$$\frac{\gamma_i \sum_n x'_{n,d} \sum_n x''_n}{(\sum_{m \neq n} x'_{m,c} + \sum_m x'_{m,d} - \sum_m x'_m)^2} \frac{\partial p}{\partial \tau_{j,c}}$$

Therefore the comparative static is given as

$$\frac{\partial \tau_{j,c}}{\partial \gamma_i} = \left(\mathbb{1}(i = j) - \frac{1}{1 + v^T A^{-1} u} \frac{\gamma_i \sum_n x'_{n,d} \sum_n x''_n}{(\sum_{m \neq n} x'_{m,c} + \sum_m x'_{m,d} - \sum_m x'_m)^2} \frac{\partial p}{\partial \tau_{j,c}} \right) \cdot \left(- \frac{\sum_n x'_{n,d}}{\sum_n x'_{n,d} + \sum_{n \neq i} x'_{n,c} - \sum_n x'_n} \right)$$

So if $\sum_n x''_n > 0$ then $\frac{\partial \tau_{j,c}}{\partial \gamma_i} > 0$ and subsidies are strategic substitutes ($\tau_{j,c}$ is less than zero so an increase makes the magnitude smaller). If $\sum_n x''_n < 0$, then $\frac{\partial \tau_{j,c}}{\partial \gamma_i} \leq 0$ for $j \neq i$ assuming demand isn't too concave ($1 + v^T A^{-1} u$ is still greater than 0). For very concave demand

$$-\sum_n x''_n \geq \frac{1}{\sum_m \frac{\gamma_i \sum_n x'_{n,d} \frac{\partial p}{\partial \tau_{m,c}}}{\left(\sum_n x'_{n,d} - x'_n + \sum_{n \neq m} x'_{n,c}\right)^2}}$$

the magnitude of the strategic interaction gets very large then flips signs again. Note this is almost the same but not identical to the pollution-tax only case indicating the threshold for "very concave" demand is slightly different for the two cases.

D.17 Derivation of Equation 20

The change in price is given by

$$\frac{\partial p}{\partial \gamma_i} = \frac{\partial p}{\partial \tau_{i,d}} \frac{\tau_{i,d}}{\gamma_i} + \frac{\partial p}{\partial \tau_{i,c}} \frac{\tau_{i,c}}{\gamma_i} = \frac{1}{\gamma_i \sum_n (x'_{n,d} + x'_{n,c} - x'_n)} \left(x'_{i,d}\tau_{i,d} + x'_{i,c}\tau_{i,c} \right).$$

Given that $\frac{1}{\gamma_i \sum_n (x'_{n,d} + x'_{n,c} - x'_n)} > 0$,

$$Sign(\frac{\partial p}{\partial \gamma_i}) = Sign(x'_{i,d}\tau_{i,d} + x'_{i,c}\tau_{i,c}).$$

D.18 Derivation of Equation 21

When policy ambition increases, the change in global pollution comes about as a result of (1) a change in the world price from an increase in the magnitude of the tax and subsidy and (2) a reduction in domestic production due to a larger wedge between the domestic dirty producer price and the world price. Mathematically, the change is

$$\begin{aligned}
\sum_n \frac{\partial x_{n,d}}{\partial \gamma_i} =& \sum_n x'_{n,d} \frac{\partial p}{\partial \gamma_i} - x_{i,d} \frac{\partial \tau_{i,d}}{\partial \gamma_i} \\
=& - x'_{i,d} \left(1 - \frac{L_i}{1-\omega_i} \right) + \sum_n (x'_{n,d}) \left(\frac{1}{\sum_n x'_{n,d} + x'_{n,c} - x'_n} \right) \left(x'_{i,d} \left(1 - \frac{L_i}{1-\omega_i} \right) - x'_{i,c} \frac{L_i}{1-\omega_i} \right) \\
=& - x'_{i,d} \left(1 - \frac{L_i}{1-\omega_i} \right) \left(1 - \frac{1}{\sum_n x'_{n,d} + x'_{n,c} - x'_n} \right) \\
& - x'_{i,c} \left(\frac{L_i}{1-\omega_i} \right) \left(\frac{1}{\sum_n x'_{n,d} + x'_{n,c} - x'_n} \right)
\end{aligned}$$